Convex geometry in SMEFT

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Oct. 23 USTC

Based on 1902.08977 with Q. Bi and S.-Y. Zhou,
2005.03047 with S.-Y. Zhou (accepted by PRL),
2009.02212 with B. Fuks, Y. Liu, S.-Y. Zhou, 2009.04490 with K. Yamashita, S.-Y. Zhou,
and other ongoing works

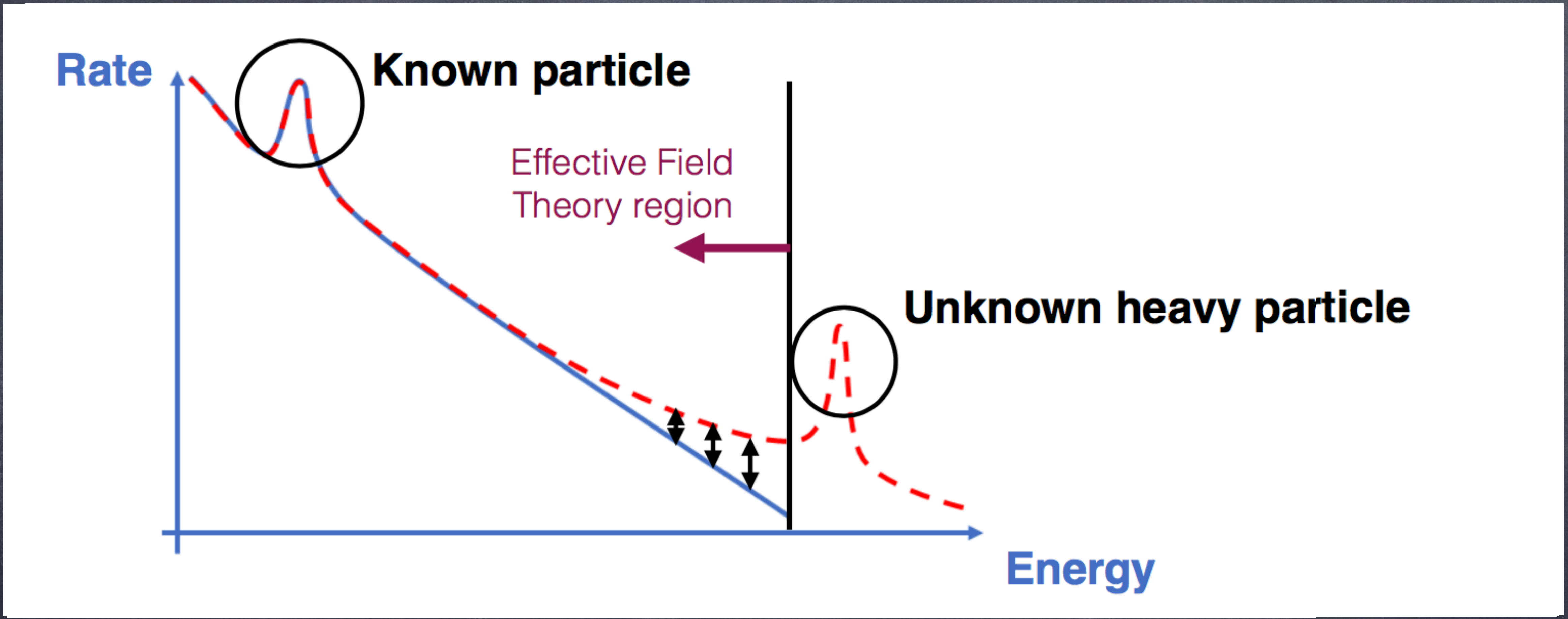
Outline

- Introduction to:
  - SMEFT
  - positivity bounds
- The positive structures in SMEFT
- Phenomenological applications

SMEFT

The SM Effective Field Theory

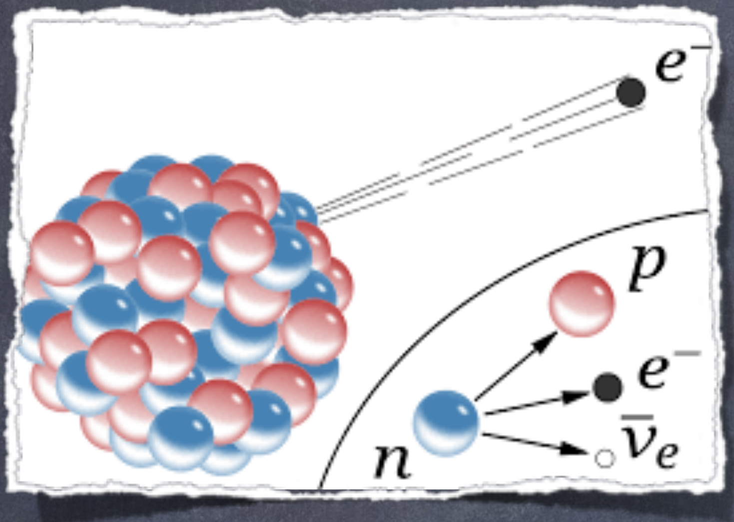
New particles? New interactions?

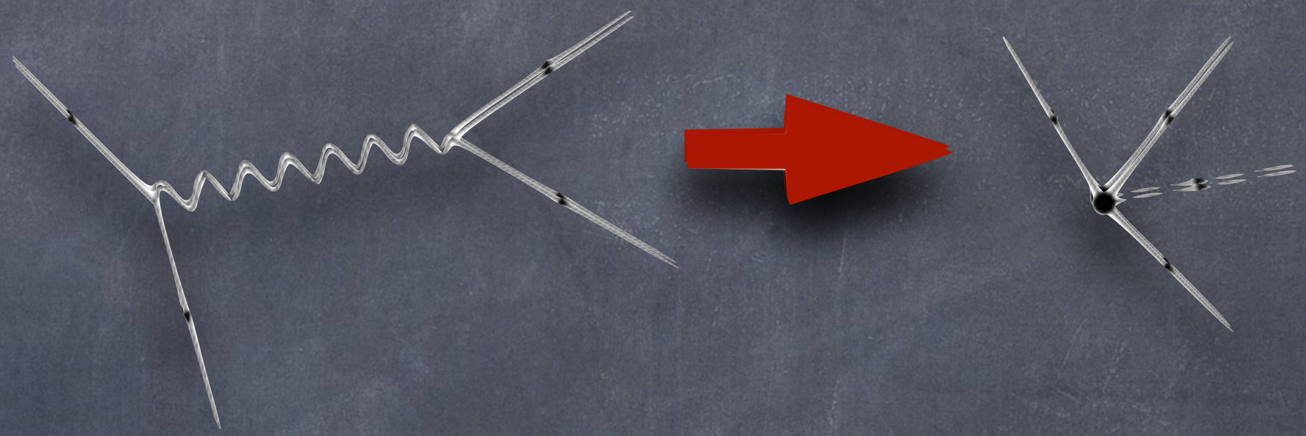


Precise measurements at low energy => probe BSM beyond the collider reach.

Slide by E. Vryonidou

The SM Effective Field Theory





Fermi's theory, as an effective theory of the weak interactions.

- Measurements at low scales could reveal information at high scales.
- A systematically improvable framework.
    - Muon lifetime has been measured with ~10^{-6} uncertainties. TH predicts at the same level, with 2-loop QED corrections.

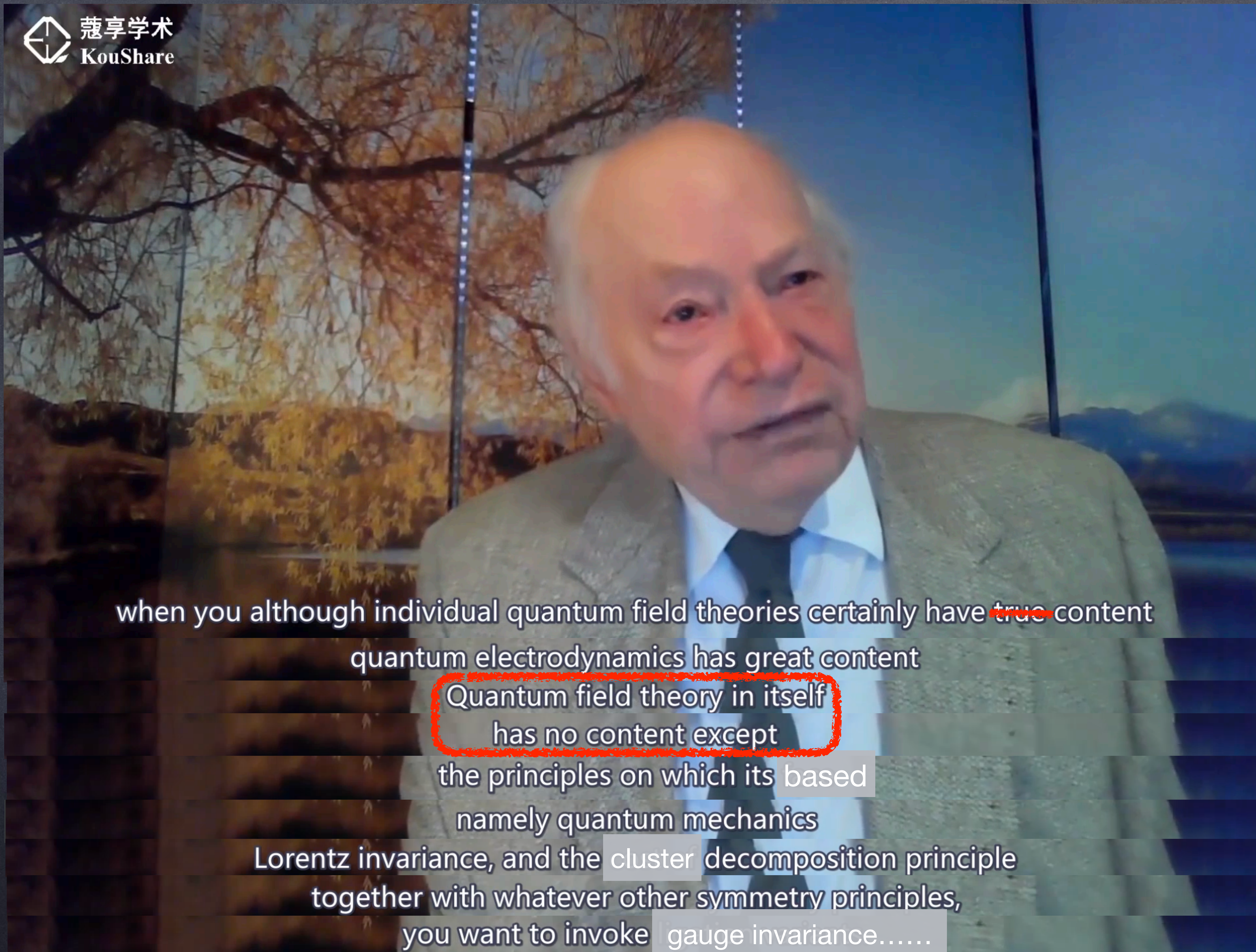
The SM Effective Field Theory

- Nature of BSM physics unknown $\rightarrow$ use EFT in a bottom-up approach.
- Write down all possible operators satisfying Lorentz and Gauge invariance

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{c_i^{(8)}}{\Lambda^4} O_i^{(8)} + \ldots$$

Λ: scale of new physics; $c_i^{(n)}$: Wilson coefficients; $O_i^{(n)}$: dim-n operators

SMEFT: parametrize high-scale New Physics in terms of Wilson coefficients. This is a **very general approach** to New Physics and allows to constrain a large class of high-scale BSM models.



From "All Things EFT Inaugural Lecture: On the Development of Effective Field Theory", Steven Weinberg https://sites.google.com/view/all-things-eft

ALL *THINGS* EFT...

ABOUT

To allow researchers in the **global Effective Field Theory community** to connect and share their work,

All Things EFT is launched as a weekly **international online seminar series** in **fall 2020,** on September 30th.

Topics include **all aspects of EFTs** such as SMEFT, HEFT, LEFT, Dark Matter EFT, EFTs of gravity, SCET, ...

The seminars will be held via zoom. To receive the link to the zoom room, please subscribe below.

FORMAT

Talks will be **weekly** on **Wednesdays** at

4pm CET (Geneva) = 10am EDT (New York) = 7am PDT (Los Angeles) = 10pm CST (Beijing)

Seminar Format: 1h plenary-style talk + discussion

PAST SEMINARS

- **Steven Weinberg** (U. Texas Austin)
- **Aneesh Manohar** (UCSD)
- **Matthias Neubert** (Mainz & Cornell)

UPCOMING SEMINARS

The seminars in this series are listed in **INSPIRE-HEP**

Confirmed speakers:

- **Henriette Elvang** (Michigan)
- **Lian-Tao Wang** (Chicago)
- **Xiaochuan Lu** (Oregon)
- **Claudia De Rham** (Imperial College London)
- **Tim Cohen** (Oregon)
- **Arsenii Titov** (Padua/Valencia)
- **Yael Shadmi** (Technion)
- **Roberto Emparan** (Barcelona)
- **Xiangdong Ji** (Maryland)
- **Walter Goldberger** (Yale)
- **Francesco Riva** (Geneva)

See also:

https://indico.ihep.ac.cn/event/12712/

https://www.youtube.com/channel/UC1_KF6kdJFoDEcLgpcegwCQ

https://inspirehep.net/seminars?sort=dateasc&size=25&page=1&start_date=upcoming&series=All%20Things%20EFT%3A%20International%20Online%20Seminar

The SM Effective Field Theory

- Dim 5 operators are lepton number (LN) violating
- There are 59 independent (B and L conserving) dim-6 operators in the SMEFT
- Counting all possible flavour configurations using 3 fermion generations: 2499
- Addional 4 B-violating operators possible (flavour not counted)

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

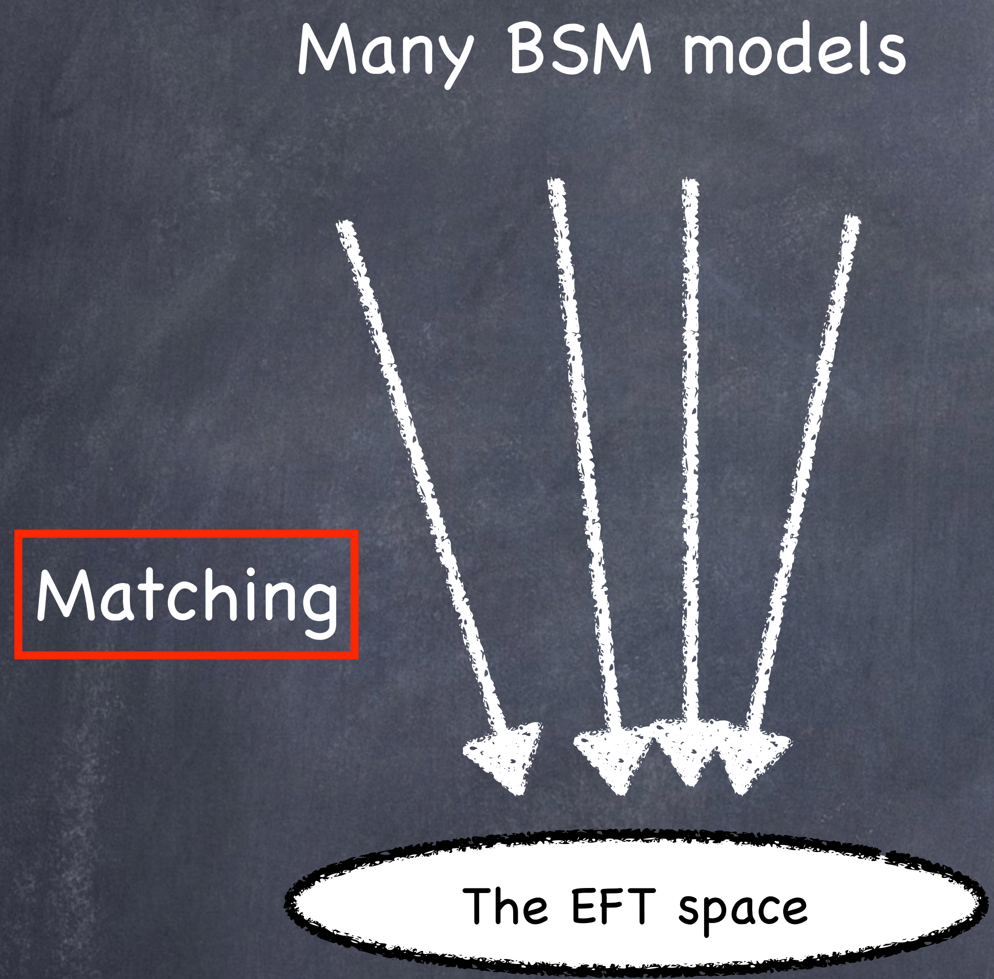
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger \varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger \varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger \varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) \varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) \varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

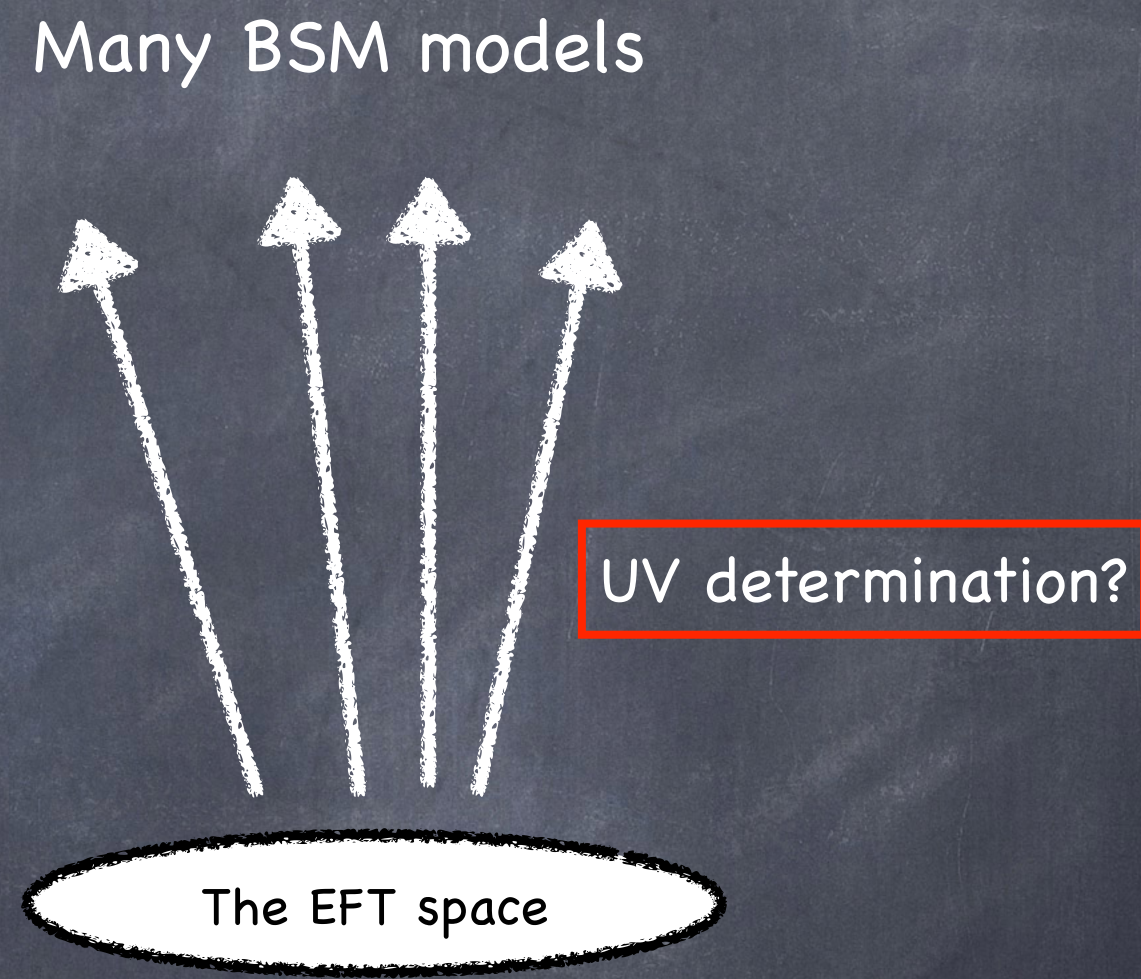
$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d^\alpha_p)^T C u^\beta_r\right]\left[(q^{\gamma j}_s)^T C l^k_t\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q^{\alpha j}_p)^T C q^{\beta k}_r\right]\left[(u^\gamma_s)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q^{\alpha j}_p)^T C q^{\beta k}_r\right]\left[(q^{\gamma m}_s)^T C l^n_t\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d^\alpha_p)^T C u^\beta_r\right]\left[(u^\gamma_s)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

[B. Grzadkowski et al., 1008.4884]

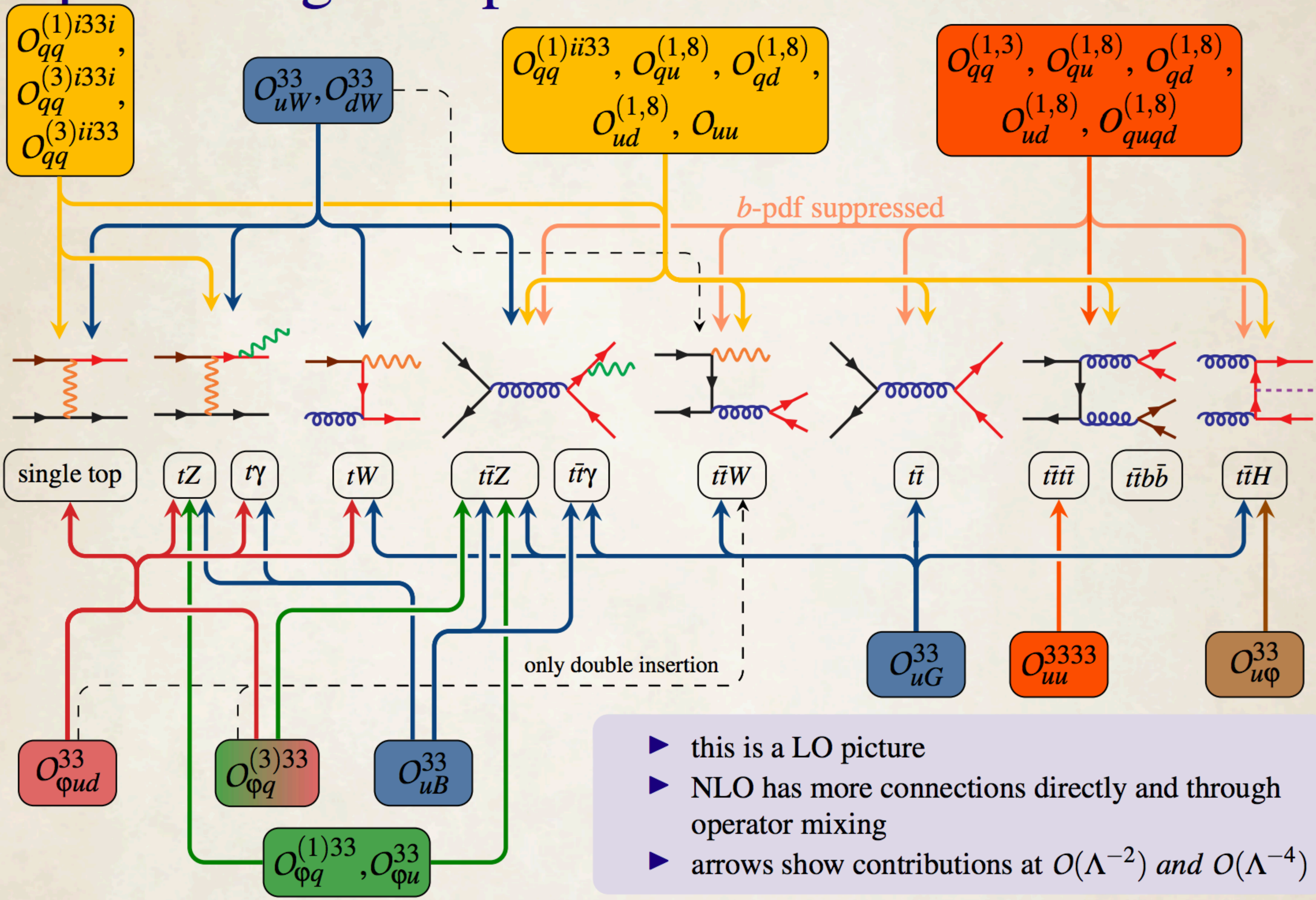
See also:
Dim-7: [L. Lehman, 14] [Liao & Ma, 16]
Dim-8: [H.-L. Li et al., 20] [C. W. Murphy, 20]
Dim-9: [H.-L. Li et al., 20] [Liao & Ma, 20]





Experiments need to be analyzed in EFT, once and for all

Top EFT: a global picture



- this is a LO picture
- NLO has more connections directly and through operator mixing
- arrows show contributions at $O(\Lambda^{-2})$ *and* $O(\Lambda^{-4})$

Theory predictions

Fitting (preliminary)

arXiv.org > hep-ph > arXiv:2008.11743

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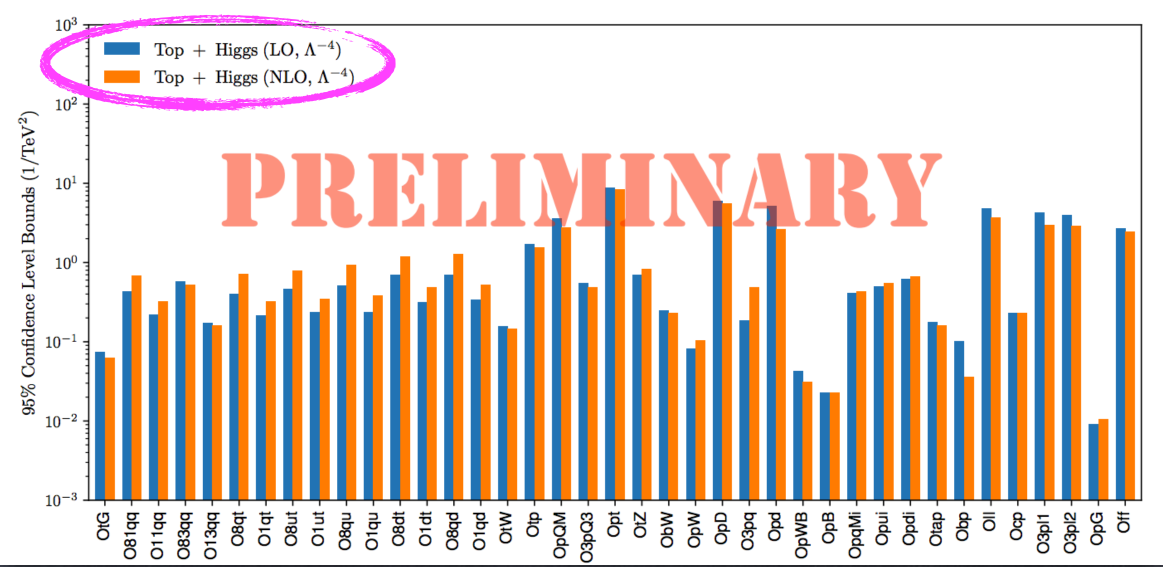
High Energy Physics – Phenomenology

[Submitted on 26 Aug 2020]

Automated one-loop computations in the SMEFT

Céline Degrande, Gauthier Durieux, Fabio Maltoni, Ken Mimasu, Eleni Vryonidou, Cen Zhang

We present the automation of one-loop computations in the standard-model effective field theory at dimension six. Our implementation, dubbed SMEFT@NLO, contains ultraviolet and rational counterterms for bosonic, two- and four-fermion operators. It presently allows for fully differential predictions, possibly matched to parton shower, up to one-loop accuracy in QCD. We illustrate the potential of the implementation with novel loop-induced and next-to-leading order computations relevant for top-quark, electroweak, and Higgs-boson phenomenology at the LHC and future colliders.



