Physics opportunities and challenges at future multi-TeV lepton colliders

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INFN
Istituto Nazionale di Fisica Nucleare

Theory and Phenomenology
of Fundamental Interactions
UNIVERSITY AND INFN · BOLOGNA

1958
University of Science and Technology of China

Our goals and the dream machines

“The party is never over”

“It is a marathon, not a sprint!”

A lot of particle physics is missing in the Standard Model

- Why Electroweak Symmetry Breaking occurs?
  What is the history of the Electroweak Phase Transition?
- The reason for the Hierarchy in Fermion Masses and their Flavor Structure
- The Nature of Dark Matter
- The origin of the Matter-Antimatter Asymmetry
- The generation of Neutrino Masses
- The cause of the Universe’s accelerated expansion - Dark Energy
- What are the quantum properties of Gravity?
- What caused Cosmic Inflation after the Big Bang?

The SM is silent about all above, BSM physics is at the core of it all

Dr. Mangano: Understanding the origin of the Higgs and EWSB is a key task, which only colliders — to the best or our current knowledge — can undertake

Colliders: current and past

Our goal is to **“Address the Big Questions”** and to **“Explore the unknown”**

- Study known phenomena at high energies looking for indirect evidence of BSM physics
  Higgs Factories ⇒ Probe TeV scale via precision measurements
- Search for direct evidence of BSM physics at the energy frontier
  Directly reach the multi-TeV scale

Colliders so far

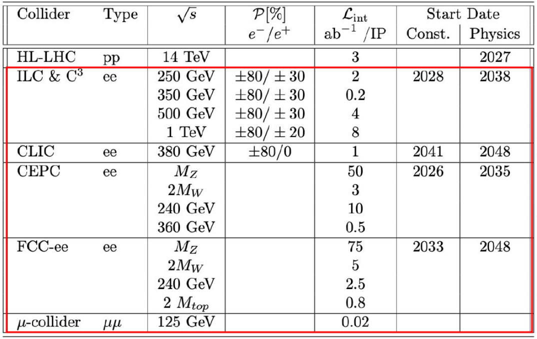
- Hadron colliders collide **composite particles** ⇒ To reach high energies
  Generate large QCD backgrounds and you use a fraction of the energy of beam for physics
- Lepton colliders collide **fundamental particles** ⇒ To reach high precisions
  Exploit the full energy and avoid large QCD backgrounds

We always want to have machines at higher energies

- Linear e^+e^- collider: CLIC, ILC
- Circular collider but with heavier beam particles: **A muon collider**

To the frontiers: machines in the plan

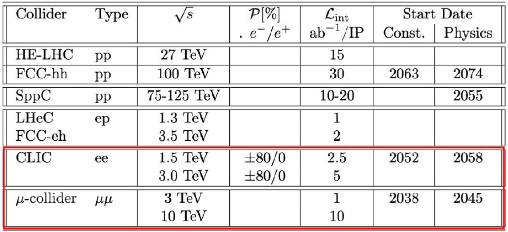
Higgs-boson factories
(up to 1 TeV c.o.m. energy)



Collider	Type	$\sqrt{s}$	$\mathcal{P}[\%]$	$\mathcal{L}_{\rm int}$	Start Date	
			e^-/e^+	$\rm ab^{-1}$ /IP	Const.	Physics
HL-LHC	pp	14 TeV		3		2027
ILC & C^3	ee	250 GeV	$\pm80/\pm30$	2	2028	2038
		350 GeV	$\pm80/\pm30$	0.2		
		500 GeV	$\pm80/\pm30$	4		
		1 TeV	$\pm80/\pm20$	8		
CLIC	ee	380 GeV	$\pm80/0$	1	2041	2048
CEPC	ee	M_Z		50	2026	2035
		$2M_W$		3		
		240 GeV		10		
		360 GeV		0.5		
FCC-ee	ee	M_Z		75	2033	2048
		$2M_W$		5		
		240 GeV		2.5		
		$2\ M_{top}$		0.8		
μ-collider	$\mu\mu$	125 GeV		0.02		

Snowmass 2021:
EF Benchmark Scenarios

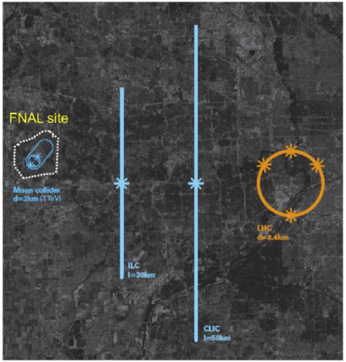
Multi-TeV colliders
(> 1 TeV c.o.m. energy)

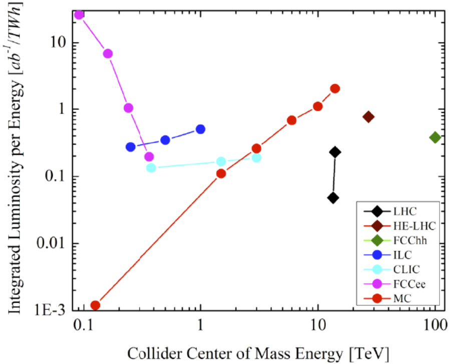


Collider	Type	$\sqrt{s}$	$\mathcal{P}[\%]$	$\mathcal{L}_{\rm int}$	Start Date	
			. e^-/e^+	$\rm ab^{-1}$/IP	Const.	Physics
HE-LHC	pp	27 TeV		15		
FCC-hh	pp	100 TeV		30	2063	2074
SppC	pp	75-125 TeV		10-20		2055
LHeC	ep	1.3 TeV		1		
FCC-eh		3.5 TeV		2		
CLIC	ee	1.5 TeV	$\pm80/0$	2.5	2052	2058
		3.0 TeV	$\pm80/0$	5		
μ-collider	$\mu\mu$	3 TeV		1	2038	2045
		10 TeV		10		

Timelines are taken from the ITF report (AF)

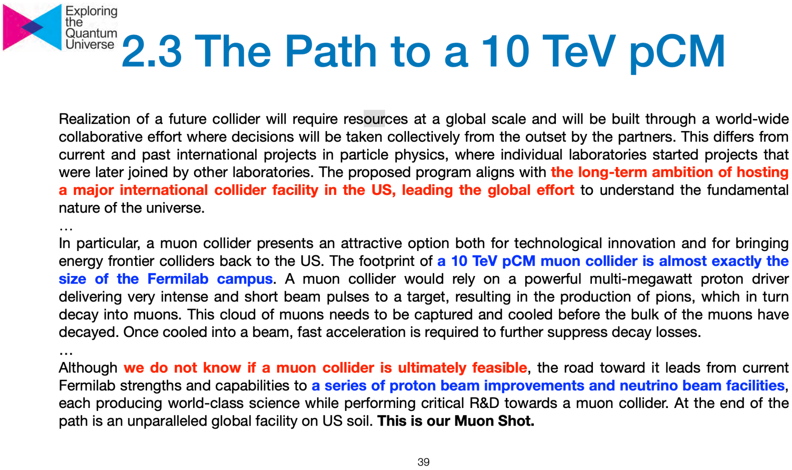
Dream machine: A possible muon collider





- Offer a precision prob of fundamental interactions in a smaller footprint
- Most power efficient machine at high energies

Good news from the Particle Physics Project Prioritization Panel (P5)



Exploring the Quantum Universe

2.3 The Path to a 10 TeV pCM

Realization of a future collider will require resources at a global scale and will be built through a world-wide collaborative effort where decisions will be taken collectively from the outset by the partners. This differs from current and past international projects in particle physics, where individual laboratories started projects that were later joined by other laboratories. The proposed program aligns with **the long-term ambition of hosting a major international collider facility in the US, leading the global effort** to understand the fundamental nature of the universe.

…

In particular, a muon collider presents an attractive option both for technological innovation and for bringing energy frontier colliders back to the US. The footprint of **a 10 TeV pCM muon collider is almost exactly the size of the Fermilab campus**. A muon collider would rely on a powerful multi-megawatt proton driver delivering very intense and short beam pulses to a target, resulting in the production of pions, which in turn decay into muons. This cloud of muons needs to be captured and cooled before the bulk of the muons have decayed. Once cooled into a beam, fast acceleration is required to further suppress decay losses.

…

Although **we do not know if a muon collider is ultimately feasible**, the road toward it leads from current Fermilab strengths and capabilities to **a series of proton beam improvements and neutrino beam facilities**, each producing world-class science while performing critical R&D towards a muon collider. At the end of the path is an unparalleled global facility on US soil. **This is our Muon Shot.**

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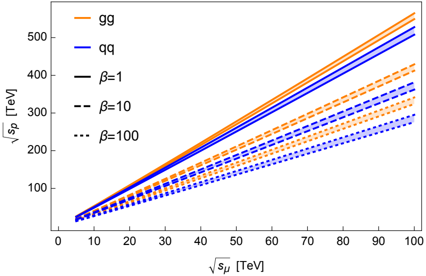
This is our Muon Shot!

