

Simulating **lattice gauge theories** with **ultracold atoms**



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University of Science and Technology of China

- **About lattice gauge theory (LGT)**
- Quantum simulation with ultracold atoms
- The toric code model and Schwinger model
- Conclusion and outlook

□ 格点规范理论的背景

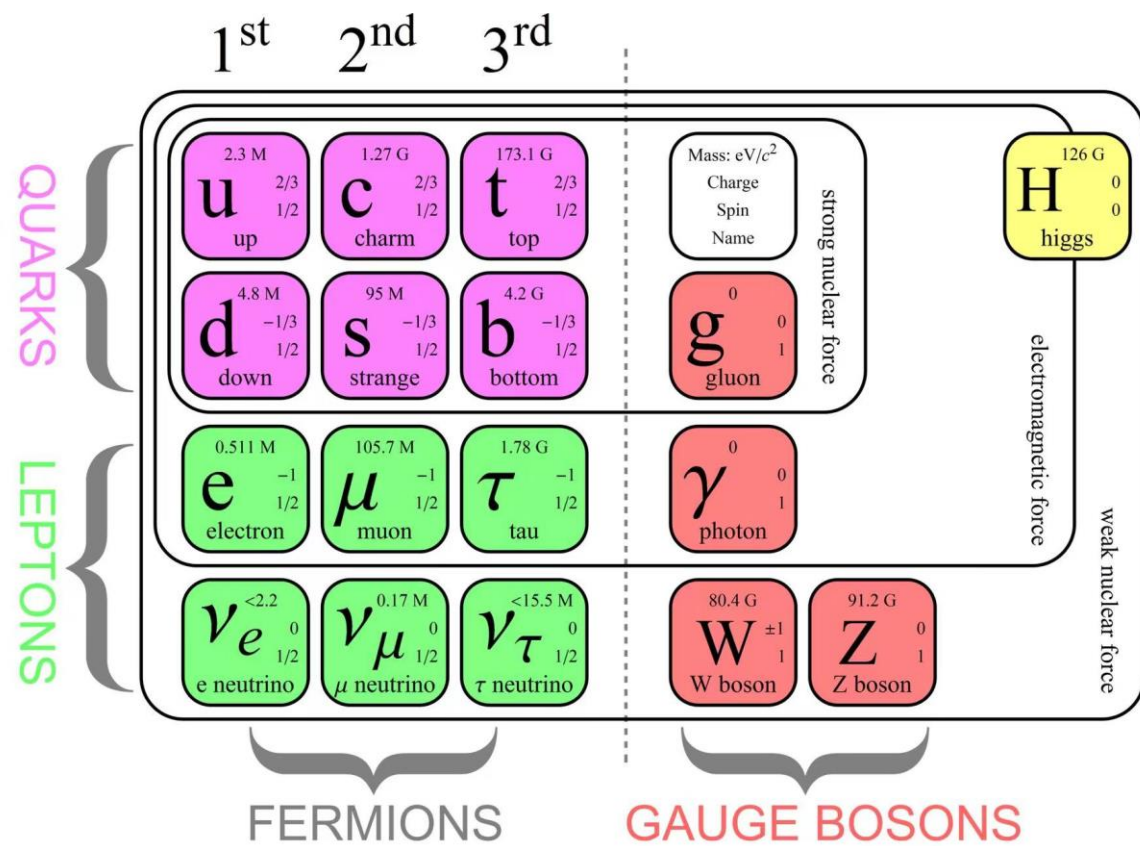
一、标准模型与规范理论

二、格点规范理论的提出和发展

三、格点规范理论的物理应用

一、标准模型与规范理论

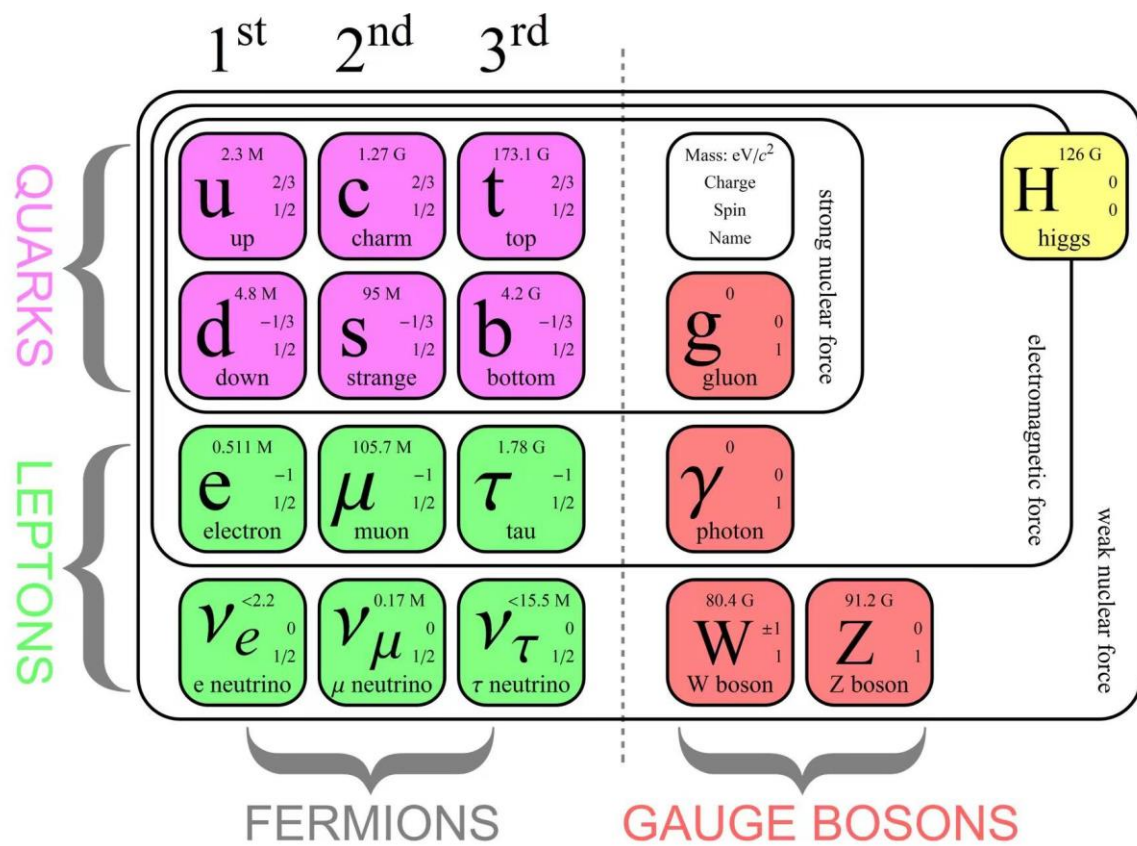
1-1. 标准模型概述 20世纪物理学最重大成就之一!



一、标准模型与规范理论

1-1. 标准模型概述

20世纪物理学最重大成就之一!



基本力

- 电磁力
- 强作用力
- 弱作用力

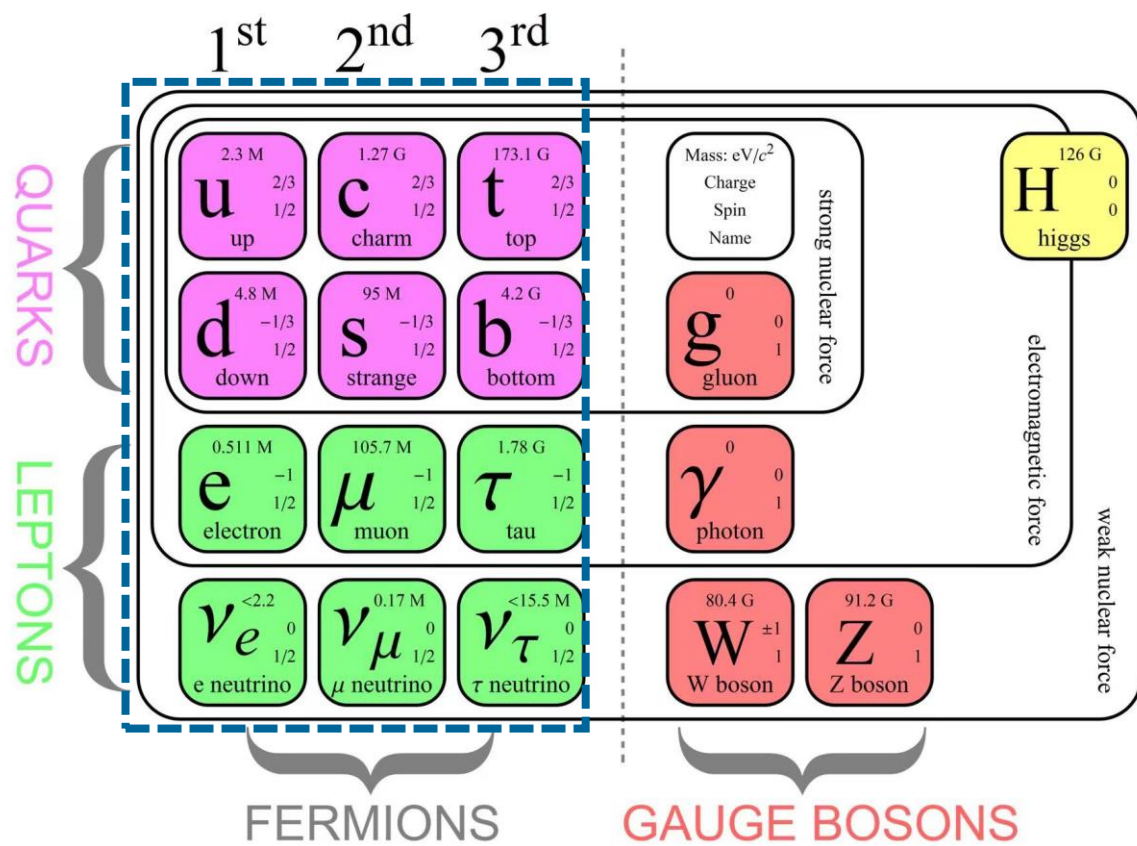
基本粒子

- 费米子
- 规范玻色子
- 希格斯玻色子

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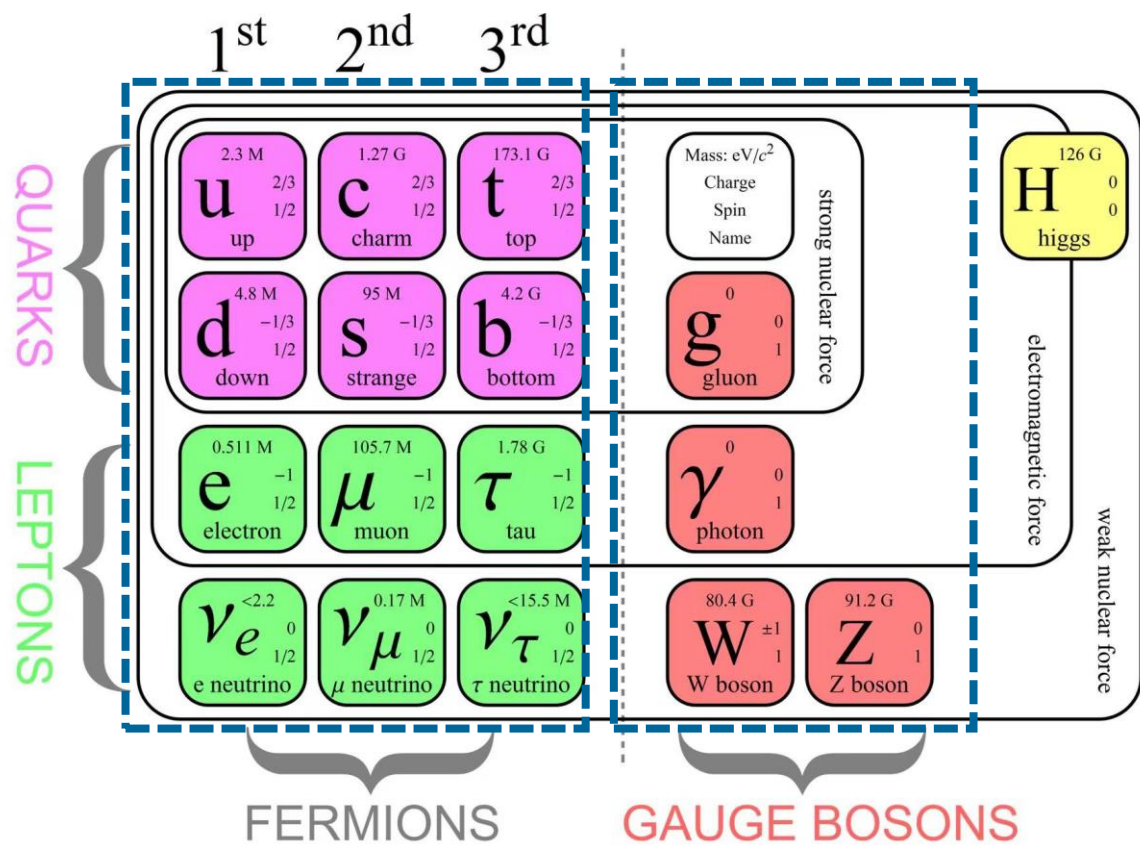
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半整数自旋
泡利不相容原理
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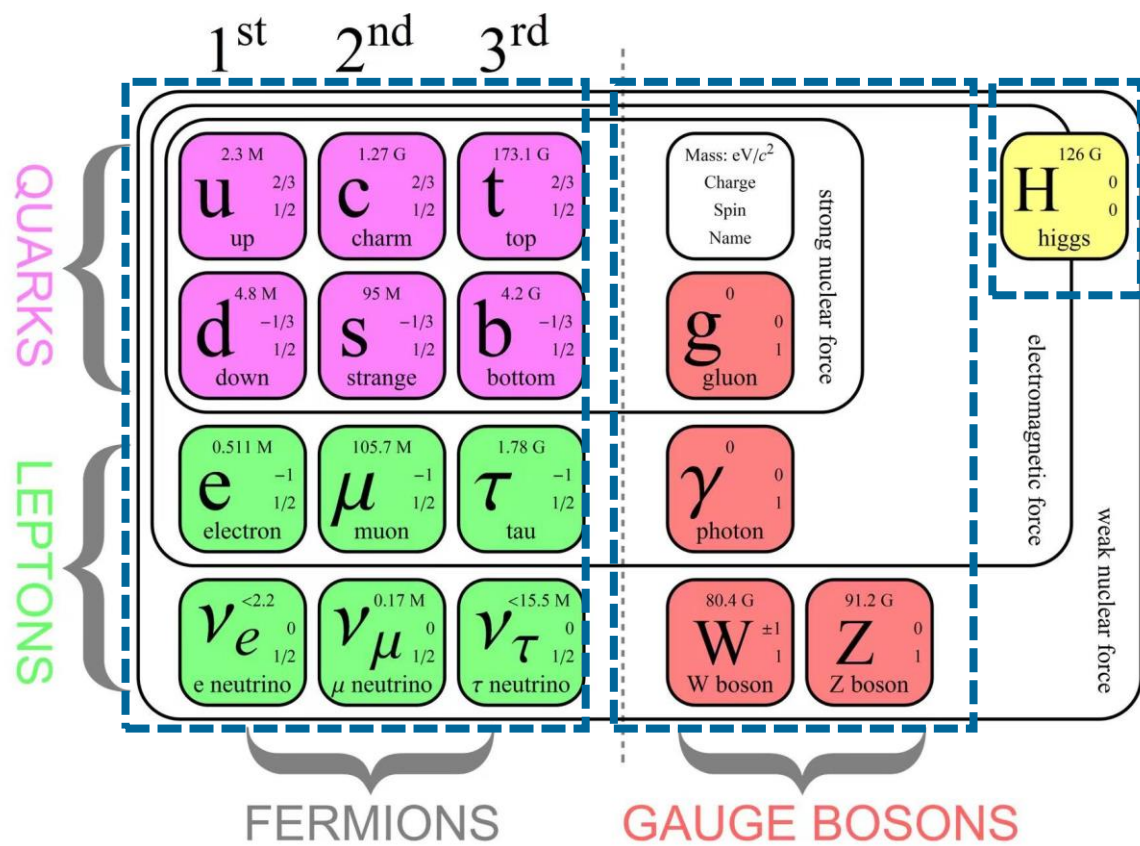
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- 希格斯玻色子
赋予规范玻色子质量

一、标准模型与规范理论

1-2. 规范对称性与规范理论 (Gauge Theory)

□ 理论的理论, 指导新理论的提出

- ✓ 确定对称性
- ✓ 构造满足该对称性的拉格朗日量

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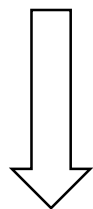
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□ 以电磁场 (阿贝尔规范场) 为例:

U(1)规范变换: $\varphi(x) \rightarrow e^{i\alpha(x)}\varphi(x)$

(local phase rotation)



规范变换: $A_\mu \rightarrow A_\mu - \frac{1}{e}\partial_\mu\alpha(x)$

引入协变导数: $D_\mu \equiv \partial_\mu + ieA_\mu(x)$

一、标准模型与规范理论

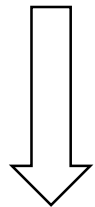
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拉格朗日量形式不变 (对称性)

$$L_{QED} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}(F_{\mu\nu})^2$$

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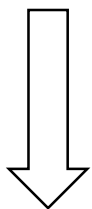
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规范对称性 (局部对称性)

物质粒子与场的耦合

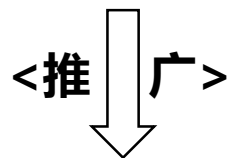
1-3. 基于规范理论诞生标准模型



C. N. Yang (1922 -) and Robert Mills (1927 - 1999)
at Stony Brook in 1999.

杨-米尔斯理论:

U(1)阿贝尔规范场论



SU(2)非阿贝尔规范场论

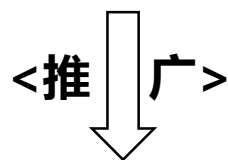
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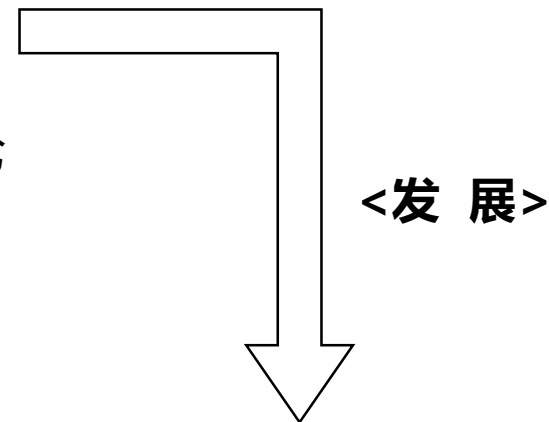
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标准模型

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一、标准模型与规范理论

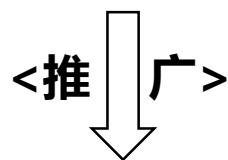
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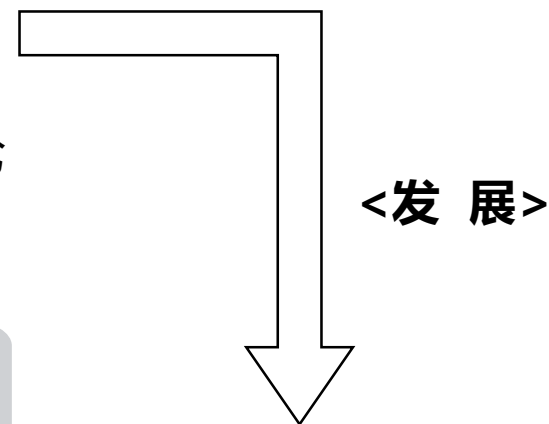
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<发展>

标准模型

=

$$SU(3) \times SU(2) \times U(1)$$

规范对称性

U(1)

SU(2)

SU(3)

规范场粒子

光子

W、Z玻色子

胶子

相互作用

电磁相互作用

弱相互作用

强相互作用

一、标准模型与规范理论

1-4. 求解规范理论时遇到的问题

□ 强耦合到渐进自由的过度

$$\alpha_s(\mu) = \frac{1}{4\pi} \frac{1}{\beta_0 \ln \frac{\mu^2}{\Lambda_{\text{QCD}}^2}}, \quad \beta_0 = \left(\frac{11N_c - 2N_f}{3} \right) / 16\pi^2$$

高能区夸克胶子渐近自由、低能区相互作用强非微扰特性。

QCD 的特殊性质

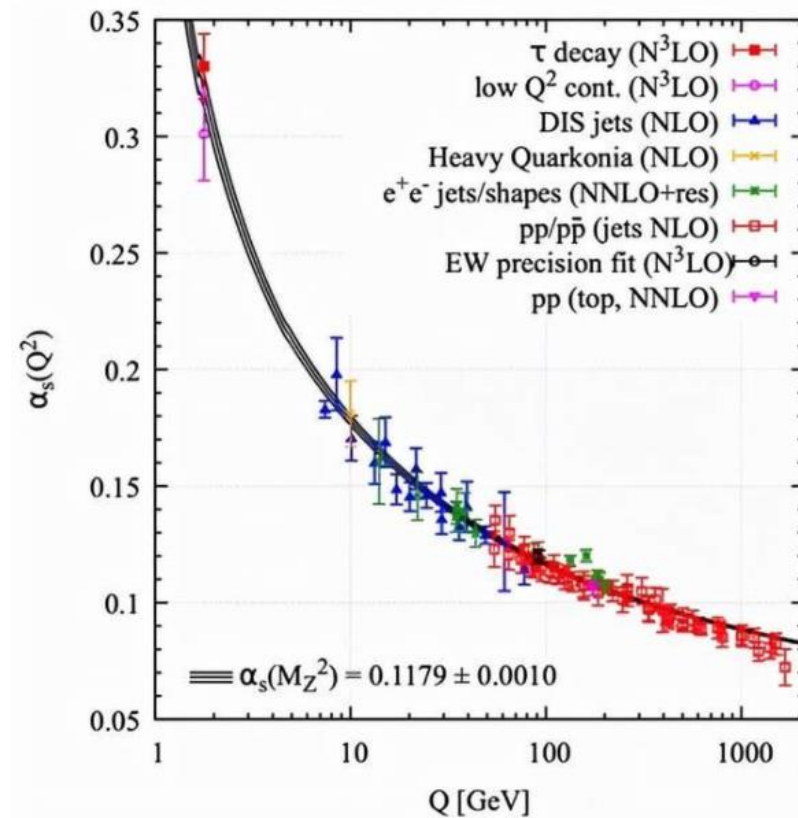
□ 色禁闭 $V(r) = A + \frac{B}{r} + \sigma r$

夸克受到QCD相互作用的强力束缚，带单个色荷的夸克不可能从核子中单个地分离出来。

□ 手征对称性自发破缺

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$

真空夸克凝聚使得手征对称性自发破缺，造就了形形色色的强子谱。



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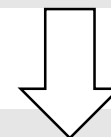
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低能强相互作用区域
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数值计算?

规范理论 ~ 连续模型

计算机无法处理无穷多个自由度!

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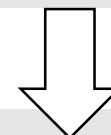
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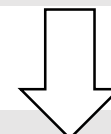
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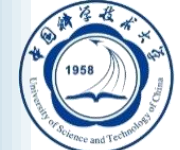


将规范理论离散化?

格点规范理论

可数值计算，解决非微扰问题

二、格点规范理论的提出和发展



-20-

2-1. 格点规范理论 (Lattice Gauge Theory) 的诞生

二、格点规范理论的提出和发展



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2-1. 格点规范理论 (Lattice Gauge Theory) 的诞生



PHYSICAL REVIEW D

VOLUME 10, NUMBER 8

15 OCTOBER 1974

Confinement of quarks*

Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.

1974年Kenneth G. Wilson在 **Confinement of quarks**, PRD 10, 2445 (1974) 中首次提出“格点规范理论”这一概念。

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- 将规范理论离散化
- 在强相互作用耦合常数趋于无穷的情况下，得到了静态夸克势随着正反夸克距离变大而变大的结果，从而明确验证了**夸克禁闭**

二、格点规范理论的提出和发展

2-2. 格点规范理论的基本思路

□ **积分** → **求和** $\int d^4x \rightarrow a^4 \sum_n$ **微分** → **差分** $\partial_\mu \psi(n) = \frac{1}{a} (\psi(n + \hat{\mu}) - \psi(n))$

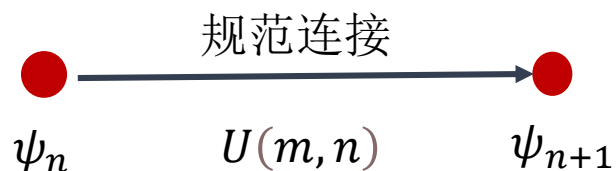
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□ **物质场之间引入规范场，重建规范对称性**

$$\bar{\psi}_n \psi_{n+1} \rightarrow \bar{\psi}_n U(m, n) \psi_{n+1}$$



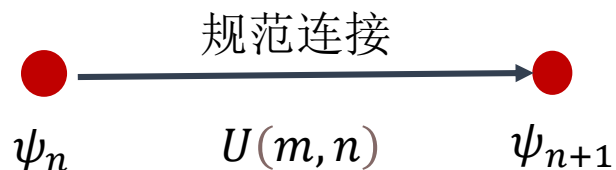
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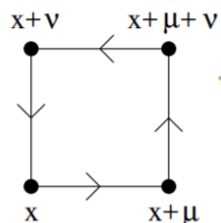
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□ **完善规范场动力学：引入Wilson loop**



$$W_\square = \text{tr} U_\mu(x) U_\nu(x + \hat{\mu}) U_\mu^\dagger(x + \hat{\nu}) U_\nu^\dagger(x) \Rightarrow \text{构成完整的格点规范理论}$$

二、格点规范理论的提出和发展



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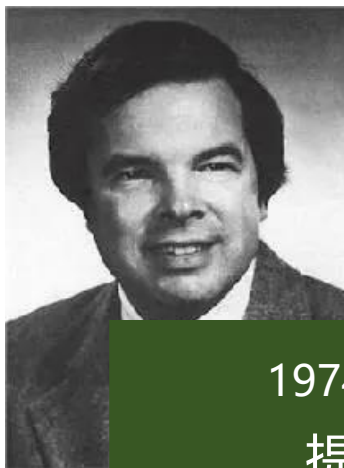
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-27-

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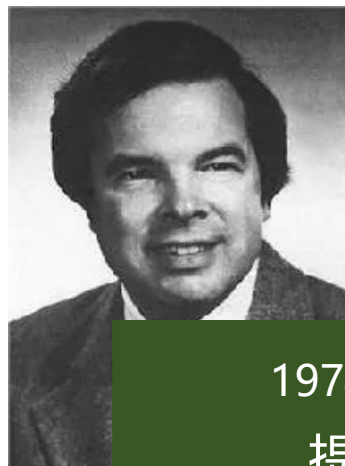
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PHYSICAL REVIEW D

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Gauge fields on a lattice. I. General outlook

R. Balian, J. M. Drouffe, and C. Itzykson

Service de Physique Théorique, Centre d'Études Nucléaires de Saclay, B.P. no. 2, 91190 Gif-sur-Yvette, France

(Received 3 June 1974)

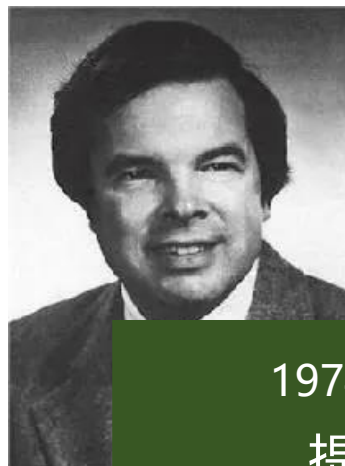
We present Wilson's model of gauge-field theory on a lattice, including a coupling to a matter field. The algebraic structure is reviewed for both commuting and noncommuting groups. Various properties are summarized. The gauge fields are shown to be the gauge fields of the gauge group.

1974, R. Balian, J. M. Drouffe and C. Itzykson
将LGT继续在非阿贝尔规范中推广

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Service de Physique Théorique, Centre d'Études Nucléaires de Saclay, B.P. no. 2, 91190 Gif-sur-Yvette, France

(Received 3 June 1974)

We present Wilson's model of gauge-field theory on a lattice, including a coupling to a matter field. The algebraic structure is reviewed for both commuting and noncommuting groups. Various properties are summarized. The gauge-invariant configurations are discussed.

1974, R. Balian, J. M. Drouffe and C. Itzykson
将LGT继续在非阿贝尔规范中推广

PHYSICAL REVIEW D

VOLUME 11, NUMBER 2

15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

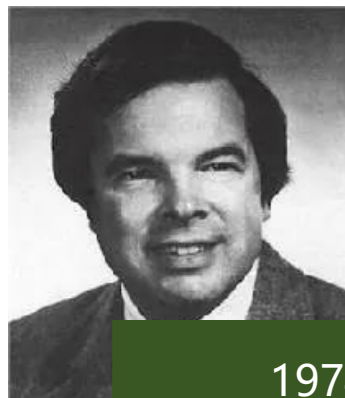
1975, Kogut and Susskind
Hamiltonian LGT

Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.

二、格点规范理论的提出和发展



2-3. 格点规范理论的发展



1974, K. G. Wilson

Light hadron spectrum on the lattice

For the first time, the lattice results appear on Sciences

21 NOVEMBER 2008 VOL 322 SCIENCE www.sciencemag.org

Ab Initio Determination of Light Hadron Masses

S. Dürr,¹ Z. Fodor,^{1,2,3} J. Frison,⁴ C. Hoelbling,^{2,3,4} R. Hoffmann,² S. D. Katz,^{2,3} S. Krieg,² T. Kurth,² L. Lellouch,⁴ T. Lippert,^{2,5} K. K. Szabo,² G. Vulvert⁴

More than 99% of the mass of the visible universe is made up of protons and neutrons. Both particles are much heavier than the quarks and gluons that they are made of. Particle physics should be able to calculate the masses of protons, neutrons, and other hadrons from the fundamental theory of the strong interaction. We present the first calculation of the pion masses down to the physical point with lattice sizes of approximately 1 fm. The results are compared with lattice QCD calculations used for a continuum extrapolation and represent a quantitative confirmation of this aspect of the Standard Model with fully controlled uncertainties.

用LGT理论解决实际物理问题

PHYSICAL REVIEW D

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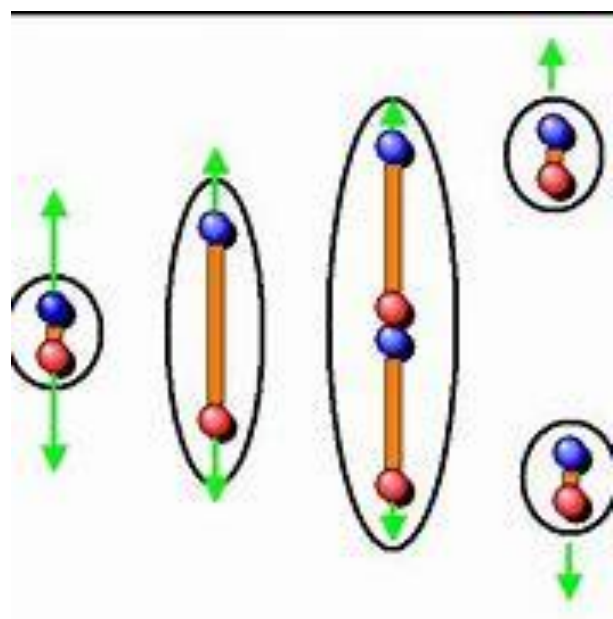
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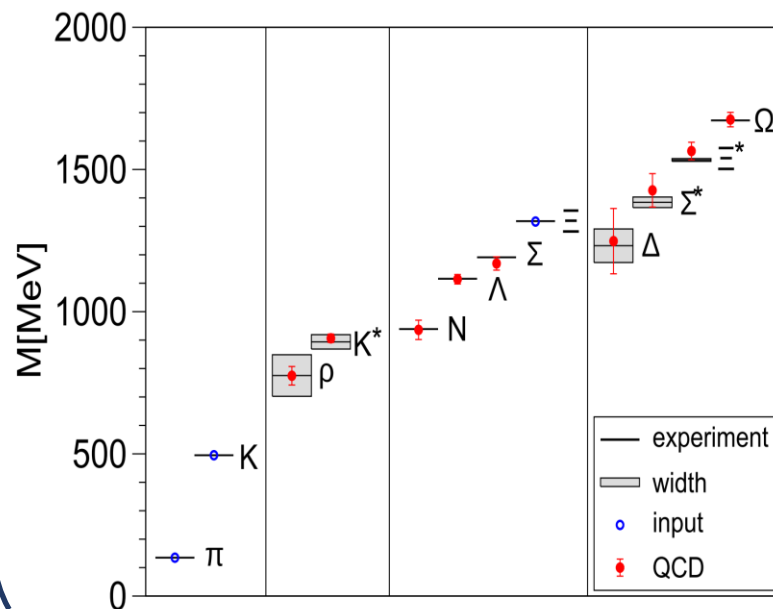
三、格点规范理论的物理应用

格点规范理论解决的问题：

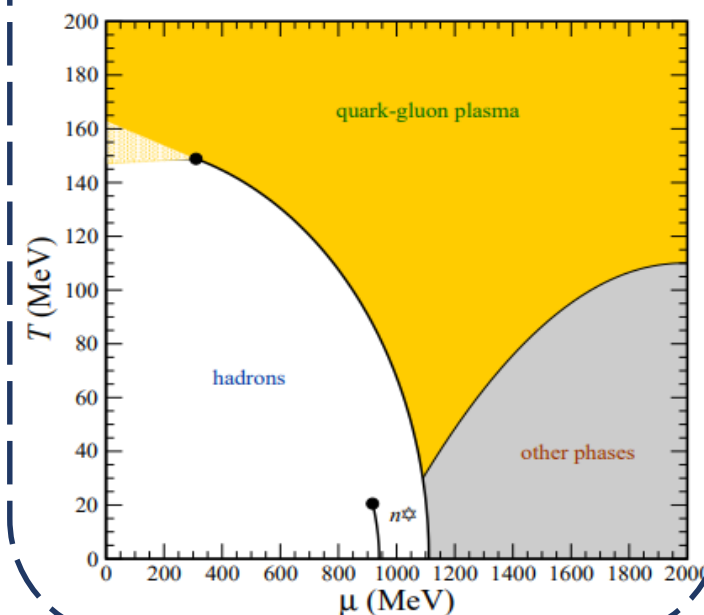
禁闭现象和渐近自由



强子谱计算



QCD热力学：解释夸克-胶子等离子态



三、格点规范理论的物理应用



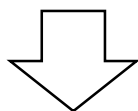
-32-

缪子反常磁矩计算

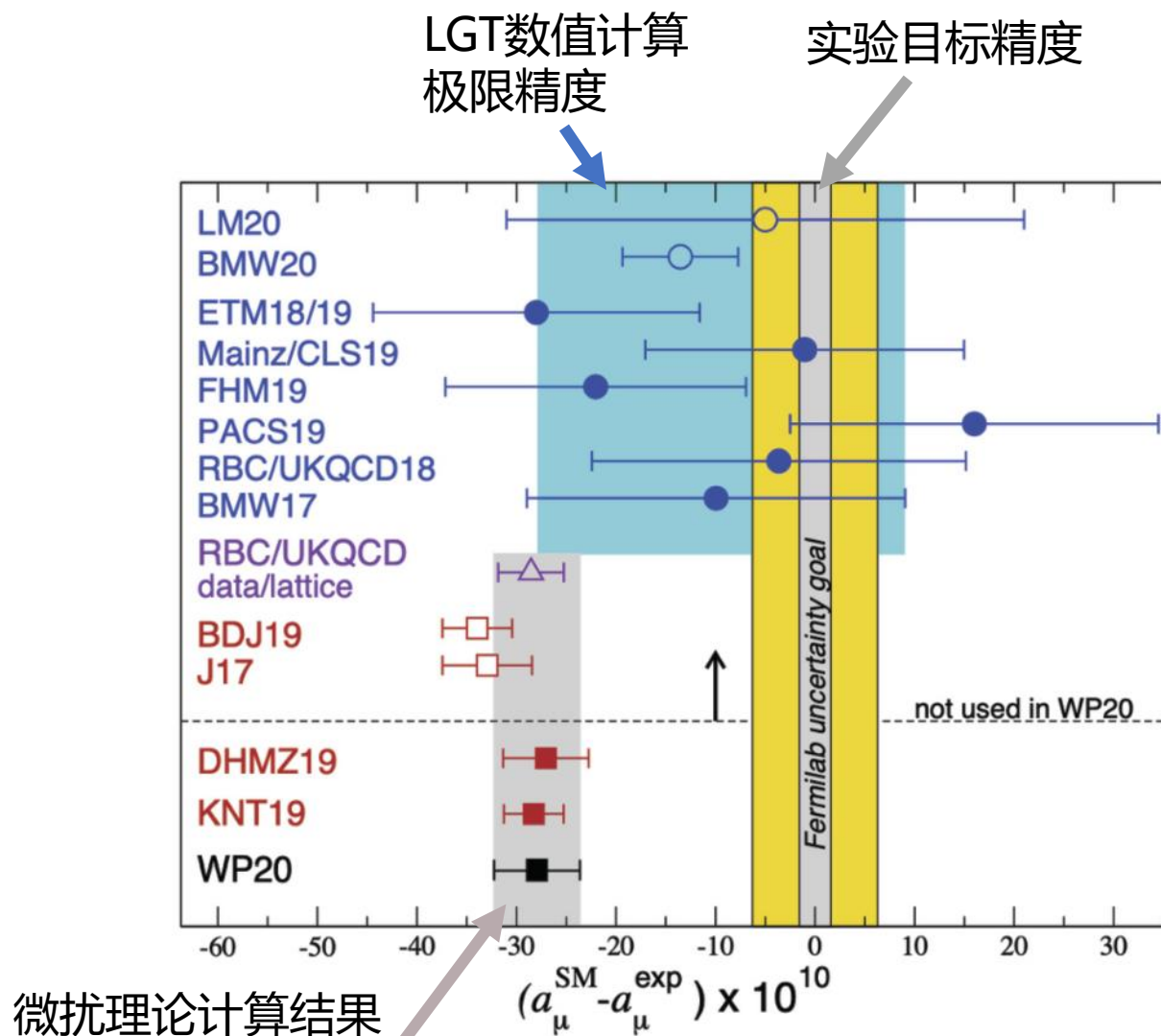
$$a_\mu = (g_\mu - 2)/2$$

标准模型与实验结果不符?

