NNU · 南京师范大学
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PPNP 134 (2024) 104094
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PRL 129 (2022) 9, 091802

Electroweak Fits and New Physics

New Wine In Old Bottles （老树新花）

武 雷

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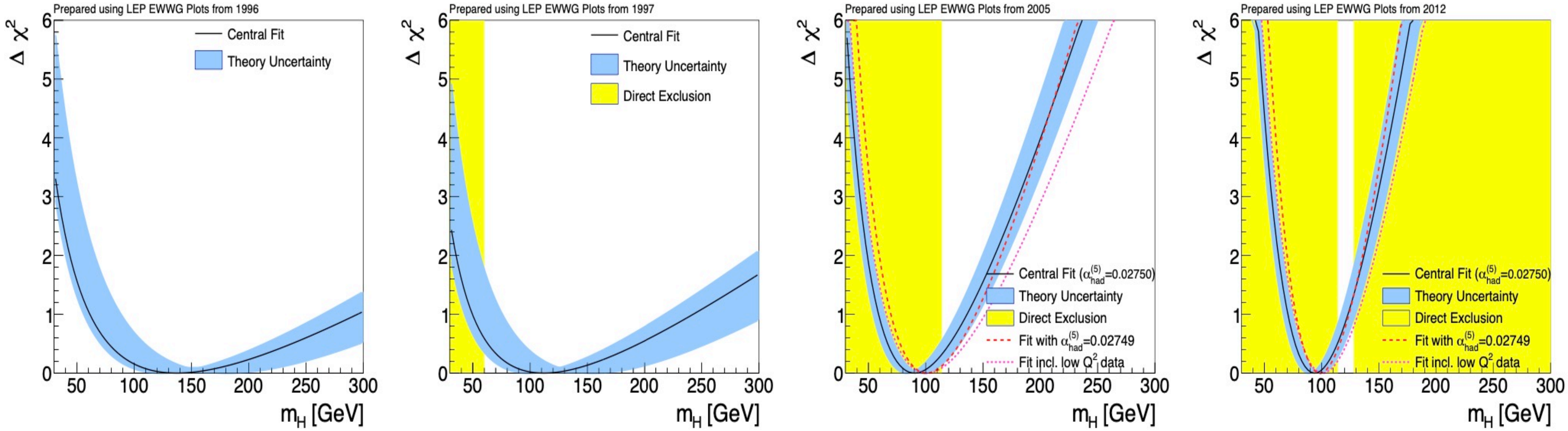
1. Higgs Era

Higgs mass?

0509008

'blue-band' plots



1. Higgs Era

July 4, 2022
CERN

QUANTUM PHYSICS | OPINION

10 Years after the Higgs, Physicists Are Optimistic for More Discoveries

The Large Hadron Collider recently reopened after upgrades and is ready to explore new territory

By Marcela Carena on July 5, 2022

nature

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A detailed m… by the ATLAS… discovery

The ATLAS Collaboration

nature

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Article | Open Access | Published: 04 July 2022

A portr… experi…

The CMS Coll…

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nature > perspectives > article

