A non-minimal $\mathrm{SU}(6)$ *GUT* *and* *high-quality Axion*

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Background: GUT

* The grand(gauge) unified theory (GUT) was first invented by Georgi & Glashow in 1974, with $SU(5)$ as the minimal simple Lie group to unify $\mathcal{G}_{\mathrm{SM}} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, with $\mathrm{rank}(\mathcal{G}_{\mathrm{SM}}) = \mathrm{rank}(\mathrm{SU}(5)) = 4$.

* 15 SM complex chiral fermions fit into irreps of $\bar{\mathbf{5}}_{\mathbf{F}}$ and $\mathbf{10}_{\mathbf{F}}$ in the $SU(5)$, which are required by the gauge anomaly cancellation.

* There was also an $SO(10)$ GUT by Fritzsch & Minkowski in 1975. This is automatically anomaly-free, and its irrep of $\mathbf{16}_{\mathbf{F}}$ even includes a ν_R.

Background: GUT

* Over 40 yrs since the $SU(5)$ and $SO(10)$, there is convincing evidence that the BSM new physics should be put forth to address:

  (1) Strong CP problem, e.g., the Peccei-Quinn mechanism
  (2) neutrino masses through the seesaw mechanism
  (3) Baryon asymmetry through the baryogenesis/leptogenesis
  (4) Dark matter

* The simplest $SU(5)$, $SO(10)$ and their varieties, do not seem to include all necessary ingredients for BSM. One often needs to put new physical ingredients into the $SU(5)$ && $SO(10)$ GUTs by hand.

Background: GUT

Topics	Authors	Publications	Date
Fourth color unification	Pati, Salam	[5]	1974
SU(5) GUT	Georgi, Glashow	[6]	1974
SO(10) GUT	Fritzsch, Minkowski	[7]	1975
Peccei-Quinn mechanism	Peccei, Quinn	[8]	1977
Seesaw mechanism	Yanagida	[9]	1979
	Gell-Mann, Ramond, Slansky	[10]	1979
KSVZ axion	Kim	[11]	1979
	Shifman, Vainshtein, Zakharov	[12]	1980
SU(N + 4) global symmetry	Dimopoulos, Raby, Susskind	[13]	1980
SUSY SU(5) GUT	Dimopoulos, Georgi	[14]	1981
DFSZ axion	Zhitnitsky	[15]	1980
	Dine, Fischler, Srednicki	[16]	1981
Axion in SU(5) GUT	Wise, Georgi, Glashow	[17]	1981
Leptogenesis	Fukugita, Yanagida	[18]	1986
...			

Table 1: Collections of the major theoretical advances in the GUT and some related BSM physical issues listed in chronological order.

Background: GUT

* We call $\mathrm{SU}(5)$ and $\mathrm{SO}(10)$ the *minimal* GUTs due to their fermion representations. The original 1974 Georgi & Glashow $\mathrm{SU}(5)$ paper assumed "*as few leptons (fermions) as possible, no unobserved leptons (fermions)…".*

* Our assumption: *a successful GUT can address as many BSM issues as possible, with the minimal set of fields unless otherwise necessary*.

* We choose to deal with the PQ-quality issue for the strong CP problem, and we get more than that (see next).

Background: Strong CP

* The strong CP problem, a topological term for the QCD vacuum $\mathscr{L}_\theta = \theta \frac{\alpha_{3c}}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu\, a}$, and experimentally from the neutron EDM: $|\bar{\theta}| \lesssim 10^{-10}$, with $\bar{\theta} = \theta + \arg\det M_q$, very different from the $\mathcal{O}(1)$ expectation of θ parameter.

* PQ mechanism: to replace θ by a periodic pseudo-scalar field $a \to a + 2\pi f_a$, f_a is known as the axion decay constant. There is a classical window of $10^8\,\mathrm{GeV} \lesssim f_a \lesssim 10^{12}\,\mathrm{GeV}$.

* Axion induced potential: $V_{\mathrm{QCD}} = -\Lambda^4_{\mathrm{QCD}} \cos(a/f_a)$.

* Invisible axion models such as KSVZ and DFSZ, a comes from a complex scalar field $\Phi = \frac{1}{\sqrt{2}}(v_a + \rho_a)\exp(ia/f_a)$.

Background: Strong CP

* PQ quality: $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry (expressed in terms of Φ) is global and put in by hand, and the gravity does not respect global symmetries, it can induce a general operator of $\mathcal{O}^{d=2m+n}_{\not{\mathrm{PQ}}} = k\frac{|\Phi|^{2m}\Phi^n}{M_{\mathrm{pl}}^{2m+n-4}} + H.c.$ ['92 Kamionkowski, March-Russell, and etc.] $\Delta\mathrm{PQ} = n$ with $\mathrm{PQ}(\Phi) = 1$.

* The $\mathcal{O}^{d=2m+n}_{\not{\mathrm{PQ}}}$ shifts the V_{QCD} minima $|\bar{\theta}| = |\langle a\rangle/f_a| \lesssim 10^{-10}$

* if $|k| \sim 10^{-2}$ and $2m+n=5,\ \Rightarrow f_a \lesssim 10\,\mathrm{GeV}$, ruled out,
else if $f_a \sim 10^{12}\,\mathrm{GeV}$ and $2m+n=5,\ \Rightarrow |k| \lesssim 10^{-55}$, very fine-tuned.