Dream machine: The physics mechanisms

Get use of the full machine energy



Discovery reach: $M \sim \frac{\sqrt{s}}{2}$

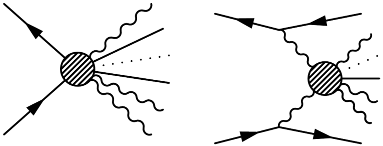
[2103.14043 "Muon Smasher's Guide"]

muC@10 TeV $\sim pp$@70 TeV

10 TeV is not the limit

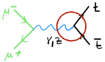
More than lepton collisions:

Two mechanisms: Annihilation VS Fusion



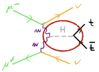
⇒ VBF collider:

$\sqrt{s} \lesssim$ 1-5 TeV



$\sigma_s \sim \frac{1}{s}$ ⬌ $\sigma_s \sim \frac{1}{M^2} \log^n \frac{s}{M}$

$\sqrt{s} \gtrsim$ 1-5 TeV



Need to resum the large Logs

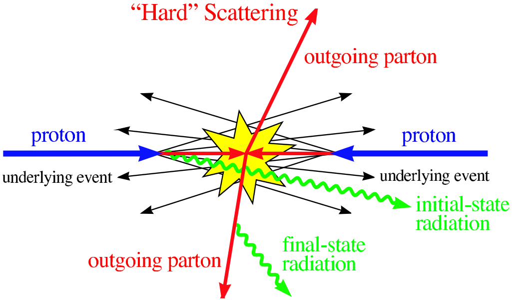
⇒ **The partonic picture is needed**

The partonic picture

[T. Han, YM, K. Xie, 2007.14300, 2103.09844]

Hadron colliders and the Parton Distribution Function (PDF)

- **Recall the hadron colliders: the $\mathrm{Sp\bar{p}S}$, the Tevatron, or the LHC**



- **Hadrons are composite**
  a, b are the “partons” from the beam particles A and B.
- **PDFs**
  $f_{a/A}, f_{b/B}$ are the probabilities to find a parton a (b) from the beam particle A (B) with a momentum fraction x_a (x_b).

- **Factorization formalism : PDFs $\otimes$ partonic cross sections**

$$\sigma(AB \to X) = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b f_{a/A}(x_a, Q) f_{b/B}(x_b, Q) \hat{\sigma}(ab \to X)$$

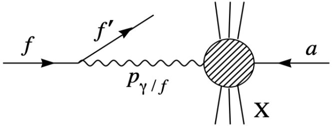
“Parton” of a lepton

Leptons are elementary particles ⇒ **“Equivalent photon approximation (EPA)”**

- Treat photon as a parton constituent in the electron

$$\sigma(\ell^- + a \to \ell^- + X) = \int \mathrm{d}x\, f_{\gamma/\ell}\hat{\sigma}(\gamma a \to X)$$

$$f_{\gamma/\ell,\mathrm{EPA}}(x_\gamma, Q^2) = \frac{\alpha}{2\pi}\frac{1+(1-x_\gamma)^2}{x_\gamma}\ln\frac{Q^2}{m_\ell^2}$$



[C. F. von Weizsacker, Z. Phys. 88, 612 (1934)]

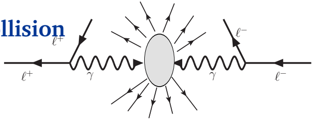
[E. J. Williams, Phys. Rev. 45, 729 (1934)]

- At lepton colliders

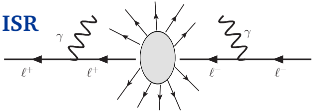
$$\sigma(\ell^+\ell^- \to F + X) = \int_{\tau_0}^{1} d\tau \sum_{ij} \frac{d\mathcal{L}_{ij}}{d\tau}\,\hat{\sigma}(ij \to F),\ \tau = \hat{s}/s$$

$$\frac{d\mathcal{L}_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}}\int_{\tau}^{1}\frac{d\xi}{\xi}\left[f_i(\xi, Q^2) f_j\left(\frac{\tau}{\xi}, Q^2\right) + (i \leftrightarrow j)\right]$$

$\gamma\gamma$ **collision**



ISR



EW physics at high energies

- At high energies, every particle become massless

$$\frac{v}{E} : \frac{v}{100\,\mathrm{TeV}} \sim \frac{\Lambda_{\mathrm{QCD}}}{100\,\mathrm{GeV}}, \frac{v}{E}, \frac{m_t}{E}, \frac{M_W}{E} \to 0!$$

- The splitting phenomena dominate due to large log enhancement
- The EW symmetry is restored: $SU(2)_L \times U(1)_Y$ unbroken
- Goldstone Boson Equivalence:

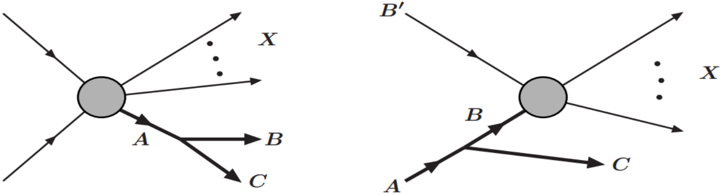
$$\epsilon_L^{\mu}(k) = \frac{E}{M_W}(\beta_W, \hat{k}) \simeq \frac{k^{\mu}}{M_W} + \mathcal{O}(\frac{M_W}{E})$$

  The violation terms is power counted as $v/E \to$ QCD higher twist effects Λ_{QCD}/Q

  [Cuomo, Wulzer, 1703.08562; 1911.12366].

- We mainly focus on the splitting phenomena, which can be factorized and resummed as the EW PDFs in the ISR, and the Fragementaions/Parton Shower in the FSR.
- Other interesting aspects: the polarized EW boson scattering, top-Yukawa coupling effect

Splitting phenomena



$$d\sigma_{X,BC} \simeq d\sigma_{X,A} \times d\mathcal{P}_{A\to B+C}, \quad E_B \approx zE_A,\ E_C \approx \bar{z}E_A,\ k_T \approx z\bar{z}E_A\theta_{BC}$$

$$\frac{d\mathcal{P}_{A\to B+C}}{dz dk_T^2} \simeq \frac{1}{16\pi^2}\frac{z\bar{z}|\mathcal{M}^{(\text{split})}|^2}{(k_T^2 + \bar{z}m_B^2 + zm_C^2 - z\bar{z}m_A^2)^2}, \quad \bar{z} = 1 - z$$

- On the dimensional ground: $|\mathcal{M}^{(\text{split})}|^2 \sim k_T^2$ or m^2
- Integrating out the k_T ends up with $\alpha_W \log\left(Q^2/M_V^2\right) P_{A\to B+C}$

[Ciafaloni et al., hep-ph/0004071; 0007096; Bauer, Ferland, Webber et al., arXiv:1703.08562;1808.08831; Manohar et al., 1803.06347; Han, Chen & Tweedie, arXiv:1611.00788]

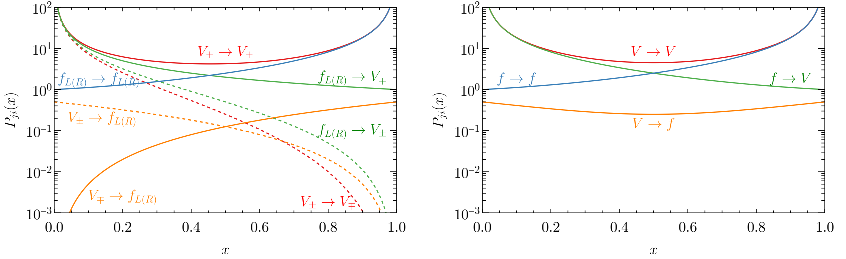
Polarizations in the EW splittings

- The EW splittings must be polarized due to the chiral nature of the EW theory

$$f_{V_+/A_+} \neq f_{V_-/A_-}, \quad f_{V_+/A_-} \neq f_{V_-/A_+},$$
$$\hat{\sigma}(V_+B_+) \neq \hat{\sigma}(V_-B_-), \quad \hat{\sigma}(V_+B_-) \neq \hat{\sigma}(V_-B_+)$$

We are not able to factorize the cross sections in an unporlarized form

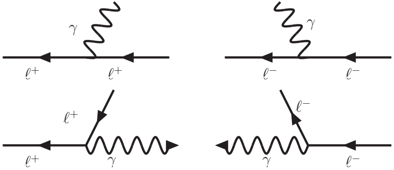
$$\sigma \neq f_{V/A} \otimes \hat{\sigma}(VB), \;\; f_{V/A} = \frac{1}{2}\sum\nolimits_{\lambda,s_1} f_{V_\lambda/A_{s_1}}, \quad \hat{\sigma} = \overline{\sum}_{\lambda,s_2} \hat{\sigma}(V_\lambda B_{s_2})$$