Perspective | Published: 04 July 2022

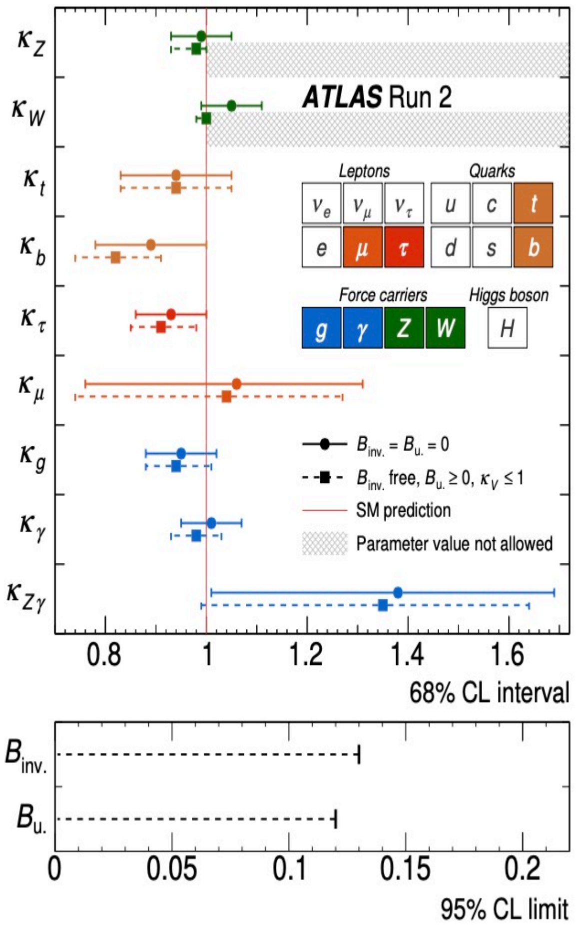
The Higgs boson turns ten

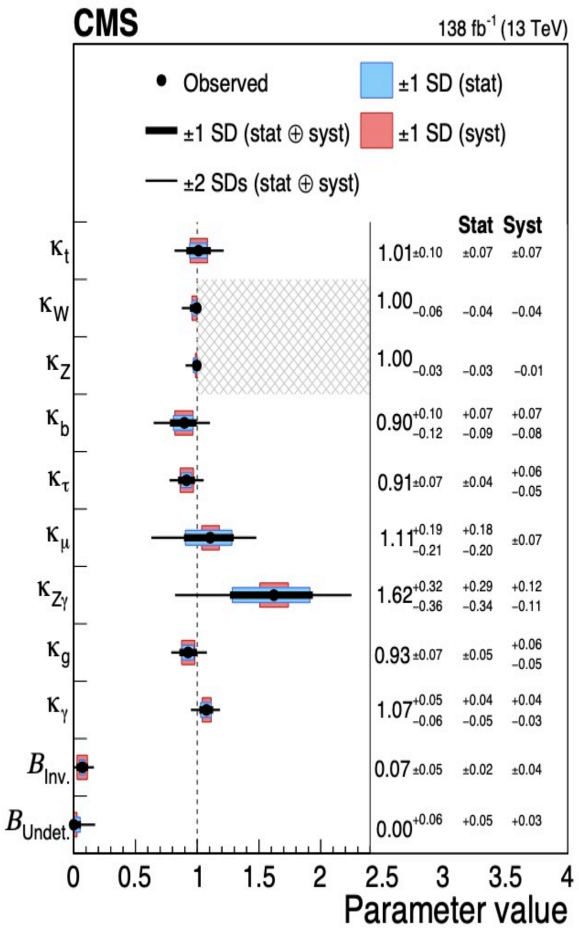
Gavin P. Salam, Lian-Tao Wang & Giulia Zanderighi

