连续场论方法计算强子结构函数的现状及挑战

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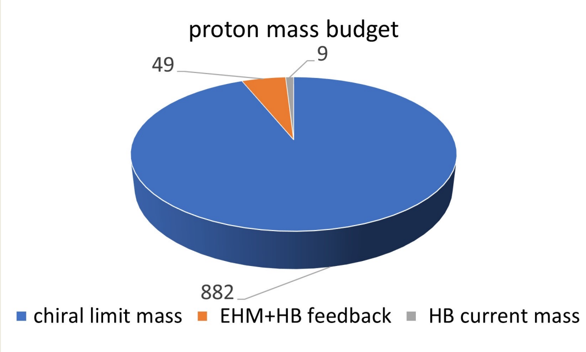
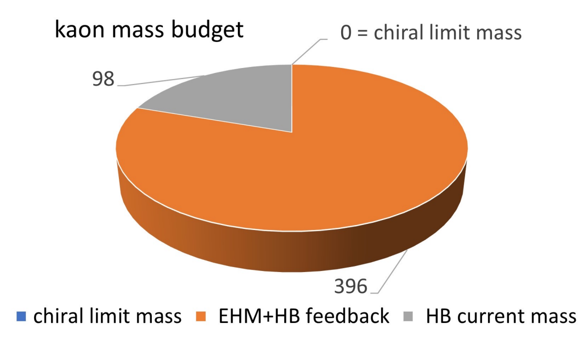
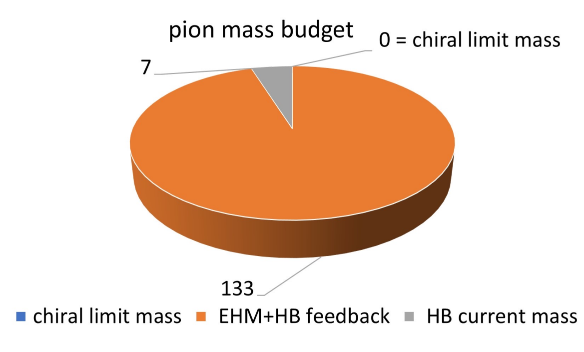
粒子物理与原子核物理学科学学术报告
2022/04/22，中科大

Mass budget







- HB current mass: Higgs-boson effects
- chiral limit mass: absence of Higgs coupling
- EHM+HB feed back: interference between emergent hadronic mass and HB current mass

Absence of Higgs(in the chiral limit)

- ✓ A very large fraction of the measured proton mass emerges as a consequence of the trace anomaly…by glue and the interactions between them;
- ✓ Pion and Kaon masses are ZERO(NG mode associated with DCSB)

Restoring Higgs boson couplings

- ✓ Sum of hadron's valence-quark current masses……..0.01 m_p

Interference………………quark condensates☺

5% for proton 80% for kaon 95% for pion

Mass puzzle

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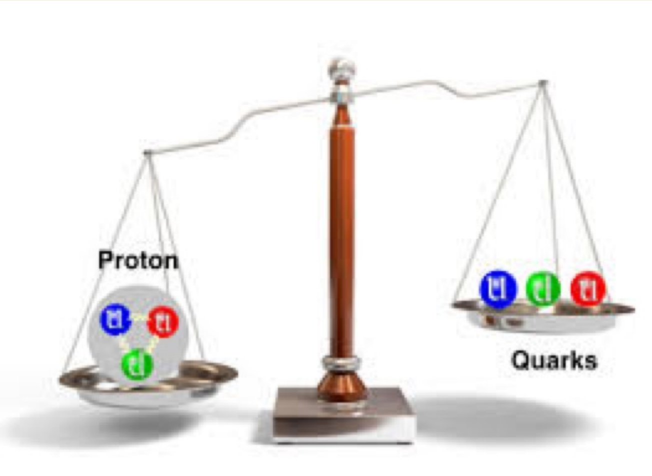


- "How does the mass of the proton arise?" will only explain one part of a great puzzle
  …simultaneously clarify Pion and Kaon
- Confinement: no gluon or quark has been seen to propagate over a length scale which exceeds the proton radius…influence of emergent mass
- In tackling these questions, opportunities are provided by studies of the properties of the SM's (pseudo-) Nambu-Goldstone modes, viz. Pions and Kaons, and Proton.

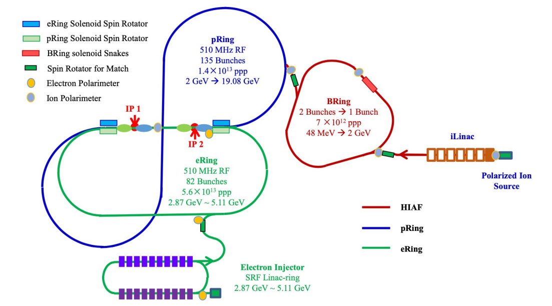


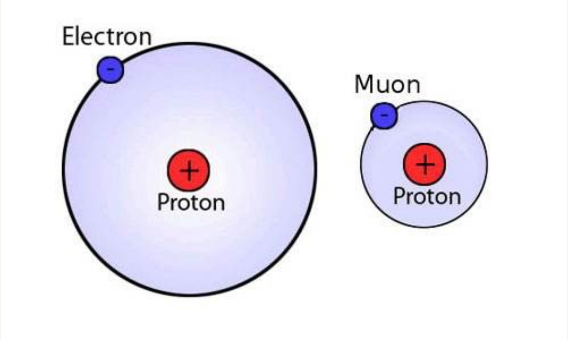
Figure 3.6: The polarization design of the EicC accelerator facility.

Pion/Kaon Proton distribution amplitudes
electromagentic form factors
structure functions

} On the same footing

Describe quark-antiquark bound-state

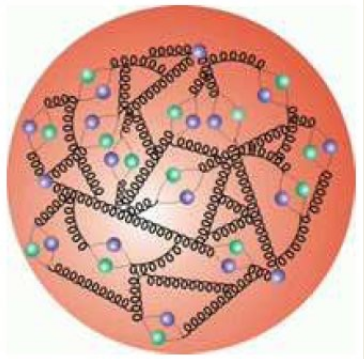
$$\Theta_0 = \beta(\alpha)\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}\,.$$



QED

Trace anomaly

- All renormalisable four-dimensional theories possess a trace anomaly;
- The size of the trace anomaly in QED must be great deal smaller than that in QCD.



QCD

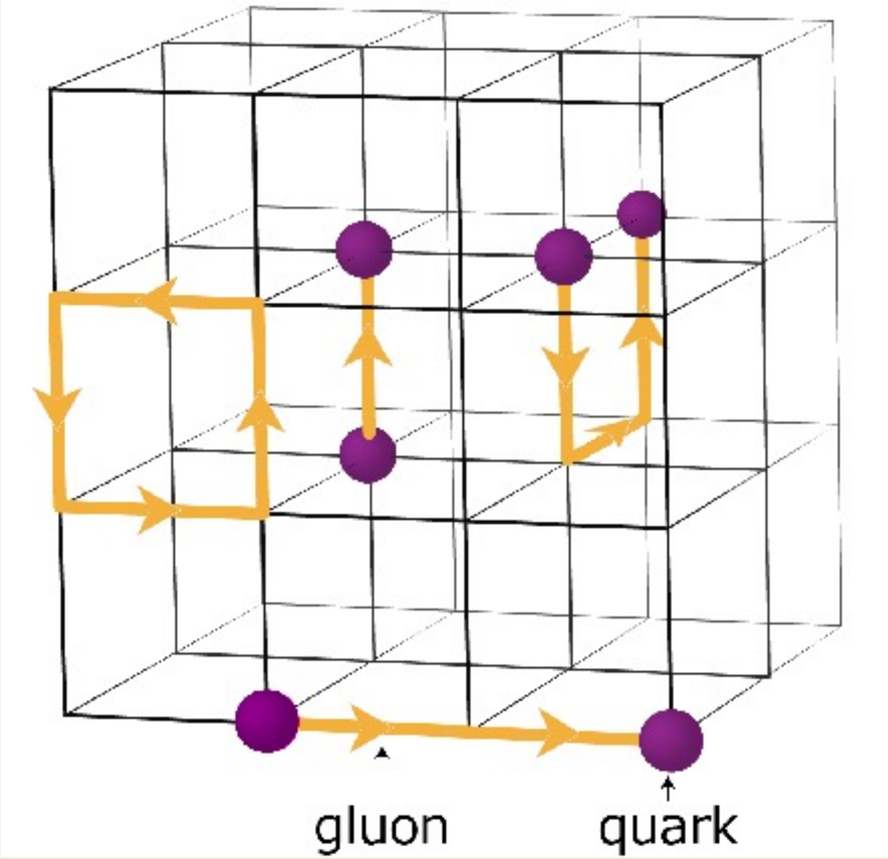
Field theory Successful:

- Nonrelativistic quantum mechanics to handle bound state;
- Perturbation theory to handle relativistic effects

Field theory not Successful yet:

- Growth of the running coupling constant in the infrared region;
- Confinement;
- Dynamical Chiral Symmetry Breaking;
- Possible nontrivial vacuum structure in hadron

Describe quark-antiquark bound-state



I. INTRODUCTION

Nowadays one sees relatively few papers on continuum non-perturbative QCD, compared to the numbers written a few years ago. Some of the reasons for this state of affairs: a general feeling that Monte Carlo simulations of lattice QCD are the best way to answer all questions, as well as an impression that no systematic non-perturbative treatment of continuum QCD is available or likely to become available.

While there is justification for this line of reasoning, it would certainly be wrong to abandon theoretical QCD in favor of the essentially experimental approach through simulations. At the same time it is hard to know what to make of models, like the bag model, which purport to mimic QCD yet have a prominent ad hoc component to them.

Cornwall, 1985

Continuum Schwinger function Method

Dyson, F. J. (1949), "The S Matrix In Quantum Electrodynamics," Phys. Rev. 75, 1736.
Schwinger, J. S. (1951), "On The Green's Functions Of Quantized Fields: 1 and 2," Proc. Nat. Acad. Sci. 37 (1951) 452; ibid 455.

Dyson-Schwinger Equations
Bethe-Salpeter Equations(Nambu)
Faddeev Equation
Ward-Takahashi identity
Scattering Problem
……

- They provide a systematic, symmetry-preserving approach to solving the bound-state problem in QCD;
- Predictions from CSM analyses are practically identical to those obtained via the lattice-regularized theory.

DSEs group

- Chang， *et al*, PLB829(2022)137078
- Cui, *et al*, EPJA 57 (2021) 5, EPJC80 (2020) 1064
- Ding, *et al*, CPC44 (2020) 031002, PRD101(2020)054014
- Binosi, *et al*, PLB790(2019)257
- Chen, *et al*, PRD98(2018) 091505
- Gao, *et al*, PRD96 (2017) 034024
- Chang, *et al*, PLB737(2014), PRL110(2013)132001,PRL111(2013)1418002

Describe quark-antiquark bound-state(incomplete. example)

A relativistic equation for Bound-State problem
Salpeter and Bethe, PR84(1951)1232

Munczek, PRD52(1995)4736

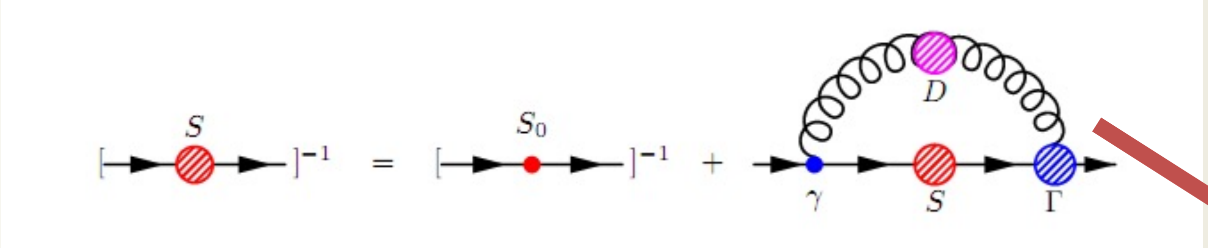
Sketching the Bethe-Salpeter kernel
Chang and Roberts, PRL103(2009)081601

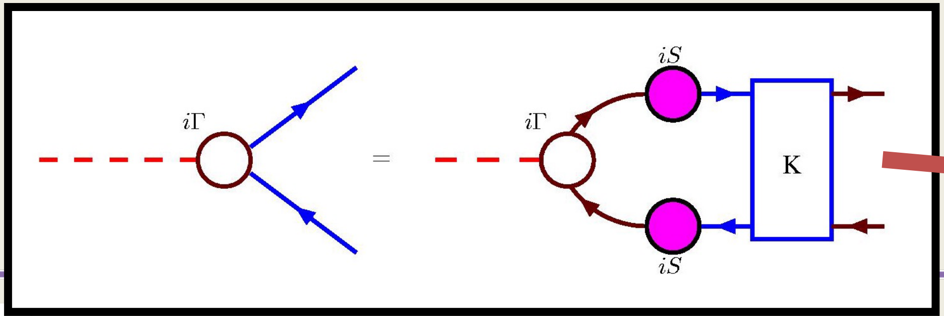
1951 — 1974/1976 — 1995 — 1998 — 2009 ……

