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从手征核力到原子核第一性原理计算

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1. 手征有效场论核力（三体力）
2. 第一性原理计算：含共振和连续谱

- 把手征核力、重整化、*ab initio*多体方法推广到复能量空间
- 弱束缚，4n体系

低能核理论的二个最基本问题：

1) **Nuclear force（核力）**

2) **Many-body correlations（多体量子关联）**

什么叫核结构第一性原理计算 (*ab initio*, first principles)？

1）从 **realistic nuclear forces**出发

2) “严格”处理量子多体关联 (“严格”— 数值计算收敛)

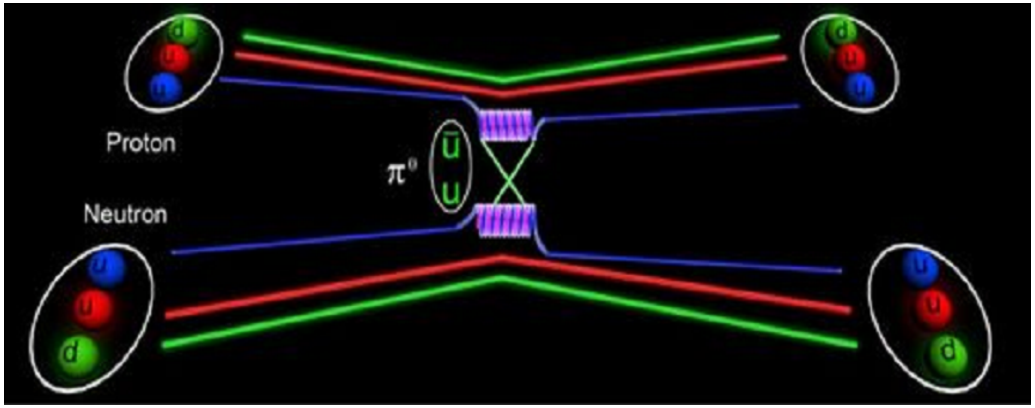
I. Realistic nuclear forces

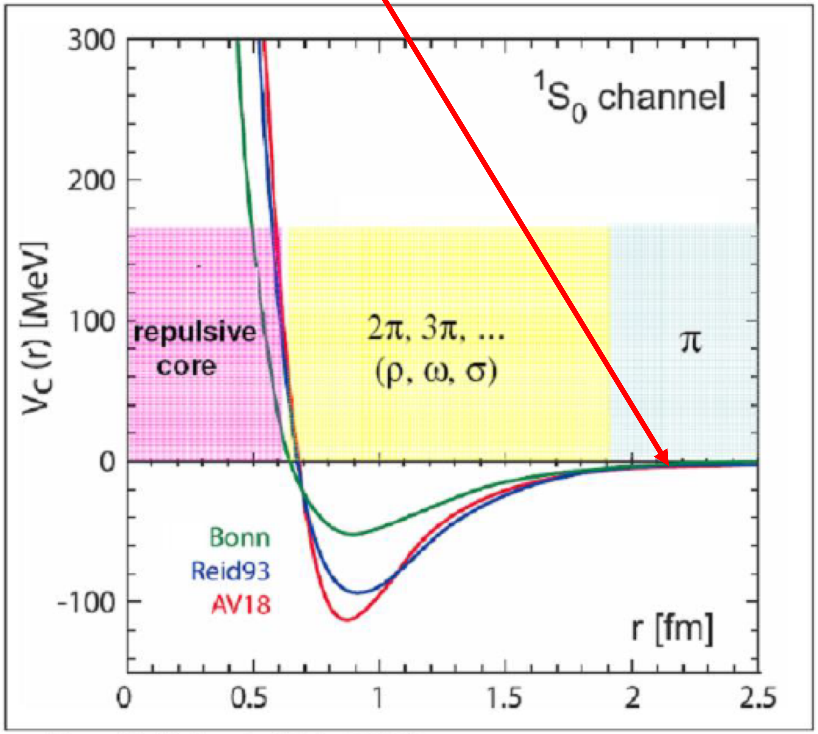
Electromagnetic force has an infinite range! $V(r) \sim \frac{1}{r}$ (交换光子，m_γ=0)

Yukawa nuclear force by π meson exchange $V(r) = -\frac{g^2}{4\pi}\frac{e^{-\frac{m_\pi c}{\hbar}r}}{r}$

交换π介子 $(m_\pi \approx 200 m_e)$

Long-range attractive，no short-range repulsive





From T. Hatsuda (Oslo 2008)

1930’s

Chadwick (1932): Neutron

Heisenberg (1932): First Phenomenology (Isospin)

Yukawa (1935): Meson Hypothesis

1940’s

Discovery of the pion in cosmic ray (1947) and in the Berkeley Cyclotron Lab (1948).

Nobelprize awarded to Yukawa (1949).

1950’s

“Pion Theories”

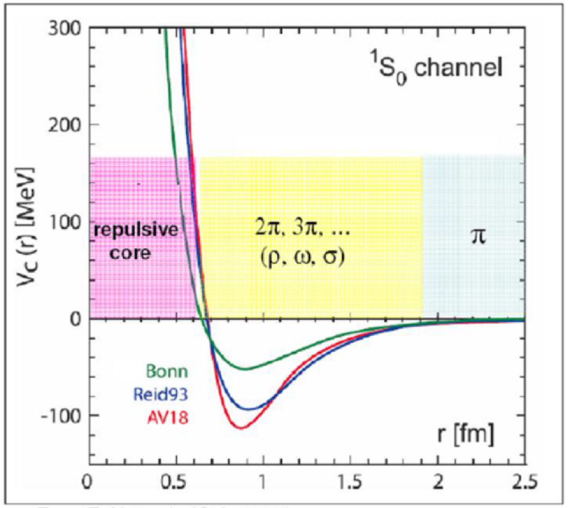
Taketani, Nakamura, Sasaki (1951): 3 ranges.

One-Pion-Exchange (OPE): o.k.

Multi-pion exchanges: Problems!

Taketani, Machida, Onuma (1952);

Brueckner, Watson (1953).



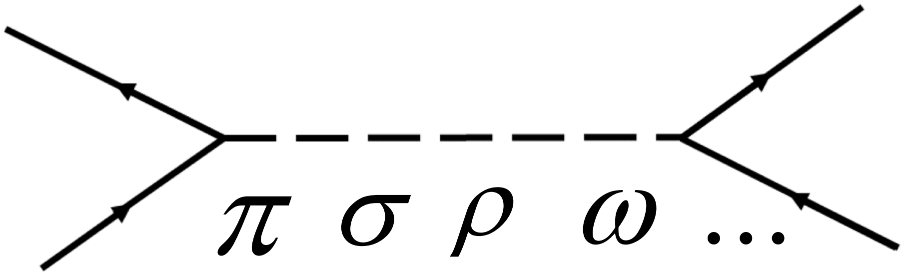
From T. Hatsuda (Oslo 2008)

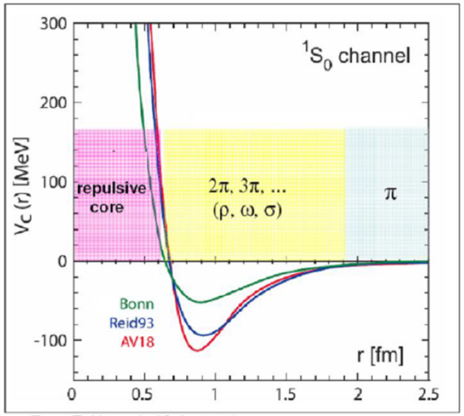
1960's

Many pions = multi-pion resonances:

$\sigma(600), \rho(770), \omega(782) \ldots$

One-Boson-Exchange Model





From T. Hatsuda (Oslo 2008)

1970's

— 80's

Refined Meson Theories

Paris Potential (Lacombe *et al.,* PRC **21**, 861 (1980))

Bonn potential (Machleidt *et al.,* Phys. Rep. **149**, 1 (1987))

1990s: 高精度核力
high-precision modern nuclear forces

$$\chi^2 = \sum_{i=1}^{i=N} \frac{\left(z_i^{theory} - z_i^{\exp}\right)^2}{\left(\Delta z_i^{\exp}\right)^2}$$

(about 6000 NN Data below 350 MeV)

$\chi^2/\mathrm{datum} \to 1$

1993 Nijmegen: high-precision phase shift analysis

1994-2001: High-precision NN potentials:

Nijmegen I, II, AV18，CD-Bonn，N^{3}LO …

Chiral EFT

1970s-1980s

QCD: Quark cluster model

从QCD做核结构计算？

1990 – today

Effective Field Theory (EFT):

Weinberg (1990); Ordonez, Ray, van Kolck (1994/96)

成功的pion theory，拥有QCD一样的对称性

特别是 chiral symmetry and broken spontaneously

QCD=quarks + gluons (symmetries: spin, isospin, parity, chiral symmetry broken spontaneously)

Chiral EFT=nucleon + pion (symmetries: spin, isospin, parity, chiral symmetry broken spontaneously)

50s: multi-pion exchange 失败， Why?

因为当时还不知道如何表达手征对称性及其破缺！写出的场论拉氏量不合适！

Weinberg (1990’s)

At low energy, the effective degrees of freedom are nucleon and pion, rather than quark and gluon!

- QCD at low energy is strong. Perturbation is inapplicable !
- Quarks and gluons are confined into colorless hadrons.
- Nuclear forces are residual color forces (similar to van der Waals forces)

Chiral EFT: $\mathscr{L}_{\text{eff}} = \mathscr{L}_{\pi\pi} + \mathscr{L}_{\pi N} + \mathscr{L}_{NN} + \cdots$

- π- π Lagrangian: $\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4}\,\mathrm{Tr}\left[\partial_\mu U\,\partial^\mu U^+ + m_\pi^2\left(U + U^+\right)\right];$

  m_π and f_π are fixed ($f_\pi = 92.4$ MeV). No free parameters.

- π -N Lagrangians: $\mathrm{L}^{(1)}_{\pi N} = \bar{N}\left[i\partial_0 - \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_0\boldsymbol{\pi}) - \frac{g_A}{2f_\pi}\boldsymbol{\tau}\cdot(\vec{\sigma}\cdot\vec{\nabla})\boldsymbol{\pi}\right]N$

  $g_A = 1.29$, no free parameters.

  $\mathrm{L}^{(2)}_{\pi N}$: 4 parameters.

  $\mathrm{L}^{(3)}_{\pi N}$: 4 parameters.

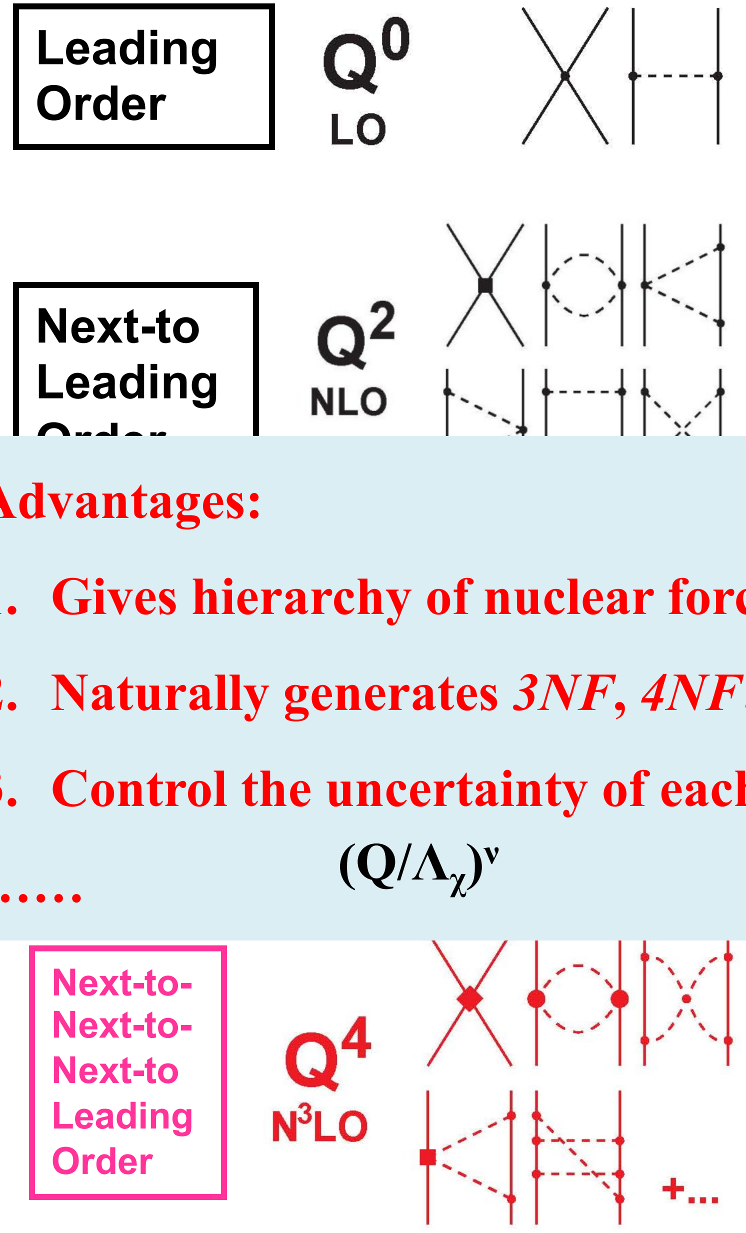
  } In principal fixed by pi-N data

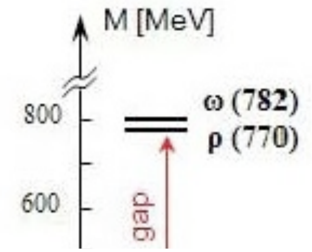
- N-N Lagrangian (“Contacts” N3LO): 24 essentially free parameters.