* NB, the renormalizable operators with $2m+n \leq 4$ are in principle possible. The discussion above considered a general Φ w.o. any gauge symmetry.

Global Symmetries

* The usual wisdom of a high-quality PQ is to have the $\mathrm{U}(1)_{\mathrm{PQ}}$ as an emergent global symmetry.

* The QCD has the global symmetry of: $\mathcal{G}_{\mathrm{global}} = \mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \otimes \mathrm{U}(1)_V \otimes \mathrm{U}(1)_A$, while QCD is vectorial.

* The chiral gauge theory w.o. Unification: to put another confining theory with the SM, e.g. $\mathrm{SU}(5) \otimes \mathcal{G}_{\mathrm{SM}}$ by Gavela, Ibe, Quilez, and Yanagida [1812.08174].

* Dimopoulos-Raby-Susskind (1980) studied a strongly-interacting theory: an anomaly-free $\mathrm{SU}(N+4)$ chiral gauge theory with N anti-fundamental fermions and one rank-2 anti-symmetric fermion, and it has $\mathcal{G}_{\mathrm{global}} = \mathrm{SU}(N) \otimes \mathrm{U}(1)$, $N \geq 2$.

Our result:

* We start from $\mathrm{SU}(6)$, and identify $\mathcal{G}_{\mathrm{global}} = \mathrm{SU}(2)_F \otimes \mathrm{U}(1)_{\mathrm{PQ}}$.

* Our finding is that a *non-minimal* $\mathrm{SU}(6)$ GUT with its minimal fermion && Higgs setup can lead to:

  (1) Automatic high-quality PQ symmetry breaking @ $10^8\,\mathrm{GeV} \lesssim f_a \lesssim 10^{10}\,\mathrm{GeV}$, with an extended symmetry of $\mathcal{G}_{331} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$
  (2) Automatic KSVZ vector-like quarks $m_D \sim f_a$, with fixed electric charge of $-1/3$.
  (3) A cosmological-safe axion model, no DW formation
  (4) Automatic Type-I seesaw mechanism with sterile neutrino mass at $\sim f_a$
  (5) Automatic Type-II 2HDM at the EW scale

The SU(6) model

The SU(6) model

* The minimal anomaly-free $\mathrm{SU}(6)$ has fermions of: $2 \times \bar{\mathbf{6}}_{\mathbf{F}} \oplus \mathbf{15}_{\mathbf{F}}$

* How to break the $\mathrm{SU}(6)$ to the $\mathcal{G}_{\mathrm{SM}} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$? The Georgi-Glashow $\mathrm{SU}(5)$ is a subgroup of the $\mathrm{SU}(6)$, while the direct breaking to the $\mathrm{SU}(5)$ is unwelcome. The sequential breaking of $\mathrm{SU}(5) \to \mathcal{G}_{\mathrm{SM}}$ leads to the proton decays with lower mass scale, hence faster decay rate.

* Alternative pattern is: $\mathrm{SU}(6) \to \mathcal{G}_{331} \to \mathcal{G}_{\mathrm{SM}}$, with $\mathcal{G}_{331} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$. This is achievable with an adjoint Higgs of $\mathbf{35}_{\mathbf{H}}$ at the GUT scale (1974 Ling-Fong Li).

The SU(6) Higgs sector

* An adjoint Higgs of $\mathbf{35_H}$ at the GUT scale.

* There is a brute-force method: to perform the tensor products of all $SU(6)$ fermions $\bar{\mathbf{6}} \otimes \bar{\mathbf{6}} = \overline{\mathbf{15}} \oplus \overline{\mathbf{21}}$, $\bar{\mathbf{6}} \otimes \mathbf{15} = \mathbf{6} \oplus \mathbf{84}$, and $\mathbf{15} \otimes \mathbf{15} = \overline{\mathbf{15}} \oplus \overline{\mathbf{105}} \oplus \overline{\mathbf{105}}'$, and include all possible Higgs fields to form gauge-invariant Yukawa couplings.

* Physical requirements: all SM Yukawa couplings should be reproduced $\Rightarrow \mathbf{6_H}$ (for d^i and ℓ^i) and $\mathbf{15_H}$ (for u^i)

* Two $\mathbf{6_H^{I\,,II}}$ are needed to respect the $SU(2)_F$.

* A $\mathbf{21_H}$ is introduced for the sterile neutrino Yukawa couplings.

The SU(6) Higgs sector

* minimal Higgs sector: $\mathbf{6}_{\mathbf{H}}^{\alpha=\mathrm{I},\mathrm{II}}$, $\mathbf{15}_{\mathbf{H}}$, $\mathbf{21}_{\mathbf{H}}$, $\mathbf{35}_{\mathbf{H}}$

* Hierarchies of Higgs VEVs: $\langle \mathbf{35}_{\mathbf{H}} \rangle \sim \Lambda_{\mathrm{GUT}}$,
$\langle \mathbf{6}_{\mathbf{H}}^{\mathrm{II}} \rangle = v_3\,, \quad \langle \mathbf{21}_{\mathbf{H}} \rangle = v_6\,, \quad v_3 \sim v_6 \sim v_{331}$
$\langle \mathbf{6}_{\mathbf{H}}^{\mathrm{I}} \rangle = v_d = v_{\mathrm{EW}} \sin\beta\,, \quad \langle \mathbf{15}_{\mathbf{H}} \rangle = v_u = v_{\mathrm{EW}} \cos\beta$
$\Lambda_{\mathrm{GUT}} \gg v_{331} \gg v_{\mathrm{EW}} = (\sqrt{2} G_F)^{-1/2} \simeq 246\,\mathrm{GeV}$

* $\mathbf{6}_{\mathbf{H}}^{\mathrm{II}}$ and $\mathbf{21}_{\mathbf{H}}$ are responsible for the $\mathcal{G}_{331} \to \mathcal{G}_{\mathrm{SM}}$ breaking.