Lane, PRD10(1974)2605
Politzer, NPB117(1976)397; Pagels…

Pion mass and decay constant
Maris, Roberts and Tandy, PLB420(1998)267

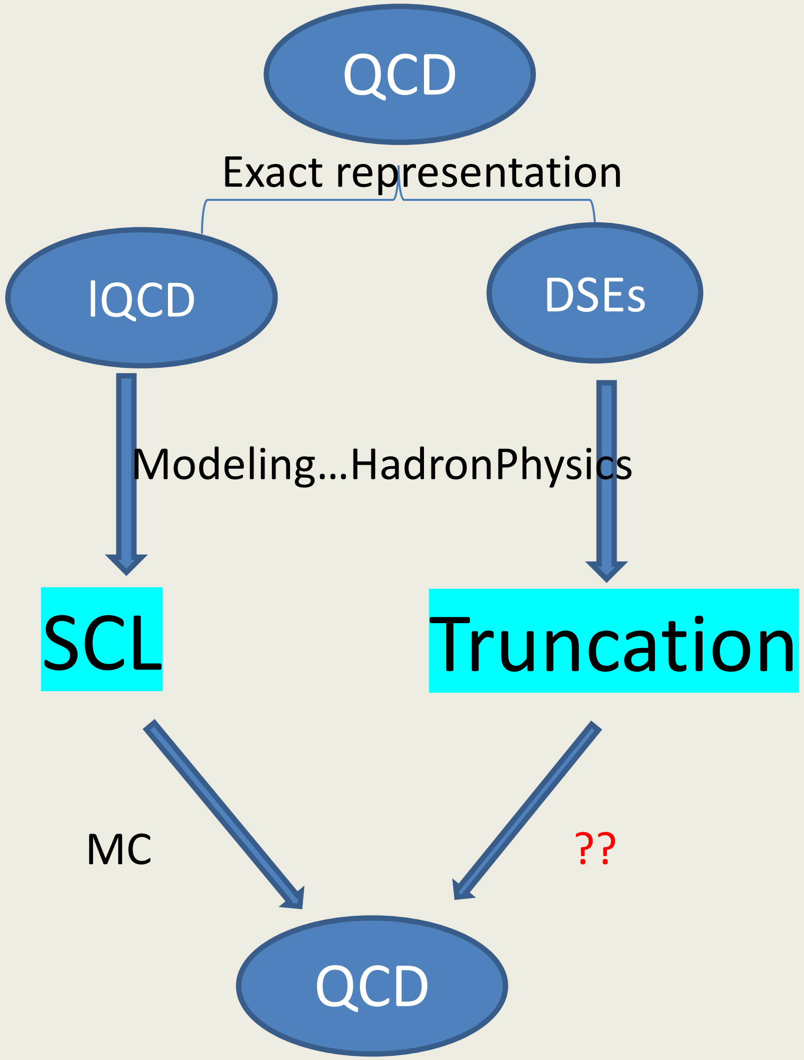
from QCD



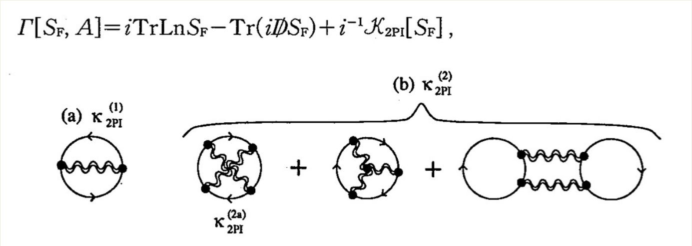


$$\mathcal{K} = \frac{\delta\Sigma}{\delta S}$$

Truncation(Put Physics at Right Place)



✓One Way: CJT approach->2PI, 3PI,…



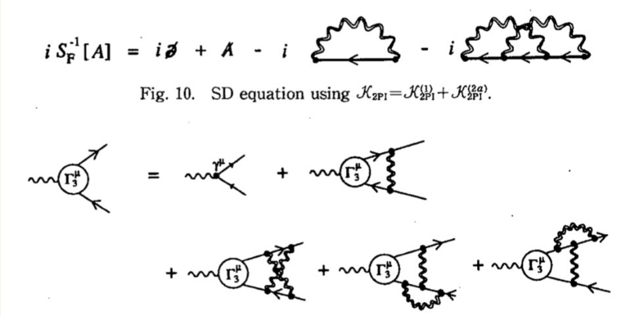


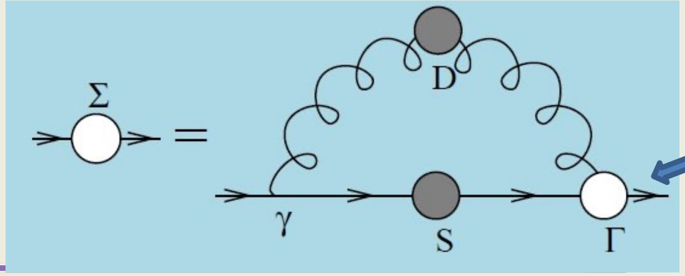
Fig. 10. SD equation using $\mathcal{K}_{2PI}=\mathcal{K}^{(1)}_{2PI}+\mathcal{K}^{(2a)}_{2PI}$.

Fig. 11. BS equation for Γ_3^μ using $\mathcal{K}_{2PI}=\mathcal{K}^{(1)}_{2PI}+\mathcal{K}^{(2a)}_{2PI}$.

✓Our Way: Minding the quark-gluon vertex

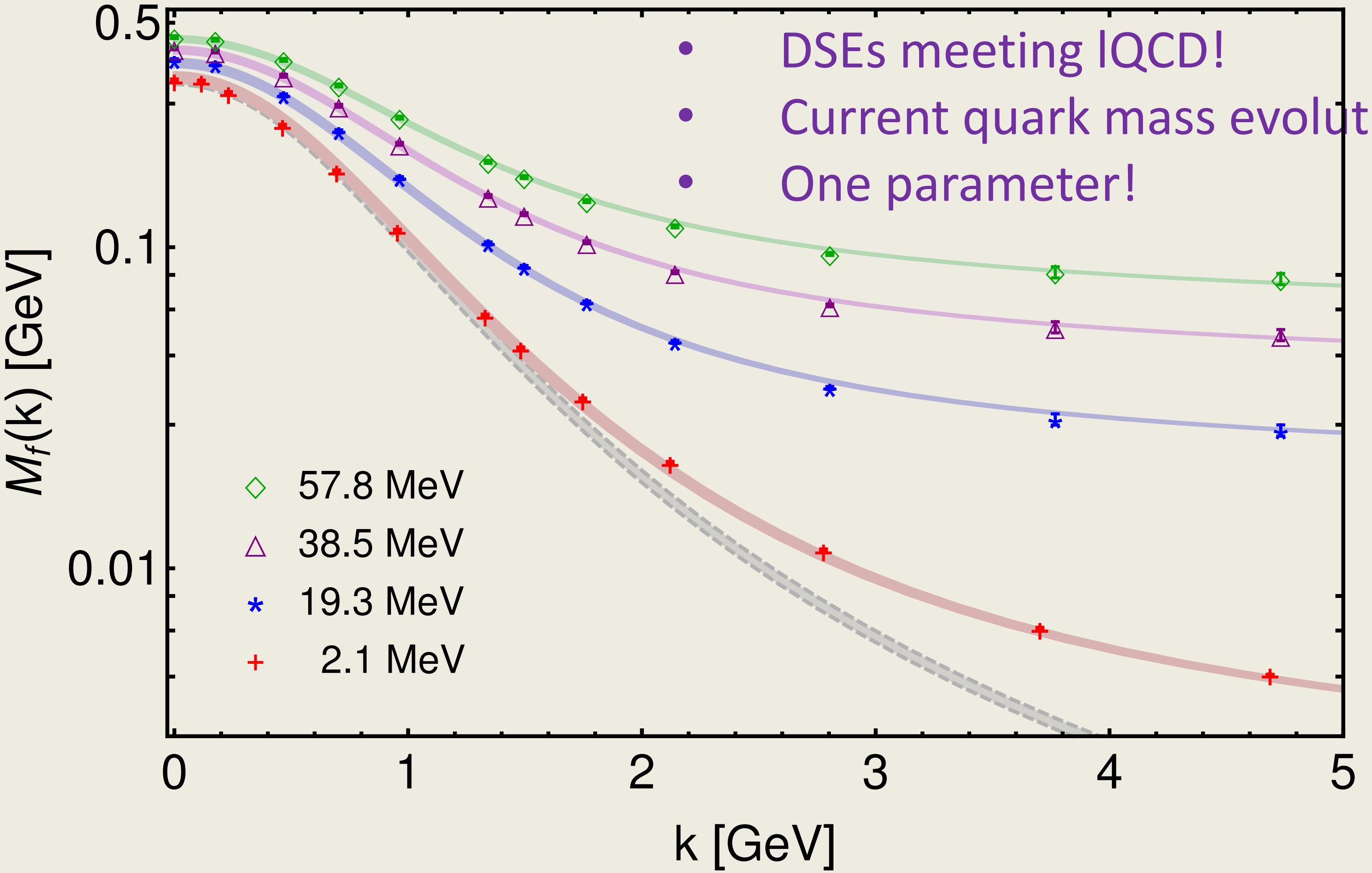
How to construct quark-gluon vertex nonperturbatively?

Symmetry!



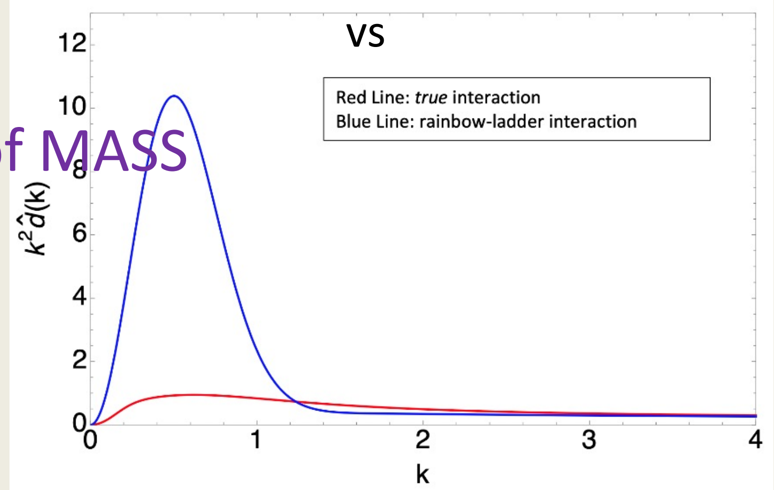
Quark Mass Generation____Dynamical Chiral Symmetry Breaking

Linking continuum and lattice quark mass functions via an effective charge,
LC, et al., PRD104(2021)094509



- DSEs meeting lQCD!
- Current quark mass evolution of MASS
- One parameter!

Rainbow-Ladder: input strong interaction by hand

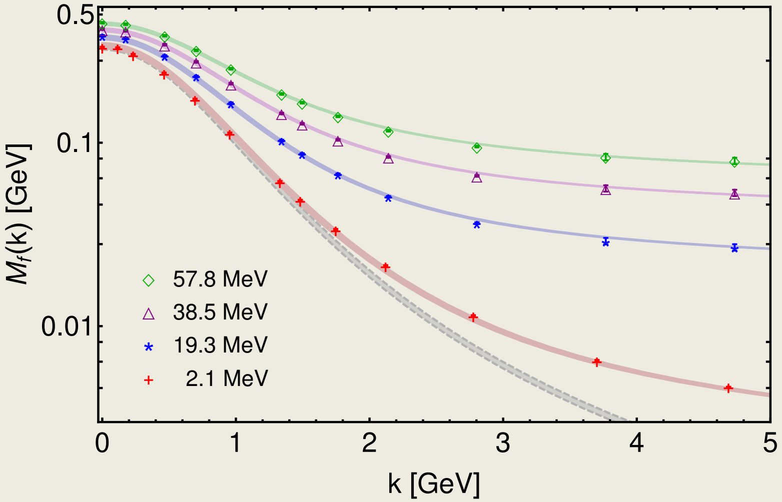


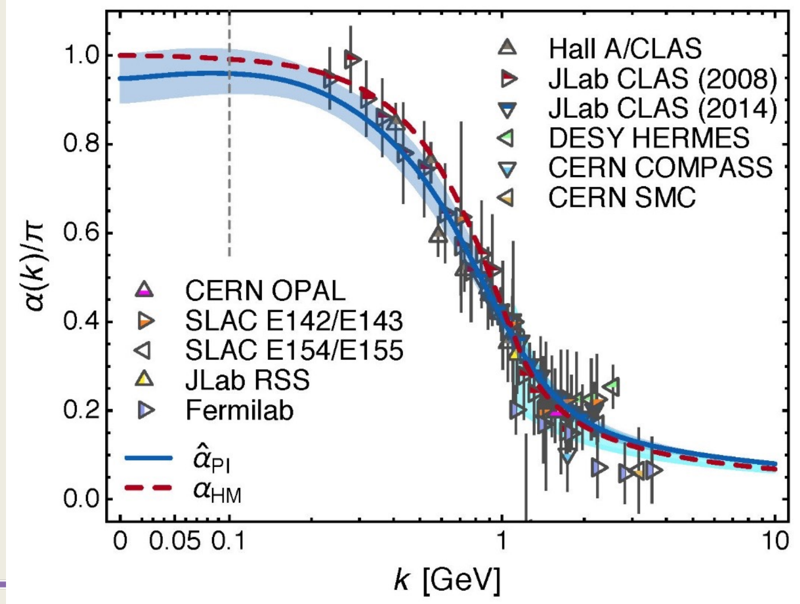
Mass scale introduced by PI coupling

Trigger quark mass

DCSB enhance quark-gluon coupling

“Constituent” quarks





- In the chiral limit, the perturbative massless quark obtain a large infrared mass through the interactions of gluon;
- M_0 is about $m_p/3$ and runs as a logarithm-corrected $1/k^2$ power-law in the ultraviolet region;
- The strong interaction of a quark with its (gluon) surrounding gives rise to a “constituent” quark with effective mass M_0;
- This consistuent quark has the finite size(B. Povh and J. Hufner, PLB245(1990)653) and finite magnetic moment;

