Inside the hadrons viewed from a light-front perspective

James P. Vary

Department of Physics and Astronomy
Iowa State University
Ames, USA

University of Science and Technology of China
Hefei, China
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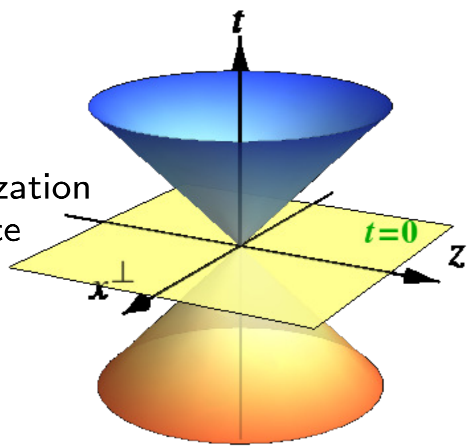
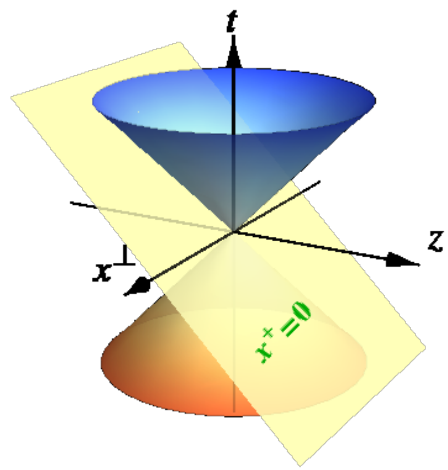
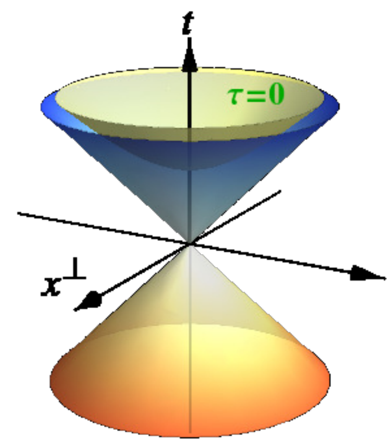
Dirac's forms of relativistic dynamics [Dirac, Rev. Mod. Phys. **21**, 392 1949]

Instant form is the well-known form of dynamics starting with $x^0 = t = 0$

$K^i = M^{0i}, \; J^i = \frac{1}{2}\varepsilon^{ijk} M^{jk}, \; \varepsilon^{ijk} = (+1,-1,0)$ for (cyclic, anti-cyclic, repeated) indeces

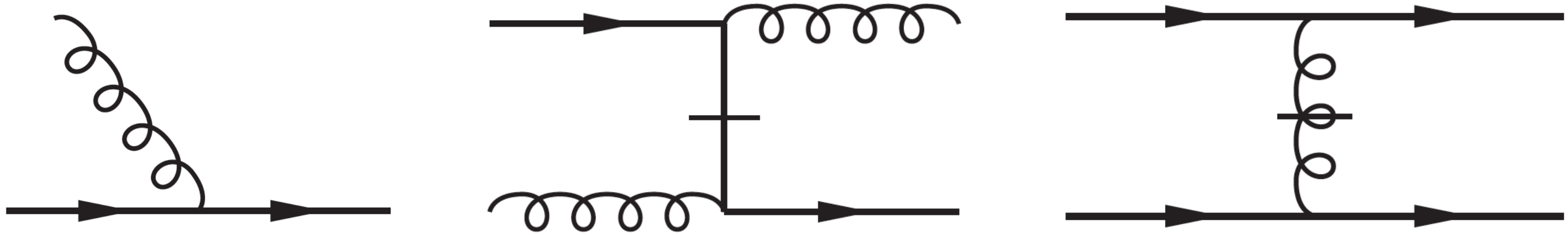
Front form defines relativistic dynamics on the light front (LF): $x^+ = x^0 + x^3 = t + z = 0$

$$P^{\pm} \triangleq P^0 \pm P^3, \; \vec{P}^{\perp} \triangleq (P^1, P^2), \; x^{\pm} \triangleq x^0 \pm x^3, \; \vec{x}^{\perp} \triangleq (x^1, x^2), \; E^i = M^{+i}, \; E^+ = M^{+-}, \; F^i = M^{-i}$$

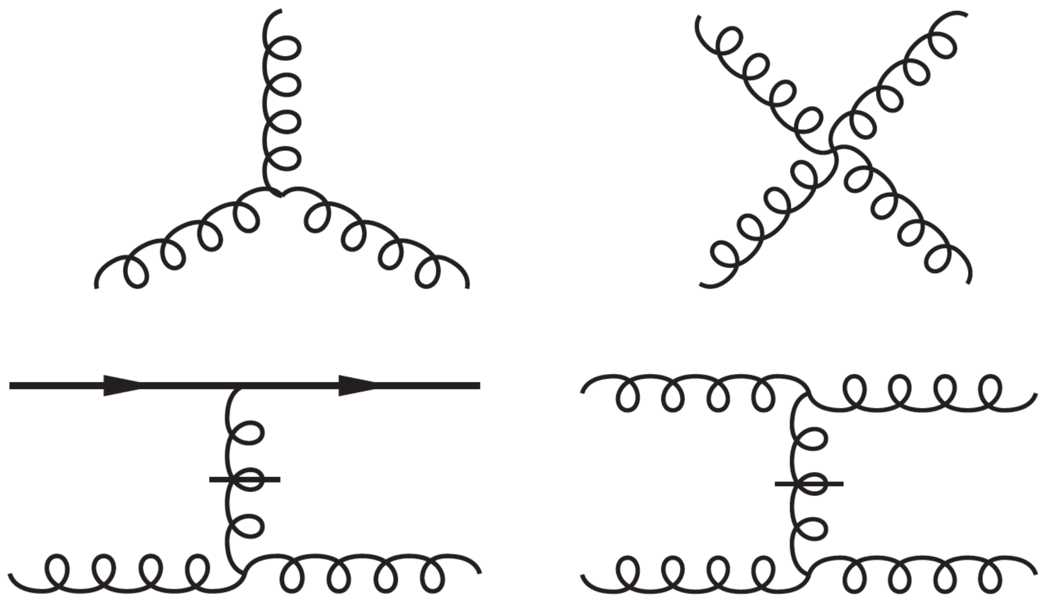
	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^{μ}
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^{\perp}, P^+, \vec{E}^{\perp}, E^+, J^-$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^{\perp}, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_{\perp}^2 + m^2)/p^+$	$p^{\mu} = m v^{\mu} \; (v^2 = 1)$

Adapted from talk by Yang Li

Light Front (LF) Hamiltonian Defined by its Elementary Vertices in LF Gauge



QED & QCD



QCD

Discretized Light Cone Quantization

[H.C. Pauli & S.J. Brodsky, PRD32 (1985)]

Basis Light Front Quantization

[J.P. Vary, et al., PRC81 (2010)]

$$\phi\left(\vec{k}_\perp, x\right) = \sum_\alpha \left[f_\alpha\left(\vec{k}_\perp, x\right) a_\alpha + f_\alpha^*\left(\vec{k}_\perp, x\right) a_\alpha^\dagger \right]$$

where $\{a_\alpha\}$ satisfy usual (anti-) commutation rules.

Furthermore, $f_\alpha(\vec{x})$ are arbitrary except for conditions:

Orthonormal: $$\int f_\alpha\left(\vec{k}_\perp, x\right) f_{\alpha'}^*\left(\vec{k}_\perp, x\right) \frac{d^2 k_\perp dx}{(2\pi)^3 2x(1-x)} = \delta_{\alpha\alpha'}$$

Complete: $$\sum_\alpha f_\alpha\left(\vec{k}_\perp, x\right) f_\alpha^*\left(\vec{k}'_\perp, x'\right) = 16\pi^3 \sqrt{x(1-x)}\,\delta^2\left(\vec{k}_\perp - \vec{k}'_\perp\right) \delta\left(x - x'\right)$$

For mesons we adopt (later extended to baryons): [Y. Li, et al., PLB758 (2016)]

$$f_{\alpha=\{nml\}}\left(\vec{k}_\perp, x\right) = \phi_{nm}\left(\vec{k}_\perp / \sqrt{x(1-x)}\right) \chi_l(x)$$

ϕ_{nm} 2D-HO functions as in AdS/QCD

χ_l Jacobi polynomials times $x^a(1-x)^b$

BLFQ
Symmetries & Constraints

Baryon number $\sum_i b_i = B$

Charge $\sum_i q_i = Q$

Angular momentum projection (M-scheme) $\sum_i (m_i + s_i) = J_z$

Longitudinal momentum (Bjorken sum rule) $\sum_i x_i = \sum_i \frac{k_i}{K} = 1$

Longitudinal mode regulator (Jacobi) $\sum_i l_i \leq L$

Transverse mode regulator (2D HO) $\sum_i (2n_i + |m_i| + 1) \leq N_{\max}$

"Internal coordinates" $\vec{k}_{i\perp} = \vec{p}_{i\perp} - x_i \vec{P}_\perp \Rightarrow \sum_i \vec{k}_{i\perp} = 0$

$H \rightarrow H + \lambda H_{CM}$

Global Color Singlets (QCD)

Light Front Gauge

Optional Fock-Space Truncation

All $J \geq J_z$ states in one calculation

Finite basis regulators

Preserve transverse boost invariance

Light-Front Wavefunctions (LFWFs)

$$|\psi_h(P,j,\lambda)\rangle = \sum_n \int [\mathrm{d}\mu_n]\, \psi_{n/h}(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_n) |\{\vec{p}_{i\perp}, p_i^+, \lambda_i\}_n\rangle$$

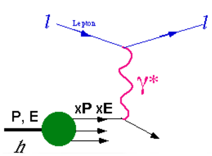
LFWFs are *frame-independent* (boost invariant) and depend only on the relative variables: $x_i \equiv p_i^+/P^+, \vec{k}_{i\perp} \equiv \vec{p}_{i\perp} - x_i \vec{P}_\perp$

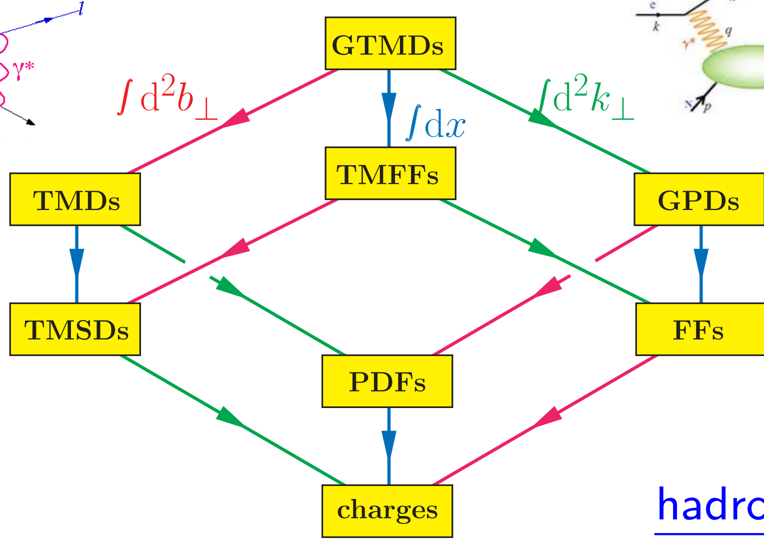
LFWFs provide intrinsic information of the structure of hadrons, and are indispensable for exclusive processes in DIS [Lepage '80]

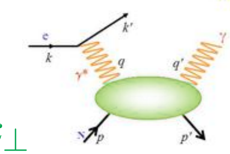
- Overlap of LFWFs: structure functions (e.g. PDFs), form factors, ...
- Integrating out LFWFs: light-cone distributions (e.g. DAs)

"Hadron Physics without LFWFs is like Biology without DNA!"

— Stanley J. Brodsky

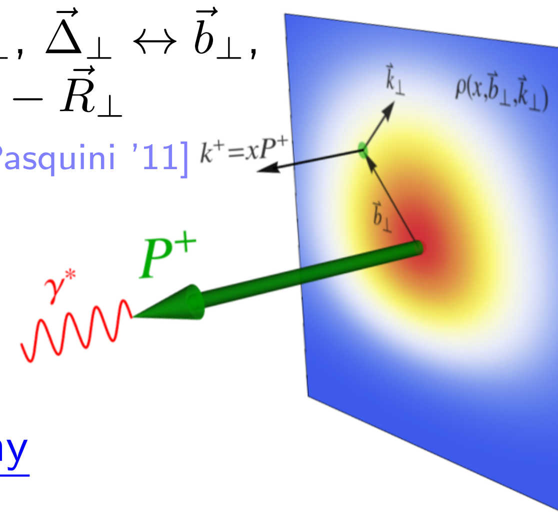






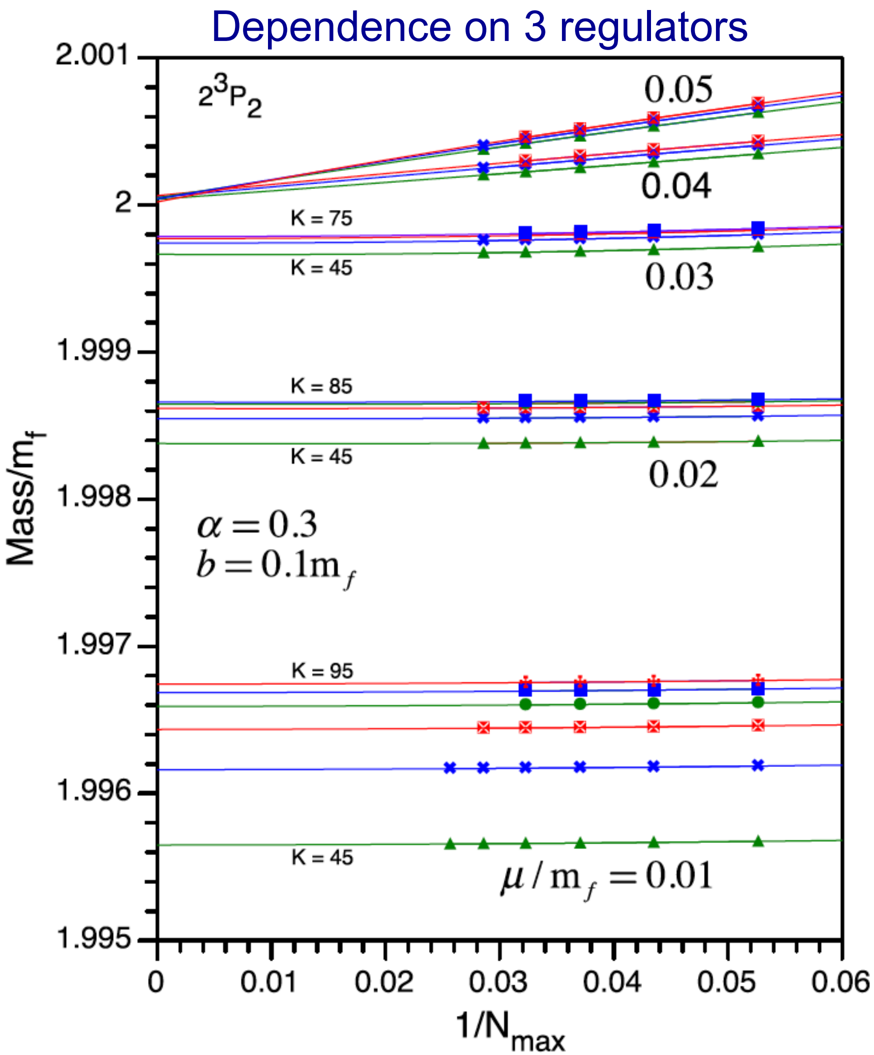
$\vec{k}_\perp \leftrightarrow \vec{r}_\perp$, $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$,
$\vec{b}_\perp = \vec{r}_\perp - \vec{R}_\perp$