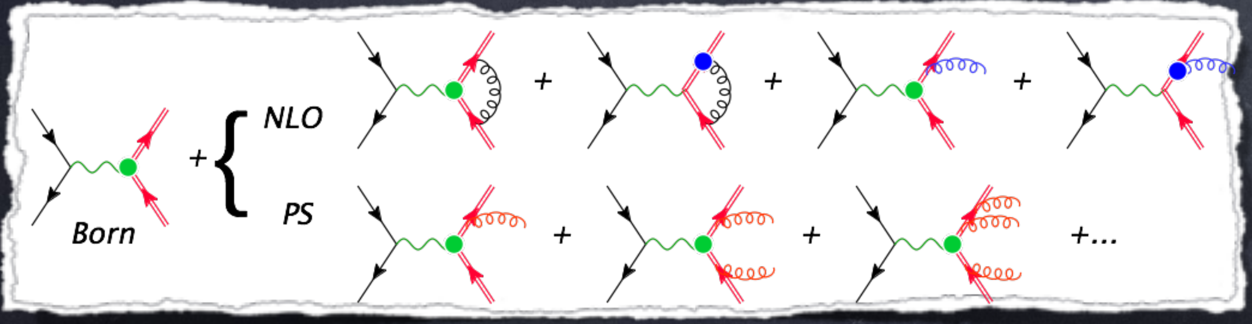
Automatic NLO+PS for all processes/operators

```
MG5_aMC>import model SMEFT
MG5_aMC>generate p p > t t~ NP=2 [QCD]
MG5_aMC>output
MG5_aMC>launch
```

		Processes						
Class	Coefficient	tt	ttV	t	tV	Hrun1	Hrun2	VV
2L2H	O81qq	81.7(82.6)	16.4(11.8)	×(×)	×(×)	0.1(0.5)	1.8(5.1)	×(×)
	O11qq	100.0(82.8)	0.0(10.5)	×(×)	×(×)	×(0.6)	0.0(6.2)	×(×)
	O83qq	48.3(39.4)	25.9(38.7)	23.9(15.5)	0.0(2.0)	0.1(0.4)	1.8(4.1)	×(×)
	O13qq	0.4(12.4)	0.0(3.9)	96.5(80.4)	3.1(1.9)	×(0.1)	0.0(1.3)	×(×)
	O8qt	56.1(55.5)	38.9(35.6)	×(×)	×(×)	0.3(0.7)	4.7(8.3)	×(×)
	O1qt	100.0(78.0)	0.0(12.4)	×(×)	×(×)	×(0.8)	0.0(8.7)	×(×)
	O8ut	97.7(90.5)	0.4(1.7)	×(×)	×(×)	0.1(0.7)	1.8(7.1)	×(×)
	O1ut	100.0(89.7)	0.0(1.5)	×(×)	×(×)	×(0.8)	0.0(8.0)	×(×)
	O8qu	88.8(80.2)	3.6(4.8)	×(×)	×(×)	0.4(1.3)	7.1(13.8)	×(×)
	O1qu	100.0(85.5)	0.0(4.0)	×(×)	×(×)	×(0.9)	0.0(9.6)	×(×)
	O8dt	95.0(89.0)	1.4(2.1)	×(×)	×(×)	0.2(0.8)	3.5(8.2)	×(×)
	O1dt	100.0(88.5)	0.0(2.2)	×(×)	×(×)	×(0.8)	0.0(8.6)	×(×)
	O8qd	94.3(83.7)	2.6(4.8)	×(×)	×(×)	0.1(1.0)	2.9(10.5)	×(×)
	O1qd	100.0(86.2)	0.0(4.2)	×(×)	×(×)	×(0.8)	0.0(8.8)	×(×)
2FB	Otp	×(×)	×(×)	×(×)	×(×)	13.7(15.9)	86.3(84.1)	×(×)
	OtG	61.1(60.3)	0.2(0.2)	×(×)	×(×)	5.9(6.0)	32.7(33.5)	×(×)
	Obp	×(×)	×(×)	×(×)	×(×)	26.6(32.5)	73.4(67.5)	×(×)
	Ocp	×(×)	×(×)	×(×)	×(×)	26.8(30.3)	73.2(69.7)	×(×)
	Otap	×(×)	×(×)	×(×)	×(×)	39.1(37.8)	60.9(62.2)	×(×)
	OtW	9.1(1.4)	0.0(0.0)	0.4(0.0)	0.2(0.0)	18.9(20.7)	71.5(77.9)	×(×)
	OtZ	×(×)	0.0(0.0)	×(×)	0.0(0.0)	21.0(21.0)	79.0(79.0)	×(×)
	ObW	×(76.0)	×(×)	×(20.2)	×(3.8)	×(×)	×(×)	×(×)
	Off	×(55.5)	100.0(0.0)	×(30.4)	×(14.1)	×(×)	×(×)	×(×)
	O3pQ3	×(0.0)	0.0(0.0)	80.0(23.4)	14.3(4.0)	1.2(13.0)	4.5(59.6)	×(×)
	OpQM	×(×)	41.8(0.1)	×(×)	0.6(0.0)	11.9(19.1)	45.7(80.8)	×(×)
	Opt	×(×)	64.5(0.1)	×(×)	0.2(0.0)	7.4(21.0)	27.9(79.0)	×(×)
B	OpG	×(×)	×(×)	×(×)	×(×)	15.3(15.7)	84.7(84.3)	×(×)
	OpB	×(×)	×(×)	×(×)	×(×)	21.0(21.1)	79.0(78.9)	×(×)
	OpW	×(×)	×(×)	×(×)	×(×)	21.0(21.2)	79.0(78.8)	×(×)
	Opd	×(×)	×(×)	×(×)	×(×)	25.4(27.4)	74.6(72.6)	×(×)
	OWWW	×(×)	×(×)	×(×)	×(×)	×(×)	×(×)	100.0(100.0)
	OW	×(×)	×(×)	×(×)	×(×)	21.1(21.1)	78.9(78.9)	0.0(0.0)
	OB	×(×)	×(×)	×(×)	×(×)	21.0(21.0)	79.0(79.0)	0.0(0.0)

Table 1: Fisher information normalized per coefficient

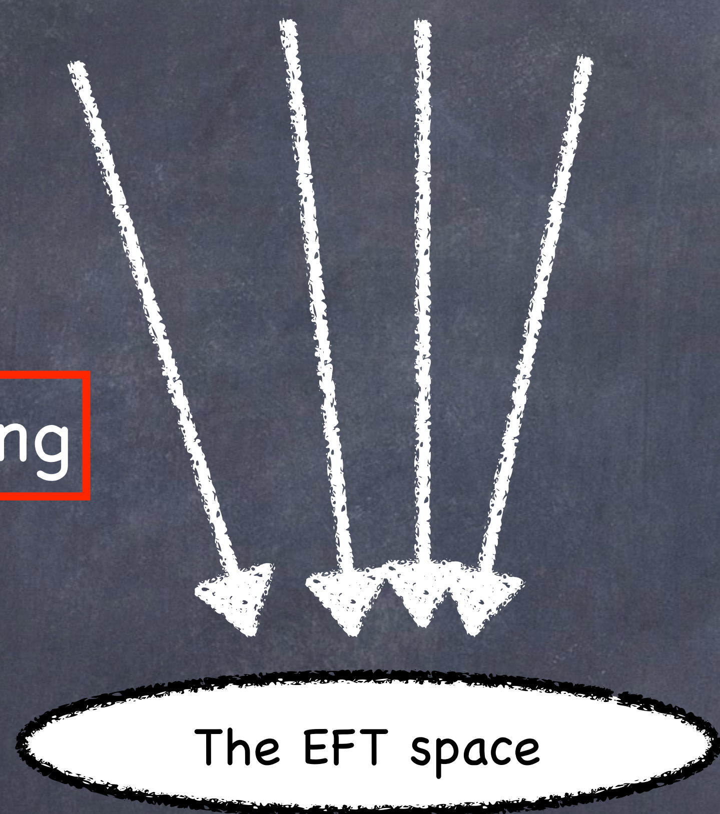
SMEFiT



See e.g. [1901.05965 N. P. Hartland et al.], [1910.03606 I. Brivio et al.]

Many BSM models

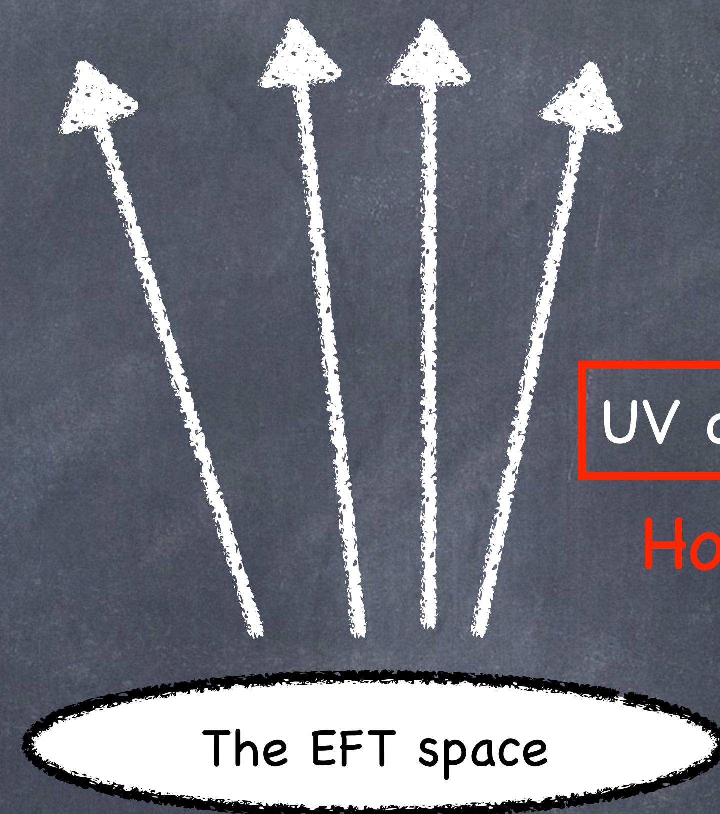
Matching



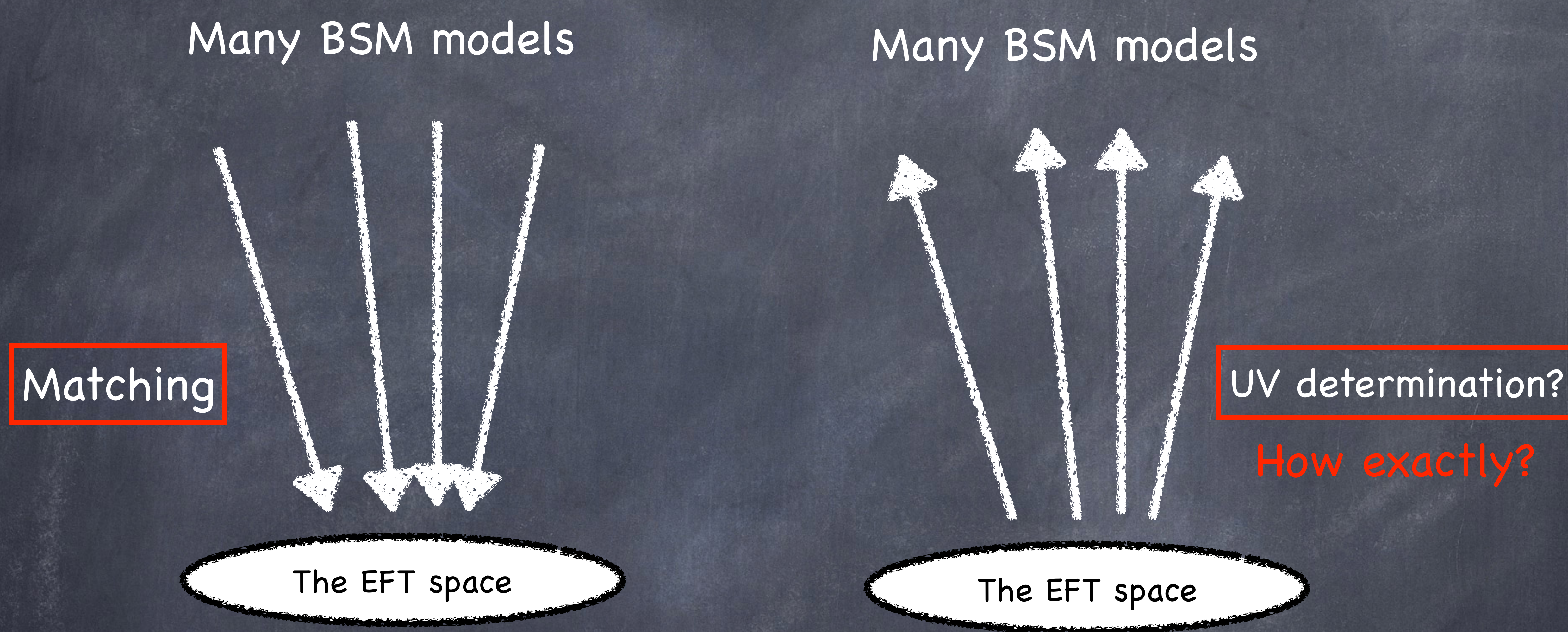
Many BSM models

UV determination?

How exactly?

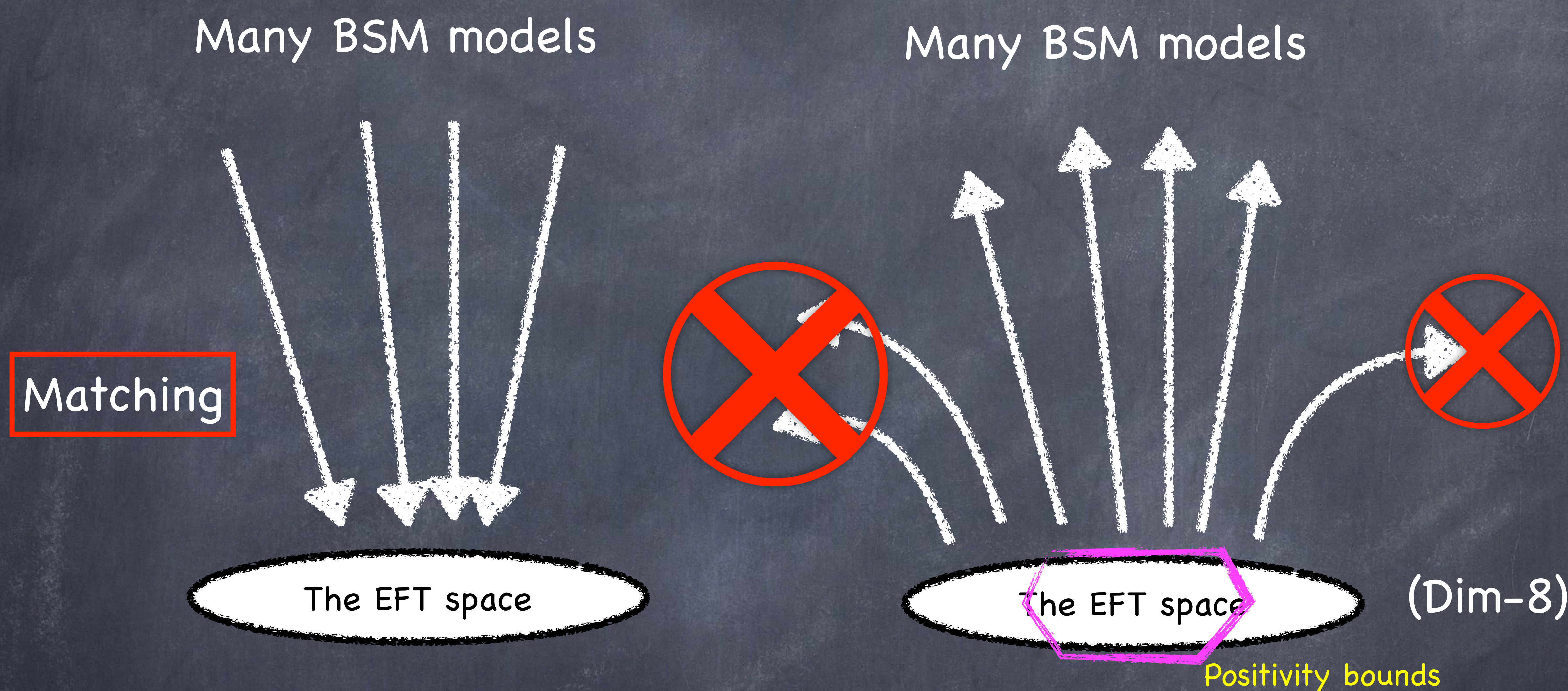


Experiments need to be analyzed in EFT, once and for all



Inverse problem: Given the measured values of the operator coefficients around the electroweak scale, to what extend can we possibly determine the nature of the new physics beyond the SM? [Gu, Wang, 2008.07551]

see also [S. Dawson et al. 2007.01296] [N. Arkani-Hamed et al. hep-ph/0512190]



- (Positivity) Not all EFTs have a UV completion!
- Using axiomatic principles of QFT, including casuality, unitarity, Lorentz symmetry, etc., we can show that if the EFT has a UV completion consistent with QFT, (positivity) bounds can be placed on Wilson coefficients.

Positivity bounds

Positivity bounds

- The second s derivative of a 2-to-2 scattering amplitude (forward and elastic), computed in the IR limit, needs to be positive. $M''(s,t=0) > 0$ at $s \to 0$.
- This can be computed in an EFT: by definition, EFT captures the IR behavior of the theory. Often just gives $c>0$.
- May go beyond elasticity (will cover in this talk) and forward limit (will not cover here, see [de Rham, Melville, Tolley & Zhou, 1706.02712]).

ijkl: particle index

$1 \le i, j, k, l \le n$

$M^2 = m_i^2 + m_j^2 + m_k^2 + m_l^2$

Forward scattering amp, at **low energy** (calculable in EFT)

$$\frac{d^2}{ds^2} M_{ij\to kl}\left(s = \frac{1}{2}M^2, t = 0\right)$$

$$= \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds\, M_{ij\to X}(s, \Pi_X) M^*_{kl\to X}(s, \Pi_X)}{\pi\left(s - \frac{1}{2}M^2\right)^3} + (j \leftrightarrow l)$$

s<->u crossing

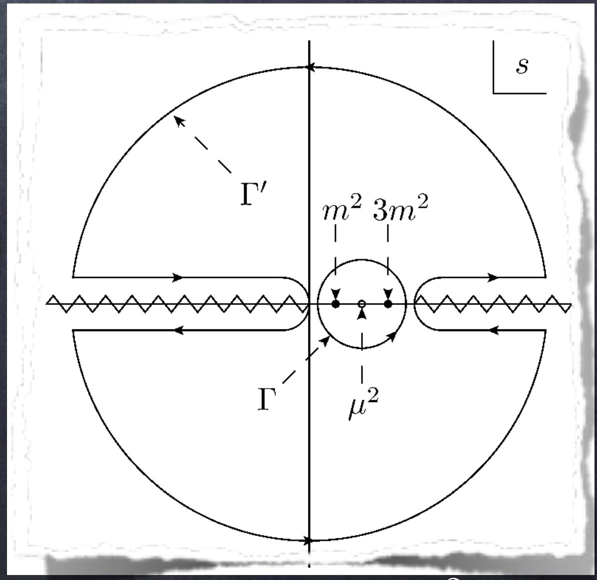
$\epsilon < 1$

X = BSM states summation & PS integration

Amplitude of SM -> X

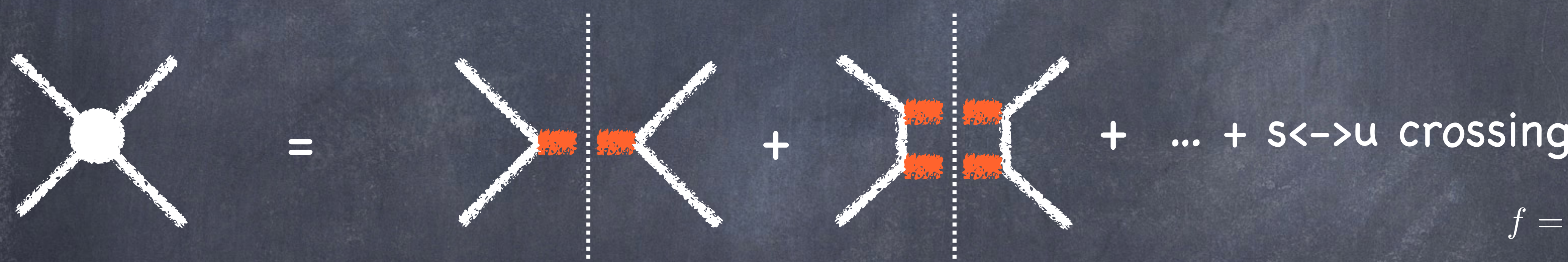
$$f = \frac{1}{2\pi i}\oint_\Gamma \mathrm{d}s \frac{A(s,0)}{(s-\mu^2)^3}$$

Dispersion relation from Cauchy's integral formula, analyticity, unitarity



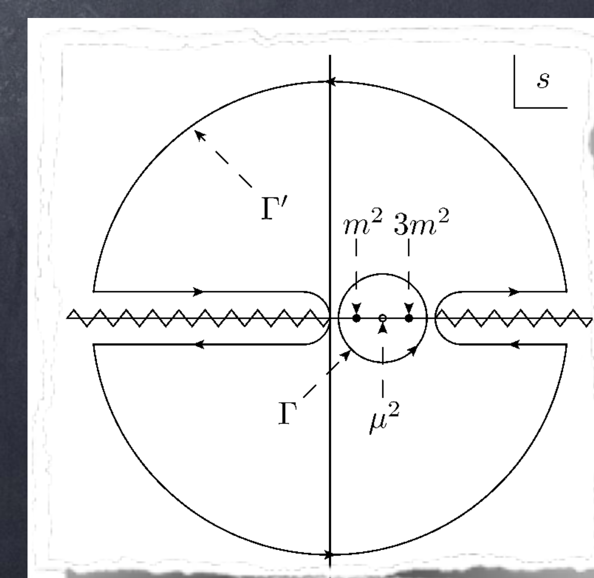
$A(s,0) \le \mathcal{O}(s \ln^2 s)$

$$\frac{d^2}{ds^2} M_{ij\to kl}\left(s = \frac{1}{2}M^2, t = 0\right)$$
$$= \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds\, M_{ij\to X}(s, \Pi_X) M^*_{kl\to X}(s, \Pi_X)}{\pi\left(s - \frac{1}{2}M^2\right)^3} + (j \leftrightarrow l)$$



$$f = \frac{1}{2\pi i}\oint_\Gamma \mathrm{d}s \frac{A(s,0)}{(s-\mu^2)^3}$$

Dispersion relation from Cauchy's integral formula, analyticity, unitarity

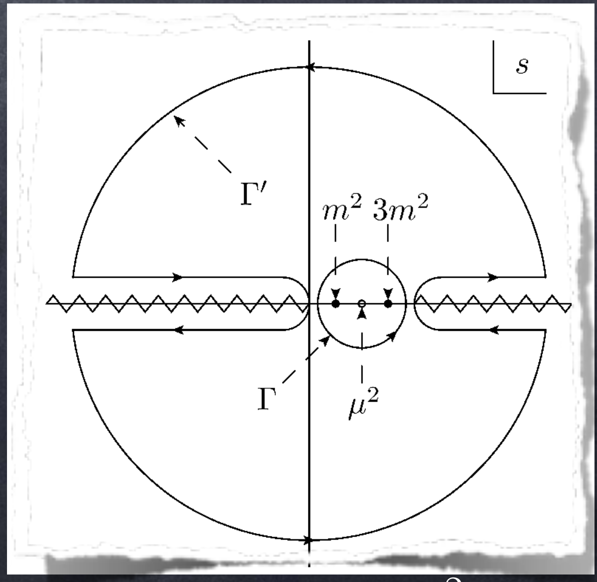


$A(s,0) \le \mathcal{O}(s \ln^2 s)$

$$\frac{d^2}{ds^2} M_{ij\to kl}\left(s=\frac{1}{2}M^2, t=0\right)$$
$$=\sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds\, M_{ij\to X}(s,\Pi_X) M^*_{kl\to X}(s,\Pi_X)}{\pi\left(s-\frac{1}{2}M^2\right)^3} + (j\leftrightarrow l)$$

$$M^{ijkl} = \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu\, {m_X}^{ij} {m_X}^{kl}}{\pi(\mu-\frac{1}{2}M^2)^3} + (j\leftrightarrow l)$$

$$f = \frac{1}{2\pi i}\oint_\Gamma \mathrm{d}s \frac{A(s,0)}{(s-\mu^2)^3}$$



$A(s,0) \le \mathcal{O}(s\ln^2 s)$

where $M^{ijkl} \equiv \frac{d^2}{ds^2} M_{ij\to kl}\left(\frac{1}{2}M^2\right), \quad m_X^{ij} \equiv M_{ij\to X}(\mu, \Pi_X)$

M and m are rank-4 and rank-2 tensors respectively.