Nature **607**, 41–47 (2022) | Cite this article

29k Accesses | **3** Citations | **349** Altmetric | Metrics

1. Higgs Era





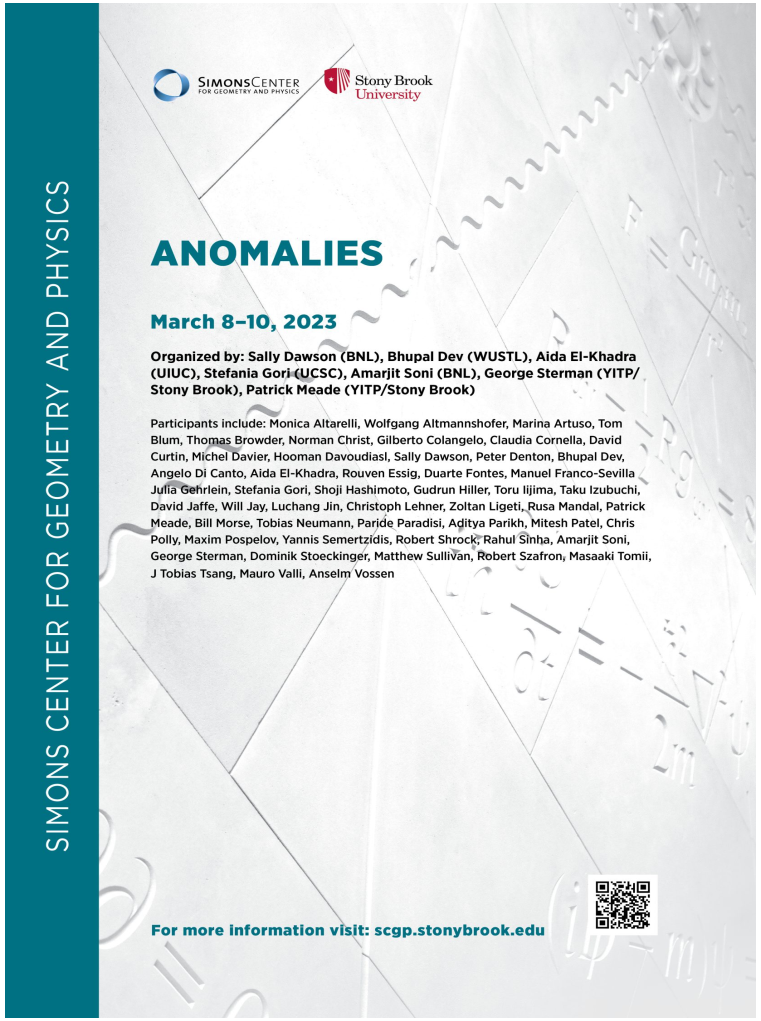
M_h=125 GeV

1902.00134, Cepeda

Process	σ(pb)@14 TeV
ggF (N^{3}LO QCD + NLO EW)	$54.72^{+4.28\%(\text{theory})+1.85\%(\text{PDF})+2.60\%(\alpha_s)}_{-6.46\%(\text{theory})-1.85\%(\text{PDF})+2.62\%(\alpha_s)}$
VBF (NNLO QCD)	$4.260^{+.45\%(\text{scale})+2.1\%(\text{PDF}+\alpha_s)}_{-.34\%(\text{scale})-2.1\%(\text{PDF}+\alpha_s)}$
Wh (NNLO QCD+NLO EW)	$1.498 \pm .51\%(\text{scale}) \pm 1.35\%(\text{PDF}+\alpha_s)$
Zh (NNLO QCD+NLO EW)	$.981^{+3.61\%(\text{scale})+1.90\%(\text{PDF}+\alpha_s)}_{-2.94\%(\text{scale})-1.90\%(\text{PDF}+\alpha_s)}$
$t\bar{t}h$ (NLO QCD + NLO EW)	$.6128^{+6.0\%(\text{scale})+3.5\%(\text{PDF}+\alpha_s)}_{-9.2\%(\text{scale})-3.5\%(\text{PDF}+\alpha_s)}$

Decay	Branching Ratio
$h \to b\bar{b}$	$.582^{+.65\%(\text{Theory})+.72\%(m_q)+.78\%(\alpha_s)}_{-.65\%(\text{Theory})-.74\%(m_q)-.80\%(\alpha_s)}$
$h \to c\bar{c}$	$.02891^{+1.20\%(\text{Theory})+5.26\%(m_q)+1.25\%(\alpha_s)}_{-1.20\%(\text{Theory})-.98\%(m_q)-1.25\%(\alpha_s)}$
$h \to \tau^+\overline{\tau}^-$	$.06272^{+1.17\%(\text{Theory})+.98\%(m_q)+.62\%(\alpha_s)}_{-1.16\%(\text{Theory})-.99\%(m_q)-.62\%(\alpha_s)}$
$h \to \gamma\gamma$	$.00227^{+1.73\%(\text{Theory})+.93\%(m_q)+.61\%(\alpha_s)}_{-1.72\%(\text{Theory})-.99\%(m_q)-.62\%(\alpha_s)}$
$h \to ZZ \to 4l(l = e, \mu, \tau)$	$.0002745 \pm 2.18\%$
$h \to WW \to l^+l^-\nu\overline{\nu}(l = e, \mu, \tau)$	$.02338 \pm 2.18\%$

2. Beyond Higgs



Topics

Light Higgs Boson?