标准模型失效, 新物理?



LGT更符合实验的结果

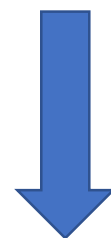


数值计算方法求解LGT中面临的难题?

数值计算方法求解LGT中面临的难题？



蒙特卡洛计算的符号问题如何解决？



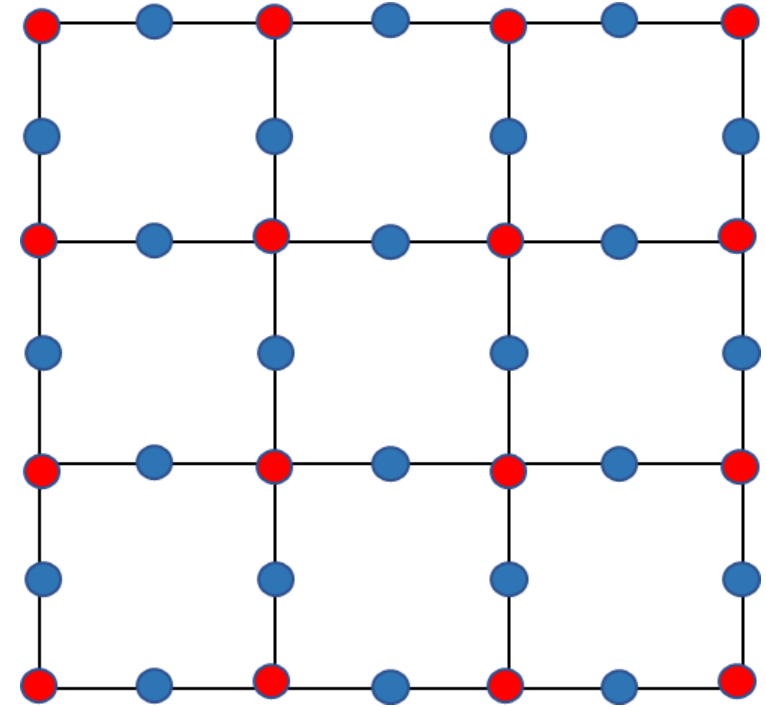
Lattice QCD
非平衡动力学演化问题？



数值误差、晶格常数等带来的精度问题？

The Hamiltonian of electromagnetic field in 2D

$$H = H_E + H_B + H_M + H_{int}$$



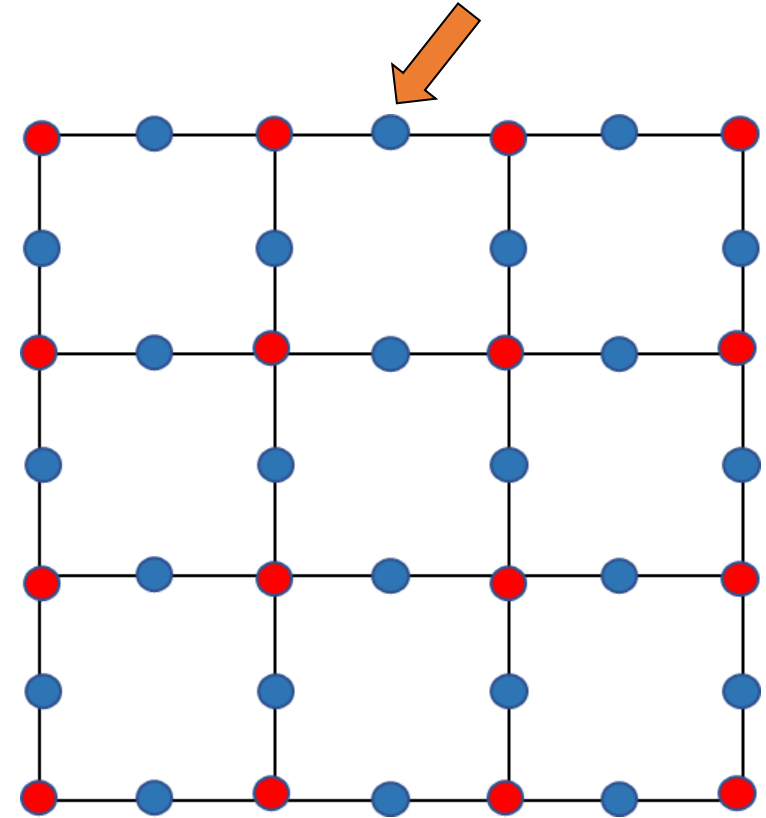
The Hamiltonian of electromagnetic field in 2D



-36-

$$H = H_E + H_B + H_M + H_{int}$$

$$H_E = \frac{g^2}{2} \sum_{\mathbf{n}, k} \mathbf{L}_{\mathbf{n}, k}^2$$

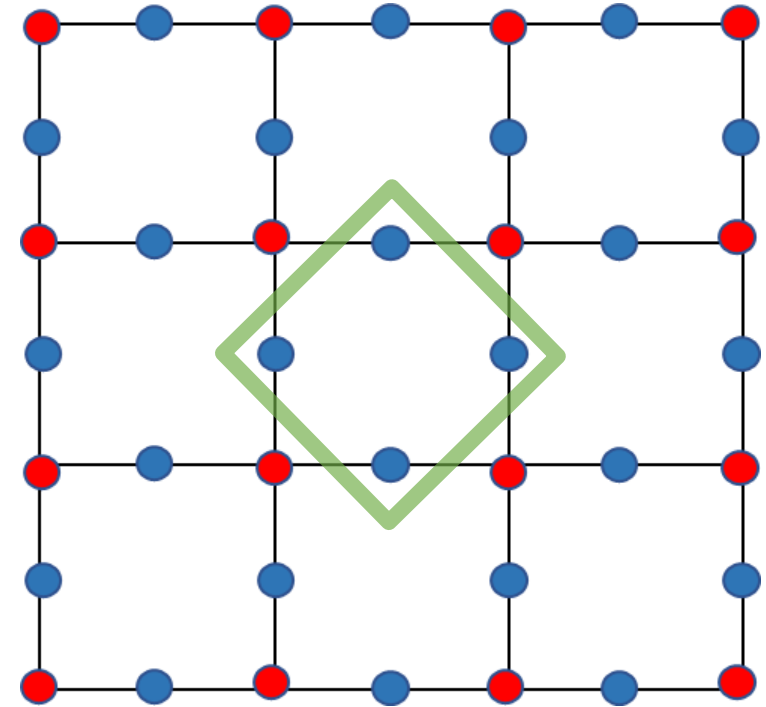


The Hamiltonian of electromagnetic field in 2D

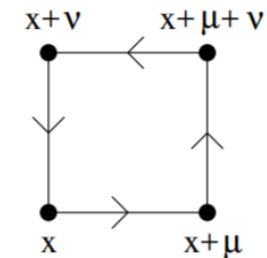
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$$H_B = -\frac{1}{g^2} \sum_{\text{plaquettes}} [\text{Tr}(U_1 U_2 U_3^\dagger U_4^\dagger) + h.c.]$$



$$W_{\square} = \text{tr} U_{\mu}(x) U_{\nu}(x + \hat{\mu}) U_{\mu}^{\dagger}(x + \hat{\nu}) U_{\nu}^{\dagger}(x)$$



Wilson Loop

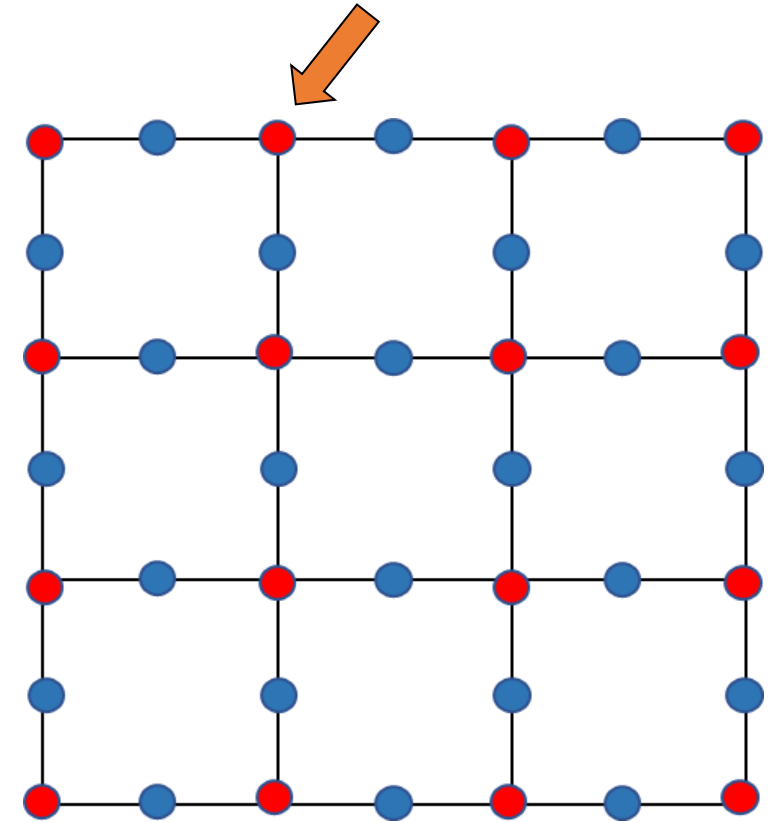
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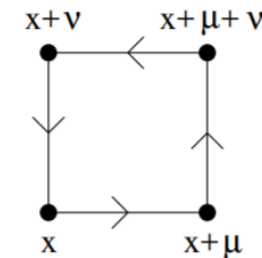
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Wilson Loop

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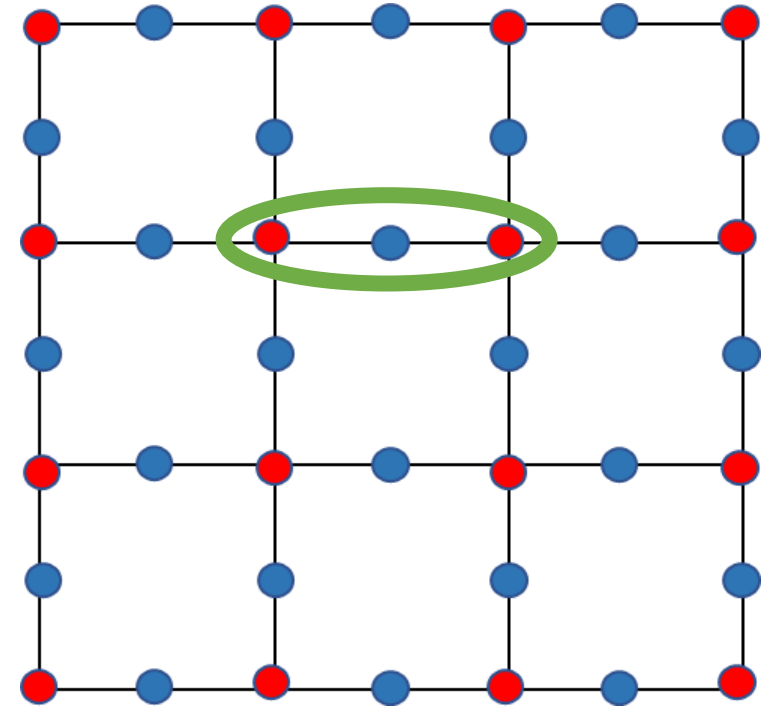
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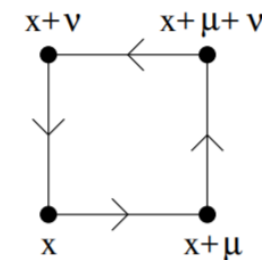
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Wilson Loop

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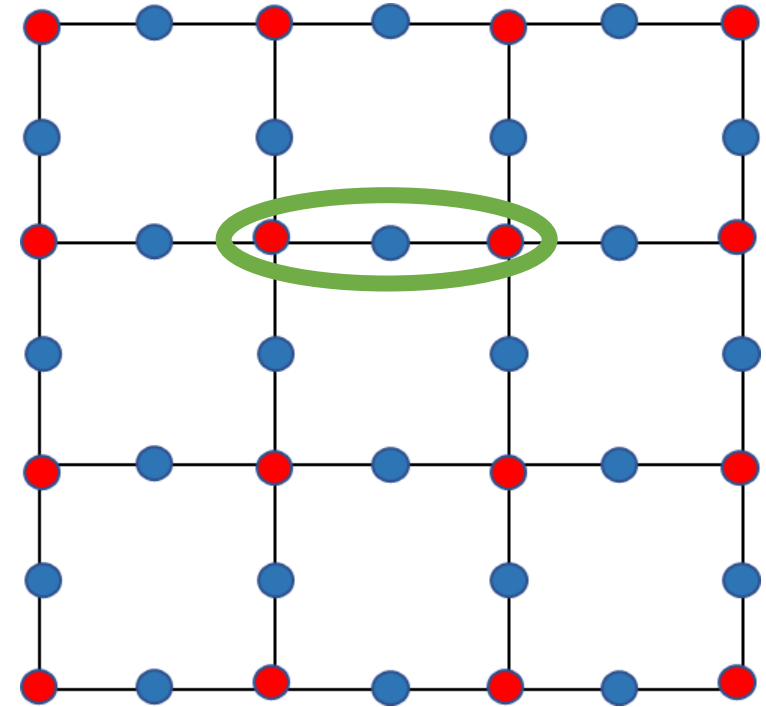
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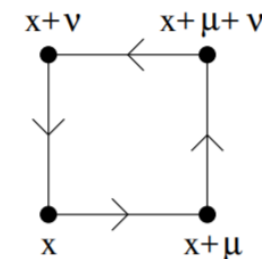
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Strongly correlated many-body system



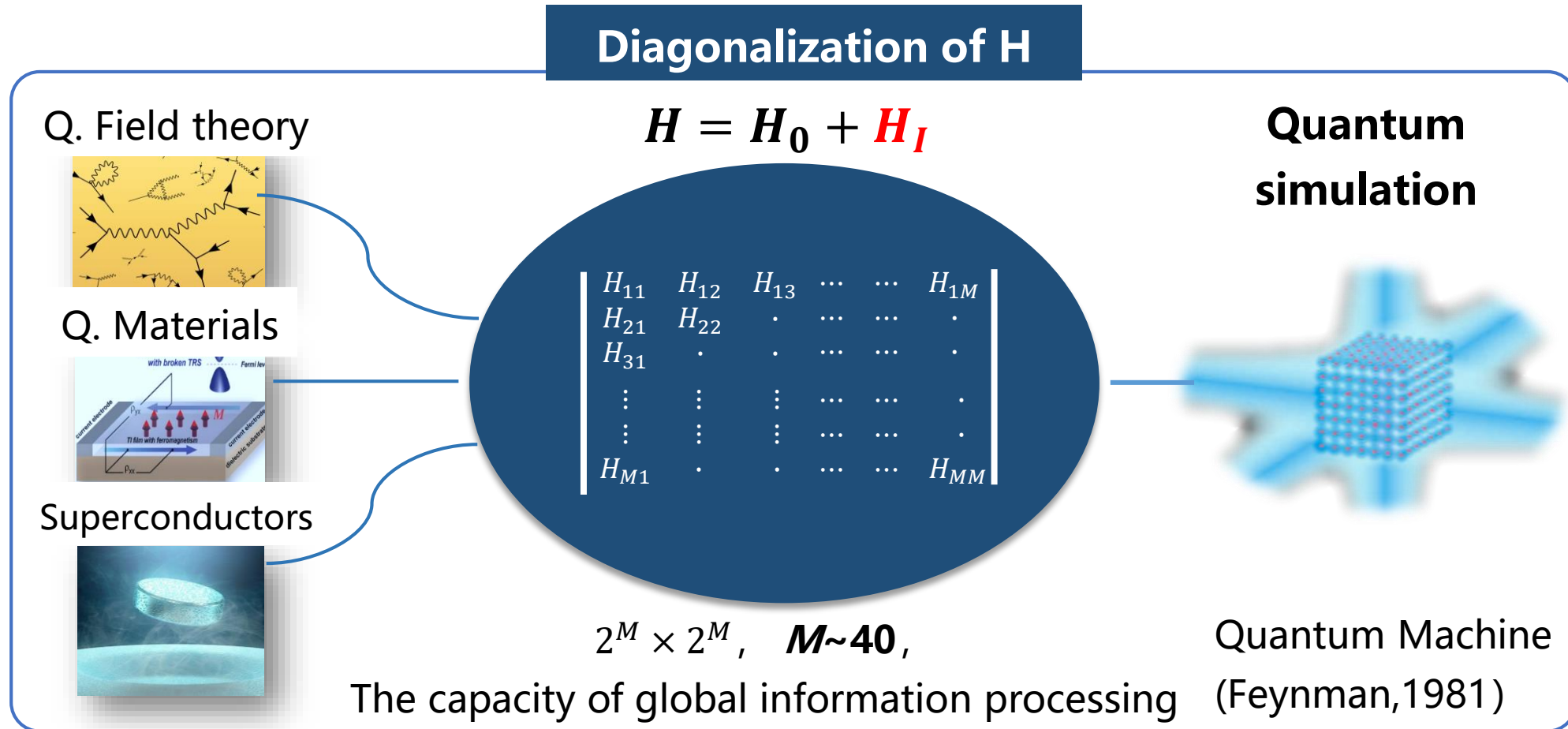
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Wilson Loop

- About lattice gauge theory (LGT)
- **Quantum simulation with ultracold atoms**
- The toric code model and Schwinger model
- Conclusion and outlook

- **Motivation:** complexity of **quantum many-body problem**



- **Requirements :** manipulation of many particles at single particle level

Superfluid-Mott Insulator transition



-43-

81, NUMBER 15

PHYSICAL REVIEW LETTERS

12 OCTOBER 1998

Cold Bosonic Atoms in Optical Lattices

D. Jaksch,^{1,2} C. Bruder,^{1,3} J. I. Cirac,^{1,2} C. W. Gardiner,^{1,4} and P. Zoller^{1,2}

¹*Institute for Theoretical Physics, University of Santa Barbara, Santa Barbara, California 93106-4030*

²*Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria*

³*Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*

⁴*School of Chemical and Physical Sciences, Victoria University, Wellington, New Zealand*

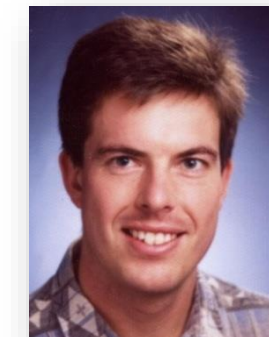
(Received 26 May 1998)

The dynamics of an ultracold dilute gas of bosonic atoms in an optical lattice can be described by a Bose-Hubbard model where the system parameters are controlled by laser light. We study the continuous (zero temperature) quantum phase transition from the superfluid to the Mott insulator phase induced by varying the depth of the optical potential, where the Mott insulator phase corresponds to a commensurate filling of the lattice (“optical crystal”). Examples for formation of Mott structures in optical lattices with a superimposed harmonic trap and in optical superlattices are presented. [S0031-9007(98)07267-6]

$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + \sum_i \epsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$



M. Fisher
PRB, 1989



D. Jaksch



P. Zoller

Superfluid-Mott Insulator transition

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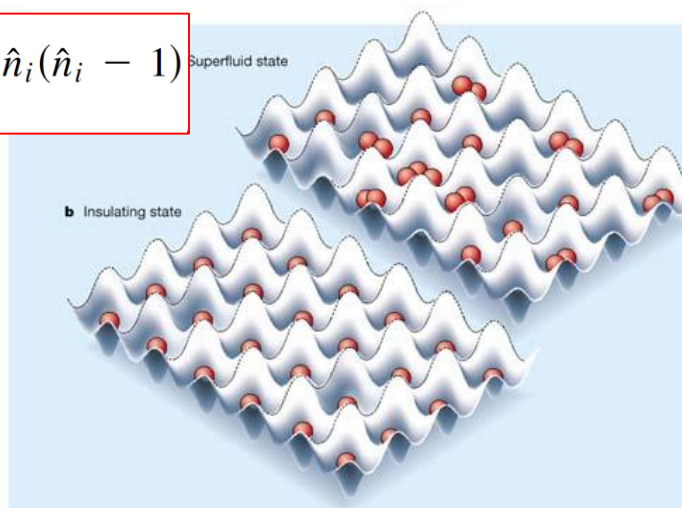
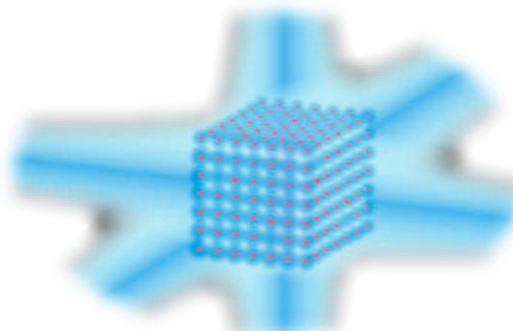
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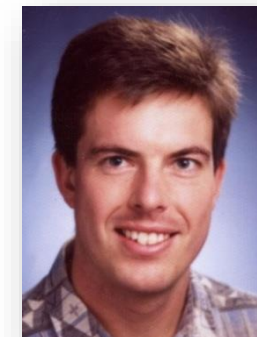
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Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

**Nature
2002**

Markus Greiner^{*}, Olaf Mandel^{*}, Tilman Esslinger[†], Theodor W. Hänsch^{*} & Immanuel Bloch^{*}

^{*} *Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany, and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany*

[†] *Quantenelektronik, ETH Zürich, 8093 Zurich, Switzerland*

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For a system at a temperature of absolute zero, all thermal fluctuations are frozen out, while quantum fluctuations prevail. These microscopic quantum fluctuations can induce a macroscopic phase transition in the ground state of a many-body system when the relative strength of two competing energy terms is varied across a critical value. Here we observe such a quantum phase transition in a Bose–Einstein condensate with repulsive interactions, held in a three-dimensional optical lattice potential. As the potential depth of the lattice is increased, a transition is observed from a superfluid to a Mott insulator phase. In the superfluid phase, each atom is spread out over the entire lattice, with long-range phase coherence. But in the insulating phase, exact numbers of atoms are localized at individual lattice sites, with no phase coherence across the lattice; this phase is characterized by a gap in the excitation spectrum. We can induce reversible changes between the two ground states of the system.

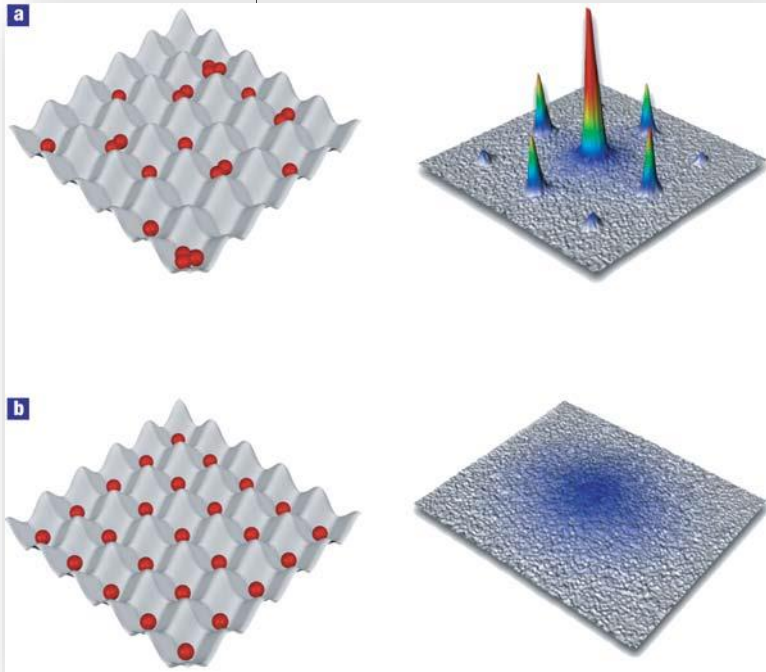
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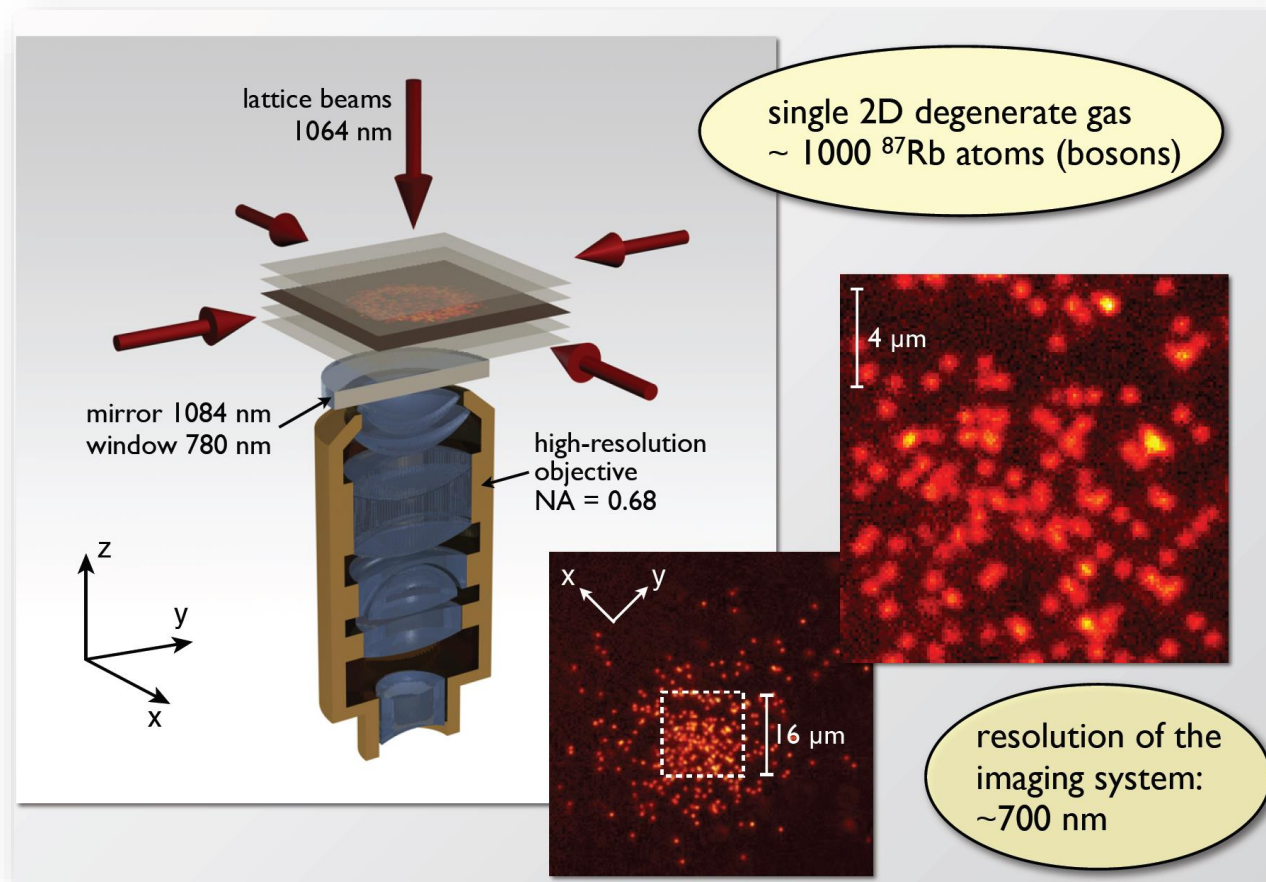
Universität, Schellingstrasse 4/III, D-80799 Munich, Germany, and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany, Switzerland



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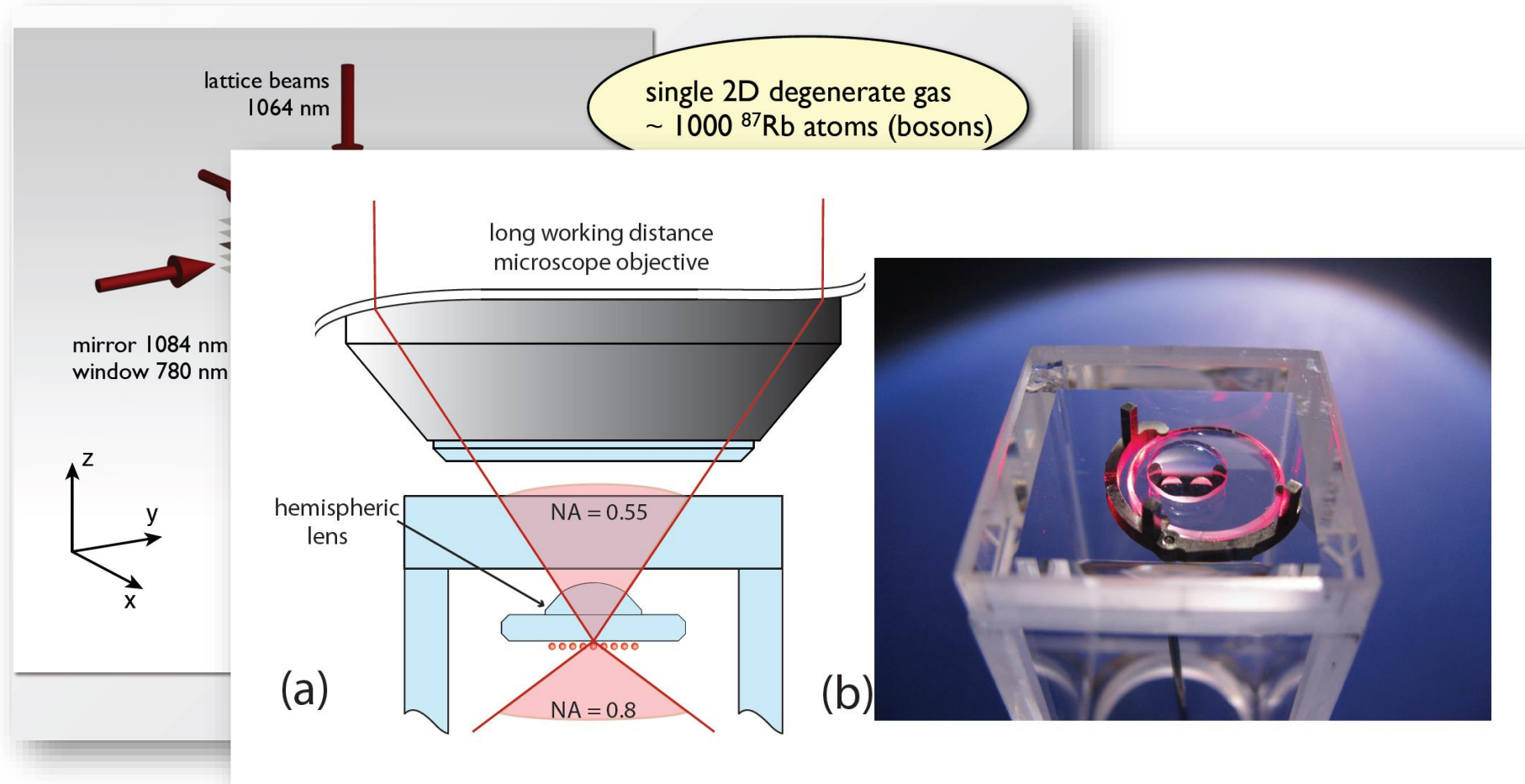
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Quantum gas microscope



I Bloch@MPQ

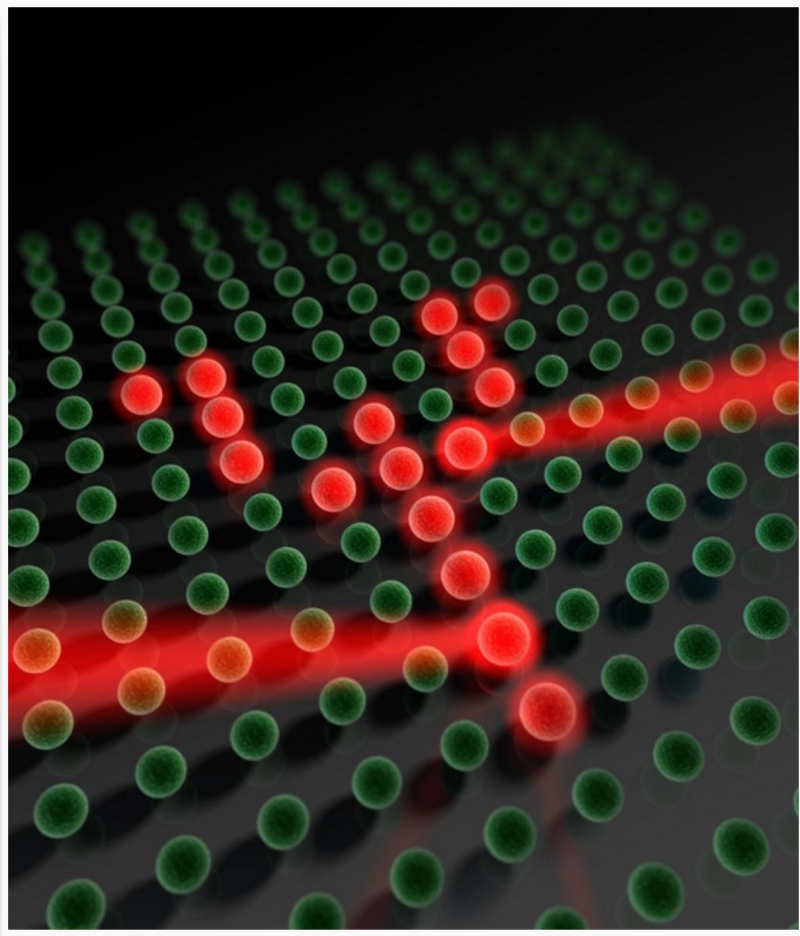
Quantum gas microscope



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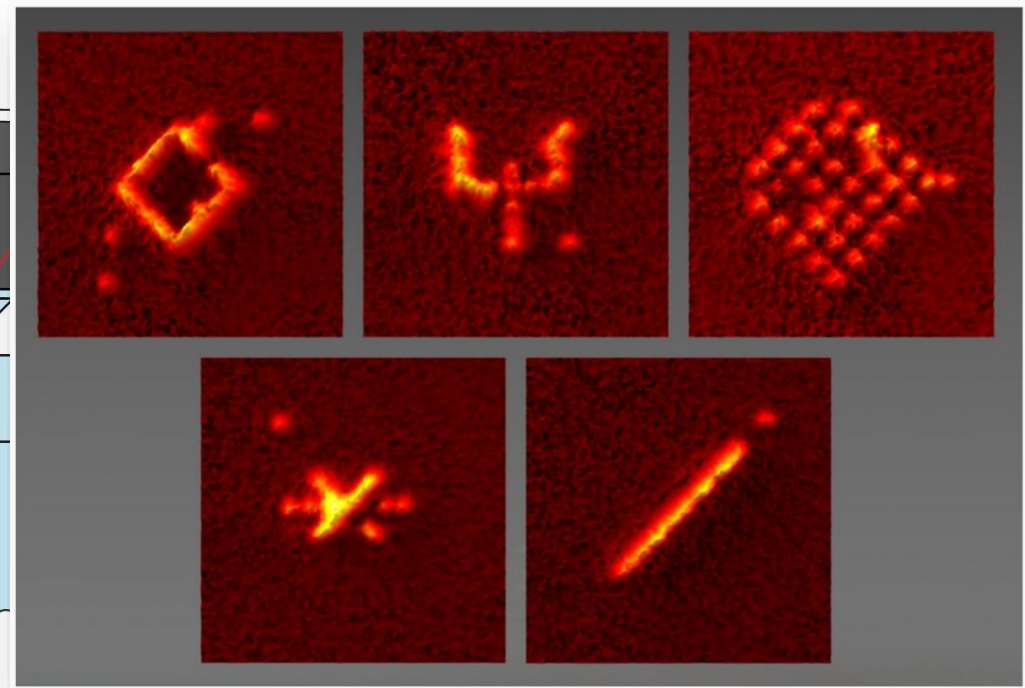
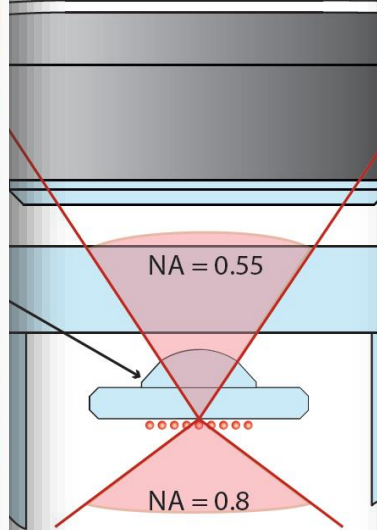
M Greiner@Harvard