M^{ijkl} calculable in SMEFT, e.g. $M^{ijkl} = \sum_\alpha C_\alpha^{(8)}/\Lambda^4 M_\alpha^{ijkl}$

$$M^{ijkl} = \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu\, {m_X}^{ij} {m_X}^{kl}}{\pi(\mu - \frac{1}{2}M^2)^3} + (j \leftrightarrow l)$$

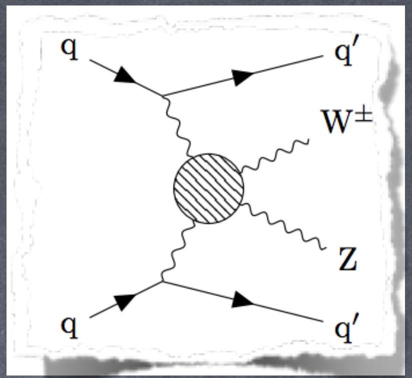
Elastic positivity is easily understood:

- When i=k, j=l, (i j -> i j),
  RHS -> $\mathrm{Tr}\left(mm^T\right) \geq 0$ i.e. $M^{ijij} \geq 0$

- Superposition: $M(|u\rangle + |v\rangle \;\rightarrow\; |u\rangle + |v\rangle) = u^i v^j u^{*k} v^{*l} \cdot M^{ijkl}$
  with superposed states: $|u\rangle = u^i\,|i\rangle\,,\; |v\rangle = v^i\,|i\rangle$
  RHS -> $|u \cdot m_X \cdot v|^2 + |u \cdot m_X \cdot v^*|^2 \geq 0$
  i.e. $u^i v^j u^{*k} v^{*l} M^{ijkl} \geq 0$

- This is however equivalent to the determination of 4-th order PSD polynomial (which is NP hard...)

- For example, the aQGC couplings are relevant in VBS measurements



- These are conventionally parameterized by

$$O_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi]$$
$$O_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi]$$
$$O_{S,2} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi]$$
$$O_{M,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times [(D_\beta\Phi)^\dagger D^\beta\Phi]$$
$$O_{M,1} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times [(D_\beta\Phi)^\dagger D^\mu\Phi]$$

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$$O_{M,3} = \left[\hat{B}_{\mu\nu}\hat{B}^{\nu\beta}\right] \times [(D_\beta\Phi)^\dagger D^\mu\Phi]$$
$$O_{M,4} = \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\mu\Phi\right] \times \hat{B}^{\beta\nu}$$
$$O_{M,5} = \frac{1}{2}\left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\nu\Phi\right] \times \hat{B}^{\beta\mu} + h.c.$$
$$O_{M,7} = \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}\hat{W}^{\beta\mu}D^\nu\Phi\right]$$

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}] \qquad O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}] \qquad O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$
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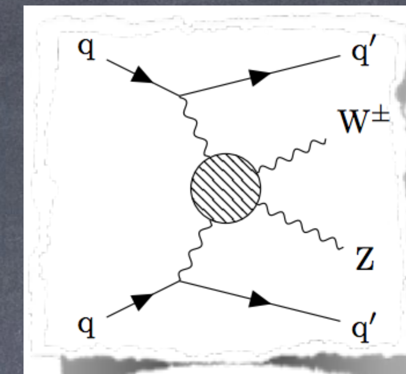
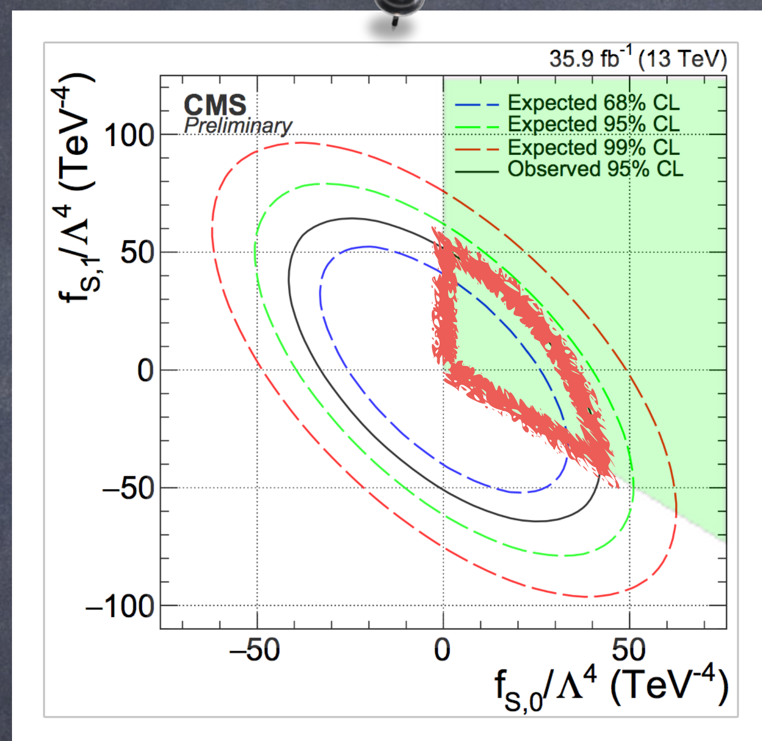
$$O_{T,7} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha} \qquad O_{T,11} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\tilde{B}^{\alpha\beta}$$
$$O_{T,8} = \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \qquad O_{T,9} = \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta}\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}$$

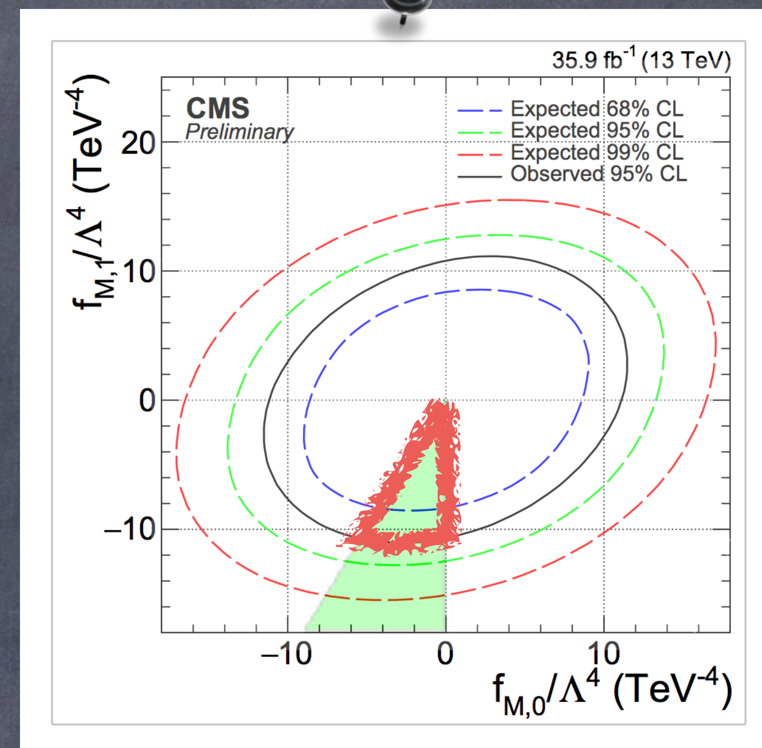
[Eboli, Gonzalez-Garcia, Mizukoshi, PRD 06]
[C. Degrande et al. Snow Mass Proceedings 13]
[Eboli, Gonzalez-Garcia, PRD 16]

- For example, the aQGC couplings are relevant in VBS measurements



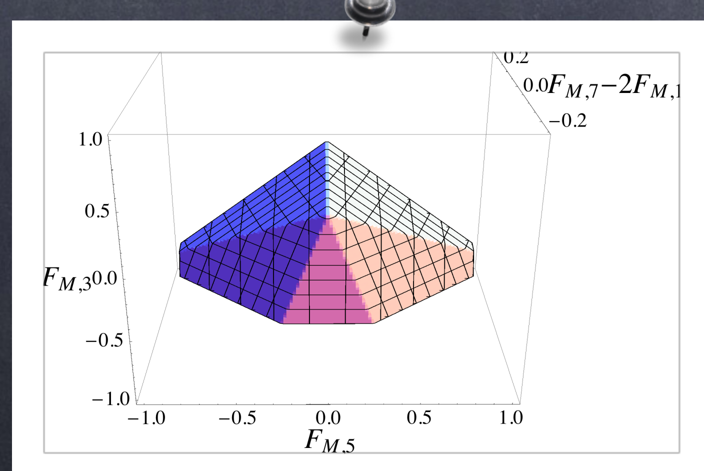
Bounds for two ops

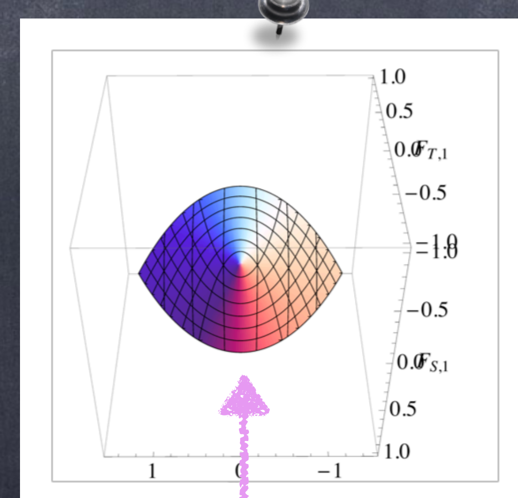


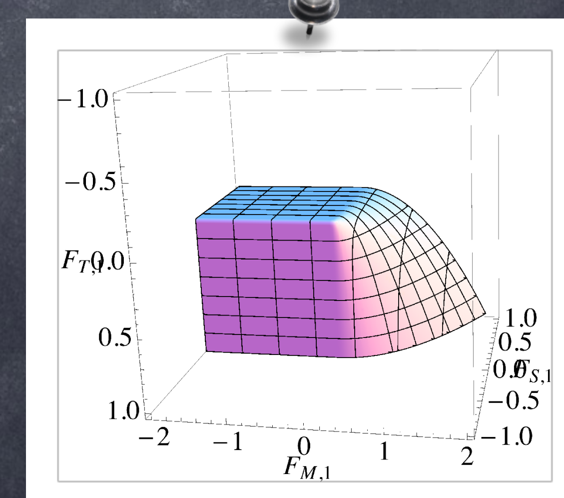


CMS-PAS-SMP-18-001

Three ops (structures?)



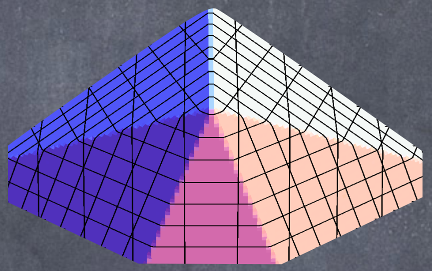




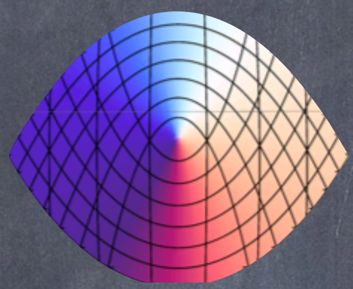
With arbitrary polarization -> arbitrary superposition of particle states

[1902.08977, Q. Bi, CZ, S.-Y. Zhou]

- What determines the structures (polytopes/cones) of the parameters space?
  - Symmetries between different modes play a role in the shapes.



vs



- What are the UV physics/models behind these structures? What do we learn from them?
- Are the conventional elastic (+superposition) approach give the best bounds that QFT principles implies?
  - E.g., going beyond t->0 limit. (But not relevant for SMEFT dim-8)
  - SMEFT is special in that **a large number of low energy modes are involved**. In this case, **elastic + superposition turns out to be insufficient.**

The positive structures in SMEFT

- Positivity has the form: $u^i v^j u^i v^j M^{ijkl} \geq 0 \;\Rightarrow\; \sum_\alpha C^{(8)}_\alpha p_\alpha(u,v) \geq 0$
- A set of linear inequality => **Convex Cone**
- Convex Cones have 2 representation:
  - **Inequality rep**: As bounded by **faces** (inequalities) which become **positivity bounds**.
  - **Extremal rep**: As **convex hull** of **extremal rays (ERs)**
    - **ER:** rays that cannot be nontrivial split into two others rays in the same cone.
  - Convex hull represents "positive generation". So ERs are "generators".
  - **Physical meaning**:
    ERs are the **generators** of all UV-completable EFTs!

Polyhedral cone

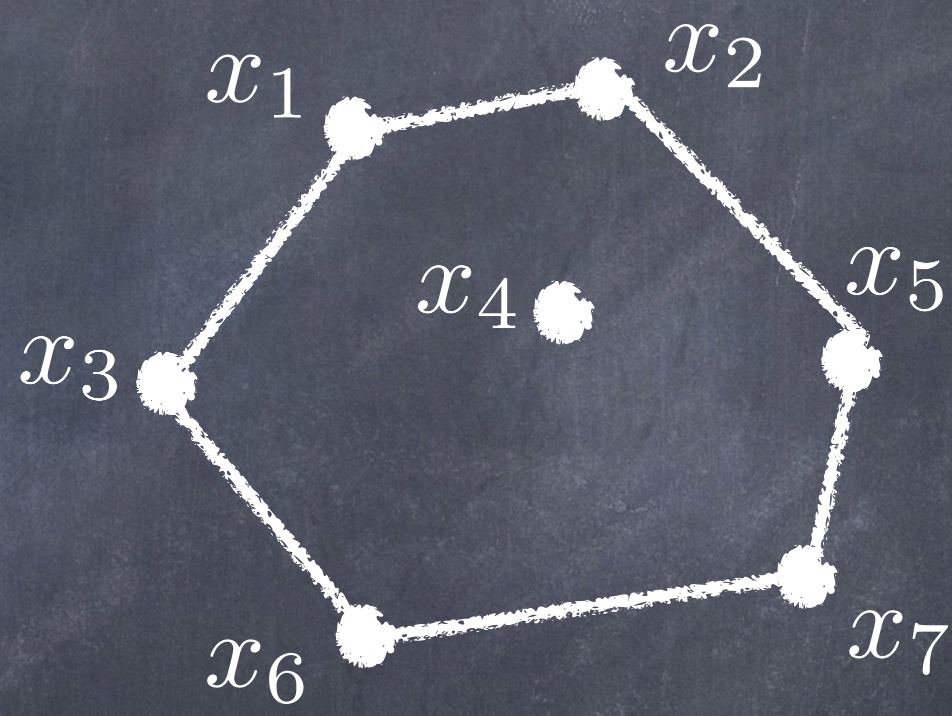
Circular cone

Convex hull: represents "positive generation"

e.g. convex hull of x_i:

$$x = \sum_i x_i w_i,$$

$$w_i \geq 0, \ \sum_i w_i = 1$$



- Consider tree-level UV completion, SM + n particles.
- Integrating out each particle gives a ray within the cone $\vec{C}_\alpha = (C_1, C_2, ...)$
- If n>1, the total coef. cannot be an ER. (ERs cannot be split)
- ER corresponds to **one-particle SM extension**!
    - From which all UV models can be generated.

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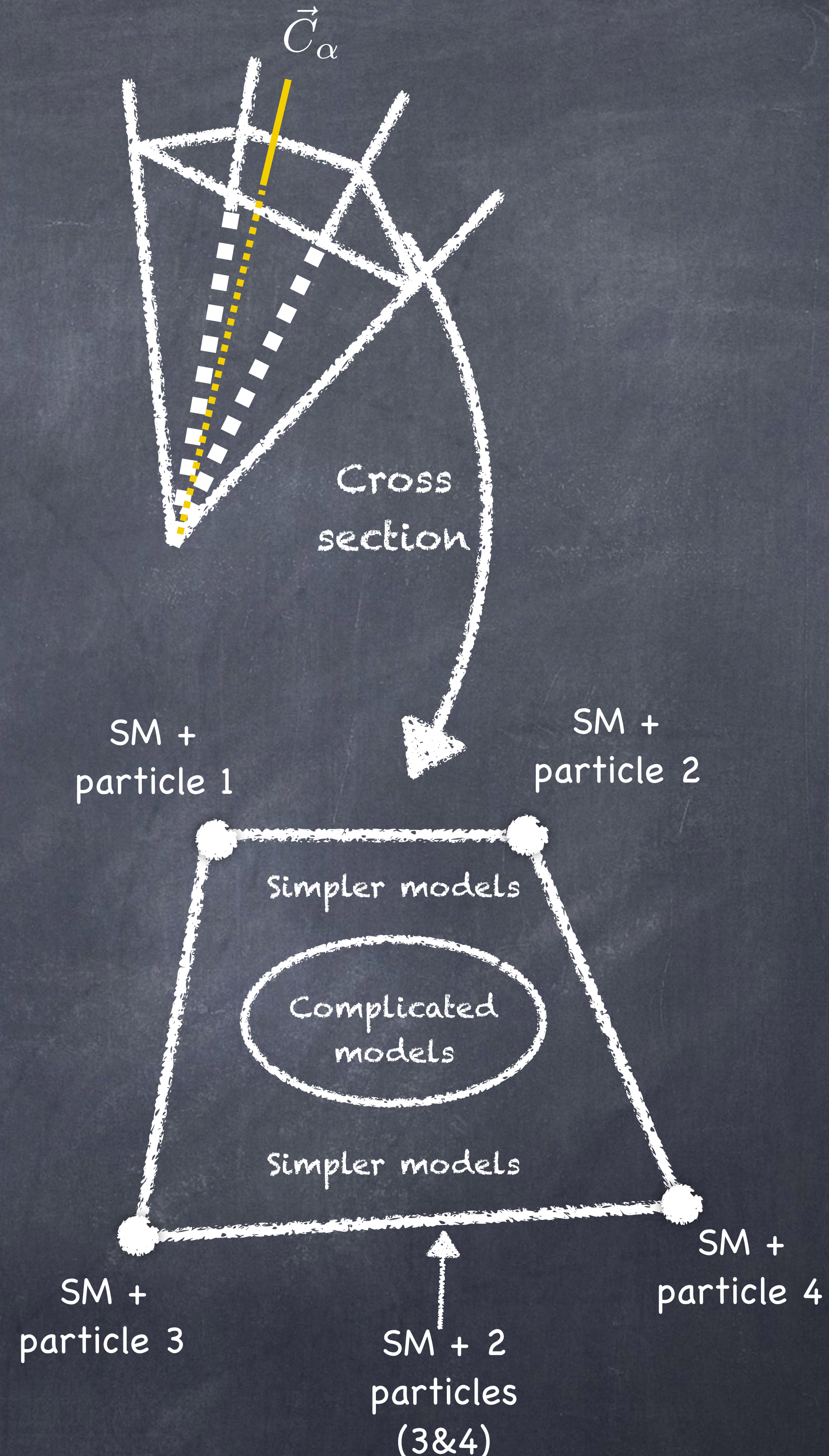
- ER corresponds to **one-particle SM extension**!
    - From which all UV models can be generated.

Heuristically,

- An ER is an almost "unique" UV.
- Points closer to an ER has less arbitrariness in UV determination.
- Similarly hold for edge, facets... or in general, a k-face, i.e. face of dim-k.
- UV-model ordered in complexity in this view.