R. Machleidt, D.R. Entemb, Phys. Rep. 503, 1 (2011)

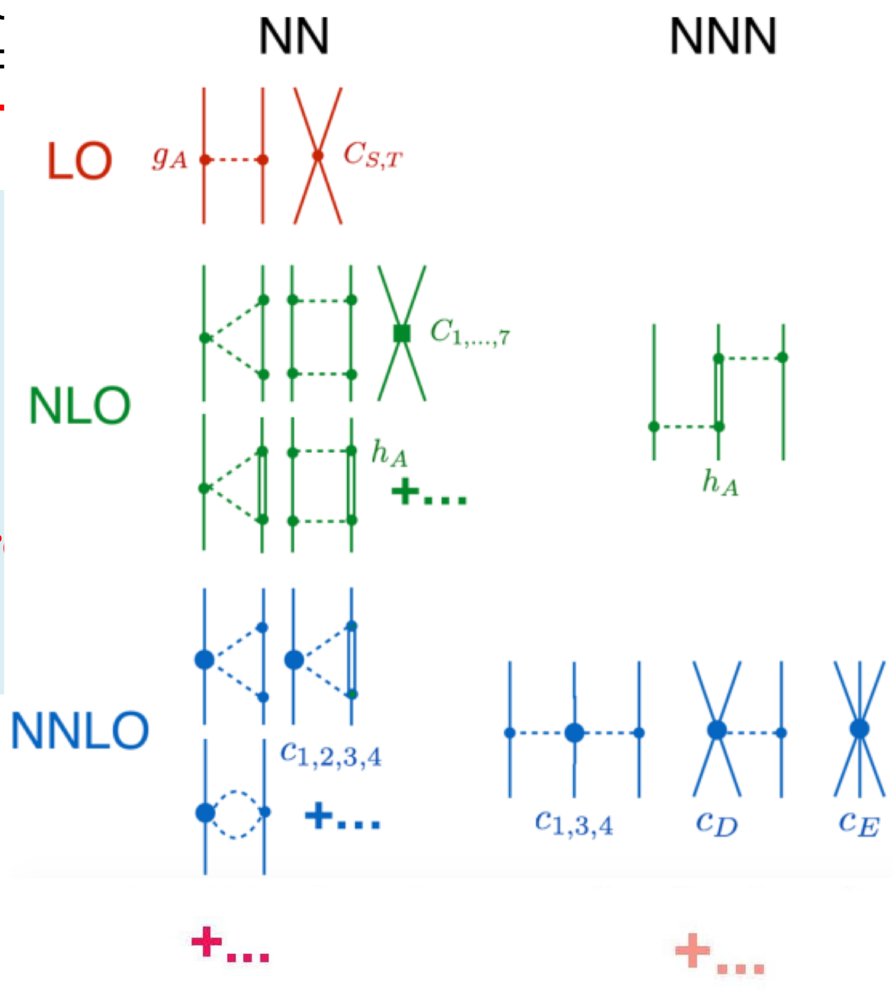
2N forces 3N forces 4N forces

Chiral EFT





200
始EF



Advantages:

1. **Gives hierarchy of nuclear force;**
2. **Naturally generates *3NF*, *4NF*…**
3. **Control the uncertainty of each hierar**

$(Q/\Lambda_\chi)^\nu$

……

NLO or NNLO, only 9 parameters

$$V_{\rm LO}^{\rm cont} = C_S + C_T \left(\vec{\sigma}_1 \cdot \vec{\sigma}_2\right)$$

$$V_{\rm LO}^{\rm OPE} = -\frac{g_A^2}{4F_\pi^2}\,\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\left(\vec{\sigma}_1 \cdot \vec{q}\right)\left(\vec{\sigma}_2 \cdot \vec{q}\right)}{q^2 + M_\pi^2}$$

$\vec{q}$ = t-channel mom. transfer

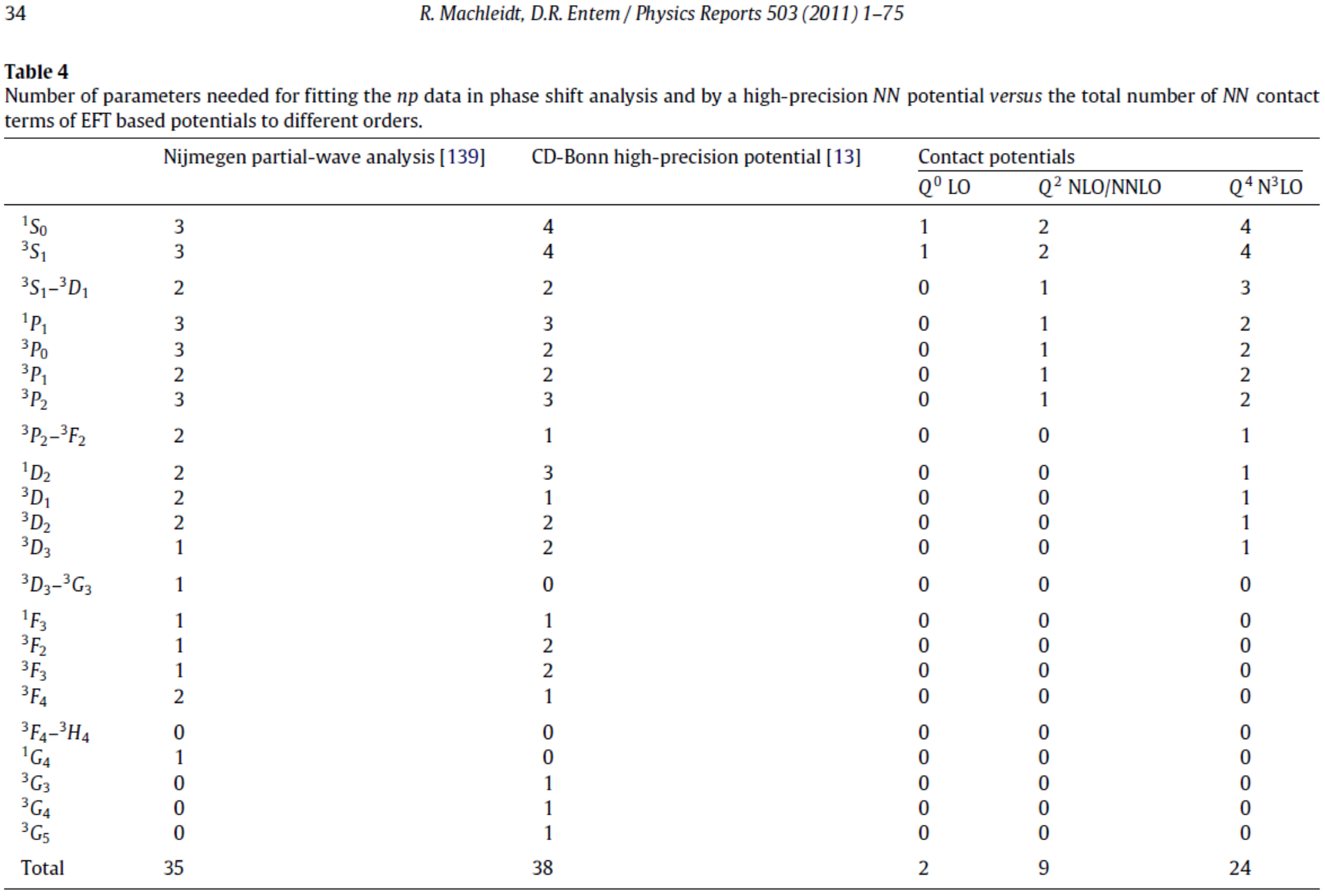
$$V_{\rm NLO}^{\rm cont} = C_1 q^2 + C_2 k^2 + \left(C_3 q^2 + C_4 k^2\right)\left(\vec{\sigma}_1 \cdot \vec{\sigma}_2\right) + iC_5 \tfrac{1}{2}\left(\vec{\sigma}_1 + \vec{\sigma}_2\right) \cdot \left(\vec{q} \times \vec{k}\right) + C_6 \left(\vec{\sigma}_1 \cdot \vec{q}\right)\left(\vec{\sigma}_2 \cdot \vec{q}\right) + C_7 (\vec{\sigma}_1 \cdot \vec{k})(\vec{\sigma}_2 \cdot \vec{k})$$

$\vec{k}$ = u-channel mom. transfer

$$V_{\rm NLO}^{\rm TPE} = -\tfrac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{384\pi^2 F_\pi^4} L(q) \Big[4M_\pi^2 \left(5g_A^4 - 4g_A^2 - 1\right) + q^2 \left(23g_A^4 - 10g_A^2 - 1\right) + \tfrac{48 g_A^4 M_\pi^4}{4M_\pi^2 + q^2}\Big] - \tfrac{3g_A^4}{64\pi^2 F_\pi^4} L(q) \left[\left(\vec{q} \cdot \vec{\sigma}_1\right)\left(\vec{q} \cdot \vec{\sigma}_2\right) - q^2 \left(\vec{\sigma}_1 \cdot \vec{\sigma}_2\right)\right]$$

2003 **N^{3}LO** : Entem and Machleidt,, PRC 68, 041001(R) (2003);

R. Machleidt, D.R. Entem, Physics Reports 503, 1 (2011)



34 R. Machleidt, D.R. Entem / Physics Reports 503 (2011) 1–75

Table 4

Number of parameters needed for fitting the *np* data in phase shift analysis and by a high-precision *NN* potential *versus* the total number of *NN* contact terms of EFT based potentials to different orders.

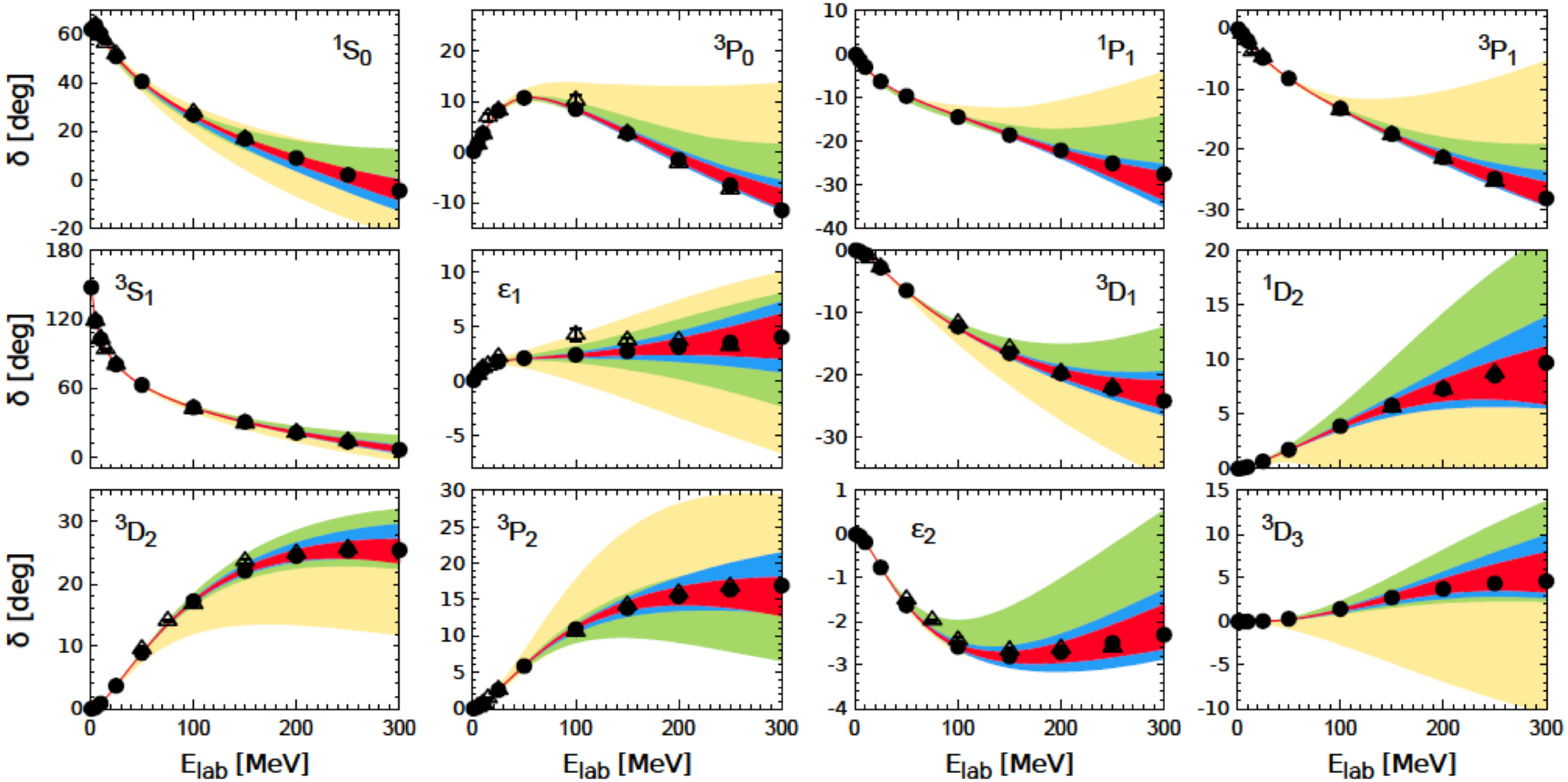
	Nijmegen partial-wave analysis [139]	CD-Bonn high-precision potential [13]	Contact potentials		
			Q^0 LO	Q^2 NLO/NNLO	Q^4 N^{3}LO
$^1S_0$	3	4	1	2	4
$^3S_1$	3	4	1	2	4
$^3S_1$–$^3D_1$	2	2	0	1	3
$^1P_1$	3	3	0	1	2
$^3P_0$	3	2	0	1	2
$^3P_1$	2	2	0	1	2
$^3P_2$	3	3	0	1	2
$^3P_2$–$^3F_2$	2	1	0	0	1
$^1D_2$	2	3	0	0	1
$^3D_1$	2	1	0	0	1
$^3D_2$	2	2	0	0	1
$^3D_3$	1	2	0	0	1
$^3D_3$–$^3G_3$	1	0	0	0	0
$^1F_3$	1	1	0	0	0
$^3F_2$	1	2	0	0	0
$^3F_3$	1	2	0	0	0
$^3F_4$	2	1	0	0	0
$^3F_4$–$^3H_4$	0	0	0	0	0
$^1G_4$	1	0	0	0	0
$^3G_3$	0	1	0	0	0
$^3G_4$	0	1	0	0	0
$^3G_5$	0	1	0	0	0
Total	35	38	2	9	24