* Two Higgs doublets from $\mathbf{6}_{\mathbf{H}}^{\mathrm{I}}$ and $\mathbf{15}_{\mathbf{H}}$ are responsible for the EWSB.

The SU(6) fermions

SU(6)	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\bar{\mathbf{6}}_{\mathbf{F}}^{\rm I}$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_{\mathbf{F}}^{\rm I}$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3})_{\mathbf{F}}^{\rm I} \; : \; d_R^c$
	$(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{F}}^{\rm I}$	$(\mathbf{1}, \mathbf{2}, -1)_{\mathbf{F}}^{\rm I} \; : \; (e_L, -\nu_L)$
		$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\rm I} \; : \; N$
$\bar{\mathbf{6}}_{\mathbf{F}}^{\rm II}$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_{\mathbf{F}}^{\rm II}$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3})_{\mathbf{F}}^{\rm II} \; : \; D_R^c$
	$(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{F}}^{\rm II}$	$(\mathbf{1}, \mathbf{2}, -1)_{\mathbf{F}}^{\rm II} \; : \; (e'_L, -\nu'_L)$
		$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\rm II} \; : \; N'$
$\mathbf{15}_{\mathbf{F}}$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_{\mathbf{F}}$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{4}{3})_{\mathbf{F}} \; : \; u_R^c$
	$(\mathbf{1}, \bar{\mathbf{3}}, +\frac{2}{3})_{\mathbf{F}}$	$(\mathbf{1}, \mathbf{2}, +1)_{\mathbf{F}} \; : \; (\nu'^c_R, e'^c_R)$
		$(\mathbf{1}, \mathbf{1}, +2)_{\mathbf{F}} \; : \; e_R^c$
	$(\mathbf{3}, \mathbf{3}, 0)_{\mathbf{F}}$	$(\mathbf{3}, \mathbf{2}, +\frac{1}{3})_{\mathbf{F}} \; : \; (u_L, d_L)$
		$(\mathbf{3}, \mathbf{1}, -\frac{2}{3})_{\mathbf{F}} \; : \; D_L$

The SU(6) Yukawa

- The most general Yukawa coupling:
$\mathbf{15_F}\bar{\mathbf{6}}^{\alpha}_{\mathbf{F}}\mathbf{6}^{\alpha\,*}_{\mathbf{H}} + \mathbf{15_F 15_F 15_H} + \bar{\mathbf{6}}^{\alpha}_{\mathbf{F}}(i\sigma_2)_{\alpha\beta}\bar{\mathbf{6}}^{\beta}_{\mathbf{F}}\mathbf{21_H} + H.c.$

- The PQ charge and a discrete $\mathbb{Z}_4$ symmetry:

	$\bar{\mathbf{6}}^{\alpha}_{\mathbf{F}}$	$\mathbf{15_F}$	$\mathbf{6}^{\alpha}_{\mathbf{H}}$	$\mathbf{15_H}$	$\mathbf{21_H}$	$\mathbf{35_H}$
$\mathrm{U}(1)_{\mathrm{PQ}}$	1	1	2	-2	-2	0
$\mathrm{SU}(2)_{\mathrm{F}}$	$\square$	1	$\square$	1	1	1
$\mathbb{Z}_4$	1	$\frac{1}{2}$	$\frac{3}{2}$	-1	-2	0

At the UV the global $\mathrm{U}(1)_{\mathrm{PQ}}[SU(6)]^2$ anomaly: $N_{\mathrm{SU}(6)} = 9$

The SU(6) Yukawa

- At the $\mathcal{G}_{331} \to \mathcal{G}_{\rm SM}$ breaking:
  $\mathbf{15_F \bar{6}_F^{II} 6_H^{II\,*}} + H.c. \supset$
  $(\mathbf{3}, \mathbf{3}, 0)_{\mathbf{F}} \otimes (\mathbf{\bar{3}}, \mathbf{1}, +\frac{1}{3})_{\mathbf{F}}^{\rm II} \otimes (\mathbf{1}, \mathbf{\bar{3}}, -\frac{1}{3})_{\mathbf{H}}^{\rm II} + H.c.$
  $(\mathbf{1}, \mathbf{\bar{3}}, +\frac{2}{3})_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{\bar{3}}, -\frac{1}{3})_{\mathbf{F}}^{\rm II} \otimes (\mathbf{1}, \mathbf{\bar{3}}, -\frac{1}{3})_{\mathbf{H}}^{\rm II} + H.c.$
  $\Rightarrow m_D \sim m_{e'} \sim m_{\nu'} \simeq \mathcal{O}(v_{331})$

  D-hadron lifetime: $\tau_D \sim m_D^{-1} \sim \mathcal{O}(10^{-36}) - \mathcal{O}(10^{-34})$ sec, Vs. the BBN constraint of $\tau_Q \lesssim 10^{-2}$ sec.