Quantum gas microscope



single 2D degenerate gas
~ 1000 ^{87}Rb atoms (bosons)

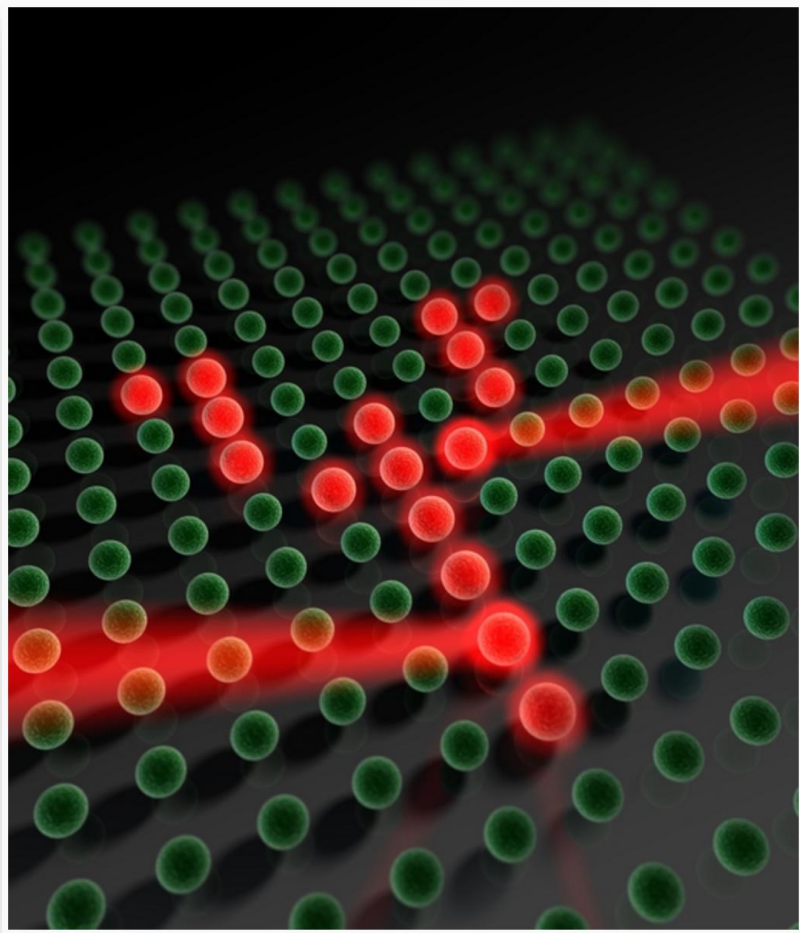
long working distance
microscope objective



I Bloch@MPQ

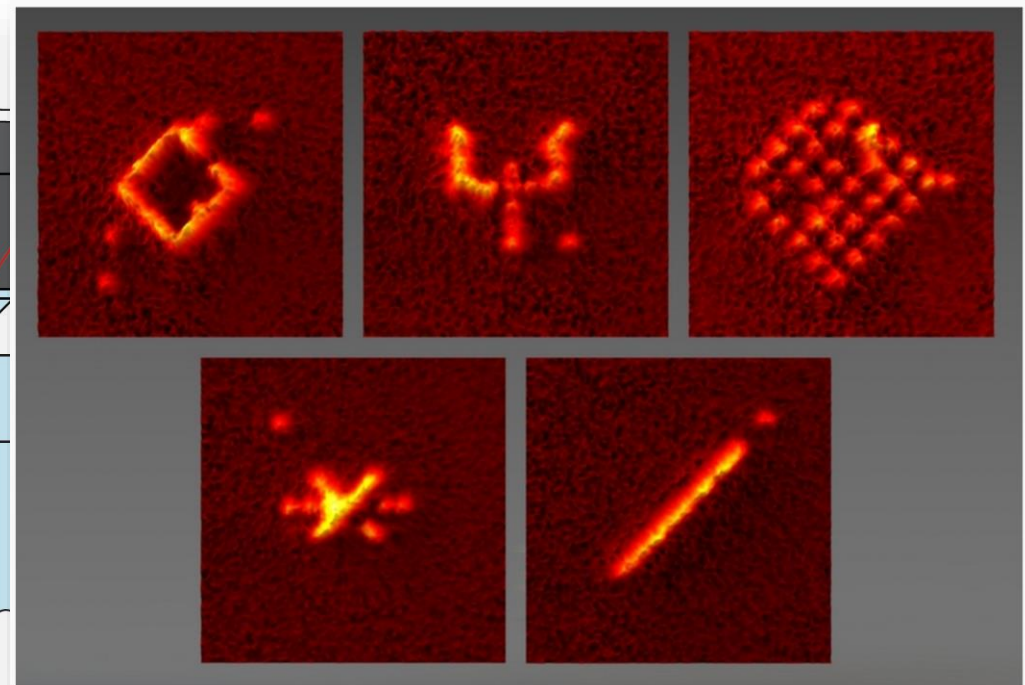
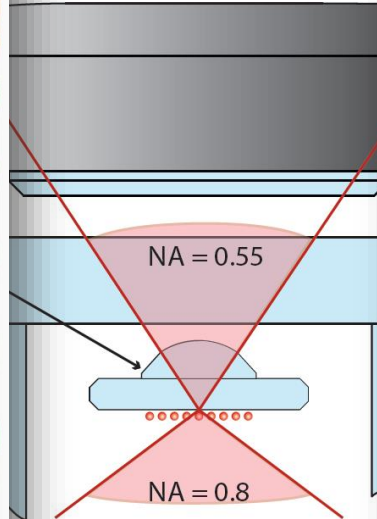
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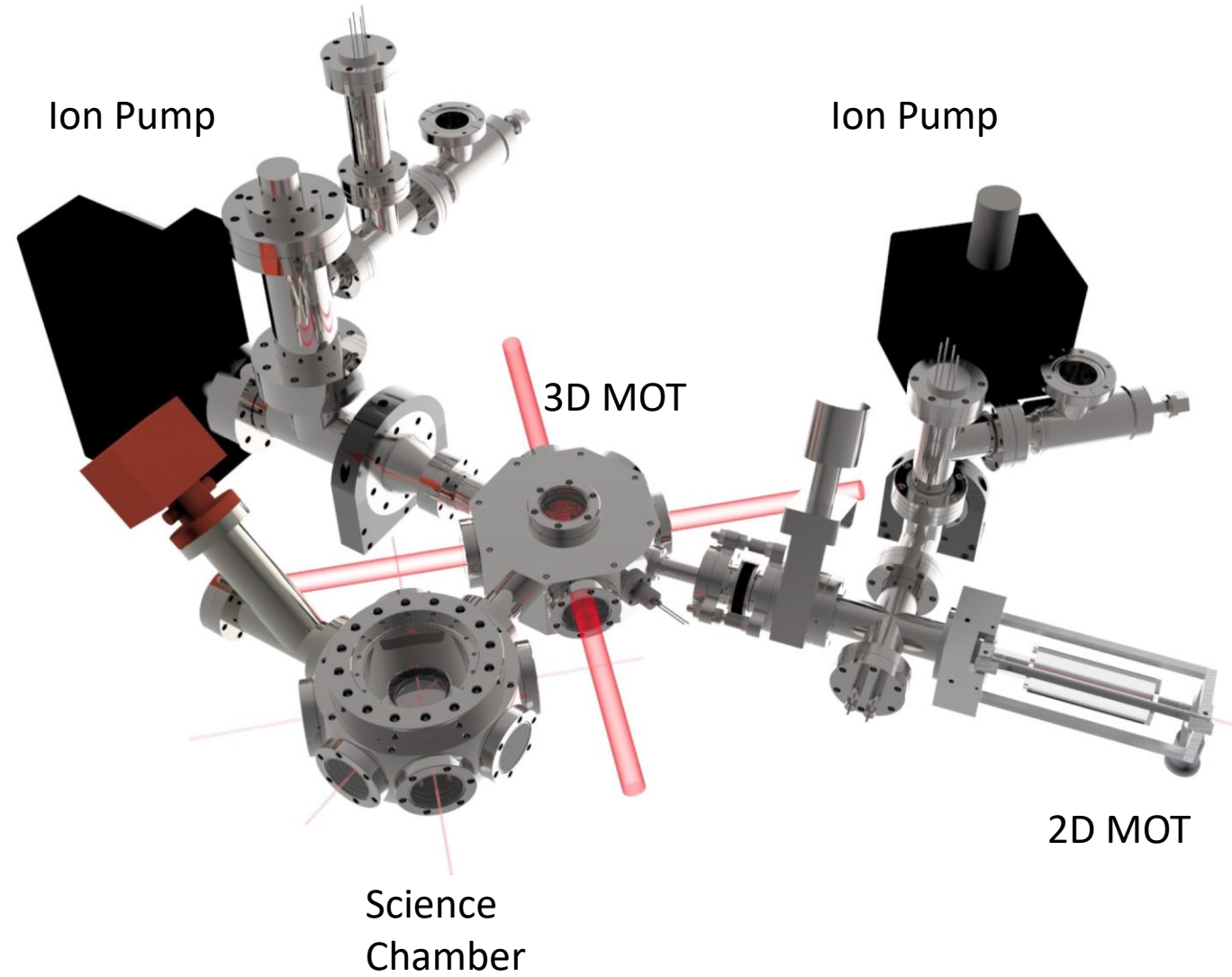
M Greiner@Harvard

S. Kuhr@Glasgow, M. Zwierlein@MIT.....

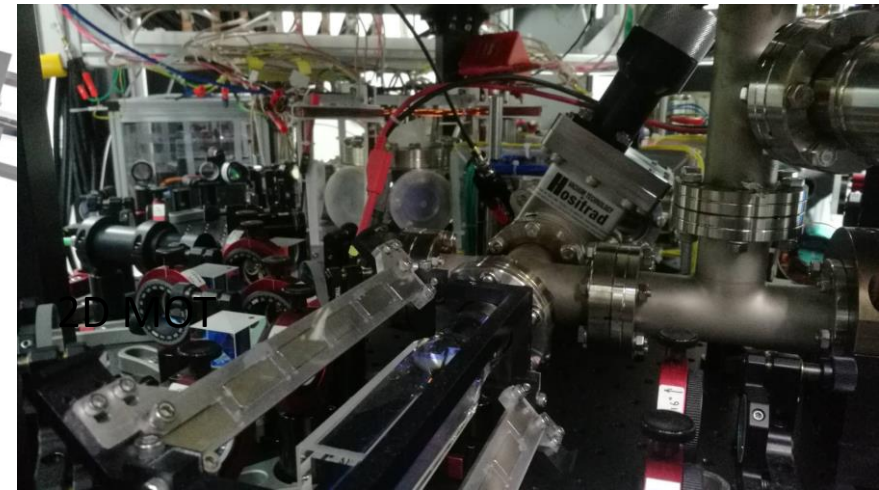
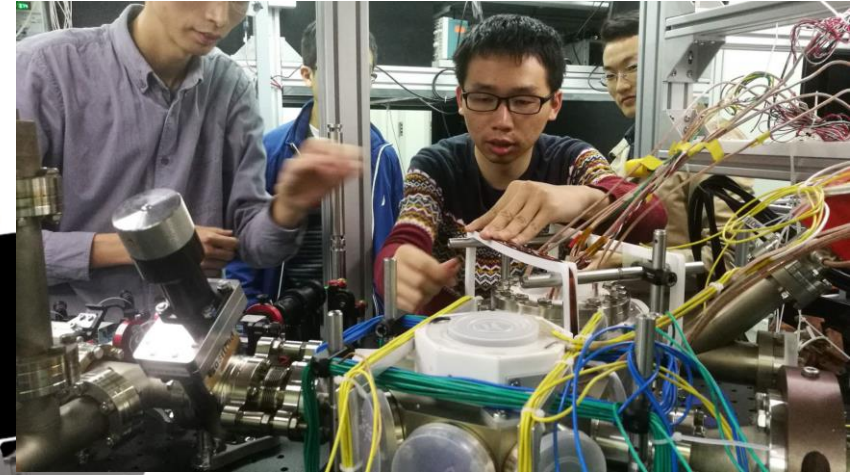
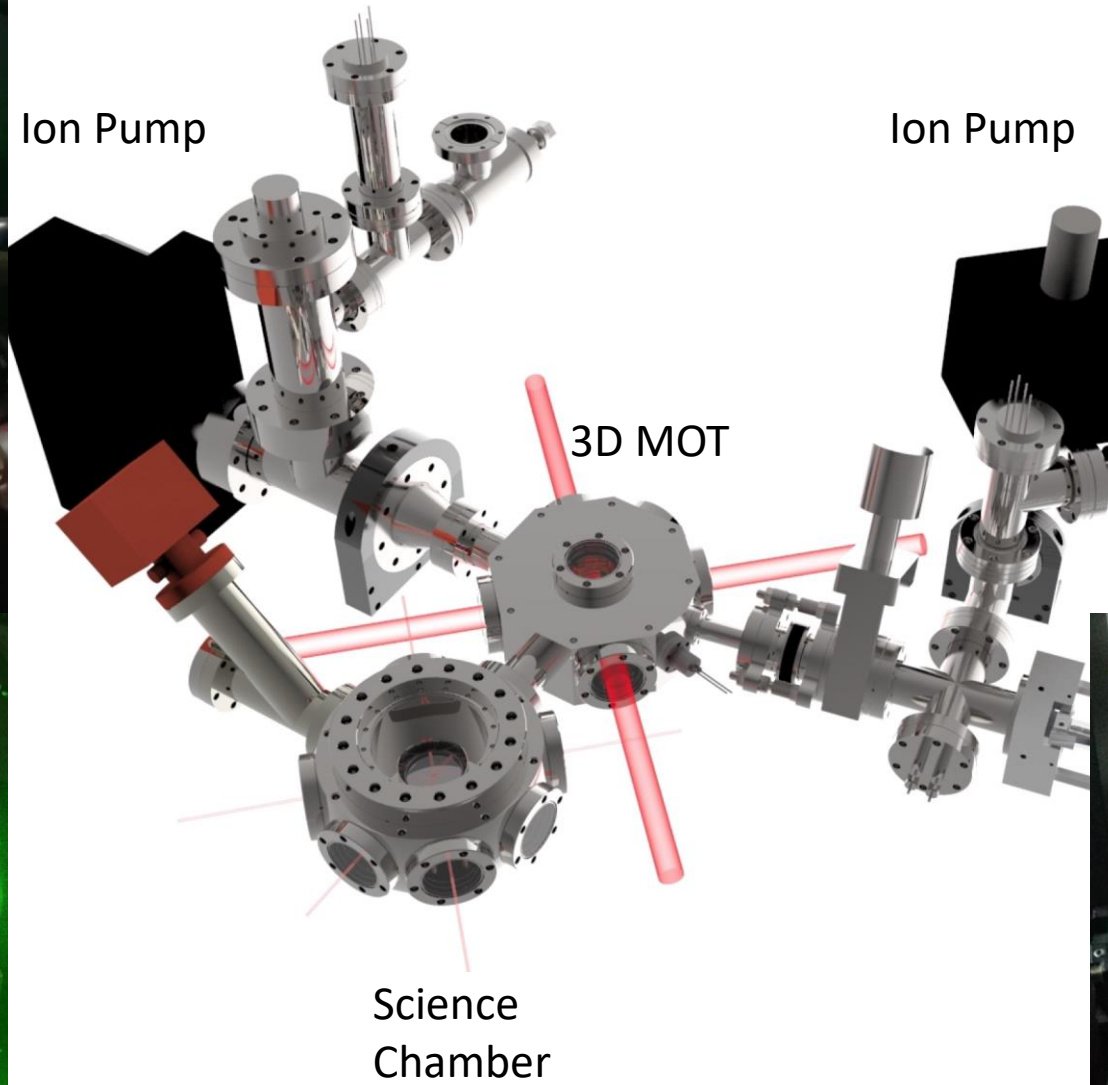
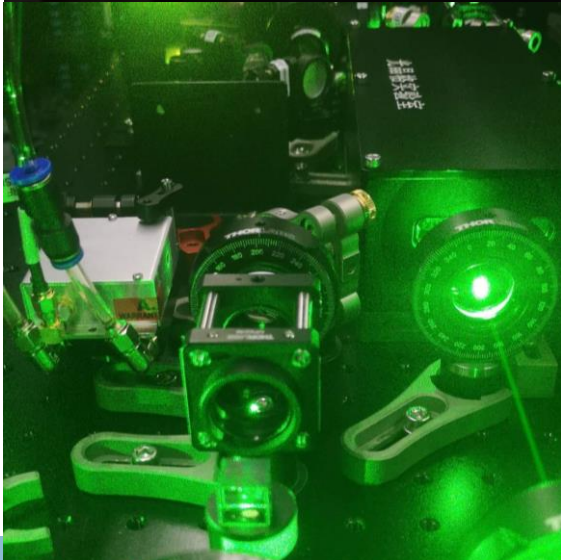
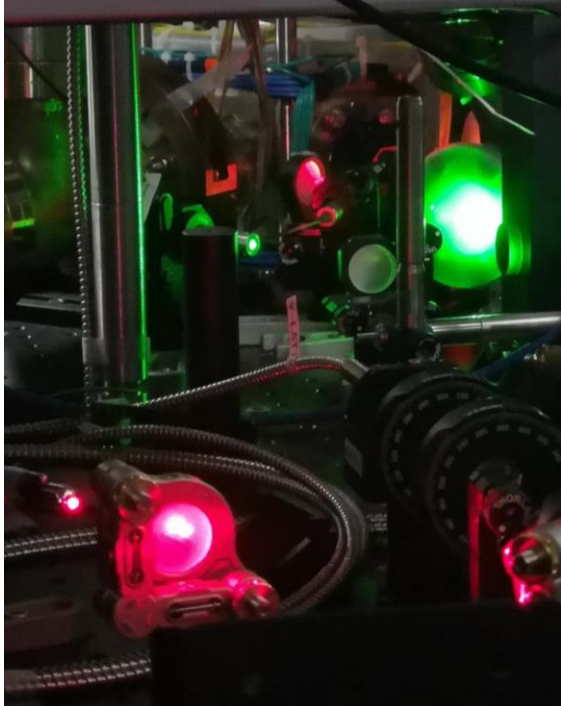
Experimental setup



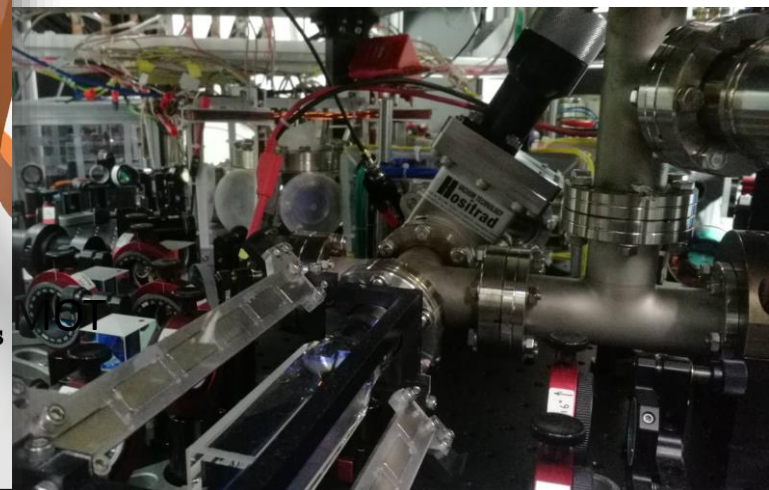
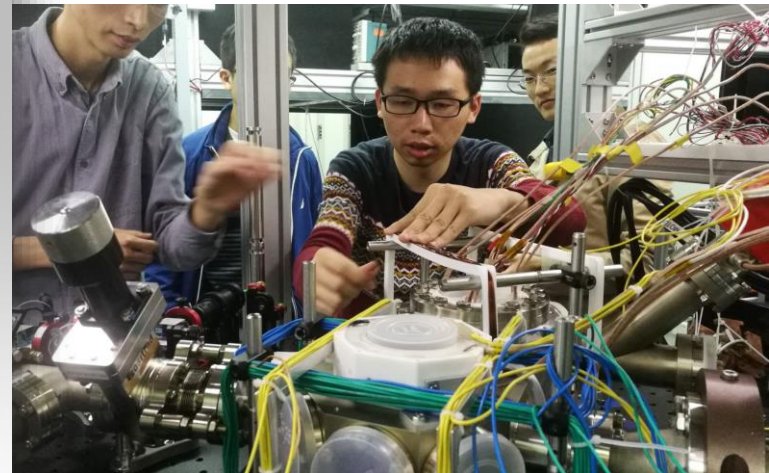
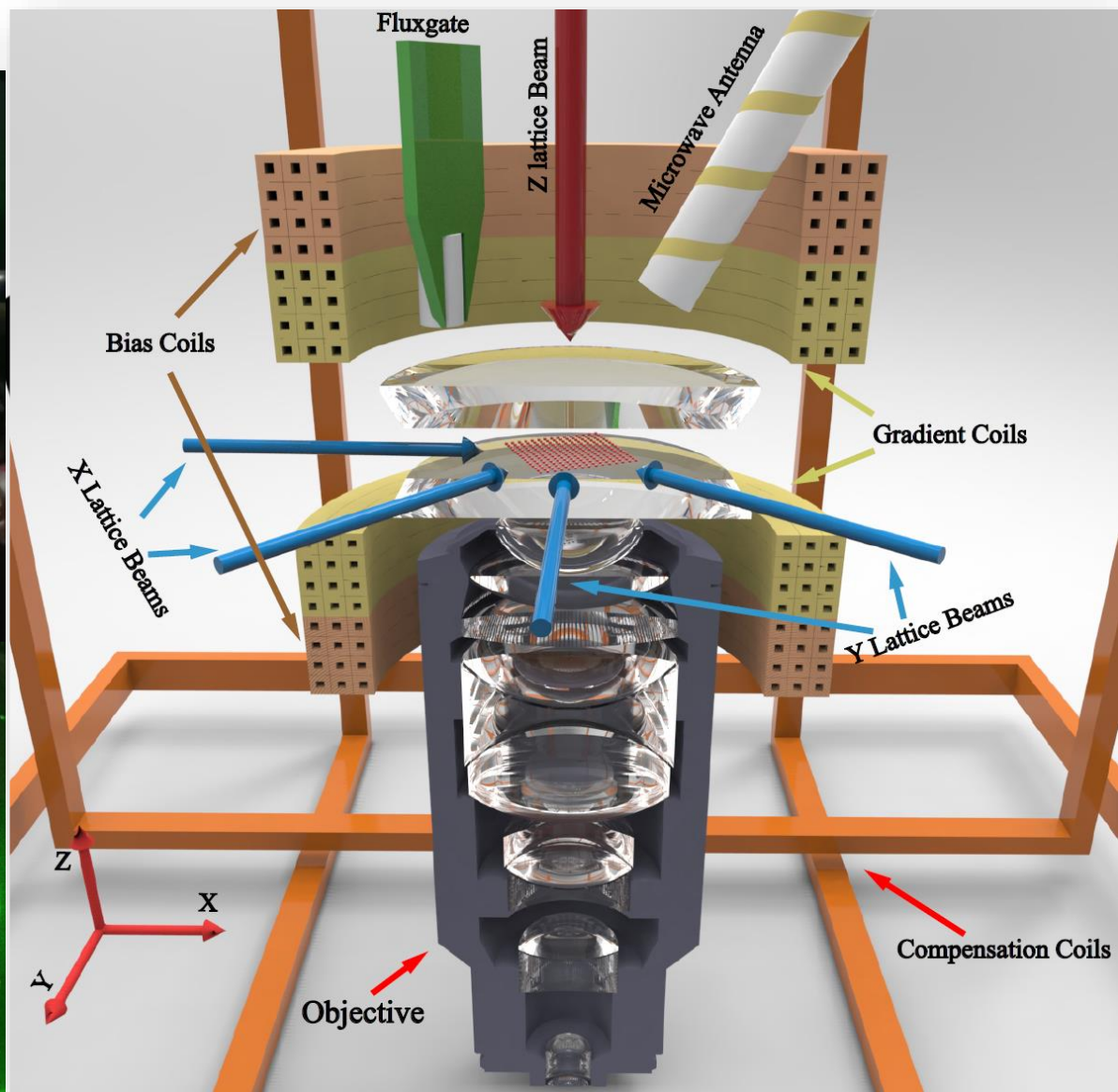
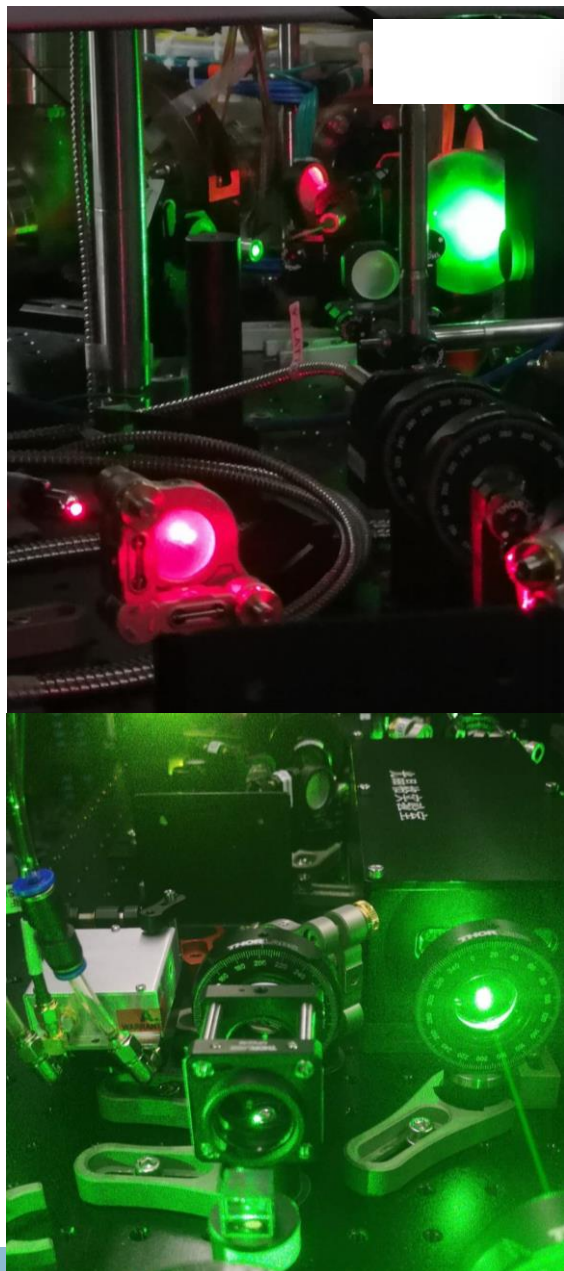
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Experimental setup



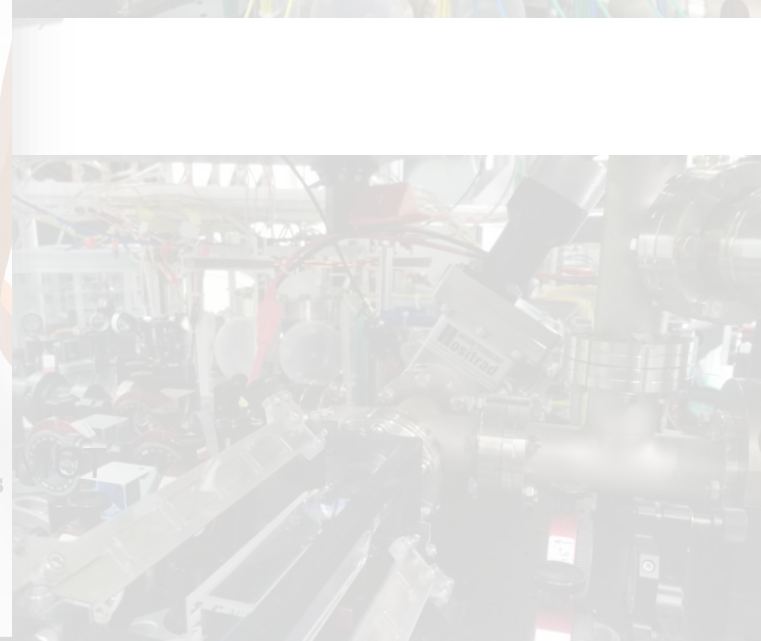
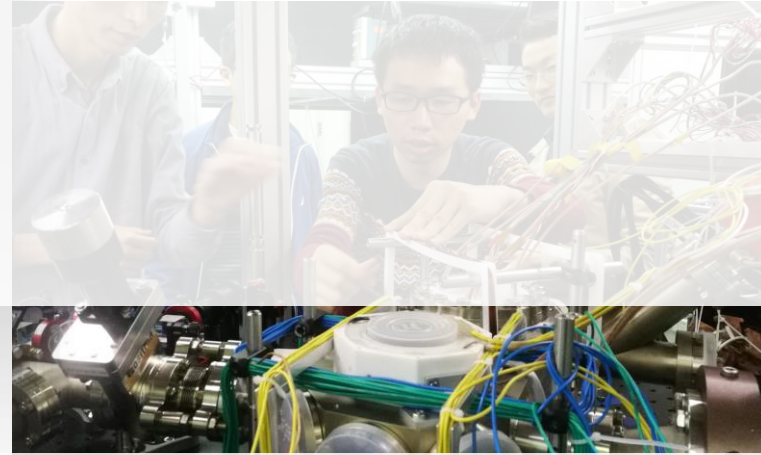
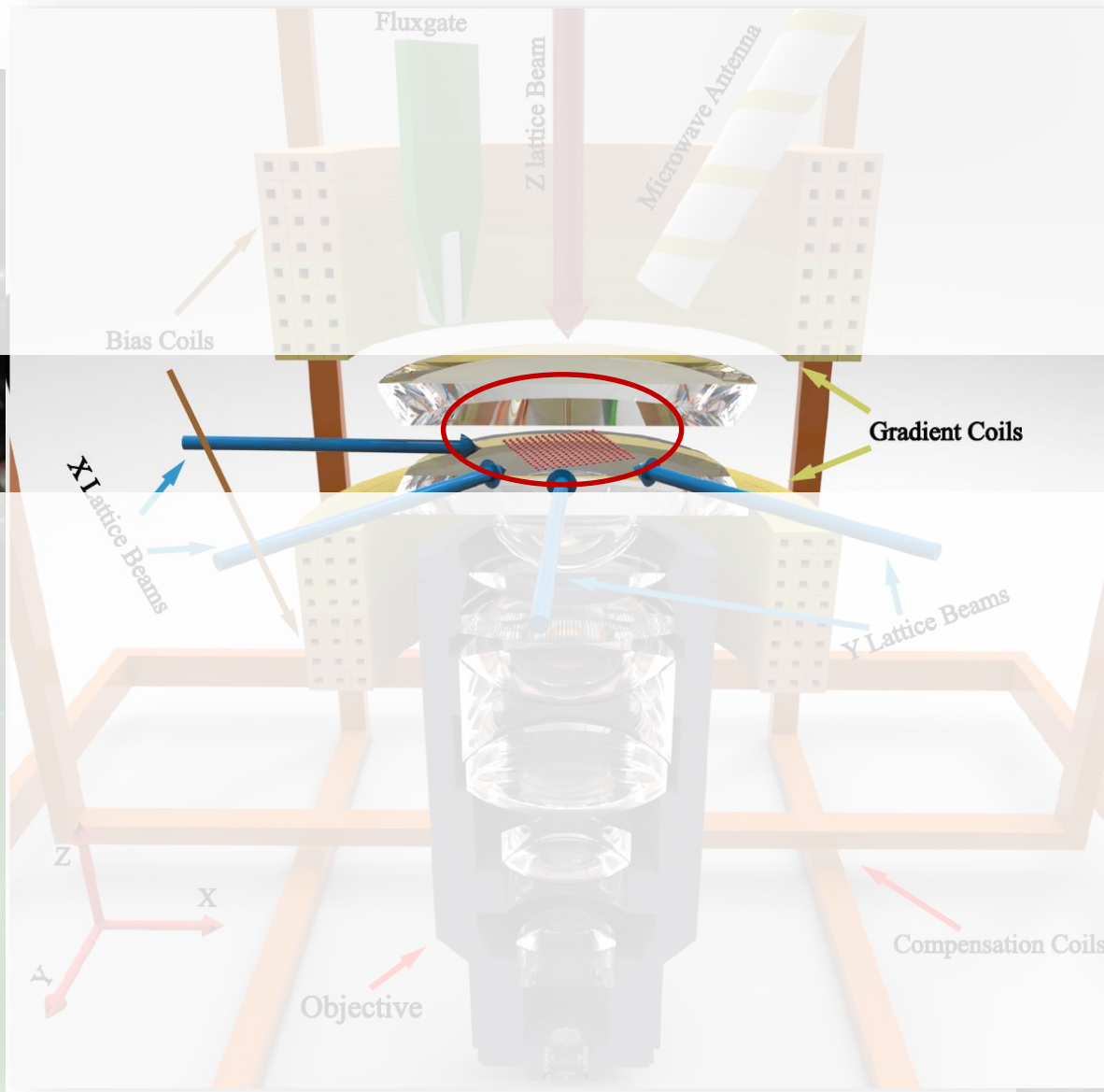
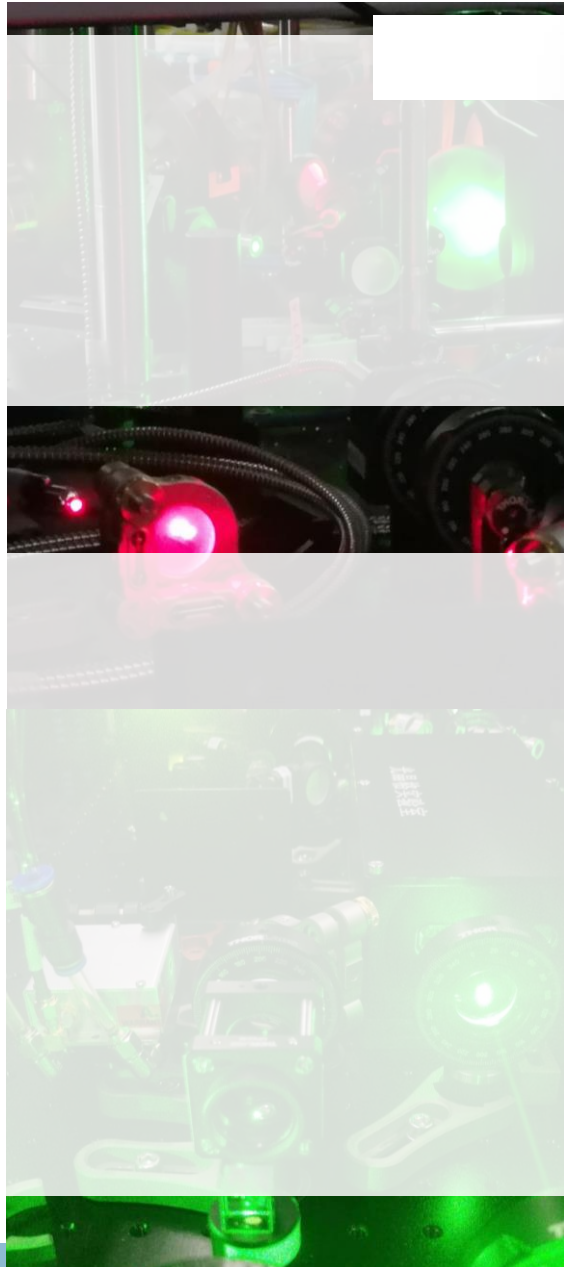
Experimental setup