PHASE SHIFTS at N4LO

⇒ Precision phase shifts with small uncertainties up to $E_{\mathrm{lab}} = 300$ MeV

$^{2S+1}L_J$

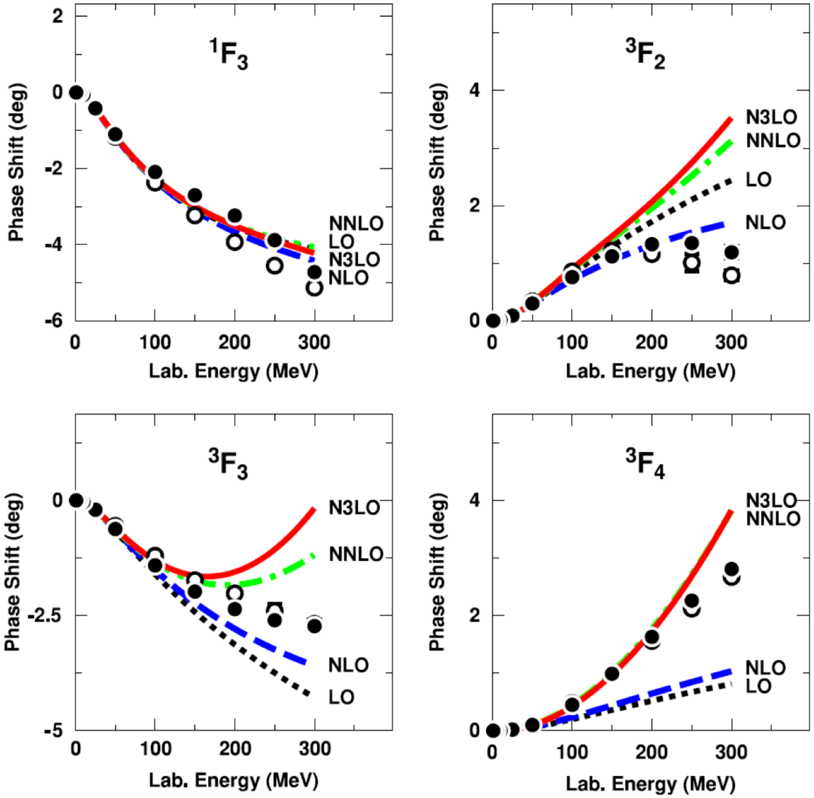
Epelbaum, Krebs, UGM, Phys. Rev. Lett. **115** (2015) 122301

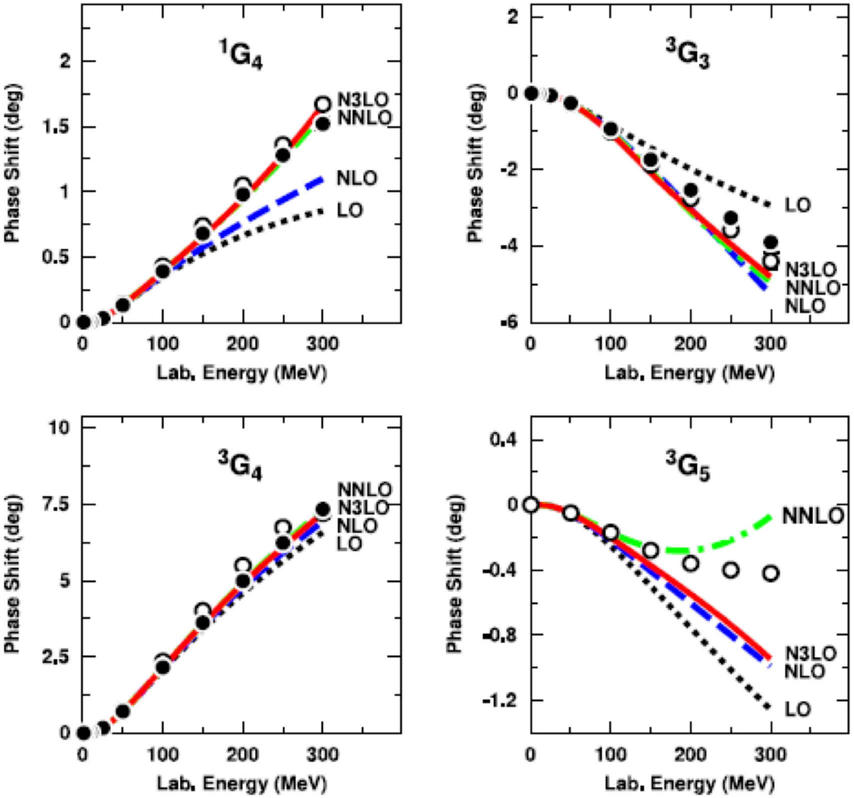


NLO N2LO N3LO N4LO

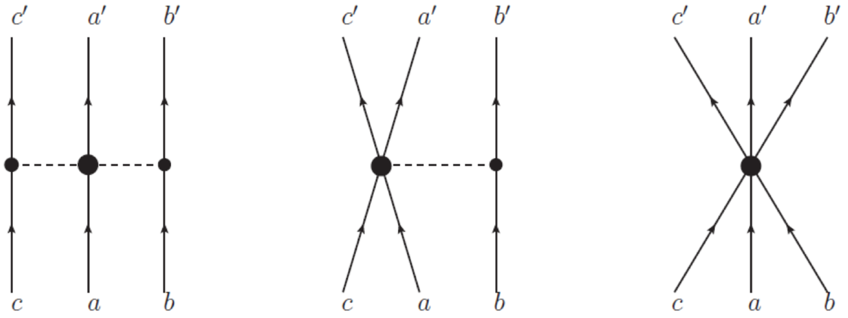
现实核力的最大特点：势参数由核子二体散射数据确定（和^{2}H），而不是核结构实验数据！

R. Machleidt, D.R. Entem / Physics Reports 503 (2011) 1–75





三体力（3NF）



$$v_{3N}^{2\pi} = \frac{1}{(2\pi)^6}\frac{g_A^2}{8f_\pi^2}\sum_{i\neq j\neq k}\frac{(\boldsymbol{\sigma}_i\cdot\boldsymbol{q}_i)(\boldsymbol{\sigma}_j\cdot\boldsymbol{q}_j)}{(q_i^2+M_\pi^2)(q_j^2+M_\pi^2)}F_{ijk}^{\alpha\beta}\tau_i^\alpha\tau_j^\beta$$

$$v_{3N}^{1\pi} = -\frac{1}{(2\pi)^6}\frac{g_A c_D}{8f_\pi^4\Lambda_\chi}\sum_{i\neq j\neq k}\frac{\boldsymbol{\sigma}_j\cdot\boldsymbol{q}_j}{q_j^2+M_\pi^2}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)(\boldsymbol{\sigma}_i\cdot\boldsymbol{q}_j)$$

$$v_{3N}^{cont} = \frac{1}{(2\pi)^6}\frac{c_E}{2f_\pi^4\Lambda_\chi}\sum_{j\neq k}\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k$$

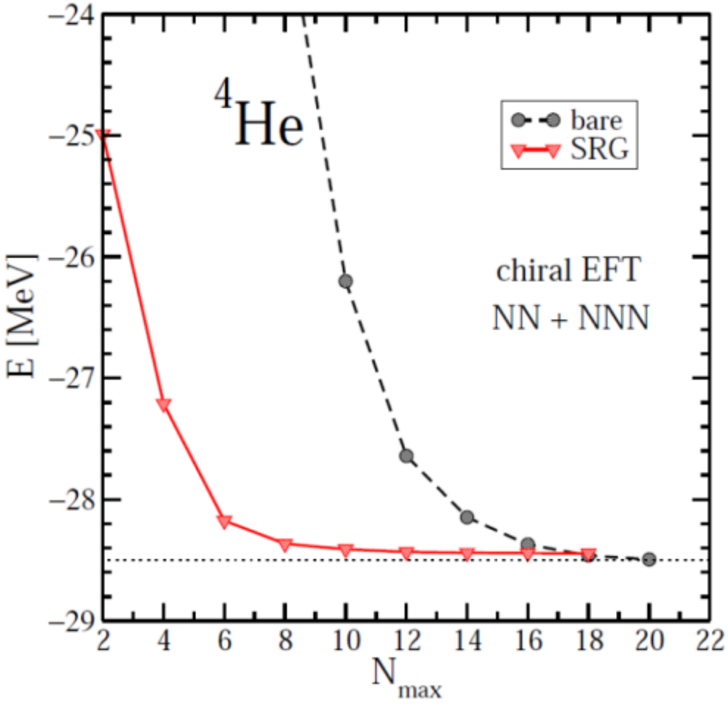
$$\begin{aligned}F_{ijk}^{\alpha\beta} =&\delta^{\alpha\beta}\left[-\frac{4c_1m_\pi^2}{f_\pi^2}+\frac{2c_3}{f_\pi^2}\boldsymbol{q}_i\cdot\boldsymbol{q}_j\right]\\ &+\frac{c_4}{f_\pi^2}\varepsilon^{\alpha\beta\gamma}\tau_k^\gamma\boldsymbol{\sigma}_k\cdot[\boldsymbol{q}_i\times\boldsymbol{q}_j],\end{aligned}$$

II. Renormalizations (softening) : G-matrix, $V_{\text{low-}k}$, OLS, SRG, UCOM…

$$\hat{H}_{int} = \sum_{i<j}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + \sum_{i<j}^{A} V_{NN,ij} + \sum_{i<j<k}^{A} V_{NNN,ijk}$$

1) 自由空间的核力软化，目的是加速数值“量子强关联多体系统”计算的收敛性

2) “无限”空间转化到有限空间，目的是缩小模型空间



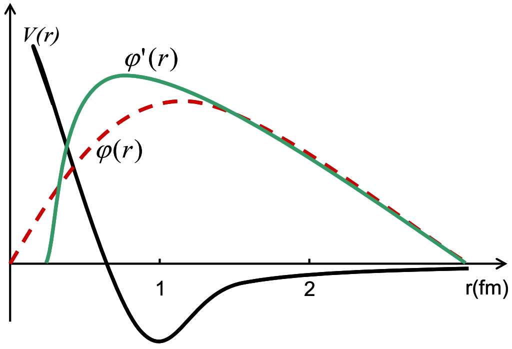
No-Core Shell Model
(NCSM)

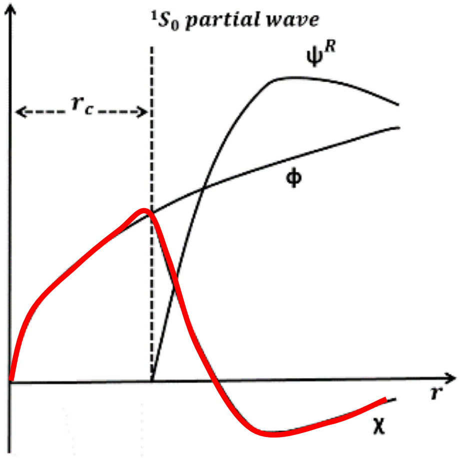
Renormalization schemes preserving all symmetries

G-Matrix：50年代初，Watson和Brueckner *et al.*

Bare force

$$\langle\varphi|G|\varphi\rangle = \langle\varphi|V_{NN}|\varphi'\rangle$$





$$\Phi = \psi^{R} + \chi$$

$$G = V_{NN} + V_{NN}\frac{Q}{\omega - QH_0Q}G$$

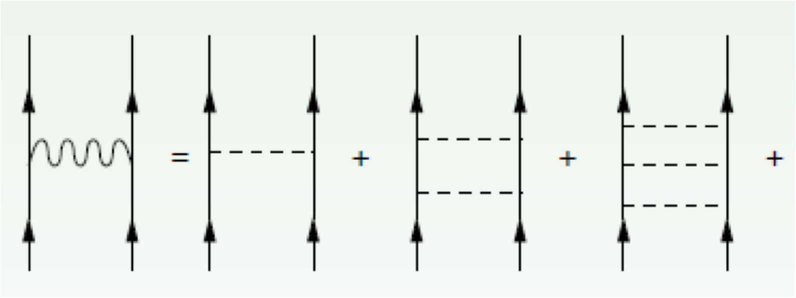
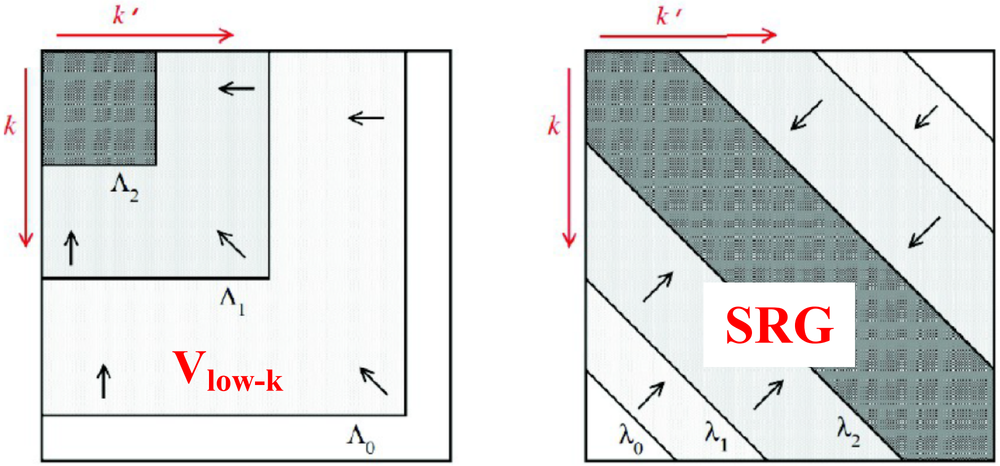