Dressed-Quark Anomalous Magnetic Moments

Lei Chang, Yu-Xin Liu, and Craig D. Roberts

Phys. Rev. Lett. **106**, 072001 (2011) - Published 16 February 2011

PI running coupling of QCD

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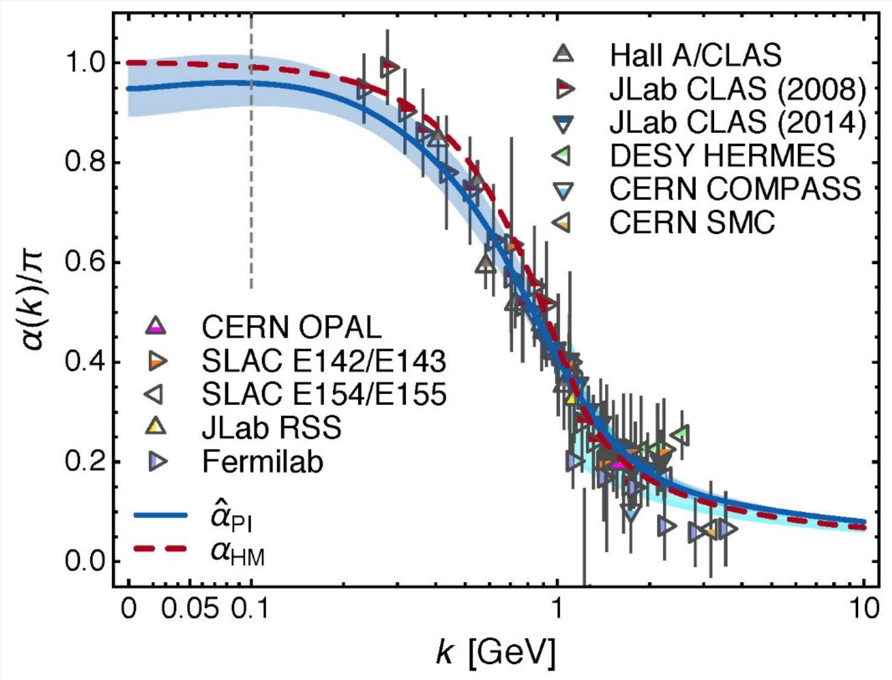


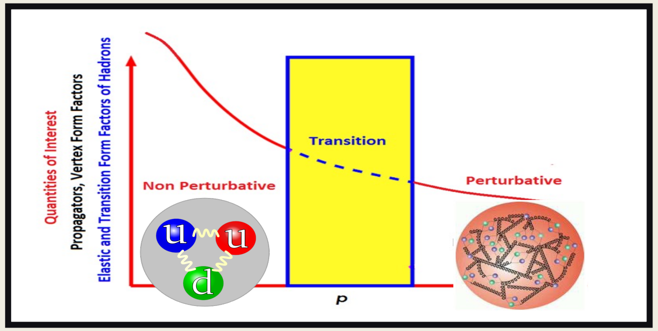
$$\hat{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{K}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]}, \quad \mathcal{K}^2(y) = \frac{a_0^2 + a_1 y + y^2}{b_0 + y}$$

Define a screening mass:

$$m_G := \mathcal{K}(k^2 = \Lambda^2_{QCD}) = 0.331\text{GeV}$$

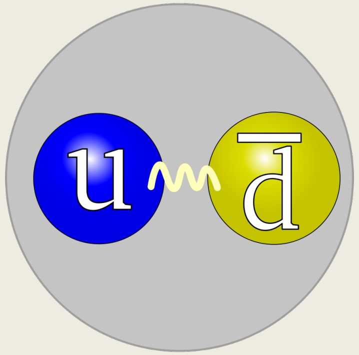
The running coupling alters at m_G so that modes with $k^2<m^2$ are screened from interactions and theory enters a practically conformal domain.

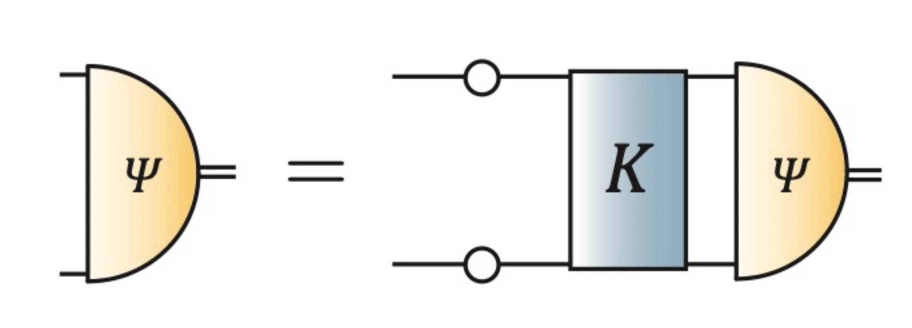
- Gluon/Quarks progressivley become more sorphisticated as experience grew with formulating and solving the quark gap equation and as computational methods and power improved for lattcie-regularised QCD.

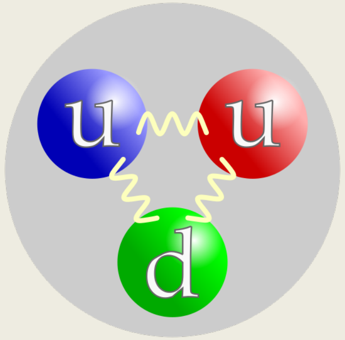


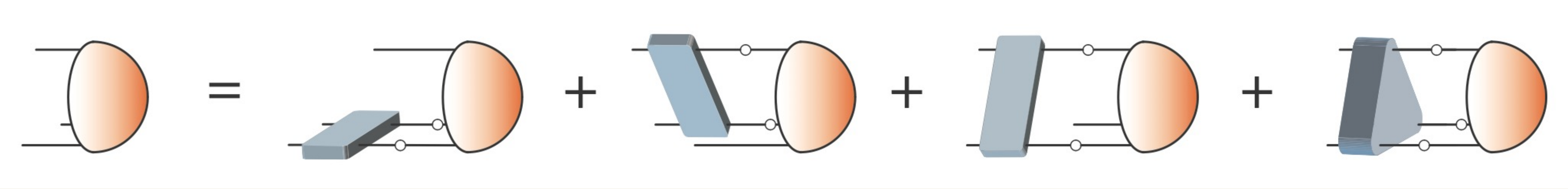
Valence Picture at Hadronic Scale!

Imagine the Hadron at the Hadronic Scale

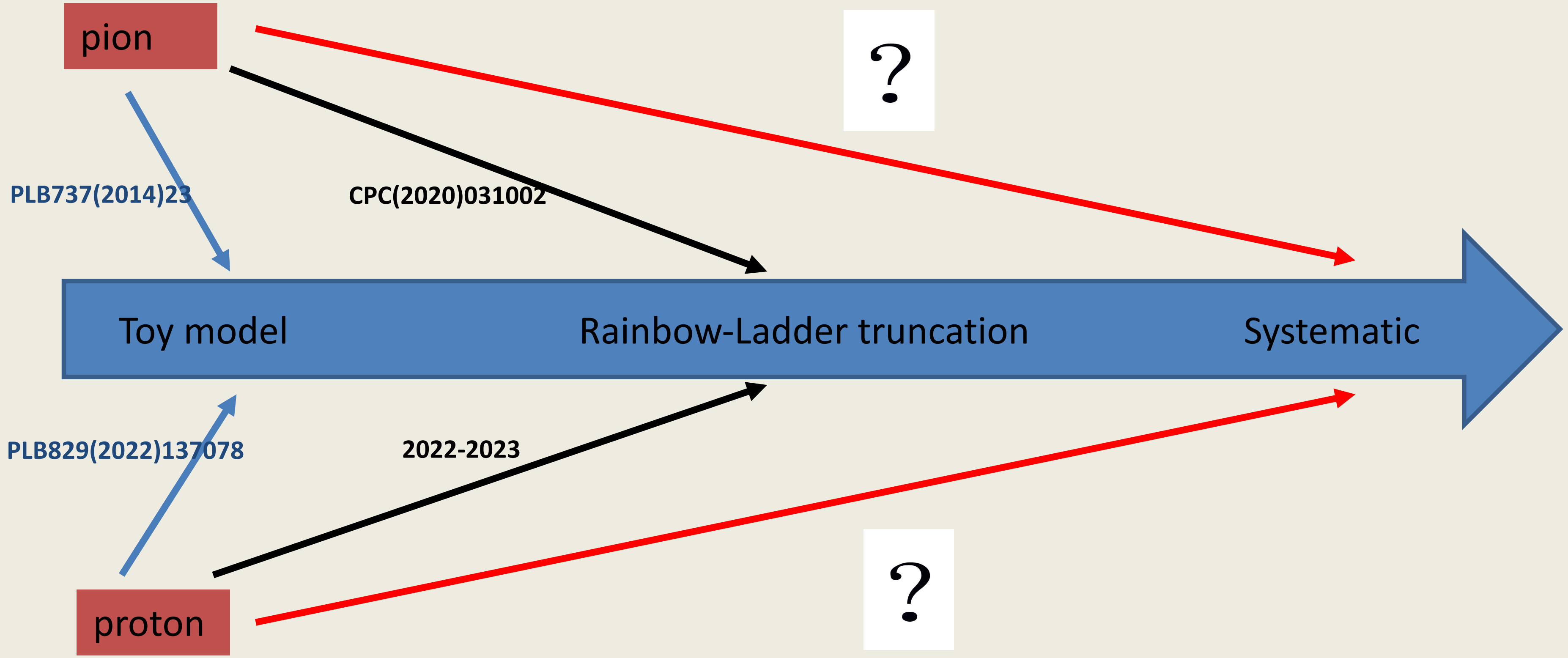






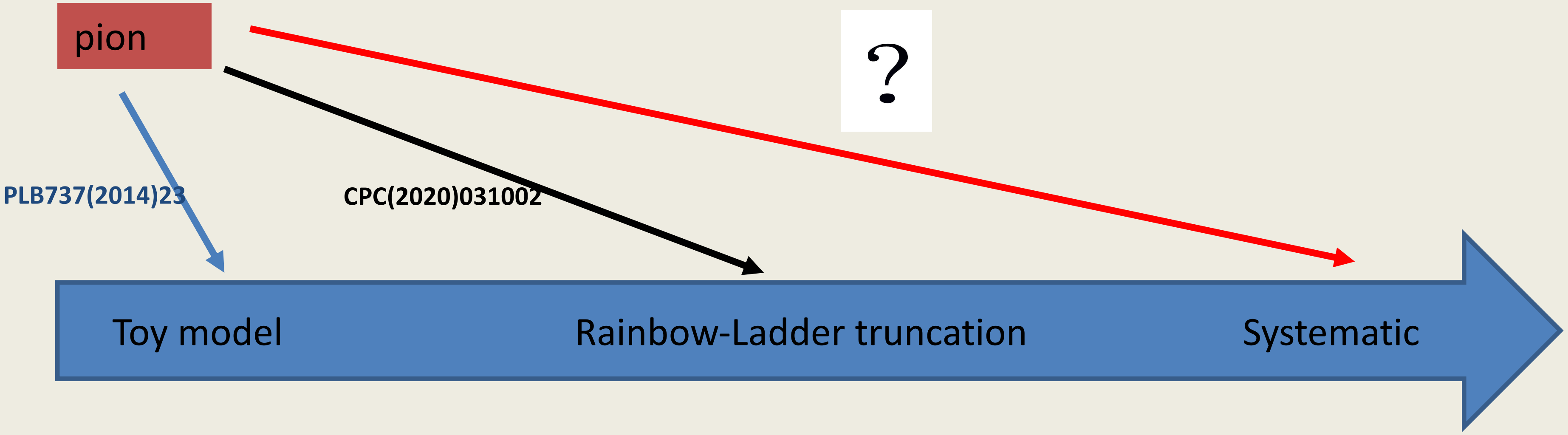


Calculate the Hadron at the Hadronic Scale

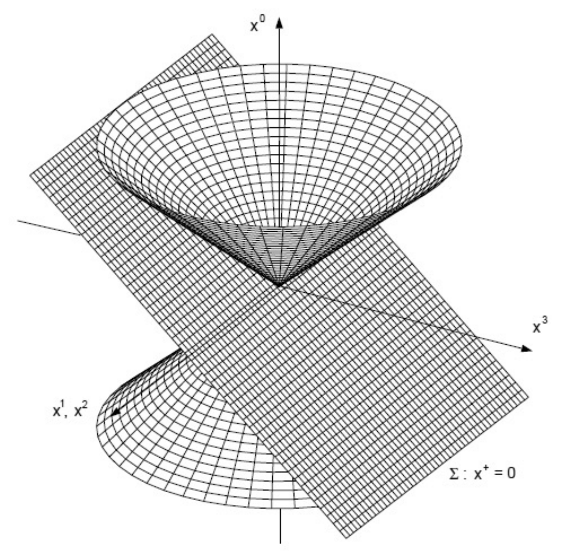


CSM developments

Calculate the Hadron at the Hadronic Scale



Story of pion



Distribution Amplitude(truncation independent)

$$f_\pi \, \varphi_\pi(x;\mu) = Z_2 \, \mathrm{tr}_{\rm CD} \int_{dk}^{\Lambda} \delta(n\cdot k - x\, n\cdot P)\, \gamma_5 \gamma\cdot n \, \chi_\pi(k;P)\,,$$