People have been doing:

- $\ell^+\ell^-$ annihilation

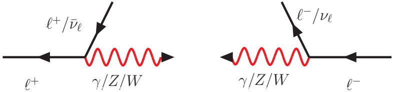
- EPA and ISR



- “Effective W Approx.” (EWA)

[G. Kane, W. Repko, and W. Rolnick, PLB 148 (1984) 367]

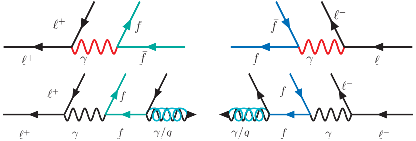
[S. Dawson, NPB 249 (1985) 42]



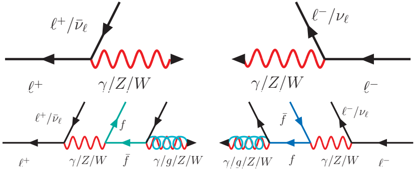
We will add [T. Han, Y. Ma, K.Xie 2007.14300, 2103.09844]

[F. Garosi, D. Marzocca, S. Trifinopoulos 2303.16964]

- Above μ_{QCD}: QED⊗QCD
  q/g emerge



- Above $\mu_{\text{EW}} = M_Z$: EW⊗QCD
  EW partons / corrections to the above



In the end, everything is parton, i.e. need the full SM PDFs.

The PDF evolution: DGLAP

- The DGLAP equations

$$\frac{\mathrm{d} f_i}{\mathrm{d}\log Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P^I_{ij} \otimes f_j$$

- The initial conditions

$$f_{\ell/\ell}(x, m_\ell^2) = \delta(1-x)$$

- Three regions and two matchings
  - $m_\ell < Q < \mu_{\mathrm{QCD}}$: QED
  - $Q = \mu_{\mathrm{QCD}} \lesssim 1$ GeV: $f_q \propto P_{q\gamma} \otimes f_\gamma, f_g = 0$
  - $\mu_{\mathrm{QCD}} < Q < \mu_{\mathrm{EW}}$: QED⊗QCD
  - $Q = \mu_{\mathrm{EW}} = M_Z$: $f_\nu = f_t = f_W = f_Z = f_{\gamma Z} = 0$
  - $\mu_{\mathrm{EW}} < Q$: EW⊗QCD.

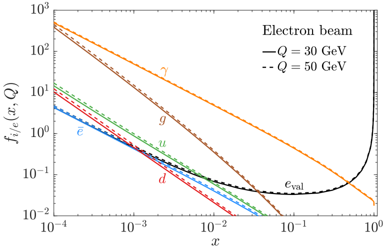
$$\begin{pmatrix} f_B \\ f_{W^3} \\ f_{BW^3} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & -2c_W s_W \\ s_W^2 & c_W^2 & 2c_W s_W \\ c_W s_W & -c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_\gamma \\ f_Z \\ f_{\gamma Z} \end{pmatrix}$$

- We work in the (B, W) basis. The technical details can be referred to the backup slides.

The QED⊗QCD PDFs for lepton colliders

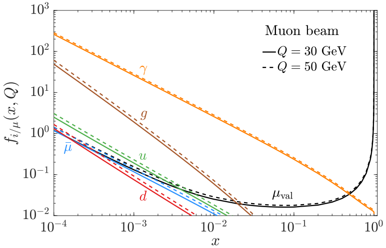
- **Electron PDFs**: $f_{e_{\rm val}},\ f_\gamma,\ f_{\ell_{\rm sea}},\ f_q,\ f_g$
- Scale uncertainty: 10% for $f_{g/e}$
- The averaged momentum fractions $\langle x_i \rangle = \int x f_i(x) \mathrm{d}x$

$Q(e^\pm)$	$e_{\rm val}$	γ	ℓsea	q	g
30 GeV	96.6	3.20	0.069	0.080	0.023
50 GeV	96.5	3.34	0.077	0.087	0.026
M_Z	96.3	3.51	0.085	0.097	0.028



- **Muon PDFs**: $f_{\mu_{\rm val}},\ f_\gamma,\ f_{\ell_{\rm sea}},\ f_q,\ f_g$
- Scale uncertainty: 20% for $f_{g/\mu}$
- The averaged momentum fractions $\langle x_i \rangle = \int x f_i(x) \mathrm{d}x$

$Q(\mu^\pm)$	$\mu_{\rm val}$	γ	ℓsea	q	g
30 GeV	98.2	1.72	0.019	0.024	0.0043
50 GeV	98.0	1.87	0.023	0.029	0.0051
M_Z	97.9	2.06	0.028	0.035	0.0062



The PDFs of a lepton beyond the EW scale

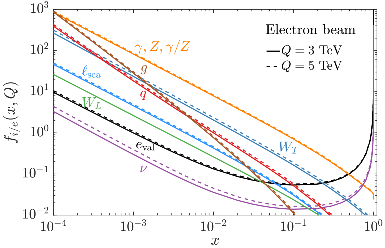
- **All SM particles are partons**

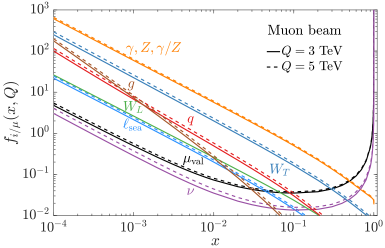
  [T. Han, Y. Ma, K.Xie 2007.14300, 2103.09844]

  - The sea leptonic and quark PDFs show up

    $$\nu = \textstyle\sum_i(\nu_i + \bar{\nu}_i),$$
    $$\ell\text{sea} = \bar{\mu} + \textstyle\sum_{i\neq\mu}(\ell_i + \bar{\ell}_i),$$
    $$q = \textstyle\sum_{i=d}^{t}(q_i + \bar{q}_i)$$

    There is even neutrino due to the EW sector
- W_L does not evolve at the leading order.
- The EW correction is not small: $\sim 50\%$ (100%) for $f_{d/e}$ $(f_{d/\mu})$ due to the relatively **large SU(2) gauge coupling**. [T. Han, Y. Ma, K.Xie 2103.09844]
- Scale uncertainty: $\sim 15\%$ (20%) between $Q = 3$ TeV and $Q = 5$ TeV





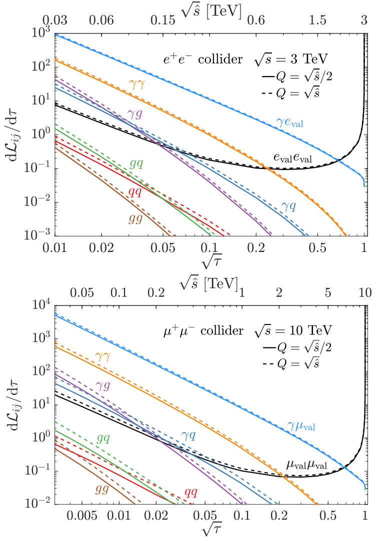
Parton luminosities at high-energy lepton colliders

A 3 TeV e^+e^- machine and a 10 TeV $\mu^+\mu^-$ machine

- Partonic luminosities for

$$\ell^+\ell^-,\ \gamma\ell,\ \gamma\gamma,\ qq,\ \gamma q,\ \gamma g,\ gq,\ \text{and}\ gg$$

- $\gamma\gamma$ gives the largest partonic luminosity
- The luminosity of $\gamma g + \gamma q$ is $\sim 50\%$ (20%) of $\gamma\gamma$
- The luminosities of qq, gq, and gg are $\sim 2\%$ (0.5%) of $\gamma\gamma$
- Given the stronger QCD coupling, **sizable QCD cross sections are expected**.
- Scale uncertainty is $\sim 20\%$ (50%) for photon (gluon) initiated processes.



The SM expectation

[T. Han, YM, K. Xie, 2007.14300, 2103.09844]

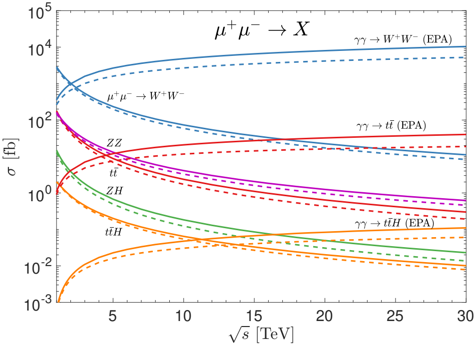
What do we get if the machine is turned on?

- What is the SM physics picture?
- What is the largest background signal?
- Where can we see the possible BSM physics?

