

QCD aspects of the pion-photon form factor

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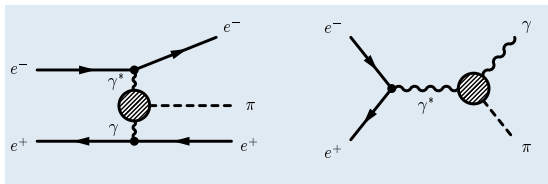
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The pion-photon transition form factor

- Theoretically simplest hadronic matrix element:

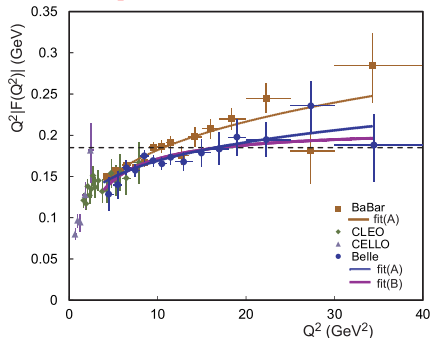
$$\langle \pi(p) | J_{\mu}^{\text{em}} | \gamma(p') \rangle = g_{\text{em}}^2 \epsilon_{\mu\nu\alpha\beta} q^{\alpha} p^{\beta} \epsilon^{\nu}(p') F_{\gamma^* \gamma \rightarrow \pi^0}(Q^2).$$

- Related to the axial anomaly at $Q^2 \equiv -(p - p')^2 = 0$.
- Golden channel to understand the QCD dynamics.
- e^+e^- collisions:



The BaBar puzzle

- Status of experimental measurements:

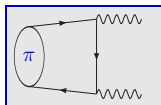


Scaling violation?

Shape of pion wave function?

The onset of QCD factorization?

- Asymptotic limit:



$$\lim_{Q^2 \rightarrow \infty} Q^2 F_{\pi\gamma\gamma^*}(Q^2) = 2 f_\pi$$

Brodsky, Lepage

QCD factorization works perfectly.

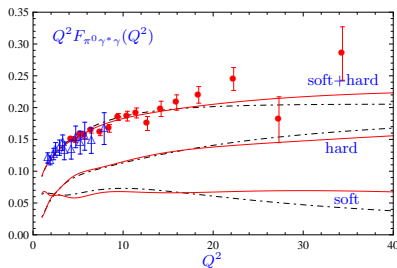
Some popular explanations:

- Non-vanishing pion wave function at the end points (Radyushkin, 2009; Polyakov, 2009).

$$F(Q^2) = \frac{\sqrt{2}}{3} \int_0^1 \frac{\varphi_\pi(x)}{xQ^2} \left[1 - \underbrace{\exp\left(-\frac{xQ^2}{2\bar{x}\sigma}\right)}_{\uparrow} \right].$$

from k_T dependence of pion wave function

- Large soft corrections at moderate Q^2 (Agaev, Braun, Offen, Porkert, 2011).

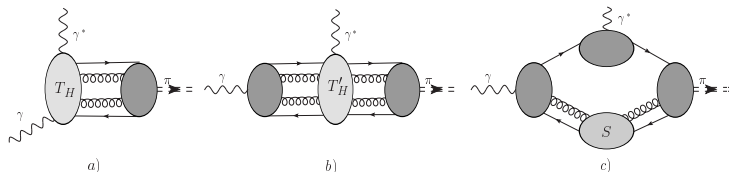


The “hard” and “soft” contributions to the $\pi^0 \gamma^* \gamma$ form factor for model I (solid curves) and model III (dash-dotted curves). The experimental data are from BaBar (full circles) and CLEO (open triangles).

- Threshold resummation generates power-like $[x(1-x)]^c(Q^2)$ distribution (Li and Mishima 2009). $c(Q^2)$ is around 1 for low Q^2 , but small for high Q^2 .

The general picture

- Schematic structure of the distinct mechanisms (Agaev, Braun, Offen, Porkert, 2011):



A: hard subgraph that includes both photon vertices

B: real photon emission at large distances

C: Feynman mechanism: soft quark spectator

$$\frac{1}{Q^2} + \frac{1}{Q^4} + \dots$$

$$\frac{1}{Q^4} + \dots$$

$$\frac{1}{Q^4} + \dots$$

- Operator definitions of different terms needed for an unambiguous classification.

Region A: Leading Twist Contribution

- QCD factorization formula:

$$F_{\gamma^* \gamma \rightarrow \pi^0}^{\text{LP}}(Q^2) = \frac{\sqrt{2}(Q_u^2 - Q_d^2)f_\pi}{Q^2} \int_0^1 dx T^\Delta(x, Q^2, \mu) \phi_\pi^\Delta(x, \mu).$$

- ▶ Renormalization-scheme dependence due to the IR subtraction.
⇒ **Verify scheme independence of $F_{\gamma^* \gamma \rightarrow \pi^0}$ at NLO (this talk).**
- ▶ NLO hard function in the NDR scheme (Braaten 1983 + many others):

$$T^{(1)} = \frac{\alpha_s C_F}{4\pi} \left\{ \frac{1}{\bar{x}} \left[-(2 \ln \bar{x} + 3) \ln \frac{\mu^2}{Q^2} + \ln^2 \bar{x} + (-1) \frac{\bar{x} \ln \bar{x}}{x} - 9 \right] + (x \leftrightarrow \bar{x}) \right\}.$$

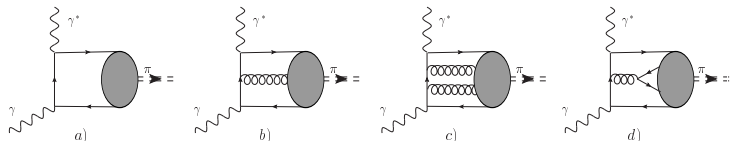
- ▶ Twist-2 pion DA (collinear matrix element):

$$\langle \pi(p) | \bar{\xi}(y) W_c(y, 0) \gamma_\mu \gamma_5 \xi(0) | 0 \rangle = -i f_\pi p_\mu \int_0^1 du e^{iup \cdot y} \phi_\pi(u, \mu) + \mathcal{O}(y^2).$$

The ERBL evolution implies Gegenbauer expansion.