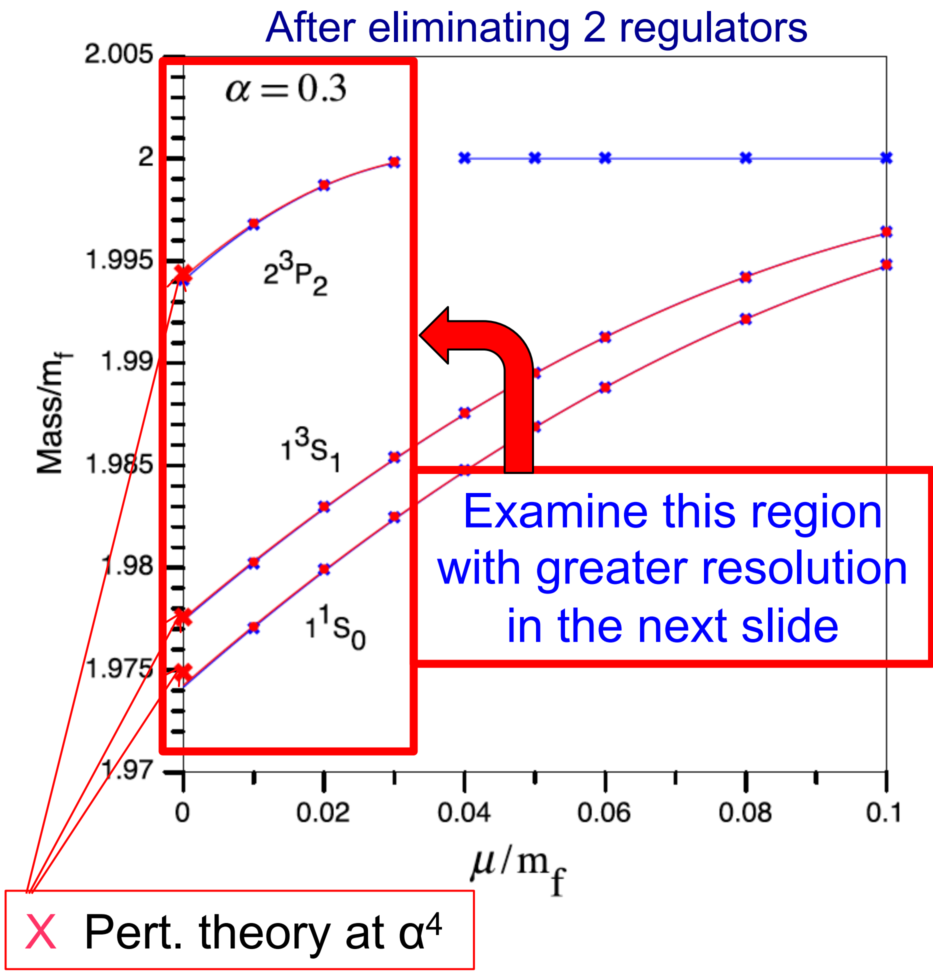
[Lorcé & Pasquini '11]



hadron tomography

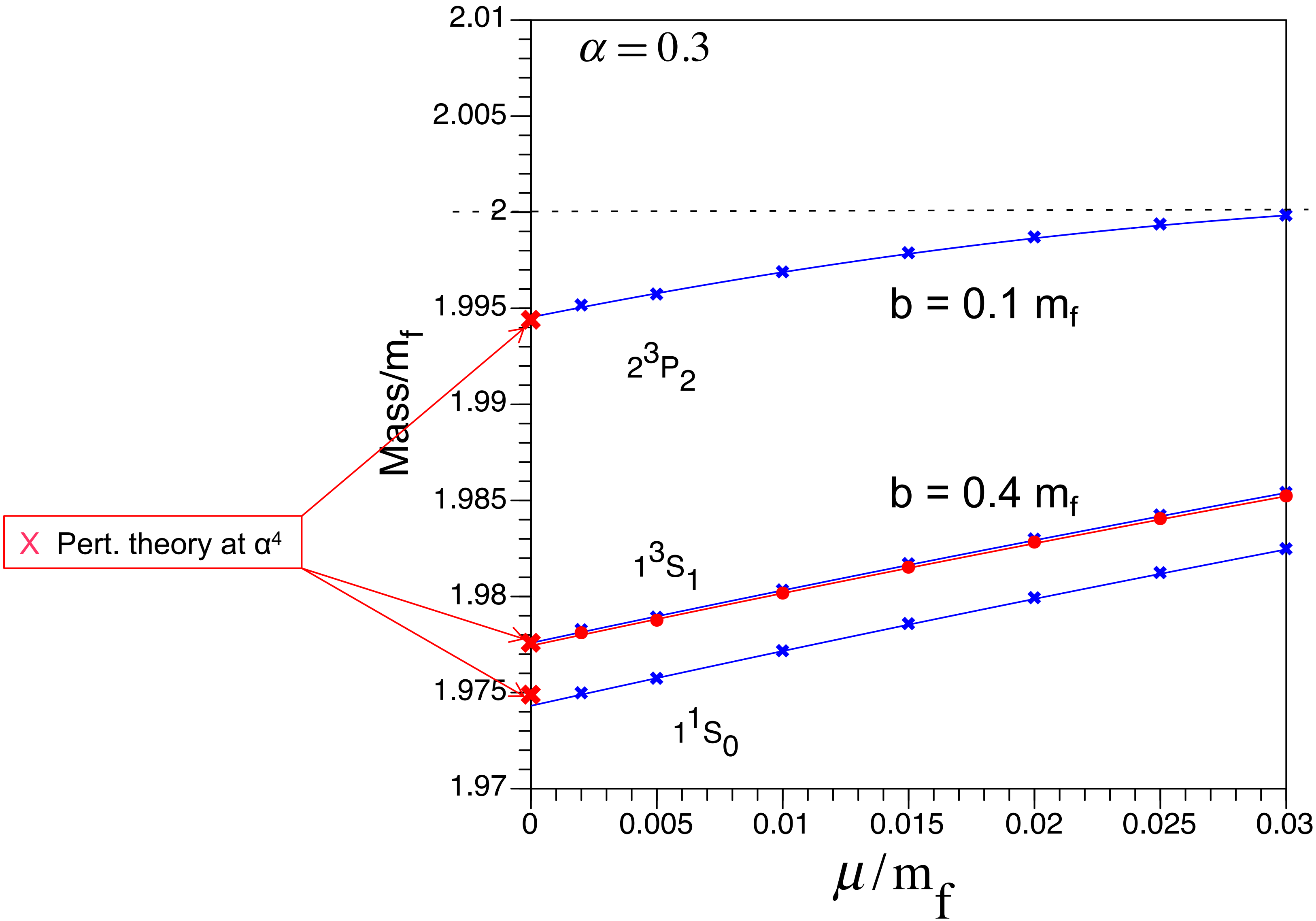
Basis Light-Front Quantization (BLFQ)
Positronium in QED at Strong Coupling (α = 0.3)
Systematic removal of regulators (b = HO momentum scale)





P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D **91**, 105009 (2015)

Positronium in QED at Strong Coupling
Covariant Basis Light-Front Quantization (BLFQ)



P. Wiecki, Y. Li, X. Zhao, P. Maris and J.P. Vary, Phys. Rev. D **91**, 105009 (2015)

Positronium with one dynamical photon: Light-front QED Hamiltonian

- QED Lagrangian $$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^{\mu}D_{\mu} - m_e)\Psi$$

- Light-front QED Hamiltonian from standard Legendre transformation

$$P^- = \int \mathrm{d}^2x^{\perp}\mathrm{d}x^- \;\; F^{\mu +}\partial_+ A_{\mu} + i\bar{\Psi}\gamma^+\partial_+\Psi - \mathcal{L}$$

Light-cone gauge: $\left(A^+ = 0\right)$

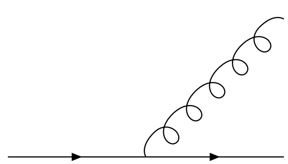
$$= \int \mathrm{d}^2x^{\perp}\mathrm{d}x^- \;\; \frac{1}{2}\bar{\Psi}\gamma^+\frac{m_e^2 + (i\partial^{\perp})^2}{i\partial^+}\Psi + \frac{1}{2}A^j(i\partial^{\perp})^2A^j$$

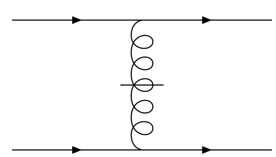
kinetic energy terms

$$+ ej^{\mu}A_{\mu} + \frac{e^2}{2}j^+\frac{1}{(i\partial^+)^2}j^+$$

vertex interaction

instantaneous photon interaction

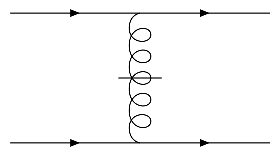
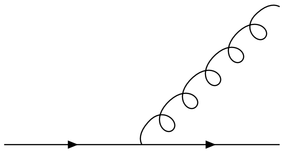
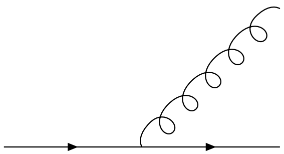




Positronium with one dynamical photon: Interaction Part Of Hamiltonian

$$\lvert\mathbf{Ps}\rangle = a\lvert e\bar{e}\rangle + b\lvert e\bar{e}\gamma\rangle + \mathrm{c}\lvert\gamma\rangle + \mathrm{d}\lvert e\bar{e}e\bar{e}\rangle + \ldots$$

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

H_{int}	$\lvert e\bar{e}\rangle$	$\lvert e\bar{e}\gamma\rangle$
$\langle e\bar{e}\rvert$		
$\langle e\bar{e}\gamma\rvert$		0 excluded by *gauge principle* [Tang et al, 1991]

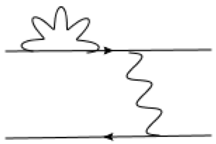
Kaiyu Fu, et al., in preparation

Mass Renormalization

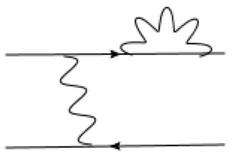
- Mass counterterm $\Delta_m = m_{bare} - m_{phys}$ is needed for fermion self-energy correction

- Mass renormalization needs to be performed on single physical electron
  - Prediction power on positronium mass

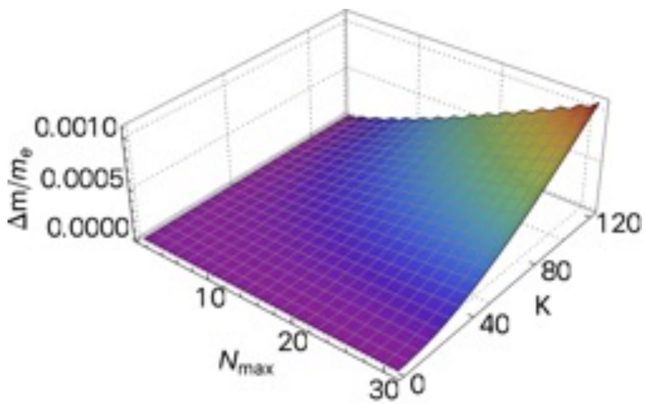
- Mass counterterm is determined by fitting single electron mass
  - Complication: Δ_m depends on UV cutoff and thus is basis dependent.
  - An extension of sector-dependent renormalization is needed: $\Delta_m(N_{max}, K)$ [Karmanov et al, 2008]



vs.



Here at α = 1/137

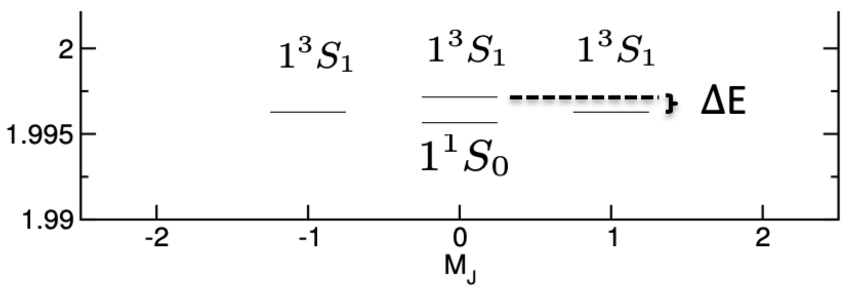


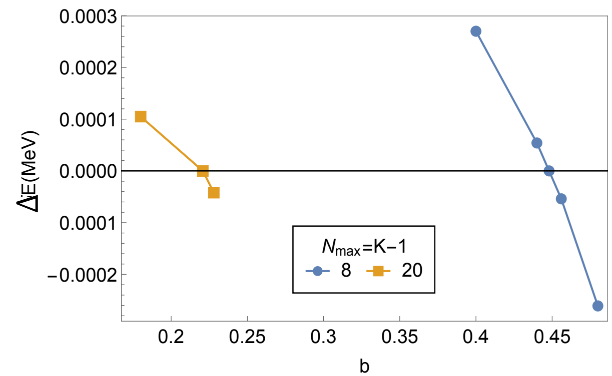
[Kaiyu Fu et al, in preparation]

- Mass counterterm is at higher order: $\Delta_m \propto \alpha m \, E_B \propto \alpha^2 m$

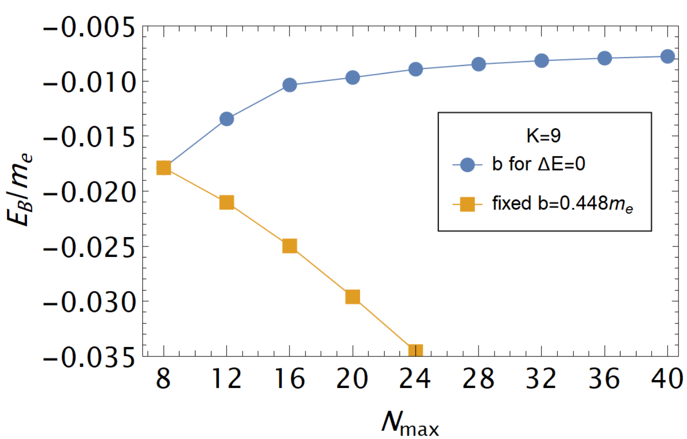
Basis Scale and Rotational Symmetry

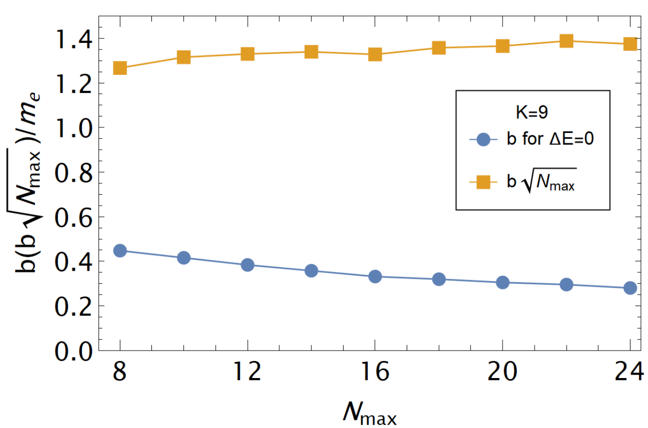
- Adjust the 2d harmonic oscillator basis scale parameter b to minimize the energy difference within the triplet 1^3S_1