Apply EPA at high-energy lepton colliders

A high-energy muon collider at first glance

What do people expect from a high-energy lepton (muon) collider?



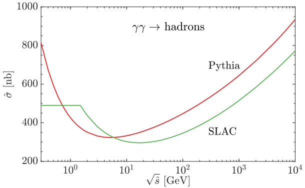
[T. Han, YM, K.Xie 2007.14300]

Some “commonsense”:

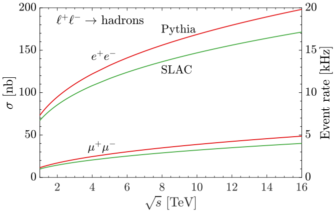
- The annihilations decrease as $1/s$.
- ISR needs to be considered, which can give over 10% enhancement.
- The fusions increase as $\ln^p(s)$, which take over at high energies.
- The large collinear logarithm $\ln(s/m_\ell^2)$ needs to be resummed, set $Q = \sqrt{\hat{s}}/2$,
- $\gamma\gamma \to W^+W^-$ production has the largest cross section.

Photon induced hadronic production at high-energy lepton colliders

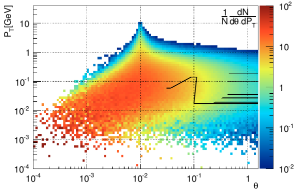
- **Model-dependent $\hat{\sigma}_{\gamma\gamma}$**



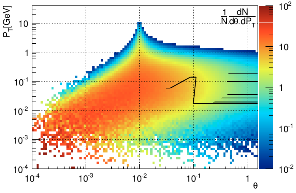
- $\sigma_{\ell\ell}$ may reach nano-barns



- The events populate at low p_T regime



(a) Pythia sample

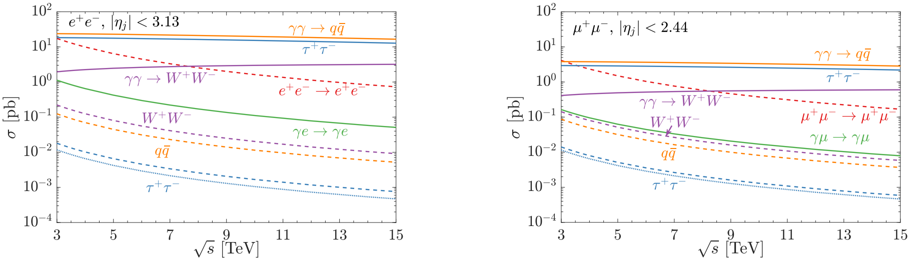


(b) SLAC sample

[T. Barklow, D. Dannheim, M. O. Sahin, and D. Schulte, LCD-2011-020]

What are the dominant processes in the high p_T range?

- Detector angle & Threshold: $\theta_{\text{cut}} = 5^\circ\ (10^\circ) \iff |\eta| < 3.13(2.44)$, $m_{ij} > 20$ GeV
- To separate from the nonperturbative hadronic production: $p_T > \left(4 + \frac{\sqrt{s}}{3\,\text{TeV}}\right)$ GeV
  - Leading-order: $\ell^+\ell^- \to \ell^+\ell^-,\ \tau^+\tau^-,\ q\bar{q},\ W^+W^-$, and $\gamma\ell \to \gamma\ell$
  - $\gamma\gamma$ scatterings: $\gamma\gamma \to \tau^+\tau^-,\ q\bar{q},\ W^+W^-$

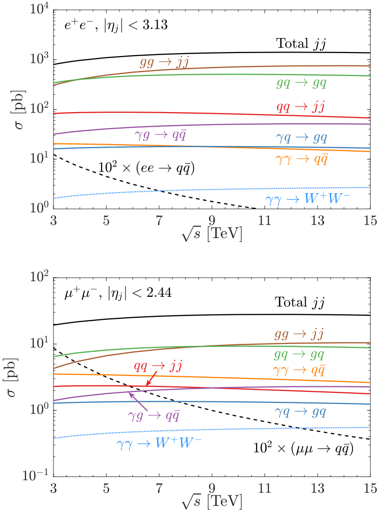


The full background: Di-Jet production at high-energy lepton colliders

- Consider all the “partons”
  $\Rightarrow$ **perturbatively computable processes**

$$\gamma\gamma \to q\bar{q},\ \gamma g \to q\bar{q},\ \gamma q \to gq,$$
$$qq \to qq\,(gg),\ gq \to gq \text{ and } gg \to gg\,(q\bar{q}).$$

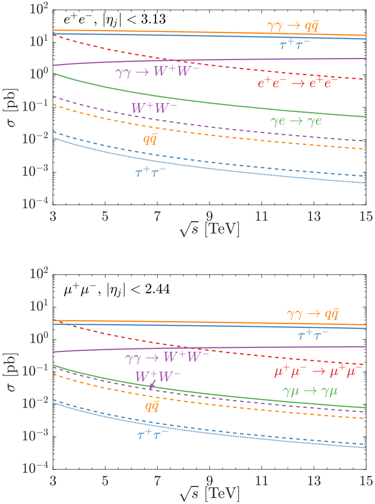
- Large $\alpha_s \ln\left(Q^2\right)$ brings a $6\% \sim 15\%$ ($30\% \sim 40\%$) enhancement if $Q = 2Q$
- The QCD contributions result in total cross section.
- gg initiated cross sections are large for the **multiplicity**
- gq initiated cross sections are large for the **luminosity**.
- $\gamma\gamma$ gives smaller cross sections than the EPA does.



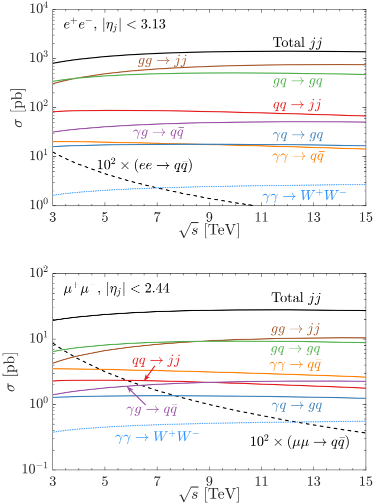
Compare the new with the old

Quark/gluon initiated jet production dominates

Before:



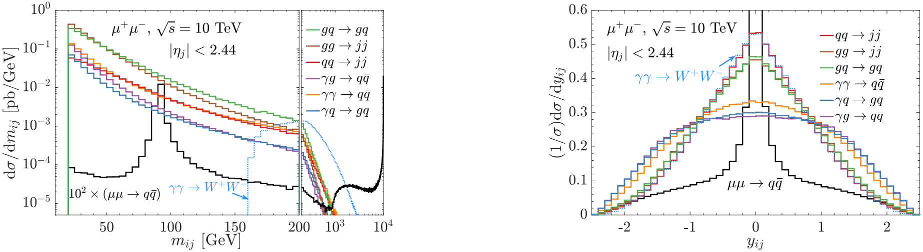
After:



Di-jet distributions at a 10 TeV muon collider

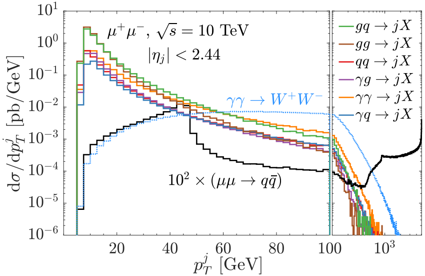
Rather a conservative set up: $\theta = 10^\circ$

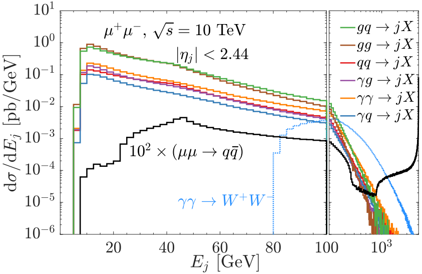
- Some physics:
  Two different mechanisms: $\mu^+\mu^-$ **annihilation** VS **Fusion processes**
  - Annihilation is more than 2 orders of magnitude smaller than fusion process.
  - Annihilation peaks at $m_{ij} \sim \sqrt{s}$;
  - Fusion processes peak near m_{ij} threshold.
  - Annihilation is very central, spread out due to ISR;
  - Fusion processes spread out, especially for γq and γg initiated ones.



Inclusive jet distributions at a 10 TeV muon collider

Important guidelines for future analysis





We expect

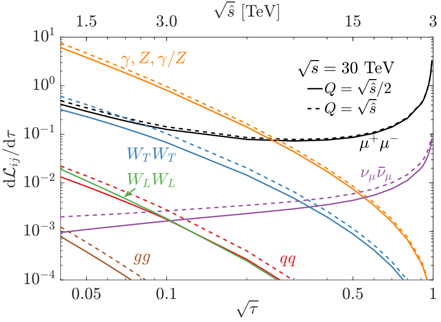
- Jet production dominates over WW production until $p_T > 60$ GeV;
- WW production takes over around energy ~ 200 GeV.