Region A: Twist-4 Contribution

- Subleading power correction at $\mathcal{O}(1/Q^4)$:



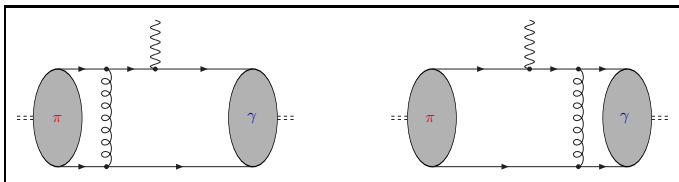
- QCD factorization at tree level (Khodjamirian, 1999):

$$F_{\gamma^* \gamma \rightarrow \pi^0}^{\text{LP}}(Q^2) = \frac{\sqrt{2} (Q_u^2 - Q_d^2) f_\pi}{Q^2} \left[\int_0^1 dx \frac{\phi_\pi(x)}{x} - \frac{80}{9} \frac{\delta_\pi^2}{Q^2} \right], \quad \delta_\pi^2 \simeq 0.2 \text{ GeV}^2.$$

- ▶ Asymptotic twist-4 contribution.
- ▶ Two-particle and three-particle twist-4 corrections related by the EOM.
- ▶ Four-particle twist-4 correction not included and assumed to be tiny.
- ▶ Asymptotic NLO twist-4 at NLO for future.

Region B: Photon emission at large distances

- Hadronic photon correction:



- QCD factorization at tree level:

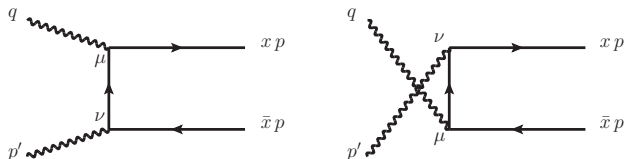
$$F_{\gamma^* \gamma \rightarrow \pi^0}^{\text{LP}}(Q^2) = \frac{\sqrt{2} (Q_u^2 - Q_d^2) f_\pi}{Q^2} \frac{16 \pi \alpha_s \chi(\mu) \langle \bar{q}q \rangle^2}{9 f_\pi^2 Q^2} \int_0^1 dx \frac{\phi_{3;\pi}^P(x)}{x} \int_0^1 dy \frac{\phi_\gamma(y)}{\bar{y}^2}.$$

Breakdown of QCD factorization due to rapidity singularities.

- QCD calculation of the hadronic photon effect beyond the VMD approximation.
 - ⇒ The NLL LCSR for the long-distance photon effect (this talk).
 - ⇒ QCD factorization for the correlation function with a pseudoscalar current.
 - ⇒ γ_5 prescription in dimensional regularization.

Part I: Leading twist contribution

- Tree diagrams:



Kinematics:

$$p'_\mu = \underbrace{\frac{n \cdot p'}{2}}_{\mathcal{O}(\sqrt{Q^2})} \bar{n}_\mu, \quad p_\mu = \underbrace{\frac{\bar{n} \cdot p}{2}}_{\mathcal{O}(\sqrt{Q^2})} n_\mu + \underbrace{\frac{n \cdot p}{2}}_{\mathcal{O}(\Lambda^2/\sqrt{Q^2})} \bar{n}_\mu.$$

- The four-point partonic matrix element at LO:

$$\begin{aligned} \Pi_\mu &= \langle q(xp) \bar{q}(\bar{x}p) | J_\mu^{\text{em}} | \gamma(p') \rangle \\ &= -\frac{i g_{\text{em}}^2 (Q_u^2 - Q_d^2)}{2\sqrt{2} n \cdot p} \varepsilon^\nu(p') \left\{ \frac{[\bar{u}(xp) \gamma_{\mu,\perp} \not{n} \gamma_{\nu,\perp} v(\bar{x}p)]}{\bar{x}} - \frac{[\bar{u}(xp) \gamma_{\nu,\perp} \not{n} \gamma_{\mu,\perp} v(\bar{x}p)]}{x} \right\} \\ &\quad + \mathcal{O}(\alpha_s^2). \end{aligned}$$

QCD matrix element free of the γ_5 ambiguity, however not the IR subtraction.

Twist-2 factorization at tree level

- Operator matching automatically:

$$\Pi_{\mu}^{(0)} = -\frac{i g_{\text{em}}^2 (Q_u^2 - Q_d^2)}{2\sqrt{2} n \cdot p} \varepsilon^{\nu}(p') \left[\frac{1}{\bar{x}'} * \langle O_{A,\mu\nu}(x,x') \rangle^{(0)} - \frac{1}{x'} * \langle O_{B,\mu\nu}(x,x') \rangle^{(0)} \right].$$

- Collinear operators in the momentum space:

$$O_{j,\mu\nu}(x') = \frac{\bar{n} \cdot p}{2\pi} \int d\tau e^{i x' \tau \bar{n} \cdot p} \bar{\xi}(\tau \bar{n}) W_c(\tau \bar{n}, 0) \Gamma_{j,\mu\nu} \xi(0).$$

$$\Gamma_{A,\mu\nu} = \gamma_{\mu,\perp} \not{\bar{n}} \gamma_{\nu,\perp}, \quad \Gamma_{B,\mu\nu} = \gamma_{\nu,\perp} \not{\bar{n}} \gamma_{\mu,\perp}.$$