New Resonance?

B anomalies?

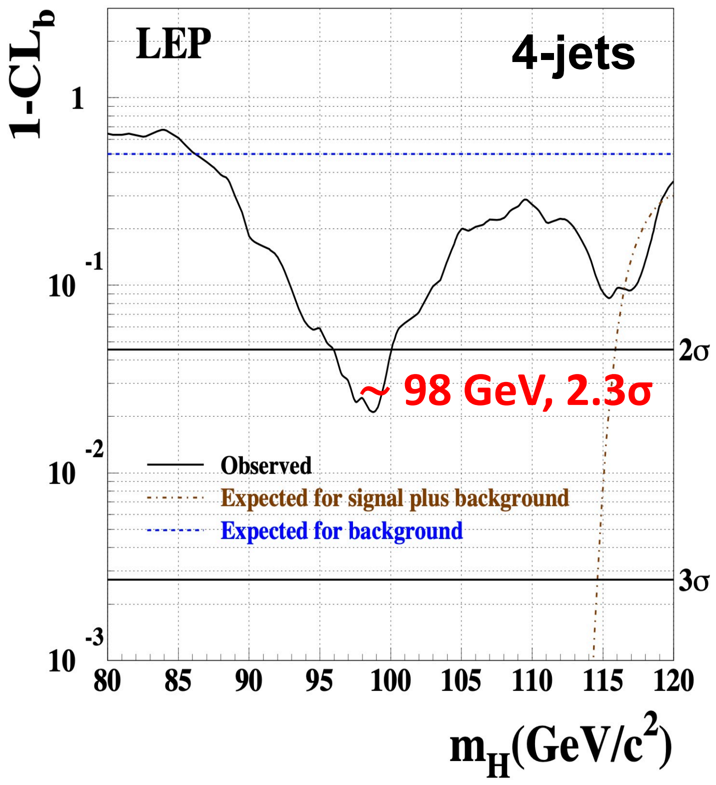
Muon g-2?

CDF-II W mass?

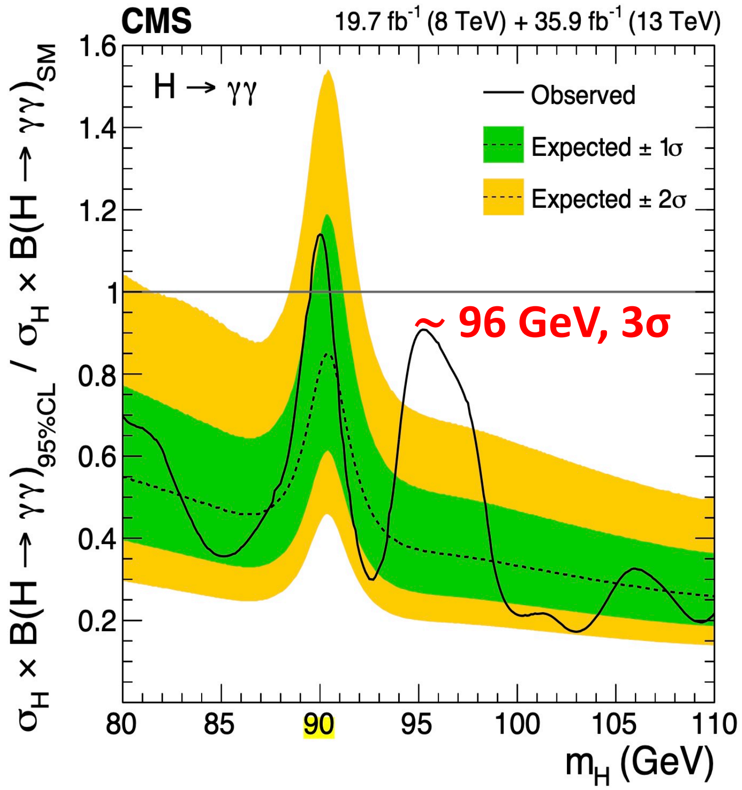
2. Beyond Higgs

"Light Higgs boson? "