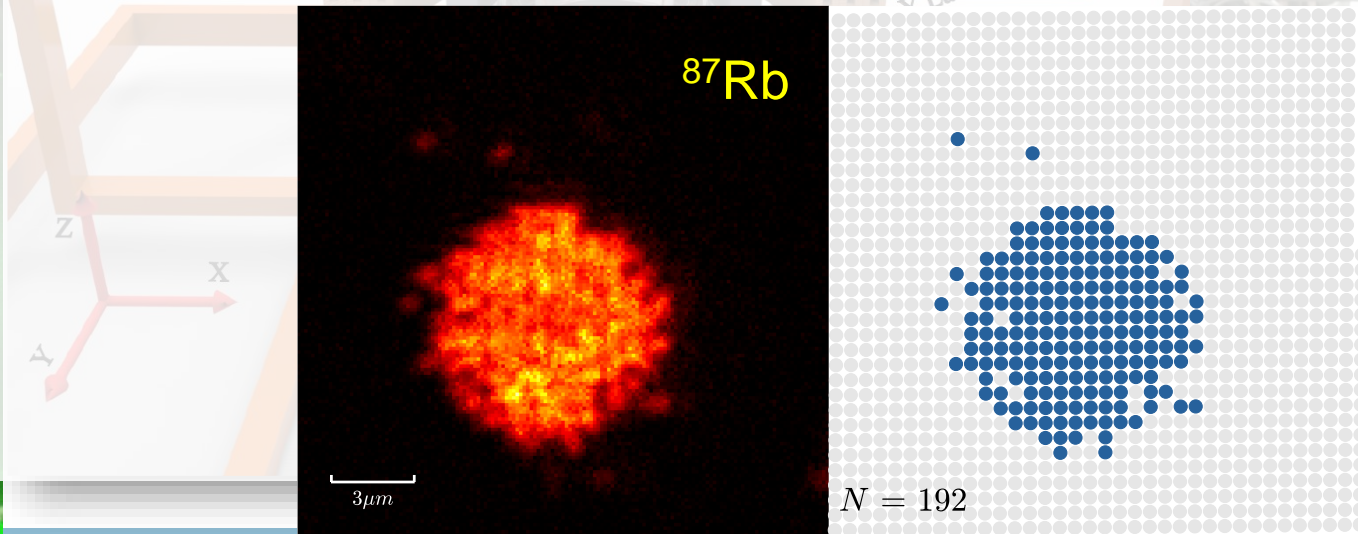
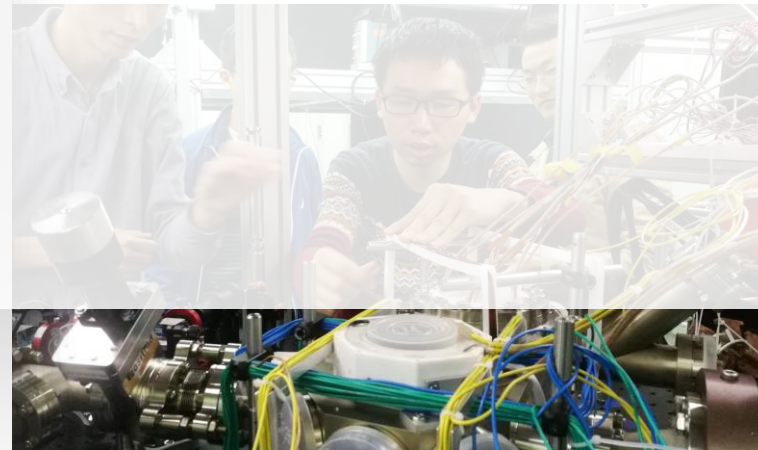
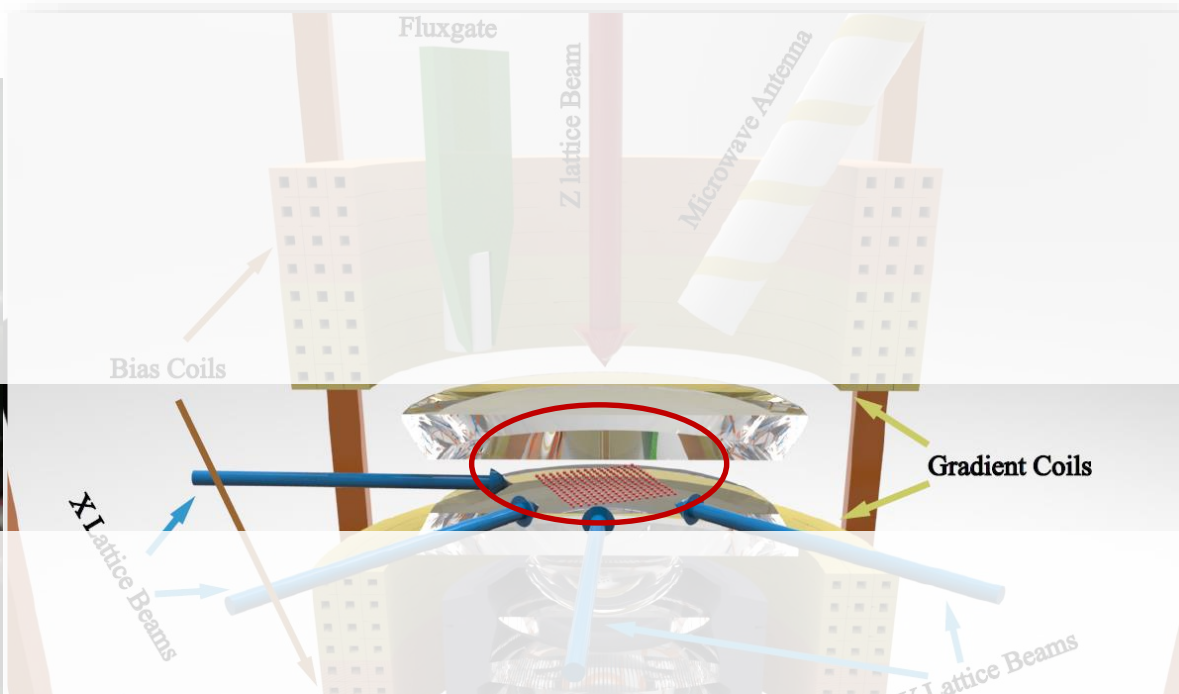
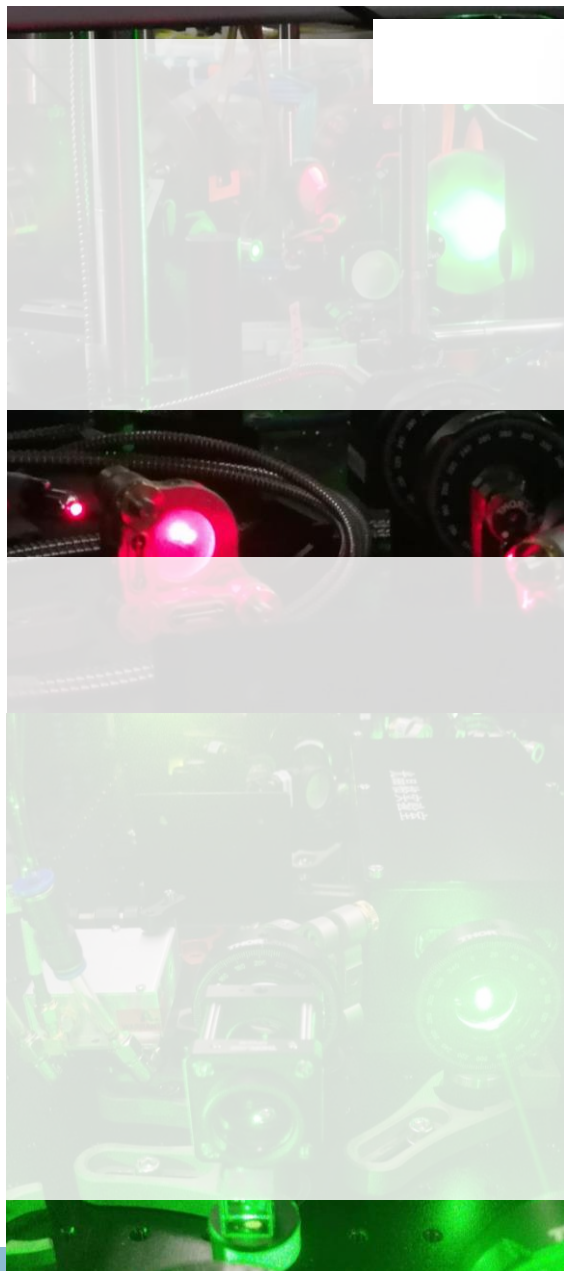
Experimental setup



-54-

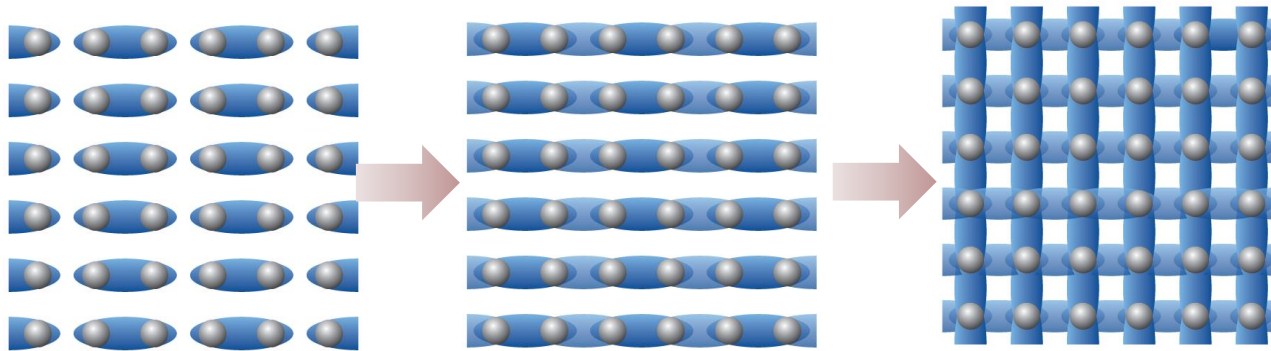


Experimental setup

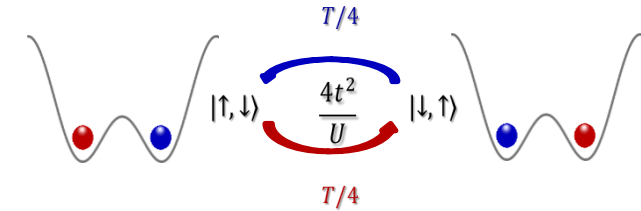


Entanglement of atoms in optical lattices

Multi-atom entanglement!



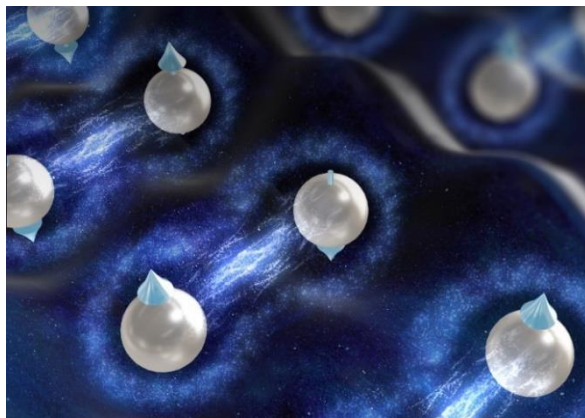
Vaucher *et al*, NJP (2008)



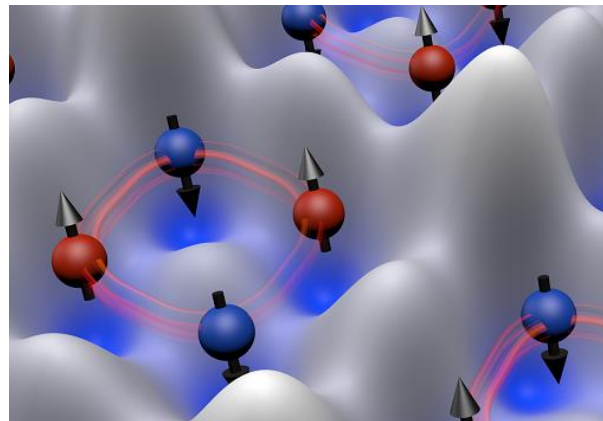
Spin exchange interaction:

Duan *et al.*, PRL 91, 090402 (2003)

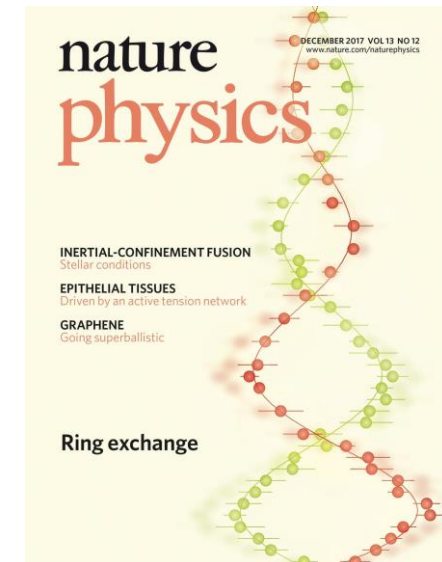
Trotzky *et al.*, Science 319, 295 (2008)



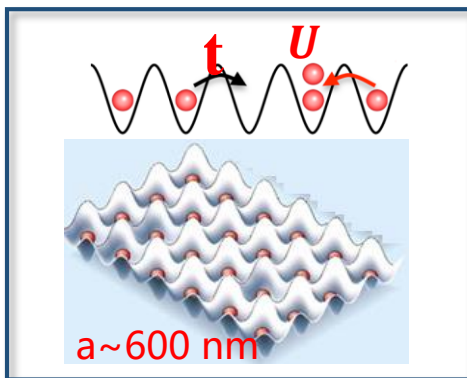
Dai *et al*, Nature Physics (2016)




Dai *et al*, Nature Physics (2017)



■ Quantum simulator : Hubbard Model



$$\hat{H} = -t \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \sum_i \mu_i \hat{n}_i$$


 t : Hopping in neighboring sites
 U : On-site interaction
 μ_i : Chemical potential

t/U competition

superfluid-insulator transition

$U \gg t$

$H \sim J_{ex} \vec{S}_i \cdot \vec{S}_j$

$J_{ex} = t^2 / U$

Spin model

$U \gg t$, spin dependency

$H \sim J_{\blacksquare} \vec{S}_1 \cdot \vec{S}_2 \cdot \vec{S}_3 \cdot \vec{S}_4$

$J_{\blacksquare} = t^4 / U^3$

Topological models

Tool kits for Quantum simulation

- About lattice gauge theory (LGT)
- Quantum simulation with ultracold atoms
- ▣ **The toric code model and Schwinger model**
- Conclusion and outlook

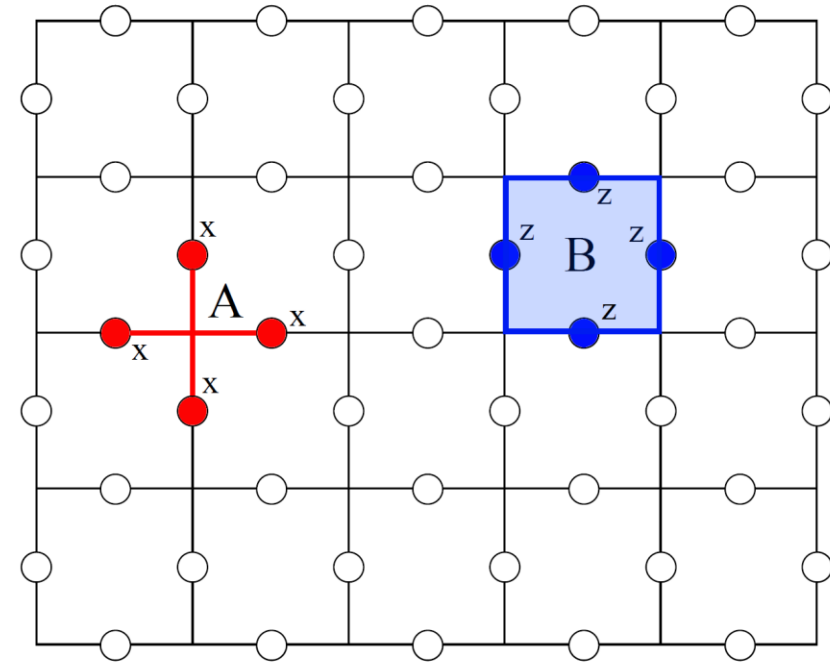
Hamiltonian:

$$H_0 = - \sum_s A_s - \sum_p B_p$$

$$A_s = \prod_{j \in \text{star}(s)} \sigma_j^x$$

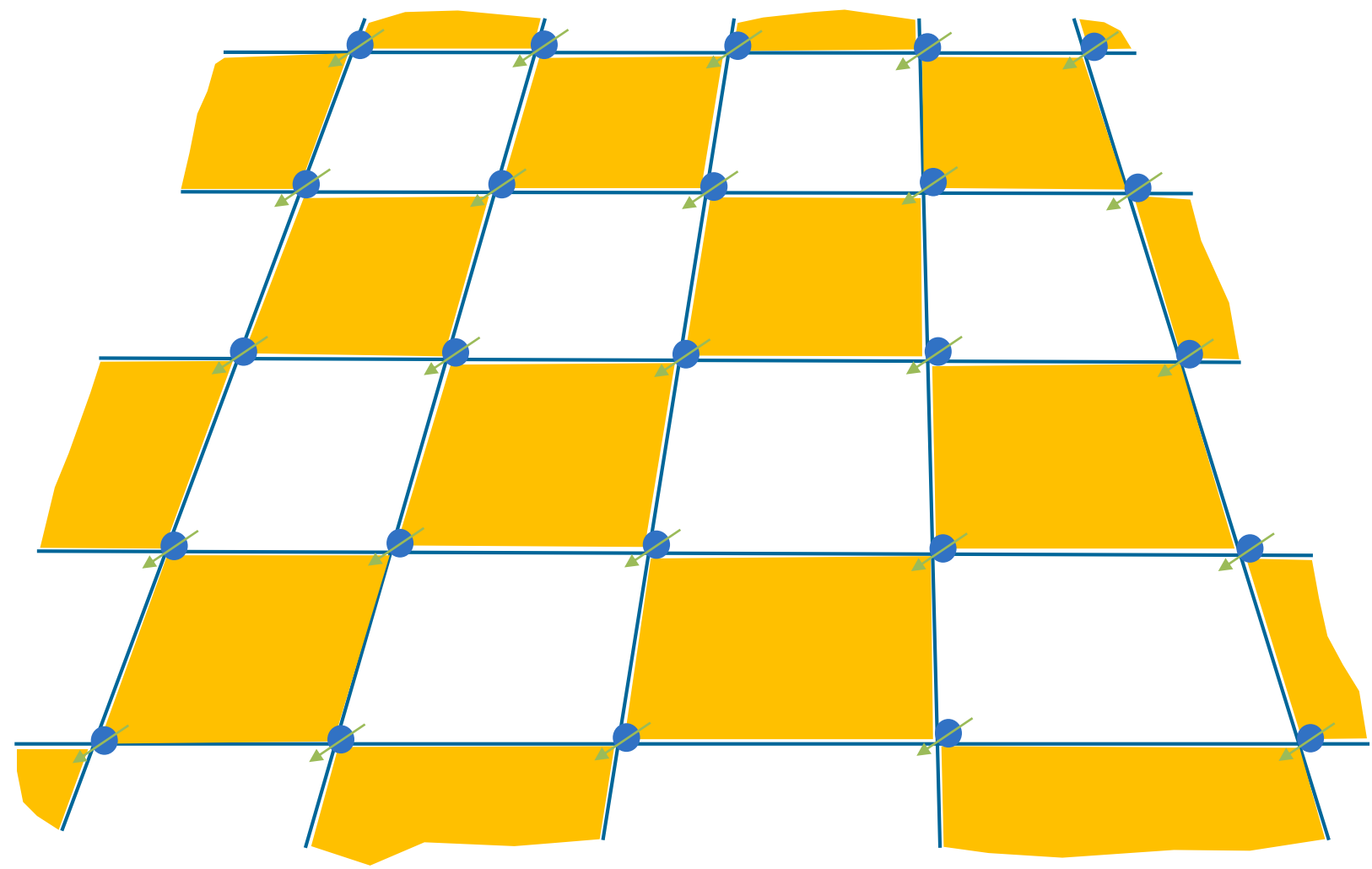
$$B_p = \prod_{j \in \text{boundary}(p)} \sigma_j^z$$

- Four-body interaction
- Abelian Anyons: e , m excitations

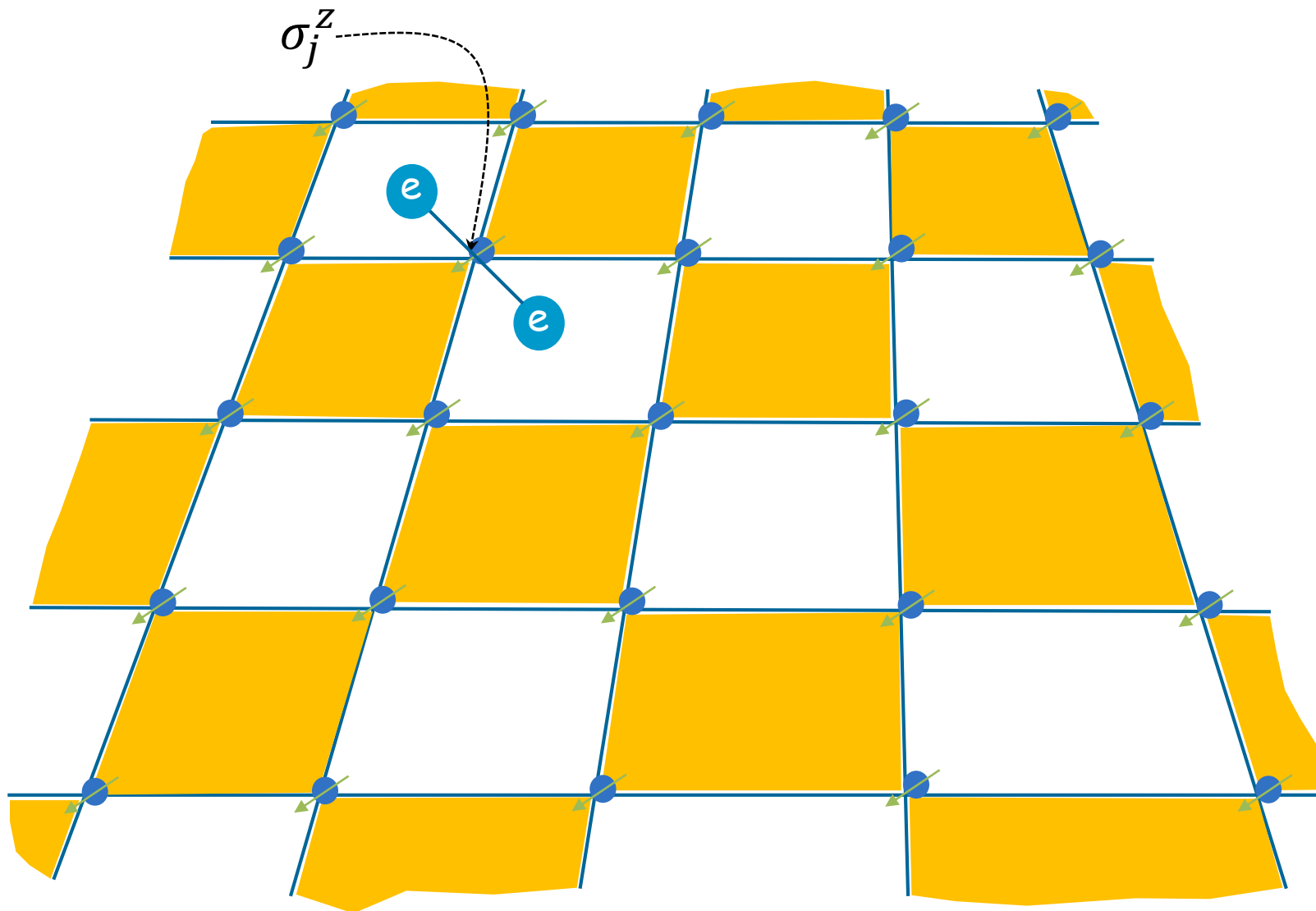


Kitaev, Annals of Physics 303, 2 (2003)