The SM EW sector, as well as any possible BSM, can only be seen in the high p_T (E_j) range.

The full picture a multi-TeV lepton collider: An electroweak Tevatron

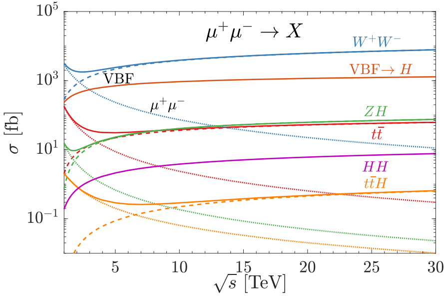
- All SM particles are partons
- We are allowed to determine the partons with their different polarizations

The EW parton luminosities of a 30 TeV muon collider



Just like in hadronic collisions:

$\mu^+\mu^- \to$ exclusive particles + remnants



Compare the "EW Tevatron" with LHC

pp VS $\mu\mu$

$$\mathcal{L}_{W^+_{\lambda_1}W^-_{\lambda_2}} = \int_\tau^1 \frac{\mathrm{d}\xi}{\xi} f_{W_{\lambda_1}}(\xi, \mu_f) f_{W_{\lambda_1}}(\frac{\tau}{\xi}, \mu_f)$$

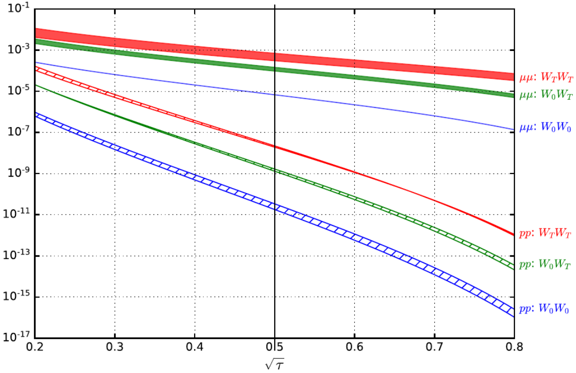
Consider the two colliders in the same ring

$$\sqrt{s}_{\mu\mu} = \sqrt{s}_{pp}$$

For $2 \to 1$ processes, take a benchmark

$$\sqrt{\tau} = \frac{M}{\sqrt{s}} = \frac{1}{2}$$

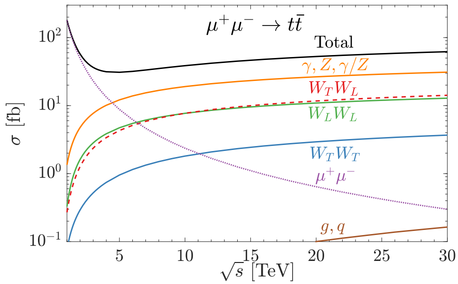
The ratio $\mu\mu/pp$ is larger than 10^4!

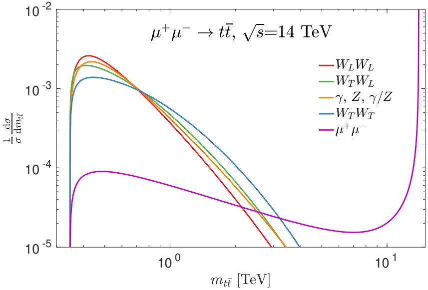


[2005.10289]

One example: $t\bar{t}$ production at a future muon collider

- Two different mechanisms: **Annihilation** and **Fusion**
- The VBF processes exceed the $\mu^+\mu^-$ annihilation at high energies
- The EW PDF formalism allows to determine different partons/polarizations
- The resummation effects lie in the tails.

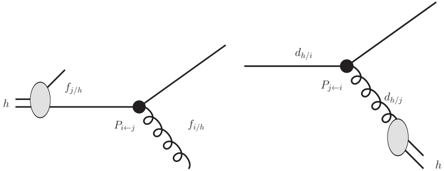




The radiation picture

[T. Han, YM, K. Xie, 2203.11129]

Final state radiation: Electroweak Fragmentations



- Backward evolution $\frac{\mathrm{d}d_i}{\mathrm{d}\log Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j d_j \otimes P^I_{ji}$
- Initial conditions for fermions [Bauer et al. 1806.10157, Han, Ma, KX, 2203.11129]

$$d^f_{f_L}(x,Q_0^2) = d^f_{f_R}(x,Q_0^2) = \delta(1-x),\;\; d^\nu_{\nu_L}(x,Q_0^2) = \delta(1-x),\; d^f_i = 0 \text{ for } i \neq f.$$

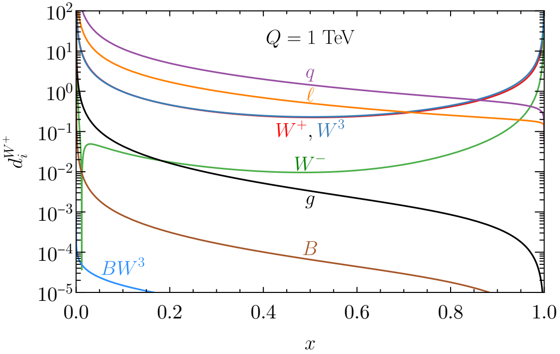
vector bosons

$$d^V_{V_+}(x,Q_0^2) = d^V_{V_-}(x,Q_0^2) = \delta(1-x),\; d^V_i(x,Q_0^2) = 0 \text{ for } i \neq V.$$

- We take the same techniques developed for PDF evolution. See backup slides for details.
- The DGLAP evolution resumms the EW logarithms, which is equivalent to the Sudakov in showering.

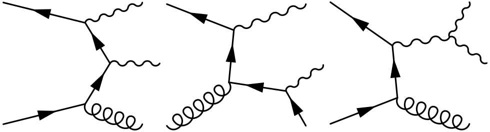
An example: a final-state W^+

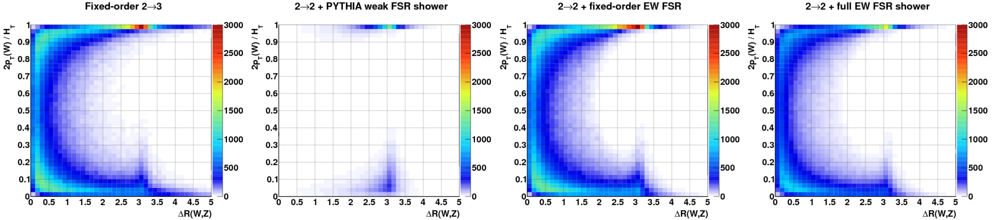
- Collinear splittings also happen to energetic final state particles $\Rightarrow$ **EW jets**
- Electroweak fragmentation function (EW FF) $d_i^{W^+}$, defined as the probability of finding a W^+ in the mother particle i (i.e., $i \to W$) [Han, Ma, KX, 2203.11129]



The parton shower approach can be found in [Han et al. 1611.00788, Herwig 2108.10817]

Applications: $pp \to WZj$



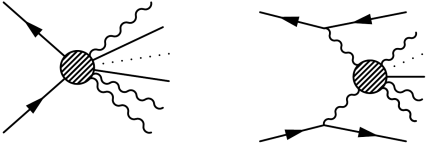


[Han et al. 1611.00788]

- $(\Delta R_{WZ}, 2p_T^W/H_T) \sim (\pi, 0), (\pi, 1)$ due to the back-to-back of W, Z
- $\Delta R_{WZ} \sim 0$ due to $W \to WZ$ splitting
- The systematic comparison between fragmentation and parton shower is still ongoing.

Muon collider is special

An electroweak Tevatron is amazing



- $\ell^+\ell^-$ annihilation **probes TeV scale directly**
- VBF **scans the new physics hints in the full spectrum**
  From the threshold to up to 2 orders of magnitude above EW scale.
- It produces a lot of H, top quarks, W/Z, ... as **a "factory" for SM precision test**
- **An "EW jet factory"**
  In addition to QCD jets, there are W/Z jet, H jet, t jet, neutrino jet, $\cdots$
  Even neutrino collision is not impossible!

Challenges:

Be careful about the radiations!

EW NLO shall be necessary, just like the NLO QCD at LHC.