- Matrix elements of the collinear operators:

$$\langle O_{j,\mu\nu}(x,x') \rangle \equiv \langle q(xp) \bar{q}(\bar{x}p) | O_{j,\mu\nu}(x') | 0 \rangle = \bar{\xi}(xp) \Gamma_{j,\mu\nu} \xi(\bar{x}p) \delta(x-x') + \mathcal{O}(\alpha_s).$$

- Operator matching with the collinear operator defining the standard pion DA:

$$O_{A,\mu\nu} = -(O_{1,\mu\nu} + O_{2,\mu\nu} + O_{E,\mu\nu}), \quad O_{B,\mu\nu} = -(O_{1,\mu\nu} - O_{2,\mu\nu} - O_{E,\mu\nu}).$$

$$\underbrace{\Gamma_{1,\mu\nu}} = g_{\mu\nu}^{\perp} \not{\bar{n}}, \quad \Gamma_{2,\mu\nu} = i \varepsilon_{\mu\nu}^{\perp} \not{\bar{n}} \gamma_5, \quad \underbrace{\Gamma_{E,\mu\nu}} = \not{\bar{n}} \left(\frac{[\gamma_{\mu,\perp}, \gamma_{\nu,\perp}]}{2} - i \varepsilon_{\mu\nu}^{\perp} \not{\bar{n}} \gamma_5 \right).$$

wrong projector

evanescent operator

Twist-2 factorization at tree level

- Operator matching with the evanescent operator:

$$\Pi_\mu = \left[\frac{i g_{\text{em}}^2 (Q_u^2 - Q_d^2)}{2\sqrt{2} n \cdot p} \varepsilon^\nu(p') \right] \sum_i T_i(x') * \langle O_{i,\mu\nu}(x, x') \rangle.$$

Expansion \Downarrow at tree level

$$T_1^{(0)}(x') = \frac{1}{x'} - \frac{1}{\bar{x}'}, \quad T_2^{(0)}(x') = T_E^{(0)}(x') = \frac{1}{x'} + \frac{1}{\bar{x}'}$$

- Hard-collinear factorization at tree level:

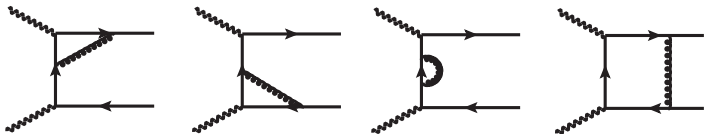
$$F_{\gamma^* \gamma \rightarrow \pi^0}^{\text{LP}}(Q^2) = \frac{\sqrt{2}(Q_u^2 - Q_d^2) f_\pi}{Q^2} \int_0^1 dx T_2^{(0)}(x) \phi_\pi(x, \mu) + \mathcal{O}(\alpha_s).$$

Evanescent operator does not mix into the physical operator at LO.

\Rightarrow Trivial IR subtraction here.

Twist-2 factorization at NLO

- The four-point partonic matrix element at NLO:



- Extracting the hard contribution with the method of regions:

$$\Pi_{\mu}^{(1)} = \frac{i g_{\text{em}}^2 (Q_u^2 - Q_d^2)}{2\sqrt{2} n \cdot p} \epsilon^{\nu}(p') \langle \mathcal{O}_{2,\mu\nu}(x, x') \rangle^{(0)} * A_{2,\text{hard}}^{(1)}(x') + \dots$$

$$A_{2,\text{hard}}^{(1)}(x') = \frac{\alpha_s C_F}{4\pi} \left\{ \frac{1}{\bar{x}'} \left[- (2 \ln \bar{x}' + 3) \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{Q^2} \right) + \ln^2 \bar{x}' + 7 \frac{\bar{x}' \ln \bar{x}'}{x'} - 9 \right] + (x' \leftrightarrow \bar{x}') \right\}.$$

Only the hard and collinear regions relevant at leading power in $1/Q^2$.

Independent of the γ_5 prescription!

- However the IR subtraction is not trivial any longer due to the operator mixing.

Twist-2 factorization at NLO

- Expanding the matching equation at NLO:

$$\left[\frac{ig_{\text{em}}^2 (Q_u^2 - Q_d^2)}{2\sqrt{2} n \cdot p} \varepsilon^{\nu}(p') \right] \sum_i A_i^{(1)}(x') * \langle O_{i,\mu\nu}(x, x') \rangle^{(0)}$$

$$= \left[\frac{ig_{\text{em}}^2 (Q_u^2 - Q_d^2)}{2\sqrt{2} n \cdot p} \varepsilon^{\nu}(p') \right] \sum_i \left[T_i^{(1)}(x') * \langle O_{i,\mu\nu}(x, x') \rangle^{(0)} + T_i^{(0)}(x') * \langle O_{i,\mu\nu}(x, x') \rangle^{(1)} \right].$$

- One-loop renormalized matrix elements of collinear operators:

$$\langle O_{i,\mu\nu} \rangle^{(1)} = \sum_j \left[M_{ij,\text{bare}}^{(1)R} + Z_{ij}^{(1)} \right] * \langle O_{j,\mu\nu} \rangle^{(0)}.$$

Vanishing bare matrix element $M_{ij,\text{bare}}^{(1)R}$ in dimensional regularization \Rightarrow

$$T_2^{(1)} = A_2^{(1)} - \sum_i T_i^{(0)} * Z_{i2}^{(1)} = \underbrace{A_2^{(1)} - T_2^{(0)} * Z_{22}^{(1)}}_{A_{2,\text{hard}}^{(1)}} - T_E^{(0)} * \underbrace{Z_{E2}^{(1)}}_{\text{operator mixing}}.$$

- The IR finite matrix element of the evanescent operator vanishes (Dugan and Grinstein, 1991).

$$Z_{E2}^{(1)} = -M_{E2}^{(1)\text{off}}.$$

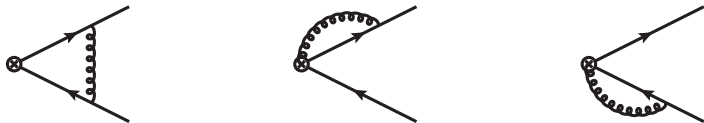
IR singularities regularized by any parameter other than the dimensions of spacetime.