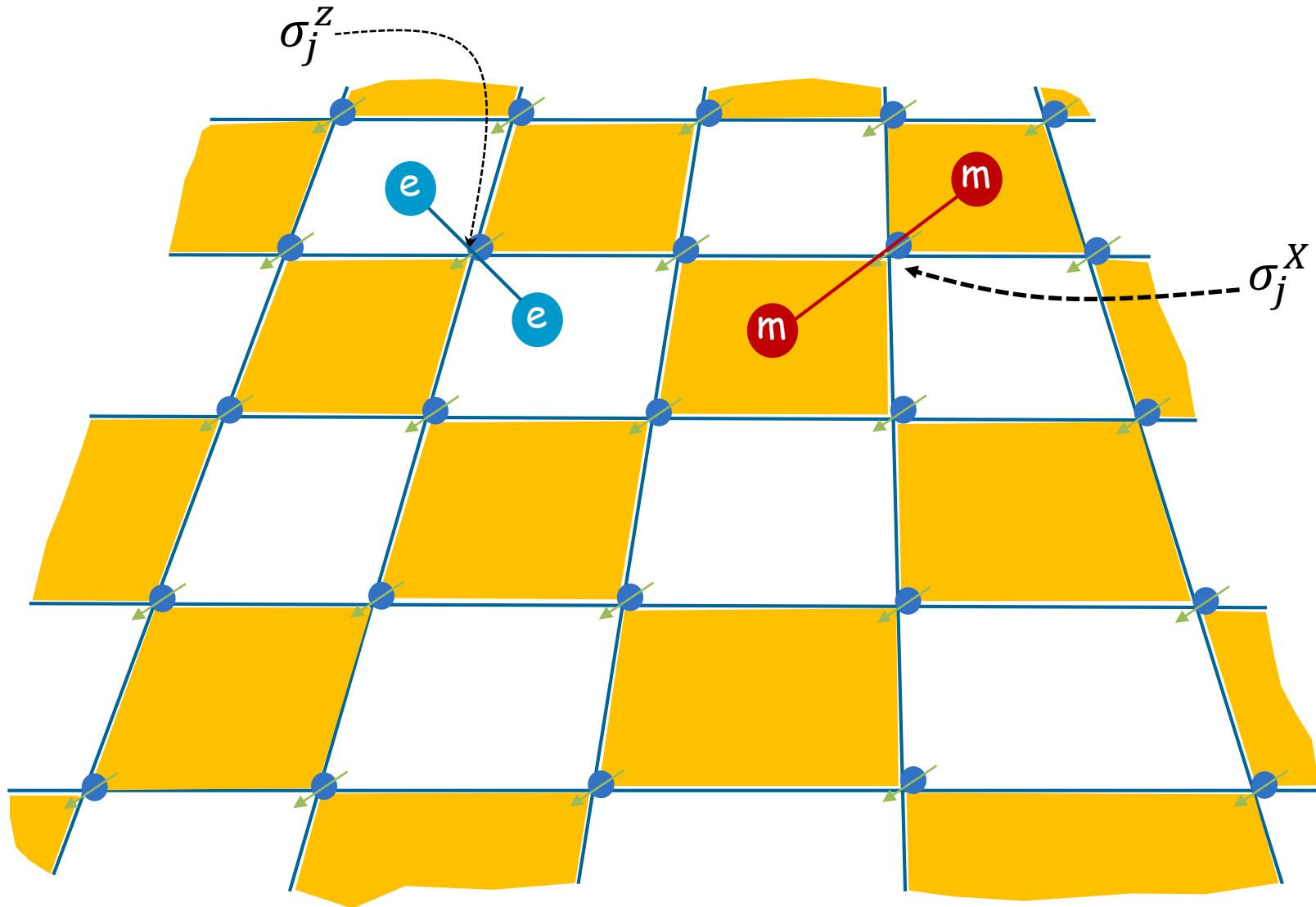
Toric code -- Braiding



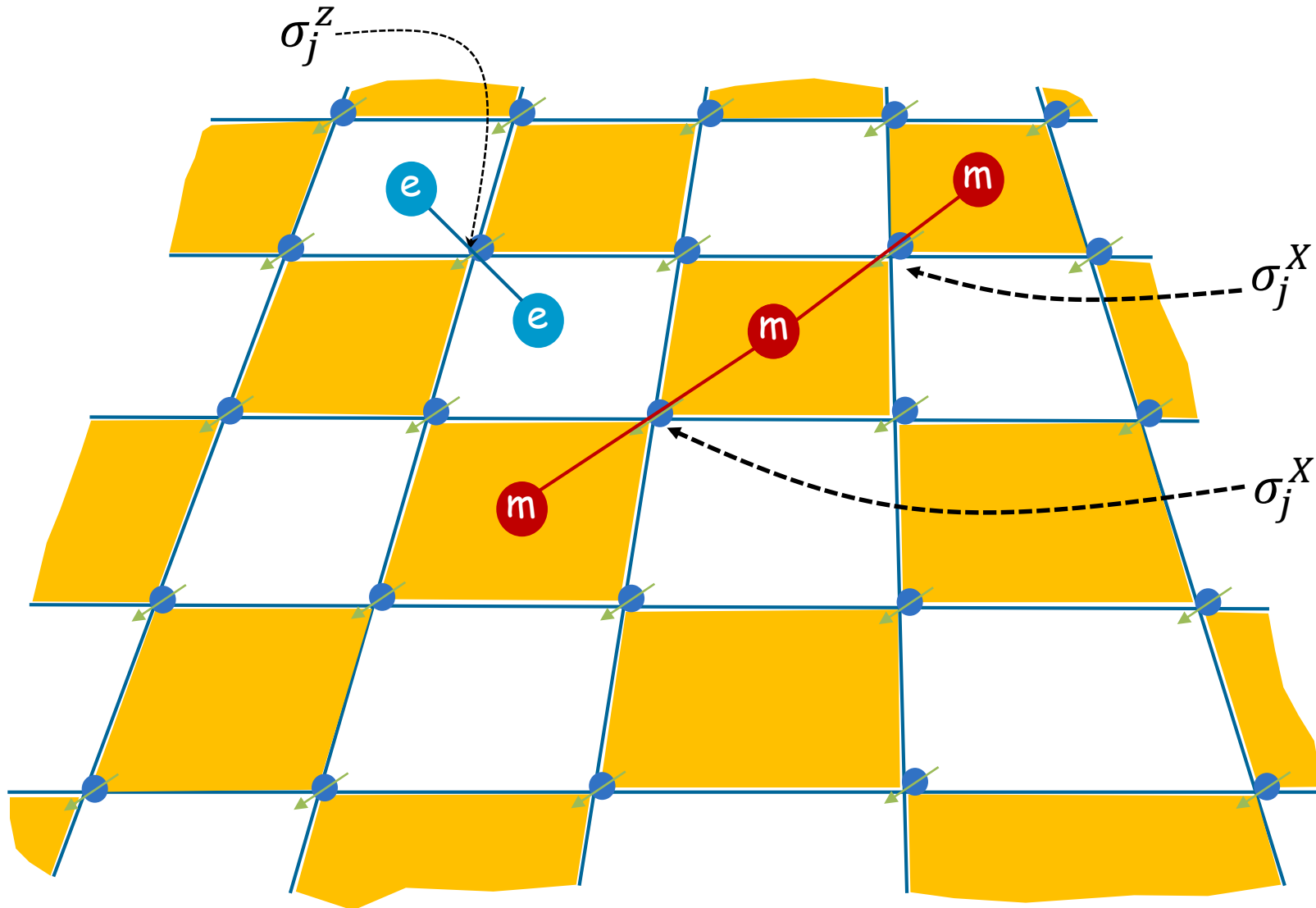
Toric code -- Braiding



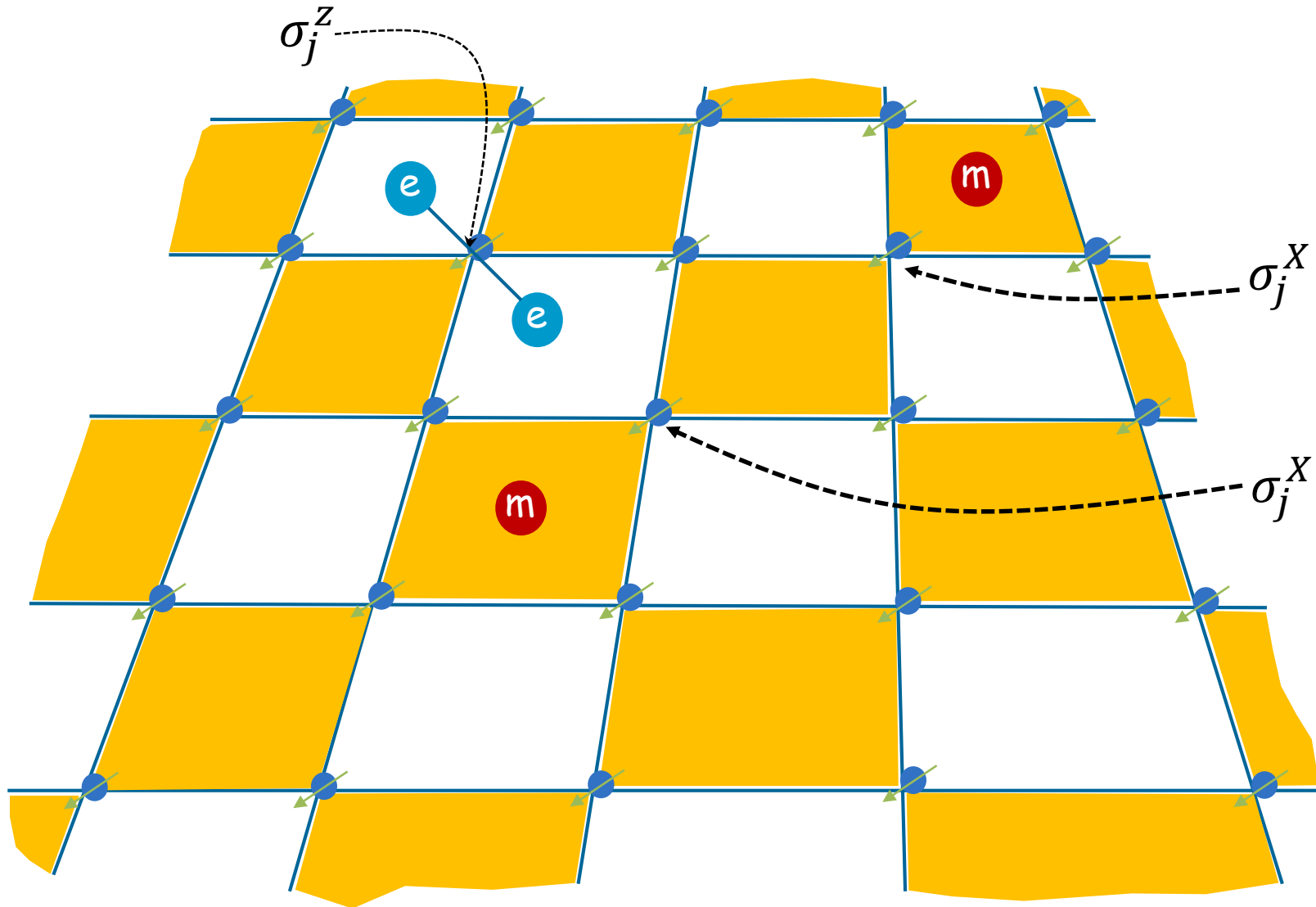
Toric code -- Braiding



Toric code -- Braiding



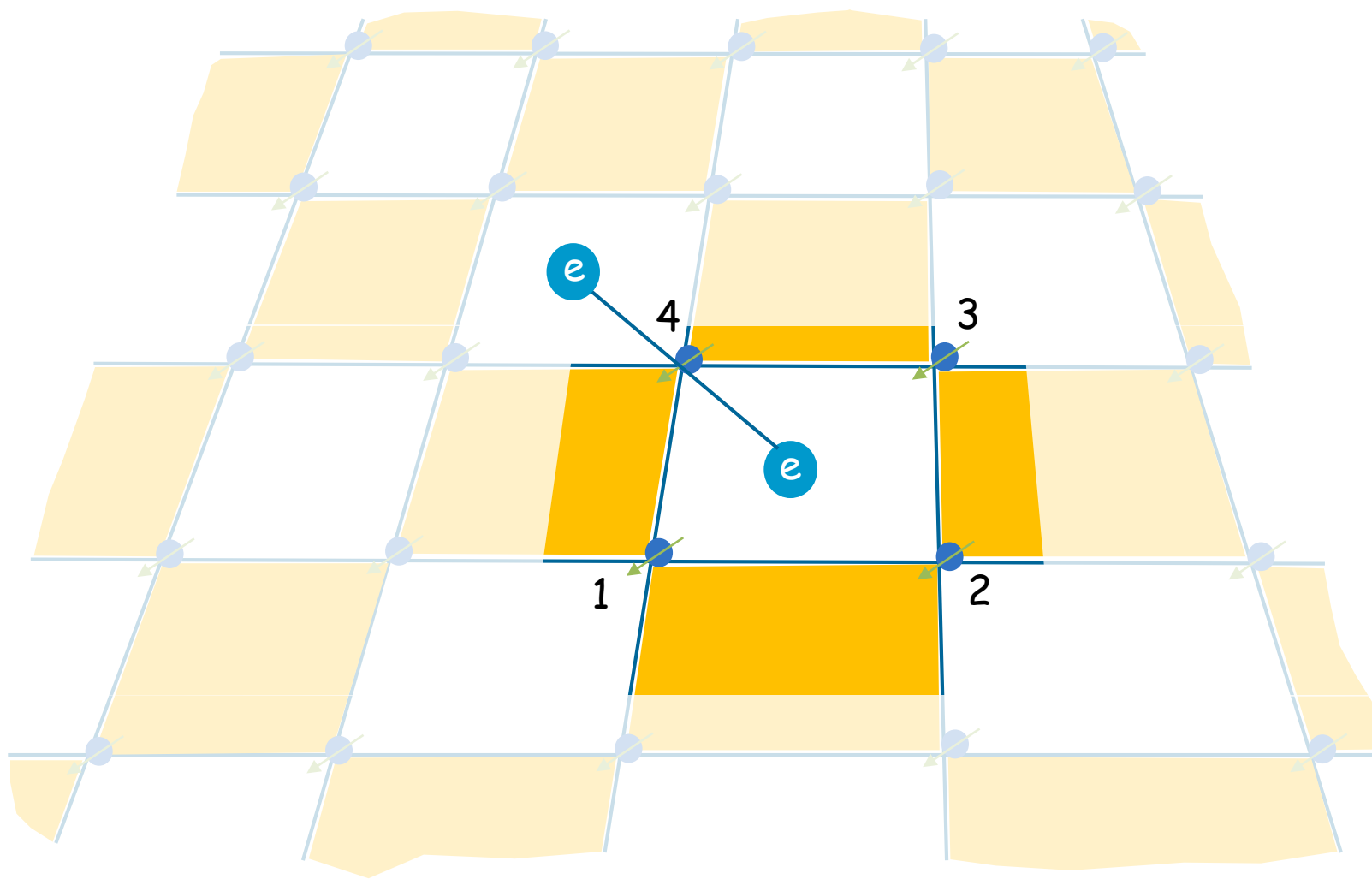
Toric code -- Braiding



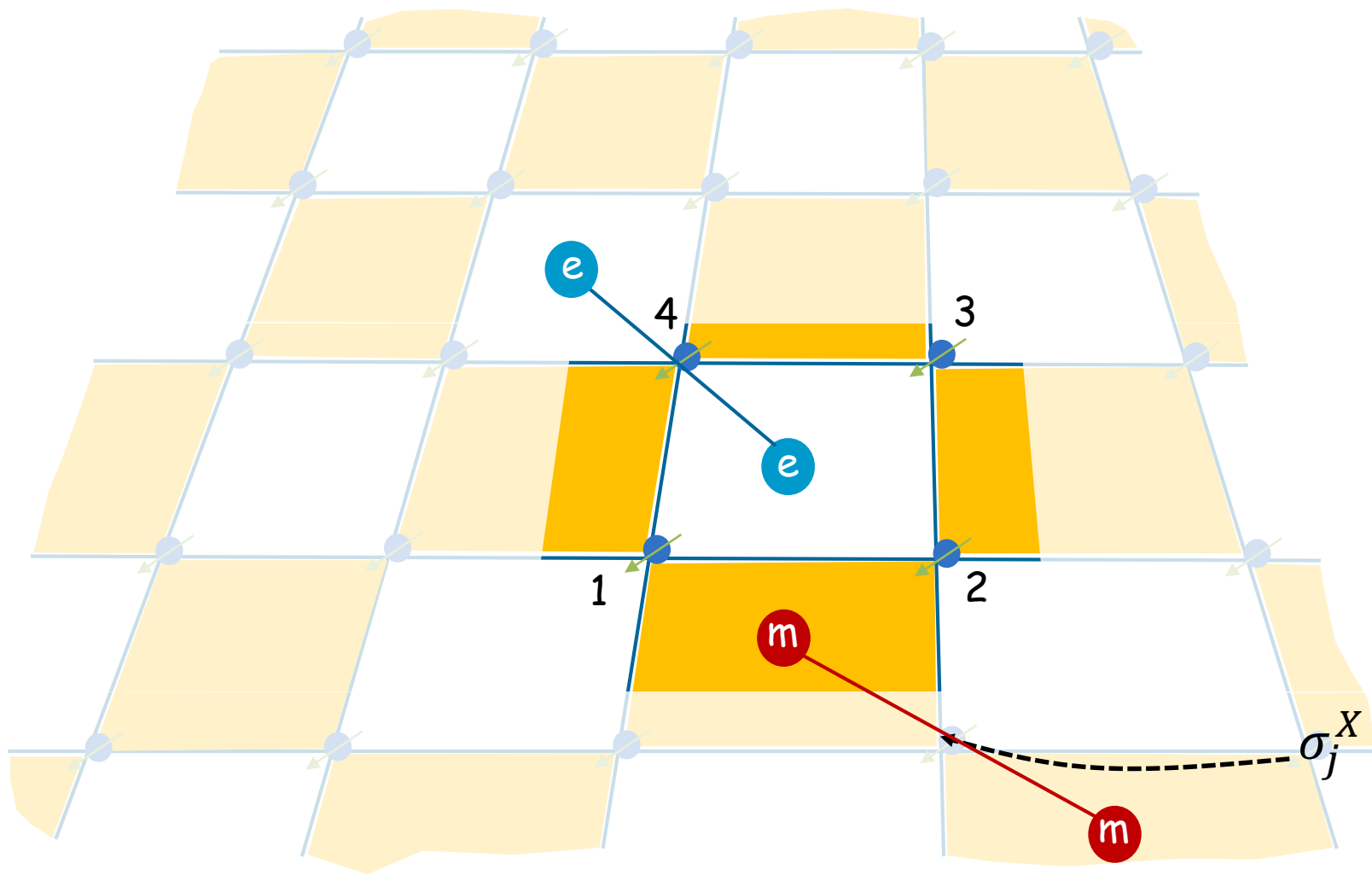
Toric code -- Braiding



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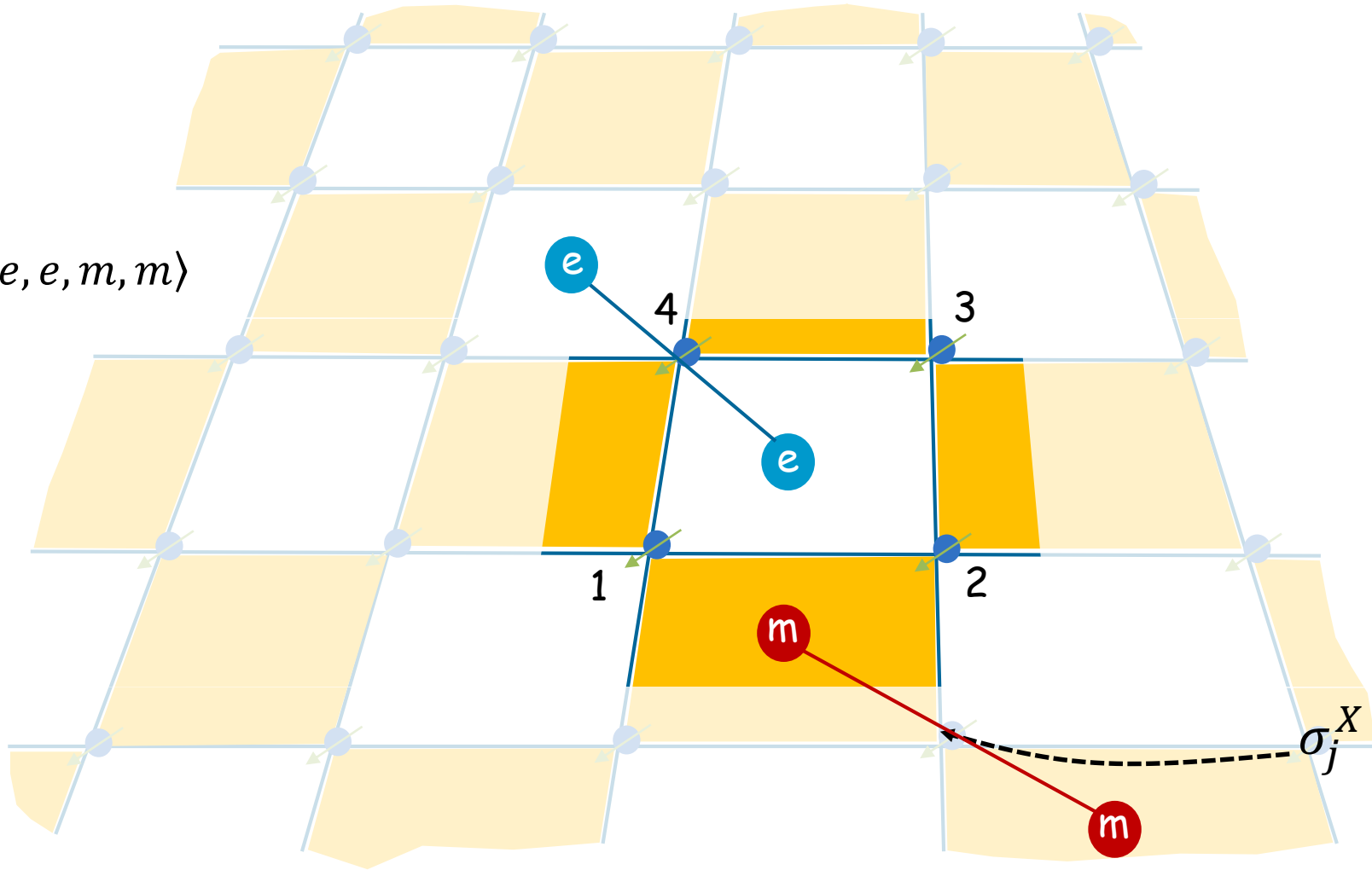


Toric code -- Braiding



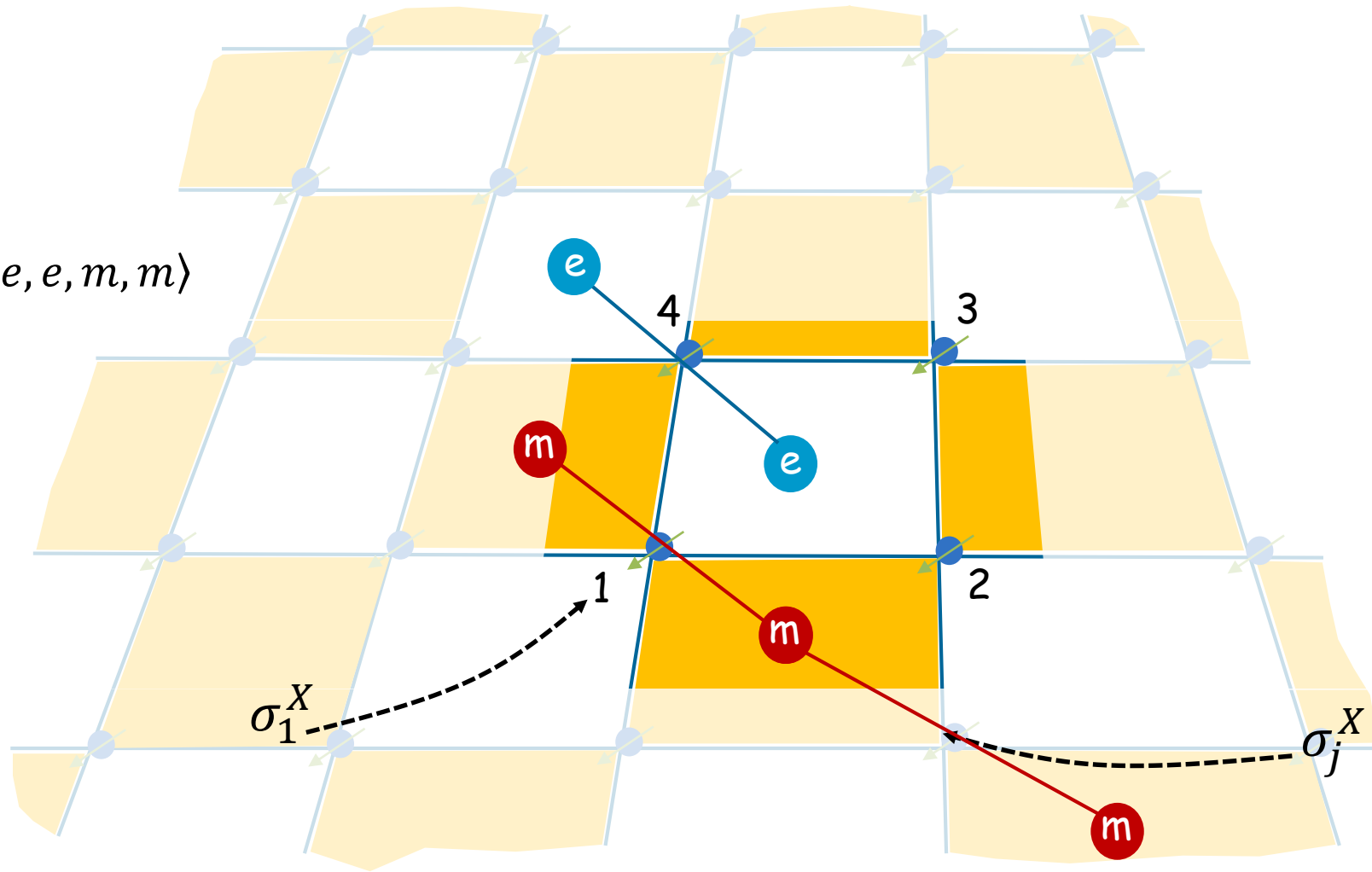
Toric code -- Braiding

$$|\varphi\rangle = |e, e, m, m\rangle$$



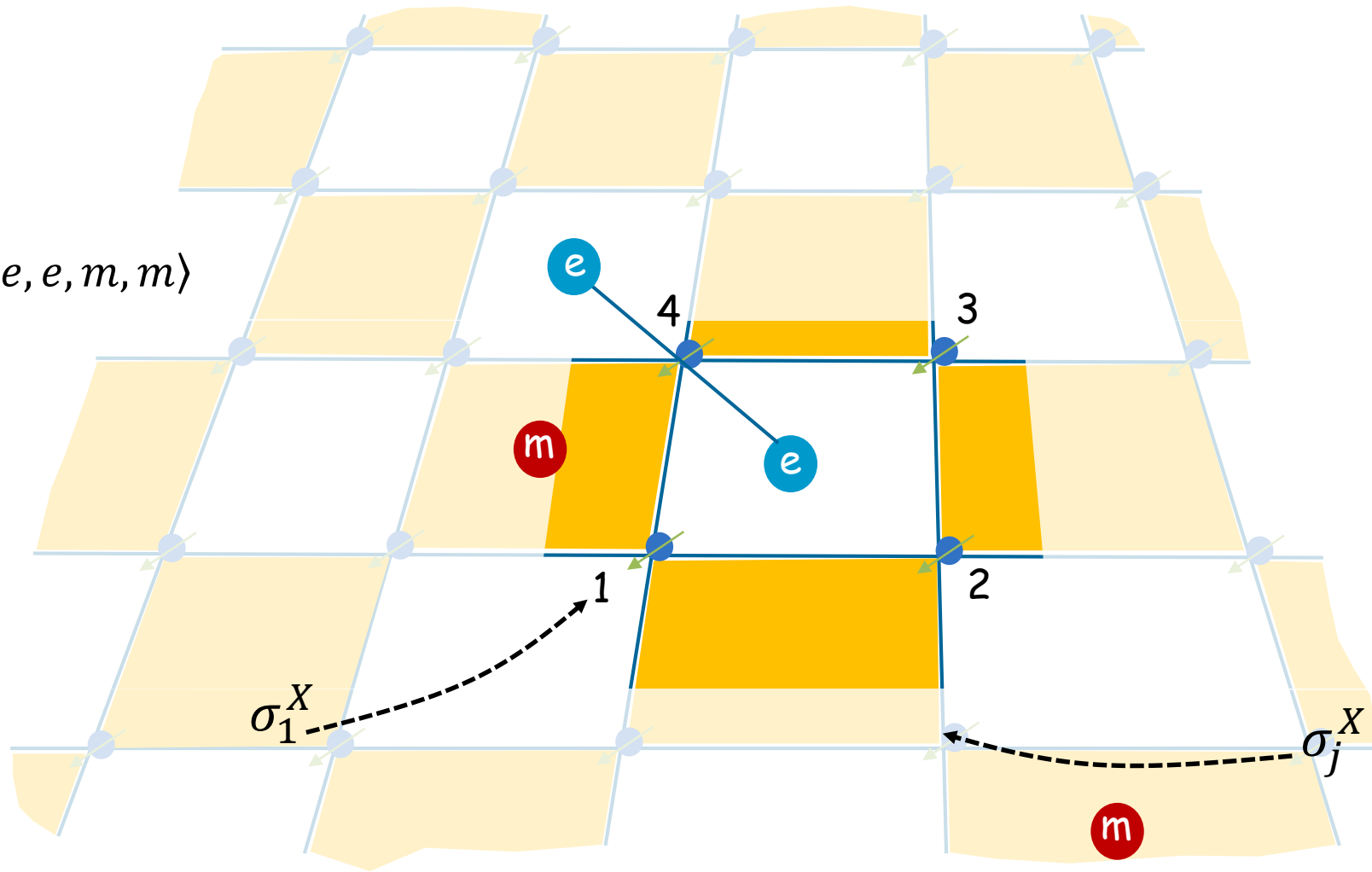
Toric code -- Braiding

$$|\varphi\rangle = |e, e, m, m\rangle$$

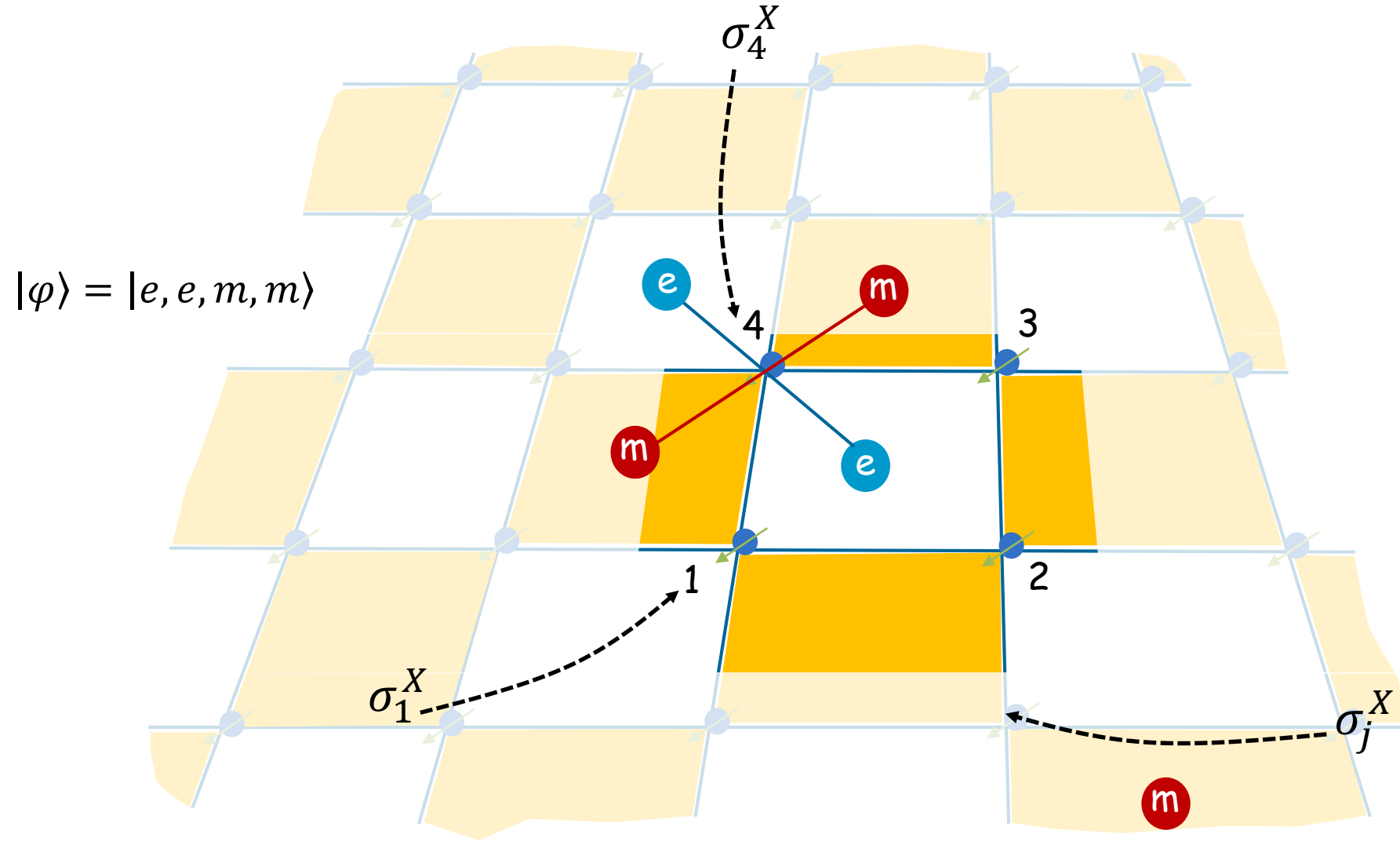


Toric code -- Braiding

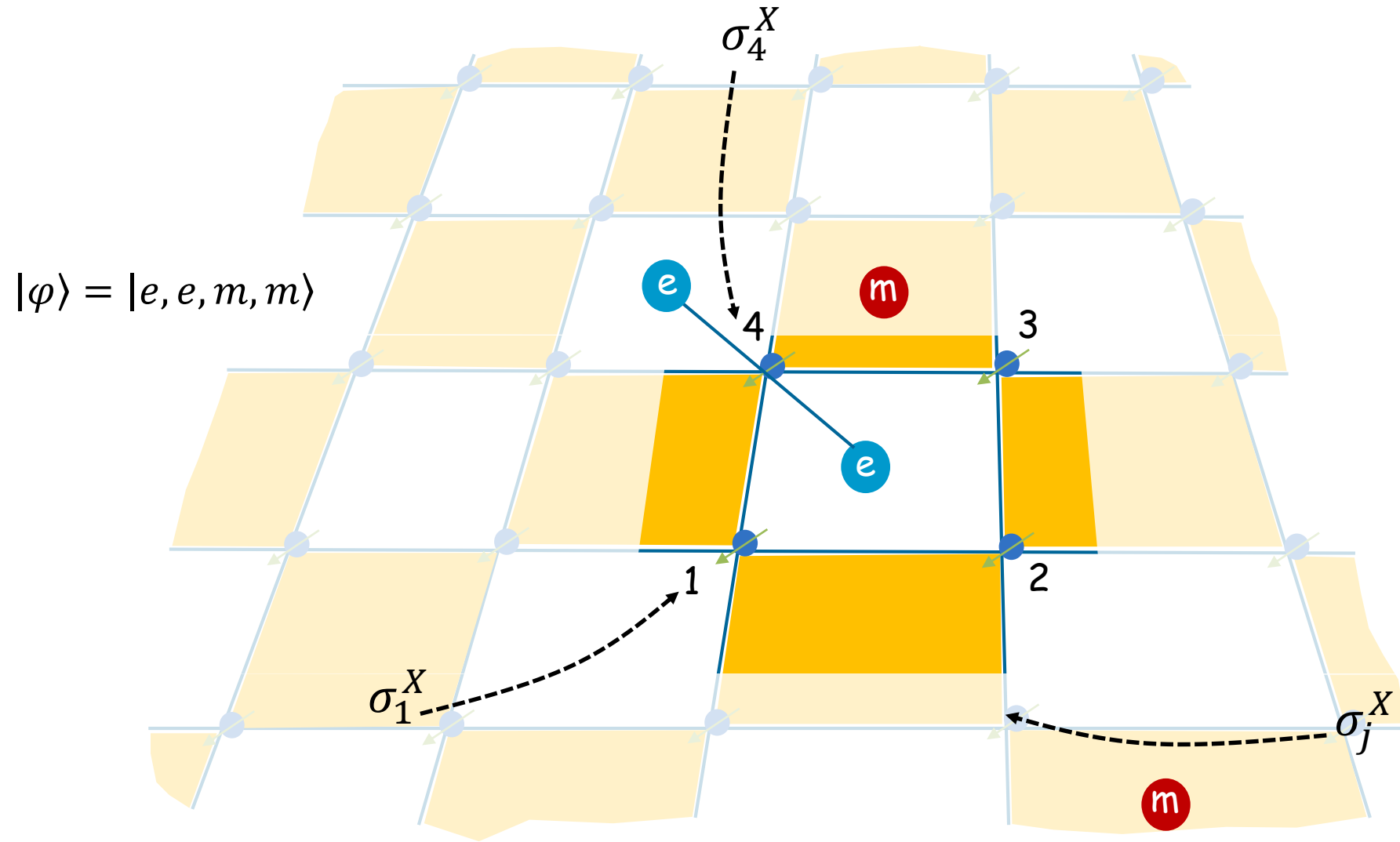
$$|\varphi\rangle = |e, e, m, m\rangle$$