Roughly, points on a k-face correspond to ~k UV particles

- Consider tree-level UV completion, SM + n particles.
- Integrating out each particle gives a ray within the cone $\vec{C}_\alpha = (C_1, C_2, ...)$
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- ER correspon
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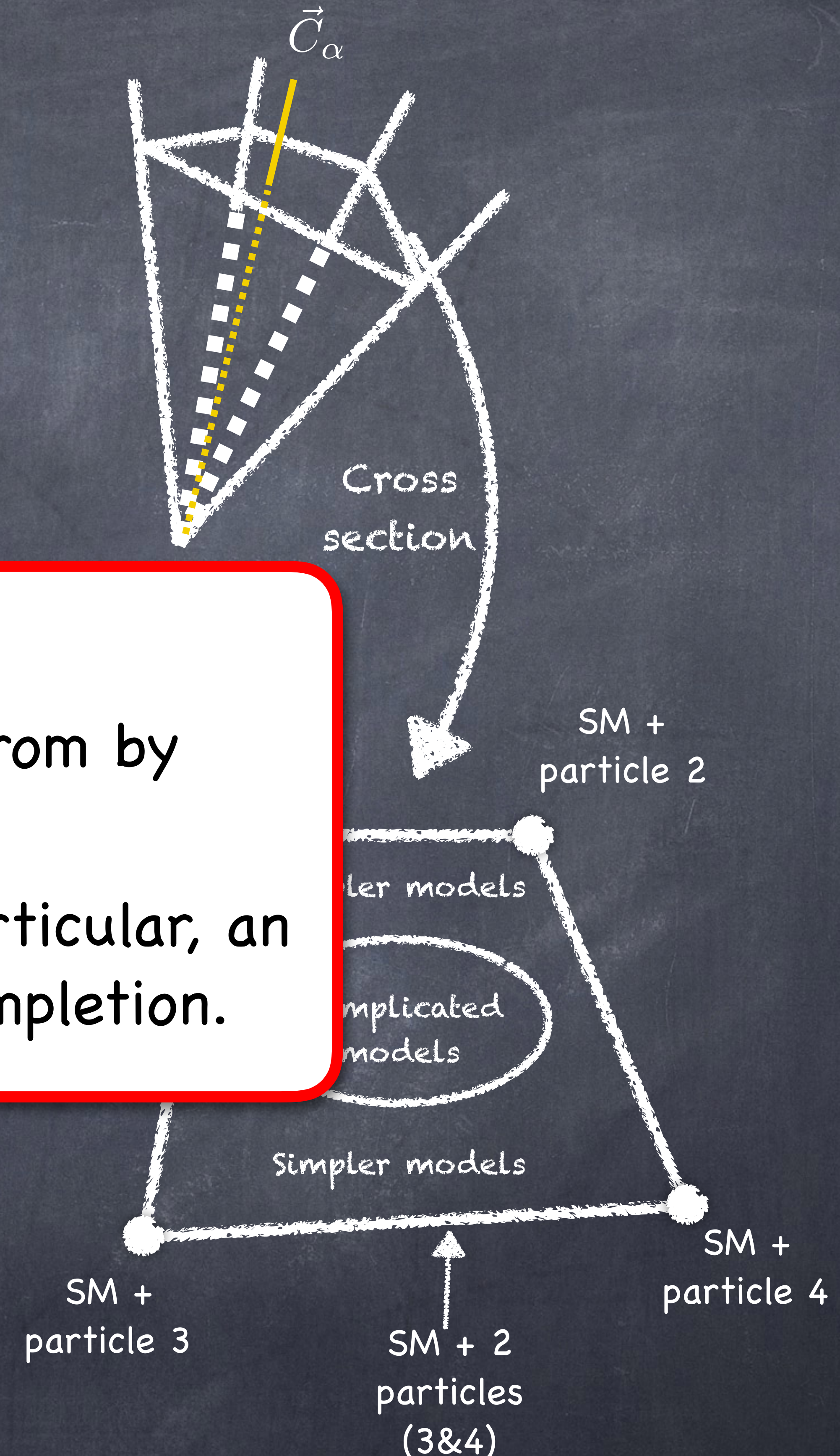
- UV-model ordered in complexity in this view.

Two interesting directions:

1. Bounds can be derived from by enumerating all the ERs
2. ERs <-> UV states. In particular, an ER has a "unique" UV completion.



Roughly, points on a k-face correspond to ~k UV particles

Finding bounds

The "extremal positivity" bounds

The conventional elastic approach:

- Inequality rep. A convex cone is the set of points satisfying a number of linear inequalities, each representing a <u>facet</u>.

The extremal approach:

- Extremal representation. A convex cone is the set of points positively generated from a number of rays, each representing an <u>edge</u>.

"Vertex enumeration"

E.g. reverse search
[Avis & Fukuda, 92']

Some of these facets are positivity bounds $u^i v^j u^{*k} v^{*l} M^{ijkl} \geq 0$

But elastic bounds are not complete.

The extremal rays are just all one particle extensions

[**CZ** and S.-Y. Zhou 2005.03047]

To formulate this approach, **symmetries of the system help** [see also 1405.2960 Bellazzini et al.] (We will also discuss cases without symmetries)

- Make use of symmetries of the problem (SM symmetries, helicities)
- Dispersion relation: $M^{ijkl} = \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu\, m_X{}^{ij} m_X{}^{kl}}{\pi(\mu - \frac{1}{2}M^2)^3} + (j \leftrightarrow l)$

Dynamics

- Becomes: $M^{ijkl} = \sum_{X\in r}{}' \int_{(\epsilon\Lambda)^2}^{\infty} d\mu \frac{|< X|M|r >|^2}{\pi\left(\mu - \frac{1}{2}M^2\right)^3} P_r^{i(j|k|l)}$

Symmetry

i(j|k|l): j,l symmetrized

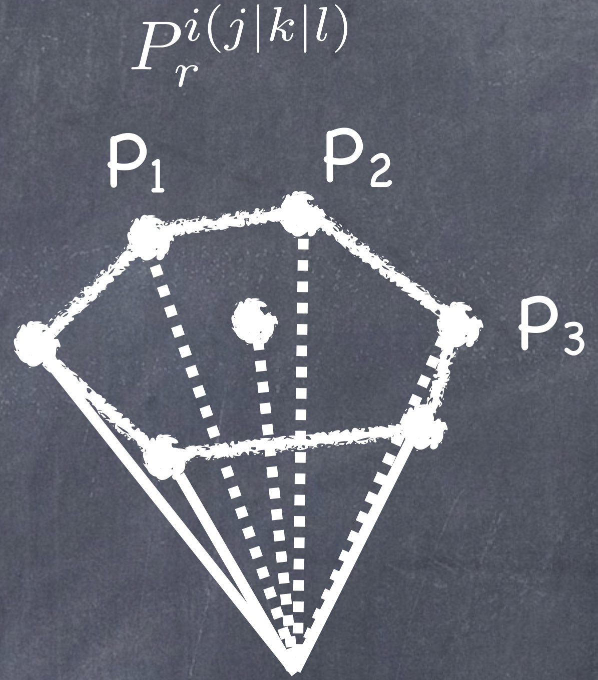
- P_r^{ijkl} is the projective operator of an irrep r, obtained by CG coefficients.

$$P_r^{ijkl} = \sum_\alpha C_{i,j}^{r,\alpha}\left(C_{k,l}^{r,\alpha}\right)^*$$

- The generators are simply (subset of) $P_r^{i(j|k|l)}$

- Computing all P's, and applying vertex enumeration leads to, in principle, the best constraints.

- Difficulty arises if an infinite number of P's are present... (due to degenerate irreps)



M (or C8) must stay inside

Toy example: WW->WW

$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}] \quad O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$

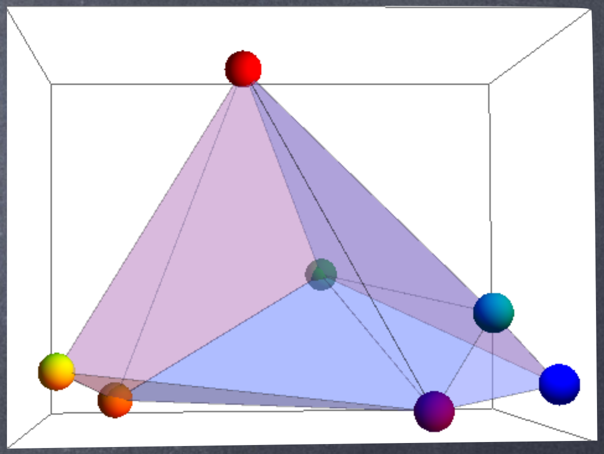
$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}] \quad O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$

+ the dim-6 WWW operator

Projectors are:

$$P^1_{\alpha\beta\gamma\sigma} = \frac{1}{N}\delta_{\alpha\beta}\delta_{\gamma\sigma}, \; P^2_{\alpha\beta\gamma\sigma} = \frac{1}{2}\left(\delta_{\alpha\gamma}\delta_{\beta\sigma} - \delta_{\alpha\sigma}\delta_{\beta\gamma}\right),$$

$$P^3_{\alpha\beta\gamma\sigma} = \frac{1}{2}\left(\delta_{\alpha\gamma}\delta_{\beta\sigma} + \delta_{\alpha\sigma}\delta_{\beta\gamma}\right) - \frac{1}{N}\delta_{\alpha\beta}\delta_{\gamma\sigma},$$



- N=3 for SU(2) adj., N=2 for polarization
- Combine both groups and cross (j<->l)
- 9 (symmetrized) projectors, 5 are independent, 8 are extremal.
- 5 D polyhedral cone with 8 edges and 9 (4D-)faces (which are bounds)
    - Numbers of 1-, 2-, 3-, and 4-faces: 8, 21, 22, 9

Toy example: WW->WW

- After mapping to operators:

$$
\begin{aligned}
&F_{T,2} \geq 0, \\
&4F_{T,1} + F_{T,2} \geq 0, \\
&F_{T,2} + 8F_{T,10} \geq 0, \\
&8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0, \\
&12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0, \\
&4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0.
\end{aligned}
$$

- To be compared with the bounds obtained from elastic and superpositions $u^i v^j u^{*k} v^{*l} M^{ijkl} \geq 0$

bounds	channel $(\lvert 1> + \lvert 2> \to \lvert 1> + \lvert 2>)$
$F_{T,2} \geq 0,$	$1: W_x^1,\ 2: W_y^2$
$4F_{T,1} + F_{T,2} \geq 0,$	$1: W_x^1,\ 2: W_x^2$
$F_{T,2} + 8F_{T,10} \geq 0,$	$1: W_x^1 + W_y^2,\ 2: W_y^1 - W_x^2$
$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$	$1: W_x^1 + W_y^2,\ 2: W_x^1 + W_y^2$

(Other channels are not independent)

Towards full set of aQGC bounds

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}]$$
$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}]$$
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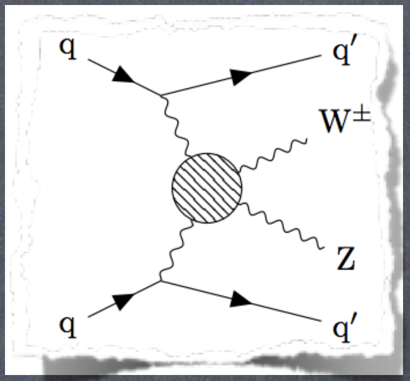
$$O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
$$O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$
$$O_{T,6} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\hat{B}_{\mu\beta}\hat{B}^{\alpha\nu}$$
$$O_{T,11} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\tilde{B}^{\alpha\beta}$$
$$O_{T,9} = \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta}\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}$$



$$O_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi]$$
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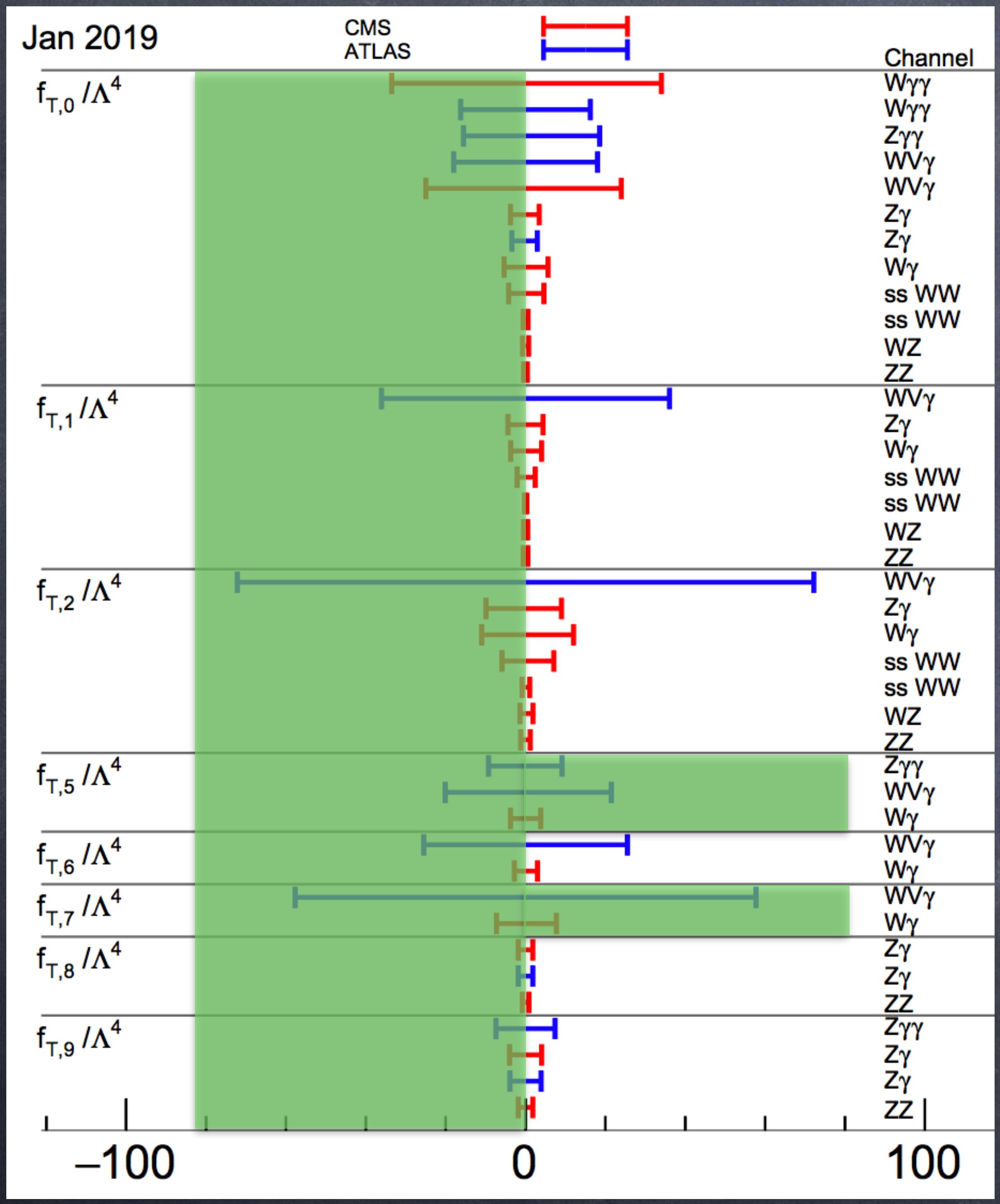
$$O_{M,7} = \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}\hat{W}^{\beta\mu}D^\nu\Phi\right]$$

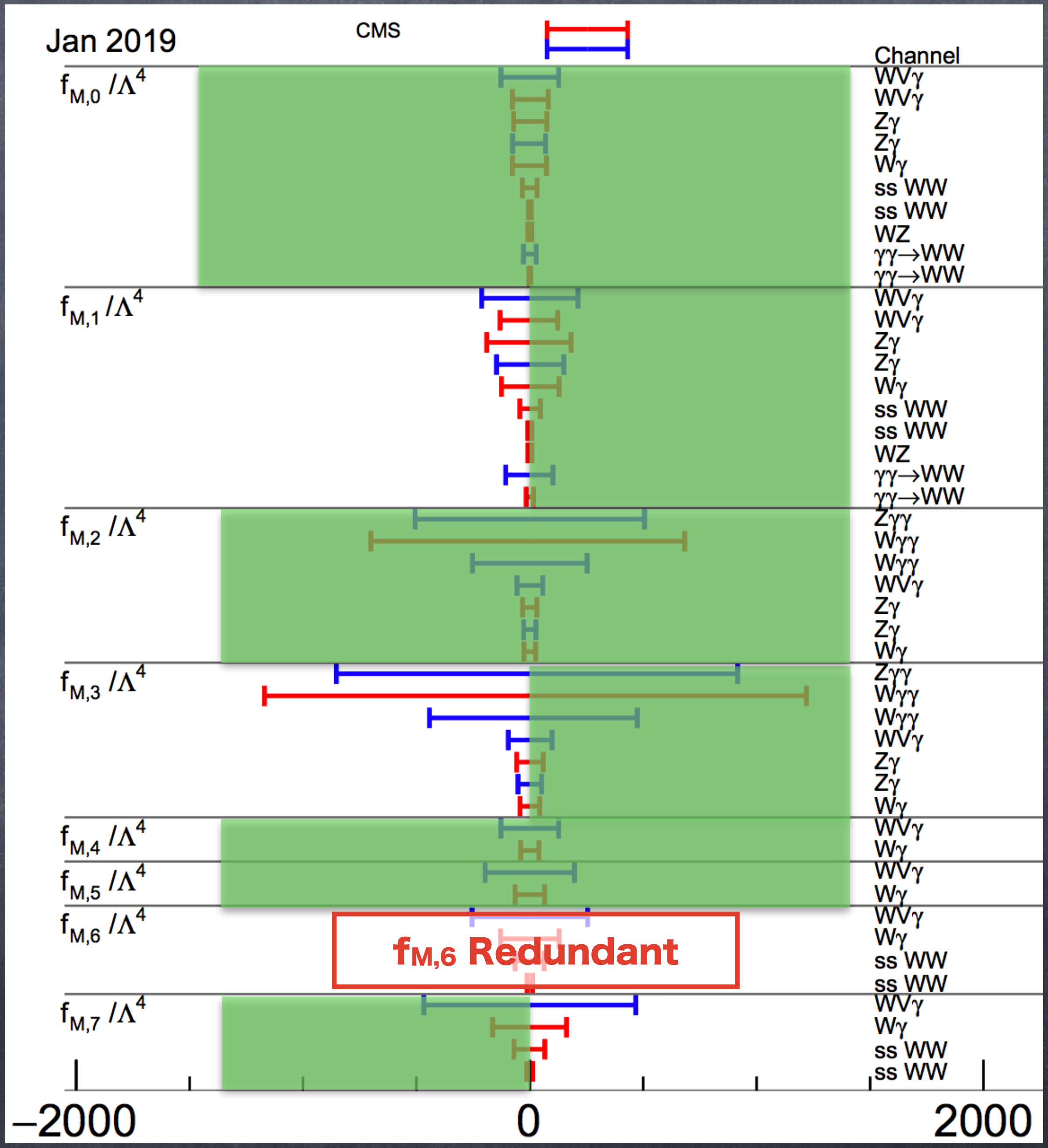
Note $O_{T,10}$ and $O_{T,11}$ have been missed in standard QGC parameterization.

Pointed out by [Remmen, Rodd, JHEP 12 (2019) 032]

See also dim-8 basis: [C. Murphy 2005.00059], [H.-L. Li 2005.00008]

Towards full set of aQGC bounds





https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSMPaTGC#aQGC_Results

Transversal aQGC

Need to consider an infinite set of (j-l symmetrized) projectors (due to degenerate irreps), continuously parametrized by r:

$$\vec{E}_1 = (1,0,0,0,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_2 = (0,1,0,0,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_3 = (0,0,1,0,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_4 = (0,0,0,1,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_5 = \left(-\frac{1}{6},\frac{1}{6},0,0,-\frac{5}{3},0,0,\frac{5}{3},0,0,\frac{5}{6},0,0\right)$$
$$\vec{E}_6 = \left(0,0,-1,1,0,-\frac{3}{4},0,0,\frac{3}{4},0,0,0,1\right)$$
$$\vec{E}_7(r) = \left(0,0,0,0,1,r,r^2,0,0,0,0,0,0\right)$$
$$\vec{E}_8(r) = \left(0,0,0,0,0,0,0,1,r,r^2,0,0,0\right)$$
$$\vec{E}_9(r) = \left(0,0,0,0,0,0,0,0,0,0,1,r,r^2\right)$$
$$\vec{E}_{10}(r) = \left(-\frac{1}{3},\frac{1}{3},-\frac{4r}{3},\frac{4r}{3},-\frac{1}{3},0,-r^2,\frac{1}{3},0,r^2,-\frac{1}{3},0,-\frac{4r}{3}\right)$$
$$\vec{E}_{11}(r) = \left(\frac{1}{2},\frac{1}{2},\frac{r^2}{2},\frac{r^2}{2},-1,-\frac{3r^2}{8},0,-1,-\frac{3r^2}{8},0,-\frac{1}{2},r,-\frac{r^2}{2}\right)$$
$$\vec{E}_{12}(r) = \left(1,0,r^2,0,-2,-\frac{3r^2}{4},0,0,0,0,1,-2r,r^2\right)$$

The question is similar in that we need to find the conical hull of all these vectors, but different in that these vectors are continuously parameterized by some real parameter r.

Analytically: a tower of linear, quadratic, cubic, … inequalities.

- So far only able to obtain the first two levels

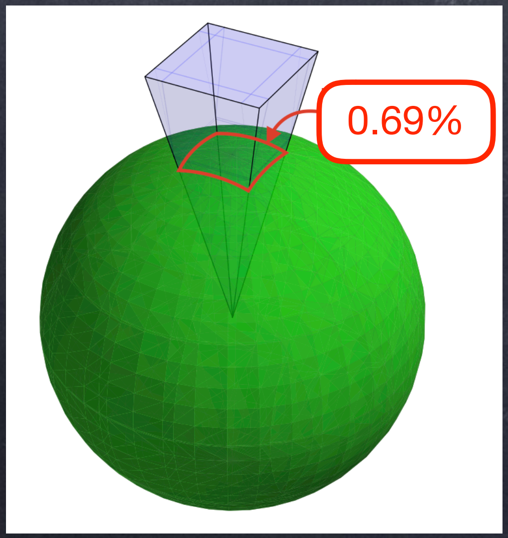
Linear:

$$F_{T,2} \geq 0$$
$$4F_{T,1} + F_{T,2} \geq 0$$
$$F_{T,2} + 8F_{T,10} \geq 0$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0$$
$$4F_{T,6} + F_{T,7} \geq 0$$
$$F_{T,7} \geq 0$$
$$2F_{T,8} + F_{T,9} \geq 0$$
$$F_{T,9} \geq 0$$

Quadratic:

$$F_{T,9}\left(F_{T,2} + 4F_{T,10}\right) \geq F_{T,11}^2$$
$$16\left(2\left(F_{T,0} + F_{T,1}\right) + F_{T,2}\right)\left(2F_{T,8} + F_{T,9}\right) \geq \left(4F_{T,5} + F_{T,7}\right)^2$$
$$32\left(2F_{T,8} + F_{T,9}\right)\left(3F_{T,0} + F_{T,1} + 2F_{T,2} + 4F_{T,10}\right) \geq 3\left(4F_{T,5} + F_{T,7}\right)^2$$
$$2\sqrt{2}\sqrt{F_{T,9}\left(F_{T,2} + 8F_{T,10}\right)} \geq \max\left(4F_{T,6} + F_{T,7} - 4F_{T,11}, F_{T,7} + 4F_{T,11}\right)$$
$$4\sqrt{\left(8F_{T,0} + 4F_{T,1} + 3F_{T,2}\right)\left(2F_{T,8} + F_{T,9}\right)} \geq \max\left(-8F_{T,5} - F_{T,7}, 8F_{T,5} + 4F_{T,6} + 3F_{T,7}\right)$$
$$4\sqrt{F_{T,9}\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left(4F_{T,6} + F_{T,7} - 4F_{T,11}, F_{T,7} + 4F_{T,11}\right)$$
$$4\sqrt{6}\sqrt{\left(2F_{T,8} + F_{T,9}\right)\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left[-3\left(8F_{T,5} + F_{T,7}\right), 3\left(8F_{T,5} + 4F_{T,6} + 3F_{T,7}\right)\right]$$
$$\sqrt{6}\sqrt{\left(4F_{T,8} + 3F_{T,9}\right)\left(6F_{T,0} + 2F_{T,1} + 3F_{T,2} + 6F_{T,10}\right)} \geq \max\left[-3\left(2F_{T,5} + F_{T,11}\right), 3\left(2F_{T,5} + F_{T,7} + F_{T,11}\right)\right]$$
$$2\sqrt{\left(12F_{T,8} + 7F_{T,9}\right)\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left(-12F_{T,5} - F_{T,7} - 2F_{T,11}, -12F_{T,5} + 4F_{T,6} - F_{T,7} - 2F_{T,11}, -12F_{T,5} - F_{T,7} + 2F_{T,11}, 12F_{T,5} + 4F_{T,6} + 5F_{T,7} + 2F_{T,11}\right)$$

- The QGC parameter space is constrained to 0.687% of the total! (Still conservative)



[2009.04490, K. Yamashita, CZ, S.-Y. Zhou]

Connecting geometry and UV physics

[**CZ** and S.-Y. Zhou 2005.03047]

- 3 HHHH operators

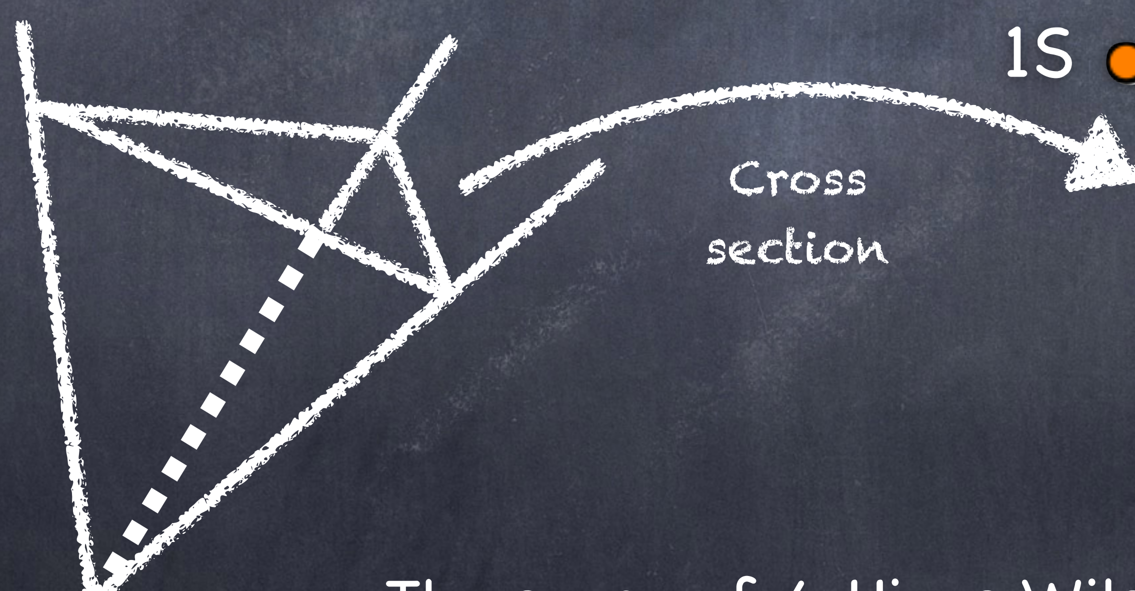
$$O_{S,0} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\mu \Phi)^\dagger D^\nu \Phi]$$
$$O_{S,1} = [(D_\mu \Phi)^\dagger D^\mu \Phi] \times [(D_\nu \Phi)^\dagger D^\nu \Phi]$$
$$O_{S,2} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\nu \Phi)^\dagger D^\mu \Phi]$$

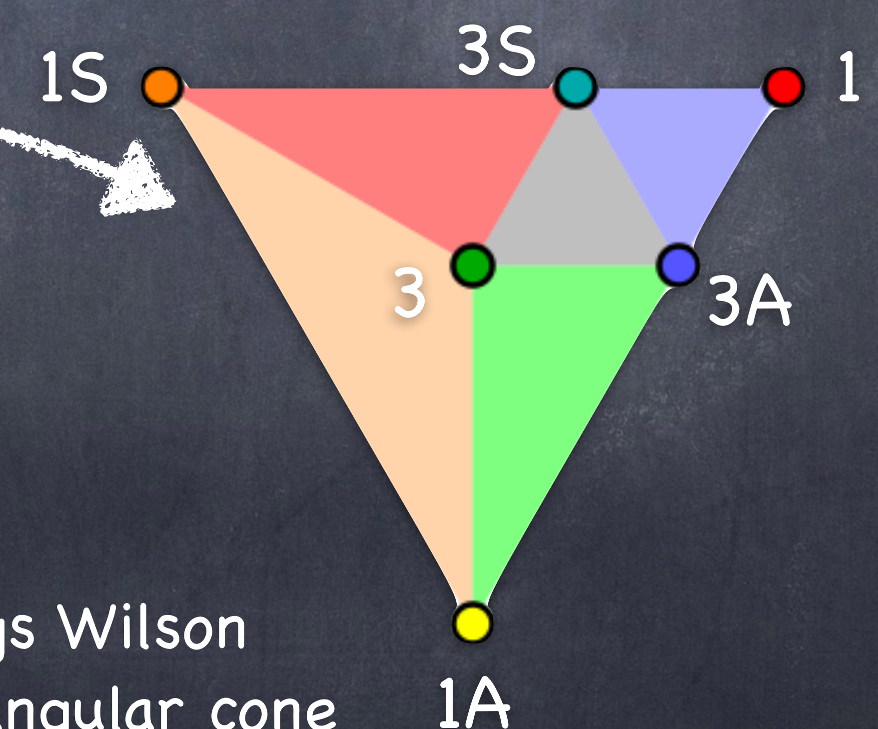
- HH can form 6 projectors.
- Each can be generated by integrating out one specific UV state.