Illustration on how the high momentum nodes are integrated out
in the Vlowk (a) and in the SRG (b) RG methods

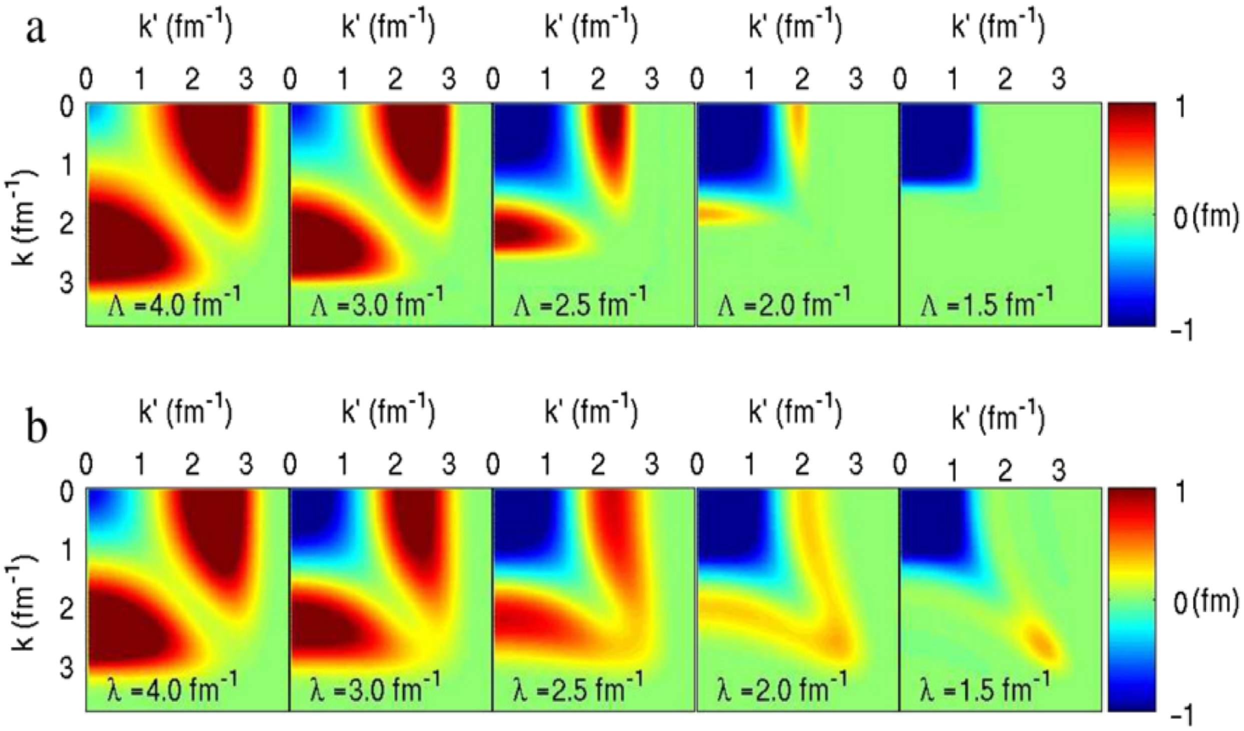
软化核力



(a) (b)

$V_{\text{low-k}}$

SRG



III. 如何“严格”求解强关联量子多体体系？*Ab initio* many-body methods

$$H_{\mathrm{int}} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|) - \frac{P^2}{2Am}$$

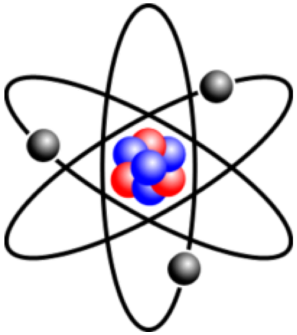
$$\vec{P} = \sum_{i=1}^{A} \vec{p}_i$$

$$\hat{H}_{int} = \sum_{i<j}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + \sum_{i<j}^{A} V_{NN,ij} + \sum_{i<j<k}^{A} V_{NNN,ijk}$$

$$\boxed{i\hbar \frac{\partial \Psi(x,t)}{\partial t} = H\Psi(x,t)}$$

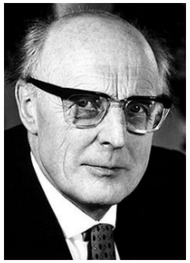
Nuclear shell model

（概念1932年由D. Ivanenko and E. Gapon提出，1949年主要由下面三人独立发展，Nobel prize 1963 ）



Eugene Paul Wigner

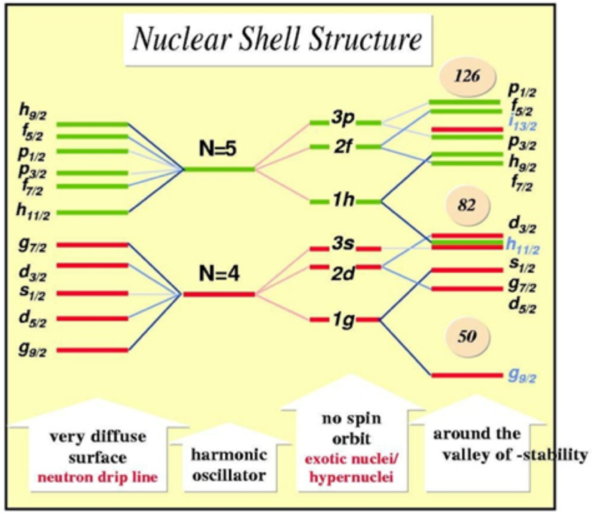
Maria Goeppert-Mayer



J. Hans D. Jensen

单粒子壳层轨道实验可观测吗？

（定量）



Spin-orbit coupling

幻数

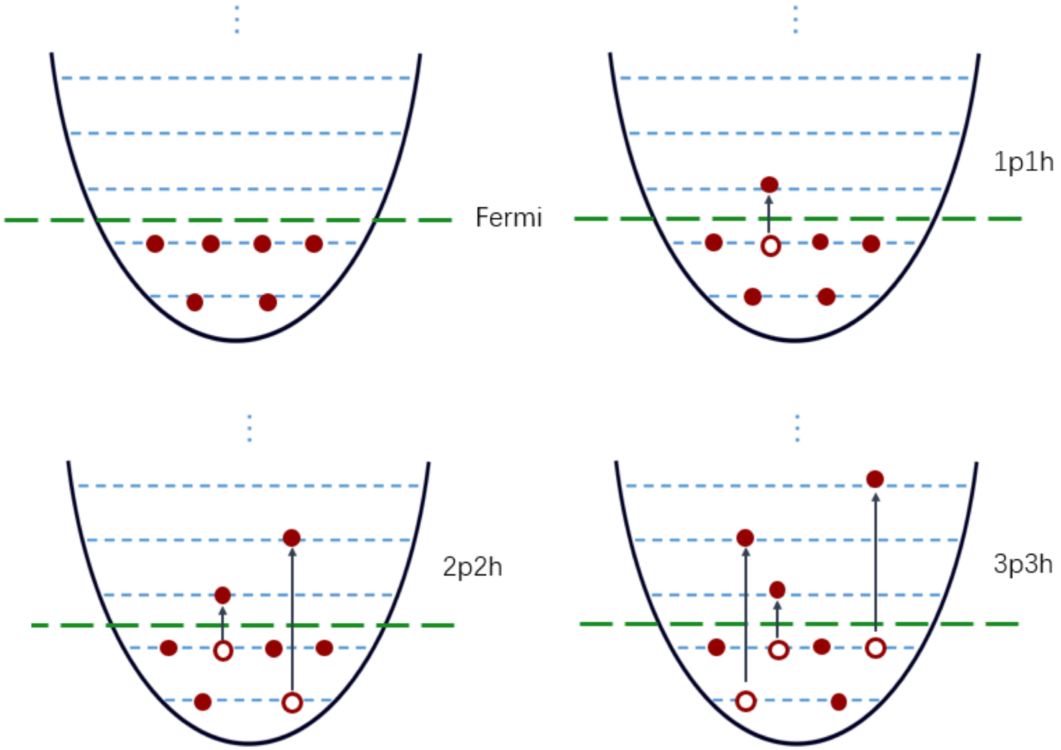
Single-particle orbits (shell)

(configuration-mixing; configuration-interaction) shell model

1950年代开始，Igal Talmi *et al.*

Many-body nucleus

原子核性质实验可观测吗？ （定量）



$N_{\max}$

$$\Psi = c_0\Phi_0 + \underbrace{\sum_{i=1}^{N}\sum_{a=N+1}^{K} c_i^a\Phi_i^a}_{\substack{\text{single}\\ \text{excitations (S)}}} + \underbrace{\sum_{i>j=1}^{N}\sum_{a>b=N+1}^{K} c_{ij}^{ab}\Phi_{ij}^{ab}}_{\substack{\text{double}\\ \text{excitations (D)}}}$$

$$+ \underbrace{\sum_{i>j>k=1}^{N}\sum_{a>b>c=N+1}^{K} c_{ijk}^{abc}\Phi_{ijk}^{abc}}_{\substack{\text{triple}\\ \text{excitations (T)}}} + \underbrace{\sum_{i>j>k>l=1}^{N}\sum_{a>b>c>d=N+1}^{K} c_{ijkl}^{abcd}\Phi_{ijkl}^{abcd}}_{\substack{\text{quadruple}\\ \text{excitations (Q)}}} + \ldots$$

我们组：

1）**Nuclear forces**

NNN done √

NN 进行中

2）核力重整化

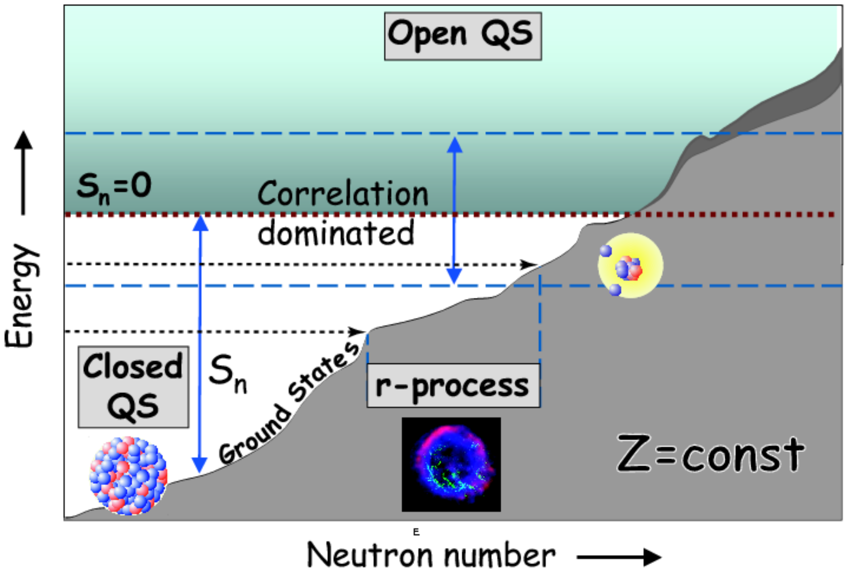
$V_{\text{low-k}}$, **Similarity Renormalization Group (SRG)**

3) Many-Body methods

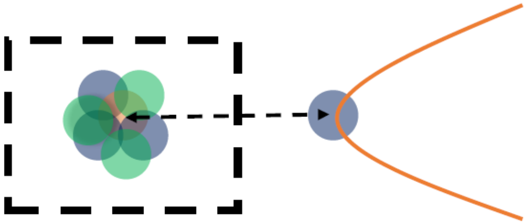
a) Configuration-interaction Gamow shell model

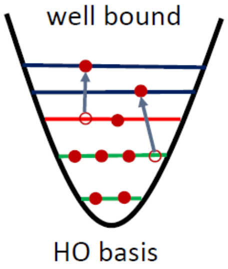
b) In-medium Similarity Renormalization Group (IM-SRG) (Gamow and conventional)

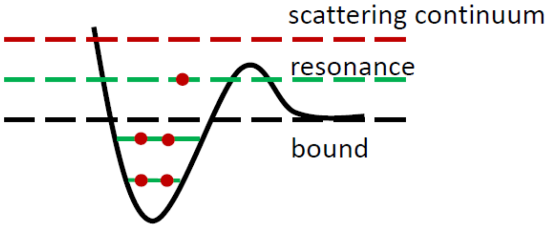
Resonance and continuum




Closed quantum system


Open quantum system





定态

$$\psi(\boldsymbol{r},t)=e^{-iEt/\hbar}\varphi_E(\boldsymbol{r})$$

$$[-\frac{\hbar^2}{2m}\nabla^2+\mathrm{V}(\mathbf{r})]\varphi_E(\boldsymbol{r})=\mathrm{E}\varphi_E(\boldsymbol{r})$$

Berggren basis: T. Berggren, Nucl. Phys. A109 (1968) 265

Berggren 60年代 “发展了"量子力学

用定态方法求解含时问题

T. Berggren, PhD in 1966 (Lund)

$$\psi(\boldsymbol{r},t)=e^{-iEt/\hbar}\varphi_E(\boldsymbol{r})$$

$$[-\frac{\hbar^2}{2m}\nabla^2+\mathrm{V}(\mathbf{r})]\varphi_E(\boldsymbol{r})=\mathrm{E}\varphi_E(\boldsymbol{r})$$

But E can be complex, and $\int\varphi_E(\boldsymbol{r})\varphi_E(\boldsymbol{r})=1$ （Berggren方法的一个关键）