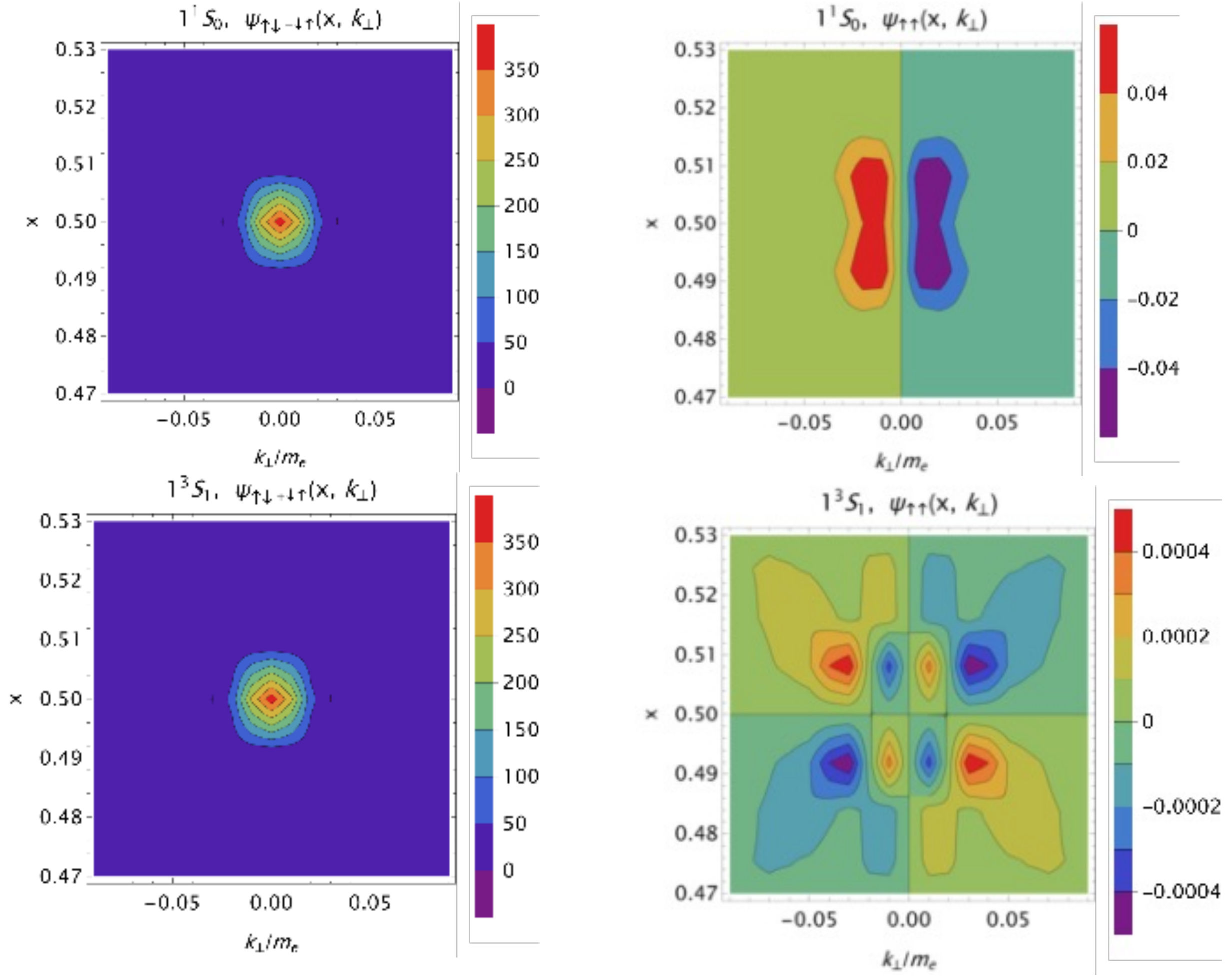
- Maintaining rotational symmetry leads to a corresponding UV cutoff





[Kaiyu Fu et al, in preparation]

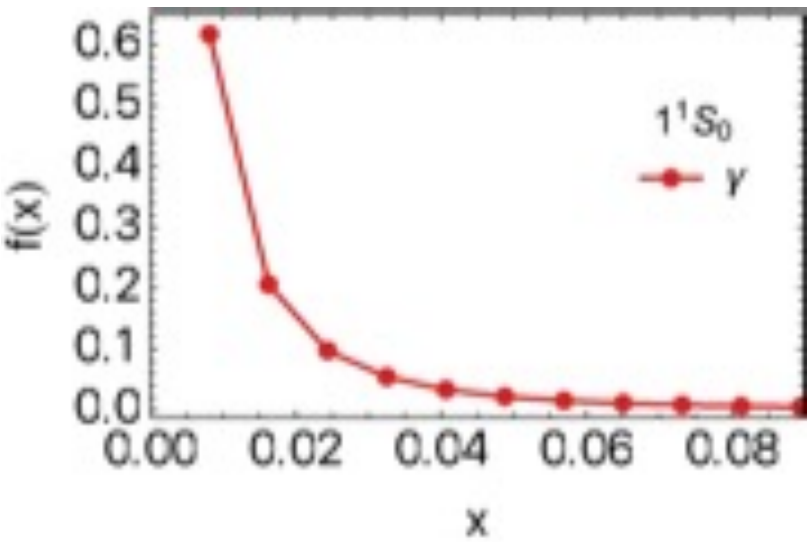
Wave Functions for S-Wave States

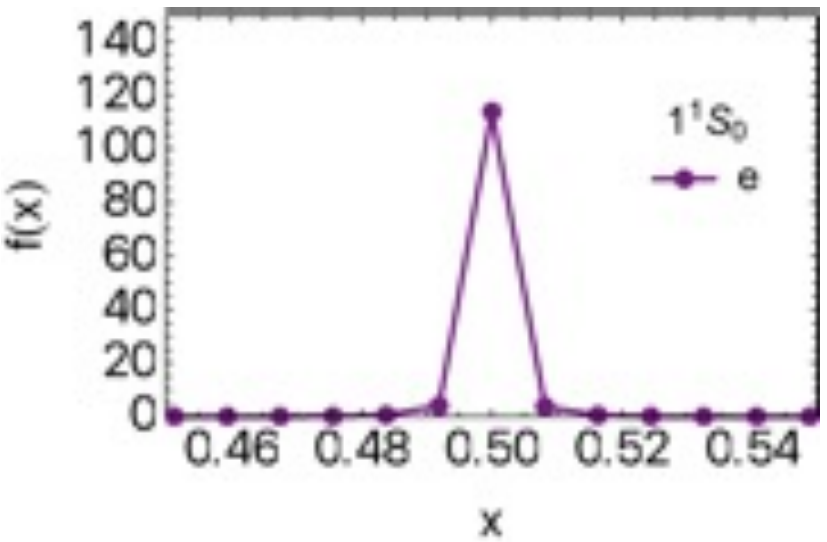


- Wave functions in $|e^+e^-\rangle$ Fock sector, dominant and non-dominant helicity component
- Nodal structure visible in non-dominant helicity component

[Kaiyu Fu et al, in preparation]

PDFs of the electron and photon





- $|e^+e^-\rangle$ Fock sector carries 99.1% probability.
- The peak of photon PDF is at small x region.

[Kaiyu Fu et al, in preparation]

Overview of BLFQ/tBLFQ applications to mesons and baryons

Common features

Transverse confinement from 2D HO (in common with LF Holography)
Longitudinal confinement (Y. Li, et al, PLB 2016, PRD 2017)
Basis states from exact solutions of this reference Hamiltonian
Compare results with experiment, lattice, DSE/BSE, . . .

Distinct features

For Veff
1) perturbative one-gluon exchange (Y. Li, et al, PLB 2016, PRD 2017)
2) NJL model for light meson applications (S. Jia, et al, PRC 2019)

For Fock space truncation
1) Valence sector
2) Valence sector plus dynamical gluon (plus sea quarks, plus …)

For observables
1) Single state properties and decays
2) Transitions between states
3) Non-perturbative probes (tBLFQ)

(Work by Meijian Li, et al)

Next Methods

BLFQ on Quantum Computers

(Work by Wenyang Qian, et al)

Heavy Quarkonia [Y.Li,PLB758,2016; PRD96,2017]

- Effective Hamiltonian in the $q\bar{q}$ sector

$$H_{\text{eff}} = \overbrace{\underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_0} + \underbrace{V_g}_{\text{one-gluon exchange}}$$

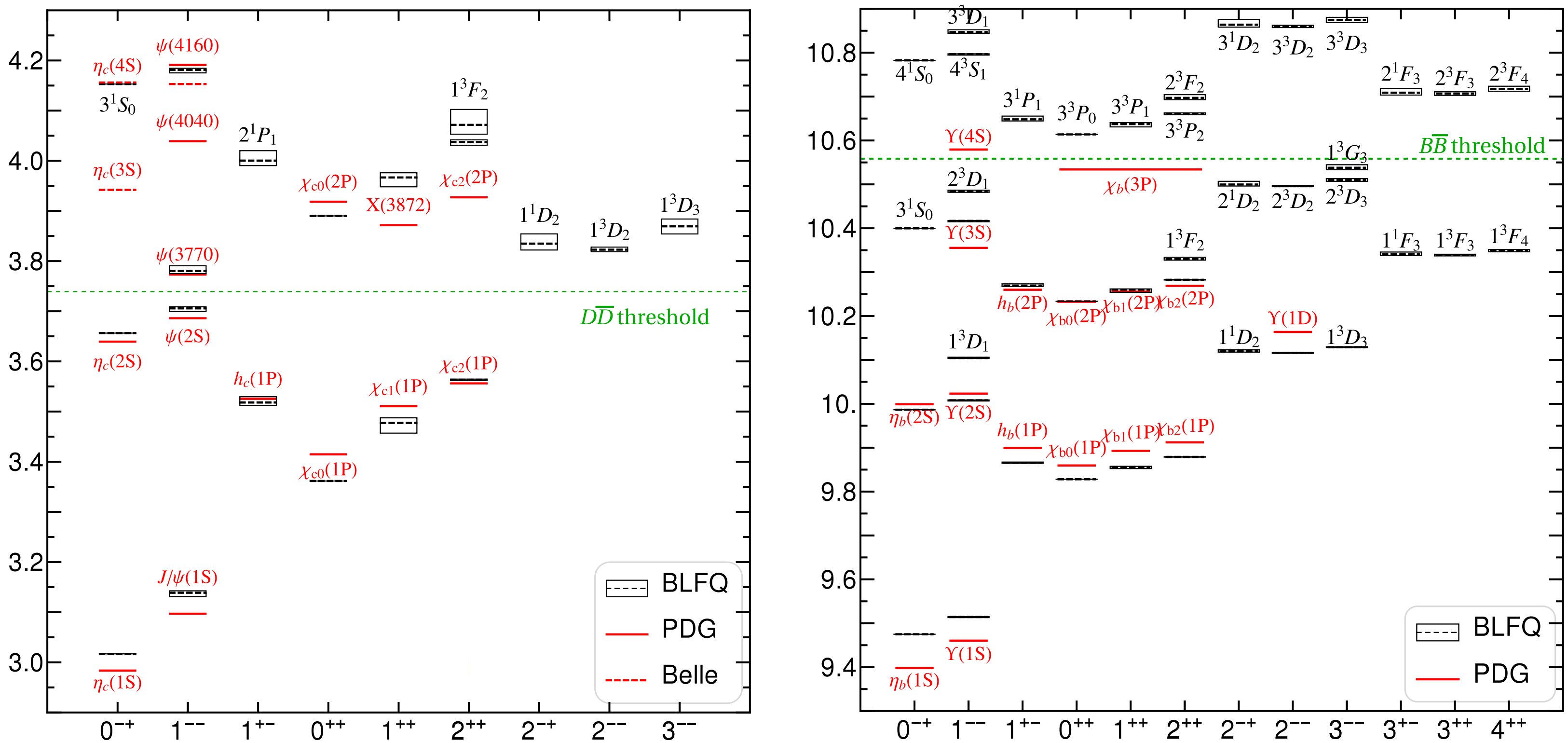
where $x = p_q^+ / P^+$, $\vec{k}_\perp = \vec{k}_{q\perp} = \vec{p}_{q\perp} - x\vec{P}_\perp = -\vec{k}_{\bar{q}\perp} = -\left(\vec{p}_{\bar{q}\perp} - (1-x)\vec{P}_\perp\right)$, $\vec{r}_\perp = \vec{r}_{q\perp} - \vec{r}_{\bar{q}\perp}$.

  - Confinement
    transverse holographic confinement [S.J.Brodsky,PR584,2015]
    longitudinal confinement [Y.Li,PLB758,2016]
  - One-gluon exchange with running coupling
    $V_g = -\frac{4}{3}\frac{4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{\sigma'}\gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$

- Basis representation
  - valence Fock sector: $|q\bar{q}\rangle$
  - basis functions: eigenfunctions of H_0 (LF kinetic energy+ confinement)

Spectroscopy

[Y. Li, et al., Phys. Letts. B 758, 118 (2016); Phys. Rev. D 96, 016022 (2017)]



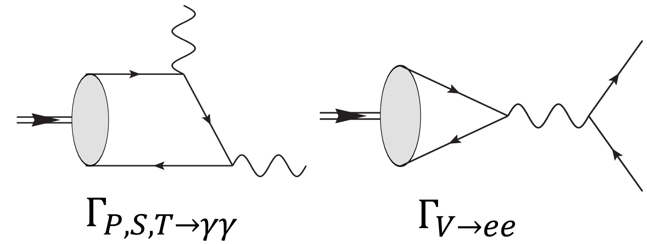
	κ (GeV)	m_q (GeV)	rms (MeV)	$\overline{\delta_J M}$ (MeV)	$N_{\max}$	basis dim.
$c\bar{c}$	0.966	1.603	31	17	32	1812
$b\bar{b}$	1.389	4.902	38	8	32	1812

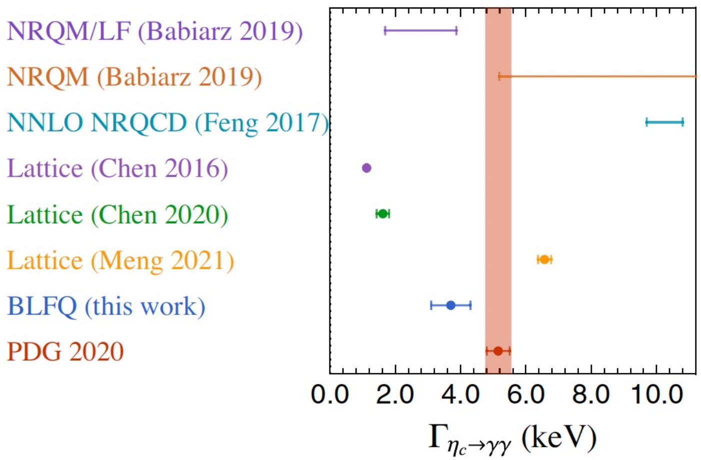
κ determined from fits to spectrum follows the HQET trajectory $\kappa_h \propto \sqrt{M_h}$, in agreement with recent LFH result [Dosch et al, PRD**95** (2017)]

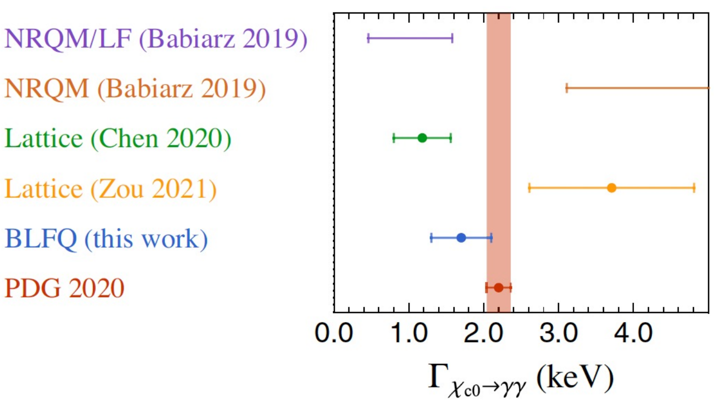
Yang Li, Meijian Li and James P. Vary, PRD 105, L071901 (2022)

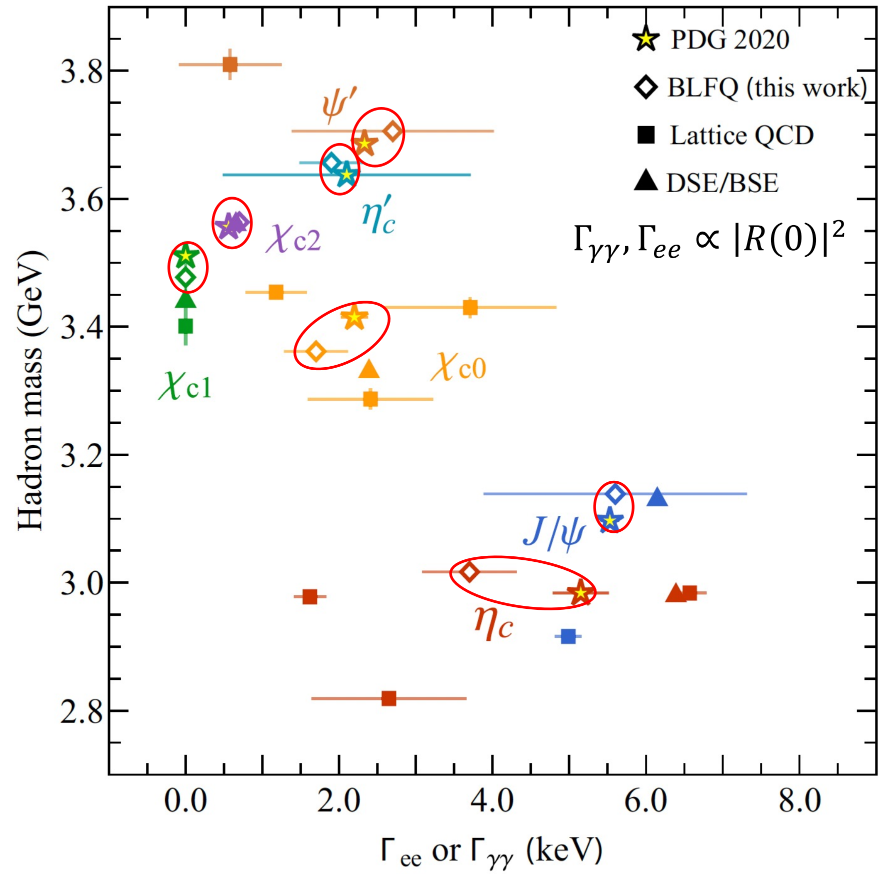
Diphoton width $\Gamma_{\gamma\gamma}$ of charmonia in BLFQ

- ✓ Notoriously challenging
- ✓ BLFQ predictions are very competetive!
  - ✓ No parameters were adjusted!