- Let's call these particles 1, 1S, 1A, 3, 3S, 3A

$$\mathcal{L} = g_1 (H^T \epsilon \overleftrightarrow{D}_\mu H) V_1^{\mu\dagger} + g_{1S} (H^\dagger H) S_1 + i g_{1A} (H^\dagger \overleftrightarrow{D}_\mu H) V_2^\mu + g_3 (H^T \epsilon \tau^I H) S_2^{I\dagger} + g_{3S} (H^\dagger \tau^I H) S_3^I + i g_{3A} (H^\dagger \tau^I \overleftrightarrow{D}_\mu H) V_3^{\mu I} + h.c.$$

- 6 projectors, 3 are linearly independent, 3 are extremal



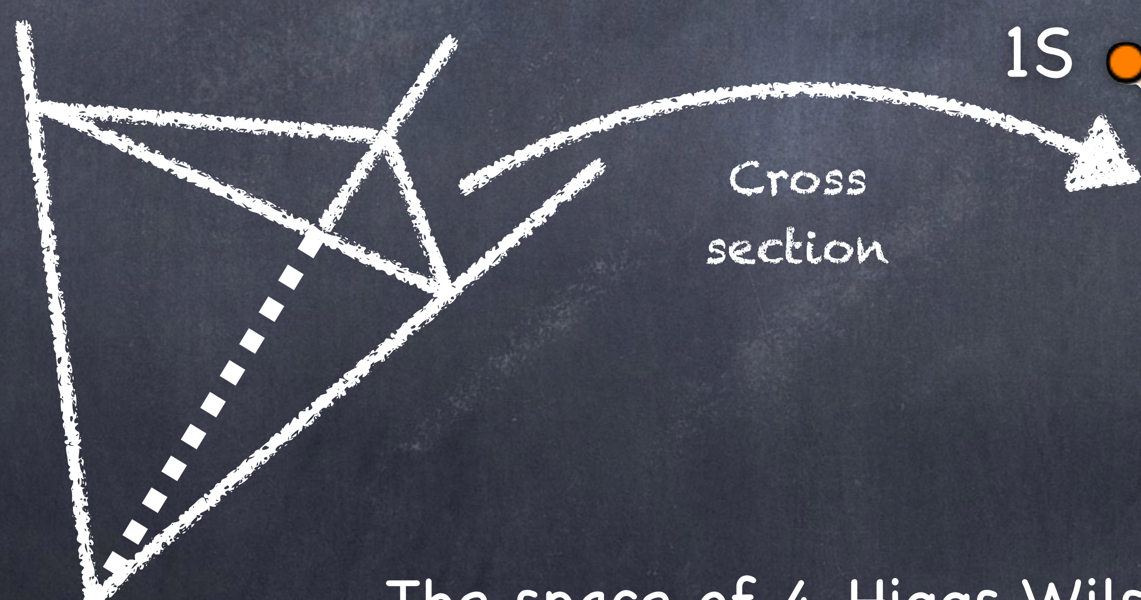


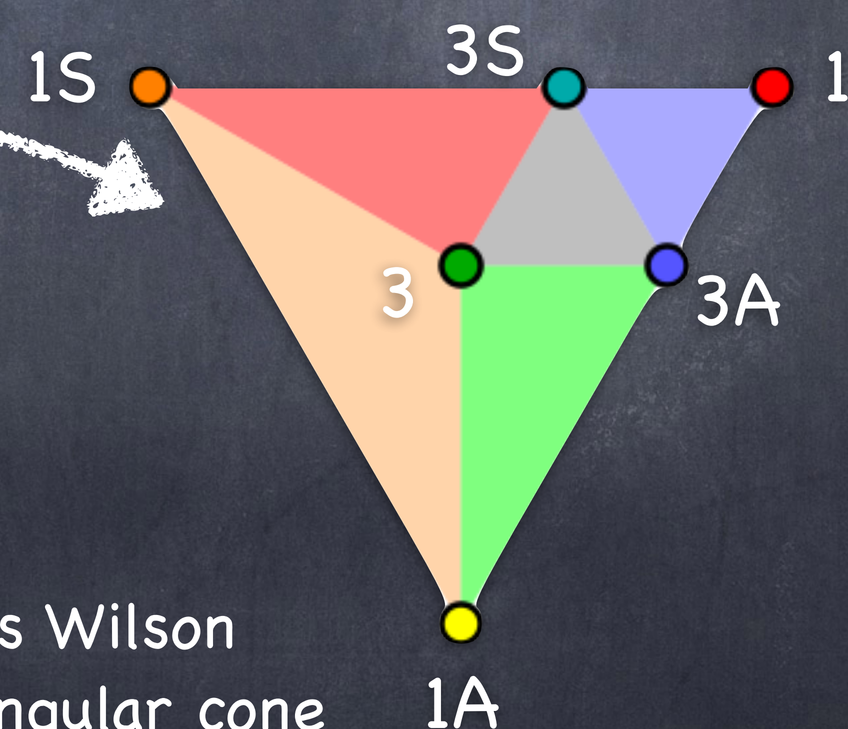
The space of 4-Higgs Wilson coefficients form a triangular cone

- How to infer UV state from the measured coefficients $C=(C_1,C_2,\ldots)$
- If C = ER => UV is **uniquely** determined.
- The closer C is to an ER, the less arbitrariness in the determination.
- Let's call these particles 1, 1S, 1A, 3, 3S, 3A

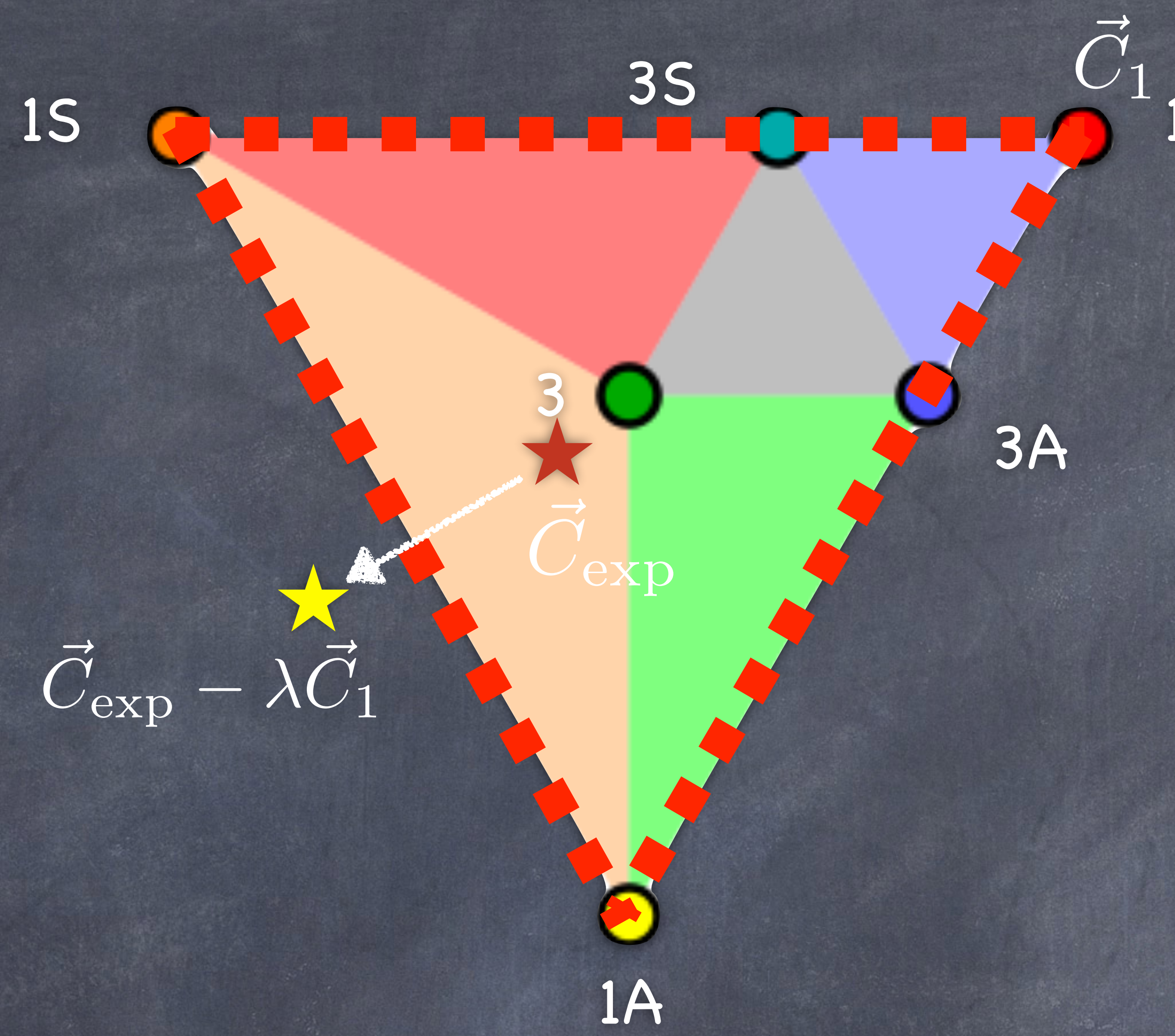
$$
\begin{aligned}
\mathcal{L} = {} & g_1 (H^T \epsilon \overleftrightarrow{D}_\mu H) V_1^{\mu\dagger} + g_{1S} (H^\dagger H) S_1 \\
& + i g_{1A} (H^\dagger \overleftrightarrow{D}_\mu H) V_2^\mu + g_3 (H^T \epsilon \tau^I H) S_2^{I\dagger} \\
& + g_{3S} (H^\dagger \tau^I H) S_3^I + i g_{3A} (H^\dagger \tau^I \overleftrightarrow{D}_\mu H) V_3^{\mu I} + h.c.
\end{aligned}
$$

- 6 projectors, 3 are linearly independent, 3 are extremal





The space of 4-Higgs Wilson coefficients form a triangular cone

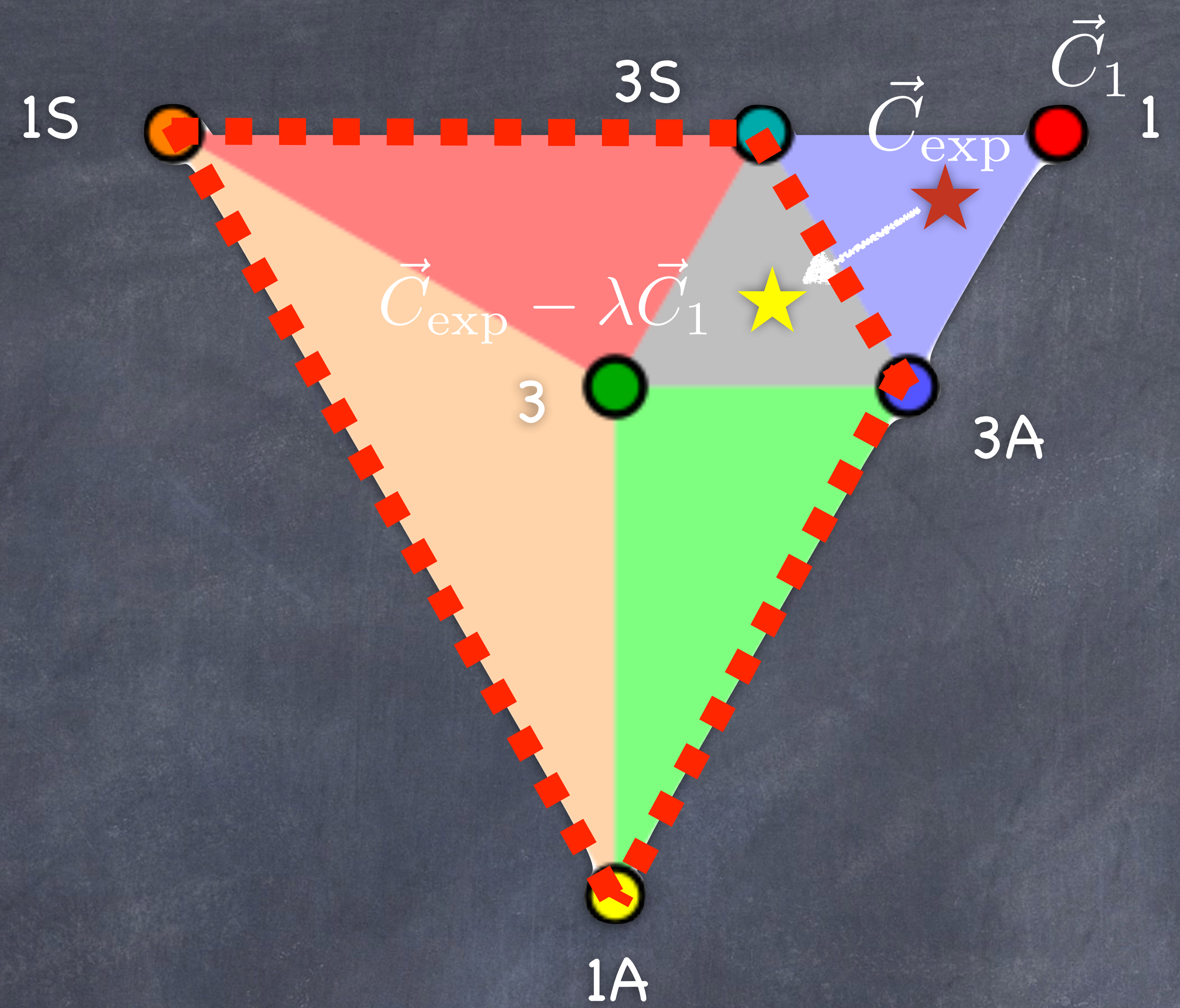


Setting upper limits on the contribution of specific UV states:

$$\vec{C}_{\rm exp} = \sum_i \lambda_i \vec{C}_i, \quad \lambda_i \geq 0 \quad \left(\lambda_i \propto \frac{g_i^2}{M_i^4} \text{ represents the size of UV state contribution} \right)$$

How: $$\vec{C}_{\rm exp} - \lambda \vec{C}_1 = \sum_{i \neq 1} \lambda_i \vec{C}_i + (\lambda_1 - \lambda) \vec{C}_1$$

The maximum λ such that $\vec{C}_{\rm exp} - \lambda \vec{C}_1 \in \mathrm{conv}\left(\{C_i\}\right)$ is an upper bound of λ_1



Setting lower limits on the contribution of specific UV states:

$$\vec{C}_{\text{exp}} = \sum_i \lambda_i \vec{C}_i, \quad \lambda_i \geq 0 \quad \left(\lambda_i \propto \frac{g_i^2}{M_i^4} \text{ represents the size of UV state contribution}\right)$$

How: $$\vec{C}_{\text{exp}} - \lambda \vec{C}_1 = \sum_{i \neq 1} \lambda_i \vec{C}_i + (\lambda_1 - \lambda)\vec{C}_1$$

The minimum λ such that $\vec{C}_{\text{exp}} - \lambda \vec{C}_1 \in \text{conv}\left(\{C_i, i \neq 1\}\right)$ is a lower bound of λ_1

- Positivity is <u>not simply a bound</u> on the coefficients. It also tells us helps to determine UV from EFT.

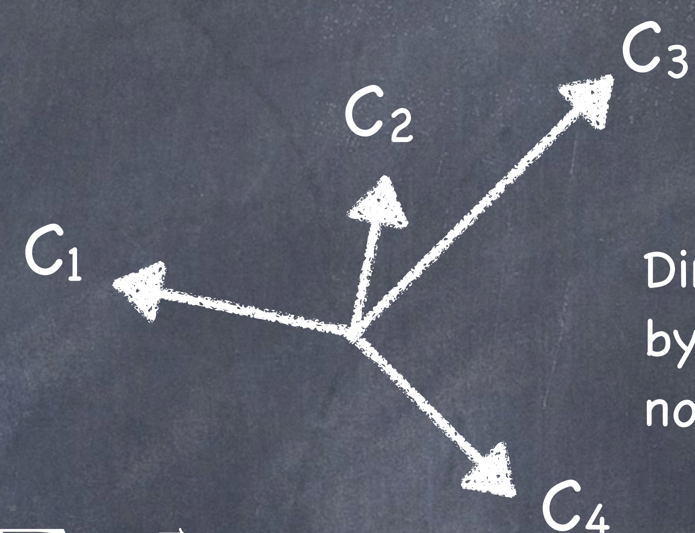
- Alternatively, the origin is also an extreme point! Which means we can "uniquely" confirm SM.

- **<u>Important for precision test of SM!</u>**

  - Dim-6: no deviation observed -> UV states may still exist (e.g. symmetries, [2008.07551 Gu & Wang])

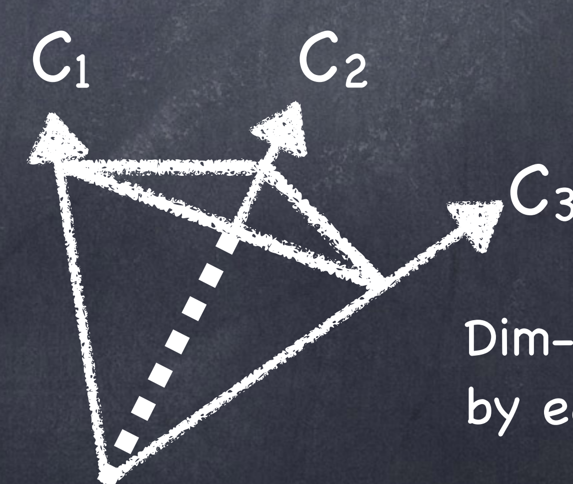
    - **<u>Dim-6 is never sufficient to confirm the SM!</u>**

  - Dim-8: no deviation observed -> UV states cannot exist. **<u>SM can be confirmed.</u>**



Dim-6 coefs generated by each UV state: not in a cone

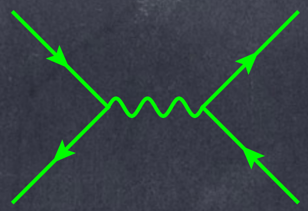
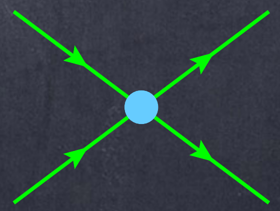
$\sum \lambda_i \vec{C}_i = 0$ <u>does not</u> imply all $\lambda_i = 0$



Dim-8 coefs generated by each UV state

$\sum \lambda_i \vec{C}_i = 0$ <u>does imply</u> all $\lambda_i = 0$

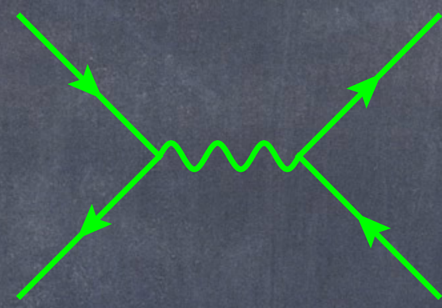
Pheno application?

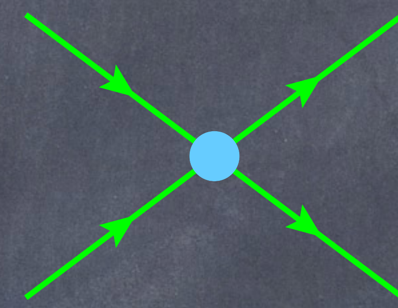
 

[2009.02212, B. Fuks, Y. Liu, **CZ**, S.-Y. Zhou]

The inverse problem in ee scattering

- Does this work in reality? Consider e+e- -> e+e- as an example.