Our first time to play with beams in another flavor

One example in precision physics: The Muon-Higgs Coupling

[T. Han, W. Kilian, N. Kreher, YM, T. Striegl, J. Reuter, and K. Xie, 2108.05362]

[E. Celada, T. Han, W. Kilian, N. Kreher, YM, F. Maltoni, D. Pagani, T. Striegl, J. Reuter, and K. Xie, in progress]

- Physics: We actually do not know whether the SM mass-generation mechanism applies just to the heavy particles, or also to the 1st/2nd generations.
- Logical possibility: Muon mass not (only) generated by SM Higgs.
  $\Rightarrow$ **Why not have an arbitrary Yukawa coupling?**

HEFT in the unitary gauge: the extended κ framework

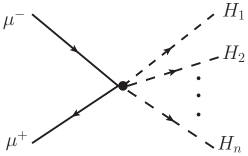
Introduce the form factors α_n, β_n

$$y_{\mu,n} = \frac{\sqrt{2}m_\mu}{v}\alpha_n, \quad f_{V,n} = \beta_n \lambda$$

In the unitary gauge, the HEFT formalism can be simplified to

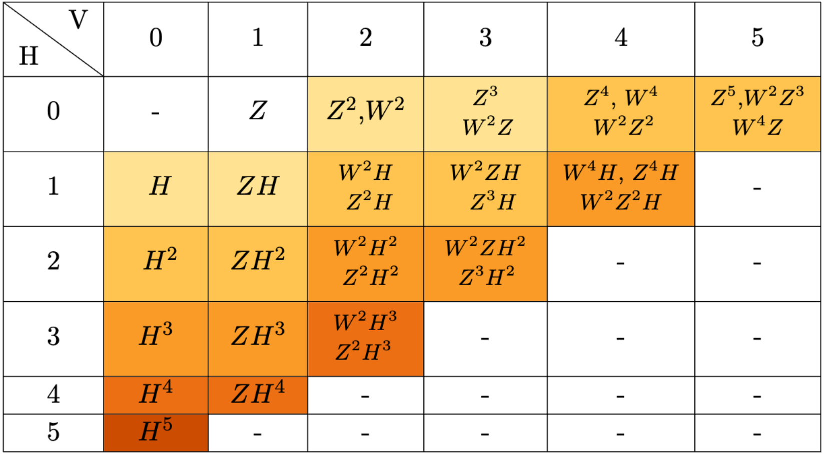
$$\mathcal{L} \supset -\frac{m_H^2}{2}H^2 - m_\mu \bar{\mu}\mu - \sum_{n=3}^{\infty} \beta_n \frac{\lambda}{v^{n-4}} H^n - \sum_{n=1}^{\infty} \alpha_n \frac{m_\mu}{v^n} H^n \bar{\mu}\mu$$

The regular "κ framework" is extended to include more vertices



$$= \frac{n!\alpha_n m_\mu}{v^n}, \quad \alpha_1 = \kappa_\mu$$

Processes in consideration



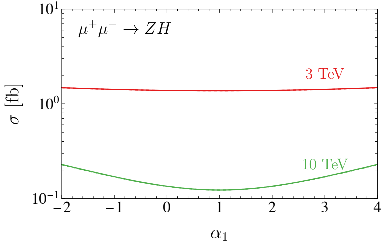
V H	0	1	2	3	4	5
0	-	Z	Z^2,W^2	Z^3 W^2Z	Z^4, W^4 W^2Z^2	Z^5,W^2Z^3 W^4Z
1	H	ZH	W^2H Z^2H	W^2ZH Z^3H	W^4H, Z^4H W^2Z^2H	-
2	H^2	ZH^2	W^2H^2 Z^2H^2	W^2ZH^2 Z^3H^2	-	-
3	H^3	ZH^3	W^2H^3 Z^2H^3	-	-	-
4	H^4	ZH^4	-	-	-	-
5	H^5	-	-	-	-	-

[E. Celada, T.Han, W.Kilian, N. Kreher, YM, F. Maltoni, D. Pagani, J. Reuter, T. Striegl, and K.Xie, coming out soon]

Measure the $\mu\mu H$ vertex

There are processes that depend on only α_1: $\mu^+\mu^- \to ZH$ and $\mu^+\mu^- \to V^3$

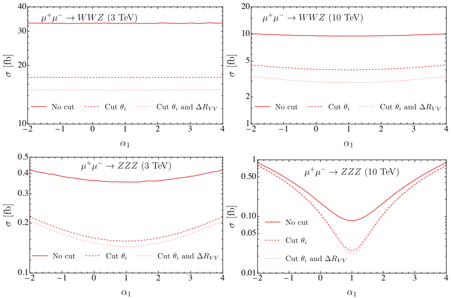
- ZH production



- Sign of Yukawa: $\alpha_1 = 1$ VS $\alpha_1 = -1$

$$\mathcal{S}^{\pm}_{3\,\mathrm{TeV}} = 1.23, \quad \mathcal{S}^{\pm}_{10\,\mathrm{TeV}} = 11.8$$
$$\mathcal{S}_{10\,\mathrm{TeV}} = 2.09, \quad |\Delta\alpha_1| \leq 0.8$$

- V^3 production

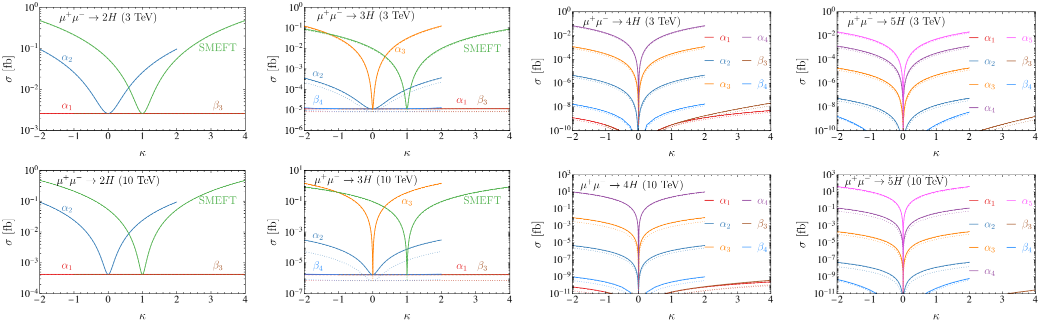


- Measure α_1 more precisely

$$\mathcal{S}^{WWZ}_{10\,\mathrm{TeV}} = 2.1, \quad |\Delta\alpha_1| \leq 0.8$$
$$\mathcal{S}^{ZZZ}_{10\,\mathrm{TeV}} = 2.2, \quad |\Delta\alpha_1| \leq 0.2.$$

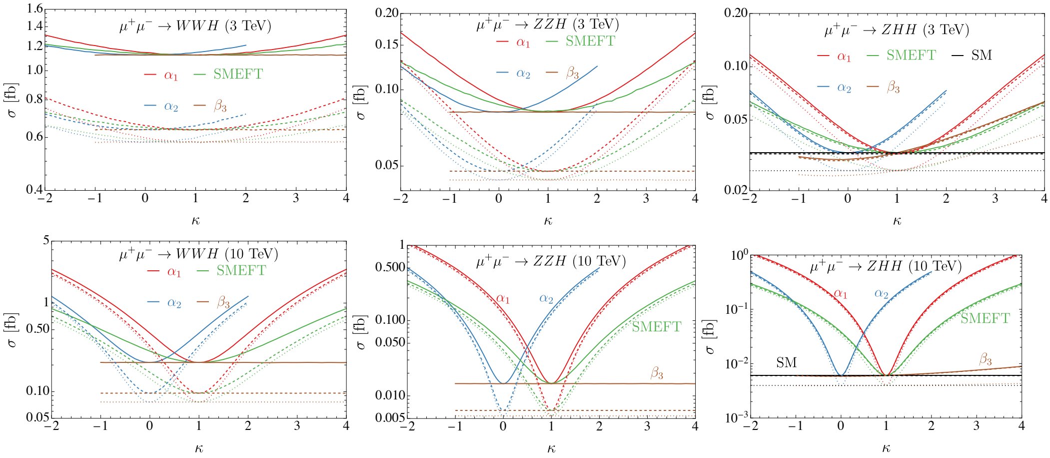
Multi-Higgs production processes: $\mu^+\mu^- \to H^n$

The cross section is solely dependent on α_n $\Rightarrow$ Measure α_n directly

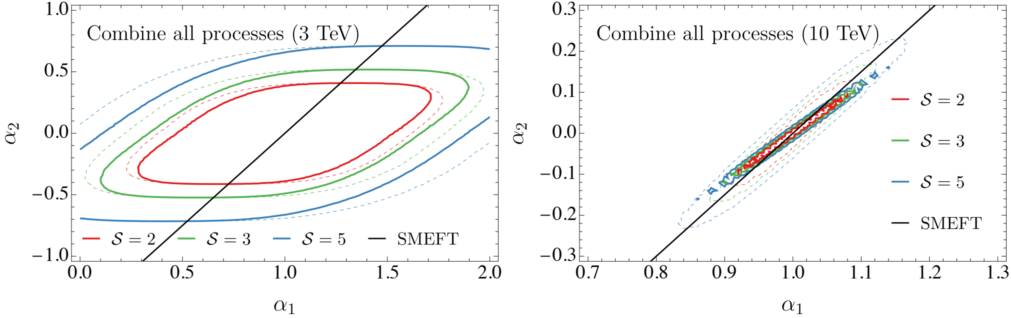


Other processes: constrain $(\alpha_1,\ \alpha_2)$ simultaneously

Example: WWH, ZZH, ZHH



Constrains on $(\alpha_1,\ \alpha_2)$



Summary and prospects

A multi TeV lepton collider is an EW version of Tevatron - full of physics opportunities