本征值：$E=E_n-i\frac{\Gamma_n}{2}$

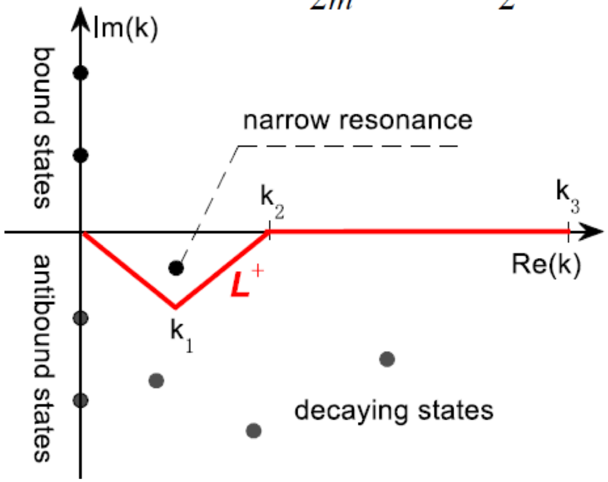
$$\psi(\boldsymbol{r},t)=e^{-iEt/\hbar}\varphi_E(\boldsymbol{r})=e^{-iE_nt/\hbar}\varphi_E(\boldsymbol{r})e^{-\Gamma_nt/2\hbar}$$

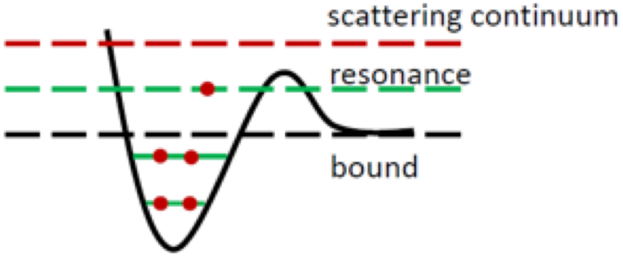
核态有衰变寿命，decay width: $T_{1/2}=\hbar\ln 2/\Gamma$

复空间求解量子力学问题要比实空间困难得多！

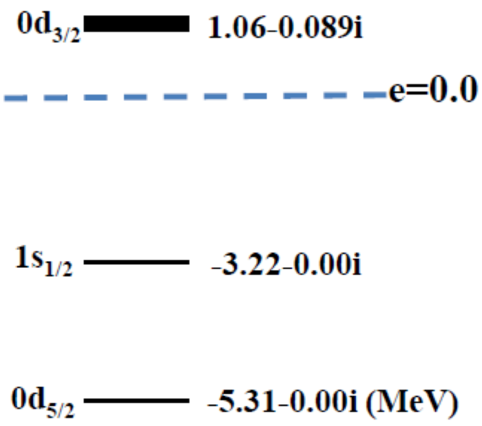
Complex-momentum space: **bound, resonance and continuum**

$$e = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$



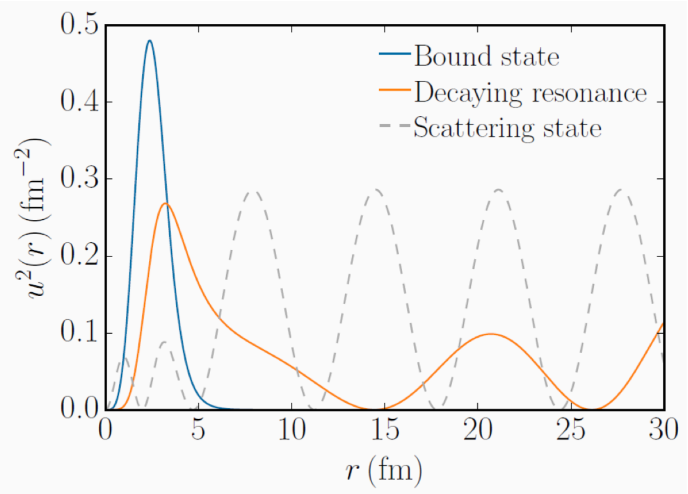


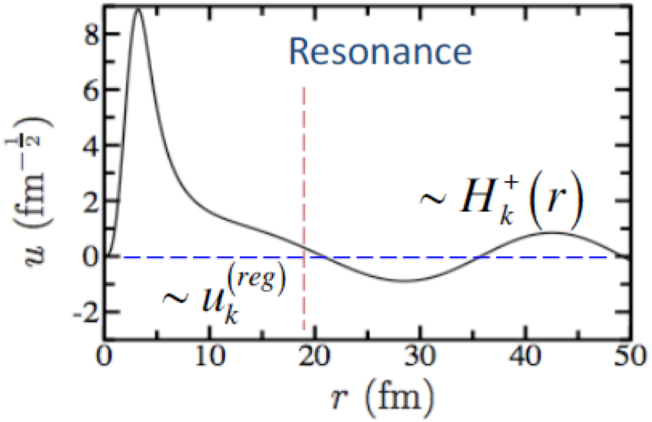
Woods-Saxon in complex-*k* space



WS SPE's

$$u(k,r) \sim C^+ H^+_{l\eta}(kr) + C^- H^-_{l\eta}(kr), \qquad r \to +\infty$$





$$\int_0^\infty dr\, u^2_{\ell,\eta}(r) = 1$$

$$C^+(k)\,C^-(k) = \frac{1}{2\pi}$$

ab initio Gamow Shell Model

我们发展了基于现实核力的 *ab initio* Gamow Shell Model

Bare forces: Strong repulsion，slow convergence

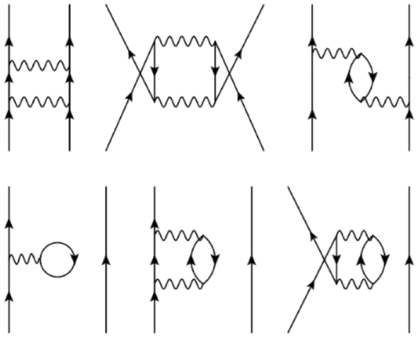
$V_{low\ k}$ or SRG

To remove hard core, but still keep good descriptions of NN scattering phase shifts

$$\hat{Q}(E) = PVP + PVQ\frac{1}{E-H}QVP,$$

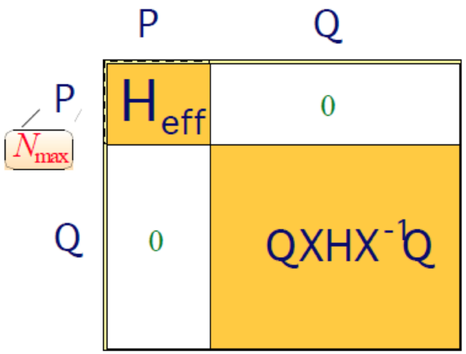
$$U(0,-\infty) = \lim_{\epsilon\to 0^+}\lim_{t'\to-\infty(1-i\epsilon)}\sum_{n=0}^{+\infty}(-i)^n\int_{t'}^{0}dt_1\int_{t'}^{t_1}dt_2\ldots\int_{t'}^{t_{n-1}}dt_n\ H_1(t_1)H_1(t_2)\ldots H_1(t_n)$$

Non-degenerate extended Kuo-Krenciglowa folded-diagram method (EKK) by Takayanagi, NPA 852, 61 (2011)

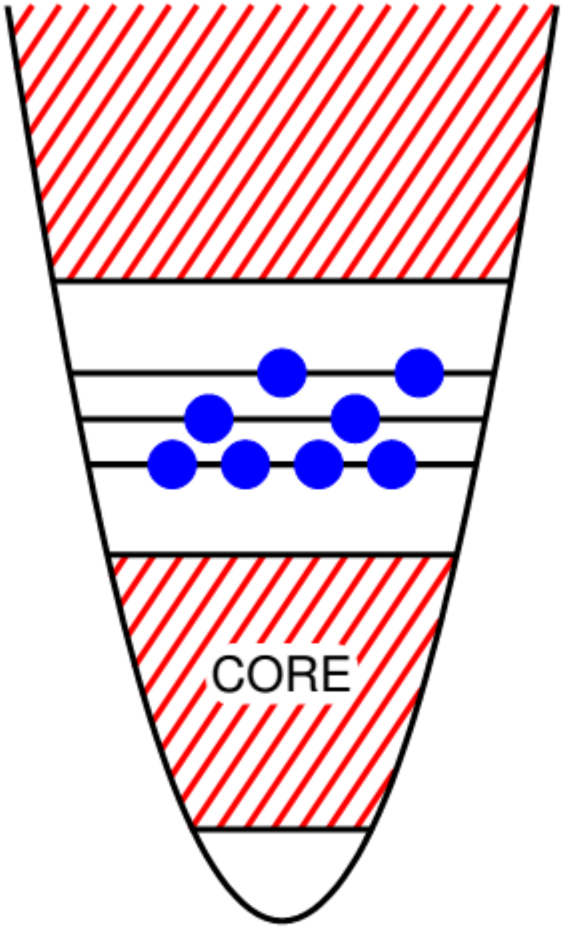


Q-box folded diagrams

S-, *Q*-box folded diagrams in complex-*k* Berggren basis



$$P + Q = 1$$



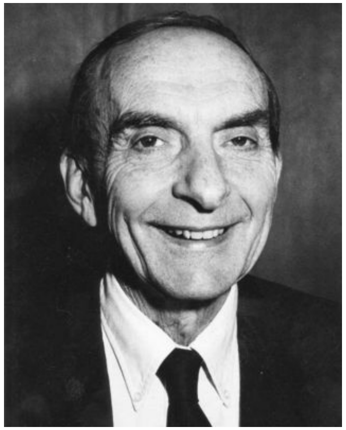
60年代以前，现实核力研究主要局限于核力本身的研究或用于一些少体问题研究，很难用于真正的核物理计算（计算核结构问题定量很差）

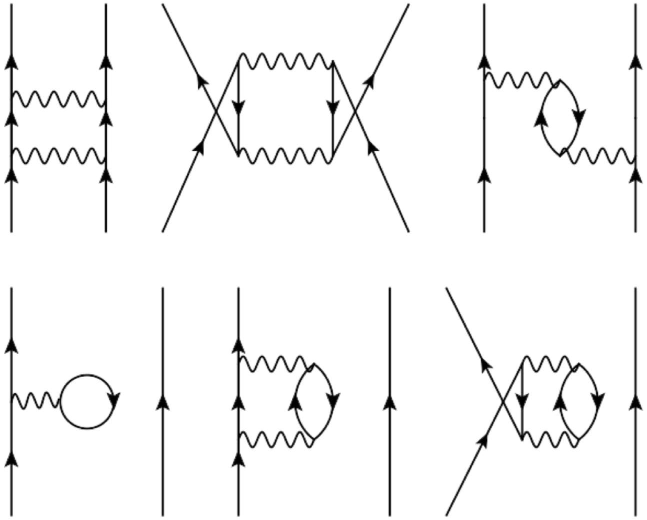
G.E. Brown 60’s: 从现实核力出发计算核结构问题，

***Q*-box folded diagrams, $V_{\text{low-k}}$**

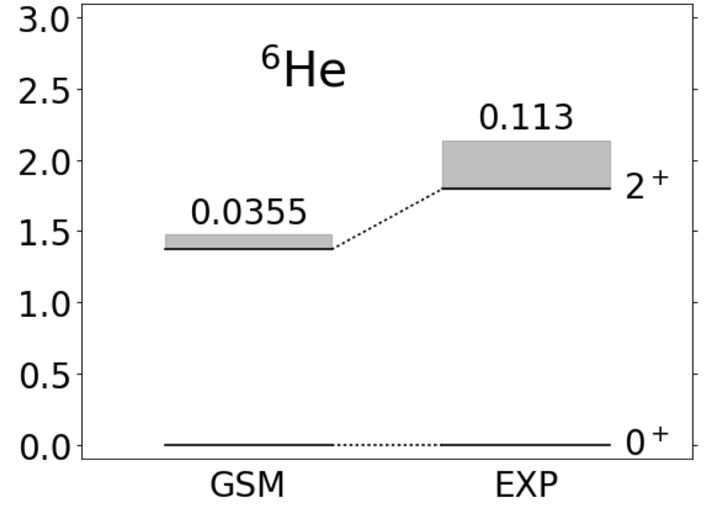
核芯极化的重要性！

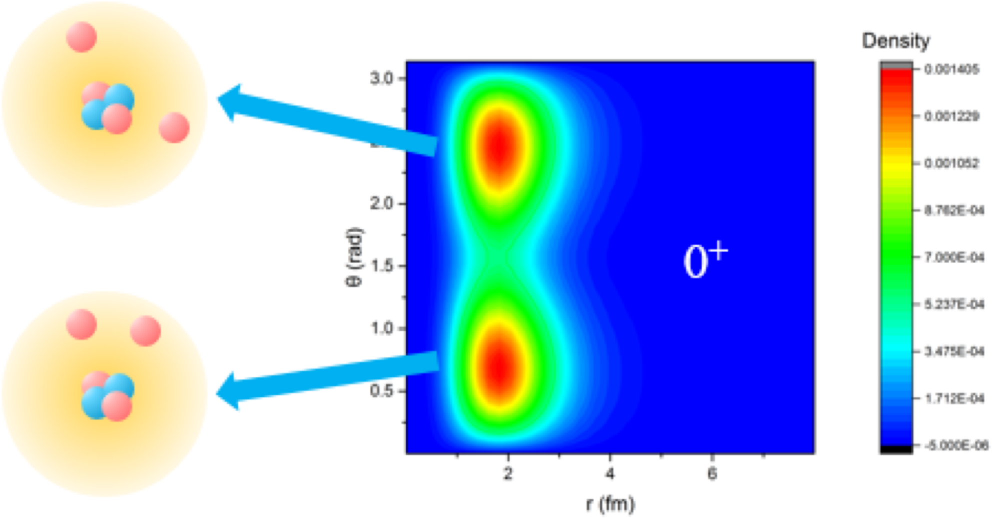
使得用现实核力计算原子核成为可能！

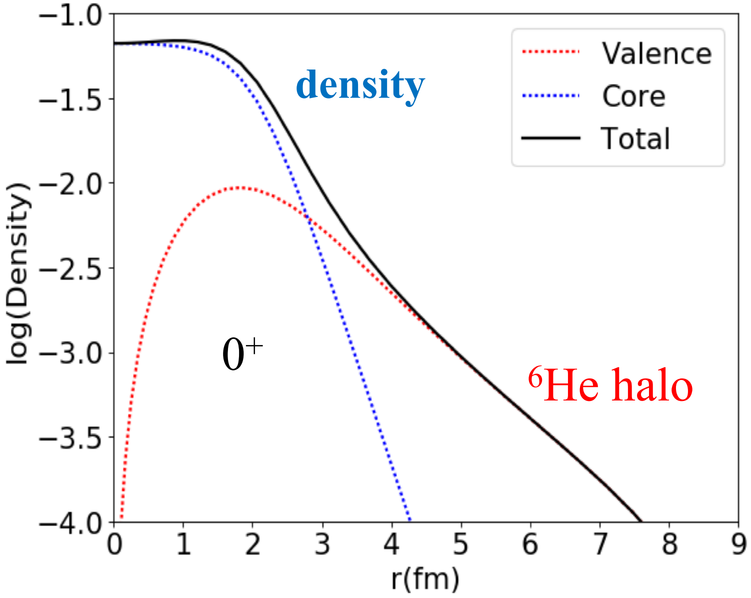




^{6}He halo







^{6}He correlated density distribution

$$\rho(r,\theta) = \langle \Psi | \delta(r_1 - r)\delta(r_2 - r)\delta(\theta_{12} - \theta) | \Psi \rangle$$