Calculate moments;
Restruct DA from moments!

$$\langle x^m \rangle := \int_0^1 dx\, x^m \varphi_\pi(x)$$

$$\langle x^m \rangle = \frac{N_c\, Z_2}{f_\pi (n\cdot P)^{m+1}} \mathrm{tr}_{\rm D} \int_{dk}^{\Lambda} (n\cdot k)^m \, \gamma_5 \gamma\cdot n \, \chi_\pi(k;P)\,.$$

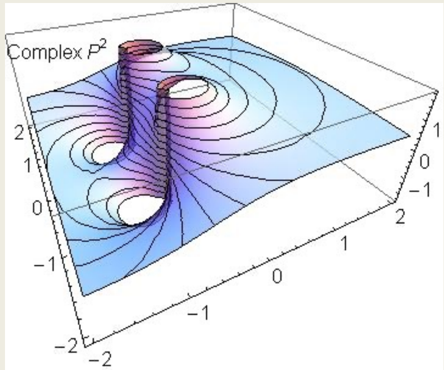
Arbitrary many moments is necessary!

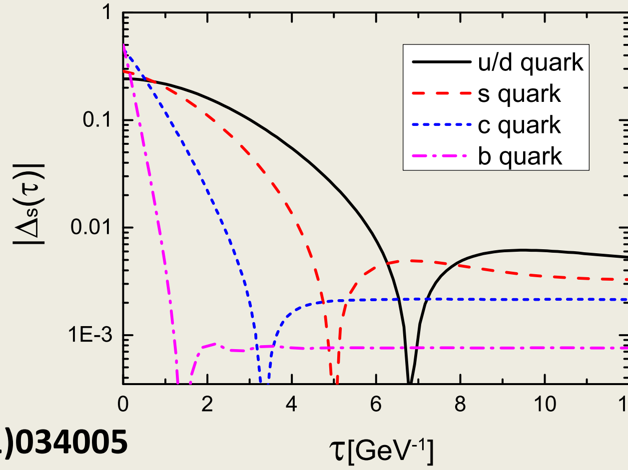
DA moments-----Method-1: Nakanishi-type representation

Imaging dynamical chiral symmetry breaking: pion wave function on the light front. LC, *et al.,* PRL110(2013)132001

✓Quark propagator

$$S(p)=\sum_{j=1}^{n_p}\left(\frac{z_j}{i\not p+m_j}+\frac{z_j^\star}{i\not p+m_j^\star}\right)$$





Zehao Zhu, *et al.,* PRD103(2021)034005

✓Bethe-Salpeter amplitude

$$\mathcal{F}_\sigma(q;P)=\int_{-1}^{1}d\alpha\int_0^\infty d\beta\sum_\gamma^{n_t}\frac{\hat\rho_\gamma(\alpha,\beta)}{(q^2+\alpha q\cdot P+\beta_0+\beta)^{n_\gamma}}$$

$$\hat\rho_\gamma(\alpha,\beta)=\rho_\gamma(\alpha)\,\delta(\beta+\beta_0-\Lambda_\gamma^2)$$

✓Standard Feynman integrals familiar from perturbation theory

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Brute force+SMP extrapolation

Leading-twist parton distribution amplitudes of S-wave heavy-qukaonia. Minghui Ding, *et al., PLB753*(2016)330;
Symmetry, symmetry breaking, and pion parton distributions. Minghui Ding, , *et al.,* PRD101(2020)054014.

$$d(k^2r^2) = 1/(1+k^2r^2)^{m/2}$$

$$M_S(z) = \frac{a_0 + a_1 z + a_2 z^2}{a_0 + b_1 z + b_2 z^2 + b_3 z^3},$$

Maximum Entropy Method

Bayesian extraction of PDA from BS wave function. Fei Gao, *et al., PLB770*(2016)551.

basis is Bayes' theorem in probability theory [12], which states the probability of an event "A", given that a condition "B" is satisfied:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \tag{4}$$

Imagine Pion global picture

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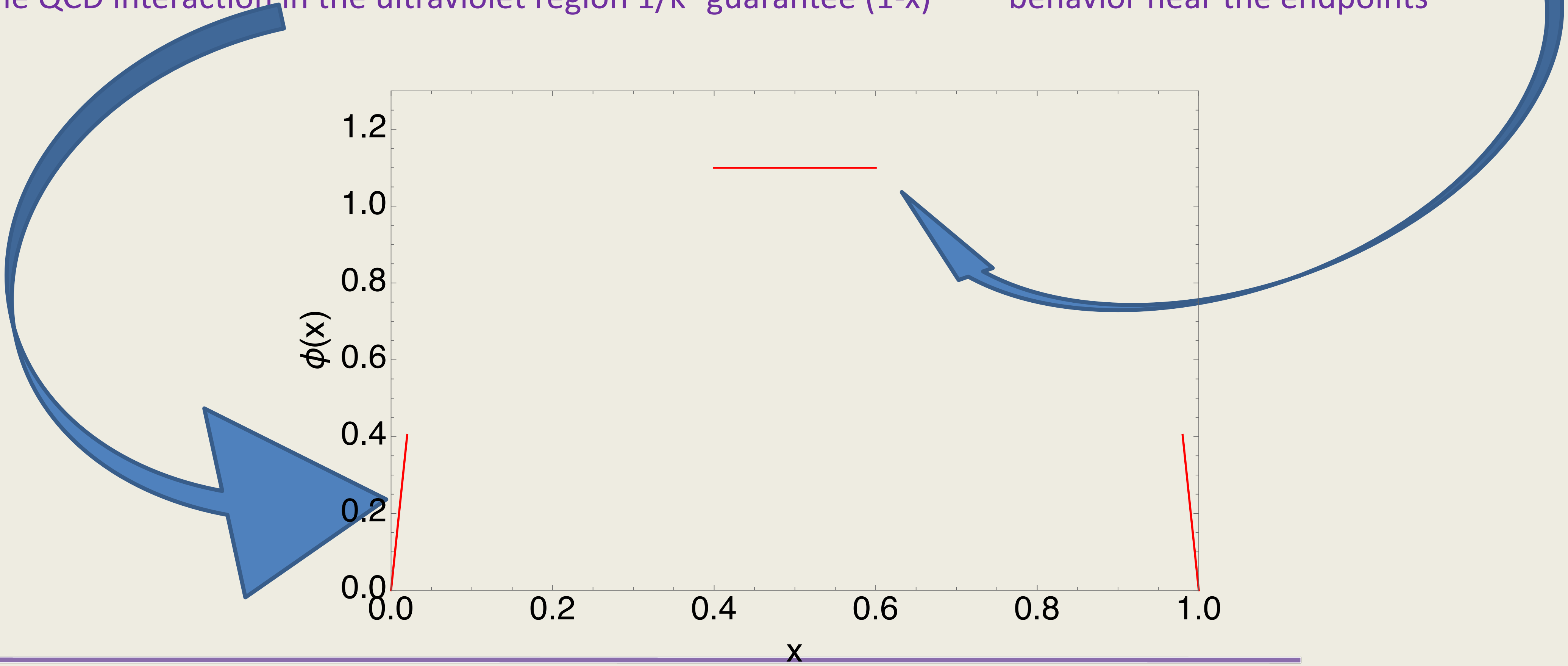
- The gluon has been hidden in the constituent quarks;
- At hadronic scale, the pion is constructed by two constituent quarks which are overlapped largely;
- Valence DA(x) is symmetric function under $x \to 1 - x$
- The screening of interaction below the hadronic scale indicates the valence DA is flat on the middle of x domain
- The QCD interaction in the ultraviolet region $1/k^2$ guarantee $(1-x)^{beta>1}$ behavior near the endpoints



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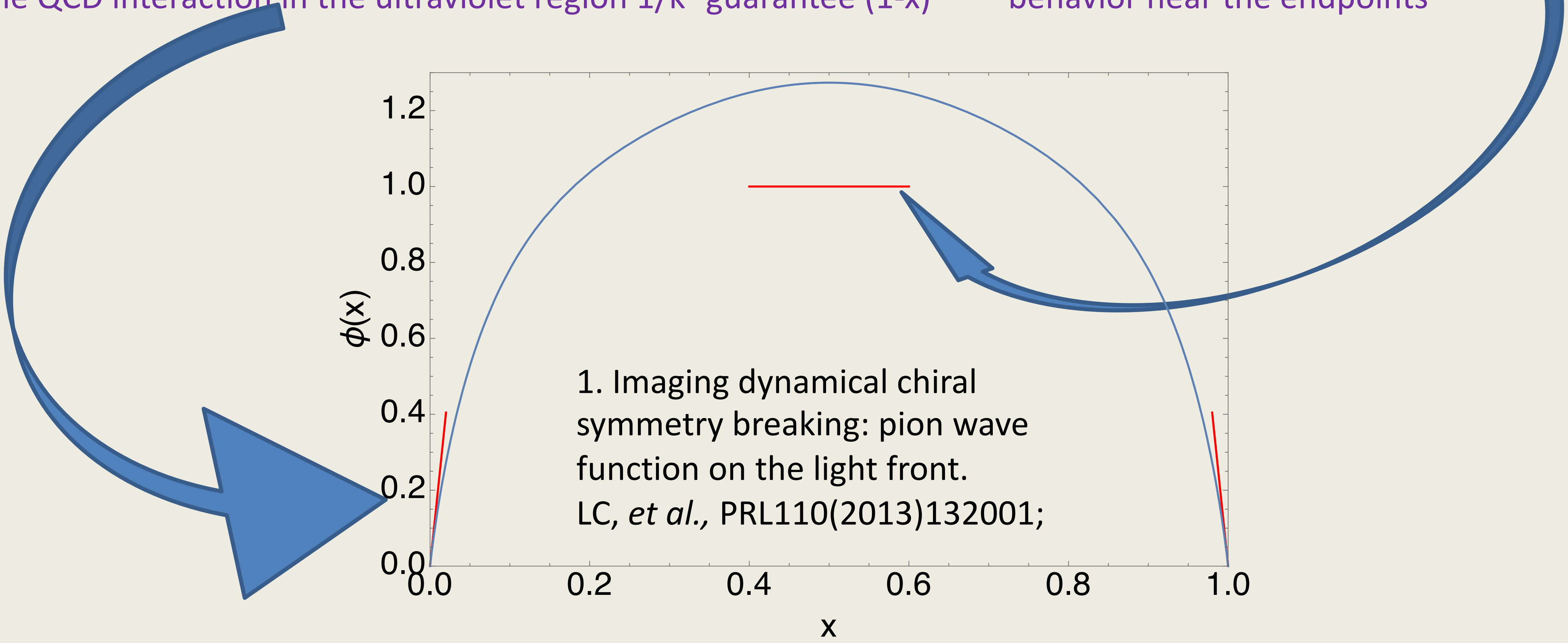