- It combines the advantages of proton and of e^+e^- colliders
- It is an amazing precision tool but also can be discovery machine

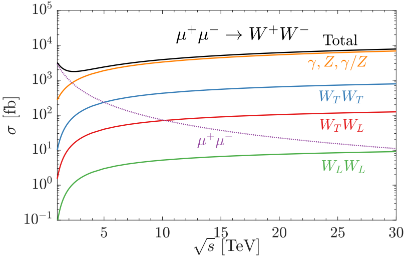
The EW partonic/jet picture - everything is parton, everything is jet

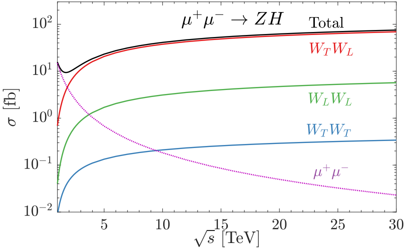
- The scale is far above the EW scale, all the SM particles are “partons”
- The EW PDF formalism allows to determine the polarization of the partons
  The PDF will be included in the main stream event generators soon!
- The FSR will be dealt with using EW shower/ EW fragmentation functions
- The main background is the (soft) jet production

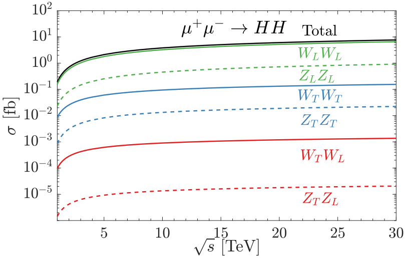
Muon collider is even more special

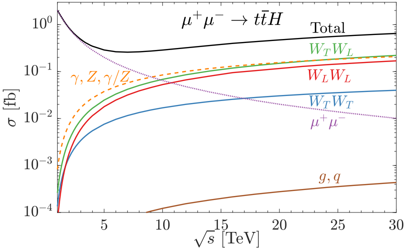
- For the first time we are considering to collide a different flavor
- The muon Yukawa coupling can be measured directly

Other processes: $W^+W^-, ZH, HH, t\bar{t}H$









Solving the DGLAP: Singlet and Non-singlet PDFs

The singlets

$$f_L = \sum_{i=e,\mu,\tau} (f_{\ell_i} + f_{\bar{\ell}_i}),\ f_U = \sum_{i=u,c} (f_{u_i} + f_{\bar{u}_i}),\ f_D = \sum_{i=d,s,b} (f_{d_i} + f_{\bar{d}_i})$$

The non-singlets

- The only non-trivial singlet $f_{e,NS} = f_e - f_{\bar{e}}$
- the leptons $f_{\ell_i,NS} = f_{\ell_i} - f_{\bar{\ell}_i}(i = 2, 3),\ f_{\ell,12} = f_{\bar{e}} - f_{\bar{\mu}},\ f_{\ell,13} = f_{\bar{e}} - f_{\bar{\tau}};$
- the up-type quarks $f_{u_i,NS} = f_{u_i} - f_{\bar{u}_i},\ f_{u,12} = f_u - f_c;$
- and the down-type quarks $f_{d_i,NS} = f_{d_i} - f_{\bar{d}_i},\ f_{d,12} = f_d - f_s,\ f_{d,13} = f_d - f_b.$

Reconstruction:

$$f_e = \frac{f_L + (2N_\ell - 1) f_{e,NS}}{2N_\ell},\ f_{\bar{e}} = f_\mu = f_{\bar{\mu}} = f_\tau = f_{\bar{\tau}} = \frac{f_L - f_{e,NS}}{2N_\ell}.$$

$$f_u = f_{\bar{u}} = f_c = f_{\bar{c}} = \frac{f_U}{2N_u},\ f_d = f_{\bar{d}} = f_s = f_{\bar{s}} = f_b = f_{\bar{b}} = \frac{f_D}{2N_d}.$$

The QED⊗QCD case

- The singlets and gauge bosons

$$\frac{\mathrm{d}}{\mathrm{d}\log Q^2}\begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix} = \begin{pmatrix} P_{\ell\ell} & 0 & 0 & 2N_\ell P_{\ell\gamma} & 0 \\ 0 & P_{uu} & 0 & 2N_u P_{u\gamma} & 2N_u P_{ug} \\ 0 & 0 & P_{dd} & 2N_d P_{d\gamma} & 2N_d P_{dg} \\ P_{\gamma\ell} & P_{\gamma u} & P_{\gamma d} & P_{\gamma\gamma} & 0 \\ 0 & P_{gu} & P_{gd} & 0 & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix}$$

- The non-singlets

$$\frac{\mathrm{d}}{\mathrm{d}\log Q^2} f_{NS} = P_{ff} \otimes f_{NS}.$$

- The averaged momentum fractions of the PDFs: $f_{\ell_{\text{val}}},\ f_\gamma,\ f_{\ell_{\text{sea}}},\ f_q,\ f_g$

$$\langle x_i \rangle = \int x f_i(x)\mathrm{d}x,\ \sum_i \langle x_i \rangle = 1$$

$$\frac{\langle x_q \rangle}{\langle x_{\ell\text{sea}} \rangle} \lesssim \frac{N_c\left[\sum_i (e_{u_i}^2 + e_{\bar{u}_i}^2) + \sum_i (e_{d_i}^2 + e_{\bar{d}_i}^2)\right]}{e_{\ell_{\text{val}}}^2 + \sum_{i\neq\ell\text{val}} (e_{\ell_i}^2 + e_{\bar{\ell}_i}^2)} = \frac{22/3}{5}$$

The DGLAP for the full SM

$$\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} f_L^{0\pm} \\ f_Q^{0\pm} \\ f_E^{0\pm} \\ f_U^{0\pm} \\ f_D^{0\pm} \\ f_B^{0\pm} \\ f_W^{0\pm} \\ f_g^{0\pm} \end{pmatrix} = \begin{pmatrix} P_{LL}^{0\pm} & 0 & 0 & 0 & 0 & P_{LB}^{0\pm} & P_{LW}^{0\pm} & 0 \\ 0 & P_{QQ}^{0\pm} & 0 & 0 & 0 & P_{QB}^{0\pm} & P_{QW}^{0\pm} & P_{Qg}^{0\pm} \\ 0 & 0 & P_{EE}^{0\pm} & 0 & 0 & P_{EB}^{0\pm} & 0 & 0 \\ 0 & 0 & 0 & P_{UU}^{0\pm} & 0 & P_{UB}^{0\pm} & 0 & P_{Ug}^{0\pm} \\ 0 & 0 & 0 & 0 & P_{DD}^{0\pm} & P_{DB}^{0\pm} & 0 & P_{Dg}^{0\pm} \\ P_{BL}^{0\pm} & P_{BQ}^{0\pm} & P_{BE}^{0\pm} & P_{BU}^{0\pm} & P_{BD}^{0\pm} & P_{BB}^{0\pm} & 0 & 0 \\ P_{WL}^{0\pm} & P_{WQ}^{0\pm} & 0 & 0 & 0 & 0 & P_{WW}^{0\pm} & 0 \\ 0 & P_{gQ}^{0\pm} & 0 & P_{gU}^{0\pm} & P_{gD}^{0\pm} & 0 & 0 & P_{gg}^{0\pm} \end{pmatrix} \otimes \begin{pmatrix} f_L^{0\pm} \\ f_Q^{0\pm} \\ f_E^{0\pm} \\ f_U^{0\pm} \\ f_D^{0\pm} \\ f_B^{0\pm} \\ f_W^{0\pm} \\ f_g^{0\pm} \end{pmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} f_L^{1\pm} \\ f_Q^{1\pm} \\ f_W^{1\pm} \\ f_{BW}^{1\pm} \end{pmatrix} = \begin{pmatrix} P_{LL}^{1\pm} & 0 & P_{LW}^{1\pm} & P_{LM}^{1\pm} \\ 0 & P_{QQ}^{1\pm} & P_{QW}^{1\pm} & P_{QM}^{1\pm} \\ P_{WL}^{1\pm} & P_{WQ}^{1\pm} & P_{WW}^{1\pm} & 0 \\ P_{ML}^{1\pm} & P_{MQ}^{1\pm} & 0 & P_{MM}^{1\pm} \end{pmatrix} \otimes \begin{pmatrix} f_L^{1\pm} \\ f_Q^{1\pm} \\ f_W^{1\pm} \\ f_{BW}^{1\pm} \end{pmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}L} f_W^{2\pm} = P_{WW}^{2\pm} \otimes f_{WW}^{2\pm}$$