- $\mathbf{\bar{6}_F^{[I} \bar{6}_F^{II]} 21_H} + H.c. \supset$
  $(\mathbf{1}, \mathbf{\bar{3}}, -\frac{1}{3})_{\mathbf{F}}^{[\rm I} \otimes (\mathbf{1}, \mathbf{\bar{3}}, -\frac{1}{3})_{\mathbf{F}}^{\rm II]} \otimes (\mathbf{1}, \mathbf{6}, +\frac{2}{3})_{\mathbf{H}} + H.c.$
  $\Rightarrow m_{N,N'} \simeq \mathcal{O}(v_{331})$

The SU(6) Yukawa

- $\mathbf{15_F}\bar{\mathbf{6}}^{\rm I}_{\mathbf{F}}\mathbf{6}^{\rm I\,*}_{\mathbf{H}} + H.c. \supset$

  $(\mathbf{3}, \mathbf{2}, +\frac{1}{3})_{\mathbf{F}} \otimes (\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3})^{\rm I}_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{2}, -1)^{\rm I}_{\mathbf{H}} + H.c.$

  $(\mathbf{1}, \mathbf{2}, -1)_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{1}, +2)^{\rm I}_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{2}, -1)^{\rm I}_{\mathbf{H}} + H.c.$

  $\Rightarrow m_{d,\ell} \simeq \mathcal{O}(v_{\rm EW})$

  $\mathbf{15_F}\mathbf{15_F}\mathbf{15_H} + H.c. \supset$

  $(\mathbf{3}, \mathbf{2}, +\frac{1}{3})_{\mathbf{F}} \otimes (\bar{\mathbf{3}}, \mathbf{1}, -\frac{4}{3})^{\rm I}_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{2}, +1)_{\mathbf{H}} + H.c.$

  $\Rightarrow m_u \simeq \mathcal{O}(v_{\rm EW})$

  The EW EFT = type-II 2HDM

The SU(6) Axion

The SU(6) Axion

* The physical axion field comes from: $\mathbf{6}_{\mathbf{H}}^{\mathrm{II}} \supset (\mathbf{1}\,, \mathbf{3}\,, +\frac{1}{3})_{\mathbf{H}}^{\mathrm{II}} \supset \frac{v_3}{\sqrt{2}} \exp(ia_3/v_3)$

  and $\mathbf{21}_{\mathbf{H}} \supset (\mathbf{1}\,, \mathbf{6}\,, +\frac{2}{3})_{\mathbf{H}} \supset \frac{v_6}{\sqrt{2}} \exp(ia_6/v_6)$

* To impose an orthogonality condition between the $\mathrm{U}(1)_{\mathrm{PQ}}$ $J^\mu_{\mathrm{PQ}} = q_3 v_3 (\partial^\mu a_3) + q_6 v_6 (\partial^\mu a_6)$ and the $\mathrm{U}(1)_N$ $J^\mu_N = \frac{1}{3} v_3 (\partial^\mu a_3) + \frac{2}{3} v_6 (\partial^\mu a_6)$ currents. Physical charge: $q \equiv c_1\, \mathrm{PQ} + c_2\, N$.

* 't Hooft global anomaly matching: $N_{\mathrm{SU}(3)_c} = N_{\mathrm{SU}(6)} \Rightarrow c_1 = 1$.

* $a_{\mathrm{phys}} = \cos\phi\, a_3 + \sin\phi\, a_6, \tan\phi = \frac{v_3}{2v_6}$.

* Axion decay const: $9 v_{331}^{-2} = \frac{1}{4} v_6^{-2} + v_3^{-2}$ and $f_a = v_{331}/18$.