Twist-2 factorization at NLO

- Master formula for the hard function:

$$T_2^{(1)} = A_2^{(1)} - T_2^{(0)} * Z_{22}^{(1)} + T_E^{(0)} * M_{E2}^{(1)\text{off}} = A_{2,\text{hard}}^{(1)} + T_E^{(0)} * M_{E2}^{(1)\text{off}}.$$

- The IR subtraction:



γ_5 -scheme dependent subtraction term:

$$T_E^{(0)} * M_{E2}^{(1)\text{off}} \Big|_{\text{NDR}} = \frac{\alpha_s C_F}{2\pi} (-4) \left(\frac{\ln \bar{x}'}{x'} + \frac{\ln x'}{\bar{x}'} \right).$$

$$T_E^{(0)} * M_{E2}^{(1)\text{off}} \Big|_{\text{HV}} = 0.$$

$M_{E2}^{(1)\text{off}}$ proportional to the spin-dependent term of the Brodsky-Lepage evolution kernel.

A simple example :

$$\gamma_\alpha \not{n} \gamma_5 \gamma^\alpha = \not{n} \gamma_5 \begin{cases} D-2, & [\text{NDR scheme}] \\ 6-D. & [\text{HV scheme}] \end{cases}$$

Twist-2 factorization at NLO

- The NLO hard matching coefficient:

$$T_2^{(1)}(x', \mu) = \frac{\alpha_s C_F}{4\pi} \left\{ \frac{1}{\bar{x}'} \left[-(2 \ln \bar{x}' + 3) \ln \frac{\mu^2}{Q^2} + \ln^2 \bar{x}' + \delta \frac{\bar{x}' \ln \bar{x}'}{x'} - 9 \right] + (x' \leftrightarrow \bar{x}') \right\}.$$

$\delta = -1$ in NDR scheme and $\delta = +7$ in HV scheme.

- The NLO factorization formula:

$$F_{\gamma^* \gamma \rightarrow \pi^0}^{\text{LP}}(Q^2) = \frac{\sqrt{2}(Q_u^2 - Q_d^2)f_\pi}{Q^2} \int_0^1 dx \left[T_2^{(0)}(x) + T_2^{(1),\Delta}(x, \mu) \right] \phi_\pi^\Delta(x, \mu) + \mathcal{O}(\alpha_s^2).$$

- Scheme dependence of the pion DA (Melic, Nizic and Passek, 2002):

$$\phi_\pi^{\text{HV}}(x, \mu) = \int_0^1 dy Z_{\text{HV}}^{-1}(x, y, \mu) \phi_\pi^{\text{NDR}}(y, \mu),$$

$$Z_{\text{HV}}^{-1}(x, y, \mu) = \delta(x-y) + \frac{\alpha_s C_F}{2\pi} 4 \left[\frac{x}{y} \theta(y-x) + \frac{\bar{x}}{\bar{y}} \theta(x-y) \right] + \mathcal{O}(\alpha_s^2).$$

↓

$$\int_0^1 dx T_2^{(0)}(x) [\phi_\pi^{\text{HV}}(x, \mu) - \phi_\pi^{\text{NDR}}(x, \mu)] = \frac{\alpha_s C_F}{2\pi} (-4) \int_0^1 dy \left(\frac{\ln \bar{y}}{y} + \frac{\ln y}{\bar{y}} \right) \phi_\pi^{\text{NDR}}(x, \mu) + \mathcal{O}(\alpha_s^2).$$

⇒ Scheme independence of $F_{\gamma^* \gamma \rightarrow \pi^0}$ at NLO.

Twist-2 factorization at NLL

- RG evolution of the pion LCDA:

$$\mu^2 \frac{d}{d\mu^2} \phi_\pi(x, \mu) = \int_0^1 dy V(x, y) \phi_\pi(y, \mu), \quad V(x, y) = \sum_{n=0} \left(\frac{\alpha_s}{4\pi} \right)^{n+1} [V_n(x, y)]_+.$$
$$V_0(x, y) = 2C_F \left[\frac{1-x}{1-y} \left(1 + \frac{1}{x-y} \right) \theta(x-y) + \frac{x}{y} \left(1 + \frac{1}{y-x} \right) \theta(y-x) \right].$$

Multiplicative renormalization at LO:

$$\phi_\pi(x, \mu) = 6x\bar{x} \sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_{V,n}^{(0)}/(2\beta_0)} a_n(\mu_0) C_n^{3/2}(2x-1), \quad a_0(\mu) = 1.$$

- NLL resummation needs two-loop evolution kernel (Mikhailov and Radyushkin, 1985; etc):

$$V_1(x, y) = 2N_f C_F V_N(x, y) + 2C_F C_A V_G(x, y) + C_F^2 V_F(x, y).$$

Triangular evolution matrix (Müller, 1994/1995 + many others):

$$a_n(\mu) = E_{V,n}^{\text{NLO}}(\mu, \mu_0) a_n(\mu_0) + \frac{\alpha_s(\mu)}{4\pi} \sum_{k=0}^{n-2} E_{V,n}^{\text{LO}}(\mu, \mu_0) d_{V,n}^k(\mu, \mu_0) a_k(\mu_0).$$

Construction from the forward anomalous dimensions and the special conformal anomaly matrix.

Twist-2 factorization at NLL

- NLL resummation improved factorization formula:

$$F_{\gamma^* \gamma \rightarrow \pi^0}^{\text{LP}}(Q^2) = \frac{3\sqrt{2}(Q_u^2 - Q_d^2)}{Q^2} f_\pi \sum_{n=0}^{\infty} a_n(\mu) C_n(Q^2, \mu) + \mathcal{O}(\alpha_s^2).$$

NNLO hard kernel in the large β_0 limit (Melic, Müller and Passek-Kumericki, 2003).