Physics World公布2022年十大年度突破性成果

Courtesy: IOP Publishing

01 迎来超冷化学的新时代

02 观测到理论预言的四中子状态

两院院士评选“2022年中国/世界十大科技进展新闻”揭晓

中国科学报 2023-01-12 11:01 发表于北京

2022年世界十大科技进展新闻

01 **首个完整人类基因组序列公布**

●

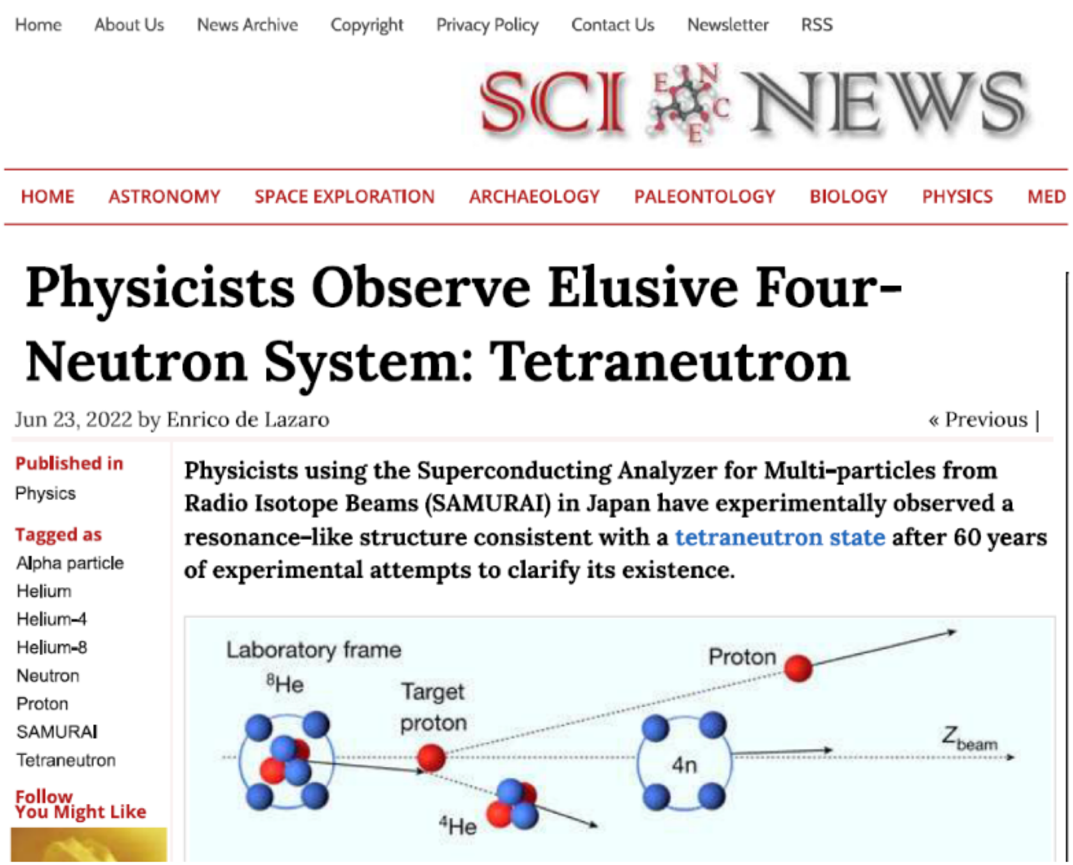
●

●

10 **科学家发现“四中子态”存在最明确证据**

四中子共振态（Tetraneutron）

（大概60年前，猜想）



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SCI NEWS

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Physicists Observe Elusive Four-Neutron System: Tetraneutron

Jun 23, 2022 by Enrico de Lazaro « Previous |

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Physics

Tagged as
Alpha particle
Helium
Helium-4
Helium-8
Neutron
Proton
SAMURAI
Tetraneutron

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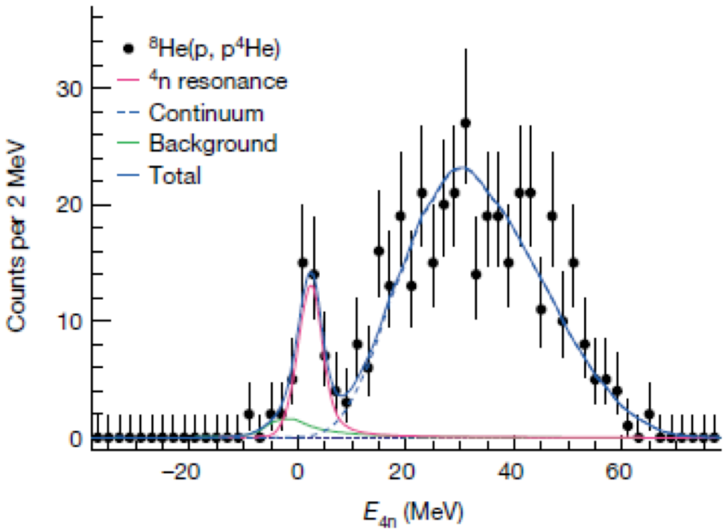
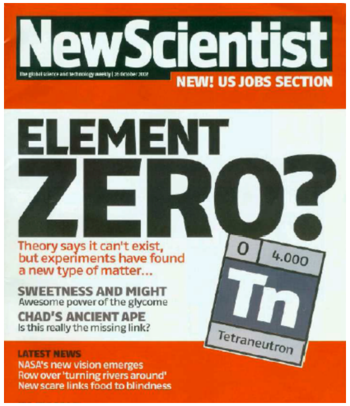
Physicists using the Superconducting Analyzer for Multi-particles from Radio Isotope Beams (SAMURAI) in Japan have experimentally observed a resonance-like structure consistent with a tetraneutron state after 60 years of experimental attempts to clarify its existence.

M. Duer *et al*., Nature 606, 678 (2022)





ScienceNews
INDEPENDENT JOURNALISM SINCE 1921

NEWS PHYSICS

Physicists may have finally spotted elusive clusters of four neutrons

If confirmed, 'tetraneutrons' could help scientists better understand nuclear forces

Four neutrons may form a short-lived agglomeration called a tetraneutron (illustrated).
SONJA BATTENBERG/TUM

By Emily Conover
JUNE 22, 2022 AT 11:00 AM

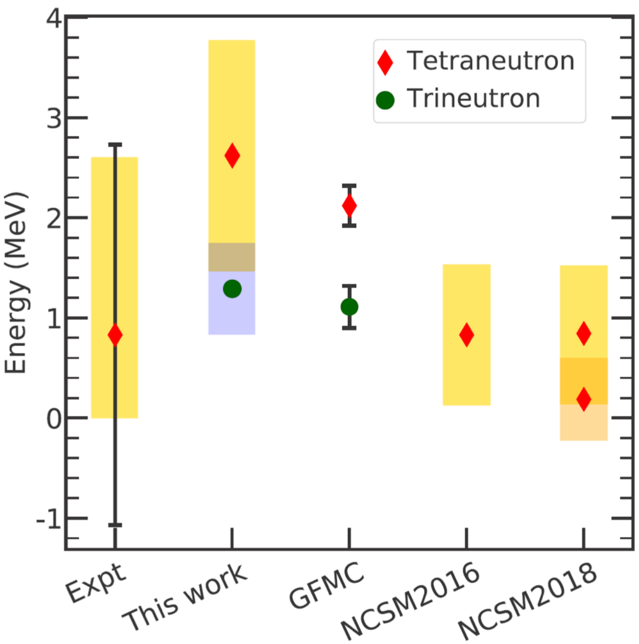
Chiral EFT N^3LO + NCGSM, 考虑更多的强关联：Gamow 自然基

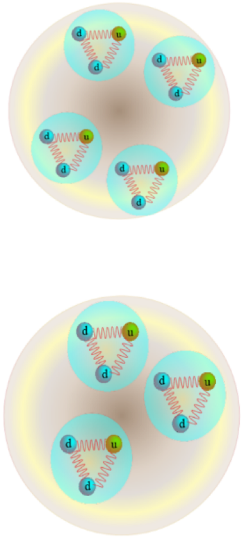
PHYSICAL REVIEW C **100**, 054313 (2019)

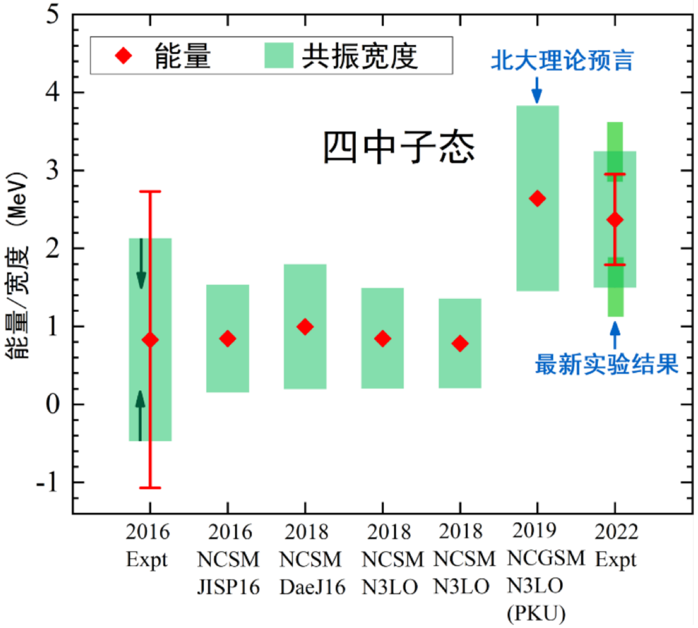
Ab initio no-core Gamow shell-model calculations of multineutron systems

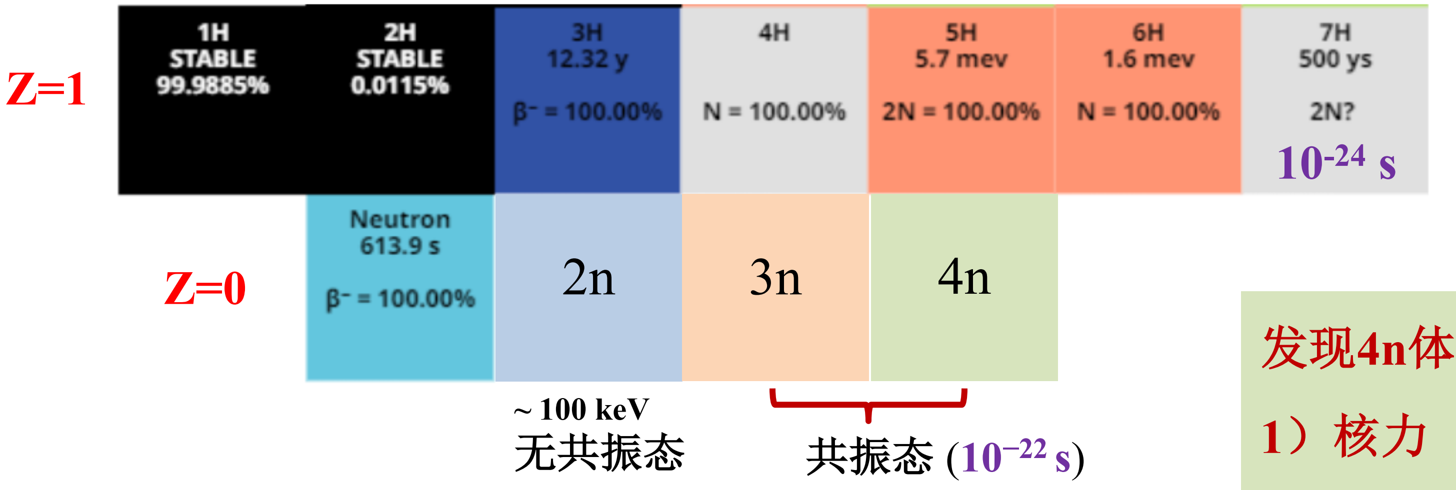
J. G. Li,[1] N. Michel,[2,3] B. S. Hu,[1] W. Zuo,[2,3] and F. R. Xu[1,*]

[1]*School of Physics, and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*
[2]*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*
[3]*School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China*









Z=1	1H STABLE 99.9885%	2H STABLE 0.0115%	3H 12.32 y β⁻ = 100.00%	4H N = 100.00%	5H 5.7 mev 2N = 100.00%	6H 1.6 mev N = 100.00%	7H 500 ys 2N? 10^{-24} s
Z=0		Neutron 613.9 s β⁻ = 100.00%	2n	3n	4n		