The PQ quality

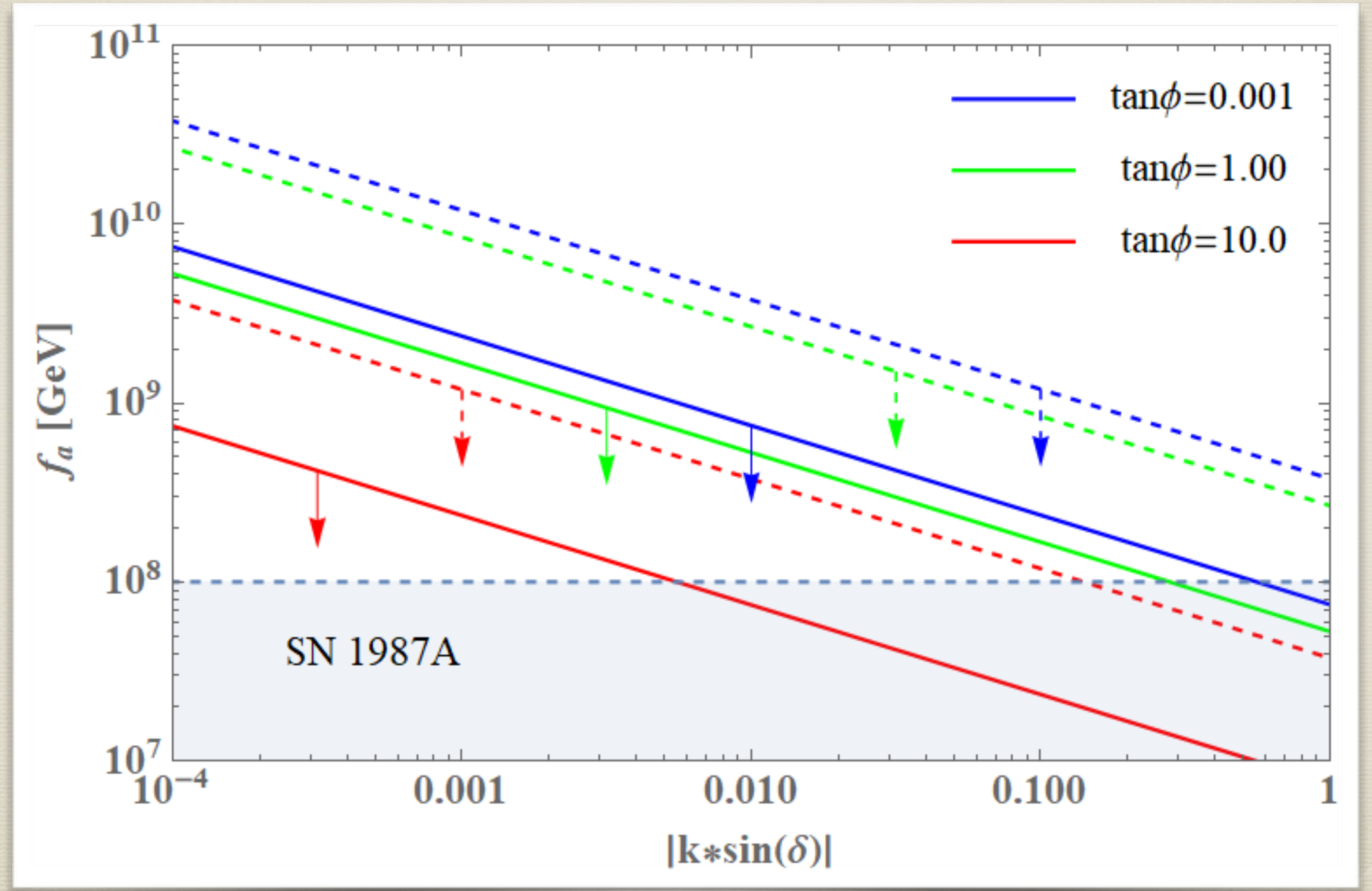
* The leading PQ-breaking operator respecting the $\mathrm{SU}(2)_F$ and $\mathbb{Z}_4$:
$$\mathcal{O}^{d=6}_{\not{PQ}} = \left[\epsilon_{\alpha\beta}\epsilon_{abc}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{a\,,\alpha}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{b\,,\beta}(\mathbf{1}\,,\mathbf{3}\,,-\frac{2}{3})^{c}\right]^2$$
if no $\mathbb{Z}_4$: $\mathcal{O}^{d=3}_{\not{PQ}} = \epsilon_{\alpha\beta}\epsilon_{abc}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{a\,,\alpha}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{b\,,\beta}(\mathbf{1}\,,\mathbf{3}\,,-\frac{2}{3})^{c}$ is dangerous in PQ-quality.

* Axion effective potential:
$$V = -\Lambda^4_{\mathrm{QCD}}\cos(\frac{a_{\mathrm{phys}}}{f_a}) + \frac{|k|(v_u v_d v_3)^2}{4M^2_{\mathrm{pl}}}\cos\left(\frac{a_{\mathrm{phys}}}{6f_a}+\delta\right)$$

* $|\bar{\theta}| \equiv \left|\frac{\langle a_{\mathrm{phys}}\rangle}{f_a}\right| \lesssim 10^{-10} \Rightarrow f_a \lesssim \frac{3.7\times 10^7\,\cos\phi}{|k\sin(\delta)|^{1/2}}(\tan\beta + \frac{1}{\tan\beta})\,\mathrm{GeV}$

This is solely determined by symmetry considerations in a GUT!