- Generating function of the Gegenbauer polynomials:

$$\frac{1}{(1-2xt+t^2)^\alpha} = \sum_{n=0}^{\infty} C_n^{(\alpha)}(x) t^n.$$

The NLO hard coefficients:

$$C_n(Q^2, \mu) = 1 + \frac{\alpha_s(\mu) C_F}{4\pi} \left\{ \left[4H_{n+1} - \frac{3n(n+3)+8}{(n+1)(n+2)} \right] \ln \frac{\mu^2}{Q^2} + 4H_{n+1}^2 - 4 \frac{H_{n+1}+1}{(n+1)(n+2)} \right. \\ \left. + 2 \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} \right] + 3 \left[\frac{1}{(n+1)} - \frac{1}{(n+2)} \right] - 9 \right\}.$$

Part II: Hadronic photon effect

- Breakdown of the direct QCD factorization for the long-distance photon effect
⇒ Construct the sum rules for the hadronic photon correction.

- Correlation function:

$$G_{\mu}(p', q) = \int d^4z e^{-iq \cdot z} \langle 0 | T \{ j_{\mu, \perp}^{\text{em}}(z), j_{\pi}(0) \} | \gamma(p') \rangle = -g_{\text{em}}^2 \varepsilon_{\mu\nu\alpha\beta} q^{\alpha} p'^{\beta} \varepsilon_{\nu}(p') G(p^2, Q^2).$$
$$j_{\pi} = \frac{1}{\sqrt{2}} (\bar{u} \gamma_5 u - \bar{d} \gamma_5 d), \quad p^2 = (p' + q)^2.$$

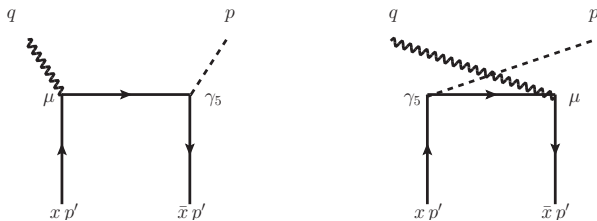
Explicit dependence on γ_5 ⇒ scheme dependence of the QCD amplitude.

- Task: QCD factorization for the transition form factor $G(p^2, Q^2)$ at NLL.
Power counting rule:

$$|n \cdot p| \sim \bar{n} \cdot p \sim n \cdot p' \sim \mathcal{O}(\sqrt{Q^2}).$$

Hadronic photon effect at tree level

- Tree diagrams:



p for the four-momentum of the pion current and q for the transfer momentum.

- The four-point QCD amplitude at LO:

$$\begin{aligned} \tilde{\Pi}_\mu &= \int d^4z e^{-iq \cdot z} \langle 0 | T \left\{ J_{\mu,\perp}^{\text{em}}(z), j_\pi(0) \right\} | q(xp') \bar{q}(\bar{x}p') \rangle \\ &= -\frac{i g_{\text{em}}}{2\sqrt{2}} \frac{\bar{n} \cdot p}{Q^2} \left[\frac{1}{xr + \bar{x}} + \frac{1}{\bar{x}r + x} \right] \sum_{q=u,d} \eta_q Q_q \bar{q}(\bar{x}p') \gamma_5 \not{n} \gamma_{\mu,\perp} q(xp'), \end{aligned}$$

where $r = -p^2/Q^2$, $\eta_u = 1$ and $\eta_d = -1$.

Hadronic photon effect at tree level

- Operator matching automatically:

$$\tilde{\Pi}_\mu^{(0)} = -\frac{i g_{em}}{2\sqrt{2}} \frac{\bar{n} \cdot p}{Q^2} \sum_{q=u,d} \eta_q Q_q \left[\frac{1}{x' r + \bar{x}'} + \frac{1}{\bar{x}' r + x'} \right] * \langle \tilde{O}_{A,\mu}(x, x') \rangle^{(0)}.$$

- (Anti)-collinear operators in the momentum space:

$$\begin{aligned} \tilde{O}_{j,\mu}(x') &= \frac{\bar{n} \cdot p'}{2\pi} \int d\tau e^{ix' \tau \bar{n} \cdot p'} \tilde{\chi}(0) W_{\bar{c}}(0, \tau n) \tilde{\Gamma}_{j,\mu} \chi(\tau n), \\ \tilde{\Gamma}_{A,\mu} &= \not{\gamma} \not{n} \gamma_{\mu,\perp}. \end{aligned}$$

- Matrix element of the (anti)-collinear operators:

$$\langle \tilde{O}_{j,\mu}(x, x') \rangle \equiv \langle 0 | \tilde{O}_{j,\mu}(x') | q(xp') \bar{q}(\bar{x}p') \rangle = \tilde{\chi}(\bar{x}p') \tilde{\Gamma}_{j,\mu} \chi(xp') \delta(x - x') + \mathcal{O}(\alpha_s).$$

- Operator matching with the effective operator defining the photon DA:

$$\begin{aligned} \tilde{O}_{A,\mu} &= \tilde{O}_{1,\mu} + \tilde{O}_{E,\mu}, \\ \tilde{\Gamma}_{1,\mu} &= \frac{n^\alpha}{2} \varepsilon_{\mu\nu\alpha\beta}^\perp \sigma^{\nu\beta}, \quad \underbrace{\tilde{\Gamma}_{E,\mu}}_{\text{evanescent operator}} = \not{\gamma} \not{n} \gamma_{\mu,\perp} - \frac{n^\alpha}{2} \varepsilon_{\mu\nu\alpha\beta}^\perp \sigma^{\nu\beta}. \end{aligned}$$

evanescent operator

Hadronic photon effect at tree level

- Operator matching with the evanescent operator:

$$\tilde{\Pi}_\mu = -\frac{i g_{\text{em}}}{2\sqrt{2}} \frac{\bar{n} \cdot p}{Q^2} \sum_{q=u,d} \eta_q Q_q \sum_i \tilde{T}_i(x') * \langle \tilde{O}_{i,\mu}(x, x') \rangle.$$