A practical way to calculate DF

LC, et al., PLB737(2014)23, arXiv: 1406.5450

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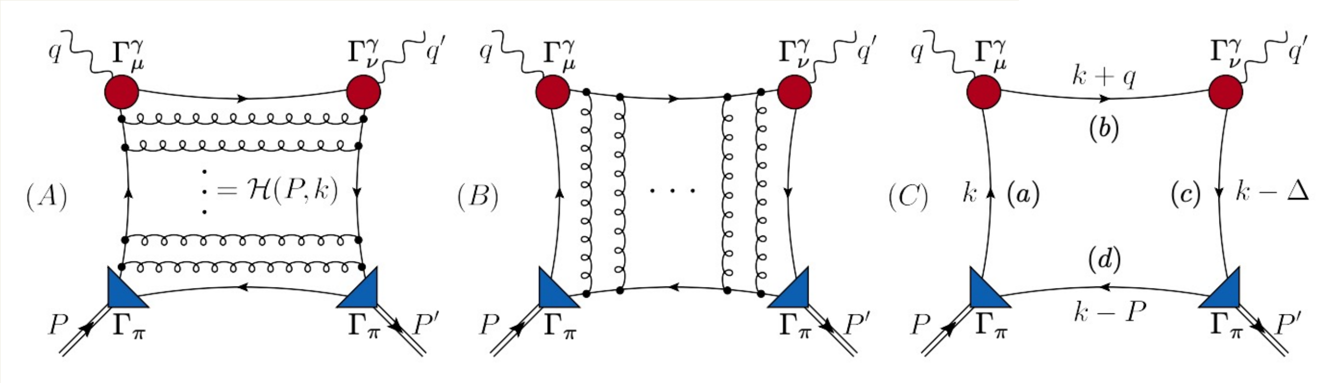
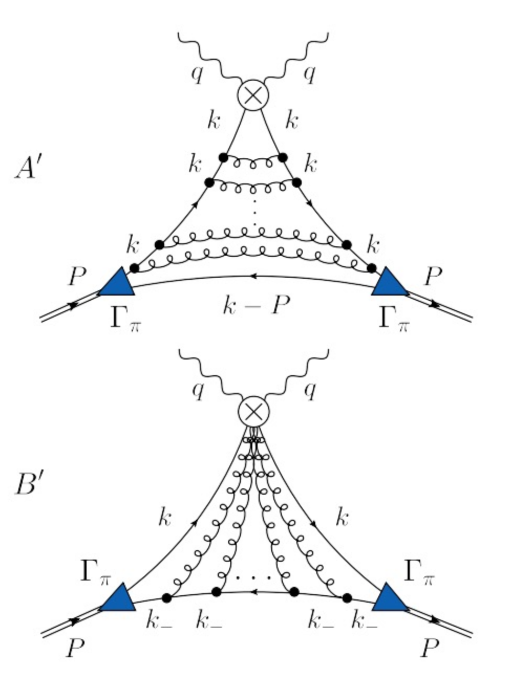
- Consider pion Compton scattering Amplitude $\mathcal{H}$

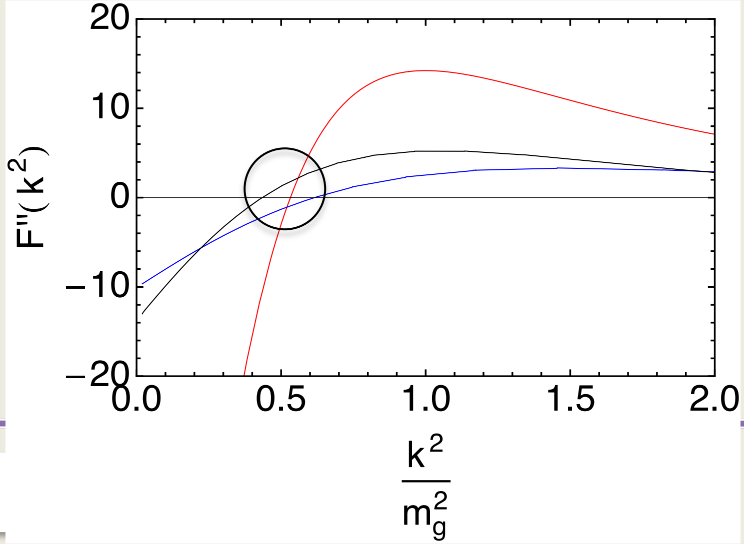
$$q_A^{\pi}(x;\zeta_H) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta)\, n\cdot\gamma\, \mathcal{H}_\pi(P,k)\,,$$



RL truncation



- Beyond Rainbow-Ladder truncation???



- Inflection points
- Red line: running gluon propagagor
- Blue line: vector part of propagator
- Black line: BSW function

$$\leq \frac{1}{\sqrt{2}} \mathrm{m}_g \sim m_G \sim \zeta_H$$

A practical way to calculate DF

LC, et al., PLB737(2014)23, arXiv: 1406.5450

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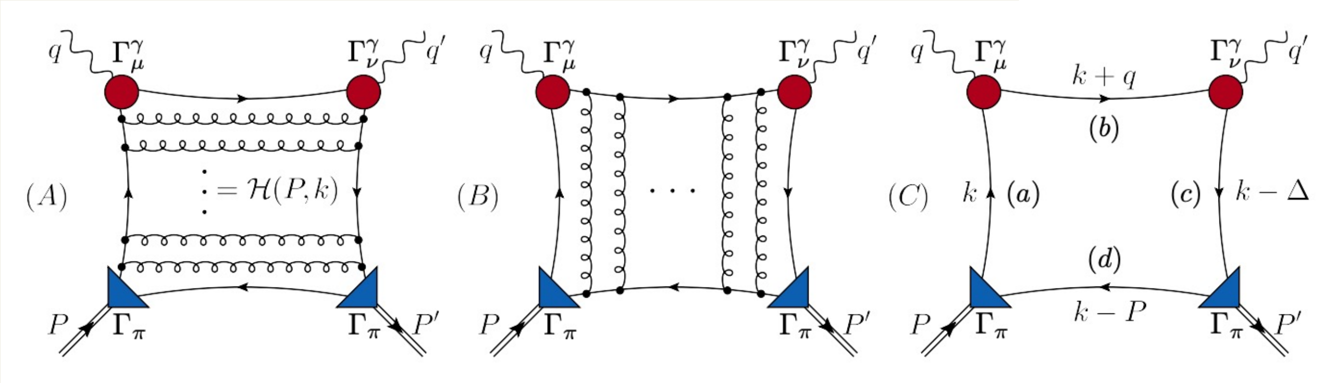
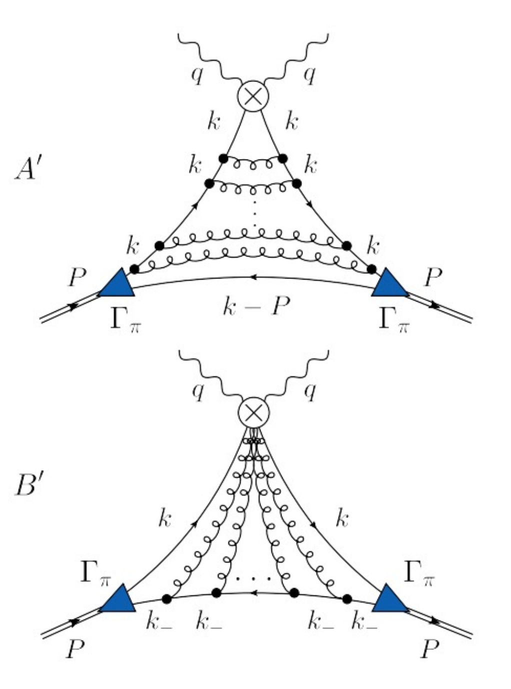


RL truncation



- Beyond Rainbow-Ladder truncation

$$\varphi_H(x;\zeta) \propto \int^\zeta d^2k_\perp\, \psi_H(x,\mathbf{k}_\perp;P),$$

$$q^H(x;\zeta) \propto \int^\zeta d^2k_\perp\, |\psi_H(x,\mathbf{k}_\perp;P)|^2,$$

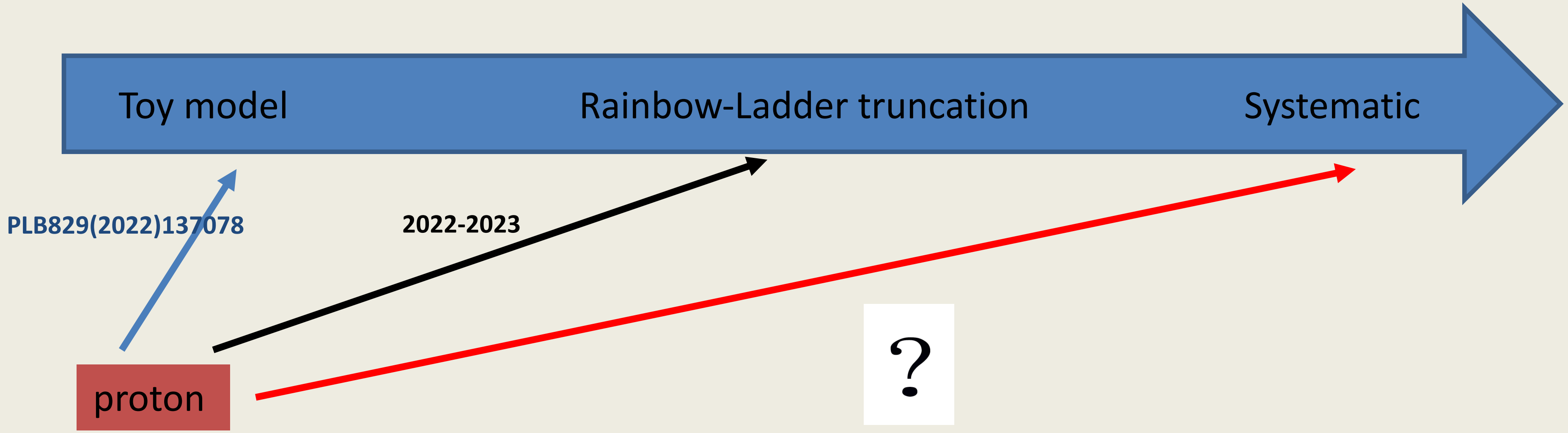
Factorization

$\zeta \to \zeta_H$

at Hadronic Scale

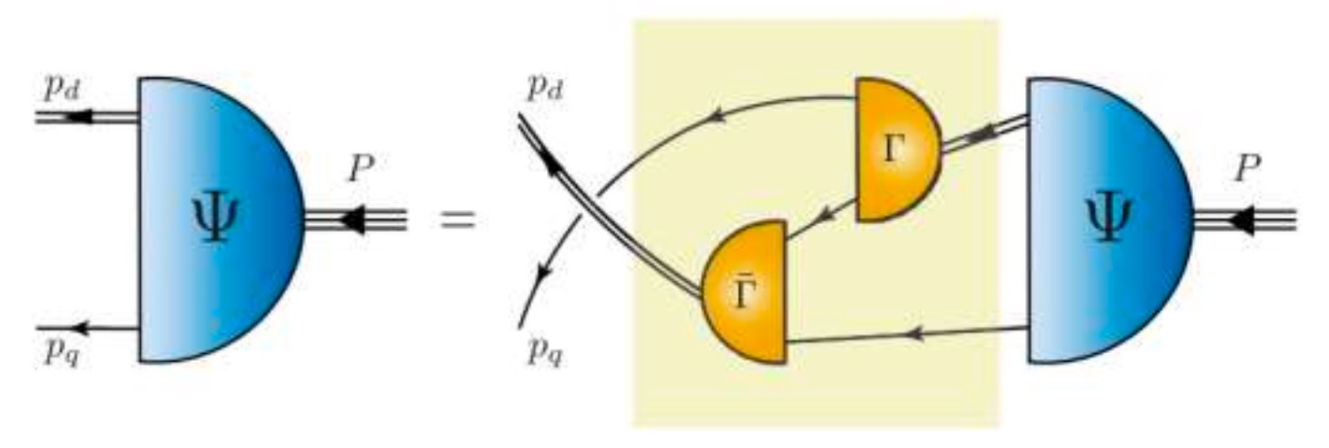
DF(x) = DA(x)^2 !

Story of proton



CSM developments

Stage-I: algebraic model



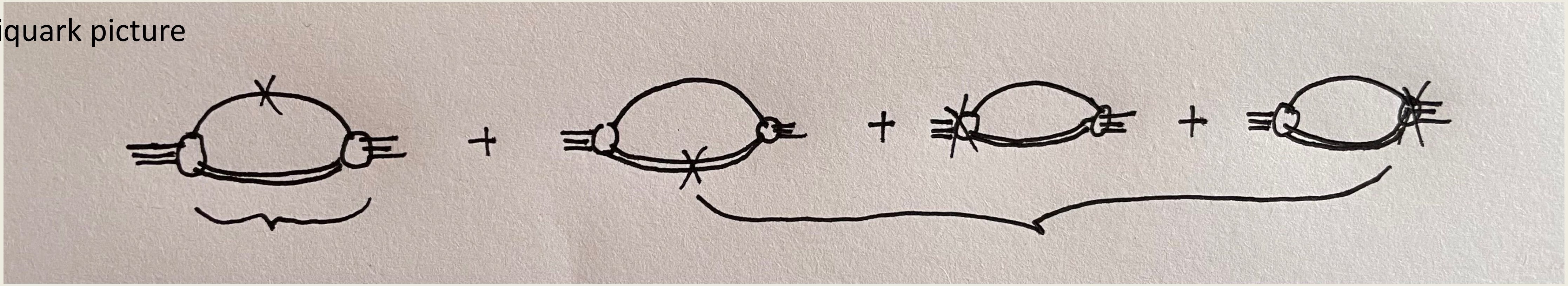
Quark+diquark Faddeev equation

$$\psi(\ell;K) = \sum_{J^P=0^+,1^+_{\{uu\}},1^+_{\{ud\}}} a_{J^P}\psi^{J^P}(\ell;K)\,.$$

Proton's wave function: [ud](isoscalar-scalar_0$^+$) and {uu},{ud}(isovector-pseudovector_1$^+$) correlations

Find quarks in the proton

Quark-diquark picture



Quark component(not sequestered with in a diquark)

u_{Vt}

diquark component(scalar+pseudovector)

s_{0^+}, s_{1^+}

Counting quarks in the diquark component(convolution)