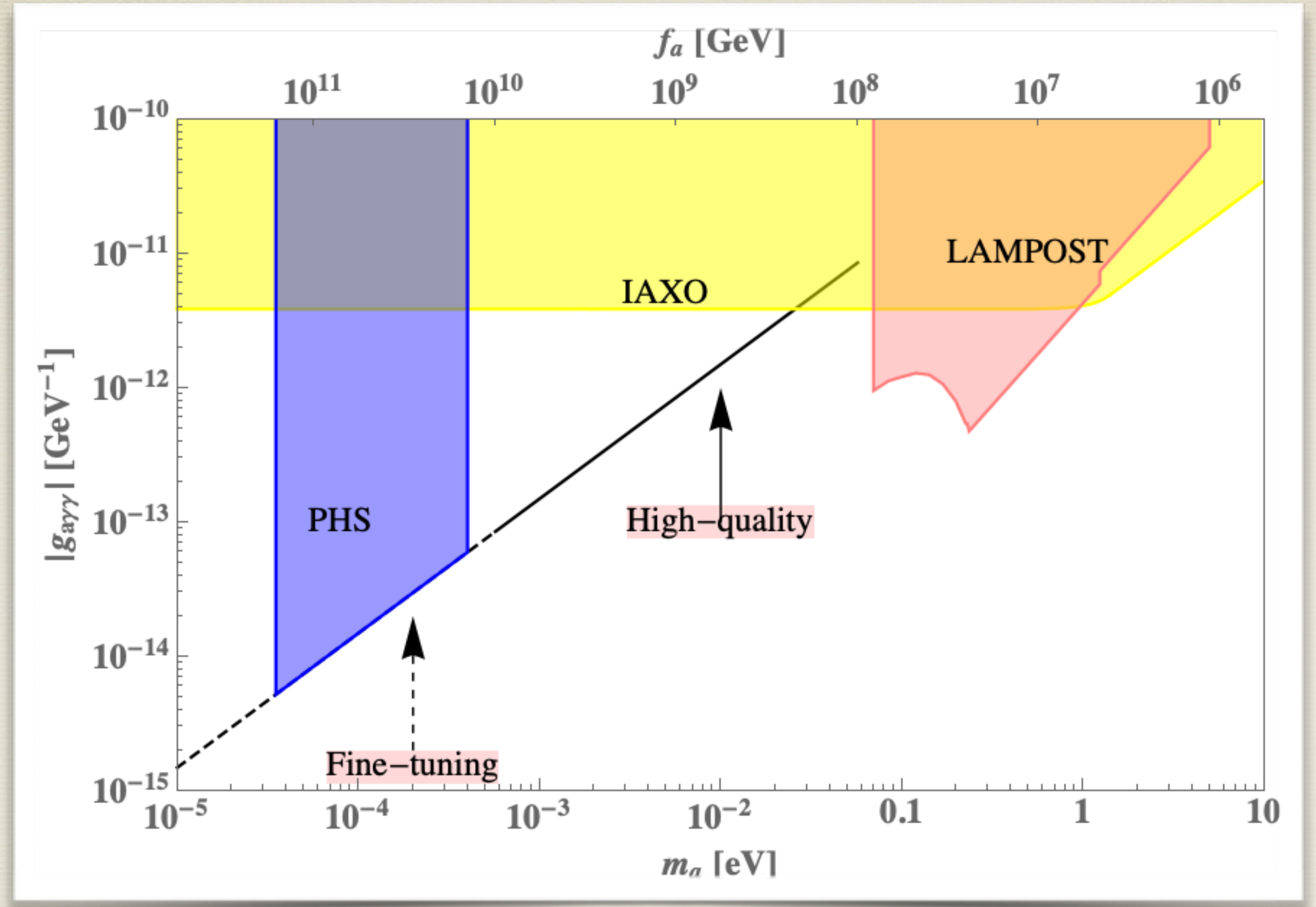
The PQ quality



$10^8\,\text{GeV} \lesssim f_a \lesssim 10^{10}\,\text{GeV}$

$$m_a = 5.70\left(\frac{10^{12}\,\text{GeV}}{f_a}\right)\mu\text{eV} \sim (10^{-4}, 10^{-2})\text{eV}$$

The axion searches



$$g_{a\gamma\gamma} = (\frac{E}{N_{SU(3)_c}} - 1.92)\left(\frac{1.14\times 10^{-3}\,\mathrm{GeV}}{f_a}\right)\mathrm{GeV}^{-1}$$

$$\mathrm{U(1)_{PQ}[U(1)_{em}]^2}\ \text{ anomaly factor : }\ E = \sum_f \mathrm{PQ}_f \dim(\mathscr{C}_f)\mathrm{Tr}q_f^2 = 24$$

The Axion domain walls

* Back to the axion effective potential:
$$V = -\Lambda_{\text{QCD}}^4 \cos\left(\frac{a_{\text{phys}}}{f_a}\right) + \frac{|k|(v_u v_d v_3)^2}{4M_{\text{pl}}^2} \cos\left(\frac{a_{\text{phys}}}{6f_a} + \delta\right)$$

* The $\cos\left(\frac{a_{\text{phys}}}{f_a}\right)$ term is periodic and has degenerate minima, this leads to the DWs.

* DWs are problematic in cosmology, with the energy density $\rho_{\text{DW}} \sim \sigma/t$. The energy densities for radiation/matter: $\rho_{\text{rad}} \propto t^{-2}$, $\rho_{\text{matt}} \propto t^{-3/2}$. DWs can overtake the Universe once they are formed.

The Axion domain walls

* The second PQ-breaking term acts as the biased term to collapse the DWs.[Vilenkin ('81), Gelmini, Gleiser, Kolb, ('89), Larsson, Sarkar, White ('96)]

* To have DWs collapse before formation: $t_{\rm dec} < t_{\rm form}$.

* In our case:

$$t_{\rm form} \sim 10^2\,{\rm sec}\left(\frac{10^{13}\,{\rm GeV}}{v_{331}}\right) \sim \mathcal{O}(10^4) - \mathcal{O}(10^6)\,{\rm sec}$$

$$t_{\rm dec} \approx \frac{\sigma_{\rm DW}}{v_{331}^4} \sim 10^{-66}\,{\rm sec}\left(\frac{M_{\rm pl} v_{331}}{v_u v_d}\right)^2 \left(\frac{10^{13}\,{\rm GeV}}{v_{331}}\right)^3$$

$$\sim \mathcal{O}(10^{-8}) - \mathcal{O}(10^{-6})\,{\rm sec}$$

The SU(6) Unification

The SU(6) unification

* The gauge couplings: $(\alpha_{3c}, \alpha_{3L}, \alpha_N)$ for the $\mathcal{G}_{331}$, and $(\alpha_{3c}, \alpha_{2L}, \alpha_Y)$ for the $\mathcal{G}_{\rm SM}$. Use $\alpha_1 = \frac{4}{3}\alpha_N$ for the $\mathcal{G}_{331}$ embedding into the $\mathrm{SU}(6)$.

* The RGEs of the $\mathrm{SU}(6)$:

$$\alpha_i^{-1}(\mu_2) = \alpha_i^{-1}(\mu_1) - \frac{b_i^{(1)}}{2\pi}\log\left(\frac{\mu_2}{\mu_1}\right) + \delta_i$$

  δ_i to account for higher-order effects: two-loop && mass threshold.