Expansion \Downarrow at tree level

$$\tilde{T}_1^{(0)}(x') = \tilde{T}_E^{(0)}(x') = \frac{1}{x' r + \bar{x}'} + \frac{1}{\bar{x}' r + x'}.$$

- Leading twist photon DA (Ball, Braun and Kivel, 2002):

$$\begin{aligned} & \langle 0 | \bar{\chi}(0) W_{\bar{c}}(0, y) \sigma_{\alpha\beta} \chi(y) | \gamma(p') \rangle \\ &= i g_{\text{em}} Q_q \chi(\mu) \langle \bar{q}q \rangle(\mu) \left[p'_\beta \varepsilon_\alpha(p') - p'_\alpha \varepsilon_\beta(p') \right] \int_0^1 du e^{-iu p' \cdot y} \phi_\gamma(u, \mu). \end{aligned}$$

- Hard-collinear factorization at LO:

$$G(p^2, Q^2) = -\frac{Q_u^2 - Q_d^2}{\sqrt{2} Q^2} \chi(\mu) \langle \bar{q}q \rangle(\mu) \int_0^1 dx \tilde{T}_1^{(0)}(x) \phi_\gamma(x, \mu) + \mathcal{O}(\alpha_s).$$

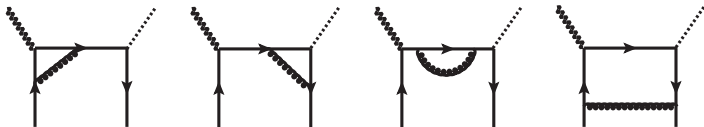
- The tree-level LCSR:

$$F_{\gamma^* \gamma \rightarrow \pi^0}^{\text{NLP}}(Q^2) = -\frac{\sqrt{2} (Q_u^2 - Q_d^2)}{f_\pi \mu_\pi(\mu)} \chi(\mu) \langle \bar{q}q \rangle(\mu) \int_{u_0}^1 \frac{du}{u} \exp \left[-\frac{\bar{u} Q^2 + u m_\pi^2}{u M^2} \right] \phi_\gamma(u, \mu) + \mathcal{O}(\alpha_s).$$

Verify the power counting of the hadronic photon correction explicitly.

Hadronic photon effect at NLO

- The four-point partonic matrix element at NLO:



- Extracting the hard contribution with the method of regions:

$$\tilde{\Pi}_\mu^{(1)} = -\frac{i g_{\text{em}}}{2\sqrt{2}} \frac{\bar{n} \cdot p}{Q^2} \sum_{q=u,d} \eta_q Q_q \langle \tilde{\mathcal{O}}_{1,\mu}(x, x') \rangle^{(0)} * \tilde{A}_{1,\text{hard}}(x') + \dots$$

Scheme dependent one-loop QCD amplitude:

$$\begin{aligned} \tilde{A}_{1,\text{hard}}(x') \Big|_{\text{NDR}} &= \frac{\alpha_s C_F}{4\pi} \left\{ \frac{1}{x' r + \bar{x}'} \left[\frac{2}{x' \bar{x}' \bar{r}} \left(((x' r - \bar{x}') \ln(x' r + \bar{x}') - x' r \ln r) \right. \right. \right. \\ &\quad \left. \left. \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{Q^2} - \frac{1}{2} \ln(x' r + \bar{x}') - \frac{1}{2} \ln r \right) \right. \right. \\ &\quad \left. \left. - \frac{1}{x' \bar{r}} (\ln r + 3) \ln(x' r + \bar{x}') - 3 \right] + (x' \leftrightarrow \bar{x}') \right\}, \\ \tilde{A}_{1,\text{hard}}(x') \Big|_{\text{HV}} &= \tilde{A}_{1,\text{hard}}(x') \Big|_{\text{NDR}} + \frac{2\alpha_s C_F}{\pi} \tilde{T}_1^{(0)}(x'). \end{aligned}$$

Hadronic photon effect at NLO

- Repeating the IR subtraction procedure as before:

$$\tilde{T}_1^{(1)} = \tilde{A}_1^{(1)} - \tilde{T}_1^{(0)} * \tilde{Z}_{11}^{(1)} + \tilde{T}_E^{(0)} * \tilde{M}_{E1}^{(1)\text{off}} = \tilde{A}_{1,\text{hard}}^{(1)} + \tilde{T}_E^{(0)} * \tilde{M}_{E1}^{(1)\text{off}}.$$

- The IR subtraction:

$$\tilde{T}_E^{(0)} * \tilde{M}_{E1}^{(1)\text{off}} \Big|_{\text{NDR}} = 0 + \mathcal{O}(\alpha_s^2), \quad \tilde{T}_E^{(0)} * \tilde{M}_{E1}^{(1)\text{off}} \Big|_{\text{HV}} = \mathcal{O}(\varepsilon \alpha_s).$$

↓

$$\tilde{T}_1^{(1)} = \tilde{A}_{1,\text{hard}}^{(1)}, \quad \text{for both NDR and HV.}$$

- Finite renormalization constant $Z_{\text{HV}}^P(\mu)$ for the HV scheme (Larin, 1993):

$$Z_{\text{HV}}^P(\mu) = 1 - \frac{2\alpha_s(\mu)C_F}{\pi} + \mathcal{O}(\alpha_s^2).$$

↓

$$Z_{\text{HV}}^P(\mu) \left[\tilde{T}_1^{(0)}(x') + \tilde{T}_1^{(1)}(x', \mu) \right]_{\text{HV}} = \left[\tilde{T}_1^{(0)}(x') + \tilde{T}_1^{(1)}(x', \mu) \right]_{\text{NDR}}$$

A non-trivial check of our results. Also reproduce $H \rightarrow J/\psi\gamma$ in the $r \rightarrow \infty$ limit.