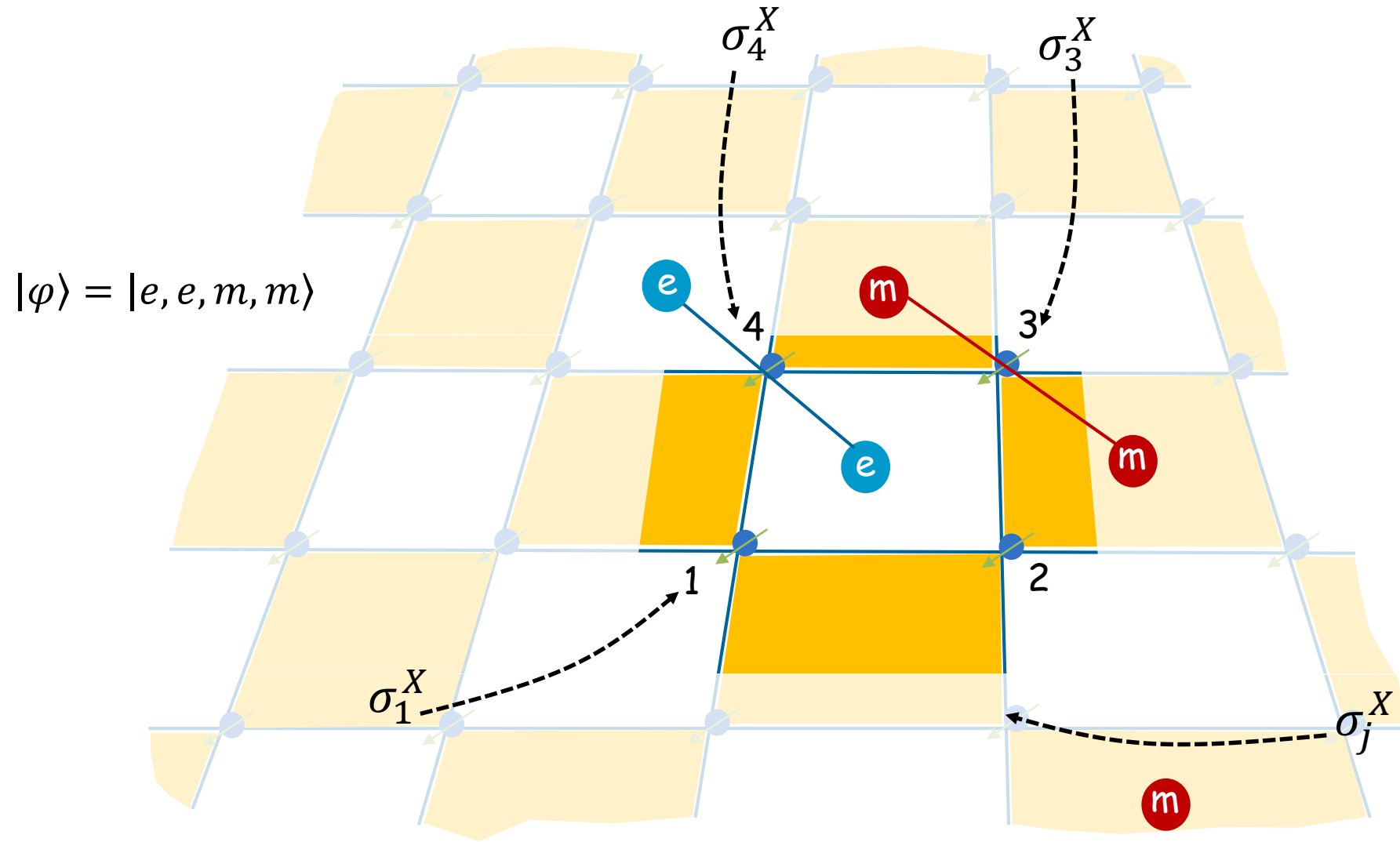
Toric code -- Braiding



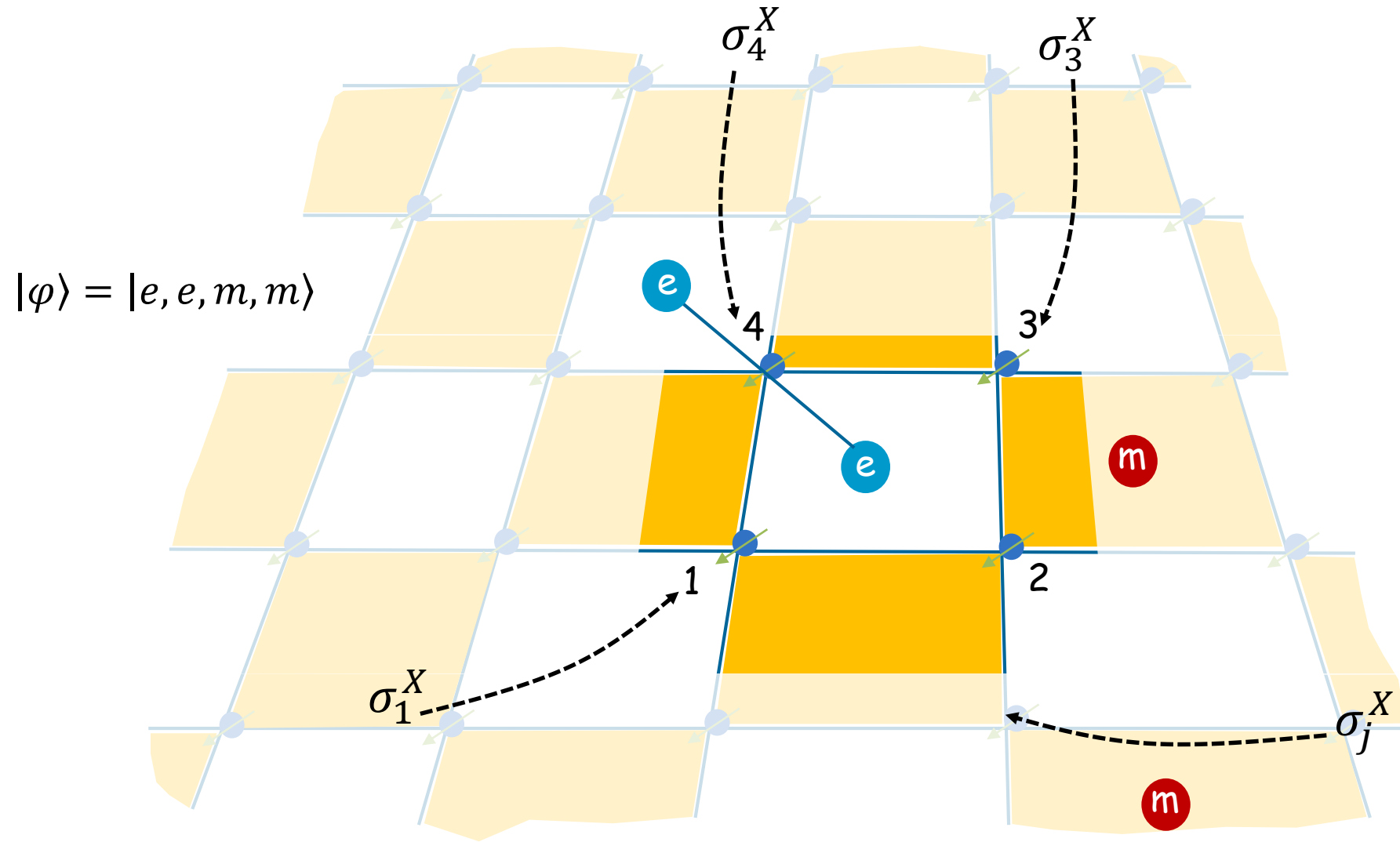
Toric code -- Braiding



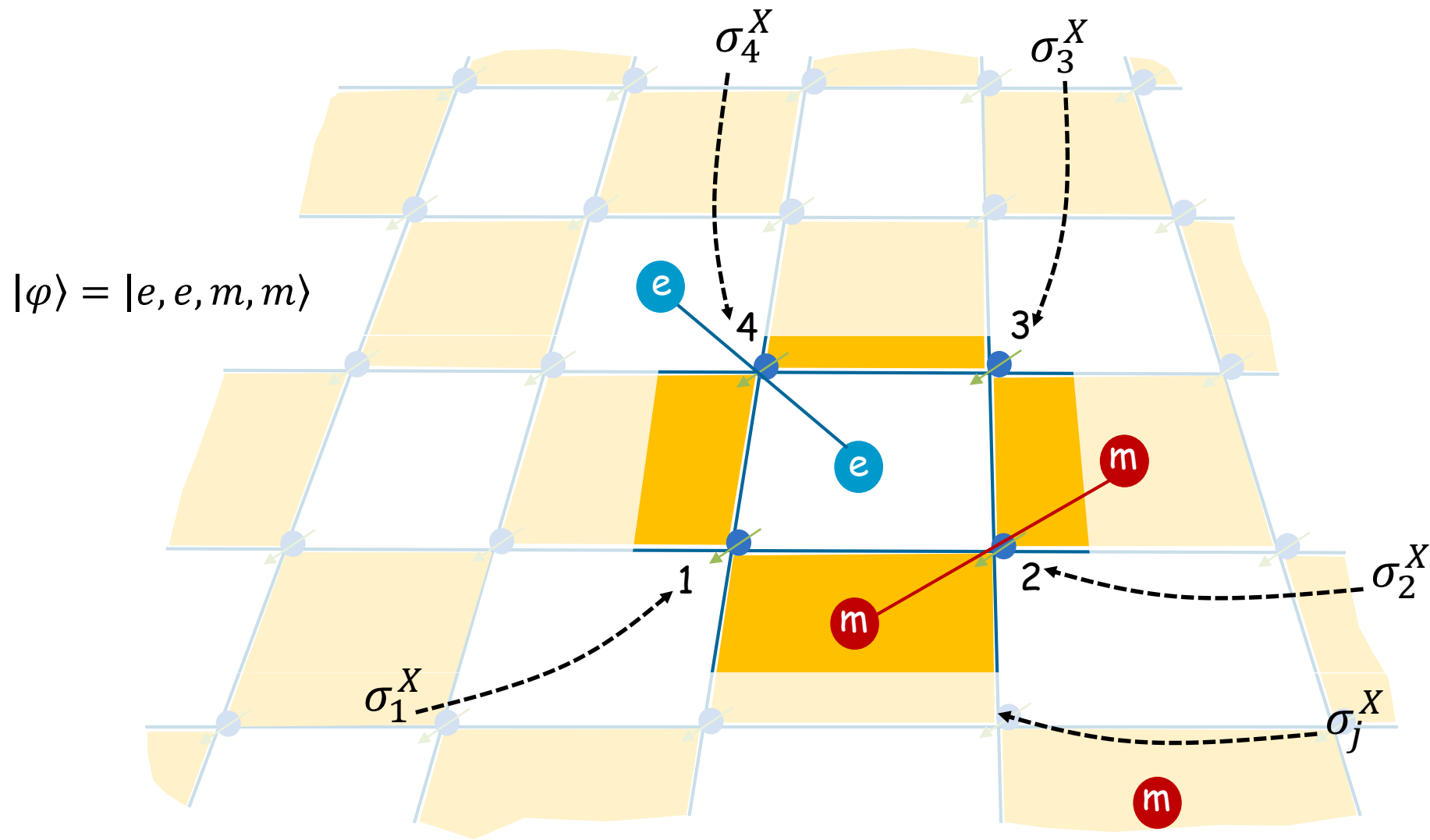
Toric code -- Braiding



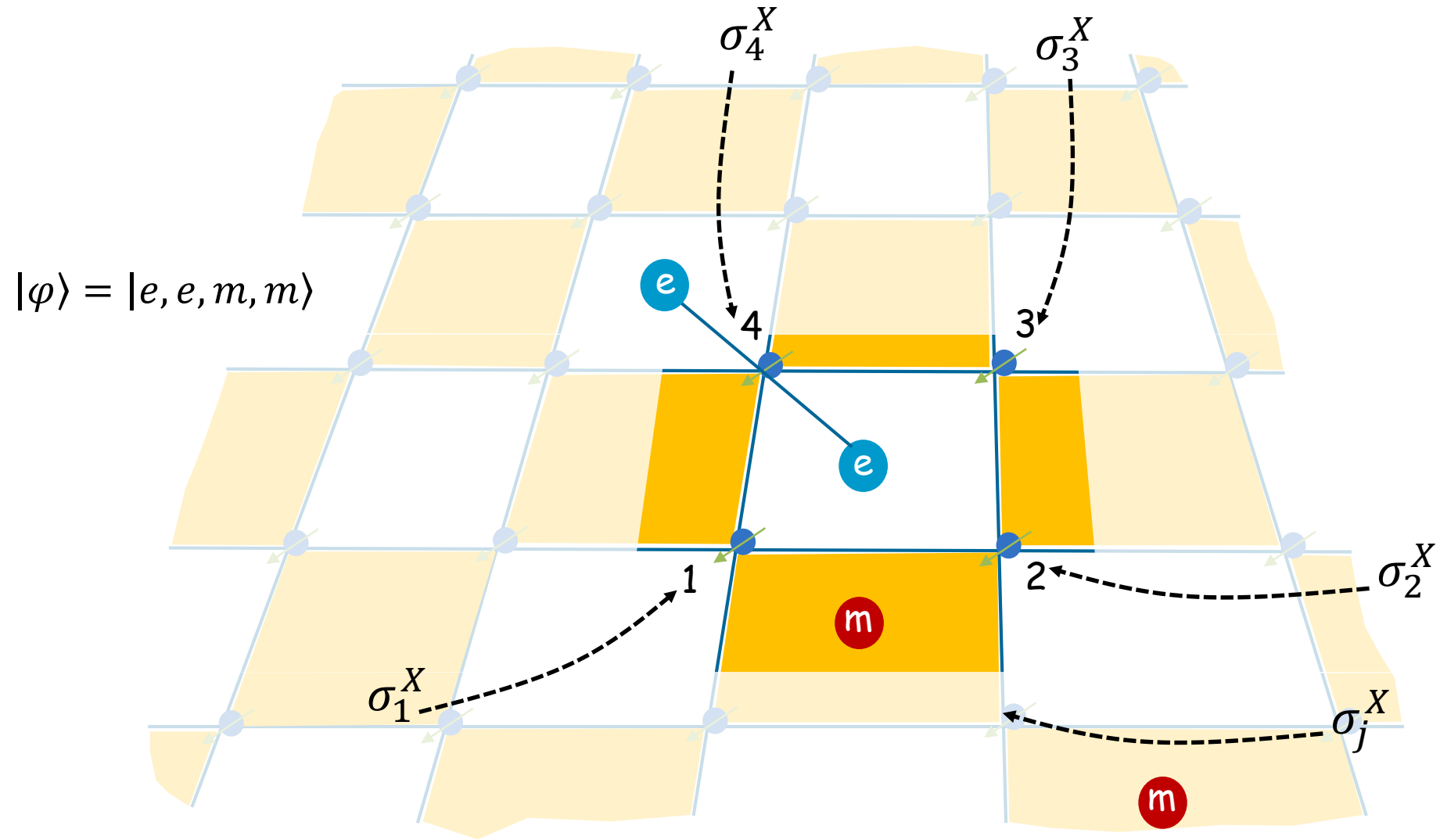
Toric code -- Braiding



Toric code -- Braiding



Toric code -- Braiding



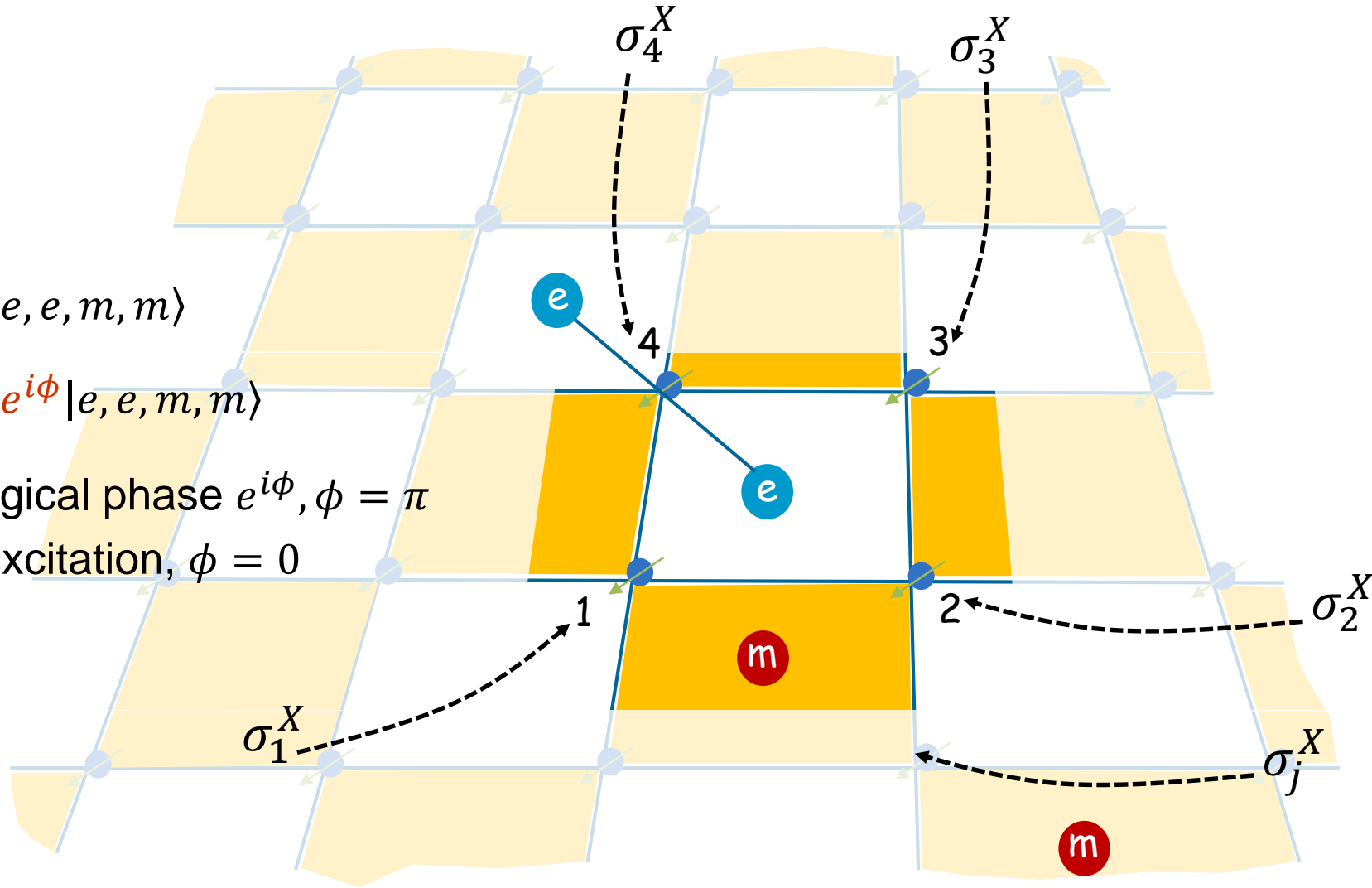
Toric code -- Braiding

$$|\varphi\rangle = |e, e, m, m\rangle$$

$$|\varphi'\rangle = e^{i\phi} |e, e, m, m\rangle$$

Topological phase $e^{i\phi}$, $\phi = \pi$

No e -excitation, $\phi = 0$



Can we build a many-body quantum system which is described by the Toric-code Hamiltonian?

Requirements:

- Create the four-body interaction
- Demonstrate the topological phase

Previous efforts:

Theory: Han *et al.*, PRL 98, 150404 (2007)

Experiments: Lu *et al.*, PRL 102, 030502 (2009). photons

Pachos *et al.*, NJP 11, 083010 (2009). photons

Barreiro *et al.*, Nature 470, 486 (2011). ions

Song *et al.*, PRL 121, 030502 (2018). superconductors

No background Hamiltonian ➡ There is no energy gap to protect the qubit!

2D-optical superlattice

BHM

$$\hat{H} = \sum_{\sigma=\uparrow,\downarrow} [-J(\hat{a}_{\sigma L}^+ \hat{a}_{\sigma R} + \hat{a}_{\sigma R}^+ \hat{a}_{\sigma L})] + U[\hat{n}_{\uparrow L} \hat{n}_{\downarrow L} + \hat{n}_{\uparrow R} \hat{n}_{\downarrow R}]$$

$J \ll U$

Super-exchange dominated :

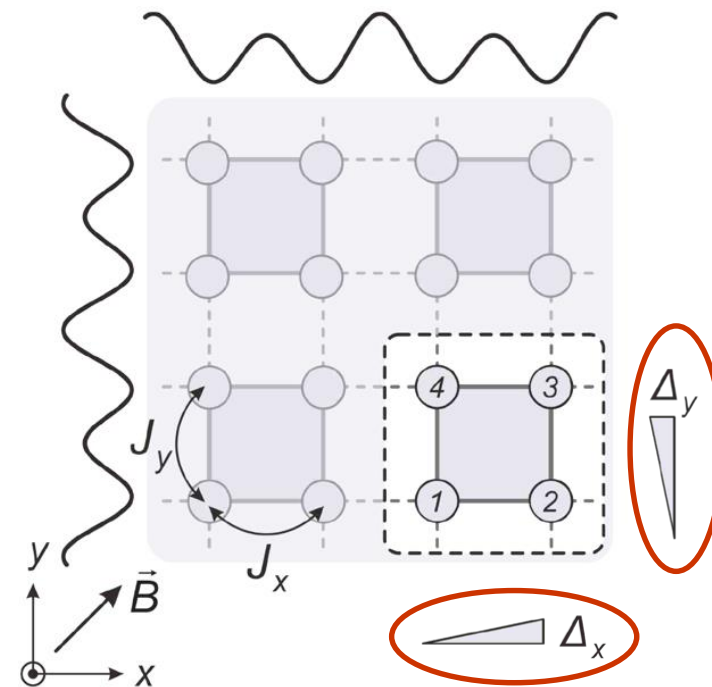
$$H_{ex} = -2J_{ex} S_L \cdot S_R$$



In isolated plaquettes, suppress super-exchange by 2D effective magnetic gradients

Ring-exchange dominated :

$$H_{\square} = -J_{\square} \hat{\sigma}_1^x \hat{\sigma}_2^x \hat{\sigma}_3^x \hat{\sigma}_4^x, J_{\square} = 40 J^4 / U^3$$

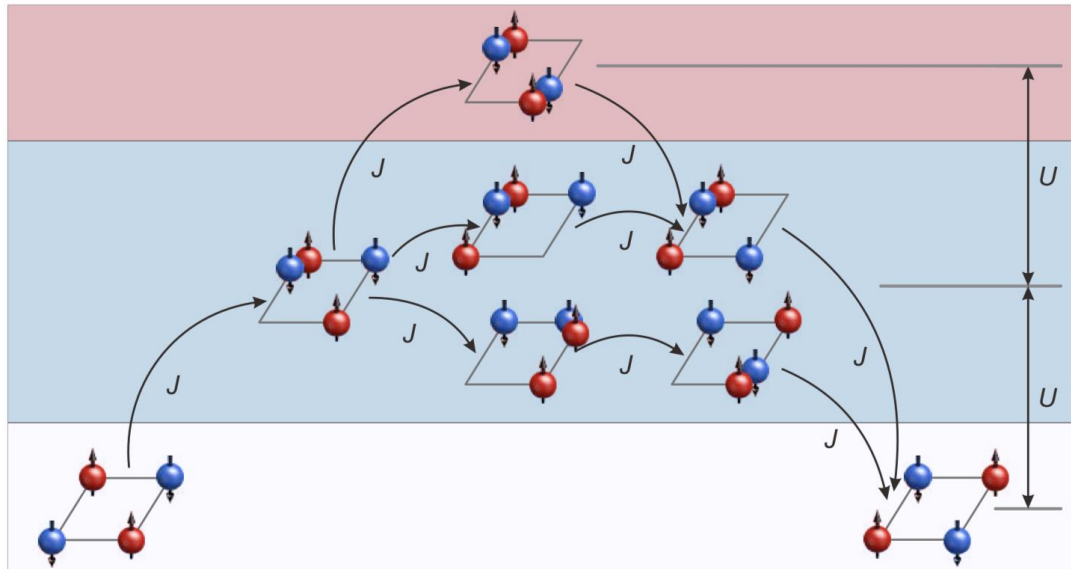


isolated plaquettes

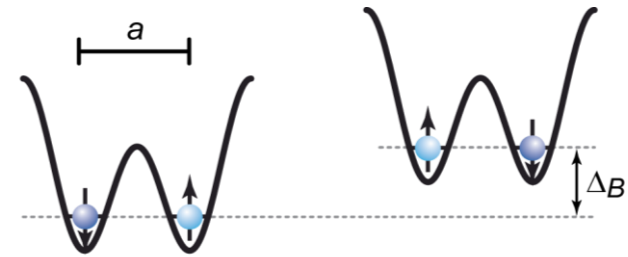
Suppress super-exchange



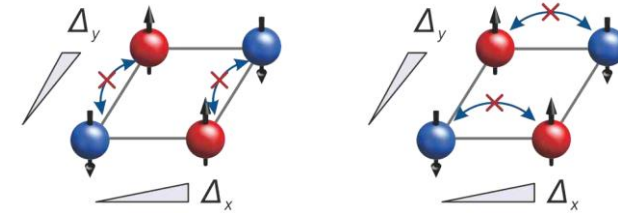
Super-exchange is dominant:
 $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ are degenerate



Low-energy state subspace:
 $\{|\uparrow\downarrow\uparrow\downarrow\rangle, |\downarrow\uparrow\downarrow\uparrow\rangle, |\uparrow\uparrow\downarrow\downarrow\rangle, |\uparrow\uparrow\downarrow\downarrow\rangle, |\uparrow\downarrow\downarrow\uparrow\rangle, |\downarrow\uparrow\uparrow\downarrow\rangle\}$

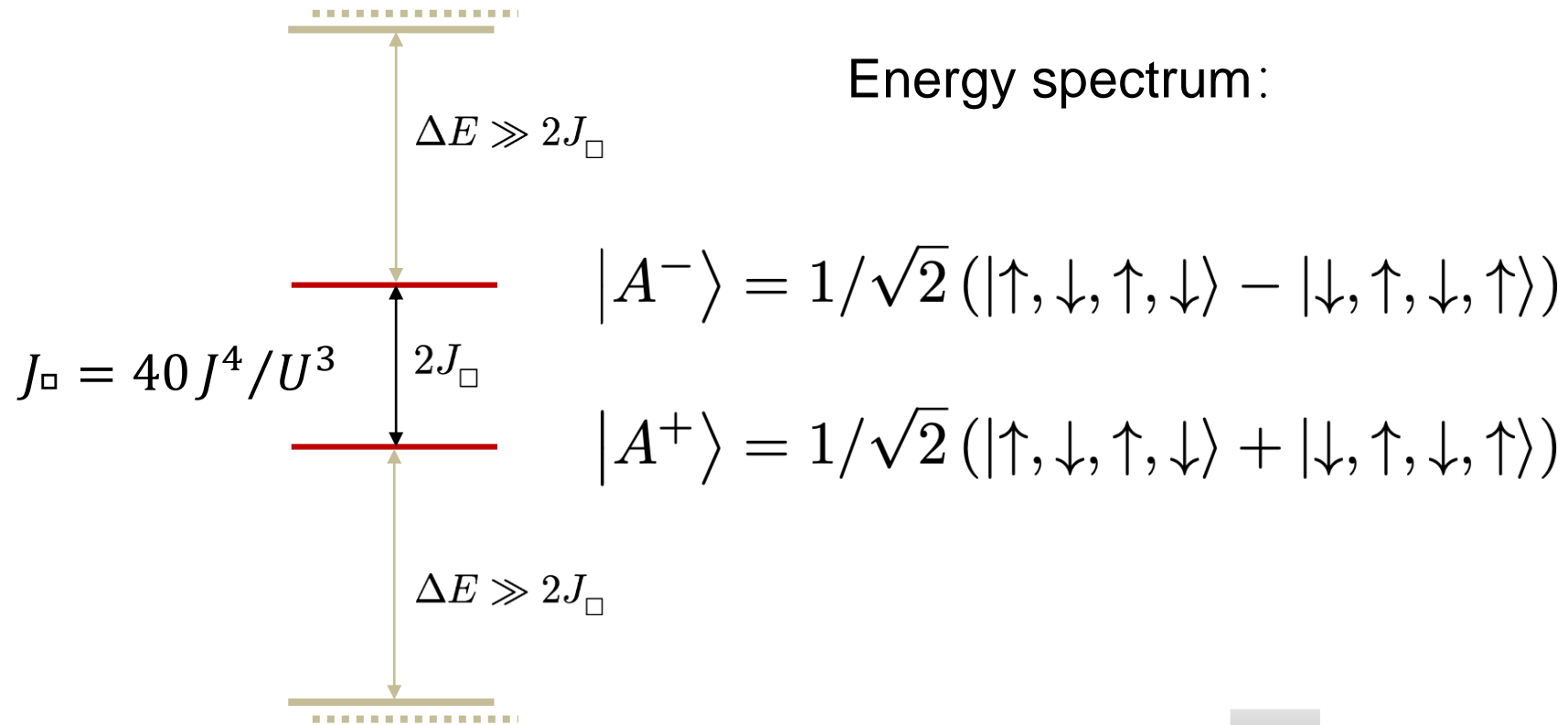


$$\Delta_{x(y)} \gg 4J^2/U$$



Super-exchange is suppressed:
 Non-degeneration due to
 effective magnetic gradients

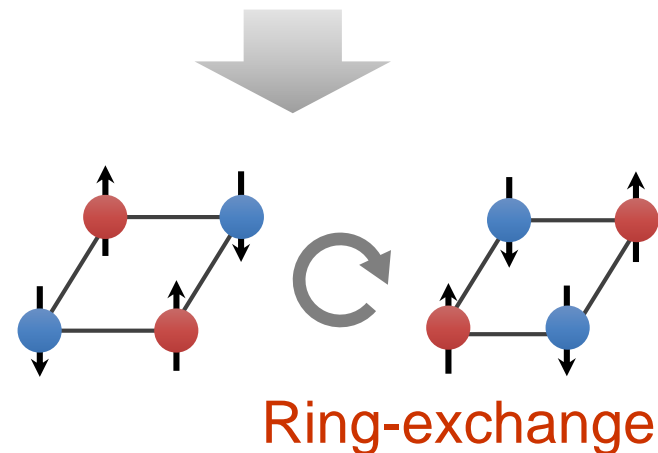
Minimum Toric code Hamiltonian



In reduced subspace $\{|\uparrow\downarrow\uparrow\downarrow\rangle, |\downarrow\uparrow\downarrow\uparrow\rangle\}$

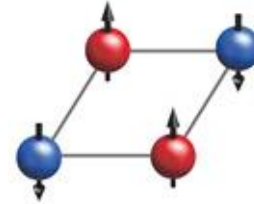
$$H = J_{\square} (|A^{-}\rangle\langle A^{-}| - |A^{+}\rangle\langle A^{+}|)$$

$$= -J_{\square} \hat{\sigma}_1^x \hat{\sigma}_2^x \hat{\sigma}_3^x \hat{\sigma}_4^x$$

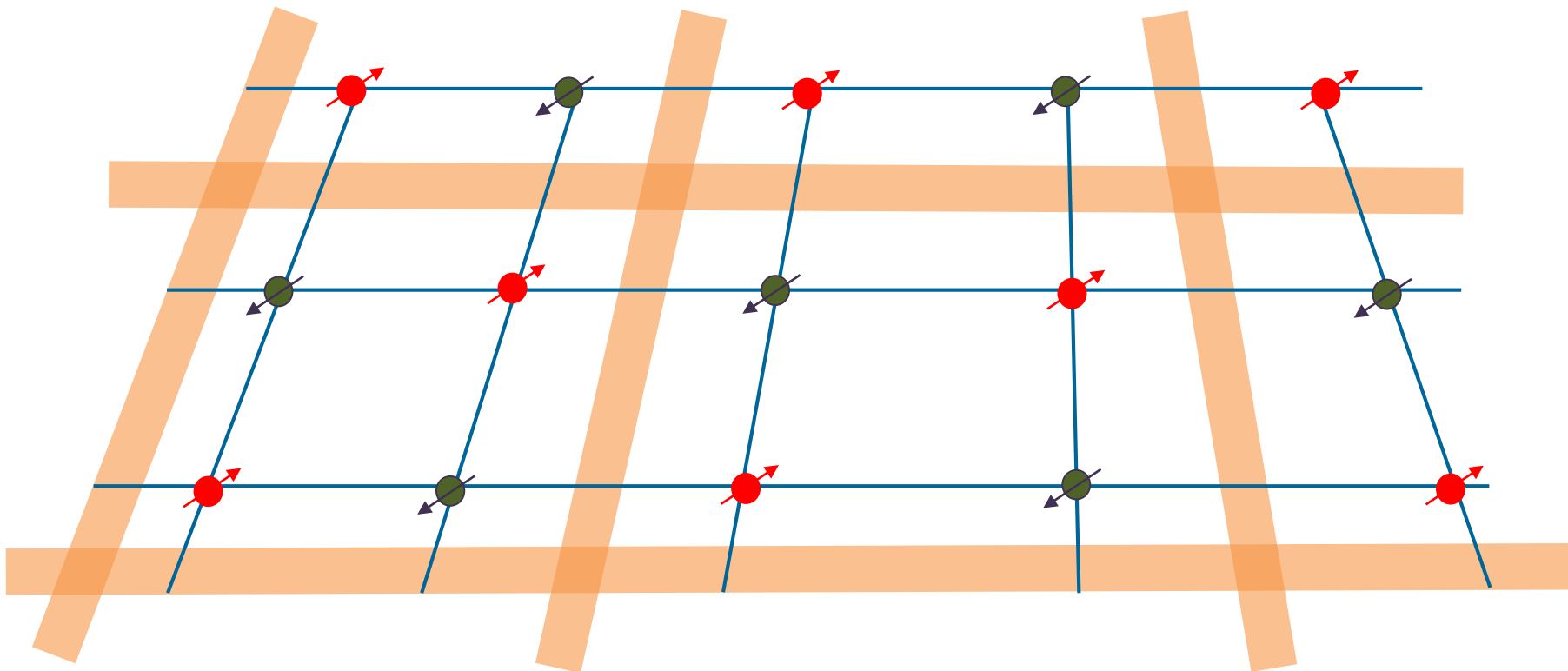


Ring exchange driven oscillation

$$\begin{array}{l}
 \text{---} |A^-\rangle = \frac{|\uparrow,\downarrow,\uparrow,\downarrow\rangle - |\downarrow,\uparrow,\downarrow,\uparrow\rangle}{\sqrt{2}} \\
 \begin{array}{c} \uparrow \\ 2J_{\square} \\ \downarrow \end{array} \\
 \text{---} |A^+\rangle = \frac{|\uparrow,\downarrow,\uparrow,\downarrow\rangle + |\downarrow,\uparrow,\downarrow,\uparrow\rangle}{\sqrt{2}}
 \end{array}$$

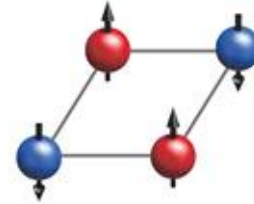


Initial state
 $= \frac{1}{\sqrt{2}} (|A^-\rangle + |A^+\rangle)$

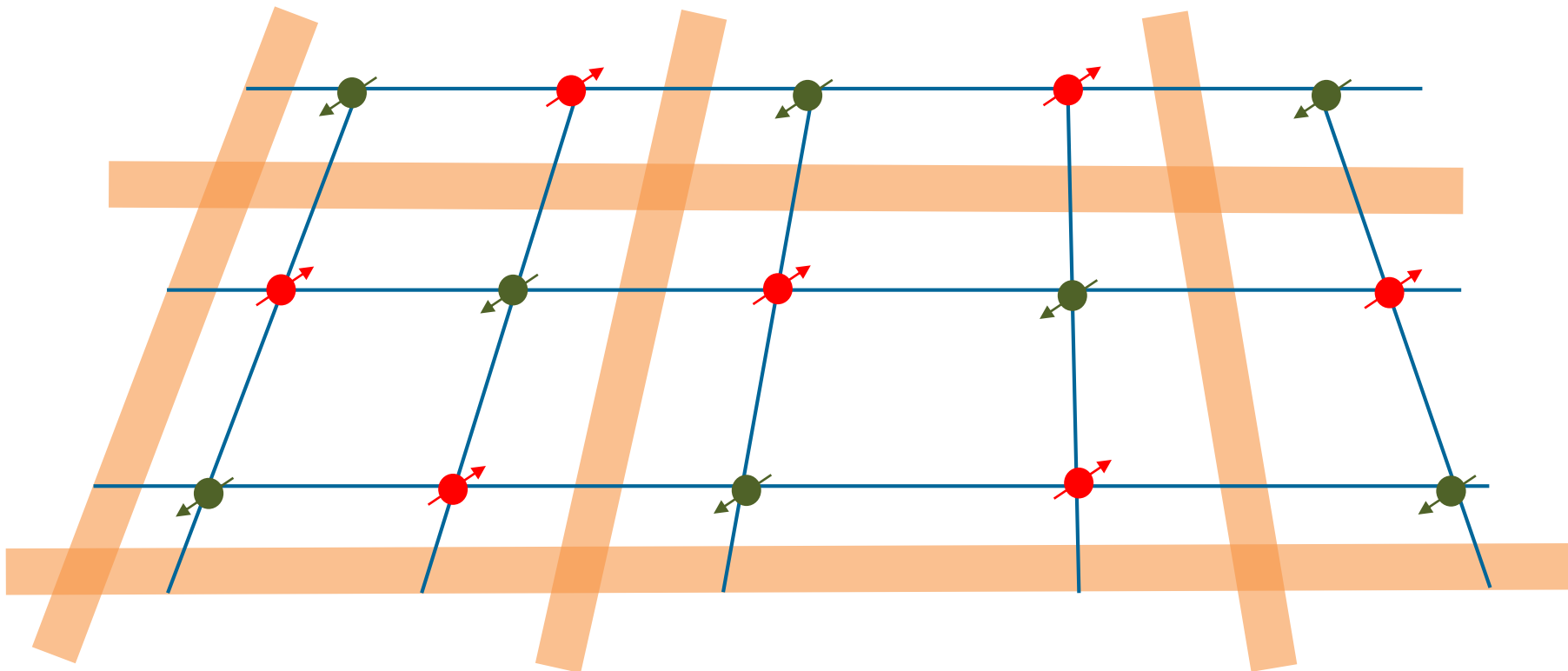


Ring exchange driven oscillation

$$\begin{array}{l}
 \text{---} |A^-\rangle = \frac{|\uparrow,\downarrow,\uparrow,\downarrow\rangle - |\downarrow,\uparrow,\downarrow,\uparrow\rangle}{\sqrt{2}} \\
 \begin{array}{c} \updownarrow \\ 2J_{\square} \\ \updownarrow \end{array} \\
 \text{---} |A^+\rangle = \frac{|\uparrow,\downarrow,\uparrow,\downarrow\rangle + |\downarrow,\uparrow,\downarrow,\uparrow\rangle}{\sqrt{2}}
 \end{array}$$

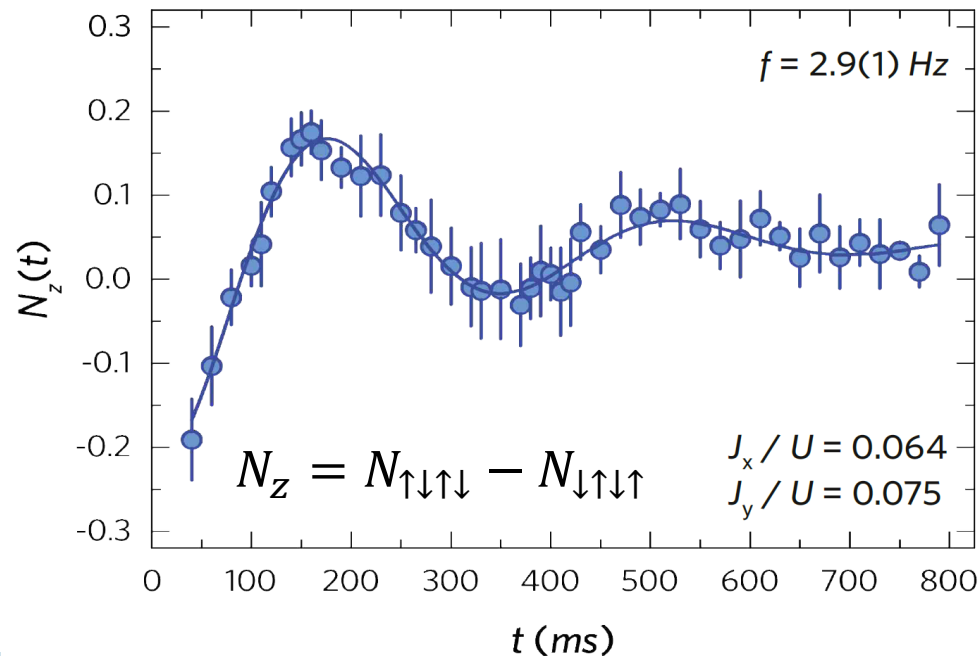
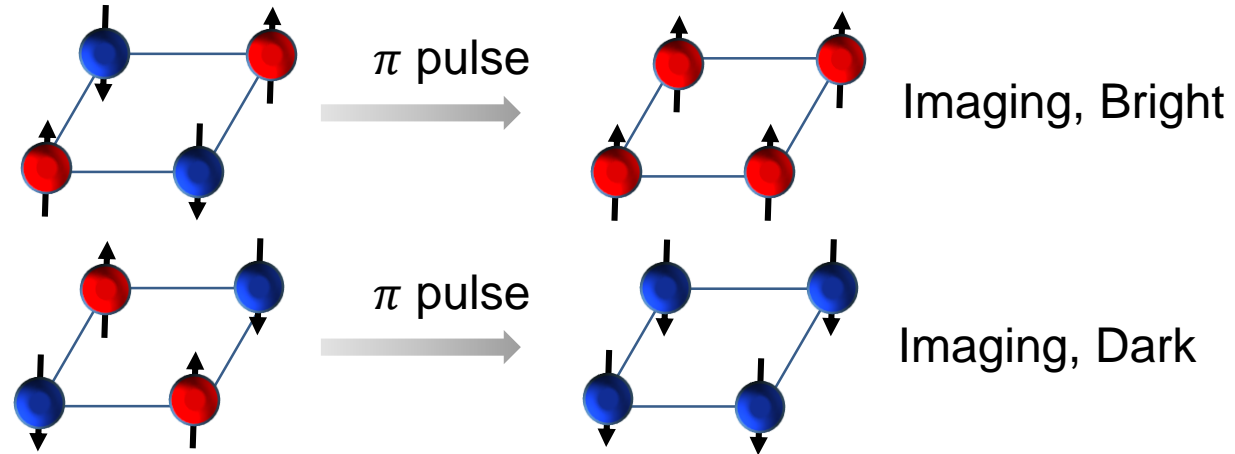