* The matching conditions: $\alpha_{3L}^{-1}(v_{331}) = \alpha_{2L}^{-1}(v_{331})$, $\alpha_1^{-1}(v_{331}) = -\frac{1}{4}\alpha_{2L}^{-1}(v_{331}) + \frac{3}{4}\alpha_Y^{-1}(v_{331})$.

The SU(6) unification

* non-SUSY: $b_i^{(1)} = -\frac{11}{3} C_2(\mathcal{G}_i) + \frac{2}{3} \sum_f T(\mathcal{R}_f^i) + \frac{1}{3} \sum_s T(\mathcal{R}_s^i)$

  SUSY: $b_i^{(1)} = -3 C_2(\mathcal{G}_i) + \sum_\chi T(\mathcal{R}_\chi^i)$

* SUSY extension: $\mathbf{21_H}$ super-multiplet is anomalous, we include a $\overline{\mathbf{21}}_\mathbf{H}$.

* The $\mathrm{SU}(6)$ SUSY extension can avoid the μ-problem, since $\mathbf{6_H^{I\,*}} \mathbf{15_H}$ is not gauge-invariant.

* non-SUSY: $m_Z \leq \mu \leq v_{331} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(2)_L}, b^{(1)}_{\mathrm{U}(1)_Y}) = (-7\,, -3\,, 7)$

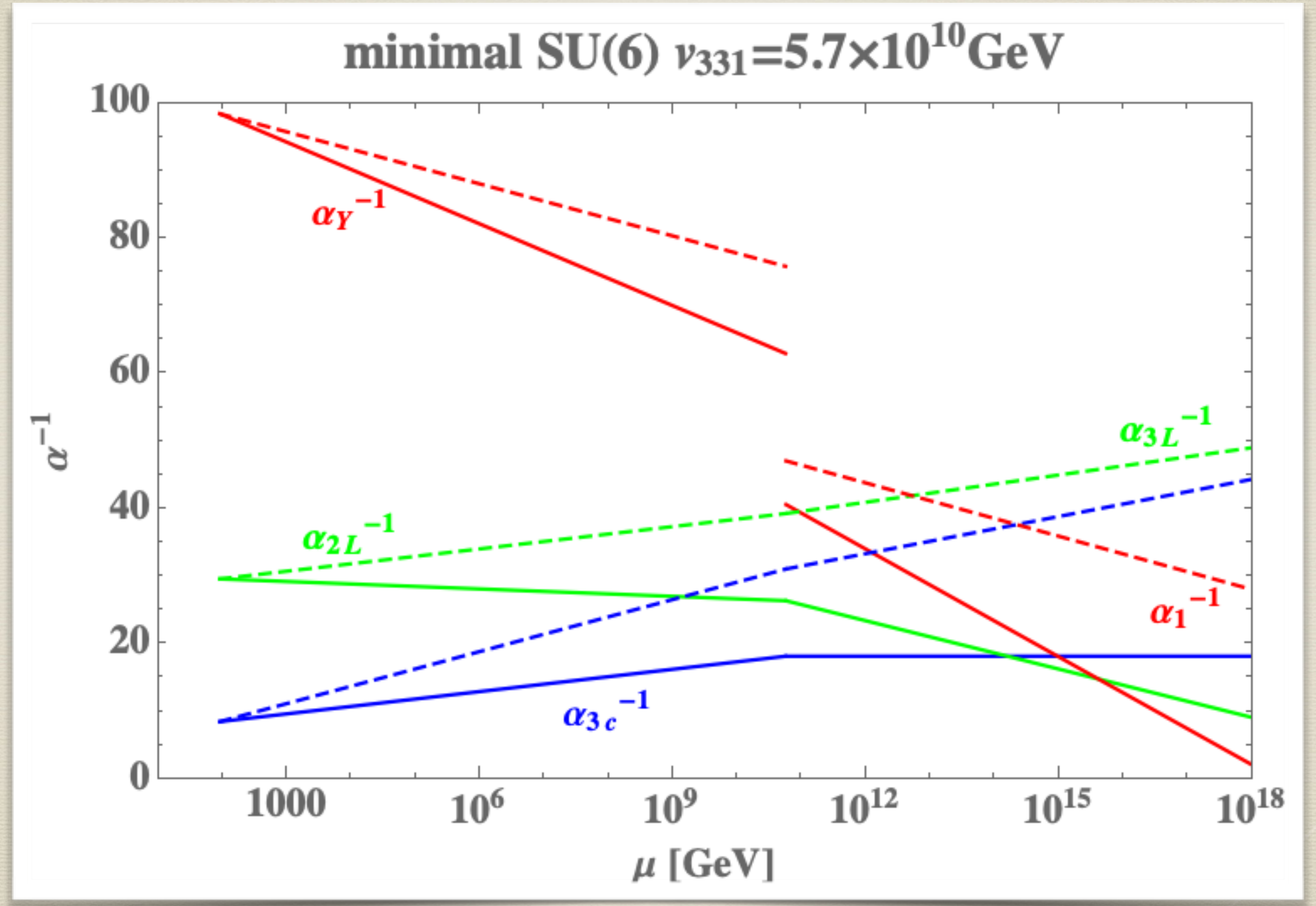
  $v_{331} \leq \mu \leq \Lambda_{\mathrm{GUT}} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(3)_L}, b^{(1)}_{\mathrm{U}(1)_1}) = (-5\,, -\frac{11}{3}\,, \frac{43}{6})$