- Operators

Dim-6

$$O_{ee} = (\bar{e}\gamma^\mu e)\ (\bar{e}\gamma_\mu e)\ ,$$
$$O_{el} = (\bar{e}\gamma^\mu e)\ (\bar{l}\gamma_\mu l)\ ,$$
$$O_{ll} = (\bar{l}\gamma^\mu l)\ (\bar{l}\gamma_\mu l)\ ,$$

Dim-8

$$O_1 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e)\ ,$$
$$O_2 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_3 = D^\alpha(\bar{e}l)\ D_\alpha(\bar{l}e),$$
$$O_4 = \partial^\alpha(\bar{l}\gamma^\mu l)\ \partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$\cancel{O_5 = D^\alpha(\bar{l}\gamma^\mu\tau^I l)\ D_\alpha(\bar{l}\gamma_\mu\tau^I l)}\ ,$$

Only 4 are independent in ee>ee

- Consider the following UV states that couple to two electrons:

Scalar		
$D \equiv \mathbf{2}_{1/2}$	$M_L \equiv \mathbf{1}_1$	$M_R \equiv \mathbf{1}_2$

Vector	
$V \equiv \mathbf{1}_0$	$V' \equiv \mathbf{2}_{-3/2}$

$$\mathcal{L}_{\rm int} = g_{Di}\bar{L}eD_i + g_{M_L i}\bar{L}^c\epsilon L M_{Li} + g_{M_R i}\bar{e}^c e M_{Ri} + g_{Vi}\left(\bar{L}\gamma^\mu L + \kappa_i \bar{e}\gamma^\mu e\right)V_{i\mu} + g_{V'i}(\bar{e}^c\gamma^\mu L)V_i^{\prime\dagger} + \text{h.c.},$$

- Each particle specie, X, generates some coefficients (C1,C2,C3,C4)
  - For example, type D (Dirac-type scalar coupling)

$$\left.\begin{aligned} D_1 &: \ \vec{c}^{\,(8)}_{D_1} = \frac{g^2_{D_1}}{M^4_{D_1}}(0,0,1,0) \\ D_2 &: \ \vec{c}^{\,(8)}_{D_2} = \frac{g^2_{D_2}}{M^4_{D_2}}(0,0,1,0) \\ &\ \ \vdots \qquad \vdots \\ D_n &: \ \vec{c}^{\,(8)}_{D_n} = \frac{g^2_{D_n}}{M^4_{D_n}}(0,0,1,0) \end{aligned}\right\} \vec{C}^{(8)}_D = w_D \vec{c}^{\,(8)}_D$$

const. vector

$$\vec{c}^{\,(8)}_D = (0,0,1,0)$$

(these are the "projectors")

"weight" of D

$$w_D = \sum_i \frac{g^2_{D_i}}{M^4_{D_i}}$$

- In total, we have 5 constant vectors (or an infinite set of, as kappa is free), and 5 **positive weights** $w_X = \sum_i \frac{g_{Xi}^2}{M_{Xi}^4} \geq 0$

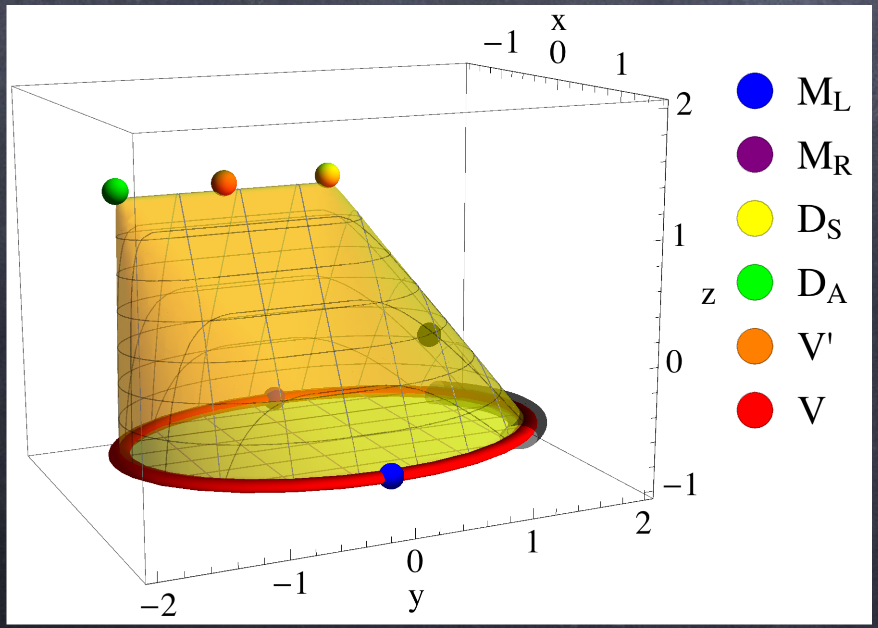
$$\vec{c}_D^{\,(8)} = (0,0,1,0), \qquad \vec{c}_{M_L}^{\,(8)} = (0,0,0,-1),$$
$$\vec{c}_{M_R}^{\,(8)} = (-1,0,0,0), \qquad \vec{c}_{V'}^{\,(8)} = (0,0,-1,2),$$
$$\vec{c}_{V(\kappa)}^{\,(8)} = (-\kappa^2/2,-\kappa,0,-1/2).$$

- Tree level positivity is simply the conical hull of the above vectors.

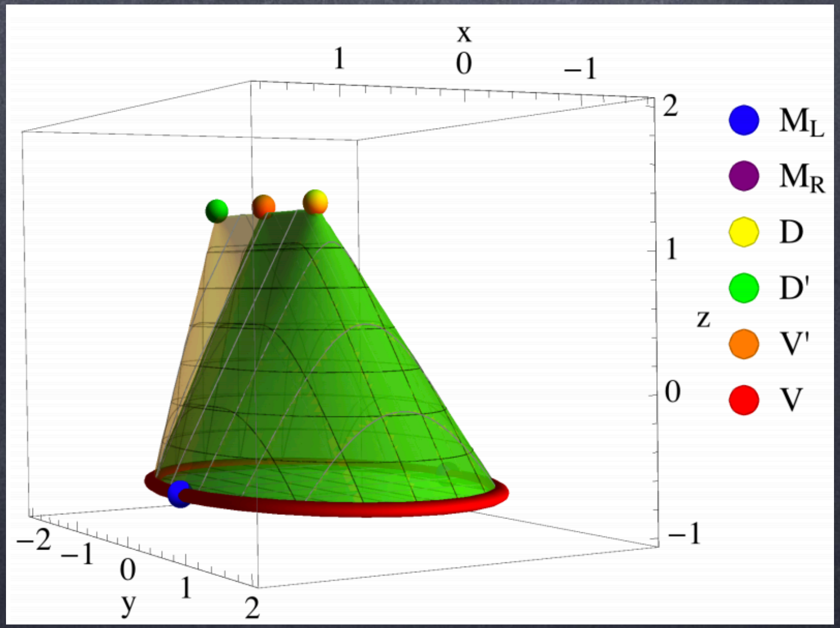
$$\vec{C}^{(8)} = \sum_X w_X \vec{c}_X^{\,(8)} \in \text{cone}\left(\left\{\vec{c}_X^{\,(8)}\right\}\right)$$

- Plot the 4D cone by its 3D cross section (ERs become extreme points)

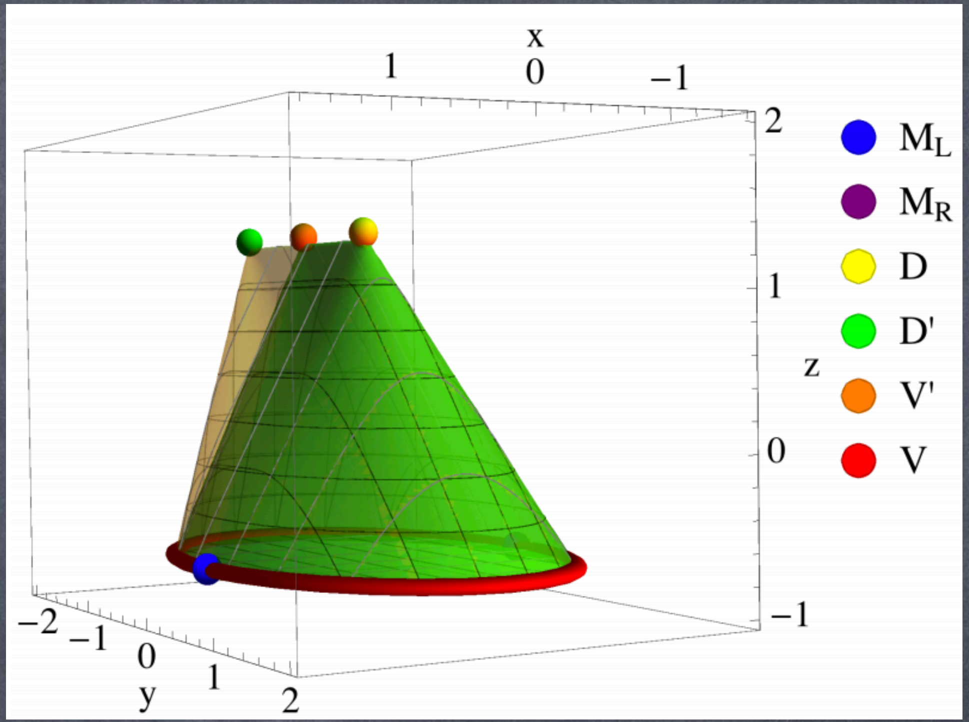
Full positivity



Green: tree level positivity

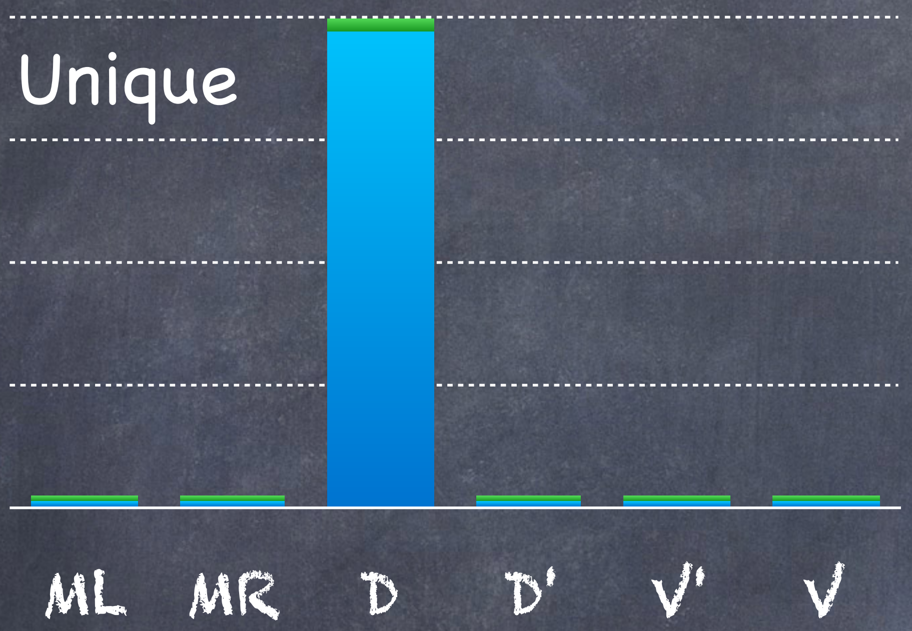


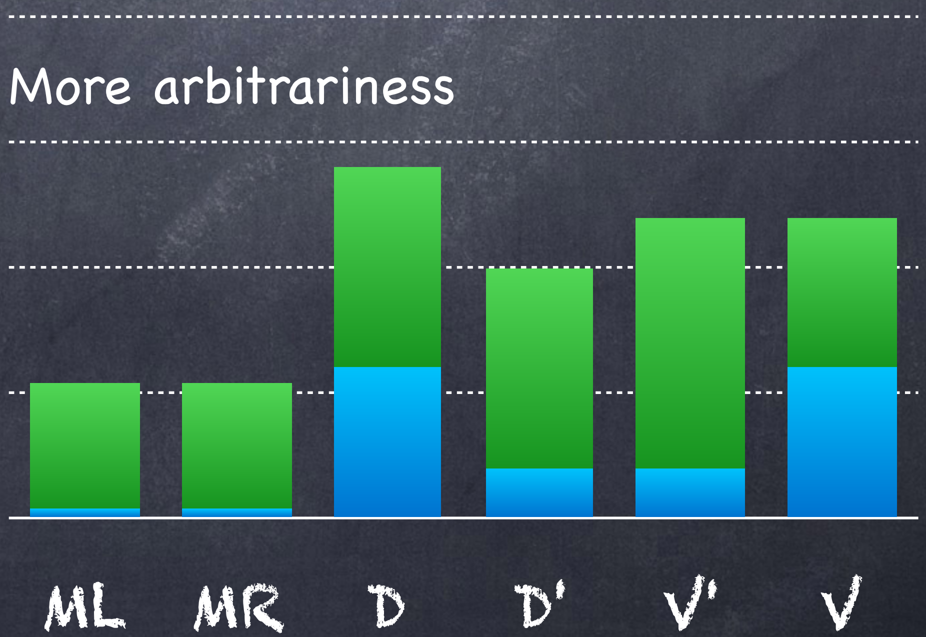
- Now positivity allows to go from EFT to UV spectrum. The goal is to determine all the weights w_X in $\vec{C}^{(8)} = \sum_X w_X \vec{c}_X^{(8)}$.



- Naively, we expect something like

$\min(w_X)$ $\max(w_X)$





- Does this work in reality? In particular with EXP uncertainties? Consider a real e+e->e+e- fit, for example, at

Scenario	Beam polarization $P(e^-, e^+)$	Runs (luminosity @ energy), $[\mathrm{ab}^{-1}]$ @ [GeV] 1	2	3	4
CEPC	None	2.6@161	5.6@240		
FCC-ee	None	10@161	5@240	0.2@350	1.5@365
ILC-500	(−80%, 30%)	0.9@250	0.135@350	1.6@500	
	(80%, −30%)	0.9@250	0.045@350	1.6@500	
ILC-1000	(−80%, 30%)	0.9@250	0.135@350	1.6@500	1.25@1000
	(80%, −30%)	0.9@250	0.045@350	1.6@500	1.25@1000
CLIC	(−80%, 0%)	0.5@380	2@1500	4@3000	
	(80%, 0%)	0.5@380	0.5@1500	1@3000	

- Consider CEPC, FCC-ee, ILC, CLIC, assuming 25 bins in cosθ , 1% systematic error. A global fit including dim-6/8 operators are carried out.

[2009.02212, B. Fuks, Y. Liu, **CZ**, S.-Y. Zhou]

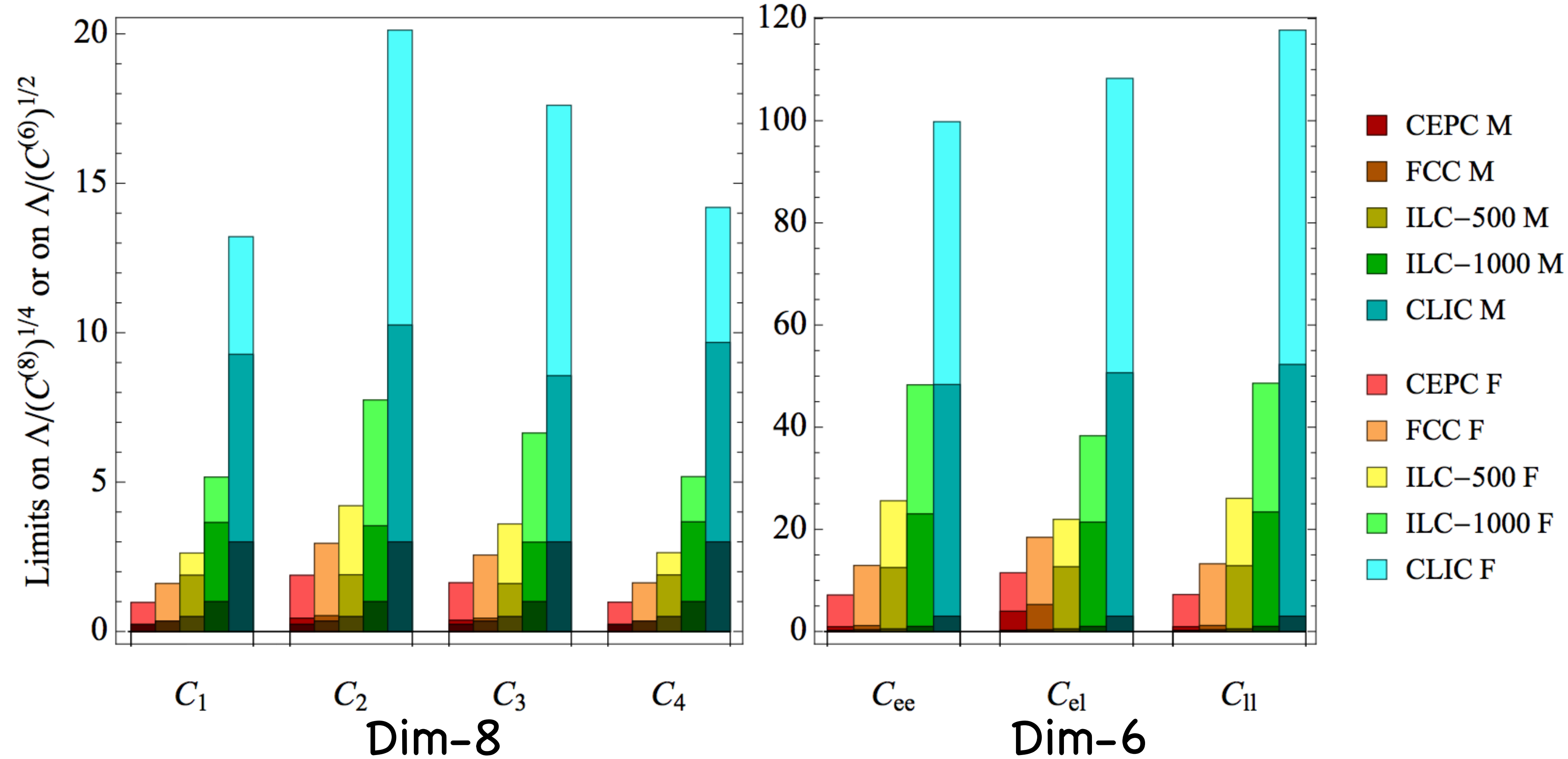


FIG. 3. Limits on the new physics characterization scale Λ_c (in TeV) for the various considered future lepton colliders. 'M' denotes marginalized limits (all other coefficients being floating) while 'F' denotes individual limits (all other coefficients being vanishing). In addition, we represent by the darkest color the largest center-of-mass energy of each collider project.

- For the dimension-8 operators, the individual sensitivities range from O(1) (CEPC) to O(10) (CLIC) TeV.
- Marginalized bounds are a factor of a few weaker when compared with the individual limits. Still, for all scenarios, the corresponding scale is sufficiently higher than the collider energy, except for the O1 and O4 operators in the CEPC and FCC-ee. (Due to an accidental flat direction.)

- Example 1: D-type scalar extension, $g_D = 0.8$, $M_D = 2$ TeV

$$\vec{C}_0^{(6)} = (0, -0.08, 0)\ , \qquad \vec{C}_0^{(8)} = (0, 0, 0.04, 0)\ .$$

- At ILC (extended with 1 TeV run), our global fit gives:

$$C_{ee} = 0 \pm 0.0024, \qquad C_{el} = -0.08 \pm 0.0035,$$
$$C_{ll} = 0 \pm 0.0023,$$
$$C_1 = 0 \pm 0.0074, \qquad C_2 = 0 \pm 0.0077,$$
$$C_3 = 0.04 \pm 0.020, \qquad C_4 = 0 \pm 0.0071.$$

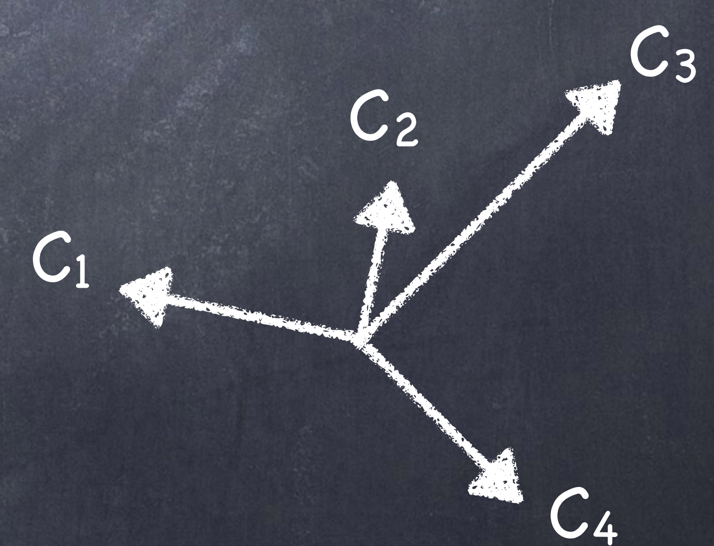
An interpretation of dim-6 EFT is useful only when we know the UV model (particle content). i.e. in a top-down approach.

- What can we learn at dim-6?
  - If assume the SM is only supplemented by D-type scalar,

$$M_D/g_D \in [2.45, 2.56]\ \mathrm{TeV}$$

  - If assume the SM is extend by D and V',

$$\frac{g_D^2}{2M_D^2} - \frac{g_{V'}^2}{M_{V'}^2} = 0.08 \pm 0.0035\ \mathrm{TeV}^{-2}$$



- If assuming more complicated models, not much to be concluded about the existence of UV states.

What about dim-8?

- Putting upper limits on the weight w_X, the contribution of each particle type

$$\text{max of } \lambda \text{ s.t. } \vec{C}_{\rm exp} - \lambda \vec{C}_X \in \mathcal{C}$$

This has EXP err

What about dim-8?

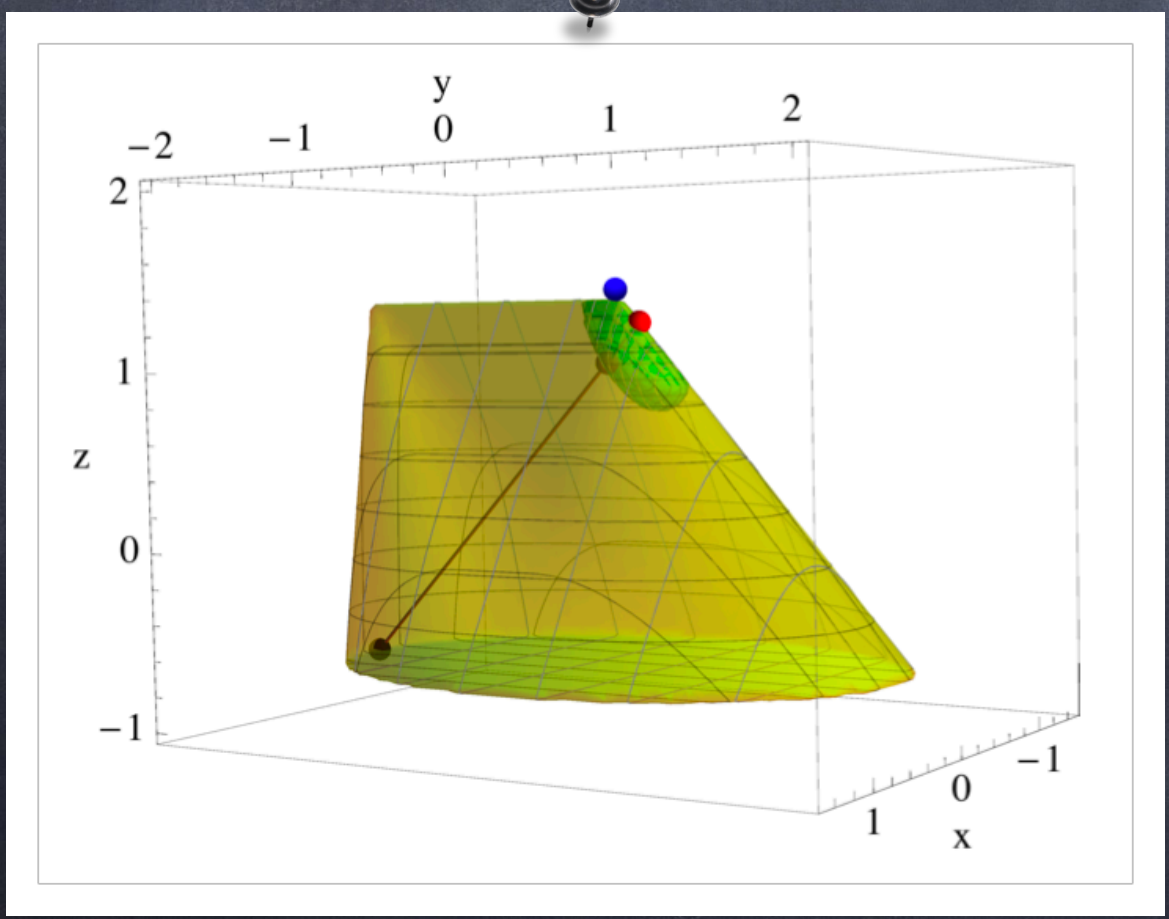
- Putting upper limits on the weight w_X, the contribution of each particle type

$$\lambda_{\max} \equiv \max_{\lambda} \left[\vec{C} - \lambda \vec{C}_X \in \mathcal{C};\ \chi^2\left(\vec{C}, \vec{C}_0\right) \leq \chi^2_c \right]$$