Initial state
 $= \frac{1}{\sqrt{2}} (|A^-\rangle + |A^+\rangle)$



Observation of ring exchange driven oscillation

Count the populations of different states



Settings:

$$V_{x1} = V_{y1} = 10 E_r$$

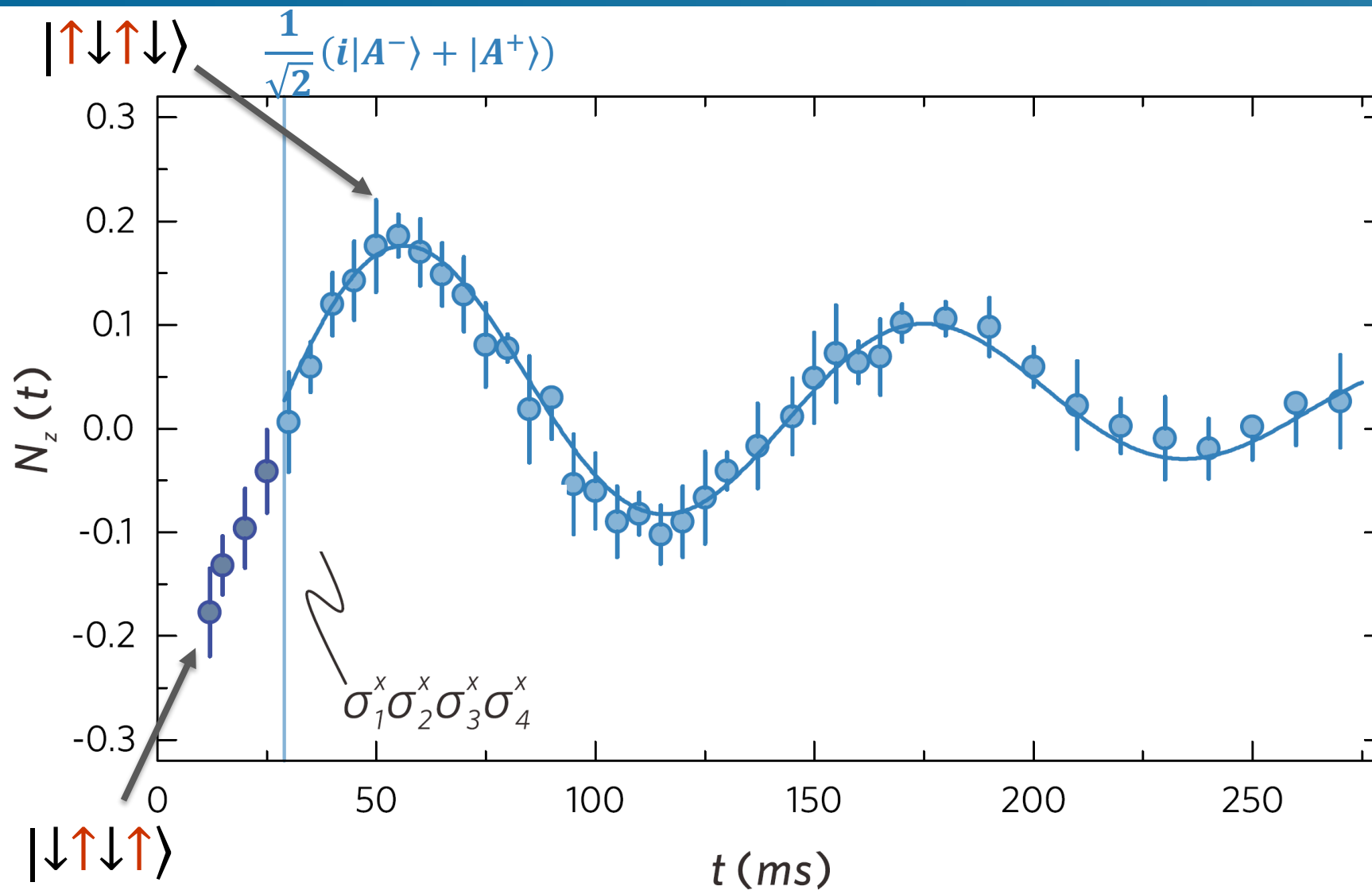
$$\Delta_x = 115(1) \text{ Hz}$$

$$V_{ys} = 18.2(1) E_r$$

$$V_{xs} = 19.2(1) E_r$$

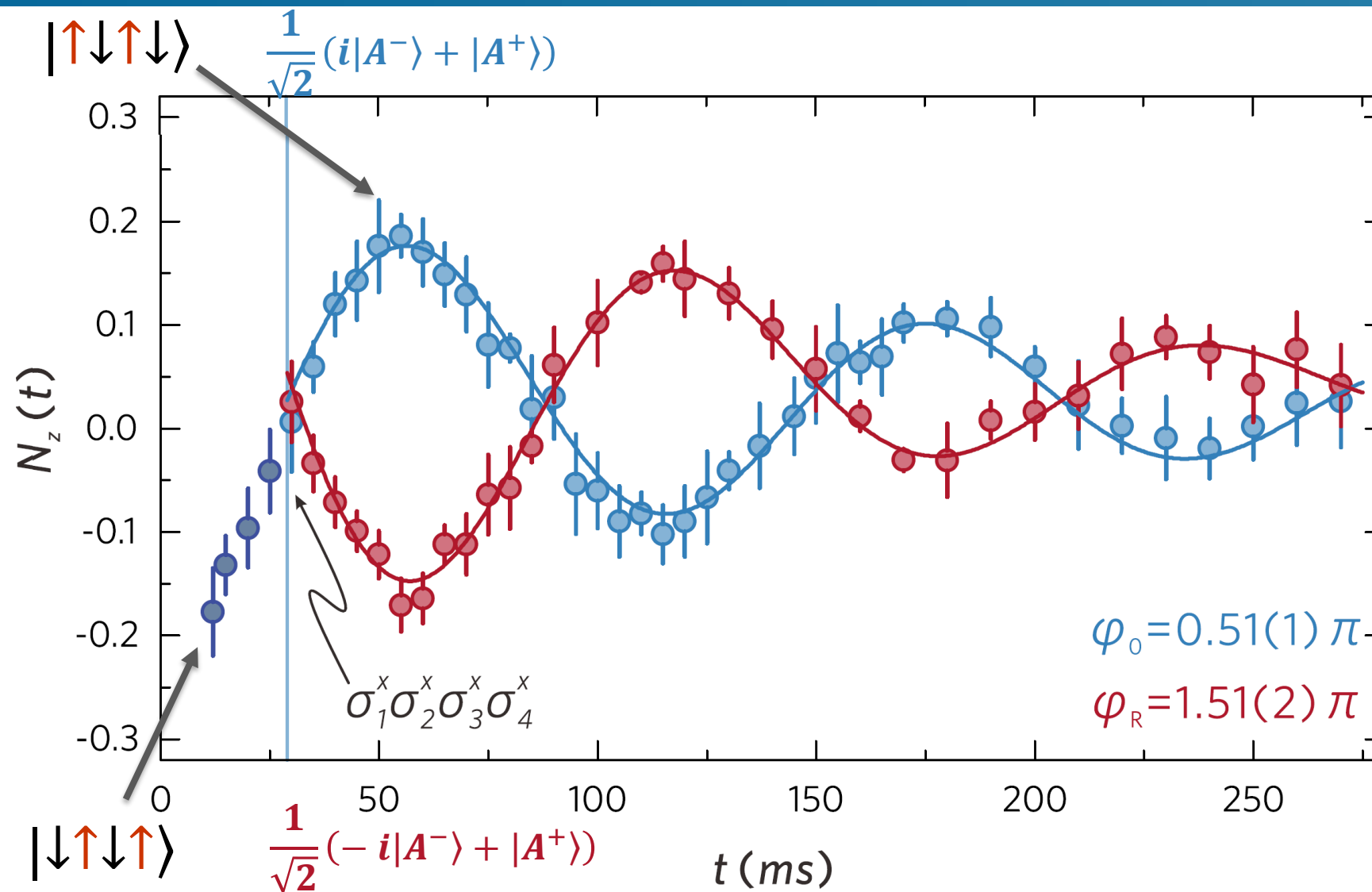
$$\Delta_y = 145(1) \text{ Hz}$$

Observation of anyonic fractional statistics



Dai *et al*, Nature Physics 13, 1195 (2017)

Observation of anyonic fractional statistics



Dai *et al*, Nature Physics 13, 1195 (2017)

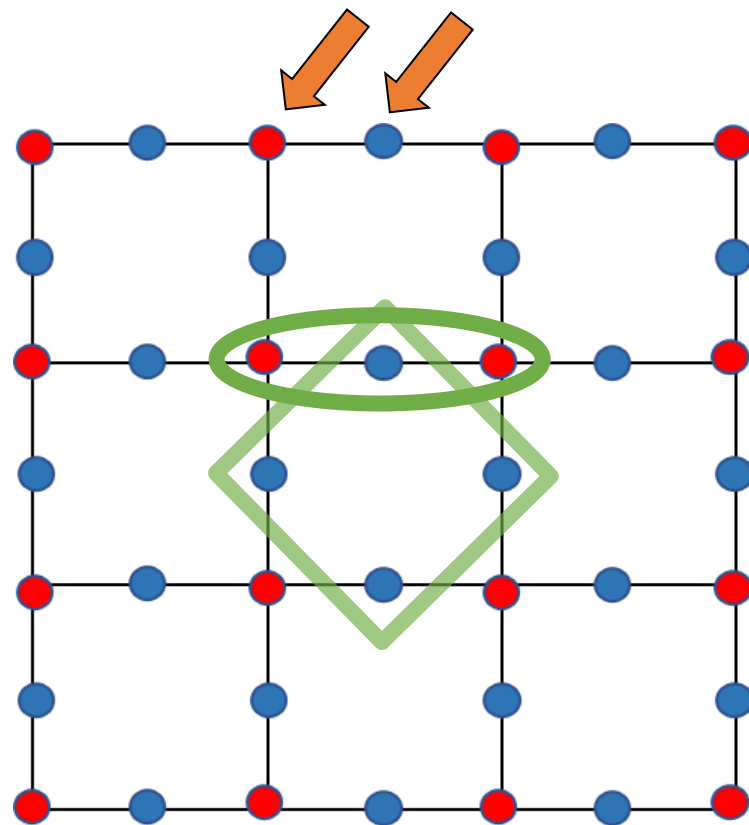
$$H = H_E + H_B + H_M + H_{int}$$

$$H_E = \frac{g^2}{2} \sum_{\mathbf{n}, k} \mathbf{L}_{\mathbf{n}, k}^2$$

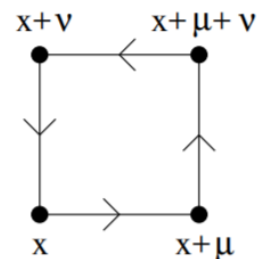
$$H_B = -\frac{1}{g^2} \sum_{\text{plaquettes}} [\text{Tr}(U_1 U_2 U_3^\dagger U_4^\dagger) + h.c.]$$

$$H_M = \sum_{\mathbf{n}, k} M \psi_{\mathbf{n}}^\dagger \psi_{\mathbf{n}}$$

$$H_{int} = \epsilon \sum_{\mathbf{n}, k} (\psi_{\mathbf{n}}^\dagger U_{\mathbf{n}, k} \psi_{\mathbf{n}+\hat{\mathbf{k}}} + h.c.)$$

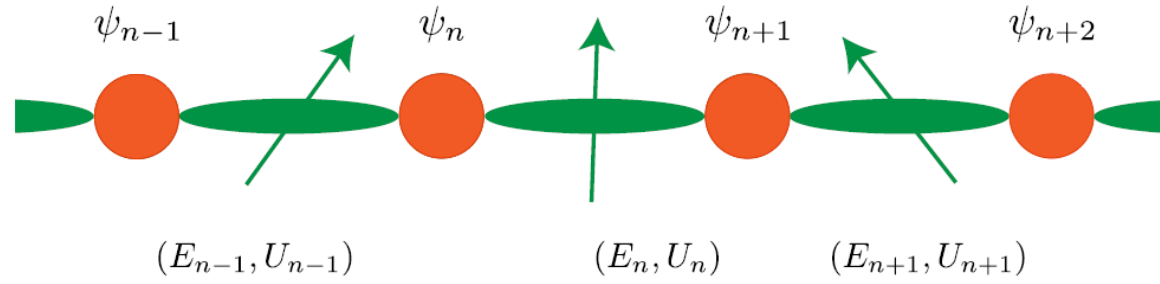


$$W_{\square} = \text{tr} U_{\mu}(x) U_{\nu}(x + \hat{\mu}) U_{\mu}^{\dagger}(x + \hat{\nu}) U_{\nu}^{\dagger}(x)$$



Wilson Loop

1D lattice Schwinger model



Electric field

Matter field

$$\hat{H}_{\text{QED}} = \frac{a}{2} \sum_l (\hat{E}_{l,l+1}^2) - \frac{i}{2a} \sum_l (\hat{\psi}_l^\dagger \hat{U}_{l,l+1} \hat{\psi}_{l+1} - \text{H.c.}) + \frac{a}{2} \sum_l m(-1)^l \hat{\psi}_l^\dagger \hat{\psi}_l$$

Static electric field

Matter—Gauge interaction

Kogut & Susskind, PRD 11, 395 (1975)

Chandrasekharan & Wiese, Nucl. Phys. B 492, 455 (1997)

1D lattice Schwinger model

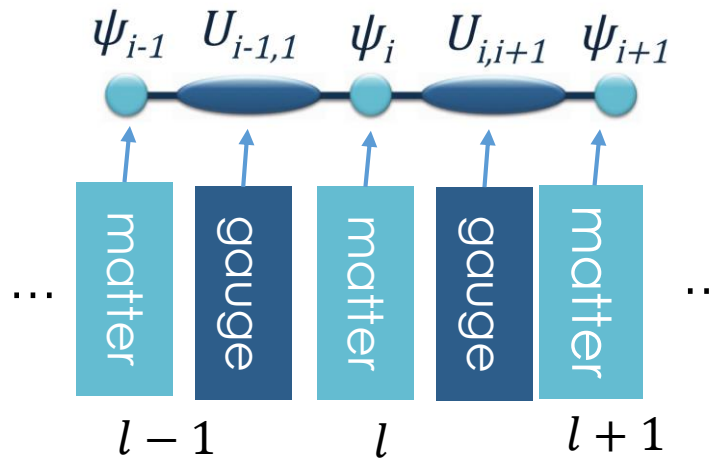
Target Hamiltonian

Gauge field

Matter field, fermionic

$$\hat{H}_{\text{LGT}} = \sum_l \left[-\frac{i\tilde{t}}{2} \underbrace{(\hat{\psi}_l \hat{U}_{l,l+1}^+ \hat{\psi}_{l+1} - \text{H. c.})}_{\text{Matter-Gauge interaction}} + m \hat{\psi}_l^\dagger \hat{\psi}_l \right]$$

Map to the
Hubbard model



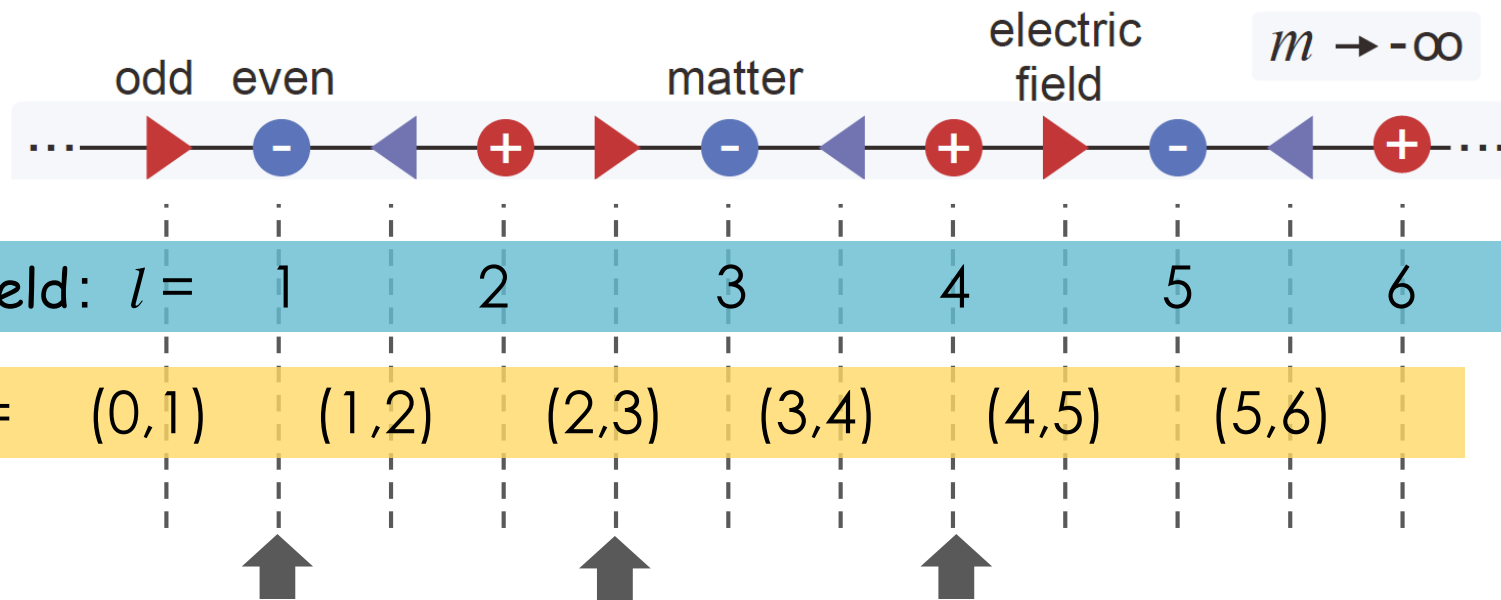
$$\hat{H}_{\text{LGT}} = \sum_l \left[\frac{\tilde{t}}{2\sqrt{2}} \left(\hat{a}_l (\hat{d}_{l,l+1}^+)^2 \hat{a}_{l+1} + \text{H. c.} \right) + m \hat{a}_l^\dagger \hat{a}_l \right]$$

- \hat{a}_l : annihilation operator on matter sites
- $\hat{d}_{l,l+1}$: annihilation operator on gauge links

Theo-Exp mapping



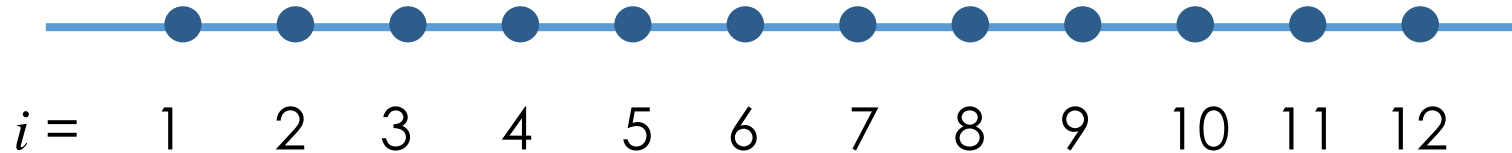
Theo:



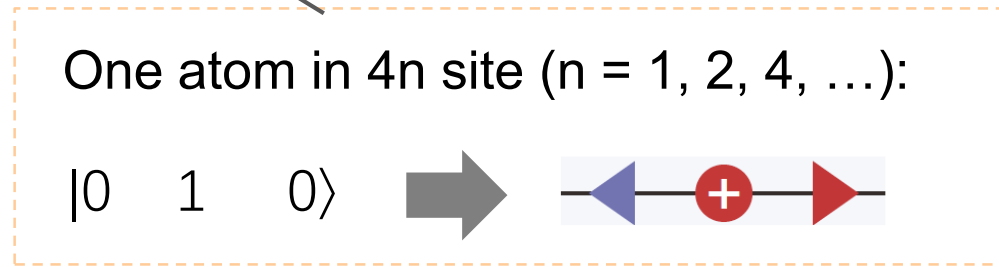
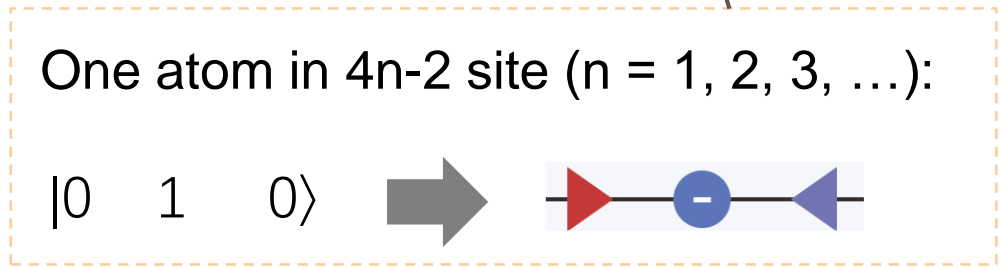
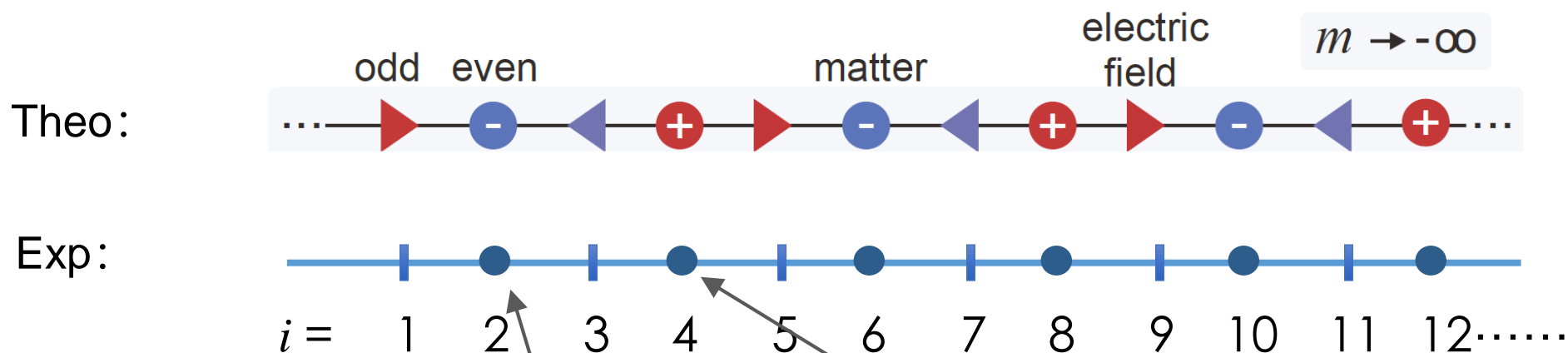
For example:

electron electric field (with different directions) positron

Exp (atom occupancy):



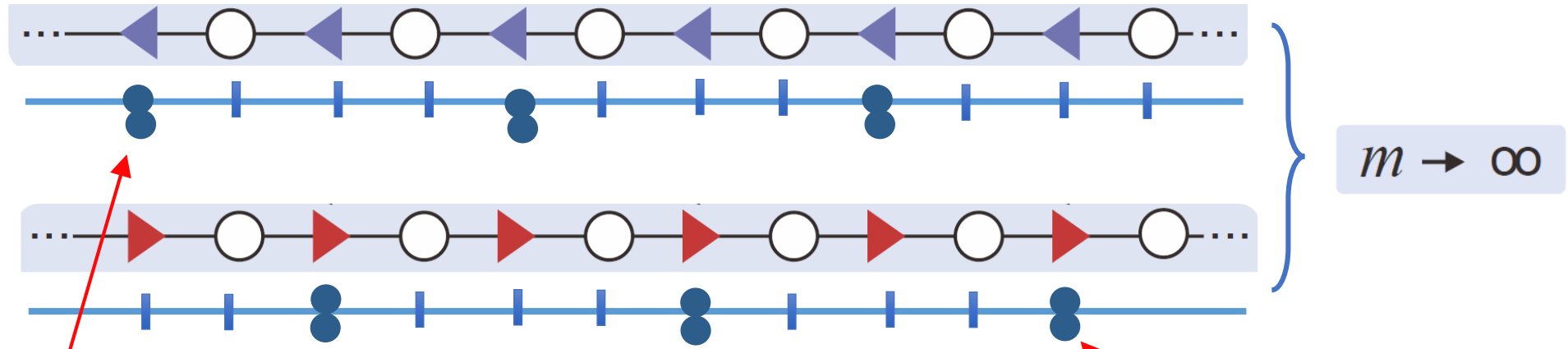
Low-energy limit ($m \rightarrow -\infty$), matter field dominates



Map atom occupancy to matter-gauge state

Theo-Exp mapping

High-energy limit ($m \rightarrow \infty$), matter field annihilated to gauge field



Map atom occupancy to matter-gauge state:

Two atoms in $4n-3$ site ($n = 1, 2, 3, \dots$):



Two atoms in $4n-1$ site ($n = 1, 2, 4, \dots$):



Experimental realization with a 71-site lattice chain

- ▶ Initial state: $|0101010101 \dots\rangle$
- ▶ Put an overall linear potential to “tilt” the whole lattice, construct the Hamiltonian

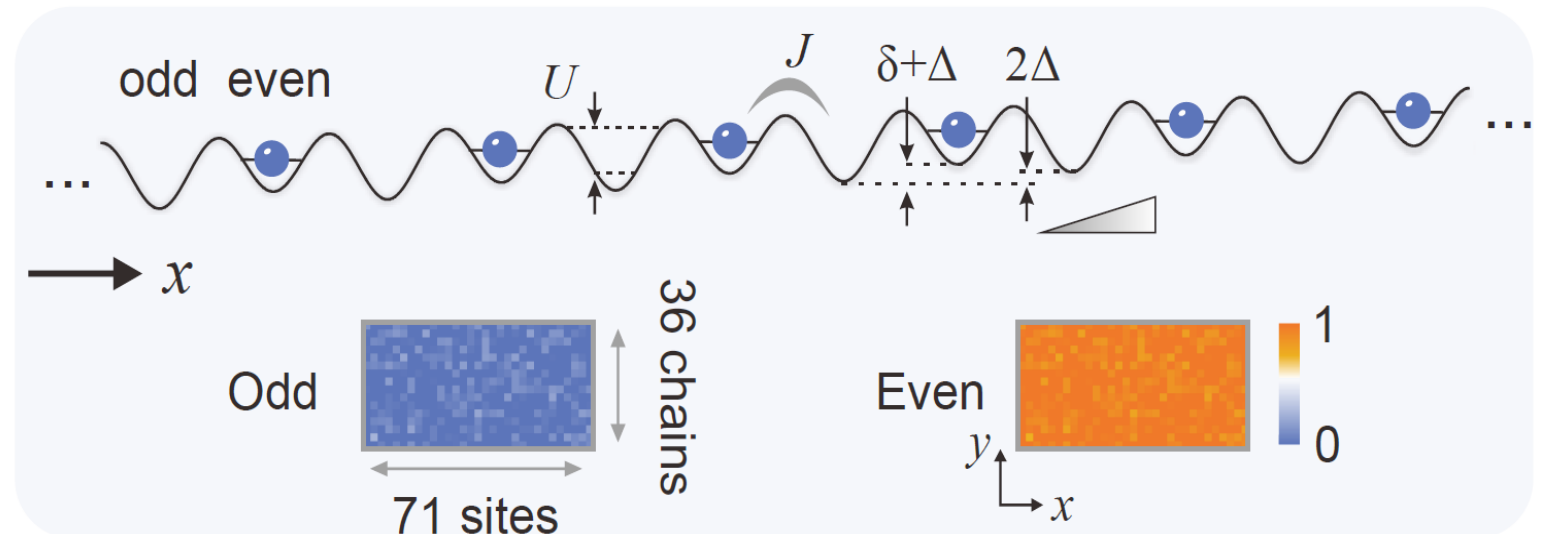
$$\hat{H}_{\text{LGT}} = \sum_l \left[\frac{\tilde{t}}{2\sqrt{2}} \left(\hat{a}_l (\hat{d}_{l,l+1}^+)^2 \hat{a}_{l+1} + \text{H. c.} \right) + m \hat{a}_l^\dagger \hat{a}_l \right]$$

$$m = \delta - U/2$$

$$\tilde{t} = 8\sqrt{2}J^2/U$$

δ intra double well

Δ inter double well

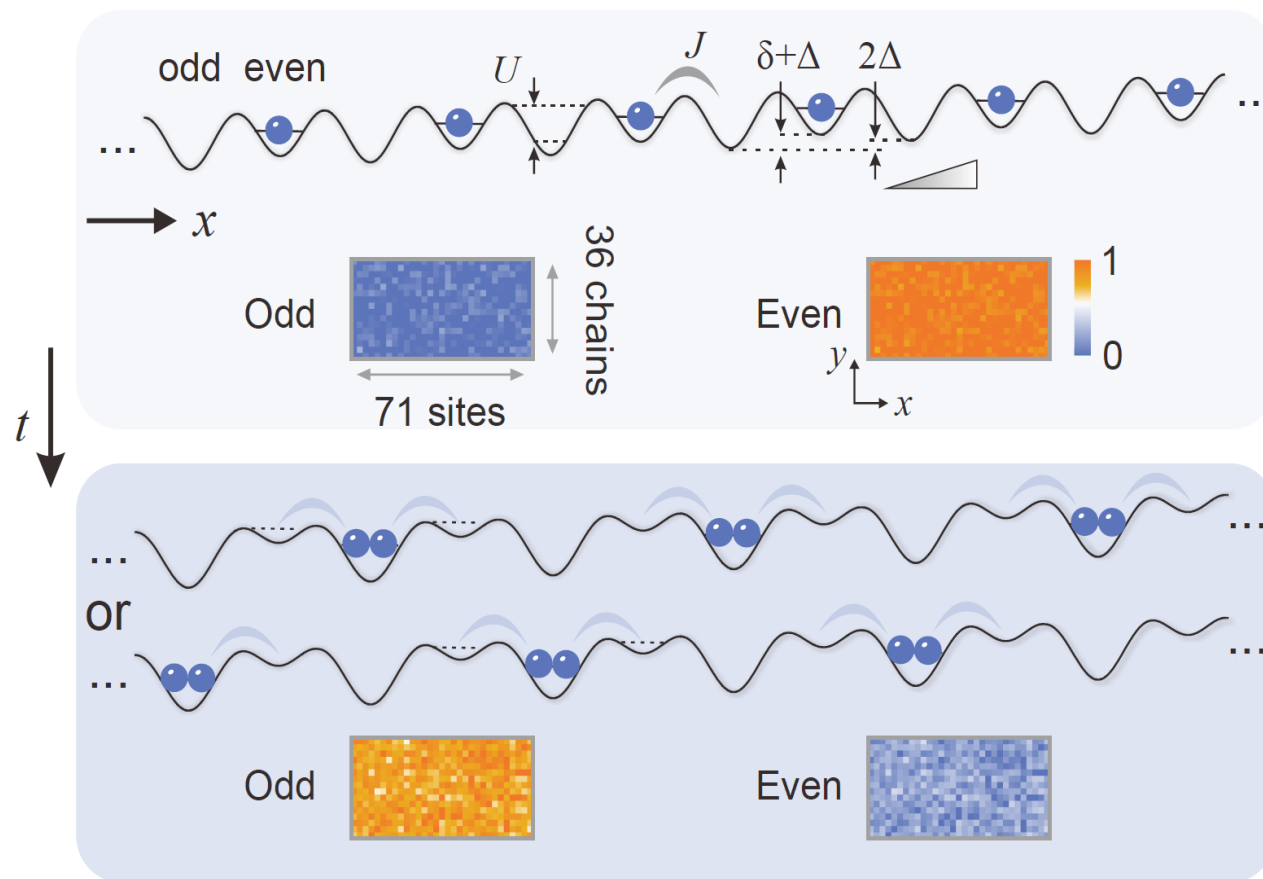


Experimental realization with a 71-site lattice chain



► Ramp the interaction U in 120ms:

$$m/\tilde{t}: -\infty \rightarrow 0 \rightarrow \infty$$



Experimental realization with a 71-site lattice chain

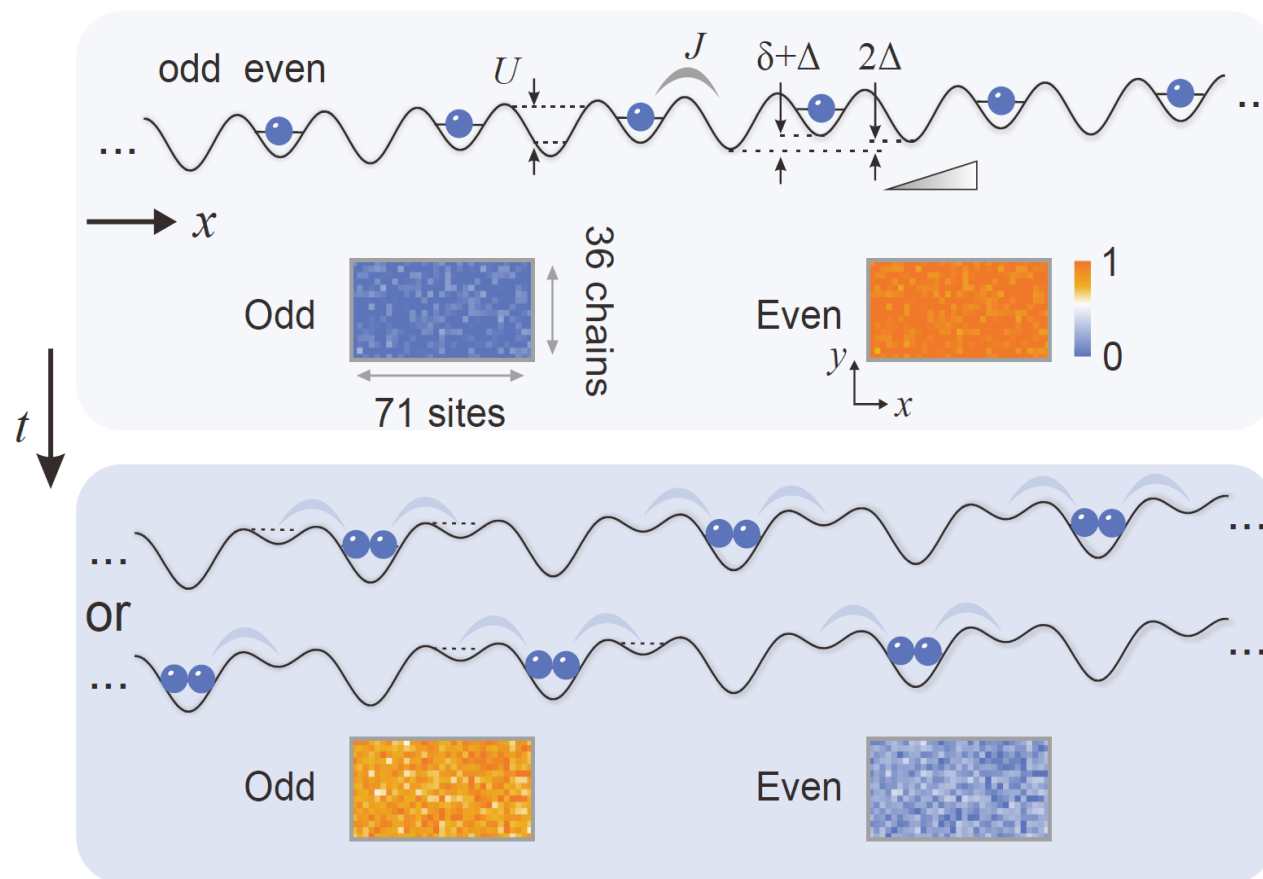
► Ramp the interaction U in 120ms:

$$m/\tilde{t}: -\infty \rightarrow 0 \rightarrow \infty$$

► Phase transition:

$$|0101010\dots\rangle$$

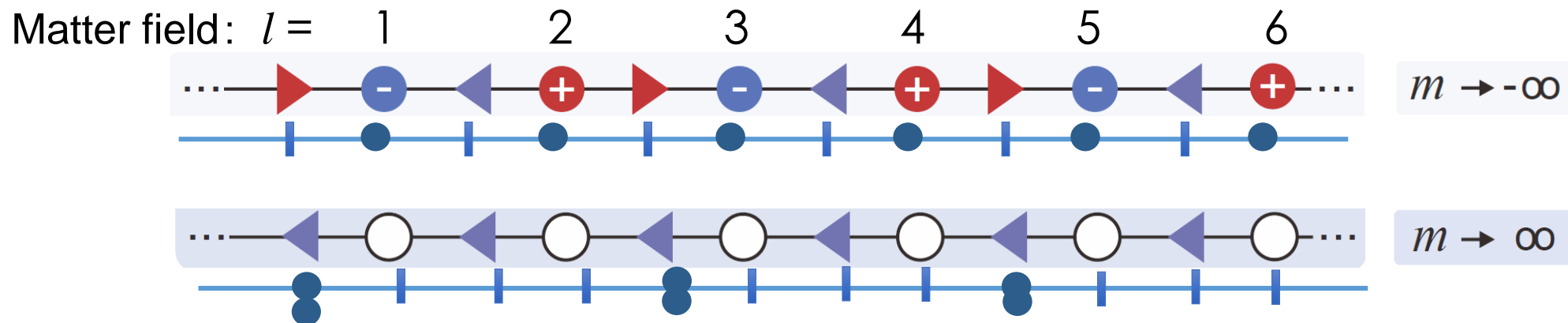
$$\rightarrow |00020002\dots\rangle \text{ or } |20002000\dots\rangle$$