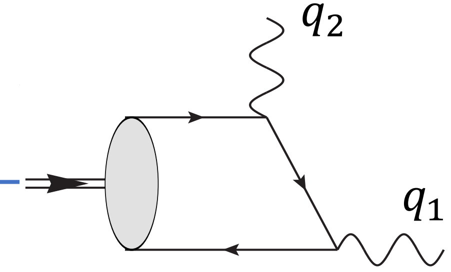




Comparison of theoretical prediction of masses and dilepton/diphoton widths combined

Lattice: Dudek '06, Chen '16, Chen '20, Meng '21, Zou '21;
NRQCD: Feng '15 & '17
NRQM: Babiarz '19 & '20

Transition form factor: η_c



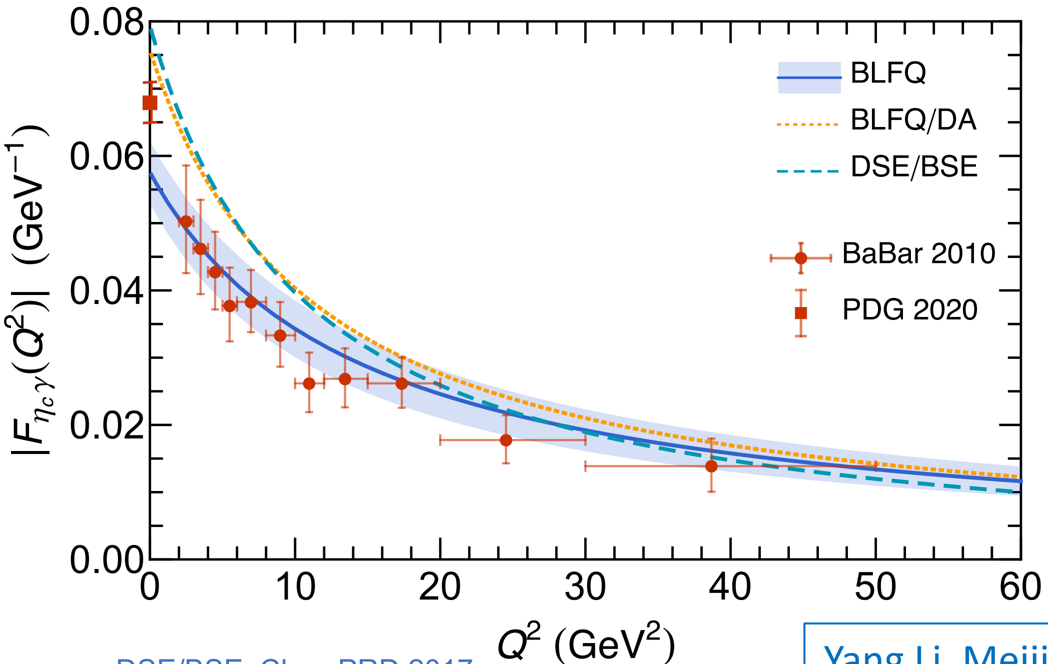
$$\mathcal{M}^{\mu\nu} = 4\pi\alpha_{em}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2, q_2^2)$$

✓ Diphoton width $\Gamma_{\gamma\gamma} = \frac{\pi}{4}\alpha_{em}^2 M_P^3 \left|F_{P\gamma\gamma}(0,0)\right|^2$

✓ Single-tag TFF $F_{P\gamma}(Q^2) = F_{P\gamma\gamma}(Q^2, 0) = F_{P\gamma\gamma}(0, Q^2)$

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_c}\int \frac{dx}{2\sqrt{x(1-x)}}\int \frac{d^2k_\perp}{(2\pi)^3}\frac{\psi_{\uparrow\downarrow-\downarrow\uparrow}(x,\vec{k}_\perp)}{k_\perp^2 + m_f^2 + x(1-x)Q^2}$$

Lepage ‘80, Feldman ‘97, Babiarz, ‘19

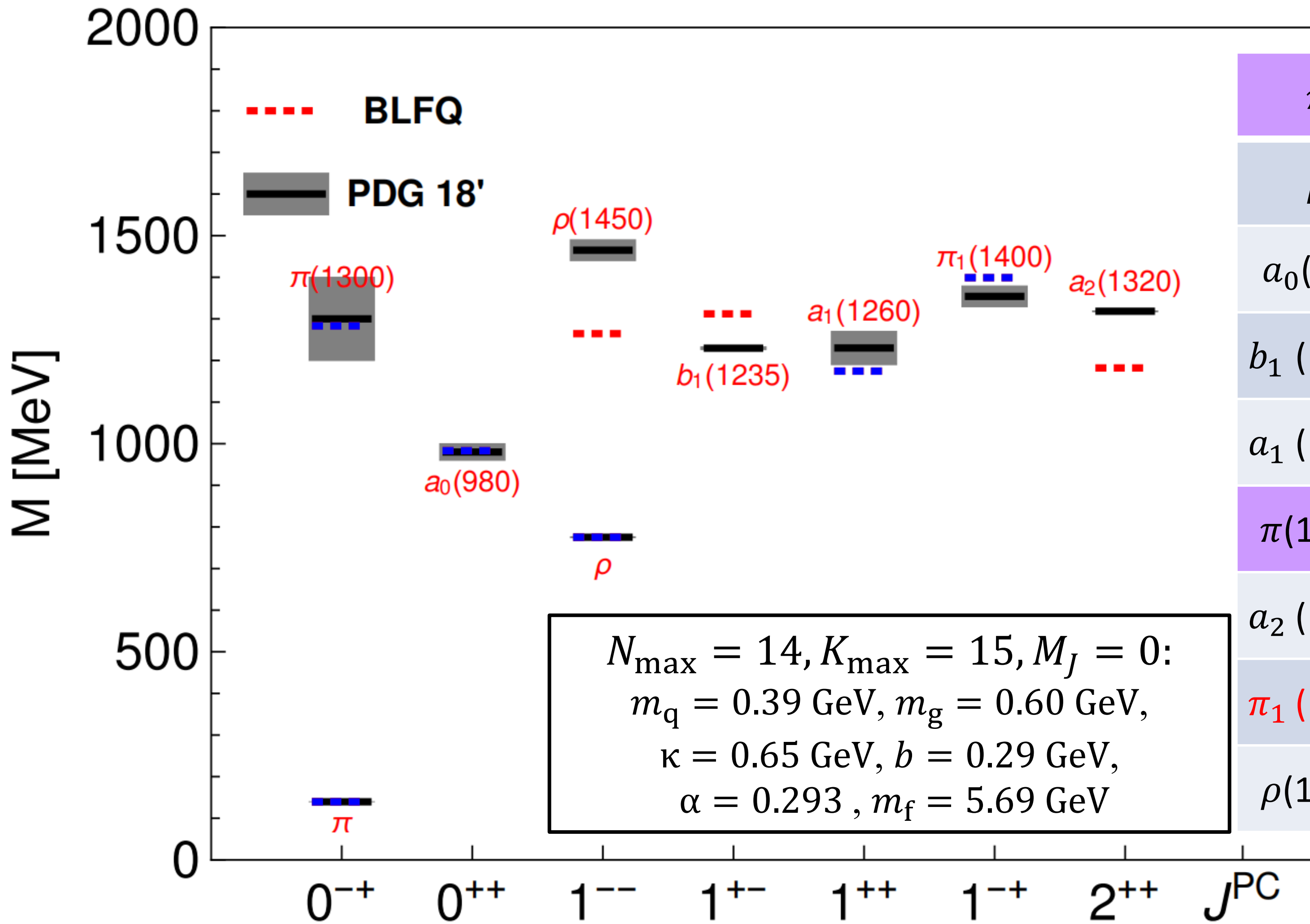


DSE/BSE: Chen PRD 2017

- BABAR data: described by monopole form with pole mass $\Lambda^2 = 8.5 \pm 0.6 \pm 0.7\ \text{GeV}^2$, and width 5.12 (53) keV
- BLFQ: using N_{max}=8 wave function corresponding to $\mu \approx 2m_c$. Basis sensitivity band is taken as the difference between the N_{max}=8,16 results.
- BLFQ/DA: prediction using the LCDA obtained from the LFWF
- Theoretical prediction in good agreement with both the width and the form factor.

Yang Li, Meijian Li and James P. Vary, PRD 105, L071901 (2022)

Light Meson Mass Spectrum Including One Dynamical Gluon



	Norm$q\bar{q}$	DC[MeV]
π	0.492	138
ρ	0.486	129
$a_0(980)$	0.370	0
b_1 (1235)	0.30	0
a_1 (1260)	0.324	11
$\pi(1300)$	0.284	53
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	46

$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

Fix the parameters by fitting six blue states

- $\pi_1(1400)$: $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: Decay Constant (DC) < π's DC

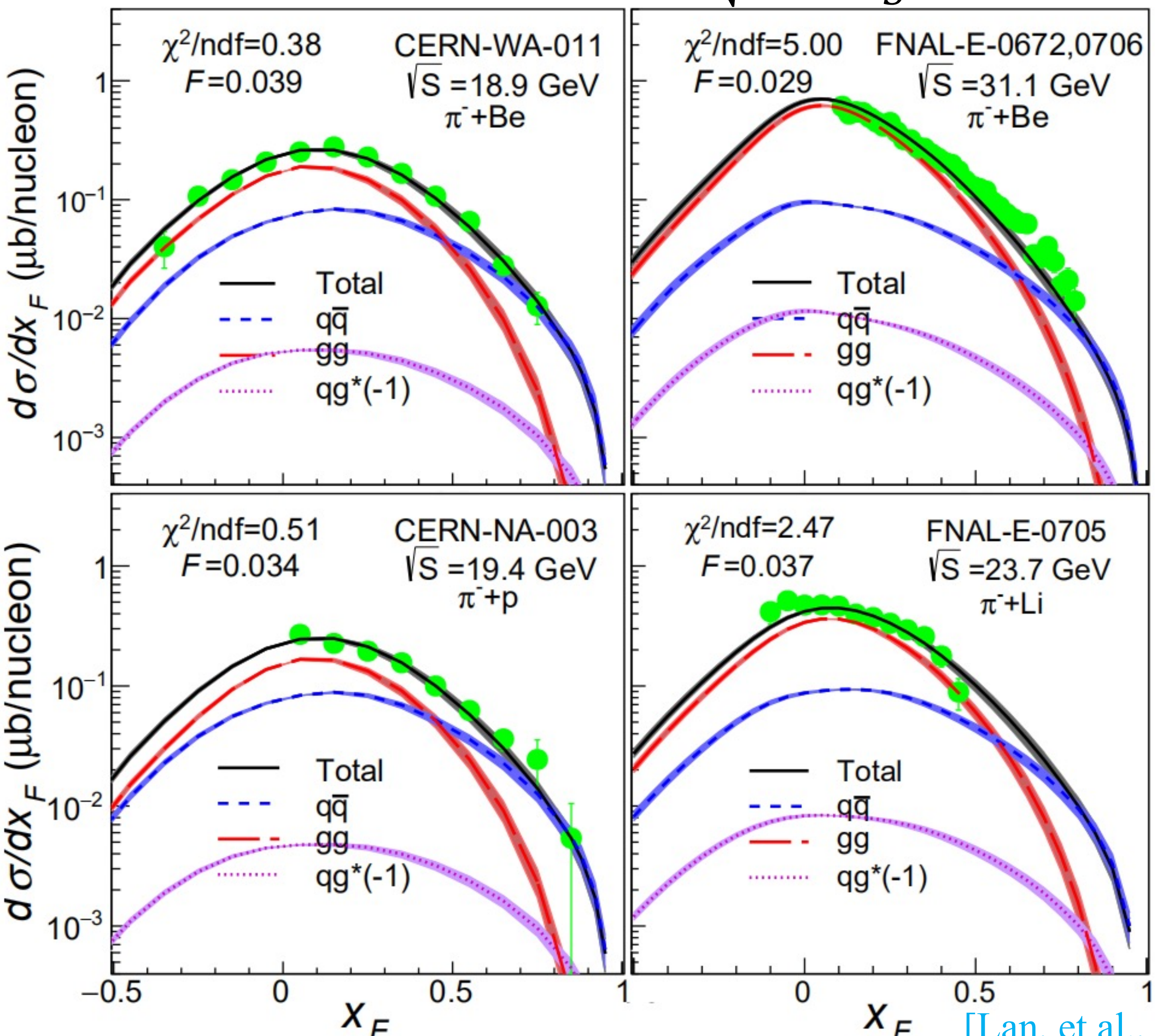
[Lan, et al., (BLFQ Collaboration) PLB 825, 136890 (2022); arXiv 2106.04954]

J/ψ production cross section

$\pi^{\pm} N \to J/\psi\ X$

$$\frac{d\sigma}{dx_F}[J/\psi] = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S\sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{S}}} \hat{\sigma}_{ij}(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^{\pm}}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

[nCTEQ 2015]



CEM

[Chang, et al, PRD 102 (2020) 054024];
[Nason, et al, NPB 303 (1988) 607];
[Mangano, et al, NPB 405 (1993) 507]

Agree with experimental data (FNAL E672, E706, E705, CERN NA3,WA11).