~100 keV
无共振态

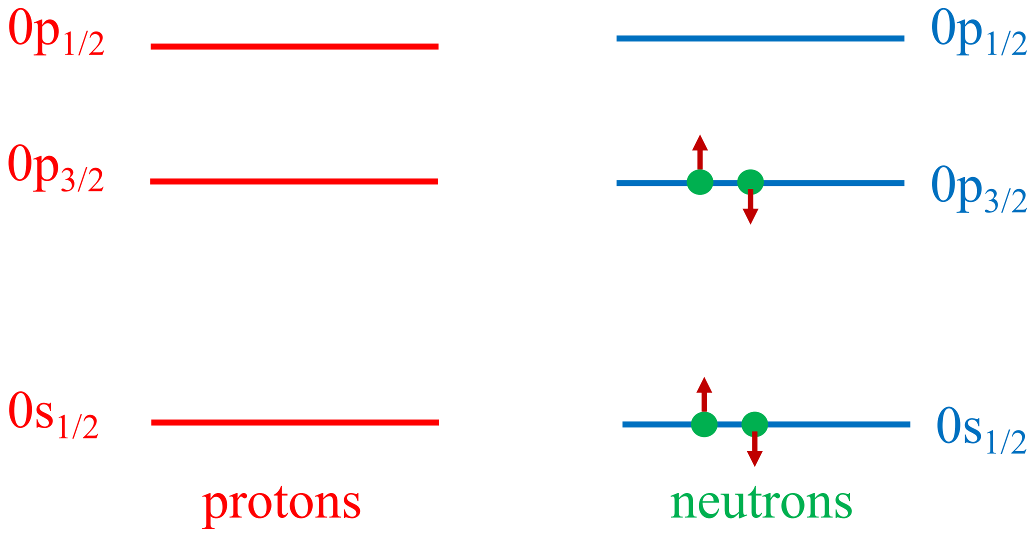
共振态 (10^{-22} s)

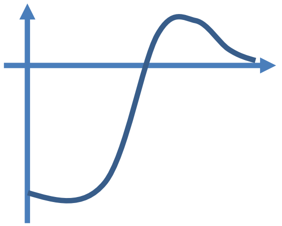
发现4n体系的重要性：

1）核力（nn）

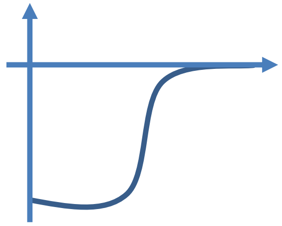
2）纯中子物质

…



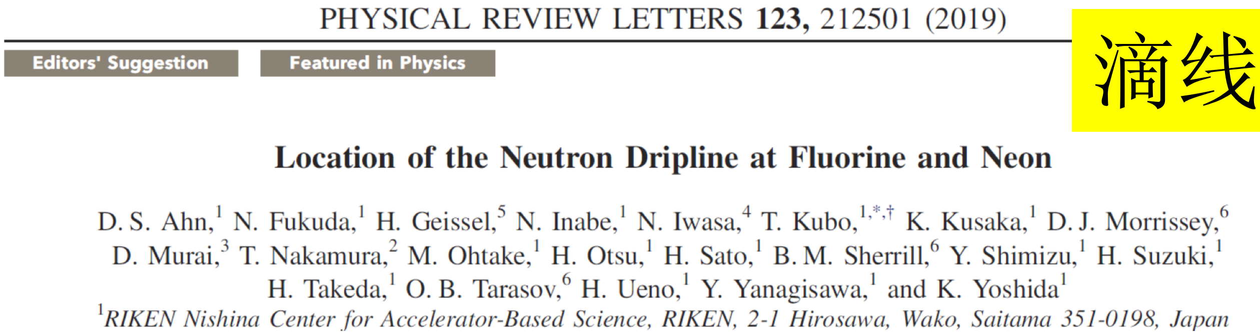


l=1离心位垒



l=0 无离心位垒

滴线区原子核



PHYSICAL REVIEW LETTERS **123,** 212501 (2019)

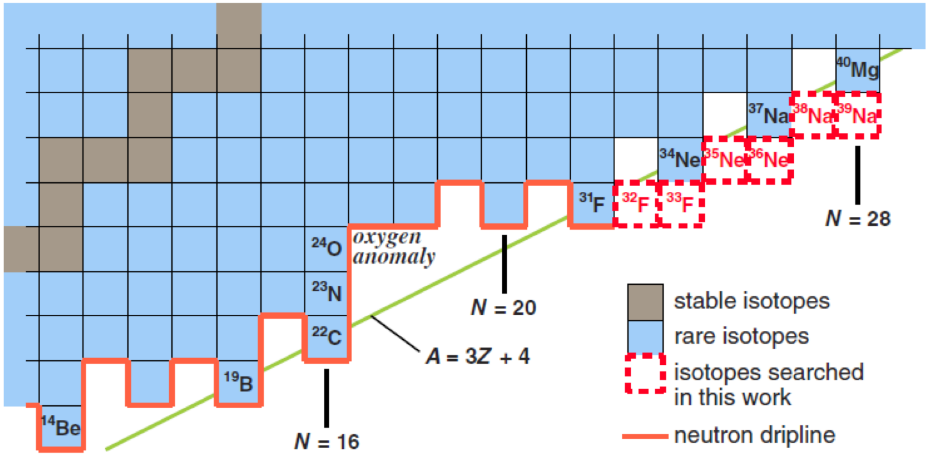
Editors' Suggestion **Featured in Physics**

Location of the Neutron Dripline at Fluorine and Neon

D. S. Ahn,[1] N. Fukuda,[1] H. Geissel,[5] N. Inabe,[1] N. Iwasa,[4] T. Kubo,[1,*,†] K. Kusaka,[1] D. J. Morrissey,[6]
D. Murai,[3] T. Nakamura,[2] M. Ohtake,[1] H. Otsu,[1] H. Sato,[1] B. M. Sherrill,[6] Y. Shimizu,[1] H. Suzuki,[1]
H. Takeda,[1] O. B. Tarasov,[6] H. Ueno,[1] Y. Yanagisawa,[1] and K. Yoshida[1]
[1]*RIKEN Nishina Center for Accelerator-Based Science, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan*

美国物理学会选出的“2019年物理学发生的十大事件”

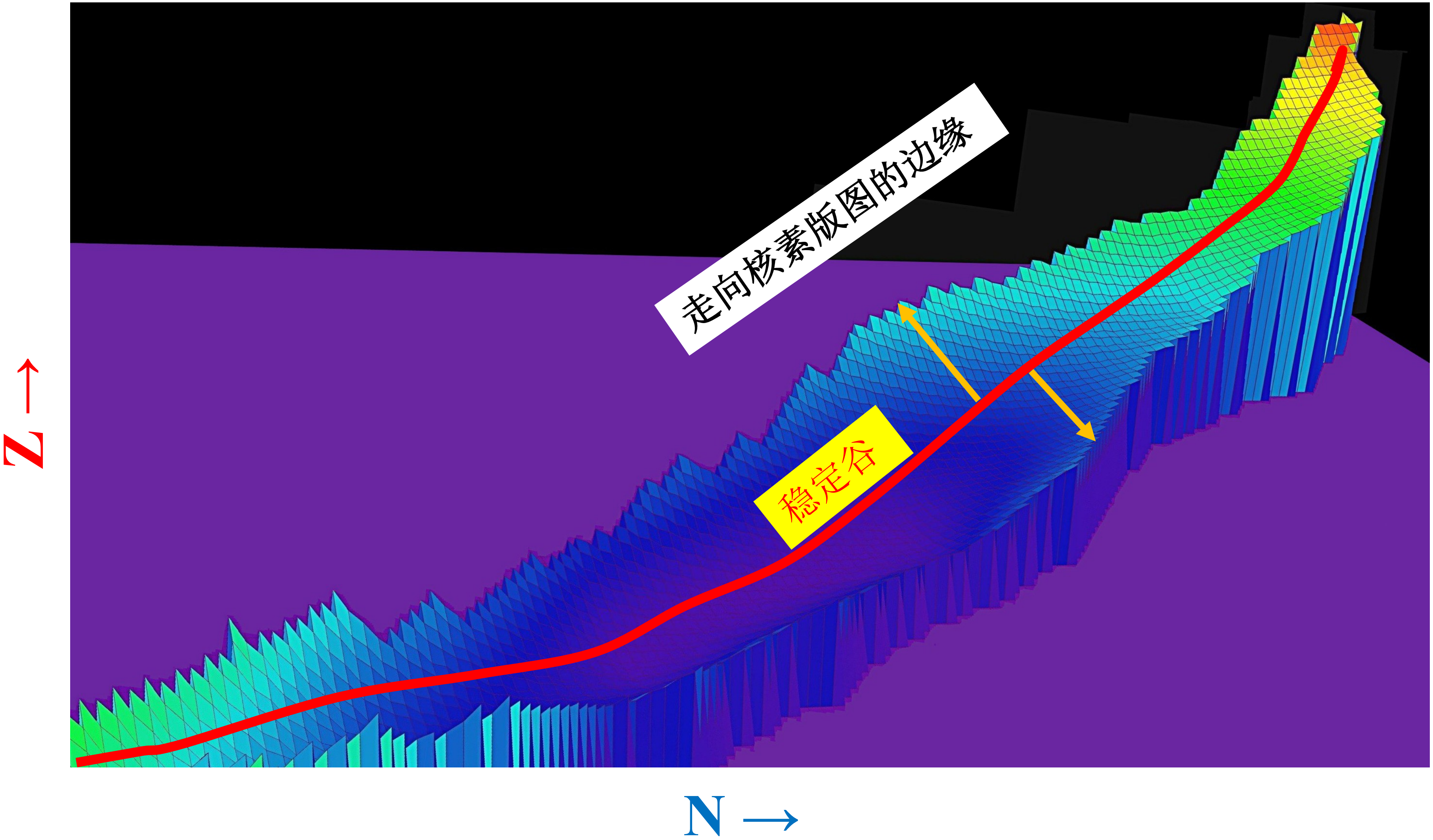
4. 中子滴线得到延伸



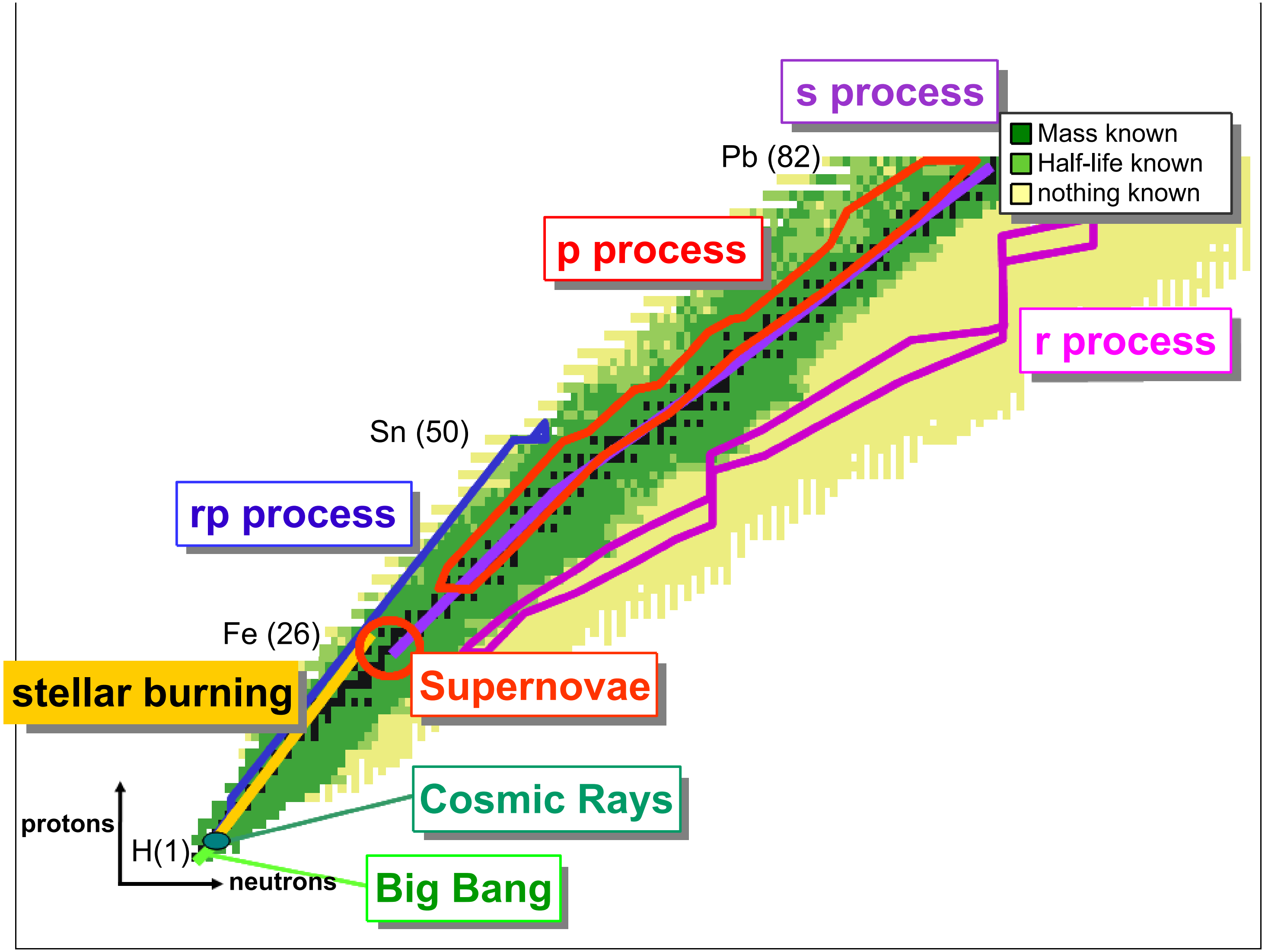
中子滴线停滞在元素周期表的前8个元素的中子滴线（**粉线**）已经20年。2019年，日本科学家将中子滴线延伸至氟（F）和氖（Ne）（**绿线**）