Upper limit on w_X

Subject to EXP bound

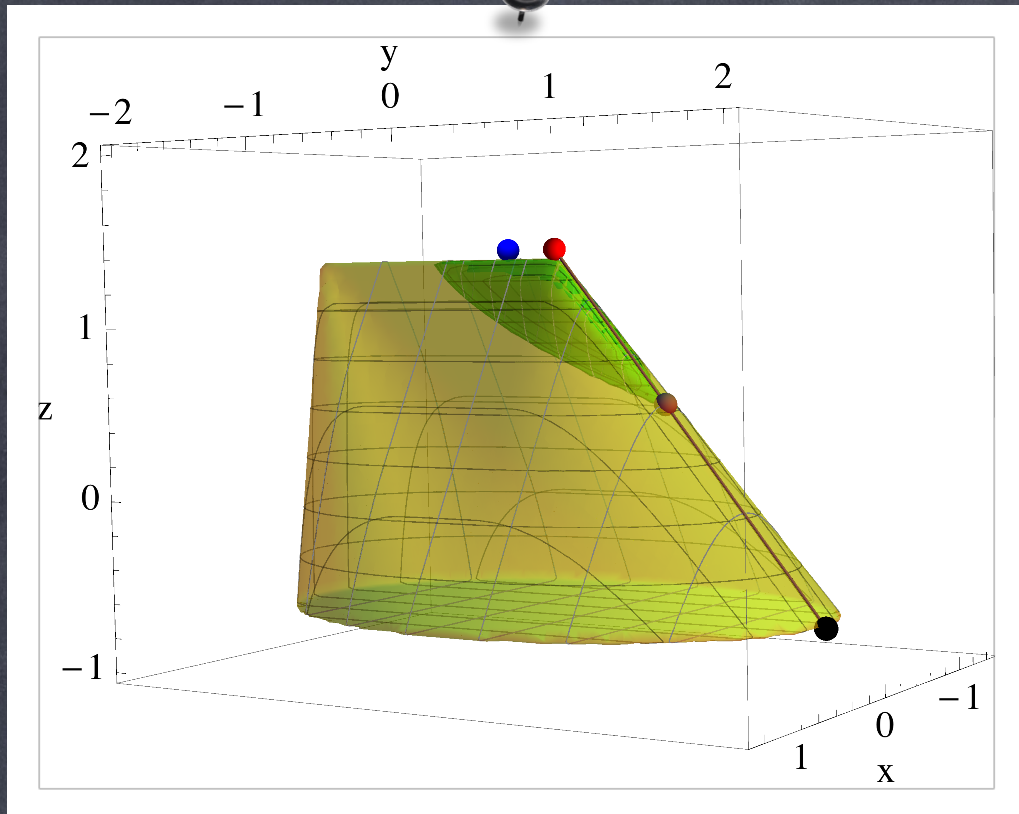
$\max(w_V)$, upper bound for SM+vector



- Positivity bound
- EXP bound
- D-type extension (assumed)
- V-type extension (to be excluded)
- $\vec{C}$ that maximizes λ
- $\vec{C} - \lambda \vec{C}_V$

Heavy vector is excluded at

$$\frac{M_V}{\sqrt{g_V}} \geq 3.7\ \mathrm{TeV}$$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−2 −1 0 1 2
z
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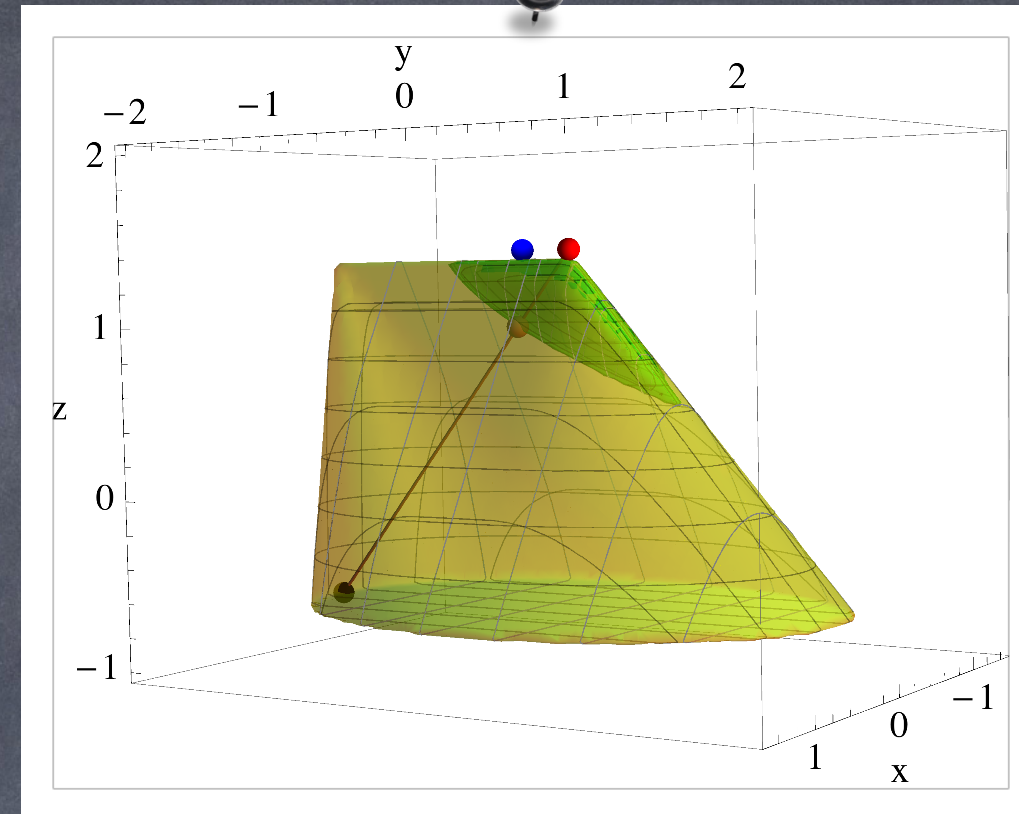
y
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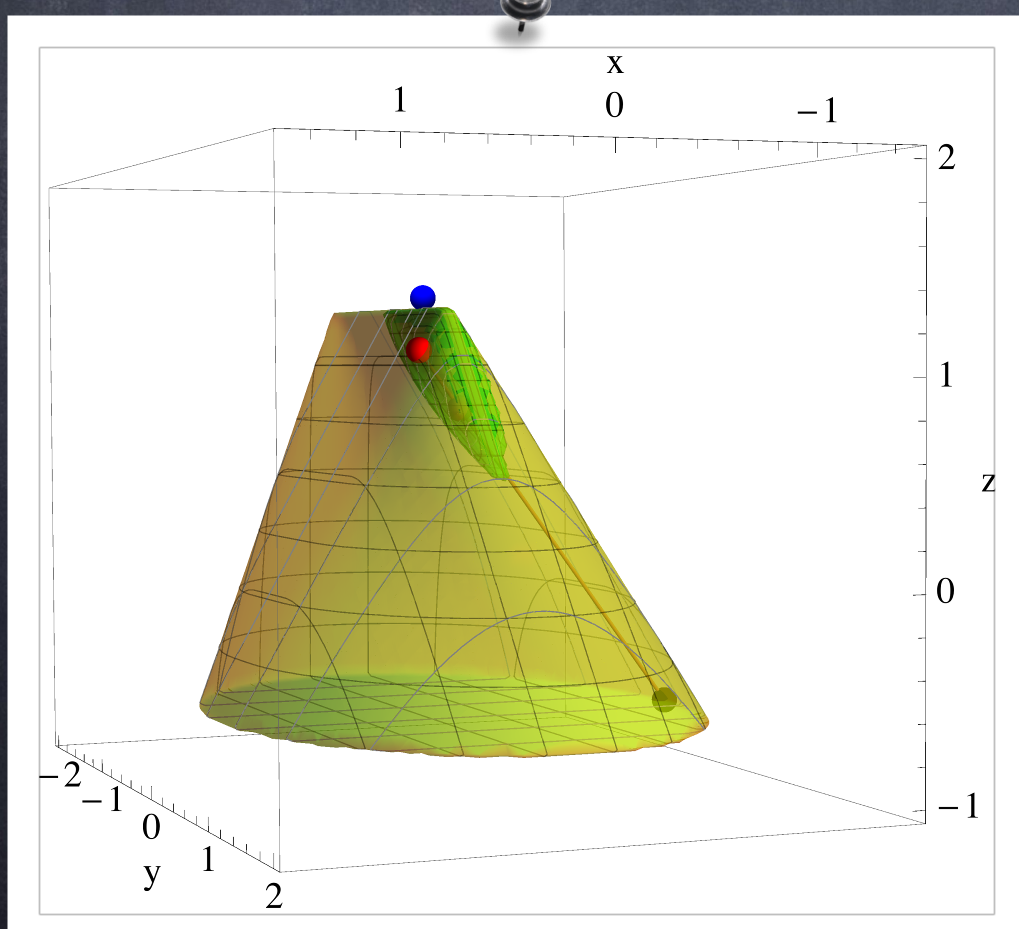
x
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2 1 0 −1
y
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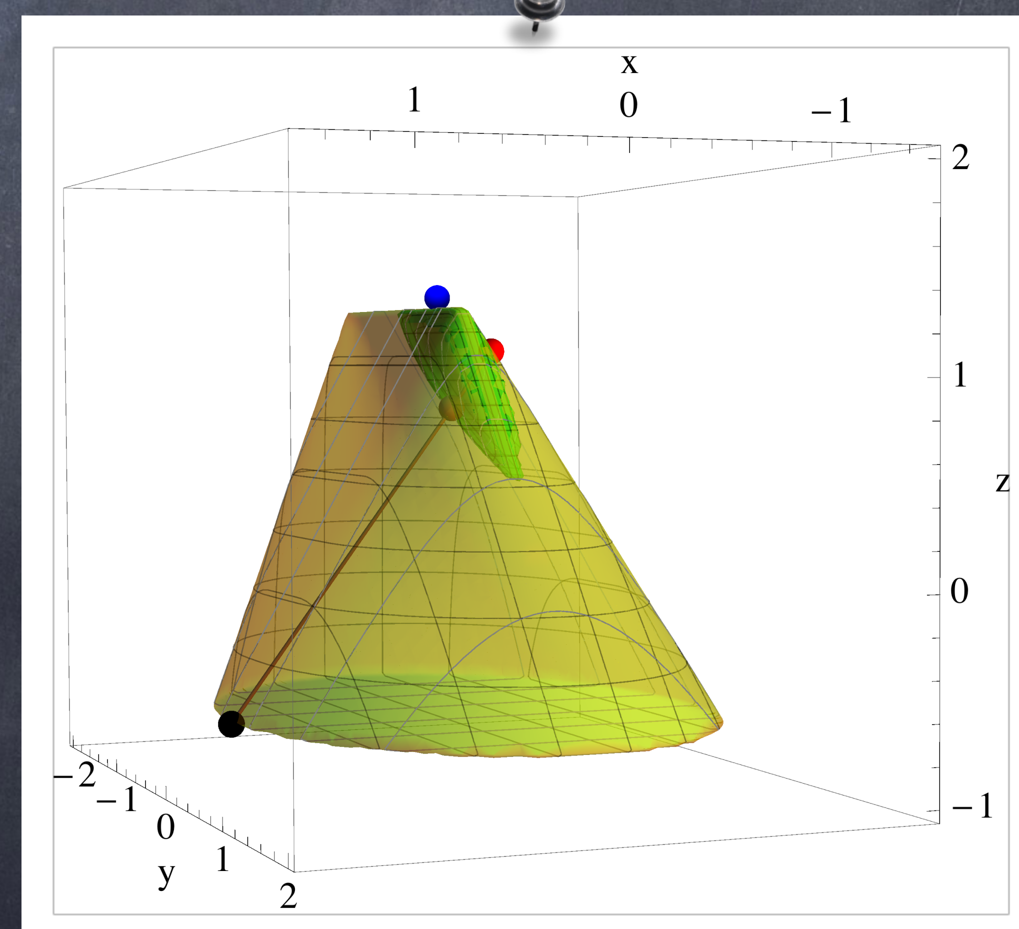
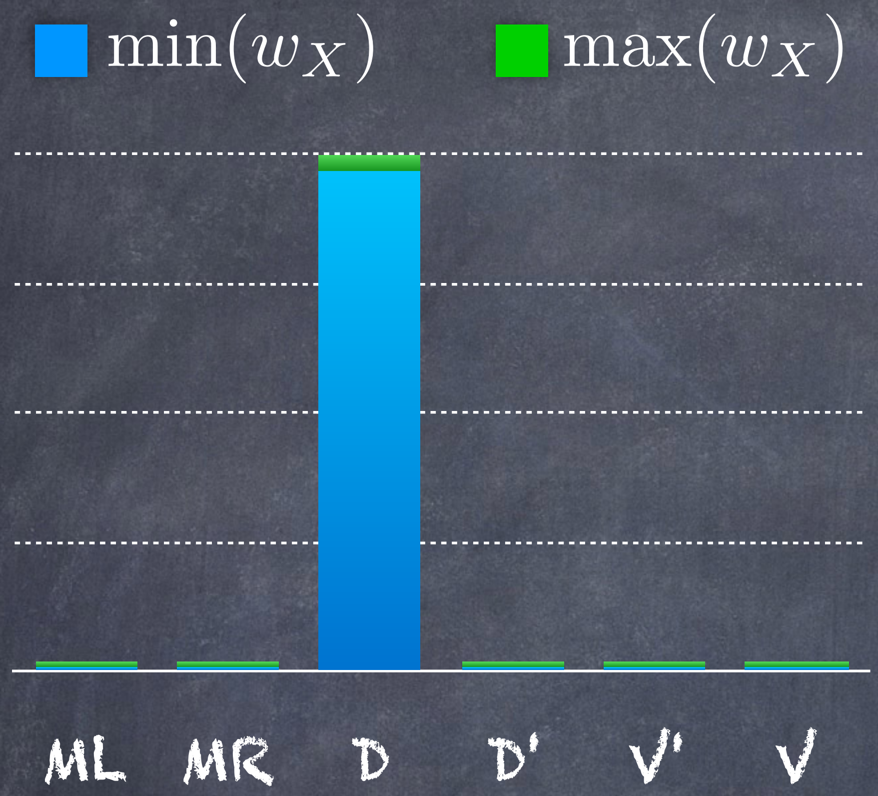
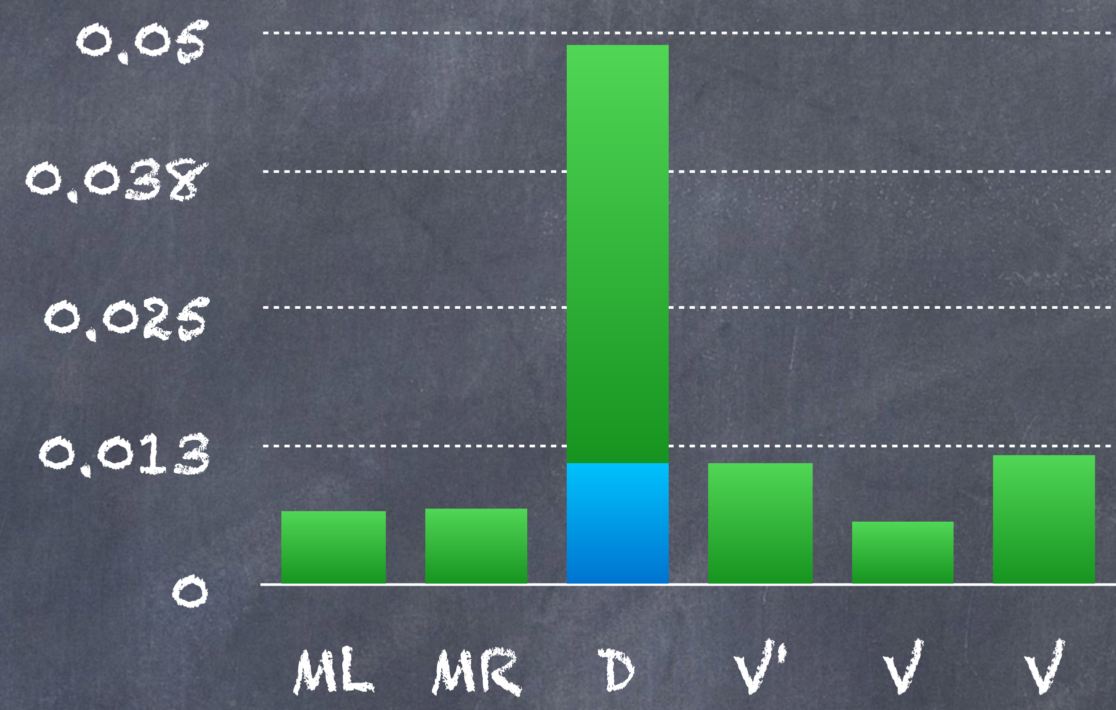

x
1 0 −1
z
2 1 0 −1
y
−2 −1 0 1 2

Ideal → With EXP uncertainty





$2.1 \leq M_D/\sqrt{g_D} \leq 3.1$ TeV

X	$\lambda_{\max}$	$M_X/\sqrt{g_X}$
M_L	0.0067	≥ 3.5 TeV
M_R	0.0069	≥ 3.5 TeV
V (vector)	0.0055	≥ 3.7 TeV
V (axial v.)	0.0116	≥ 3.0 TeV
V'	0.0109	≥ 3.1 TeV

- Heavy states are excluded above ~4 TeV (from a 1 TeV experiment).
- These limits **do not depend on the UV model setup.**

Example 2: precision test of SM?

- If all measurement agrees with SM with all **dim-6** coefs.->0: **cannot confirm the SM**, because various UV states may exist in such a way that their dim-6 coefs. cancel out.

- Going to **dim-8** allows to ultimately confirm the SM. **Positivity implies that at least some dim-8 operators have to be positive**.
    - In practice, with EXP errors, the same method as example 1 can be adopted to derive exclusion limits on each UV state.
    - E.g. 1 TeV e+e- measurement excludes all states up to 4 TeV.

X	$\lambda_{\rm max}$	$M_X/\sqrt{g_X}$
D_S	0.0076	≥ 3.4 TeV
M_L	0.0053	≥ 3.7 TeV
M_R	0.0054	≥ 3.7 TeV
V (vector)	0.0056	≥ 4.0 TeV
V (axial v.)	0.004	≥ 4.0 TeV
V'	0.0041	≥ 4.0 TeV

Summary

- While SMEFT has proved useful as a phenomenological tool for LHC physics, some questions are still left open.
- Positive structures arise at the dim-8 level in SMEFT space, as a consequence of axiomatic QFT principles. A convex geometric perspective helps to understand these structures, and reveals its connection with UV physics.
  - We have proposed new approaches to derive the best positivity bounds
  - The ERs correspond to UV states (as living in irreducible representations). This connection may provide some partial answer to the inverse problem: the determination of UV physics from EFT measurements.
- More to be studied: general convex structure of EFT space, better ways to find bounds, applications in phenomenology… Discussions are welcome!

Backups

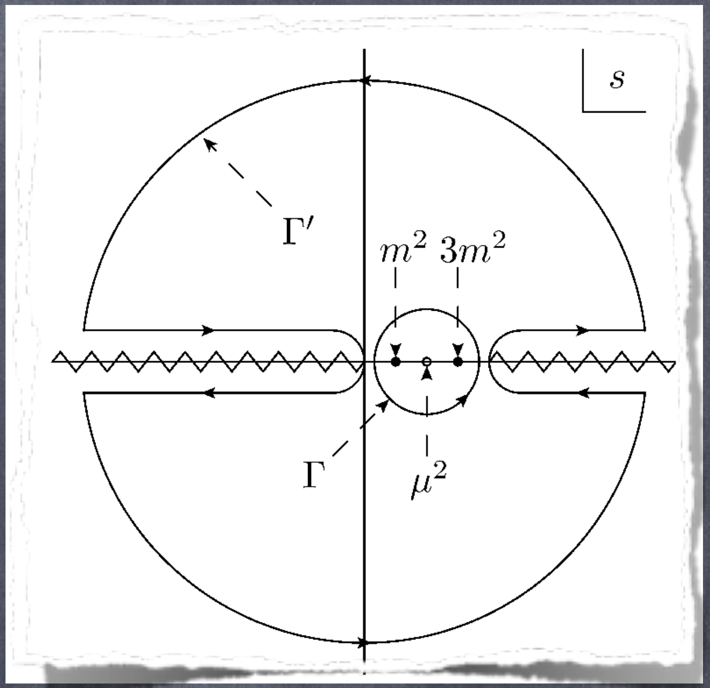
Dispersion relation

- Consider 2-to-2 forward elastic scattering amplitude, A(s,t=0)

- The contour integral $f = \frac{1}{2\pi i}\oint_\Gamma \mathrm{d}s \frac{A(s,0)}{(s-\mu^2)^3}$

- Deform Γ to Γ'. Boundary contributions vanish due to Froissart bound. (Froissart, 1961)

$$A(s,0) \leq \mathcal{O}(s \ln^2 s)$$



$$f = \frac{1}{2\pi i}\oint_\Gamma \mathrm{d}s \frac{A(s,0)}{(s-\mu^2)^3} = \frac{1}{2\pi i}\left(\int_{-\infty}^{0} + \int_{4m^2}^{\infty}\right)\mathrm{d}s \frac{\mathrm{Disc}A(s,0)}{(s-\mu^2)^3}$$

IR

Can be calculated in EFT.
In SMEFT gives C8+C6²

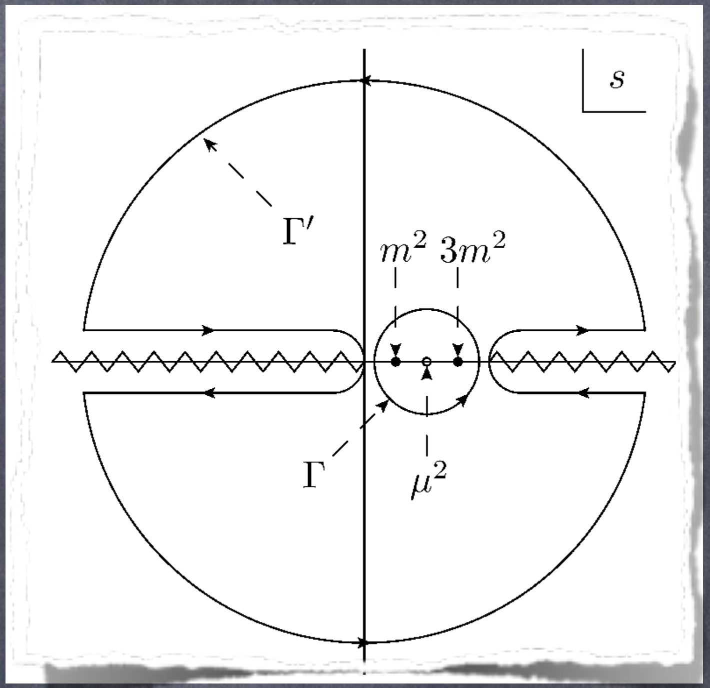
UV

These are unknown
but positive

Positivity bounds

- For the discontinuity in the + real axis: use optical theorem

$$\mathrm{Disc}A(s,0) = 2i\mathrm{Im}A(s,0)$$
$$\mathrm{Im}A(s,0) = s\sigma(s)\sqrt{1-4m^2/s} \geq 0$$



- For the discontinuity in the - real axis: use crossing then optical theorem

$$A(s,0) \;\rightarrow\; A'(u,0) = A'(4m^2-s,0)$$

- (Disc at large s is where NP enters)

- f>0: $$f \approx \frac{\mathrm{d}^2 A(\mu^2)}{\mathrm{d}s^2} = \sum_i c_i^{(8)} x_i + \sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j} \geq 0$$

if computed in SMEFT at tree level

- The E^4/Λ^4 operators (dim-8 SMEFT operators) need to satisfy "positivity bounds", for a UV completion to exist (with causality, locality, Lorentz invariance...) Certain linear combinations of dim-8 coefficients must be positive.

- Not new: [A. Adams, A. Arkani-Hamed, S. Dubovsky, A. Nicolis, R. Rattazzi, JHEP 06] already gave longitudinal quartic-gauge-boson couplings (QGC)

There are similar positivity conditions in more familiar effective field theories in particle physics. Consider for instance the $SU(2)$ chiral Lagrangian, parametrized by the unitary field $U = e^{i\pi^a\sigma^a}$,

$$\mathcal{L} = f^2\,\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + L_4\left[\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U)\right]^2 + L_5\left[\mathrm{tr}(\partial_\mu U^\dagger \partial_\nu U)\right]^2 + \cdots \tag{52}$$

Of course the pion chiral Lagrangian follows from QCD which is a local quantum field theory, so these conditions must neccessarily be satisfied. The situation is perhaps more interesting for the electroweak chiral Lagrangian governing the dynamics of the longitudinal components of the W/Z bosons. While it is most likely, given precision electroweak constraints, that the UV completion involves Higgses and a linear sigma model, there may also be more exotic possibilities, including in the extreme case a low fundamental scale close to the electroweak scale. This physics should manifest itself through the higher-dimension operators in the effective Lagrangian, and assuming custodial $SU(2)$ is a good approximate symmetry, the constraint on the electroweak chiral Lagrangian is the same $L_{4,5} > 0$ (with the derivatives covariantized for the $SU(2)\times U(1)$ gauge symmetry $\partial_\mu \to D_\mu$). These operators are not associated with the well-known constraints of precision electroweak physics— instead, in unitary gauge $U = 1$, they represent anomalous quartic couplings for the W/Z, which must be positive.