[Lan, et al., PLB 825, 136890 (2022)]

BLFQ Basis States

- BLFQ basis: expansion in Fock space

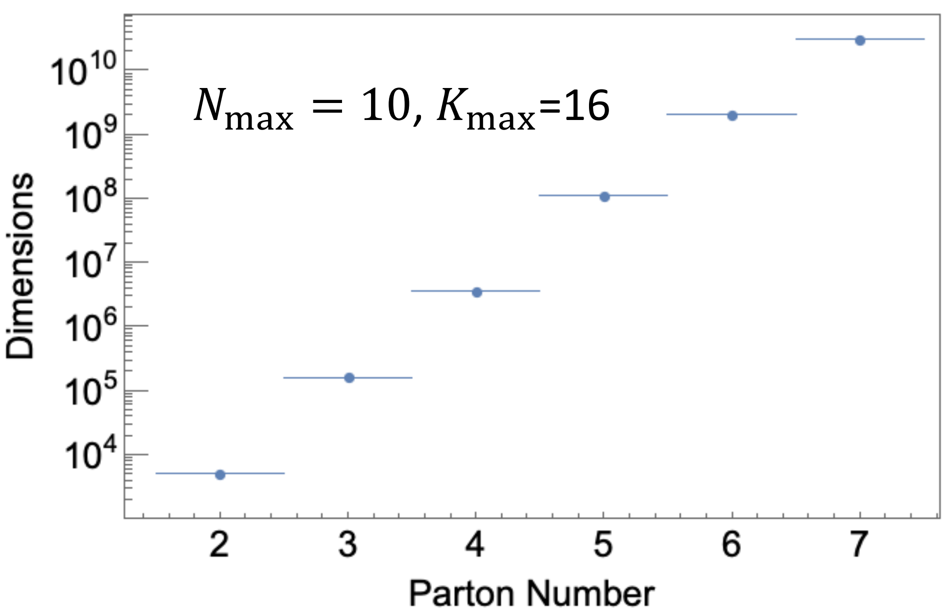
$$|\beta_{\text{meson}}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |gg\rangle + |q\bar{q}\ q\bar{q}\rangle + |q\bar{q}\ gg\rangle + |q\bar{q}\ q\bar{q}\ g\rangle + |q\bar{q}q\bar{q}gg\rangle + \cdots$$

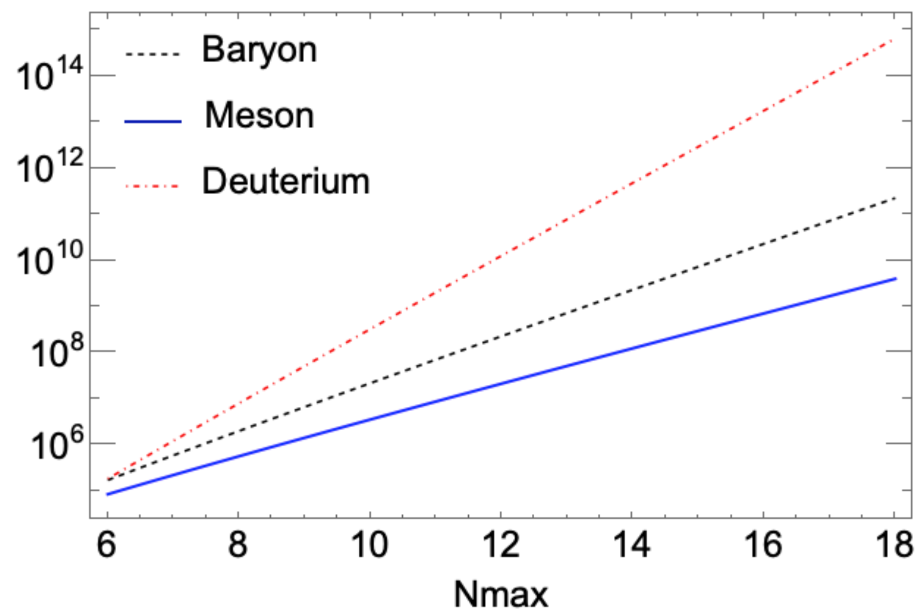
$$|\beta_{\text{baryon}}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\ q\bar{q}\rangle + |qqq\ gg\rangle + |qqq\ q\bar{q}g\rangle + |qqqq\bar{q}gg\rangle + \cdots$$

$$|\beta_{\text{deuterium}}\rangle = |qqq\ qqq\rangle + |qqq\ qqq\ g\rangle + |qqq\ qqq\ q\bar{q}\rangle + |qqq\ qqq\ gg\rangle + \cdots$$

- Dimension of basis states increases with number of Fock sectors

  => motivation for quantum computing





Baryons with one dynamical gluon

$$|P_{baryon}\rangle = \Psi_1|qqq\rangle + \Psi_2|qqqg\rangle$$

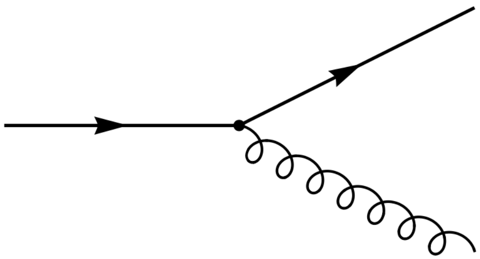
$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

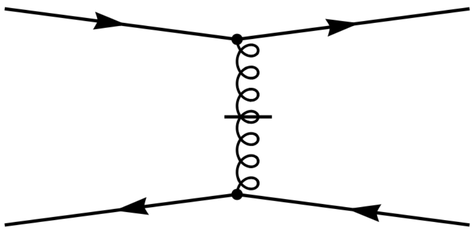
$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$H_{trans} \sim \kappa_T^4 r^2$ -- Brodsky, Teramond arXiv: 1203.4025

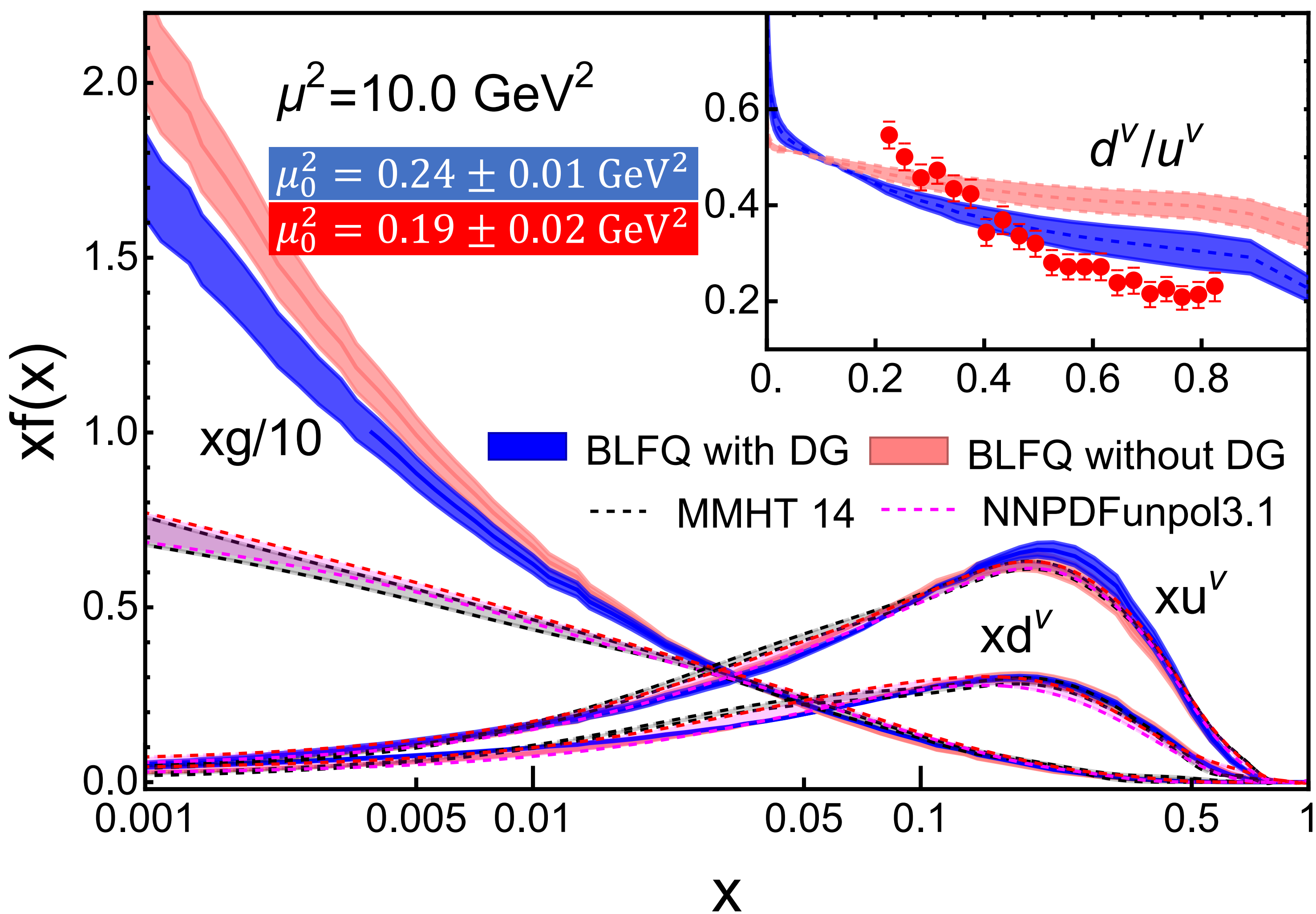
$H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i}\left(x_i x_j \partial_{x_j}\right)$ ---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

$$H_{Interact} = H_{Vertex} + H_{inst} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\frac{1}{(i\partial^+)^2}j^+$$





Unpolarized Parton Distribution Functions



The data are extracted from MARATHON data

Including the One Dynamical Gluon Fock Sector, the gluon distribution is closer to the global fit.

S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph]. [EPJC 77 (2017) 663]

Nucleon Spin with BLFQ

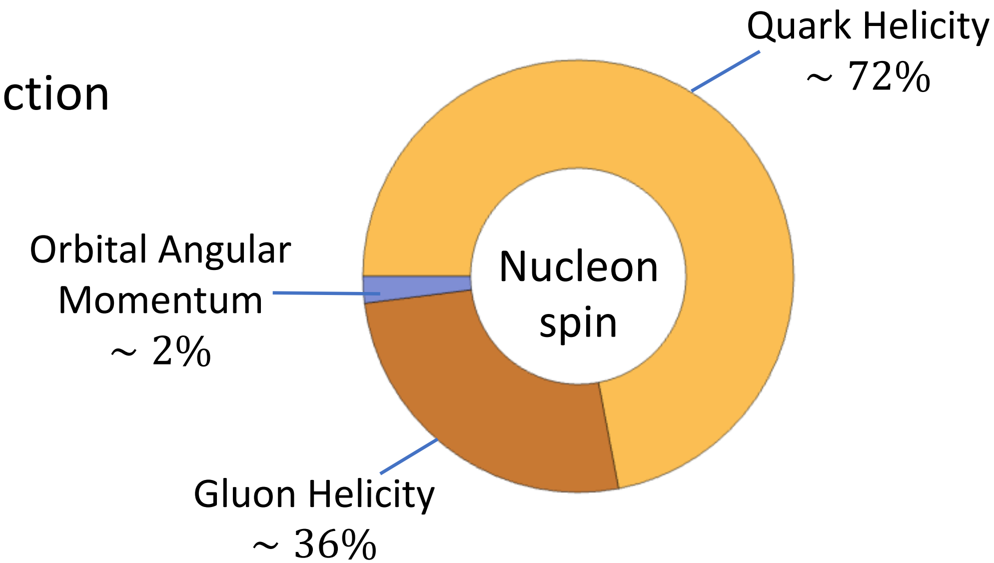
➢ Obtain observables from wave function

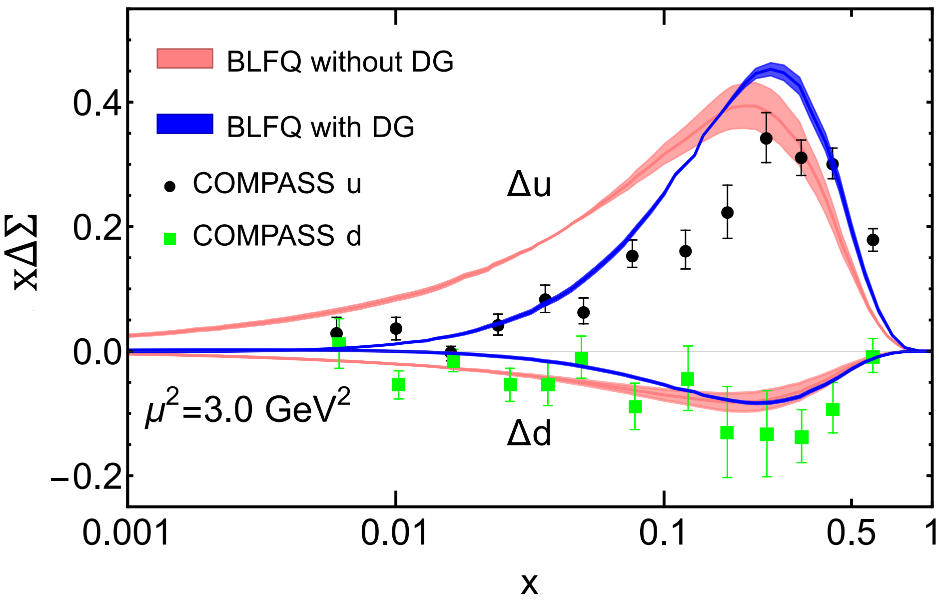
$$O \equiv \langle \beta', \Lambda' | \hat{O} | \beta, \Lambda \rangle$$

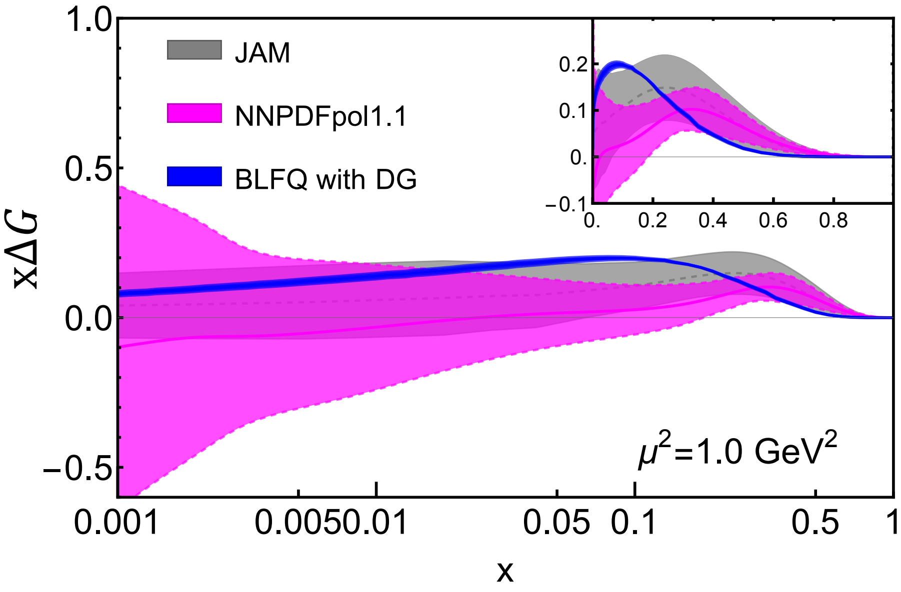
➢ Spin decomposition in BLFQ

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

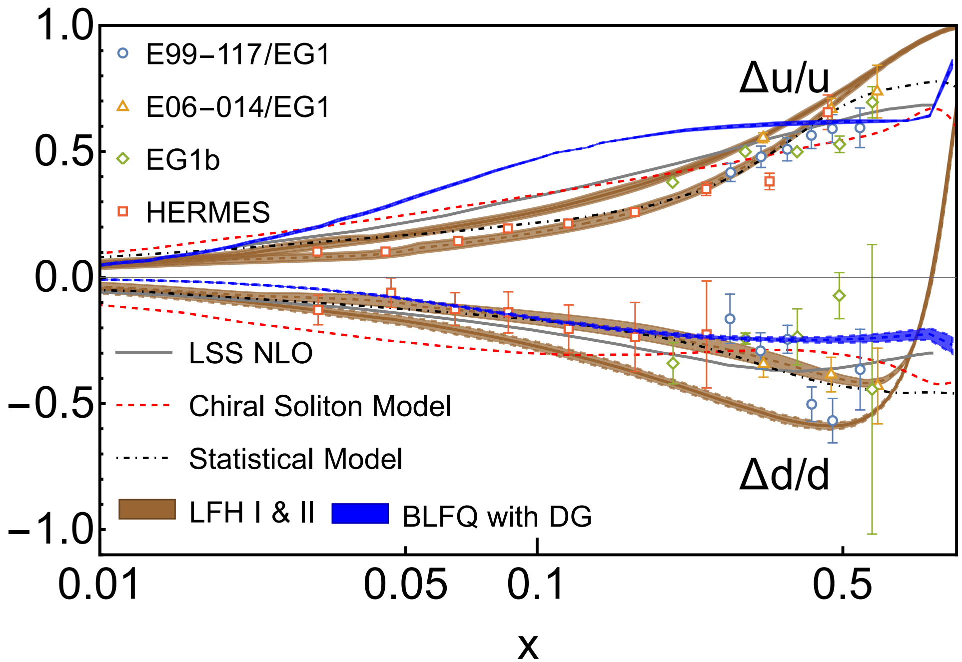
S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