Similar expression for scalar and pseudovector diquarks

$$u^p_{VD_0}(x;\zeta_{\mathcal{H}}) = a^2_{0^+}\int_x^1 dy\, s^p_{0^+}(y;\zeta_{\mathcal{H}})u_V^{0^+}(x/y;\zeta_{\mathcal{H}}),$$

Quarks in proton= quarks in diquark ⊗ Diquark in proton

$$u^p_V(x;\zeta_{\mathcal{H}}) = \sum_{t=Q,D_0,D_1} u^p_{Vt}(x;\zeta_{\mathcal{H}}).$$

Models

- Propagators for quark and diquarks

$$S(\ell) = (-i\gamma\cdot\ell + M)\sigma_M(\ell^2)\,,\ \sigma_M(s) = 1/[s+M^2]\,,$$
$$\Delta^{0^+}(\ell) = \sigma_{M_0}(\ell^2)\,,\ \Delta^{1^+}_{\sigma\rho}(\ell) = \delta_{\sigma\rho}\sigma_{M_1}(\ell^2)\,,$$

- Faddeev amplitudes

$$\psi^{0^+}(\ell;K) = \mathbb{I}\int_{-1}^{1} dz\,\omega(z)\hat{\sigma}_\Lambda(\ell_z^2)^2,$$
$$\psi^{1^+}_\rho(\ell;K) = \tfrac{1}{\sqrt{3}}\gamma_5\gamma^K_\rho\int_{-1}^{1} dz\,\omega(z)\hat{\sigma}_\Lambda(\ell_z^2)^2,$$

- $\omega(z) = \frac{1-2z}{2}$
- $1/k^4$ behaviors

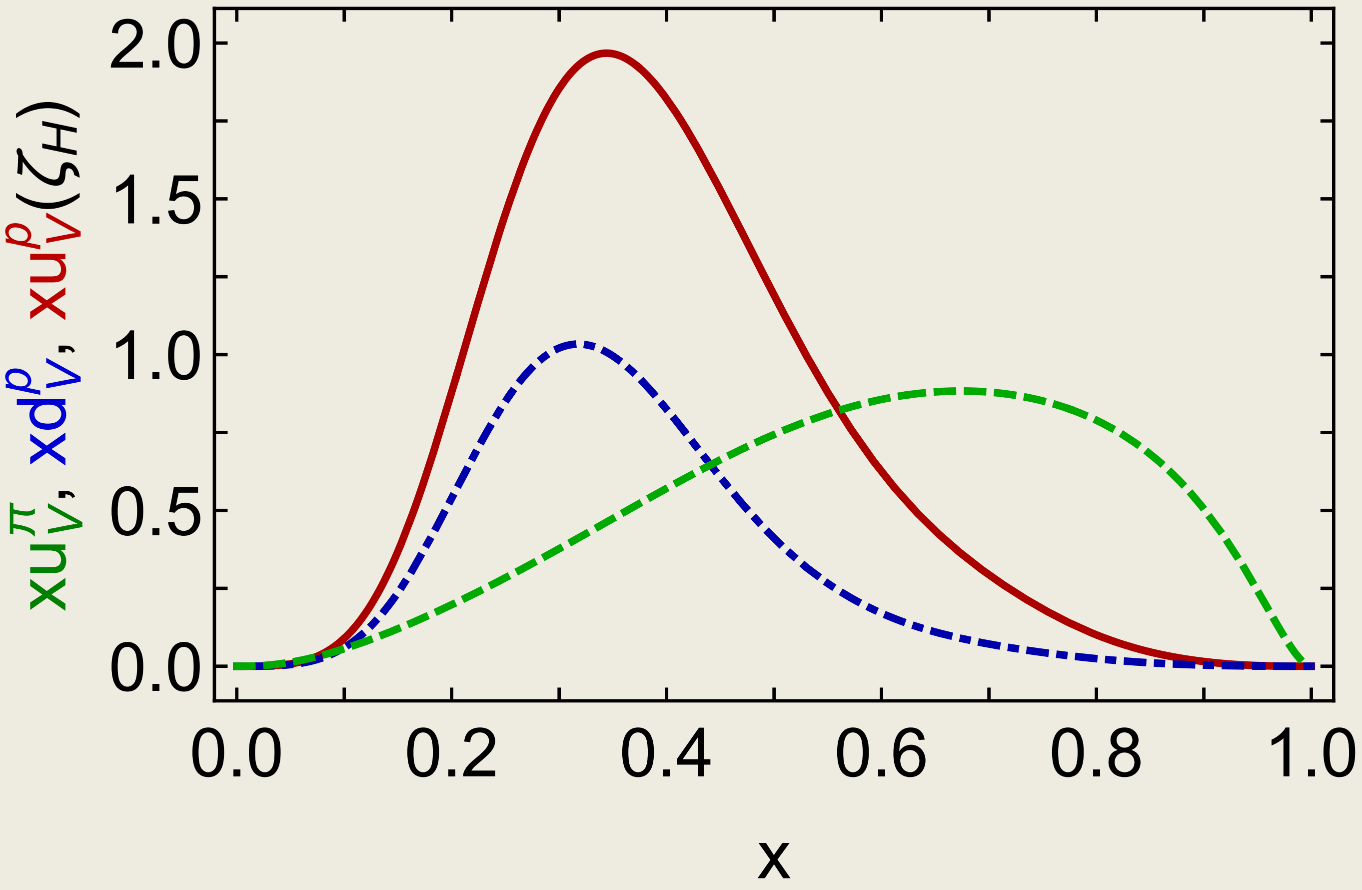
- Quark distribution in the diquarks

$$u_V^{0^+,1^+}(x;\zeta_{\mathcal{H}}) = q_{01}(x;\zeta_{\mathcal{H}}) = n_q x^2(1-x)^2 e^{20x(1-x)-1}\,,$$

- *My choice*

- Four parameters: M=0.4GeV, M_0=0.78GeV, M_1=0.92
  $r_{10} = a_{1^+}/a_{0^+}$

Distributions at the hadronic scale



- Valence quarks carry all the momentum of hadron at the hadroni scale

$$\langle x\rangle_{u_p}^{\zeta_{\mathcal{H}}} = 0.687\,,\ \langle x\rangle_{d_p}^{\zeta_{\mathcal{H}}} = 0.313\,,\ \langle x\rangle_{u_\pi}^{\zeta_{\mathcal{H}}} = 0.5\,,$$

- Diquark correlations $u_V(x) \neq d_V(x)$ in the proton
- Pion is the Nature's most dilated PDF

Large-x behavior

$$d^p(x;\zeta_{\mathcal{H}}), u^p(x;\zeta_{\mathcal{H}}) \overset{x\simeq 1}{\propto} (1-x)^3 ,$$
$$\bar{d}^\pi(x;\zeta_{\mathcal{H}}), u^\pi(x;\zeta_{\mathcal{H}}) \overset{x\simeq 1}{\propto} (1-x)^2 ;$$

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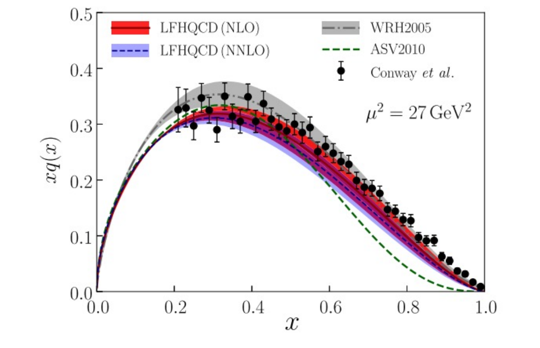
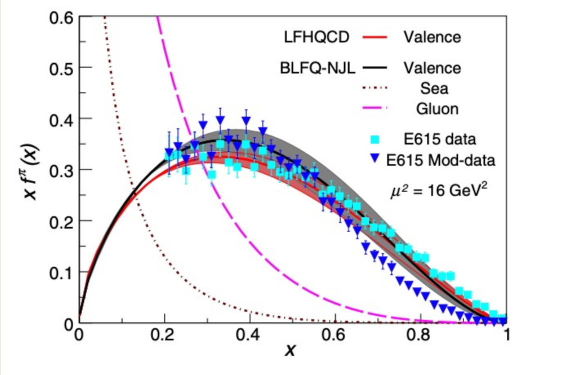


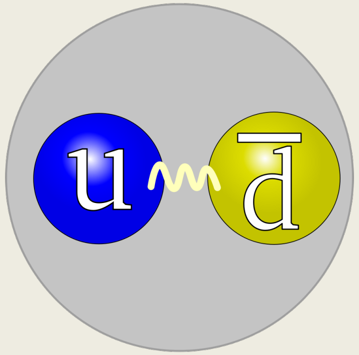
PHYSICAL REVIEW LETTERS **120,** 182001 (2018)

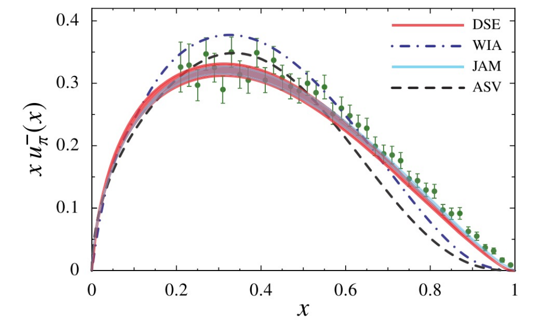
Guy F. de Teramond, *et al*, LFHQCD



PHYSICAL REVIEW LETTERS **122,** 172001 (2019)

Jiangshan Lan, *et al*, Light front Hamitonian





PHYSICAL REVIEW LETTERS **124,** 042002 (2020)

Kyle D. Bendner, *et al*, DSEs

Large-x behavior

$$d^{p}(x;\zeta_{\mathcal{H}}),\, u^{p}(x;\zeta_{\mathcal{H}}) \overset{x\simeq 1}{\propto} (1-x)^{3}\,,$$
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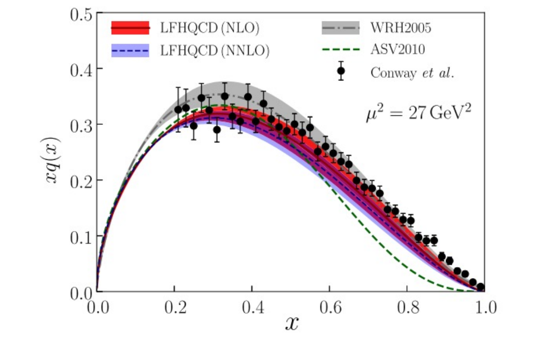


PHYSICAL R

Guy F. de Tera

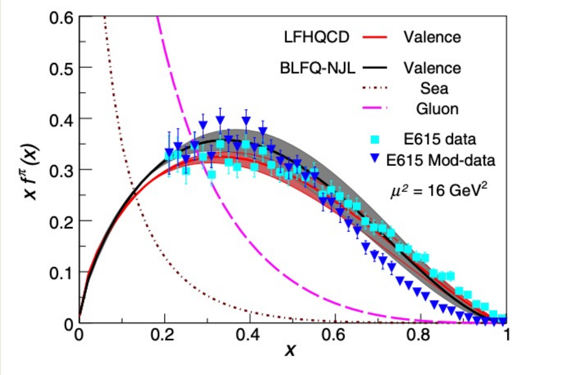
Pion Parton Distribution Function in Light-Front Holographic QCD

#24

Lei Chang (Nankai U.), Khépani Raya (Nankai U.), Xiaobin Wang (Nankai U.) (Jan 21, 2020)

Published in: *Chin.Phys.C* 44 (2020) 11, 114105 • e-Print: 2001.07352 [hep-ph]

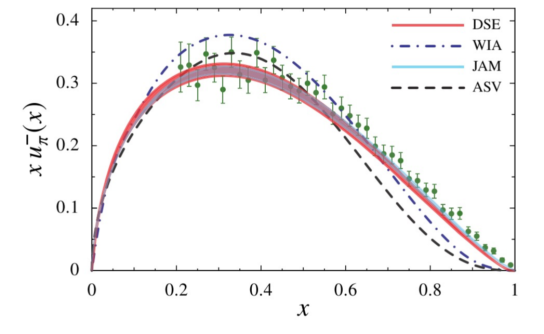
pdf DOI cite 16 citations



PHYSICAL REVIEW LETTERS **122,** 172001 (2019)

Jiangshan Lan, *et al*, Light front Hamitonian

NJL-like!



PHYSICAL REVIEW LETTERS **124,** 042002 (2020)

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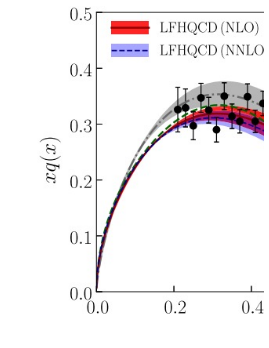
Limited moments!