Hadronic photon effect at NLL

- The NLO factorization formula:

$$G(p^2, Q^2) = -\frac{Q_u^2 - Q_d^2}{\sqrt{2} Q^2} \chi(\mu) \langle \bar{q}q \rangle(\mu) \int_0^1 dx \left[\tilde{T}_1^{(0)}(x) + \tilde{T}_1^{(1)}(x, \mu) \right]_{\text{NDR}} \phi_\gamma(x, \mu) + \mathcal{O}(\alpha_s^2).$$

- RG evolution of the photon LCDA (Lepage and Brodsky, 1979):

$$\mu^2 \frac{d}{d\mu^2} [\chi(\mu) \langle \bar{q}q \rangle(\mu) \phi_\gamma(x, \mu)] = \int_0^1 dy \tilde{V}(x, y) [\chi(\mu) \langle \bar{q}q \rangle(\mu) \phi_\gamma(y, \mu)],$$
$$\tilde{V} = \sum_{n=0} \left(\frac{\alpha_s}{4\pi} \right)^{n+1} \tilde{V}_n, \quad \tilde{V}_0 = 2 C_F \left[\frac{\bar{x}}{\bar{y}} \frac{1}{x-y} \theta(x-y) + \frac{x}{y} \frac{1}{y-x} \theta(y-x) \right]_+ - C_F \delta(x-y).$$

⇒ Factorization-scale independence of $G(p^2, Q^2)$ at one loop.

- NLL resummation needs two-loop evolution kernel (Mikhailov and Vladimirov, 2009 + others):

$$\tilde{V}_1(x, y) = \frac{N_f}{2} C_F \tilde{V}_N(x, y) + C_F C_A \tilde{V}_G(x, y) + C_F^2 \tilde{V}_F(x, y).$$

$$\chi(\mu) \langle \bar{q}q \rangle(\mu) b_n(\mu) = E_{T,n}^{\text{NLO}}(\mu, \mu_0) b_n(\mu_0) + \frac{\alpha_s(\mu)}{4\pi} \sum_{k=0}^{n-2} E_{T,n}^{\text{LO}}(\mu, \mu_0) d_{T,n}^k(\mu, \mu_0) b_k(\mu_0).$$

Hadronic photon effect at NLL

- Dispersion form of the NLL factorization formula:

$$G(p^2, Q^2) = -\frac{\sqrt{2} (Q_u^2 - Q_d^2)}{Q^2} \chi(\mu) \langle \bar{q}q \rangle(\mu) \int_0^\infty \frac{ds}{s - p^2 - i0} \left[\rho^{(0)}(s, Q^2) + \frac{\alpha_s C_F}{4\pi} \rho^{(1)}(s, Q^2) \right].$$

The NLO spectral function:

$$\begin{aligned} \rho^{(1)}(s, Q^2) = & 2 \int_0^1 du \frac{\phi_\gamma(u, \mu)}{\bar{u}} \left\{ \theta \left(u - \frac{Q^2}{Q^2 + s} \right) \frac{Q^2}{Q^2 + s} \left[\frac{\bar{u} - u}{u} \ln \left(\frac{\mu^2}{us - \bar{u}Q^2} \right) + \frac{3}{2} \frac{\bar{u}}{u} \right] \right. \\ & \left. + \ln \left(\frac{\mu^2}{s} \right) \left[\frac{Q^2}{Q^2 + s} - \mathcal{P} \frac{\bar{u}Q^2}{\bar{u}Q^2 - us} \right] \right\} \\ & + \frac{Q^2}{Q^2 + s} \int_0^1 du \theta \left(u - \frac{Q^2}{Q^2 + s} \right) \left\{ 2 \ln \left(\frac{us - \bar{u}Q^2}{Q^2} \right) \left[\ln \left(\frac{\mu^2}{us - \bar{u}Q^2} \right) \right. \right. \\ & \left. \left. + \ln \left(\frac{\mu^2}{Q^2} \right) + \frac{3}{2} \right] - \ln^2 \left(\frac{\mu^2}{Q^2} \right) + \ln^2 \left(\frac{\mu^2}{s} \right) - \frac{\pi^2}{3} + 3 \right\} \frac{d}{du} \phi_\gamma(u, \mu). \end{aligned}$$

- The NLL LCSRs for the hadronic photon effect:

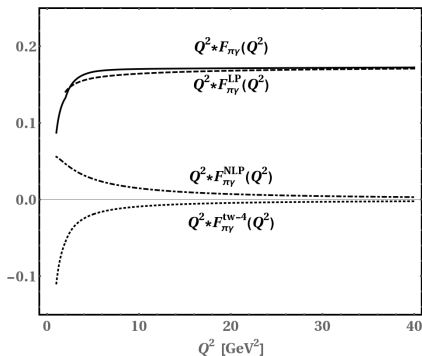
$$\begin{aligned} F_{\gamma^* \gamma \rightarrow \pi^0}^{\text{NLP}}(Q^2) = & -\frac{\sqrt{2} (Q_u^2 - Q_d^2)}{f_\pi \mu_\pi(\mu) Q^2} \chi(\mu) \langle \bar{q}q \rangle(\mu) \int_0^{s_0} ds \exp \left[-\frac{s - m_\pi^2}{M^2} \right] \\ & \times \left[\rho^{(0)}(s, Q^2) + \frac{\alpha_s C_F}{4\pi} \rho^{(1)}(s, Q^2) \right] + \mathcal{O}(\alpha_s^2). \end{aligned}$$