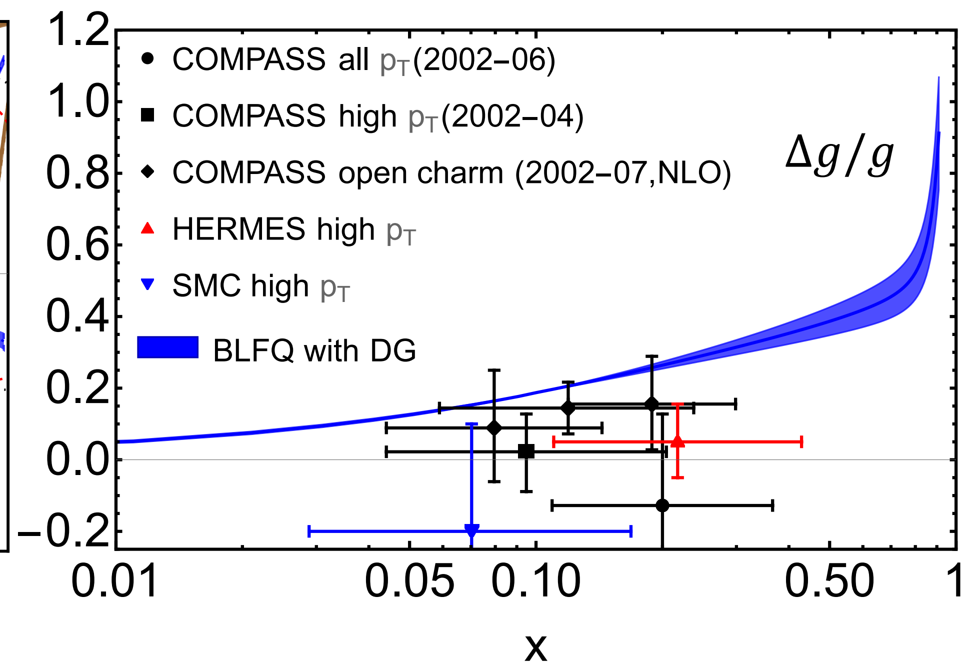






Helicity Parton Distribution Functions





- $\Delta G = \int_0^1 \mathrm{d}x \Delta g(x) = 0.131 \pm 0.003$, is sizeable to the proton spin.
- PHENIX Collaboration: $\Delta G^{[0.02,0.3]} = 0.2 \pm 0.1$ PRL 103 (2009) 012003

The sea quarks' contributions come from the DGLAP evolution

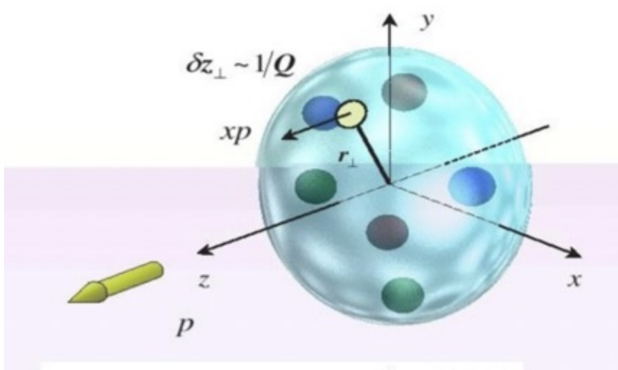
S. Xu, C. Mondal, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

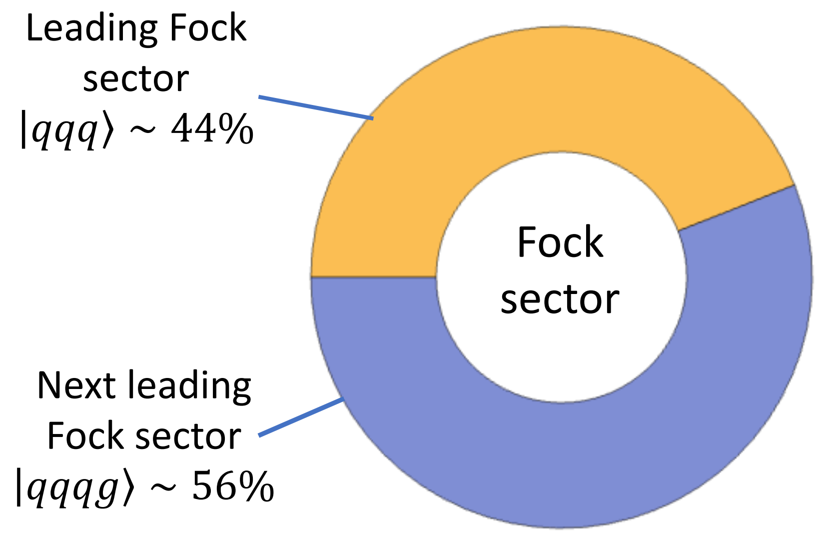
N. Sato et al. [JAM], PRD93 (2016); E. R. Nocera et al. [NNPDF], NPB 887 (2014).

3-Dimension Structure of Nucleon

- Obtain observables from wave function

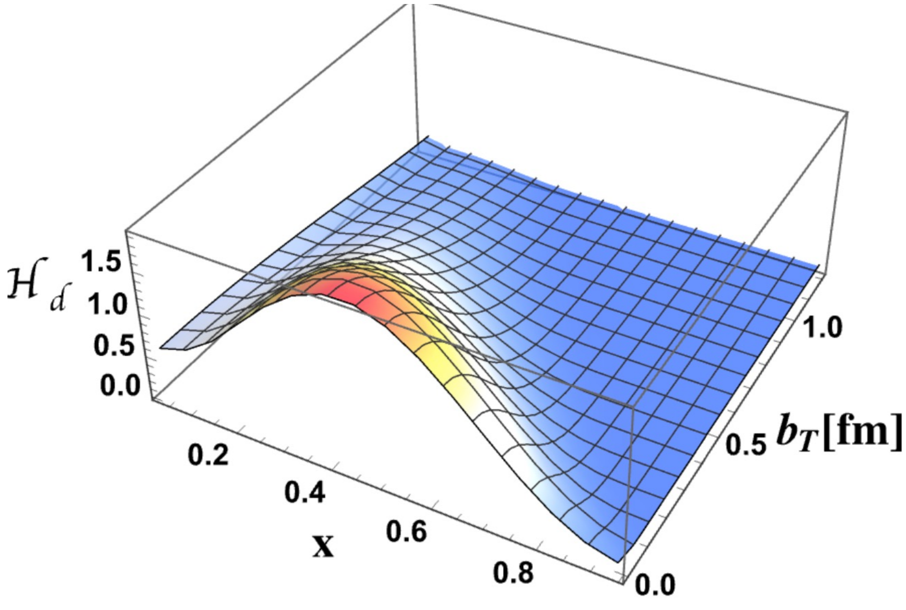
$O \equiv \langle\beta|\hat{O}|\beta\rangle \qquad |\beta_{\text{nucleon}}\rangle = |qqq\rangle + |qqqg\rangle$

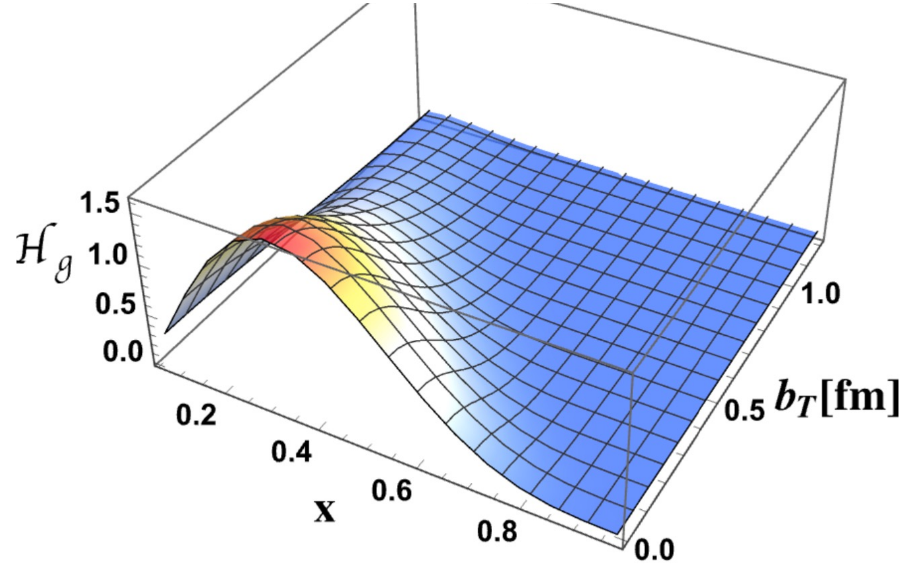




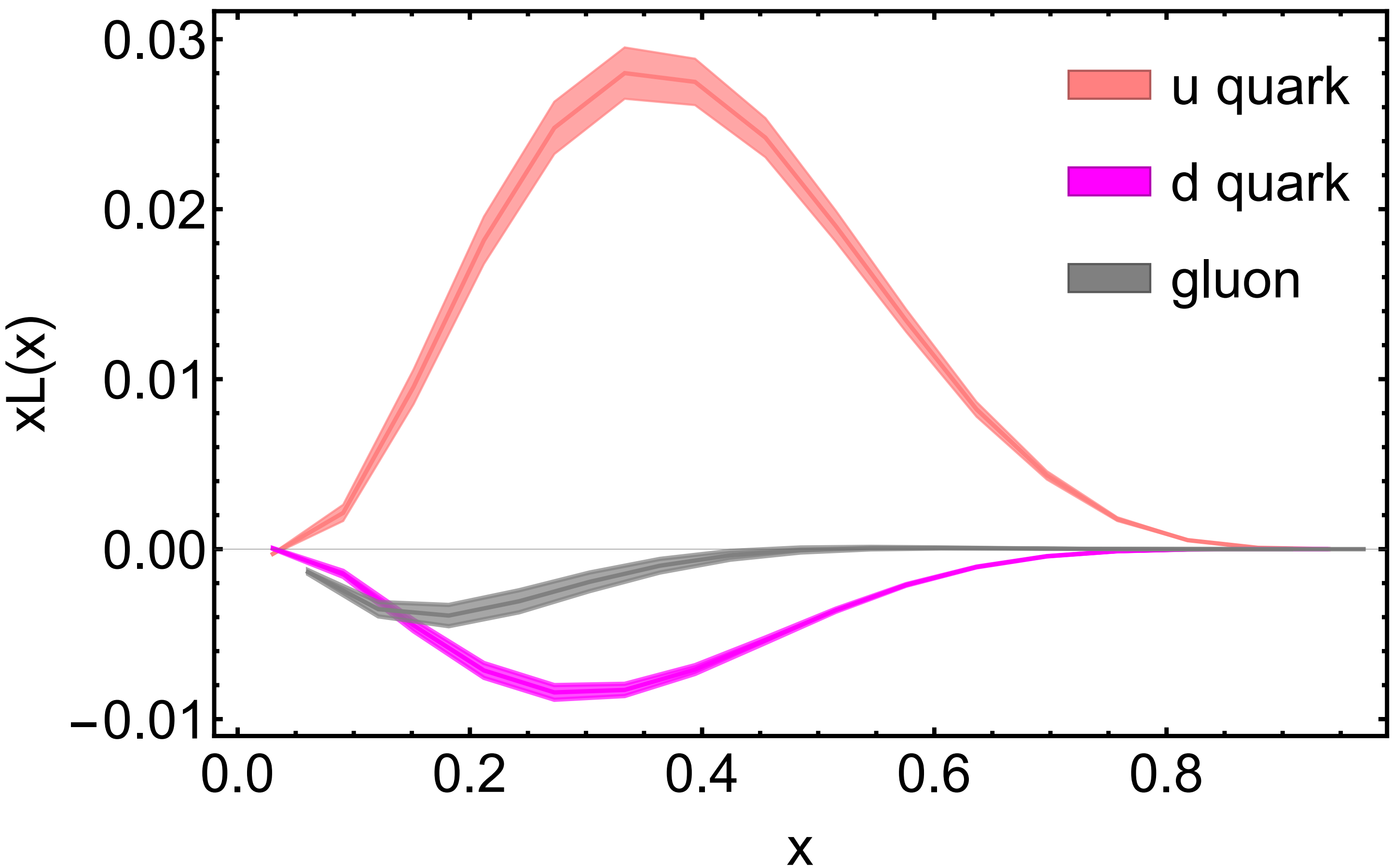
[arXiv:2209.08584 [hep-ph]]

[In preparation, Bolang Lin, Siqi Xu, C. Mondal *et.al*]





Orbital angular momentum distributions



Canonical: $\ell_d = -0.0114 \pm 0.0004 \quad \ell_u = 0.0327 \pm 0.0013 \quad \ell_g = -0.0065 \pm 0.0005$

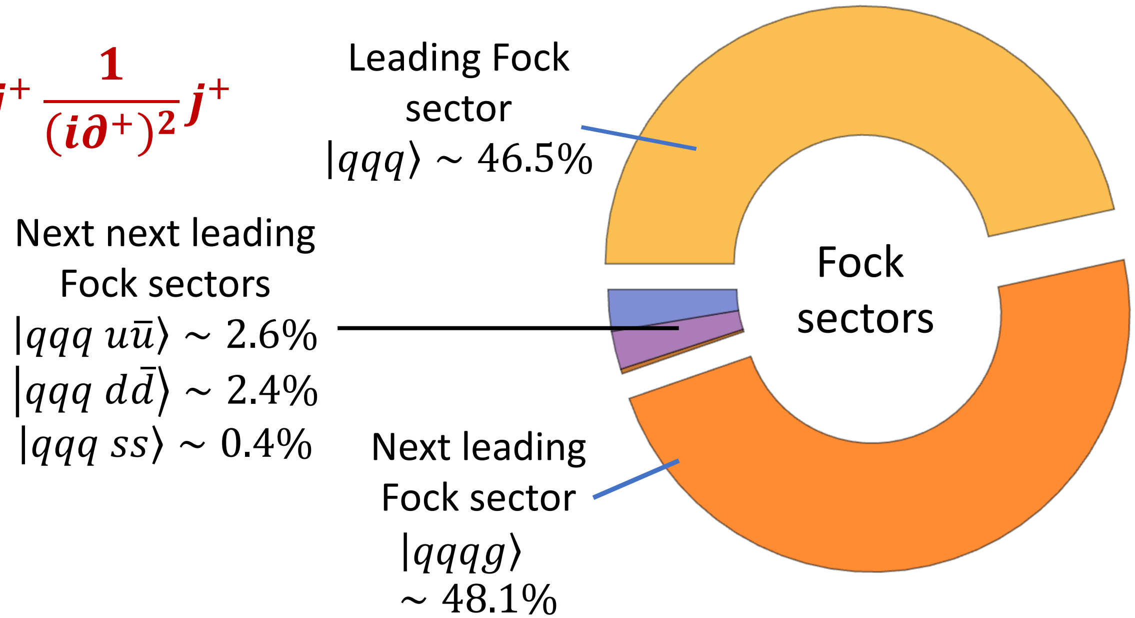
At the LC gauge : $\frac{1}{2}\Delta\Sigma = 0.359 \pm 0.002 \quad \Delta G = 0.131 \pm 0.003$