* SUSY:

  $m_Z \leq \mu \leq v_{331} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(2)_L}, b^{(1)}_{\mathrm{U}(1)_Y}) = (-3\,, 1\,, 11)$

  $v_{331} \leq \mu \leq \Lambda_{\mathrm{GUT}} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(3)_L}, b^{(1)}_{\mathrm{U}(1)_1}) = (0\,, \frac{13}{2}\,, \frac{29}{2})$

The SU(6) unification



$\alpha_{3c}(m_Z)\,,\alpha_{\rm em}(m_Z)\,,\sin^2\theta_W(m_Z)$ as inputs

The SU(6) unification

* To impose the unification condition at the UV:
$\alpha_{3c}^{-1}(\Lambda_{\rm GUT}) = \alpha_{3L}^{-1}(\Lambda_{\rm GUT}) = \alpha_{1}^{-1}(\Lambda_{\rm GUT}) = \alpha_{\rm GUT}^{-1}(\Lambda_{\rm GUT})$

* Benchmark $v_{331} = 5.7 \times 10^{10}\,{\rm GeV}$, we find:
$\Lambda_{\rm GUT} \approx 7.8 \times 10^{15}\,{\rm GeV}\,, \quad \alpha_{\rm GUT}^{-1}(\Lambda_{\rm GUT}) = 18.14$
$\sin^2\theta_W(m_Z) = 0.22923$
PDG: $\sin^2\theta_W(m_Z) = 0.23117$

* Proton lifetime:
$$\tau[p \to e^+\pi^0] \sim 10^{36}\,{\rm yrs}\left(\frac{\alpha_{\rm GUT}^{-1}}{35}\right)^2\left(\frac{\Lambda_{\rm GUT}}{10^{16}\,{\rm GeV}}\right)^4$$
$\approx 9.8 \times 10^{34}\,{\rm yrs}$
Super-Kamionkande: $\tau_p \gtrsim 2.4 \times 10^{34}\,{\rm yrs}$

Summary

* We show a *non-minimal* $\mathrm{SU}(6)$ GUT model with the minimal setup to achieve a high-quality axion by identifying the $\mathrm{U}(1)_{\mathrm{PQ}}$ as the Abelian component of the emergent global symmetries.

* The axion decay constant: $10^8\,\mathrm{GeV} \lesssim f_a \lesssim 10^{10}\,\mathrm{GeV}$ w.o. much fine-tuning of the EFT parameter.

* The GUT spectrum contains vector-like KSVZ D-quarks, and heavy leptons && singlet neutrinos, type-I seesaw.

* Safe from the cosmological constraints.

Outlook

* Why $\mathrm{SU}(6)$? Beyond the $\mathrm{SU}(6)$? $\mathrm{SU}(7)$? No answer now.
* $\mathrm{SU}(7) \rightarrow \mathrm{SU}(4)_{\mathrm{PS}} \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$, there can be small instanton effect.
* The discrete symmetries from the SUSY?
* The RGE analysis, usually involves many details, two-loops, mass threshold effects.
* String theory? $\mathrm{SU}(6) \otimes \mathrm{SU}(2) \subset E_6 \subset E_8$

Backups

The SU(6) Yukawa

* $\bar{\mathbf{6}}_{\mathbf{F}}^{[\mathrm{I}}\bar{\mathbf{6}}_{\mathbf{F}}^{\mathrm{II}]}\mathbf{15}_{\mathbf{H}} + H.c. \supset$
  $(\mathbf{1}, \mathbf{2}, -1)_{\mathbf{F}}^{[\mathrm{I}} \otimes (\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\mathrm{II}]} \otimes (\mathbf{1}, \mathbf{2}, +1)_{\mathbf{H}} + H.c.$
  $\Rightarrow m_{\nu N'} \sim m_{\nu' N} \simeq \mathcal{O}(v_{\mathrm{EW}})$

* Type-I seesaw: $(\nu, N')\mathcal{M}(\nu, N')^T \Rightarrow m_\nu \sim \frac{v_{\mathrm{EW}}^2}{v_{331}}, m_{N'} \sim v_{331}$

The SU(6) fermions

* To obtain the spectrum: $N = \frac{1}{3}\mathrm{diag}(-\mathbb{I}_3\,, +\,\mathbb{I}_3)$ for the $\mathrm{SU}(6)$ fundamental, and $Y = \mathrm{diag}(\frac{1}{3} + 2N\,, \frac{1}{3} + 2N\,, -\frac{2}{3} + 2N)$ for the $\mathrm{SU}(3)_L$ fundamental.
* In the $\mathrm{SU}(5)$ GUT: $\bar{\mathbf{5}}_{\mathbf{F}} = d_R^c \oplus \ell_L$, $\mathbf{10}_{\mathbf{F}} = q_L \oplus u_R^c \oplus e_R^c$
* In the $\mathrm{SU}(6)$ GUT: $\bar{\mathbf{6}}_{\mathbf{F}}^{\mathrm{I}} \supset d_R^c \oplus \ell_L$, $\mathbf{15}_{\mathbf{F}} \supset q_L \oplus u_R^c \oplus e_R^c$
* Additional $\mathrm{SU}(6)$ fermions are vectorial: $(D_L\,, D_R^c)$, $(N\,, N')$, $(\ell'_L\,, \ell'^c_R)$