Final expression of the pion-photon form factor

- Adding up different contributions together:

$$F_{\gamma^* \gamma \rightarrow \pi^0}(Q^2) = F_{\gamma^* \gamma \rightarrow \pi^0}^{\text{LP}}(Q^2) + F_{\gamma^* \gamma \rightarrow \pi^0}^{\text{NLP}}(Q^2) + F_{\gamma^* \gamma \rightarrow \pi^0}^{\text{tw-4}}(Q^2).$$

- Breakdown of various contributions:



Input parameters:

$$a_2(1.0 \text{ GeV}) = 0.21_{-0.06}^{+0.07},$$

$$a_4(1.0 \text{ GeV}) = -\left(0.15_{-0.09}^{+0.10}\right),$$

$$\delta_\pi^2(1.0 \text{ GeV}) = 0.20 \pm 0.04 \text{ GeV}^2,$$

$$\chi(1.0 \text{ GeV}) = (3.15 \pm 0.3) \text{ GeV}^{-2},$$

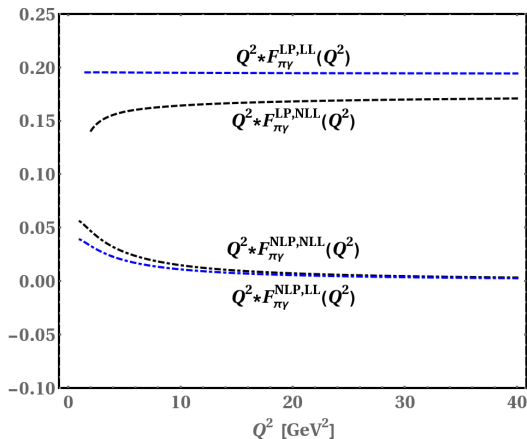
$$b_2(1.0 \text{ GeV}) = 0.07 \pm 0.07.$$

Will discuss the model dependence of the pion DA.

Large cancellation between the hadronic photon correction and the twist-4 effect.
Smallness of the power corrections at large Q^2 numerically.

QCD corrections to the pion-photon form factor

- NLO QCD corrections to both the twist-2 and the hadronic photon contributions:



- ▶ \mathcal{O} (25 %) QCD correction to the twist-2 contribution (almost Q^2 independent).
- ▶ (20-30) % QCD correction to the hadronic photon effect, but not relevant at large Q^2 .

Model dependence of the pion-photon form factor

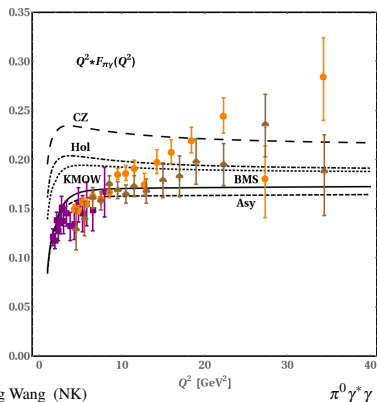
- Models of the pion DA:

$$a_2(1.0\text{GeV}) = 0.21^{+0.07}_{-0.06}, \quad a_4(1.0\text{GeV}) = -\left(0.15^{+0.10}_{-0.09}\right), \quad [\text{BMS}, 2004];$$

$$a_2(1.0\text{GeV}) = 0.17 \pm 0.08, \quad a_4(1.0\text{GeV}) = 0.06 \pm 0.10, \quad [\text{KMOW}, 2011];$$

$$a_n(1.0\text{GeV}) = \frac{2n+3}{3\pi} \left(\frac{\Gamma[(n+1)/2]}{\Gamma[(n+4)/2]} \right)^2, \quad [\text{Holographic}],$$

$$a_2(1.0\text{GeV}) = 0.5, \quad a_{n>2}(1.0\text{GeV}) = 0, \quad [\text{CZ}].$$



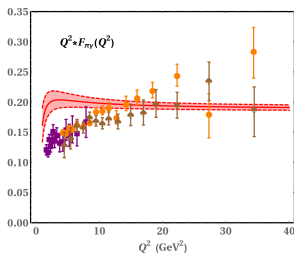
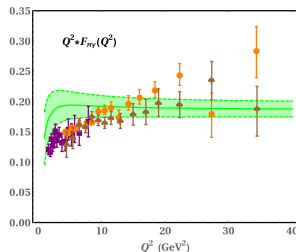
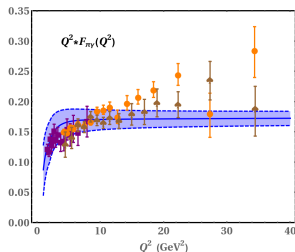
Purple squares from CLEO.
Orange circles from BaBar.
Brown bins from Belle.

KMOW and Holographic models appear to describe the data at $Q^2 \geq 10\text{GeV}^2$.

“Soft” physics more important at low Q^2 and the nonperturbative modification of the spectral function works better.

Model dependence of the pion-photon form factor

Theory predictions with BMS, KMOW and Holographic models:



Concluding Remarks

- **Scheme independence of the leading twist contribution at one loop explicitly.**
 - ▶ Operator matching with evanescent operators and the γ_5 prescription.
 - ▶ \mathcal{O} (25 %) QCD correction to the twist-2 contribution (almost Q^2 independent).
- **Hadronic photon effect at NLO.**
 - ▶ Construction of the NLL LCSRs due to the violation of the direct QCD factorization.
 - ▶ QCD factorization for the correlation function at NLO.
 - ▶ Operator matching with both the NDR and HV schemes.
 - ▶ (20-30) % QCD correction, but not relevant at large Q^2 .
- **Predictions with different models of the pion DAs confronted with the data.**
 - ▶ "Exotic" end-point behavior not needed.
 - ▶ More precise data needed (BES III at low Q^2 , Belle II?).
- **Future work:**
 - ▶ A complete NNLO twist-2 calculation.
 - ▶ The NLO twist-4 calculation (including the 3-particle DA effect).
 - ▶ Yet higher twist contributions (no correspondence between the twist and power expansion).