[In preparation, Siqi Xu, C. Mondal *et.al*]

Light-Front QCD Hamiltonian

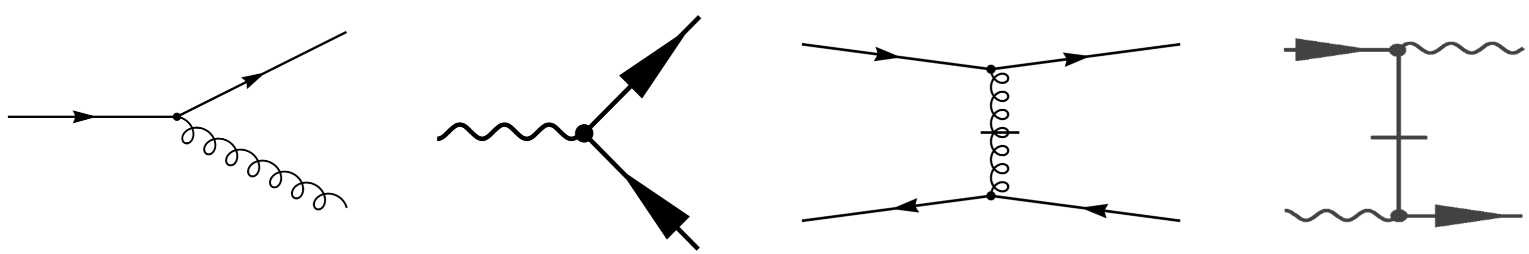
$$|P_{baryon}\rangle = \Psi_1|qqq\rangle + \Psi_2|qqqg\rangle + \Psi_{31}|qqq\,u\bar{u}\rangle + \Psi_{32}|qqq\,d\bar{d}\rangle + \Psi_{33}|qqq\,s\bar{s}\rangle$$

$$\boldsymbol{H_{Interact} = g\bar{\psi}\,\gamma^{\mu}T^{a}\,\psi\,A_{\mu}^{a} + \frac{g^2 C_F}{2}\,j^{+}\frac{1}{(i\partial^{+})^2}j^{+}}$$



$$\boldsymbol{H_{Interact} = g\bar{\psi}\,\gamma^{\mu}T^{a}\,\psi\,A_{\mu}^{a} + \frac{g^2 C_F}{2}\,j^{+}\frac{1}{(i\partial^{+})^2}j^{+} + \frac{g^2 C_F}{2}\,\bar{\psi}\gamma^{\mu}A_{\mu}\frac{\gamma^{+}}{i\partial^{+}}A_{\nu}\gamma^{\nu}\psi}$$

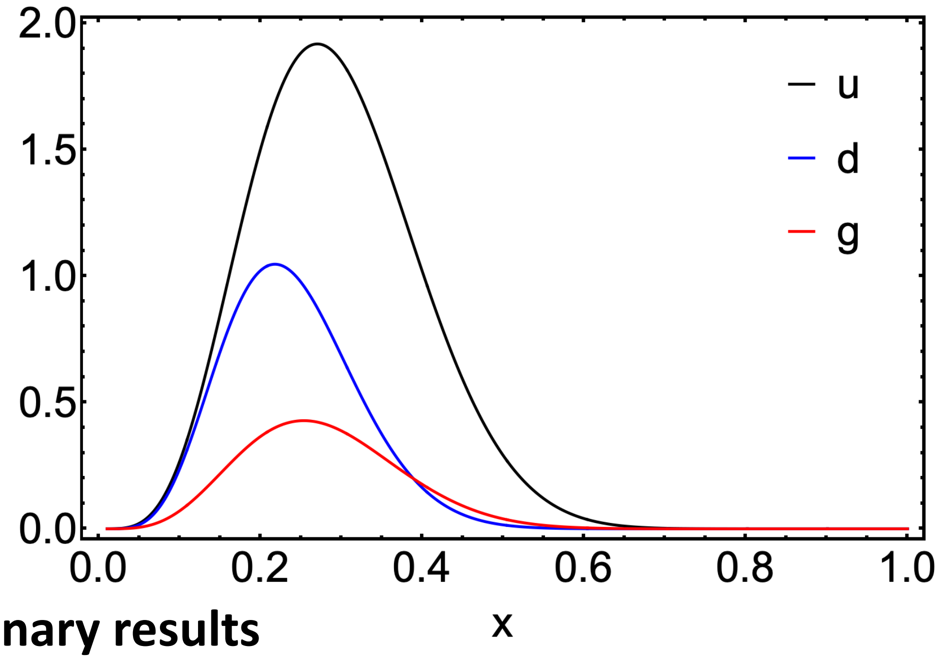
Preliminary results
Siqi Xu, et al, in prep



Parton Distribution Function

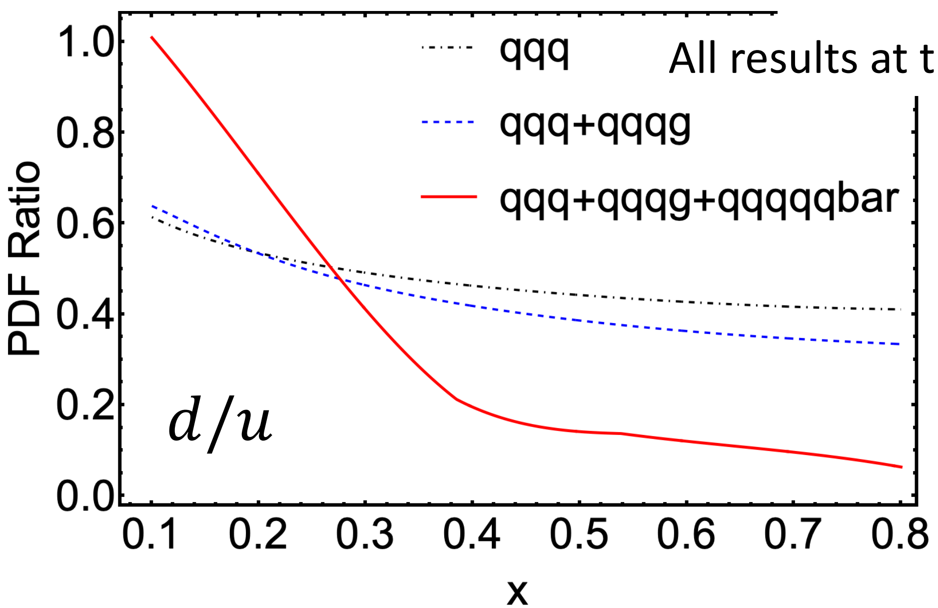
- Parton distribution functions with five Fock sectors

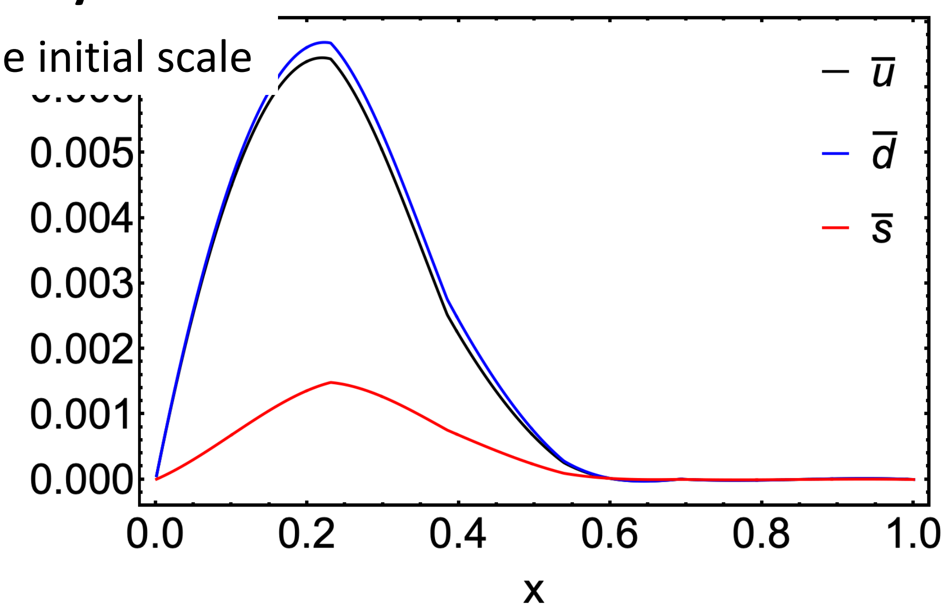
- One diagonalization, got the distribution of **valence quark**, **sea quark** and **gluon**
- PDF ratio d/u$= 0.04$ at $x \to 1$



Preliminary results

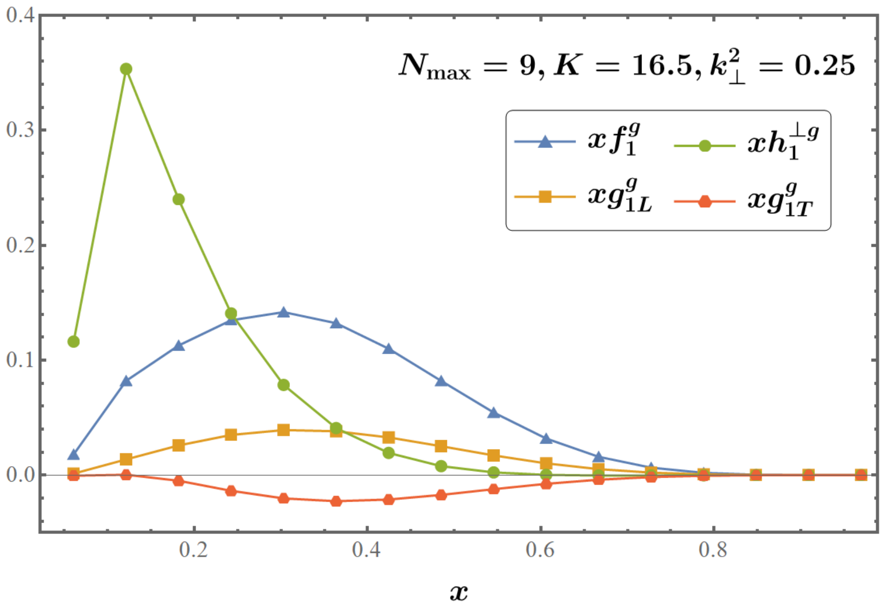
All results at the initial scale



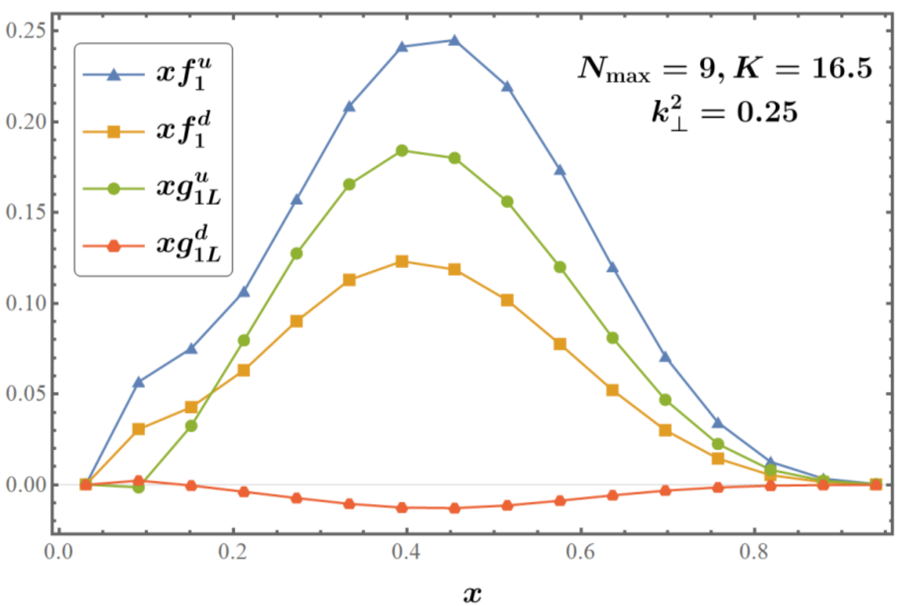


Now return to applications to baryons solved in the qqq + qqqg sectors

Transverse-momentum dependent distribution



Gluon T-even TMD

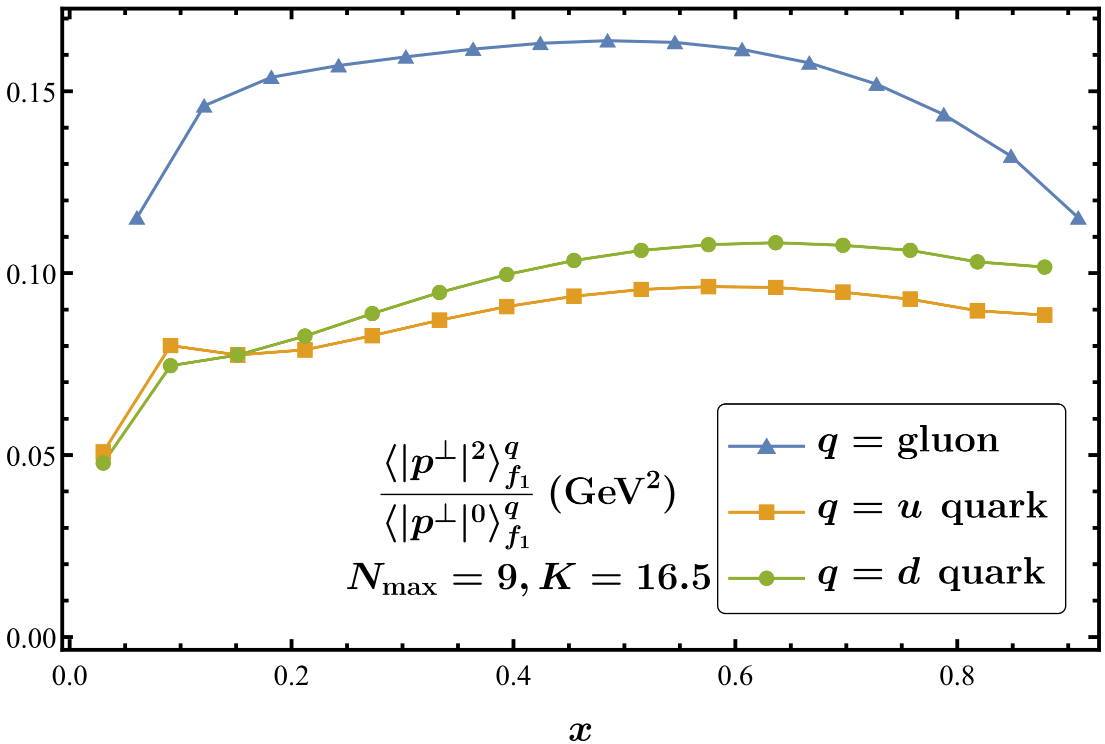


Quark T-even TMD

- Within the Basis Light-front Quantization (BLFQ) we expand proton to $|qqq\rangle + |qqqg\rangle$ Fock sector, obtain the corresponding LFWFs and calculate T-even TMDs of gluon and quark

Zhi Hu, et al., in preparation

Average transverse momentum of quark and gluon



- After integrating over x, we further obtain
  g: 0.156 (GeV)^2
  u: 0.082 (GeV)^2
  d: 0.083 (GeV)^2