The splitting functions can be found in [Chen et al. 1611.00788, Bauer et al. 1703.08562,1808.08831]

EFT parameterizations

- Nonlinear HEFT gives $\kappa_\mu = \frac{v}{\sqrt{2}m_\mu} y_1$ [Coleman et al., PR1969, Weinberg, PLB1980, $\cdots$]

$$\begin{aligned}\mathcal{L}_{UH} =& \frac{v^2}{4}\,\mathrm{Tr}\left[D_\mu U^\dagger D^\mu U\right] F_U(H) + \frac{1}{2}\partial_\mu H \partial^\mu H - V(H) \\ &- \frac{v}{2\sqrt{2}}\left[\bar{\ell}^i_L \tilde{Y}^{ij}_\ell (H) U \left(1-\tau_3\right) \ell^j_R + \text{ h.c.}\right]\end{aligned}$$

with $F_U, V, \tilde{Y}$ expanded as

$$F_U(H) = 1 + \sum_{n\geq 1} f_{U,n}\left(\frac{H}{v}\right)^n, V(H) = v^4 \sum_{n\geq 2} f_{V,n}\left(\frac{H}{v}\right)^n, \tilde{Y}^{ij}_\ell(H) = \sum_{n\geq 0} \tilde{Y}^{ij}_{\ell,n}\left(\frac{H}{v}\right)^n$$

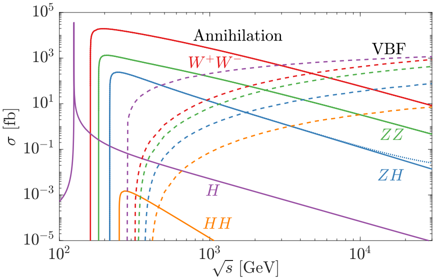
- Linear SMEFT [Weinberg PRL1979, Abbott & Wise PRD1980, $\cdots$]

$$\mathcal{L} \supset -\sum_{n=1}^{\infty} \frac{c_\varphi^{(2n+4)}}{\Lambda^{2n}}\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right)^{n+2} - \sum_{n=1}^{\infty} \frac{c_{\ell\varphi}^{(2n+4)}}{\Lambda^{2n}}\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right)^{n} (\bar{\ell}_L \varphi e_R + \text{ h.c.})$$

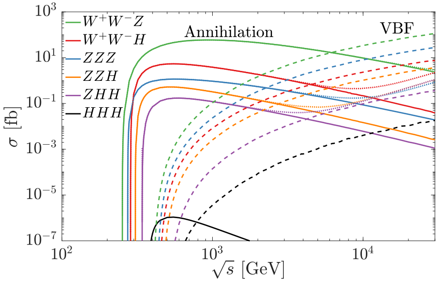
Multi-boson final states and the Muon-Higgs coupling

- **SM**: $\lambda(\mathrm{Muon}-\mathrm{Higgs}) \sim y_\mu^{\mathrm{SM}} = \sqrt{2} m_\mu^{\mathrm{SM}}/v$
- **Possible BSM physics**: $m_\mu = m_\mu^{\mathrm{SM}}$, $\lambda(\mathrm{Muon}-\mathrm{Higgs}) \sim \kappa_\mu y_\mu^{\mathrm{SM}}$, e.g. $\kappa_\mu = 0$

Two-boson final states

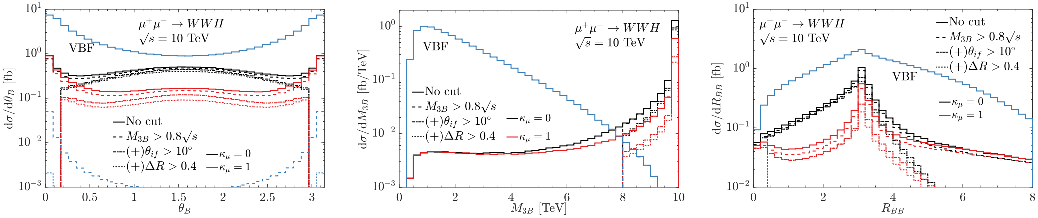


Three-boson final states



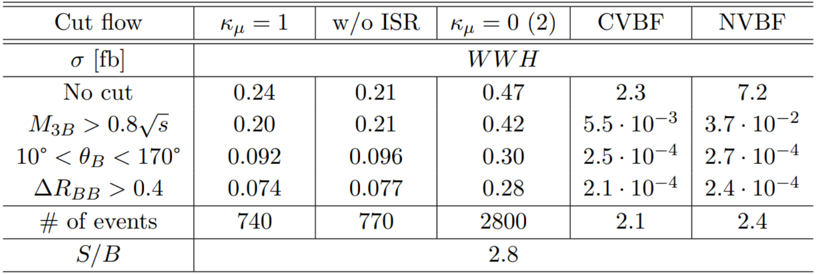
New physics signal shows up in the high energy region

WWH at a 10 TeV muon collider: Kinematics



- Background (VBF) is much larger than signal (annihilation)
- VBF events accumulate around threshold, and mostly forward
- Annihilation in the rest frame (central, and $M \sim \sqrt{s}$ spread by ISR)
- Annihilation also has forward dominance, due to the gauge splitting $W \to WH$

WWH at a 10 TeV muon collider: Cuts

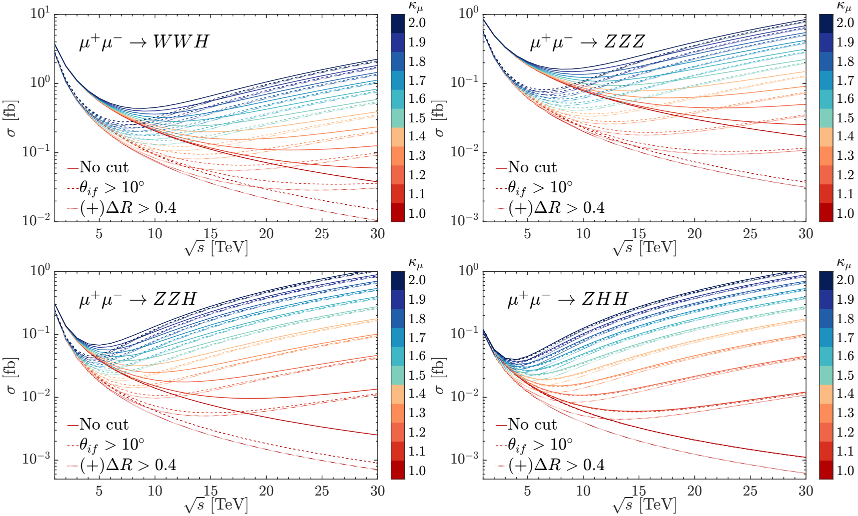


Cut flow	$\kappa_\mu = 1$	w/o ISR	$\kappa_\mu = 0$ (2)	CVBF	NVBF
σ [fb]	WWH				
No cut	0.24	0.21	0.47	2.3	7.2
$M_{3B} > 0.8\sqrt{s}$	0.20	0.21	0.42	$5.5 \cdot 10^{-3}$	$3.7 \cdot 10^{-2}$
$10° < \theta_B < 170°$	0.092	0.096	0.30	$2.5 \cdot 10^{-4}$	$2.7 \cdot 10^{-4}$
$\Delta R_{BB} > 0.4$	0.074	0.077	0.28	$2.1 \cdot 10^{-4}$	$2.4 \cdot 10^{-4}$
# of events	740	770	2800	2.1	2.4
S/B	2.8				

- Integrated luminosity $\mathcal{L} = (\sqrt{s}/10\ \text{TeV})^2 \cdot 10\,\text{ab}^{-1}$ [1901.06150]
- $S = N_{\kappa_\mu} - N_{\kappa_\mu=1},\ B = N_{\kappa_\mu=1} + N_{\text{VBF}}.$
- VBF and ISR are mostly excluded by invariant mass cut.
- Angular cut also weaken VBF further.

Scan triboson over $\sqrt{s}$ and κ_μ

[Han et al., 2108.05362]



Test the muon Yukawa: statistical sensitivity

- The most sensitive channels are ZHH and ZZH, similar probes due to GBET.
- Taking $S = 2$ criterion, we can test the muon-Higgs coupling up to 10% (1%) precision at a 10 (30) TeV muon collider, corresponding to new physics scale $\Lambda_{\mathrm{NP}} \sim 30 - 100$ TeV.