Large-x behavior



JLAB-THY-22-3592

Complementarity of experimental and lattice QCD data on pion parton distributions

P. C. Barry,[1] C. Egerer,[1] J. Karpie,[2] W. Melnitchouk,[1] C. Monahan,[1,3] K. Orginos,[1,3] Jian-Wei Qiu,[1,3] D. Richards,[1] N. Sato,[1] R. S. Sufian,[1,3] and S. Zafeiropoulos[4]

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[2]*Physics Department, Columbia University, New York City, New York 10027, USA*

[3]*Department of Physics, William & Mary, Williamsburg, Virginia 23185, USA*

[4]*Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France*

Jefferson Lab Angular Momentum (JAM) and HadStruc Collaborations

(Dated: April 4, 2022)

Abstract

We extract pion parton distribution functions (PDFs) in a Monte Carlo global QCD analysis of experimental data together with reduced Ioffe time pseudo-distributions and matrix elements of current-current correlators generated from lattice QCD. By including both experimental and lattice QCD data, our analysis rigorously quantifies both the uncertainties of the pion PDFs and systematic effects intrinsic to the lattice QCD observables. The reduced Ioffe time pseudo-distributions significantly decrease the uncertainties on the PDFs, while the current-current correlators are limited by the systematic effects associated with the lattice. Consistent with recent phenomenological determinations, the behavior of the valence quark distribution of the pion at large momentum fraction is found to be $\sim (1-x)^{\beta_{\rm eff}}$ with $\beta_{\rm eff} \approx 1.0 - 1.2$.

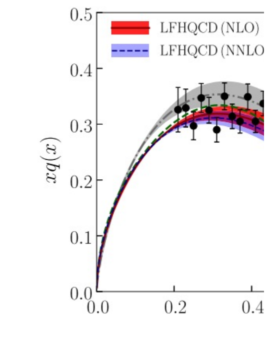
19)

n

ETTERS **124,** 042002 (2020)

al, DSEs

Large-x behavior



We are preparing a paper to answer WHY based on the ultraviolet analysis of BS amplitude of massless pion.

JLAB-THY-22-3592

Complementarity of experimental and lattice QCD data on pion parton distributions

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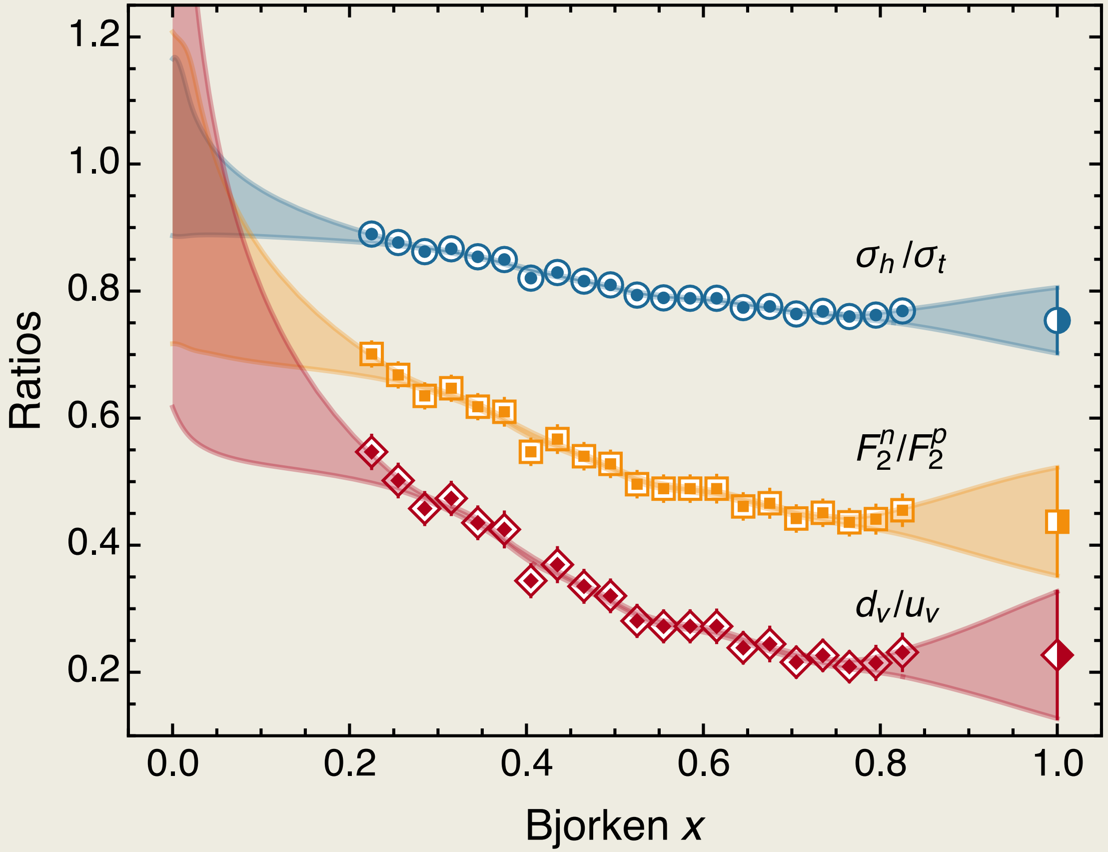
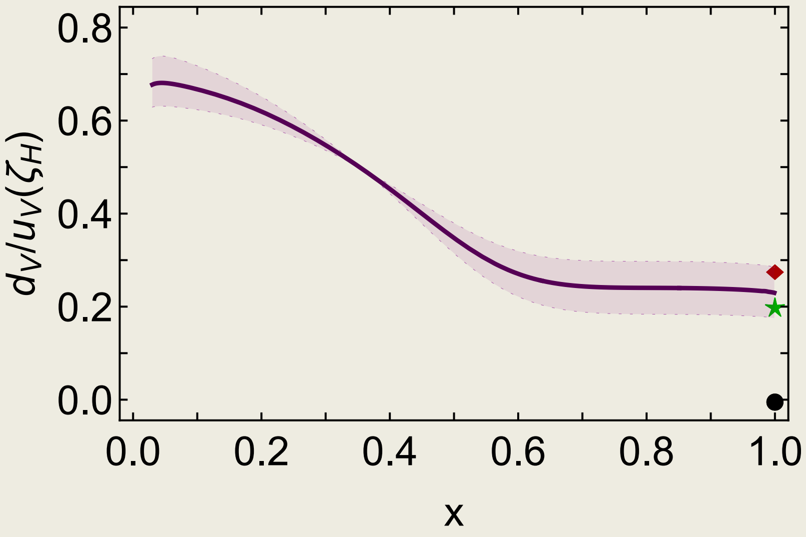


- Cui, *et al.*, CPL39(2022)041401(Schlessinger Point Method extroplating MARATHON experiment data)
- Our model

$$u_V^p(x;\zeta_\mathcal{H}) \overset{x\simeq 1}{\sim} (1-x)^3\,,$$
$$\lim_{x\to 1}\frac{d_V^p(x;\zeta_\mathcal{H})}{u_V^p(x;\zeta_\mathcal{H})} = \frac{2\eta r_{10}^2}{1+\eta r_{10}^2}\,,$$

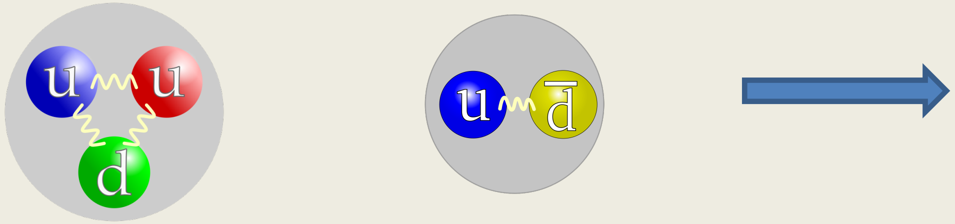
$\frac{1}{k^4}$ asymptotic behavior of Faddeev amplitudes guarantee

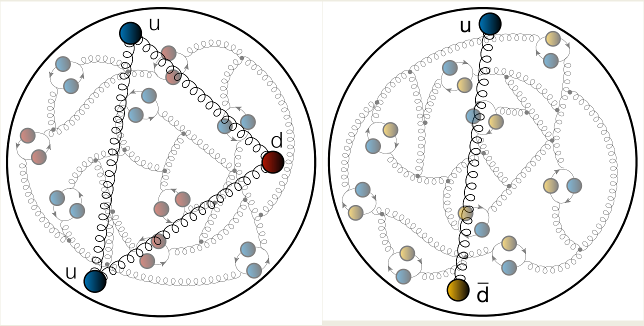
- $r_{10} = 0, \;\; 0.47(6)$



Evolution

- The gluon has been hidden in the constituent quarks;
- At hadronic scale, the pion is constructed by two constituent quarks which are overlapped largely;
- Let gluon/sea show up!





DGLAP with the effective charge

$$\frac{\partial q^{NS}}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} P_{qq} \otimes q^{NS} \, ,$$

$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} q^S \\ g \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi} \begin{pmatrix} P_{qq} & 2n_f \, P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q^S \\ g \end{pmatrix} ,$$

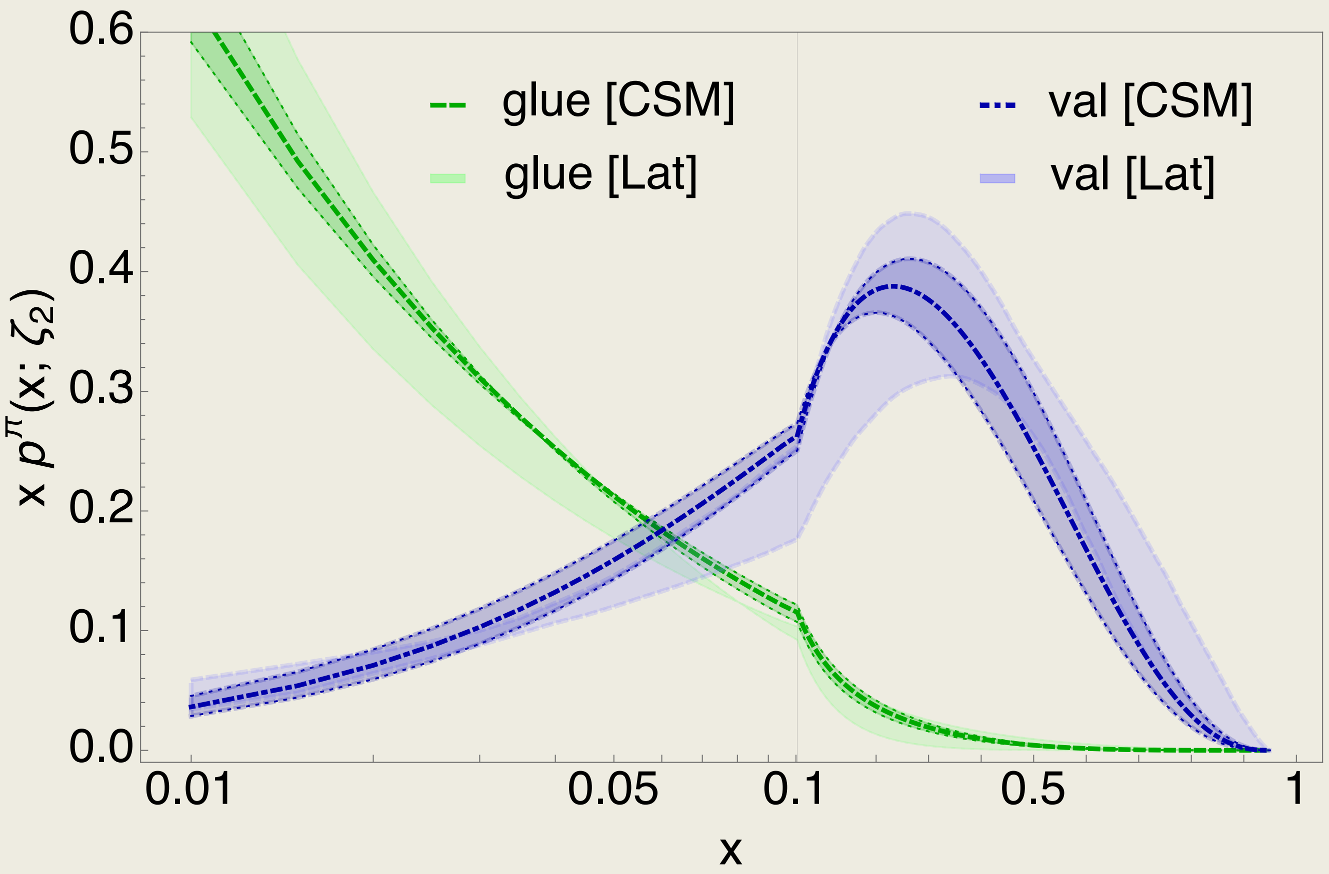
$$\tilde{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln \left[\frac{\mathcal{K}^2(k^2)}{\Lambda^2_{\rm QCD}} \right]} \, ,$$

$$\mathcal{K}^2(y) = \frac{a_0^2 + a_1 y + y^2}{b_0 + y}$$

with $\{a_0, \; a_1, \; b_0\} = \{0.104(1), 0.0975, 0.121(1)\}$

The sea quarks can arise from gluon splitting, xS(x) is expected to follow the trend of xg(x).

Breaking News!