Experimental observation

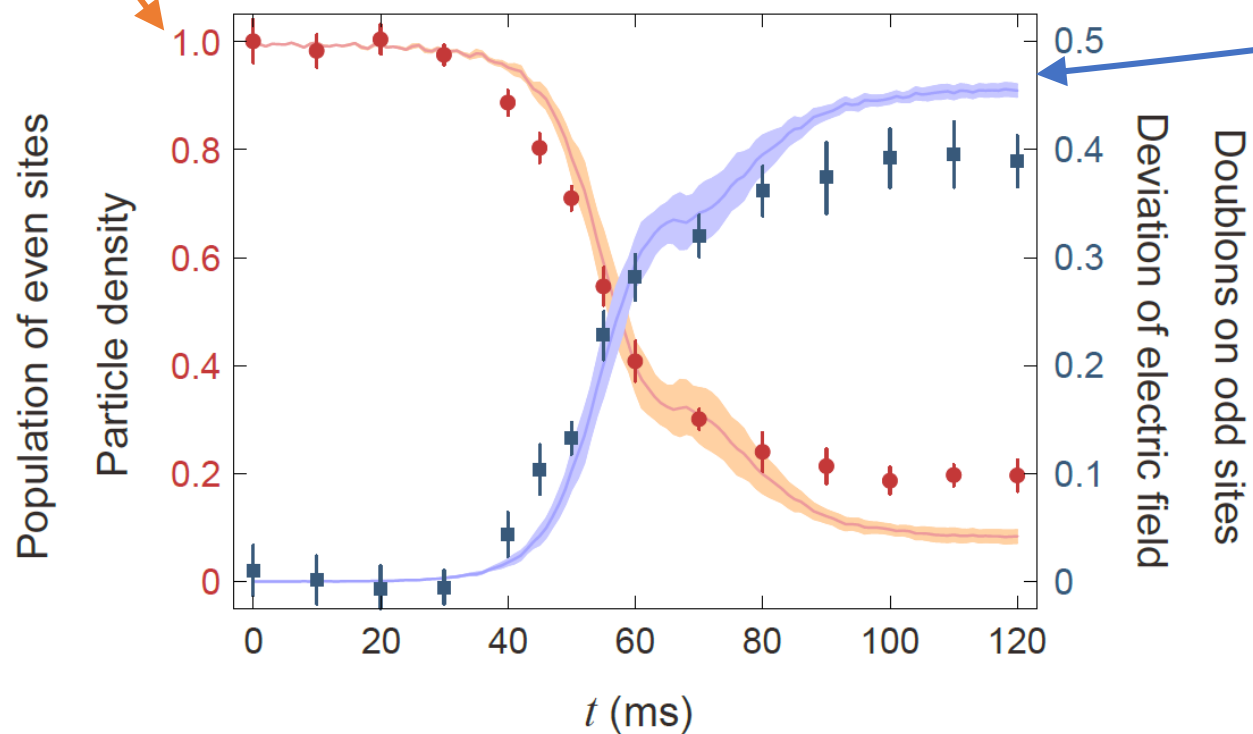
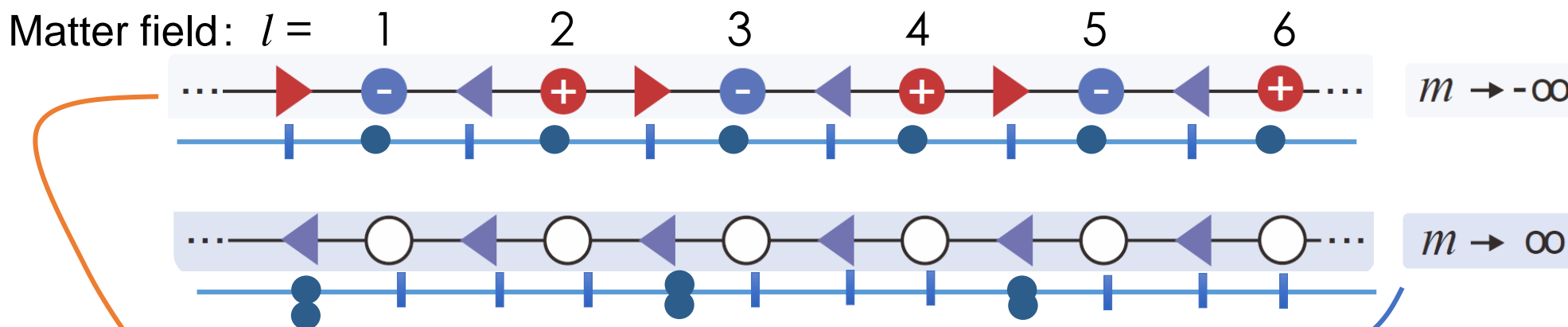


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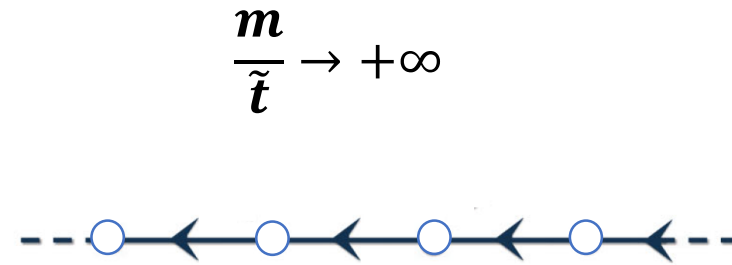
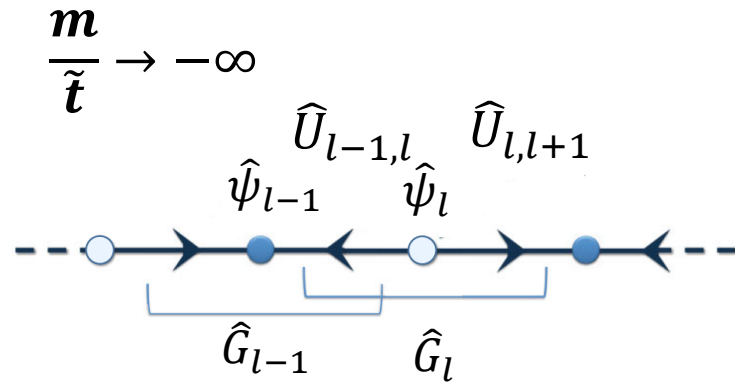
Observed transition from matter field dominated phase to gauge field dominated phase

Experimental observation

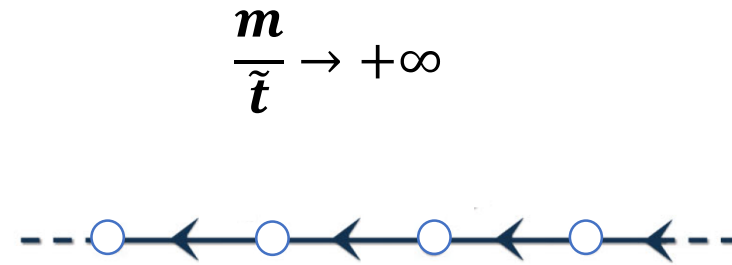
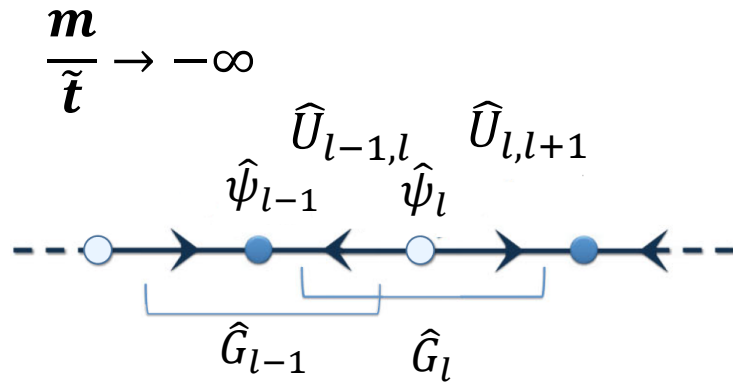


Observed transition from matter field dominated phase to gauge field dominated phase

Gauge invariance and Gauss's law

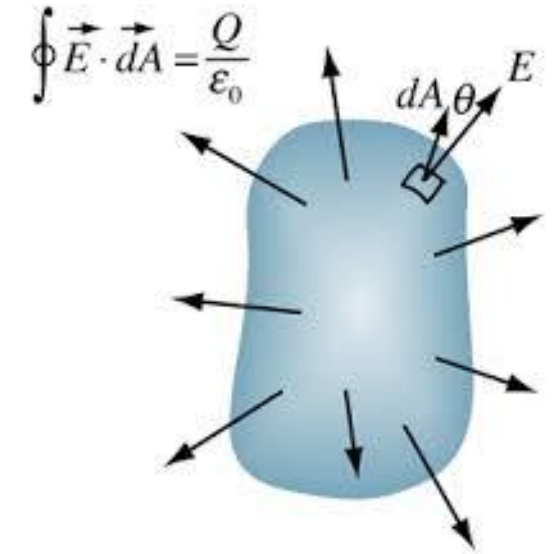
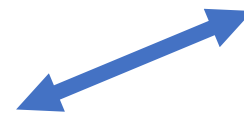


Gauge invariance and Gauss's law

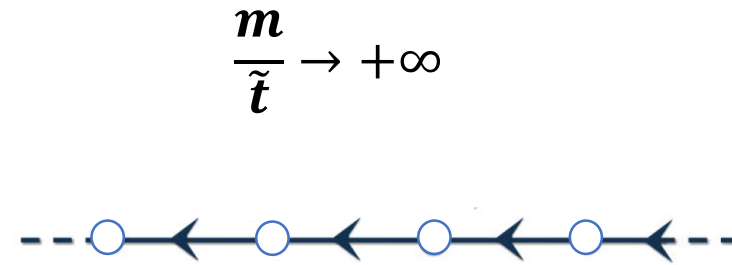
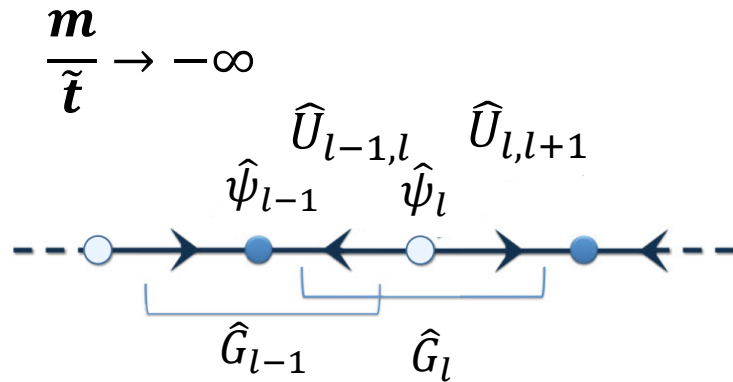


Gauge invariance: $\hat{G}_l = (-1)^{l+1}(\hat{U}_{l,l+1} + \hat{U}_{l-1,l} + \hat{\psi}_l^+ \hat{\psi}_l)$

$$[\hat{G}_l, \hat{H}_{\text{QLM}}] = 0$$



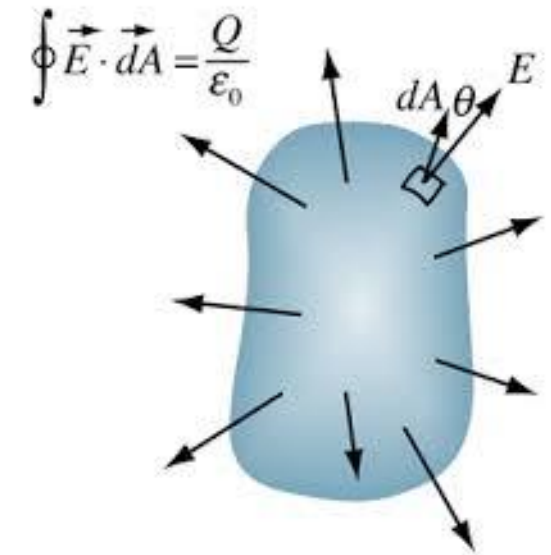
Gauge invariance and Gauss's law



Gauge invariance: $\hat{G}_l = (-1)^{l+1}(\hat{U}_{l,l+1} + \hat{U}_{l-1,l} + \hat{\psi}_l^+ \hat{\psi}_l)$

$$[\hat{G}_l, \hat{H}_{\text{QLM}}] = 0$$

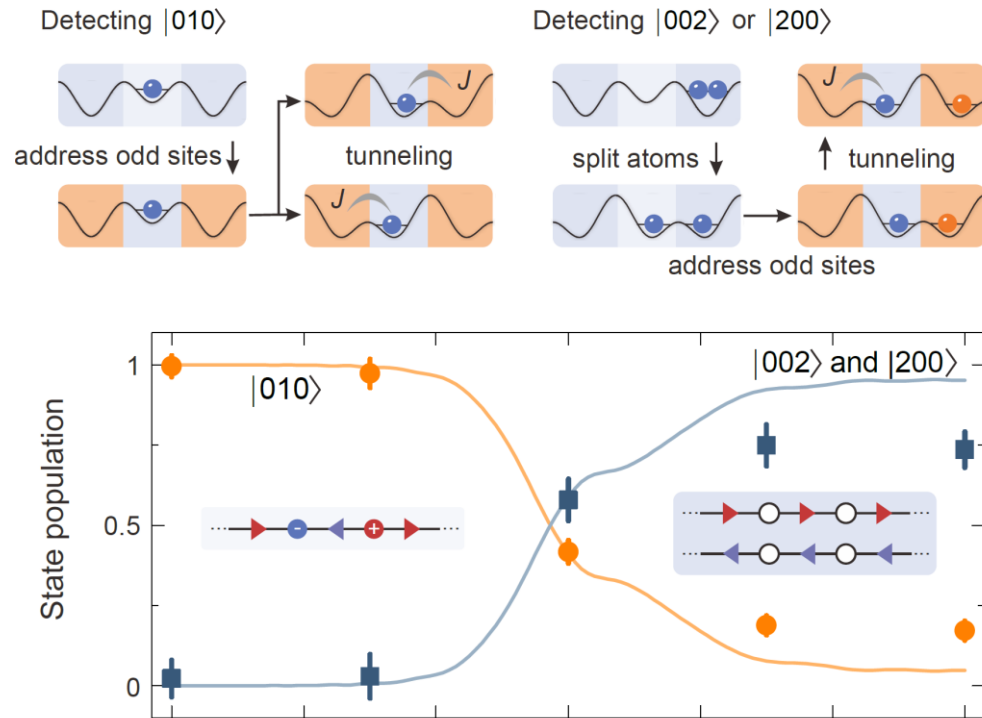
Observable: $\hat{P}_l = |010\rangle\langle 010| + |002\rangle\langle 002| + |200\rangle\langle 200|$



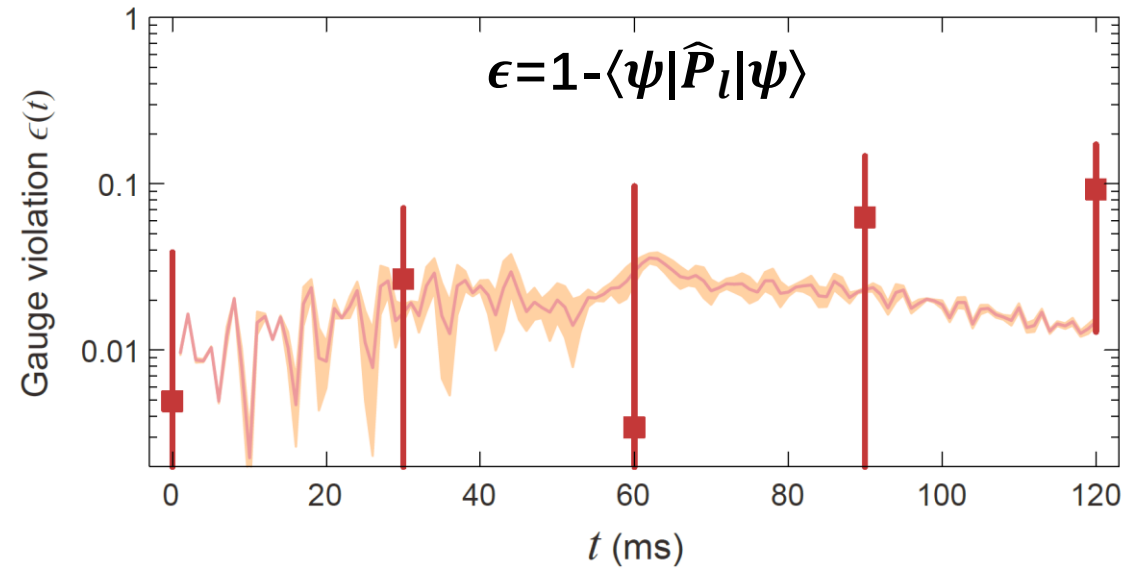
Observation of the gauge invariance



Population of atoms: $\hat{P}_l = |010\rangle\langle 010| + |002\rangle\langle 002| + |200\rangle\langle 200|$



Violation of Gauss's law: $\epsilon = 1 - \langle \psi | \hat{P}_l | \psi \rangle$



Data points show the violation of Gauss's law, the curve is from a t-DMRG calculation

Illustration of the experiment



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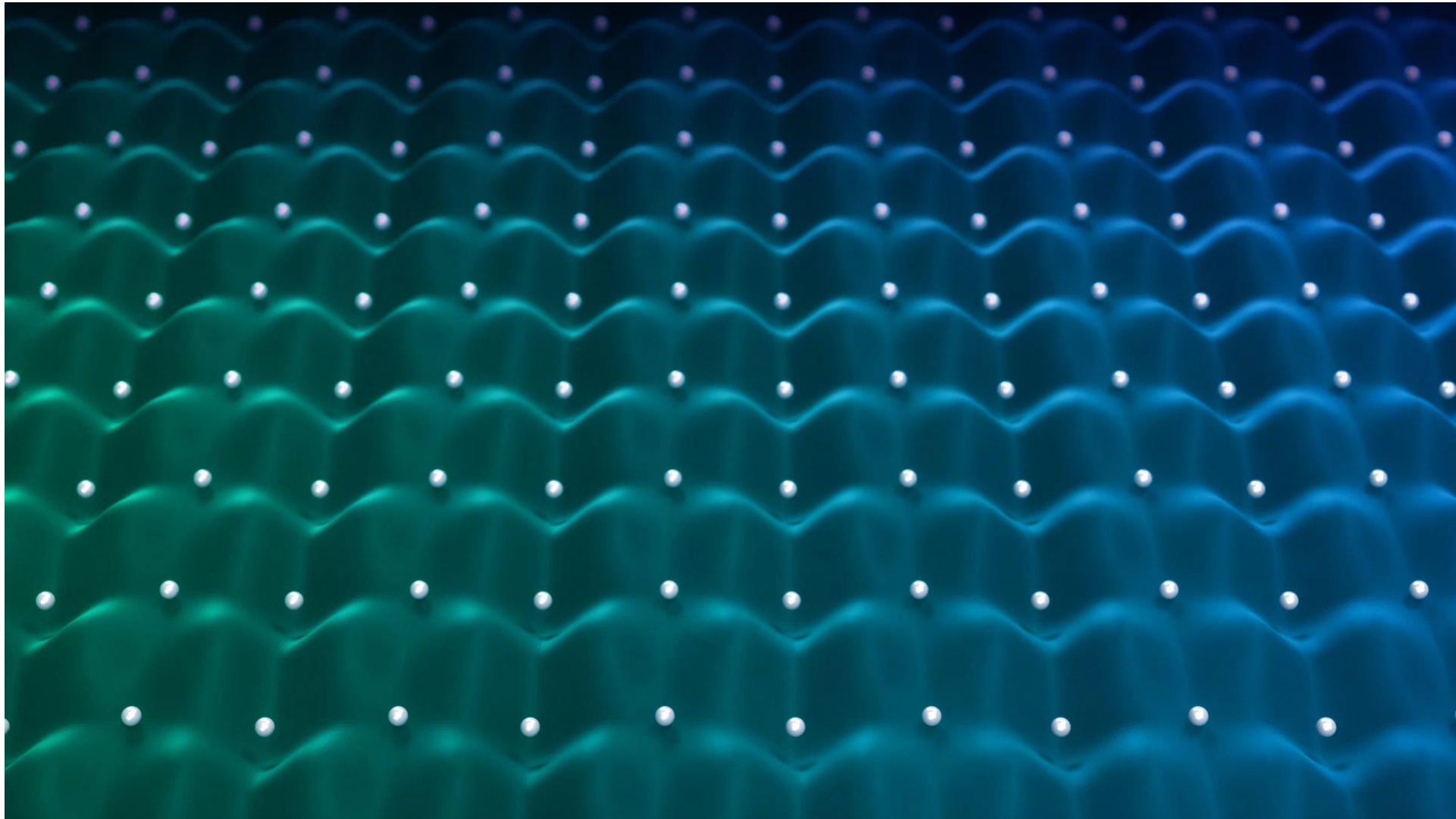
Yang B,Yuan Z -S, Hauke P, Pan J- W, Nature 2020

动画：梁琰、石千惠、苑震生等

Illustration of the experiment



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Yang B,Yuan Z -S, Hauke P, Pan J- W, Nature 2020

动画：梁琰、石千惠、苑震生等

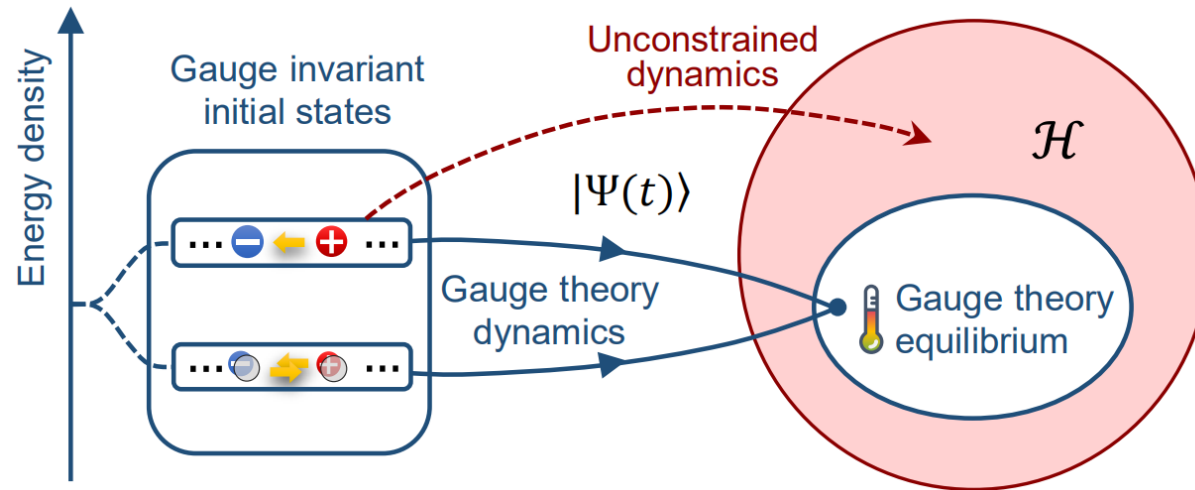
Thermalization of the Lattice Gauge Theory



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Questions:

- Does a LGT system out-of-equilibrium thermalize to a steady state?
- How do two different initial states with the same energy evolve?

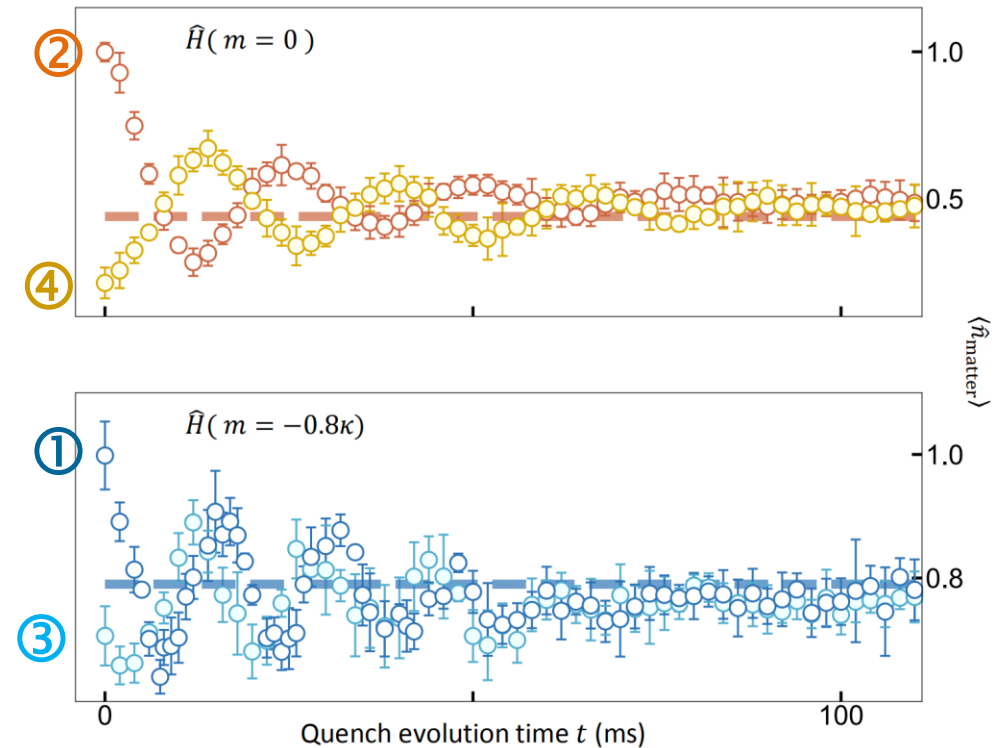
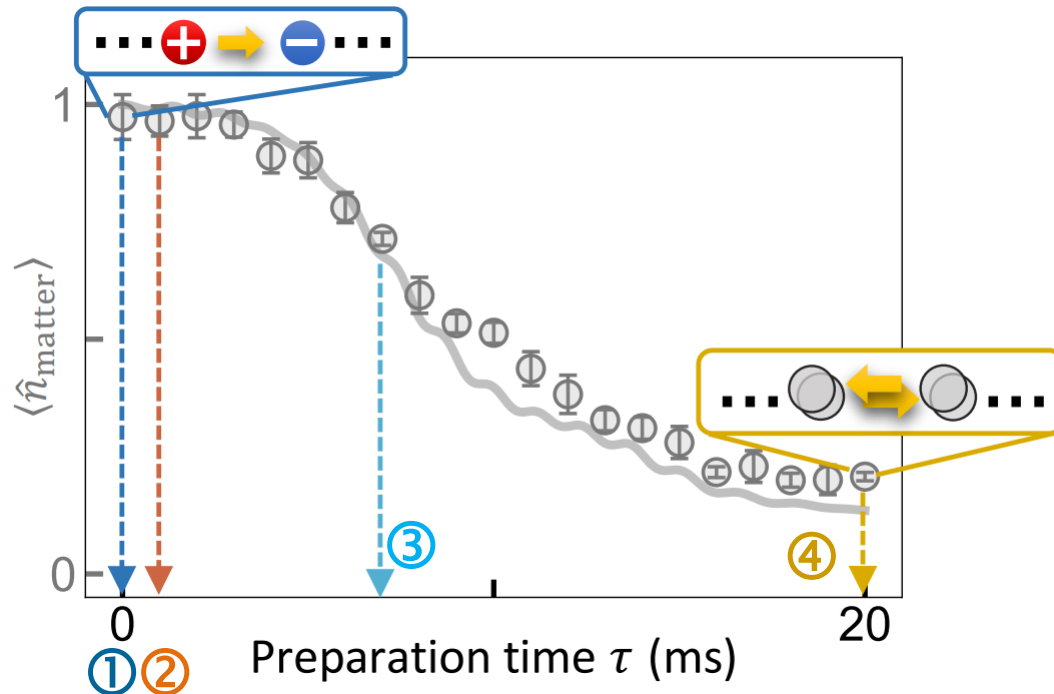


Thermalization of the Lattice Gauge Theory



Preparing different initial states with the same **energy density**

- The system thermalizes to a steady state with an effective temperature.
- Different initial states with the same energy density evolve to an identical effective temperature.



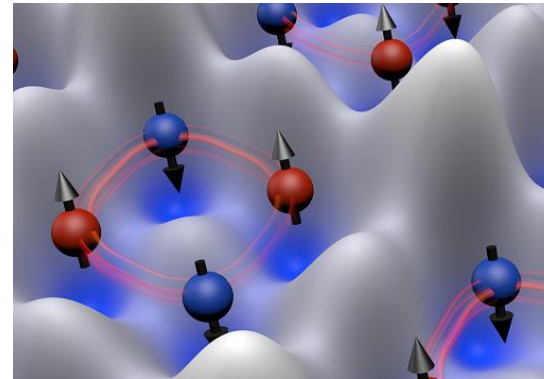
Zhou et al, Science 2022

- About lattice gauge theory (LGT)
- Quantum simulation with ultracold atoms
- The toric code model and Schwinger model
- **Conclusion and outlook**

Conclusion and outlook



- Minimum instance of the toric code model



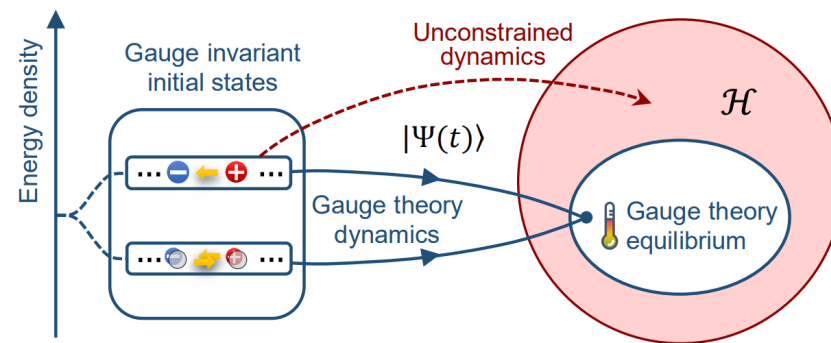
H-N. Dai et al, Nature Physics 2017

- Simulating the 1D Schwinger model with 71-site optical lattice



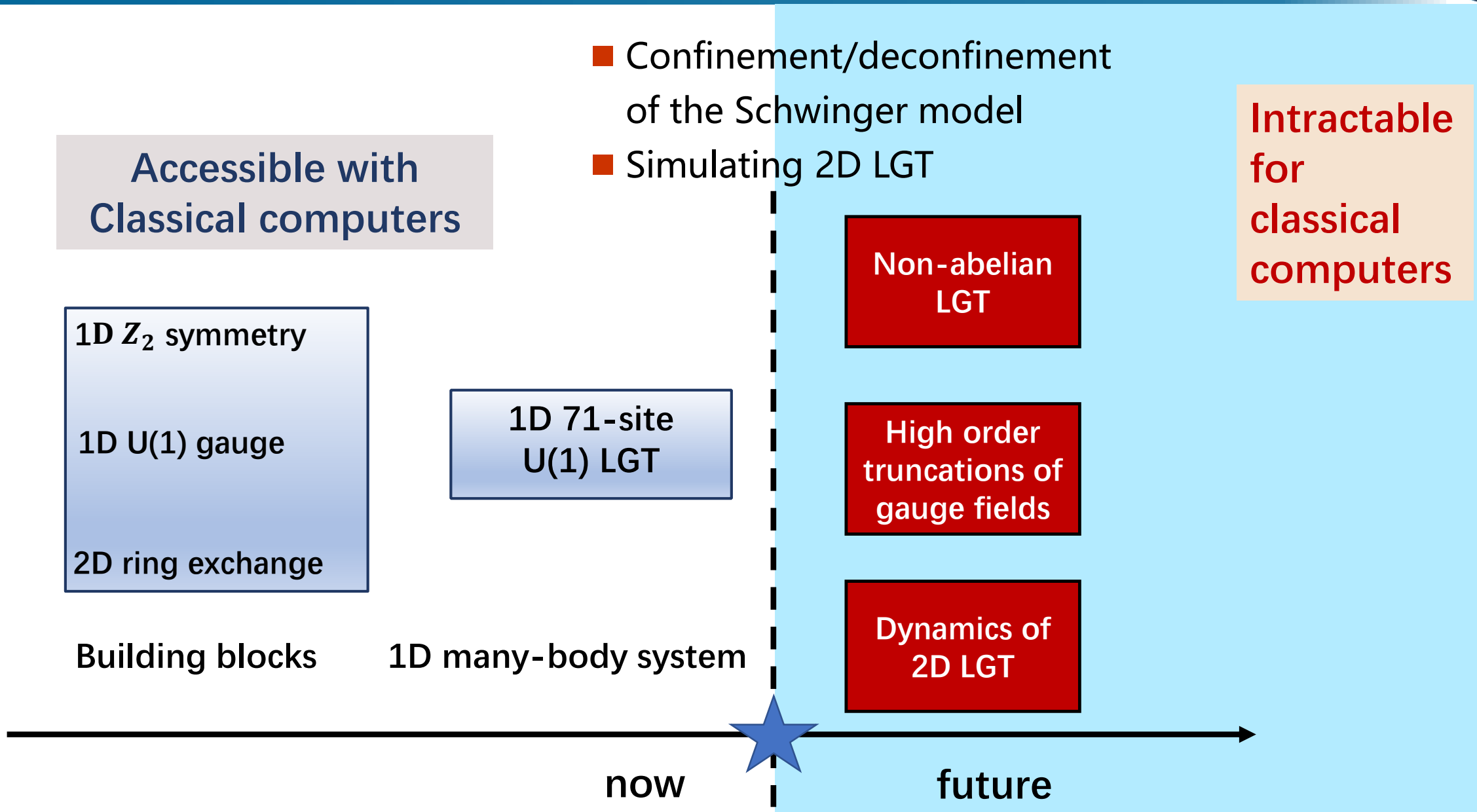
B Yang et al, Nature 2020

- Thermal dynamics of a lattice gauge theory



Z-Y Zhou et al, Science 2022

Conclusion and outlook



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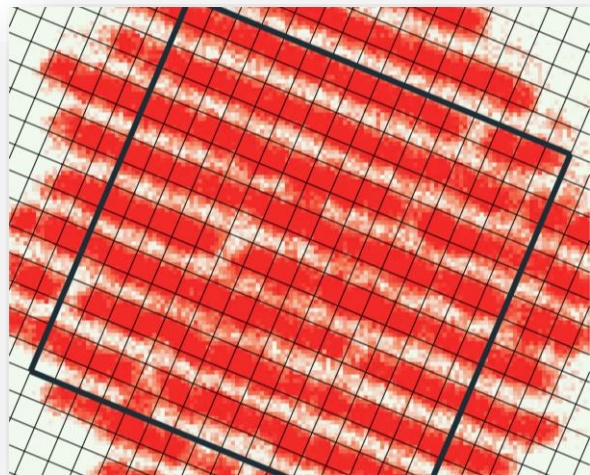


Prof. Hui Zhai
(Tsinghua)

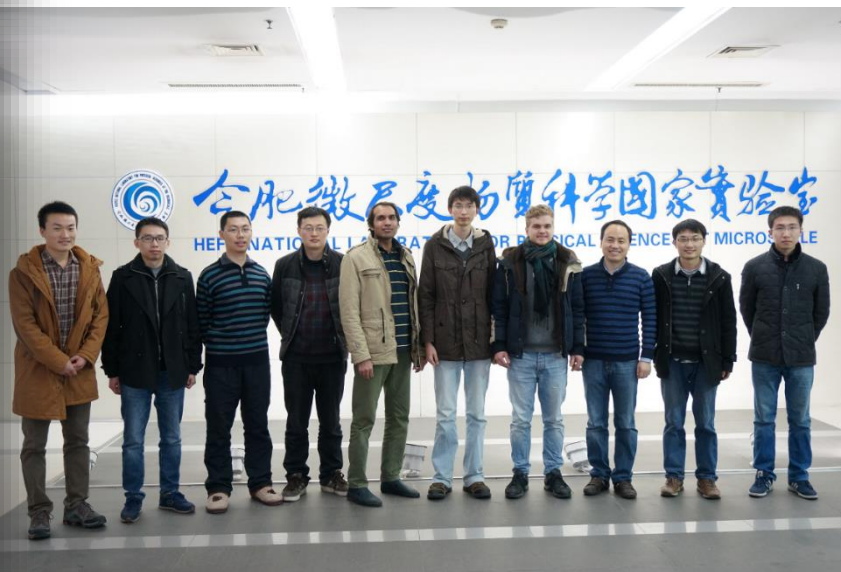
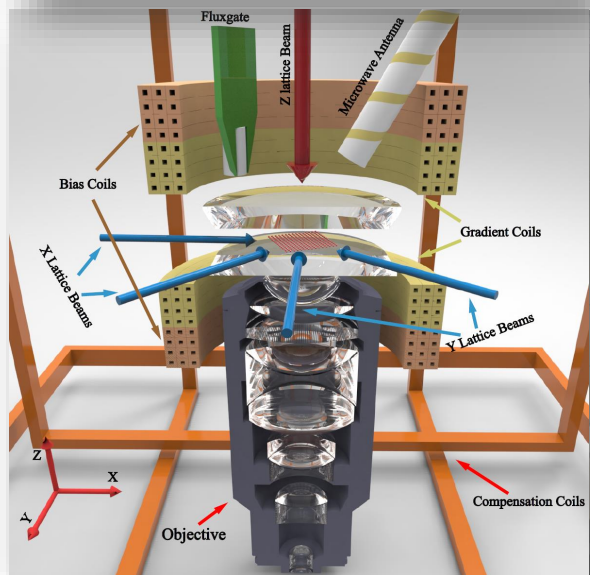
Acknowledgement—Quantum gas microscope



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