Positivity bounds: polarization dependence

- With FS, FM, FT operators, the spin of VV' can take any direction.
- Amplitude depends on external polarization. We will use

$$\epsilon^\mu(V_1)=\sum_{i=1}^{3}a_i\epsilon^\mu_{(i)}=\left(a_3\frac{p_1}{m_{V_1}},a_1,a_2,a_3\frac{E_1}{m_{V_1}}\right),$$

$$\epsilon^\mu(V_2)=\sum_{i=1}^{3}b_i\epsilon^\mu_{(i)}=\left(b_3\frac{p_2}{m_{V_2}},b_1,b_2,b_3\frac{E_2}{m_{V_2}}\right),$$

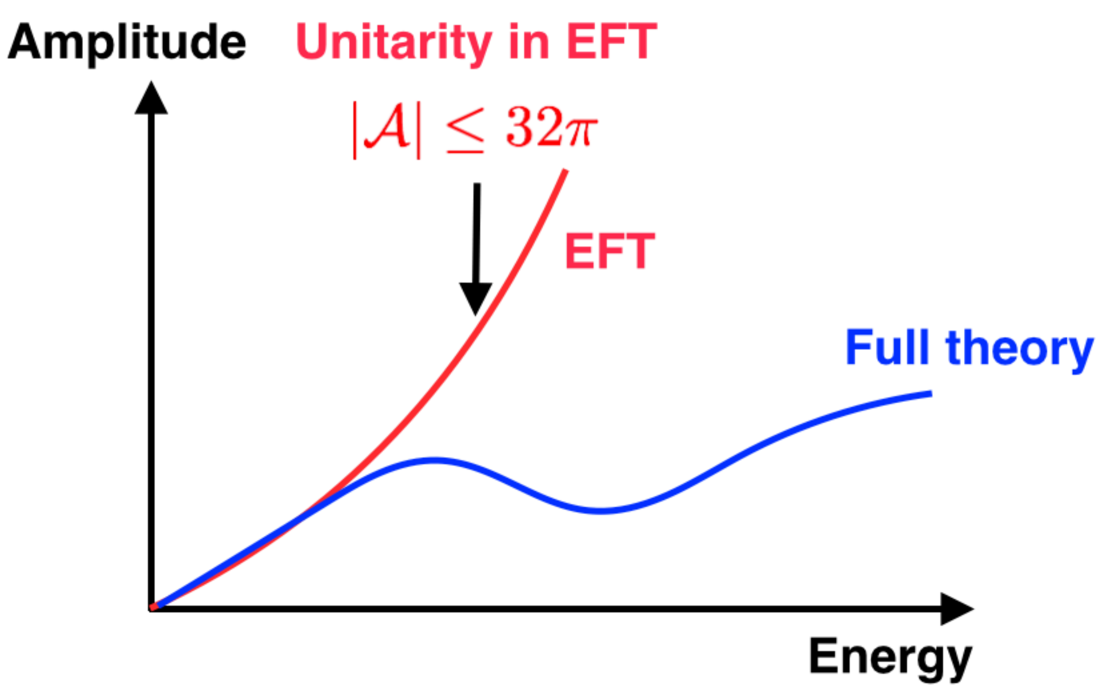
- For example, from WW channel: (Similar in all other channels)

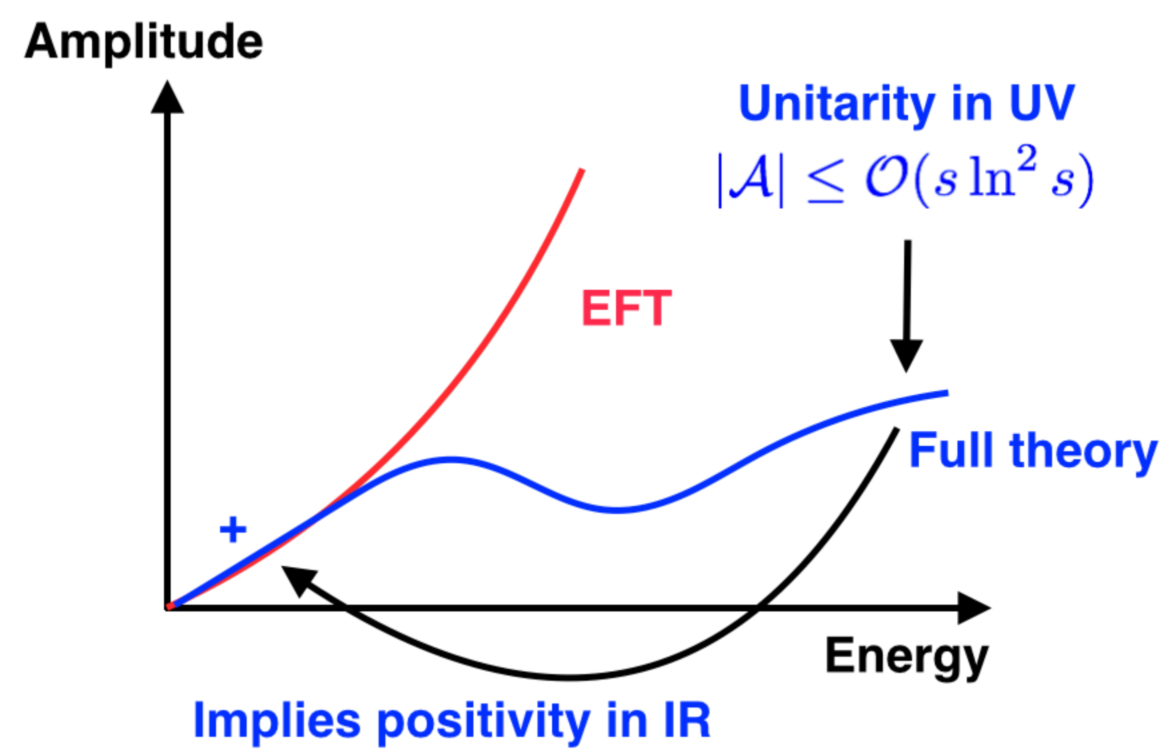
$$\boxed{\begin{aligned}&2A_1(8F_{T,0}+12F_{T,1}+5F_{T,2})+6A_2F_{T,2}+(A_3+A_3')(-2F_{M,1}+F_{M,7})\\&+2A_4(8F_{T,1}+F_{T,2})+2A_4'(8F_{T,0}+4F_{T,1}+F_{T,2})+A_5'(4F_{M,0}-F_{M,1}+F_{M,7})\\&+4A_6(2F_{S,0}+F_{S,1}+F_{S,2})>0\end{aligned}}$$

$$\begin{aligned}
A_1&\equiv|a_1|^2|b_1|^2+|a_2|^2|b_2|^2, & A_4&\equiv a_1a_2^*b_1b_2^*+c.c.,\\
A_2&\equiv|a_1|^2|b_2|^2+|a_2|^2|b_1|^2, & A_4'&\equiv a_1a_2^*b_1^*b_2+c.c.,\\
A_3&\equiv(|b_1|^2+|b_2|^2)|a_3|^2, & A_5&\equiv(a_1b_1+a_2b_2)a_3^*b_3^*+c.c.,\\
A_3'&\equiv(|a_1|^2+|a_2|^2)|b_3|^2, & A_5'&\equiv-(a_1b_1^*+a_2b_2^*)a_3^*b_3+c.c.\\
A_3''&\equiv|b_1|^2|a_3|^2 & A_6&\equiv|a_3|^2|b_3|^2,
\end{aligned}$$

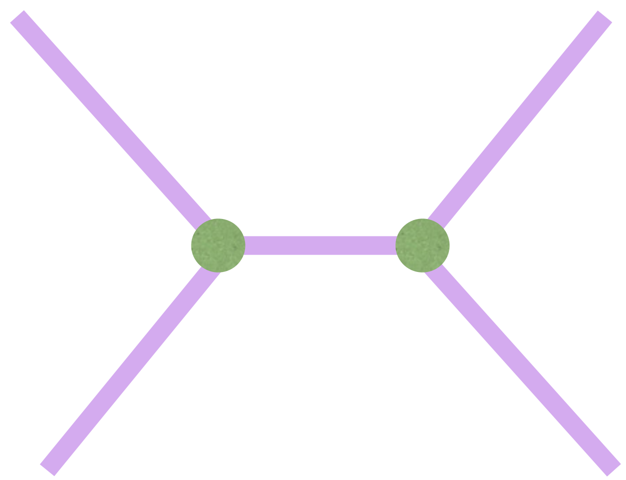
Has to hold for any a,b

Unitarity bounds $\neq$ Positivity bounds





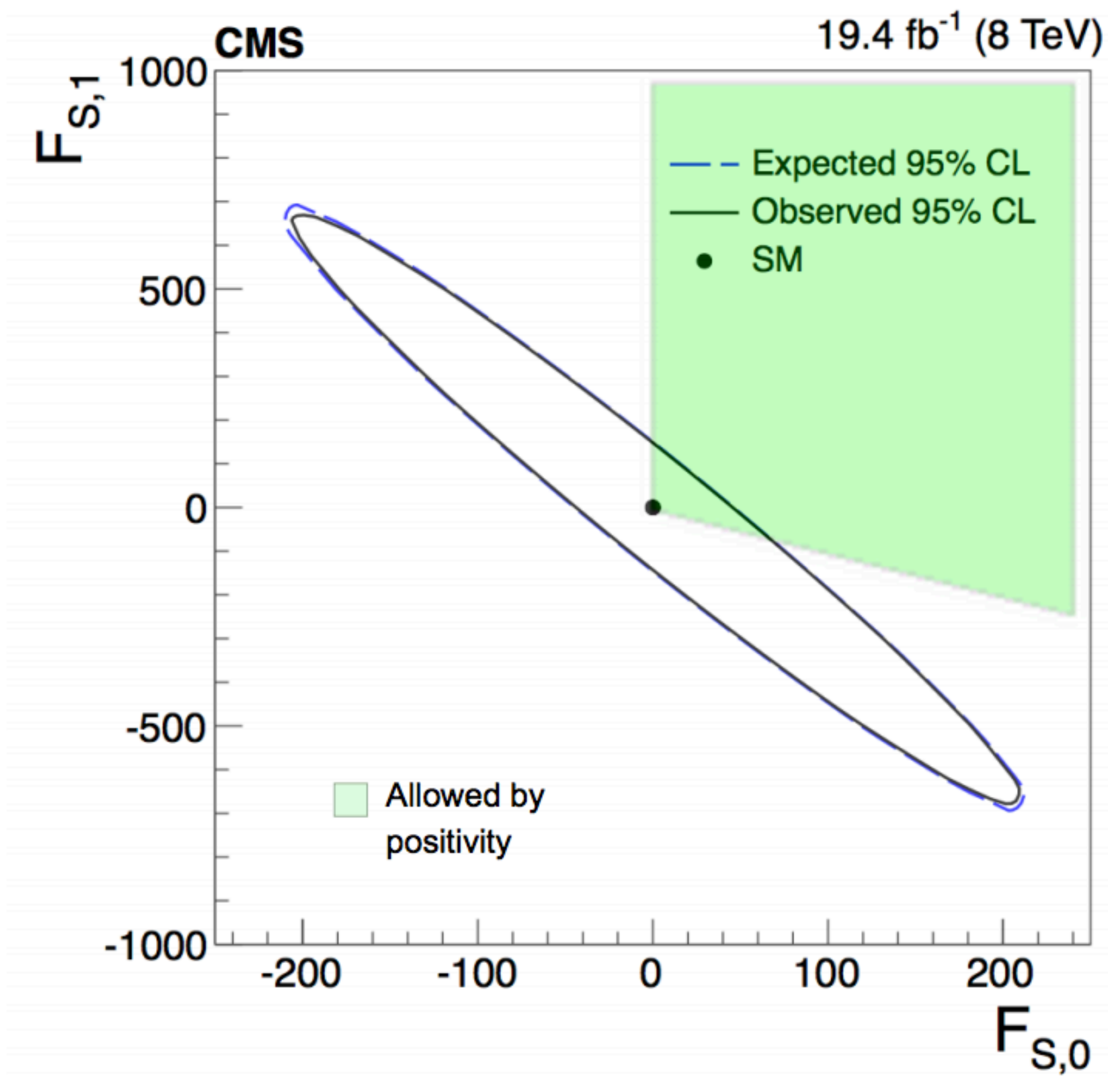
Dim-6 contributions



$$F_{T,2} \geq 0,$$
$$4F_{T,1} + F_{T,2} \geq 36\bar{a}_W^2,$$
$$F_{T,2} + 8F_{T,10} \geq 36\bar{a}_W^2,$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0,$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 72\bar{a}_W^2.$$

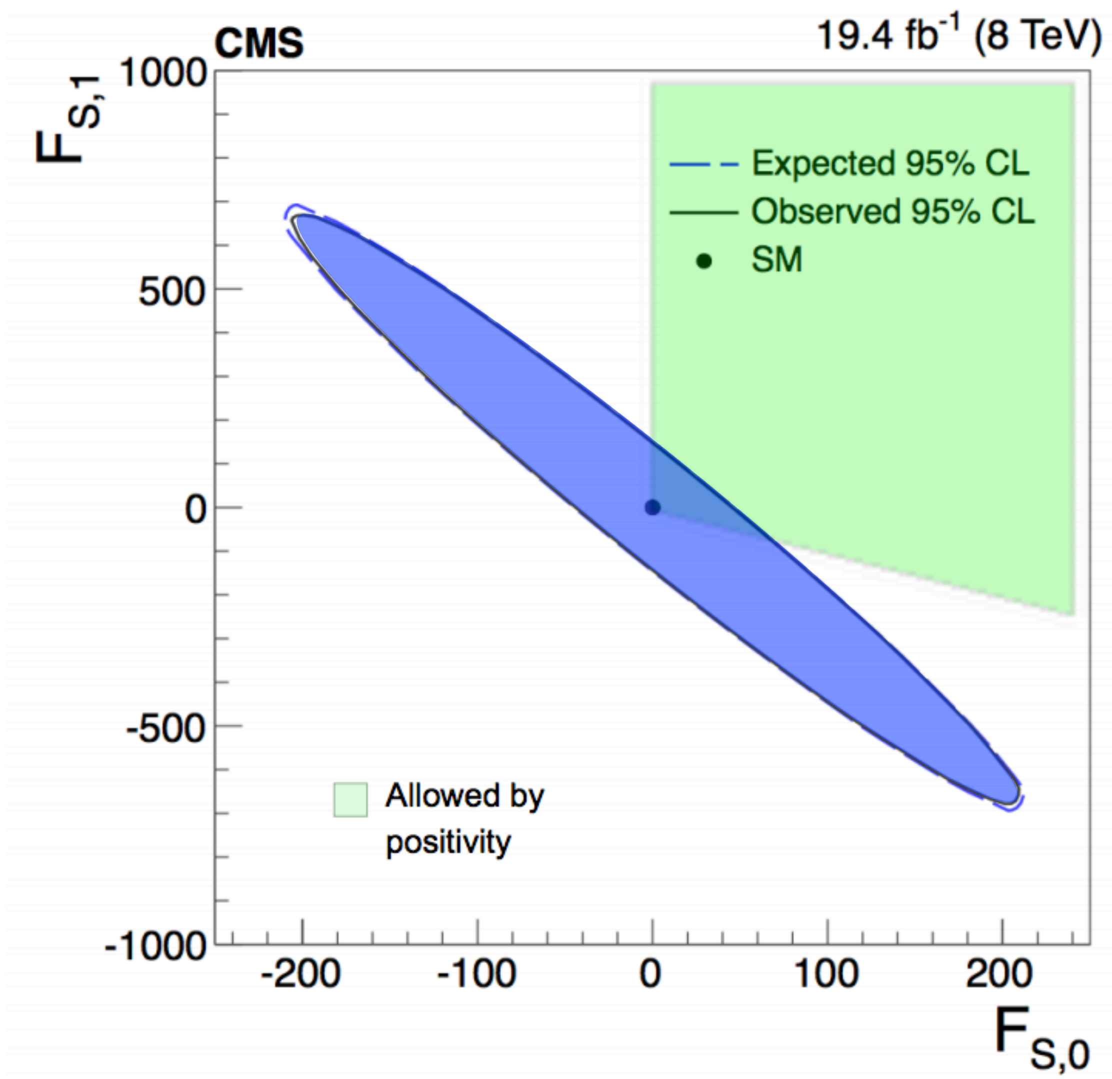
Dim-6 squares can be safely removed without affecting the inequalities

Affect limits by modifying the prior



[CMS, PRL 15]

Affect limits by modifying the prior

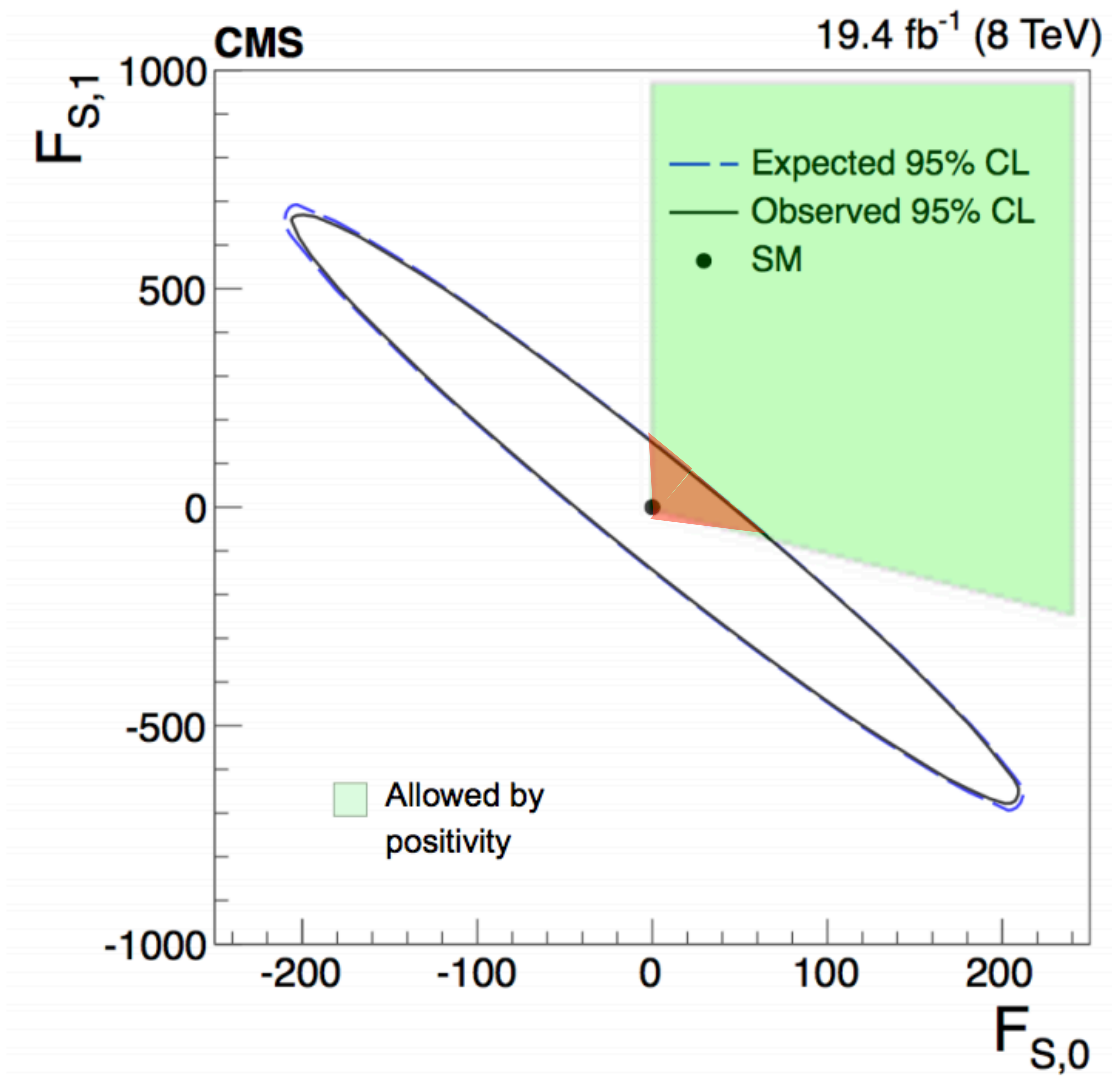


[CMS, PRL 15]

Without positivity

$$\int dF_{S,1} dF_{S,2} L = 95\%$$

Affect limits by modifying the prior



[CMS, PRL 15]

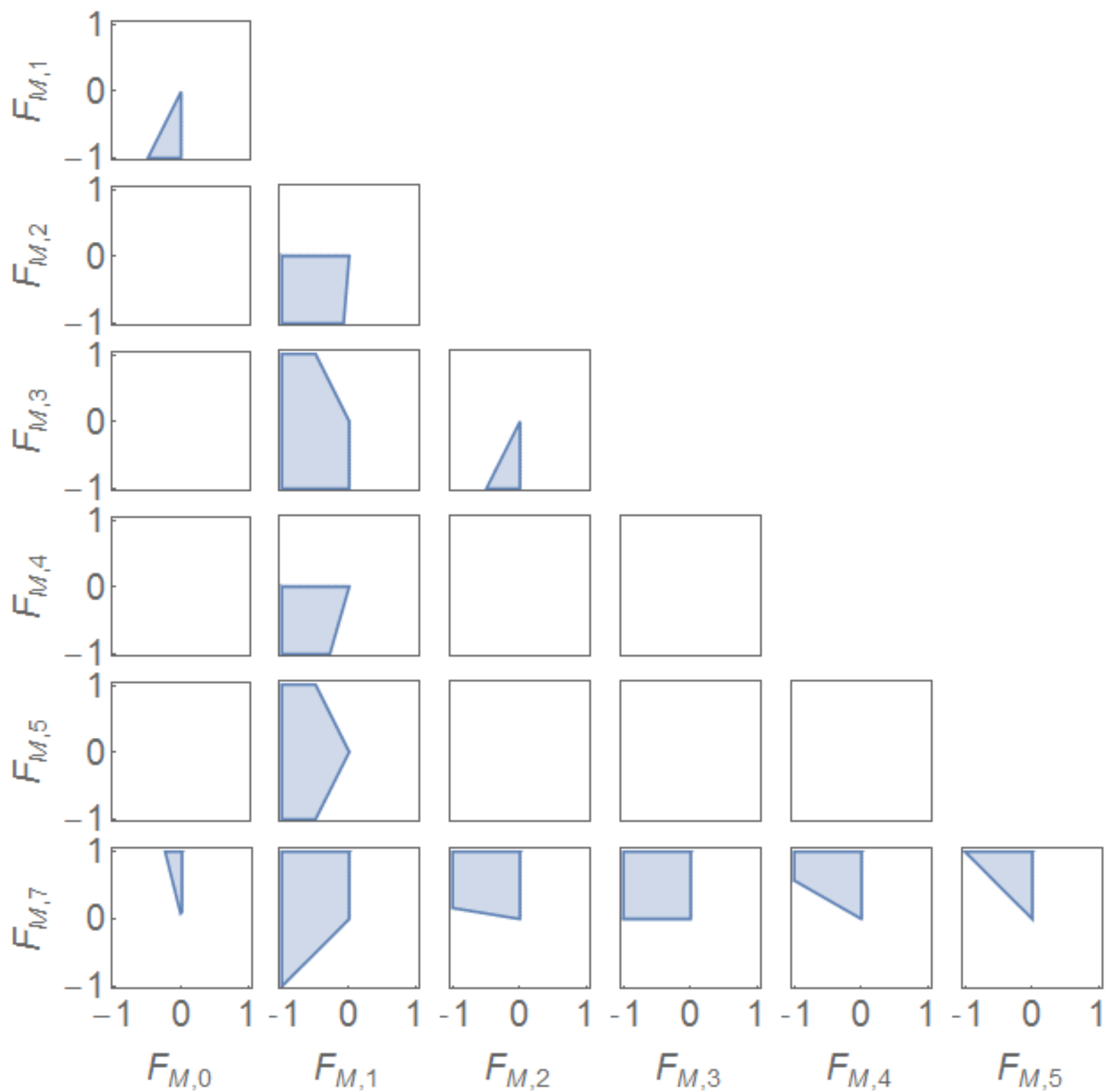
Without positivity

$$\int dF_{S,1} dF_{S,2} L = 95\%$$

With positivity

$$\frac{\int dF_{S,1} dF_{S,2} L}{\int dF_{S,1} dF_{S,2} L} = 95\%$$

aQGC all 2D subspaces



aQGC all 2D subspaces