物理学家希望下一个元素的滴线不要再等上20年。下一代稀有同位素装置计划在两年内投入使用，可能会将滴线延伸至镁元素，即元素周期表中的第12号元素。

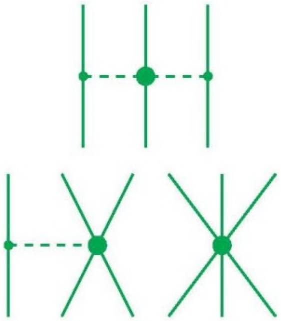
核素版图 Nuclear Landscape



核天体物理：Element production



Chiral EFT 3NF (NNLO) → Gamow 3NF



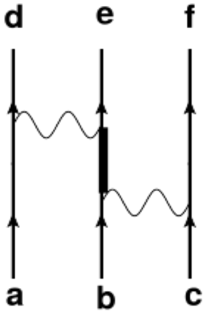
Chiral EFT 3NF

1. E. Epelbaum *et al*., PRC 66, 064001 (2002);
2. P. Navratil *et al*., PRL 99, 042501(2007)

Phenomenological 3NFs:

- **Urbana (1995)**

- **Tucson-Melbourne (1975-1999)**
- **Illinois (2001-2010)**
- **CD-Bonn + Δ (Deltuva, Sauer, 2003)**



Δ-resonance ($m \sim 1232$MeV, $t \sim 10^{-24}$ s)

isobar degrees of freedom贡献很小，

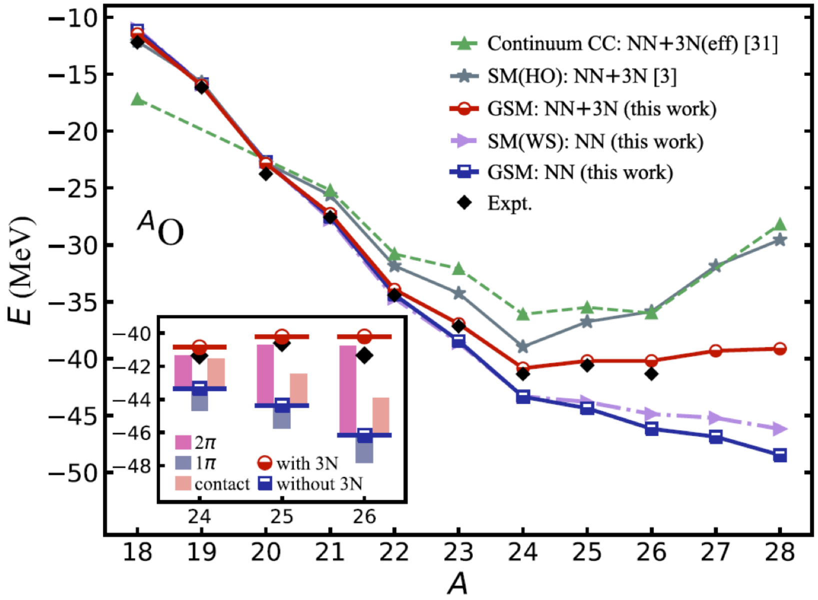
在Lagranian中可以不考虑

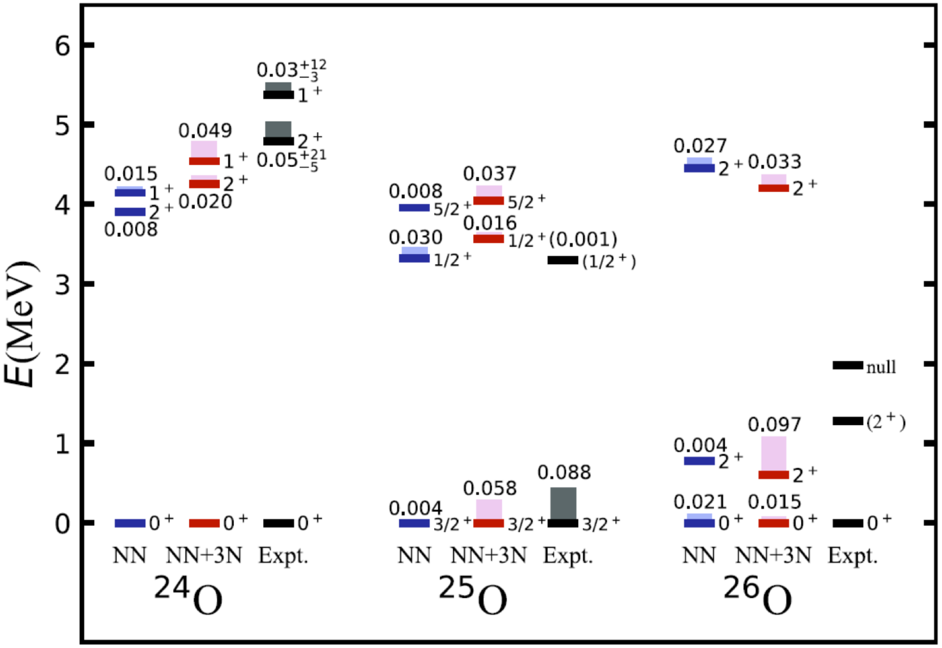
(m_n=939.6 MeV; m_p=938.3 MeV)

GSM with ^{16}O core: N^{3}LO(NN) + N^{2}LO(NNN)

S_{2n} (MeV)	NN	NN+3N	Expt.
^{24}O	9.110	7.038	6.925
^{25}O	6.254	3.568	3.453
^{26}O	3.362	−0.150	−0.018

Ma, Xu, Coraggio, Hu, Li, Fukui, Angelis, N.Itaco, Gargano, PLB 802, 135257 (2020)





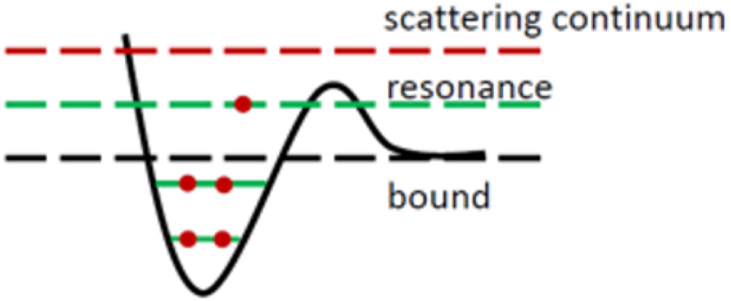
[3] Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL 105 (2010) 032501.
[31] Hagen, Hjorth-Jensen, Jansen, Machleidt, Papenbrock, PRL108 (2012) 242501.

N^{3}LO(NN): Entem and Machleidt, PRC **66**, 014002 (2002).

N^{2}LO(NNN): c_D=-1, c_E=-0.34, Navrátil, Gueorguiev, Vary, Ormand, A. Nogga, PRL 99, 042501 (2007).

Summary

Berggren complex space



Ab initio nuclear structure calculations with resonance and continuum considered

1. **Complex EFT 3NF at N^2LO**

2. ***Ab initio* Gamow shell model with a core**

   calculating complex *S*-box and *Q*-box folded diagrams

Both the continuum coupling and 3NF are important in nuclei around driplines

Thank you for your attention

科大 2023.04.20

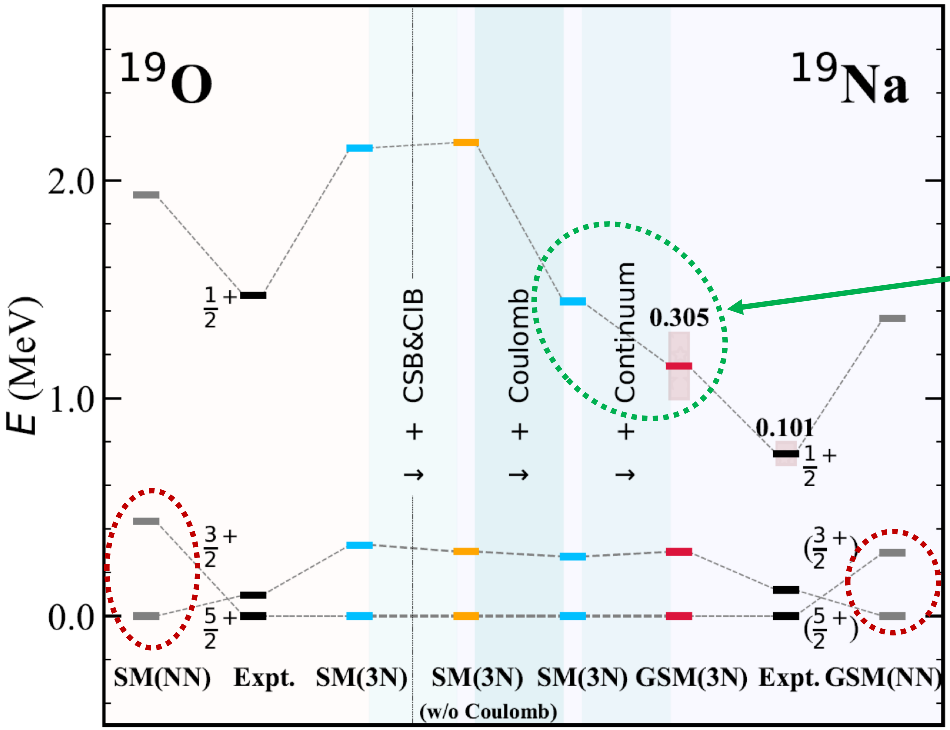
胡柏山, 马远卓, 孙中浩, 张爽, 吴强, 袁琪…

李健国, Michel, 左维

Coraggio, Fukui， …

Thomas-Ehrman shift (TES) in mirror pair

Both continuum effect and 3NF are important to make the observed TES



Continuum effect

If no 3NF, calculation cannot give the correct order of the levels.

CIB: charge independence breaking, a violation of rotation invariance in isospin space.

T=1 *NN* interaction: Tz=+1 (pp), 0 (np) and -1 (nn)

The main reasons: $m_p \neq m_n$, π^0, $\pi^\pm$ mass splitting

CIB is more significant than CSB

CSB: charge symmetry breaking, a violation of rotation invariance by 180°

Only for *pp* and *nn*

S. Zhang, Y.Z. Ma, J.G. Li, B.S. Hu, Q. Yuan, Z.H. Cheng, F.R. Xu, PLB 827, 136958 (2022)

Q-box folded diagrams

Full space 本征值问题

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

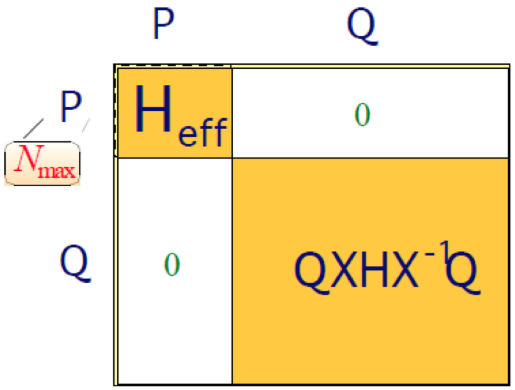
$$H = H_0 + H_1$$

$$H_0 = \sum_{i=1}^{A}(t_i + U_i)$$ One-body

$$H_1 = \sum_{i<j=1}^{A} V_{ij}^{NN} - \sum_{i=1}^{A} U_i$$ Residual two-body

$$P + Q = 1$$

$$P = \sum_{i=1}^{d} |\Phi_i\rangle\langle\Phi_i|$$



求解无限空间→有限空间多体问题

$$H_{\text{eff}}(E_v)P|\Psi_v\rangle = E_v P|\Psi_v\rangle$$

$$H_{\text{eff}}(E_v) = P(H_0 + H_1)P + P(H_0 + H_1)Q\frac{1}{E_v - H_{QQ}}Q(H_0 + H_1)P$$

$$= PH_0P + PH_1P + (\underline{PH_0Q} + PH_1Q)\frac{1}{E_v - H_{QQ}}(QH_0P + QH_1P)$$

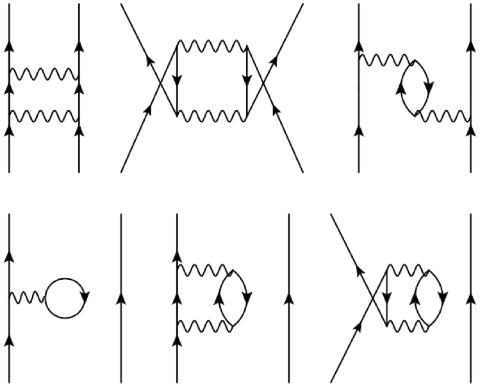
$$= H_0\underline{PQ}$$

$$= 0 \qquad (\because P, Q\text{与}H_0\text{对易})$$

$$H_0 = \sum_{i=1}^{A}(t_i + U_i)$$ unperturbed one-body

$$V_{\text{eff}}(E_v) = PH_1P + PH_1\frac{Q}{E_v - QH_0Q}H_1P + PH_1\frac{Q}{E_v - QH_0Q}H_1\frac{Q}{E_v - QH_0Q}H_1P + \cdots$$

2nd order perturbation 3rd order perturbation



$$H_{\text{eff}}^{(n)} = PH_0P + \hat{Q}(E) + \sum_{k=1}^{\infty}\frac{1}{k!}\frac{d^k\hat{Q}(E)}{dE^k}\{H_{\text{eff}}^{(n-1)} - E\}^k$$

$$\hat{Q}_k(E) = \frac{1}{k!}\frac{d^k\hat{Q}(E)}{dE^k}$$
$$= (-1)^k PVQ\frac{1}{(E - QHQ)^{k+1}}QVP$$