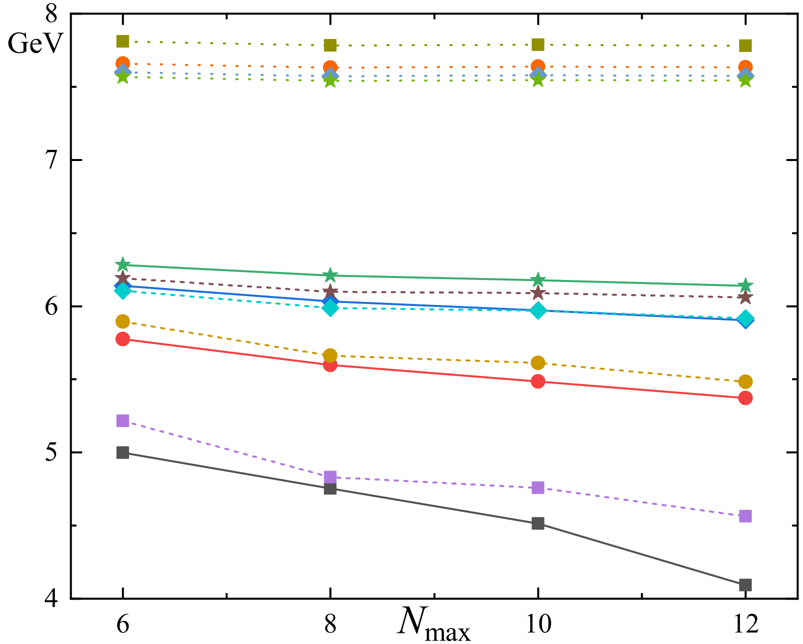
- Define $\langle|p^\perp|^n\rangle^q_{f_1} = \int d^2p^\perp\, |p^\perp|^n \times f_1^q$ then we know that $\frac{\langle|p^\perp|^2\rangle^q_{f_1}}{\langle|p^\perp|^0\rangle^q_{f_1}}$ would be the average transverse momentum of flavor q
- Average transverse momentum of d quark is slightly larger than that of u, the same as our $|qqq\rangle$ Fock sector conclusion.
- With $|qqqg\rangle$ Fock sector we now also know that transverse momentum of gluon is larger than that of quark

Zhi Hu, et al., in preparation

All-charm tetraquark using BLFQ

- New issues compared with mesons and baryons
  - Cluster decomposition principle for interactions
  - Identical particles issue
  - More than one color singlet
- Hamiltonian
  - Transverse confining potential like in AdS/QCD
  - Longitudinal confining potential (Głazek et al. PLB773, 172-178 (2017), different than $BLFQ_0$)
  - One-gluon-exchange spin-dependent potential (Wiecki et al.)
- Problem with negative M^2 solved by ad hoc modification of the Hamiltonian (which breaks cluster decomposition principle)

Kamil Serafin, et al., Phys Rev. D 105, 094028 (2022)



$K = 6$ ■, 10 ●, 14 ◆, 18 ★

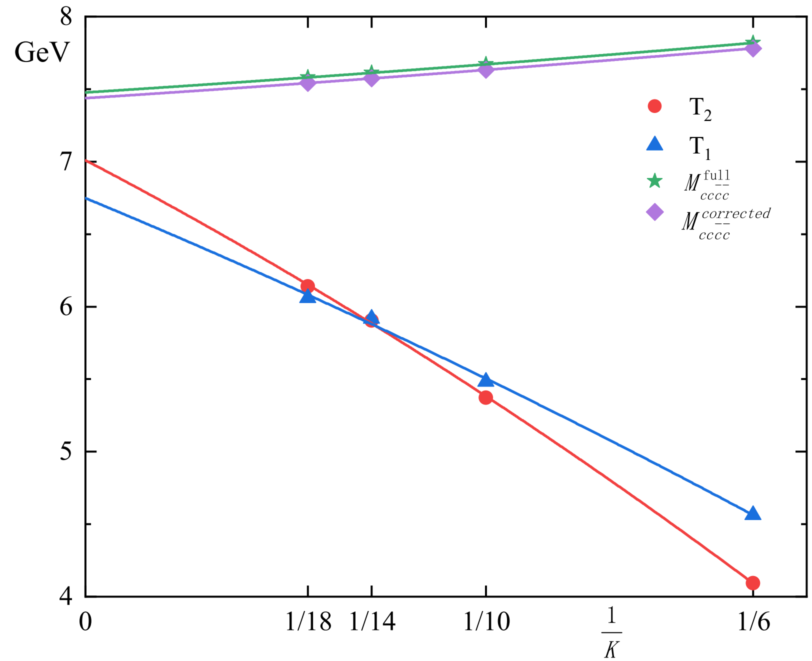
Dotted – $M^{\mathrm{corrected}}_{cc\bar{c}\bar{c}}$

Solid – T_2

Dashed – T_1

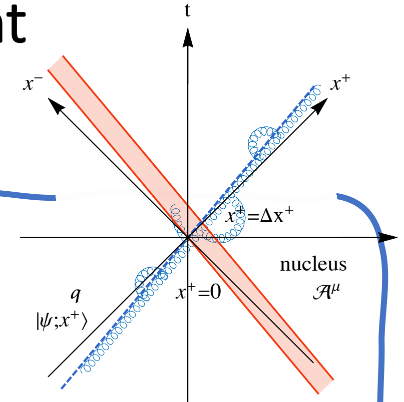
Extrapolations $K \to \infty$

$N_{\mathrm{max}} = 12$



Kamil Serafin, et al., Phys Rev. D 105, 094028 (2022)

Forward quark jet-nucleus scattering in a light-front Hamiltonian approach



Time-dependent Basis Light-Front Quantization (tBLFQ)

❖ First-principles:

In the light-front Hamiltonian formalism, the state obeys the time-evolution equation, and the Hamiltonian is derived from the QCD Lagrangian

$$\frac{1}{2}P^-(x^+)|\psi(x^+)\rangle = i\frac{\partial}{\partial x^+}|\psi(x^+)\rangle$$

❖ Nonperturbative treatment:

The time evolution operator is divided into many small timesteps, each timestep is evaluated numerically and intermediate states are accessible,

$$|\psi(x^+)\rangle = \mathcal{T}_+ \exp\left[-\frac{i}{2}\int_0^{x^+} dz^+ P^-(z^+)\right]|\psi(0)\rangle$$

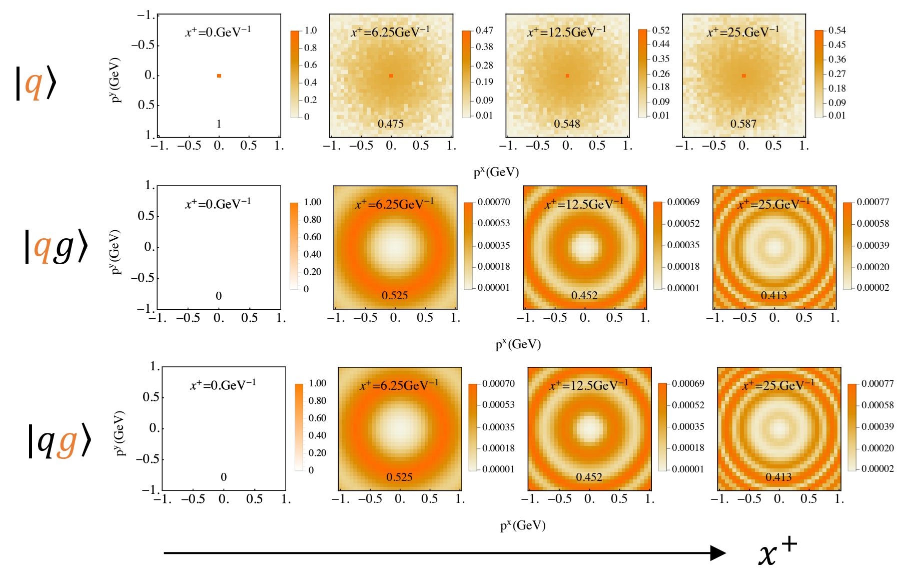
$$= \lim_{n\to\infty}\prod_{k=1}^{n}\mathcal{T}_+ \exp\left[-\frac{i}{2}\int_{x_{k-1}^+}^{x_k^+} dz^+ P^-(z^+)\right]|\psi(0)\rangle$$

❖ Basis representation:

Optimal basis has the same symmetries of the system, and it is the key to numerical efficiency

We consider scattering of a high-energy quark moving in the positive z direction, on a high-energy nucleus moving in the negative z direction.

Time evolution of a quark state in the $|q\rangle + |qg\rangle$ Fock space observed from the transverse momentum space

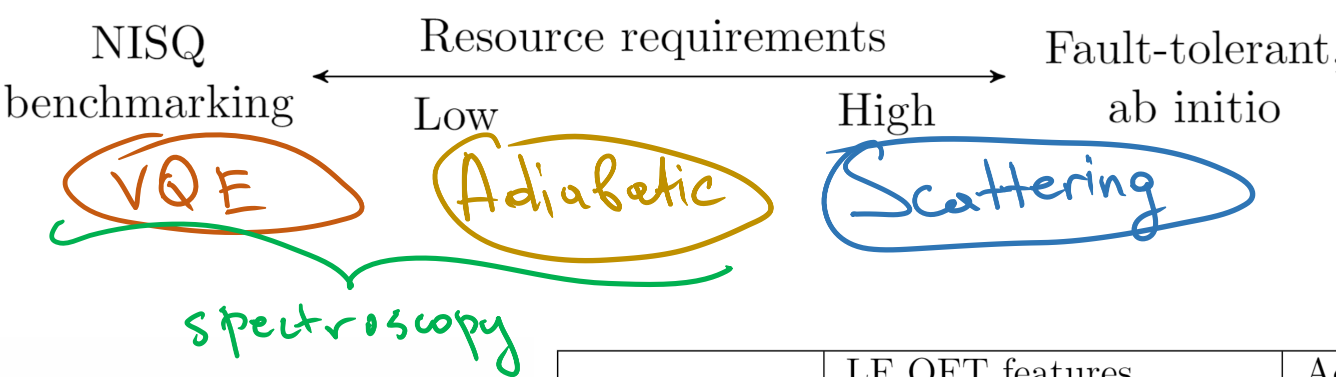


M. Li, T. Lappi and X. Zhao, Phys. Rev. D 104, 056104 (2021)

Wenyang Qian, et al

Quantum Simulation of QFT in the Front Form

2002.04016, 2105.10941, 2011.13443, 2009.07885



$$\mathfrak{Q}_{\text{Direct}} = O(K \log K)$$

$$\mathfrak{Q}_{\text{Compact}} = O(\sqrt{K} \log K)$$

	Trotter	Oracle
Direct	✓	✓
Compact	✗	✓

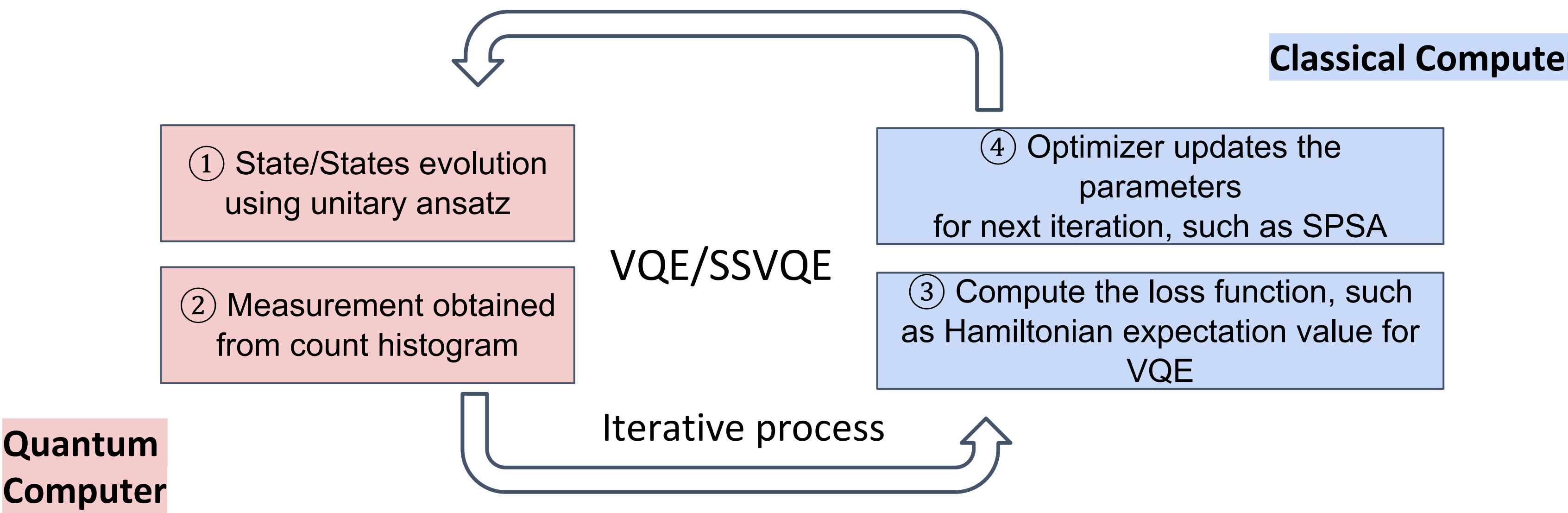
$$O_F\,|x, i\rangle = |x, y_i\rangle \ ,$$

$$O_H\,|x, y, 0\rangle = |x, y, H_{xy}\rangle$$

	LF QFT features	Advantages for QC
Resources	No ghost fields Linear EoM	Low qubit count
	LF momentum > 0	Efficient encoding
Evolution	Sparse Hamiltonians	Using sparsity-based methods
Measurement	LF wavefunction $\to$ $\to$ static quantities; Simple form of operators in the second-quantized formalism	Simple form of measurement operators
Other	Trivial vacuum, fewer cut-offs, no fermion doubling, form invariance of H	

Light front approach to hadrons on quantum computers

- Quantum computers: New tool to simulate many-body quantum system. (quantum mechanical nature and high scalability)
- In the Noisy Intermediate-Scale Quantum (NISQ) era, the Variational Quantum Eigensolver (VQE) and Subspace-search VQE (SSVQE) [Nakanishi, 1810.09434] approaches are promising tools to solve nuclear physics problems.
- Advantages of light front Hamiltonian formalism are directly applicable
- We first formulate the problem on the light front and then map the Hamiltonian to qubits (quantum bits)



Wenyang Qian, et al. Phys. Rev. Research 4, 043193 (2022)

Formulating the problem on qubits

- We adopt the Hamiltonian used in a previous work: [Qian, 2005.13806]

$$H_{\mathrm{eff},\gamma_5} = \overbrace{\underbrace{\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\mathbf{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_{\mathrm{eff}}} + V_g + H_{\gamma_5}$$

[Vary, 0905.1411]

- Basis representation (BLFQ) is key to represent the Hamiltonian on qubits.
- Small-size Hamiltonians (4-by-4 and 16-by-16) are used. [Seeley, 1208.5986]
- Direct encoding and compact encoding are compared. [Kreshchuk, 2002.04016]

	N_f	$\alpha_s(0)$	κ (MeV)	m_q (MeV)	$N_{\max}$	$L_{\max}$	Matrix dimension
$H_{\mathrm{eff}}^{(1,1)}$	3	0.89	560 ± 10	300 ± 10	1	1	4 by 4
$H_{\mathrm{eff}}^{(4,1)}$					4	1	16 by 16

$$H_{\mathrm{direct}}^{(1,1)} = 2269462\,\mathrm{IIII} - 284243\,(\mathrm{ZIII} + \mathrm{IIZI}) - 850488\,(\mathrm{IZII} + \mathrm{IIIZ}) + 12714\,(\mathrm{XZXI} + \mathrm{YZYI}) - 7883\,(\mathrm{IXZX} + \mathrm{IYZY}),$$

$$H_{\mathrm{compact}}^{(1,1)} = 1134731\,\mathrm{II} - 566245\,\mathrm{IZ} + 4831\,\mathrm{XI} + 20598\,\mathrm{XZ}$$

Wenyang Qian, et al. Phys. Rev. Research 4, 043193 (2022)

Summary and Outlook

Basis Light Front Quantization approach to mesons and baryons yields competitive descriptions and predictions

- Positronium test applications found successful
- Bound states and transitions of hadrons are described
- Time-dependent scattering applications are advancing
- Plan: continue to expand the Fock spaces (e.g. more gluons)
- Plan: continue to develop renormalization & counterterms
- Efficient utilization of supercomputing resources
- Well-positioned to exploit advances in quantum computing

Thank you for your attention