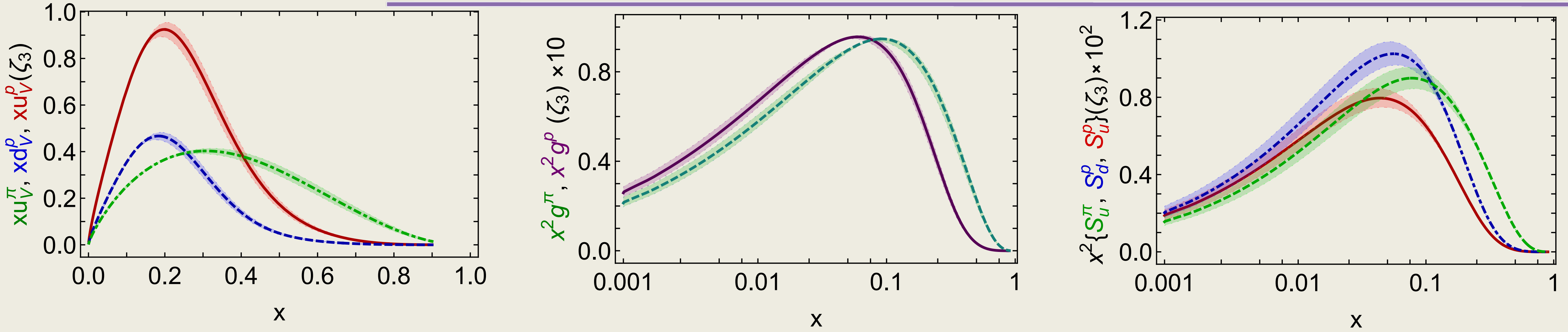
- Within uncertainties, there is pointwise agreement between the two results on the entire depicted domains

- Val[Lat] Sufian *et al.*, arXiv: 1901.03921
(valence DF: using lattice-calculated matrix element obtained through spatially separated current-current correlations in coordinate space)
- Glue/5[Lat] Fan *et al.*, arXiv: 2104.06372
(Glue DF: using pseudo-PDF approach(Balitsky, Morris and Radyushkin,arXiv:1910.13963))
- CSM see short review: LC and C.D.Roberts, *Chin.Phys.Lett.38(2021)081101.*

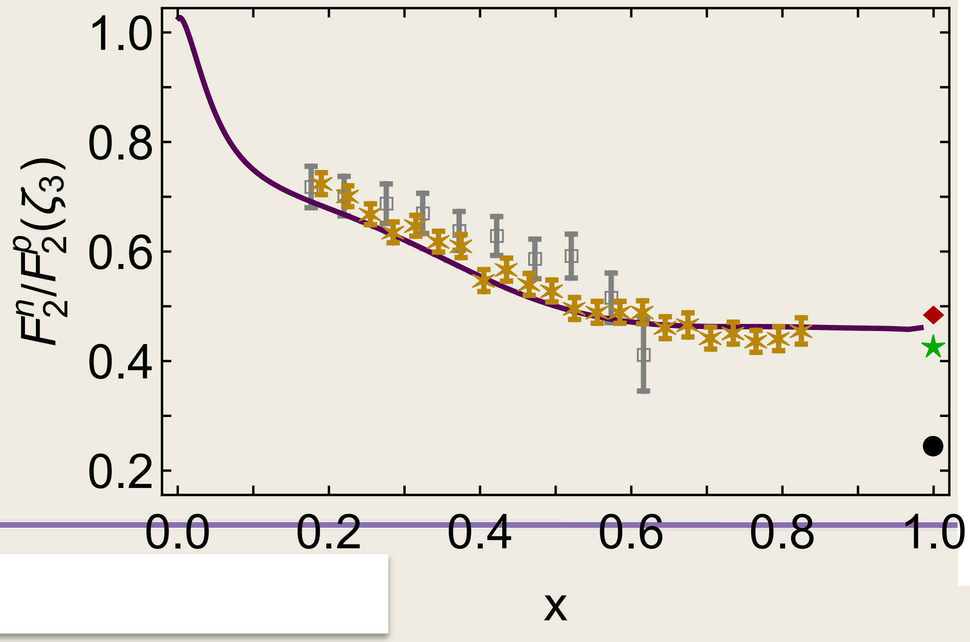
- Lattice methods: moments(…)
  LaMET(Ji)
  good lattice cross section(Qiu…)
  pseudo-PDF(Radyushkin…)

Continuum QCD approach
A long story from 2013

Evolution ($\zeta_3 = m_{J/\psi} = 3.097GeV$)



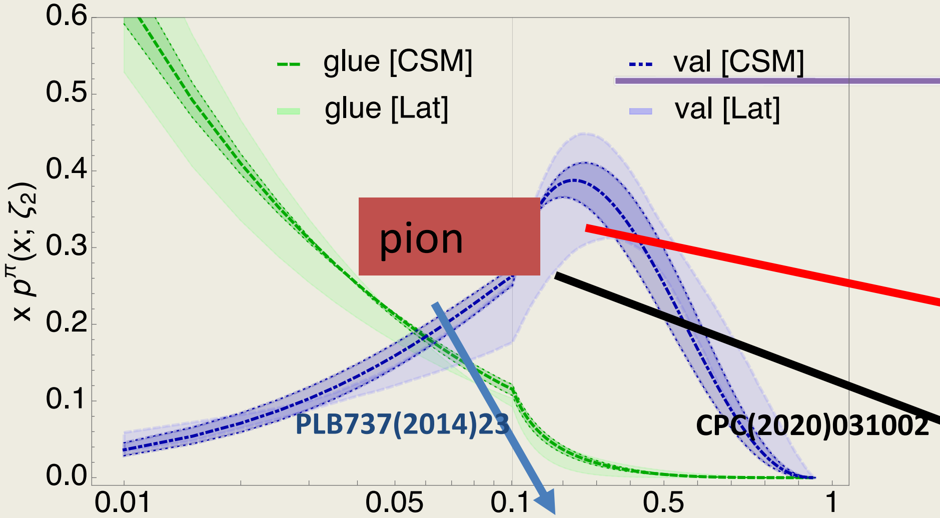
Pion possess significantly more support on the valence domain than those in the proton!



Using our results for the valence and sea DFs, it is straightforward to calculate the neutron-proton structure function ratio:

$$\frac{F_2^n(x;\zeta)}{F_2^p(x;\zeta)} = \frac{\mathcal{U}(x;\zeta) + 4\mathcal{D}(x;\zeta) + \Sigma(x;\zeta)}{4\mathcal{U}(x;\zeta) + \mathcal{D}(x;\zeta) + \Sigma(x;\zeta)}, \quad (12)$$

where, in terms of quark and antiquark DFs, $\mathcal{U}(x;\zeta) = u(x;\zeta) + \bar{u}(x;\zeta)$, $\mathcal{D}(x;\zeta) = d(x;\zeta) + \bar{d}(x;\zeta)$, $\Sigma(x;\zeta) = s(x;\zeta) + \bar{s}(x;\zeta) + c(x;\zeta) + \bar{c}(x;\zeta)$. The $\zeta = \zeta_3$ prediction is drawn in Fig. 4B: in comparison with modern data [69, MARATHON], it yields χ^2/degree-of-freedom = 1.3. Notably, both data and calculation indicate the presence of a significant axial-vector diquark component in the proton wave function [83, 84].



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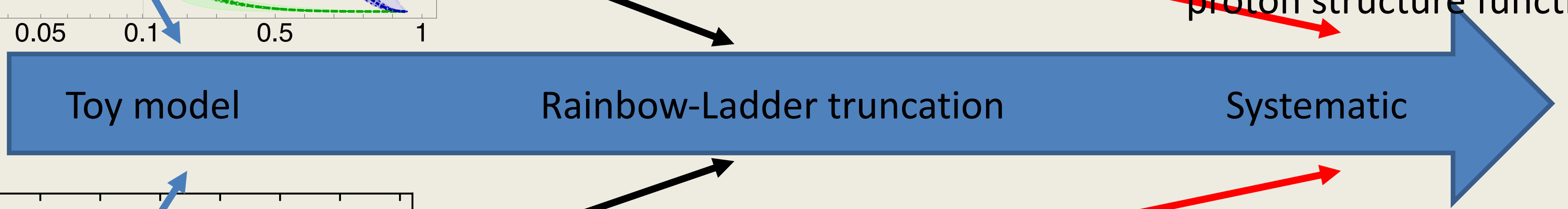
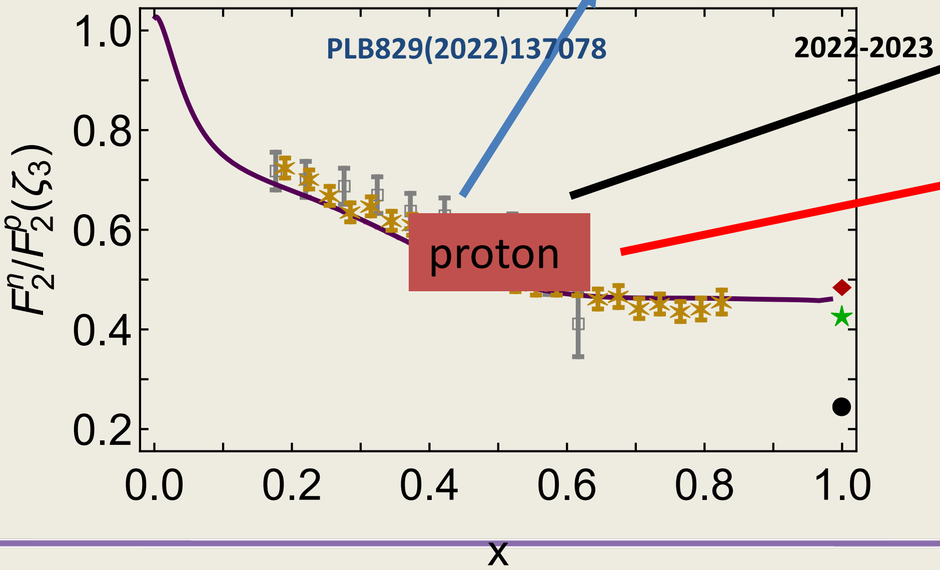
- Ending debate on the large-x behavior of pion and wait for the experiments
- Develop RL truncation of proton techniques and calculate the proton structure function directly

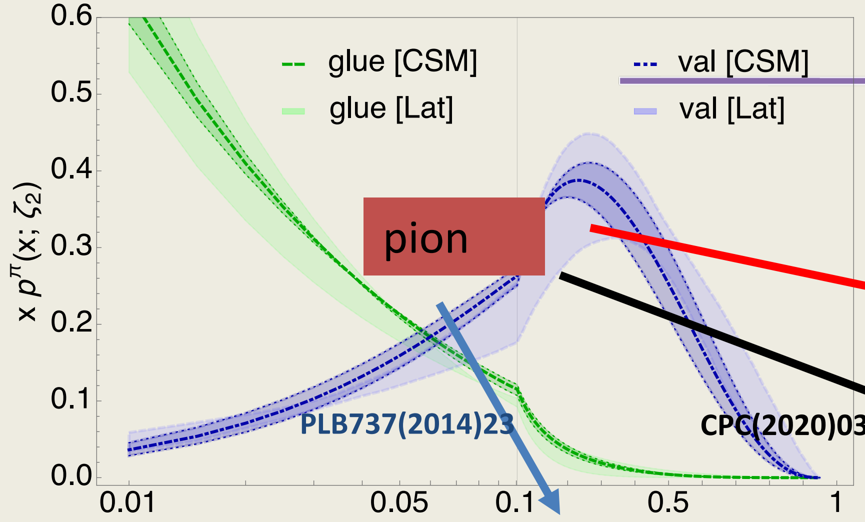




?

- Systematic error controls

Nankai University



?

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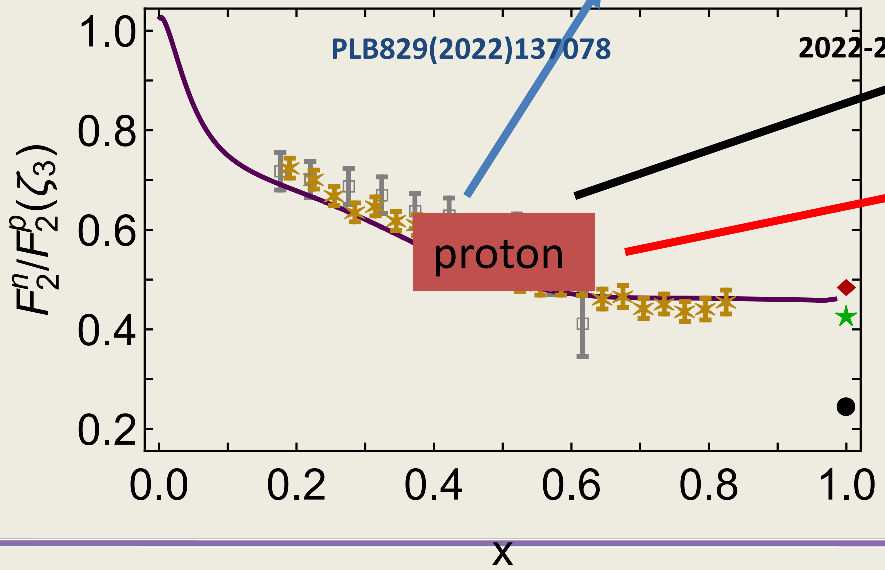
Toy model → Rainbow-Ladder truncation → Systematic



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