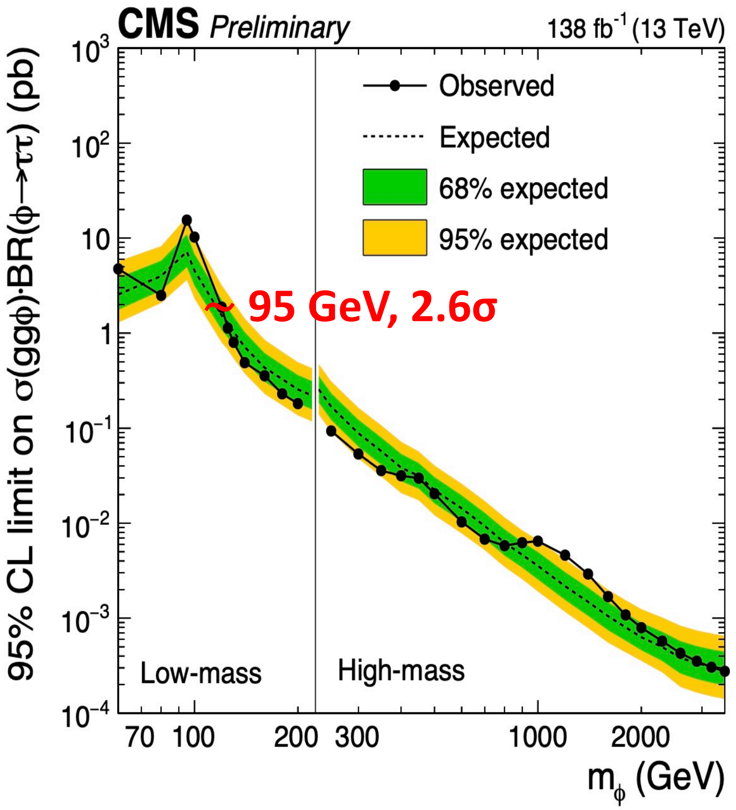
0306033



1811.08459, Sirunyan



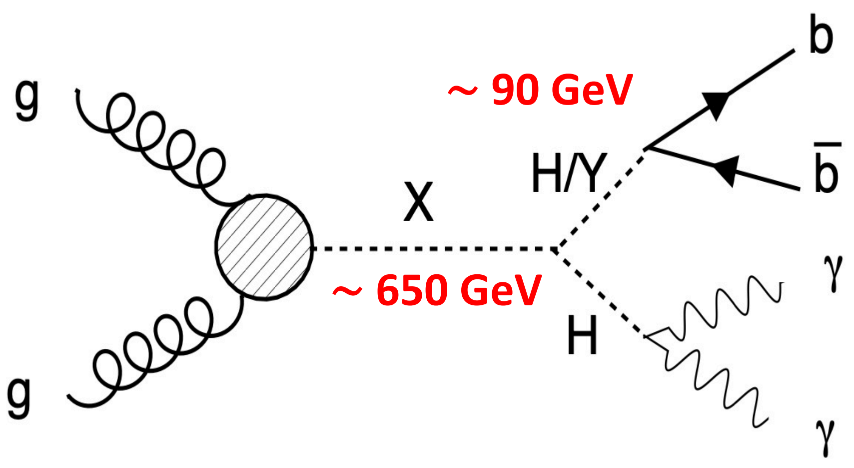
CMS PAS HIG-21-001

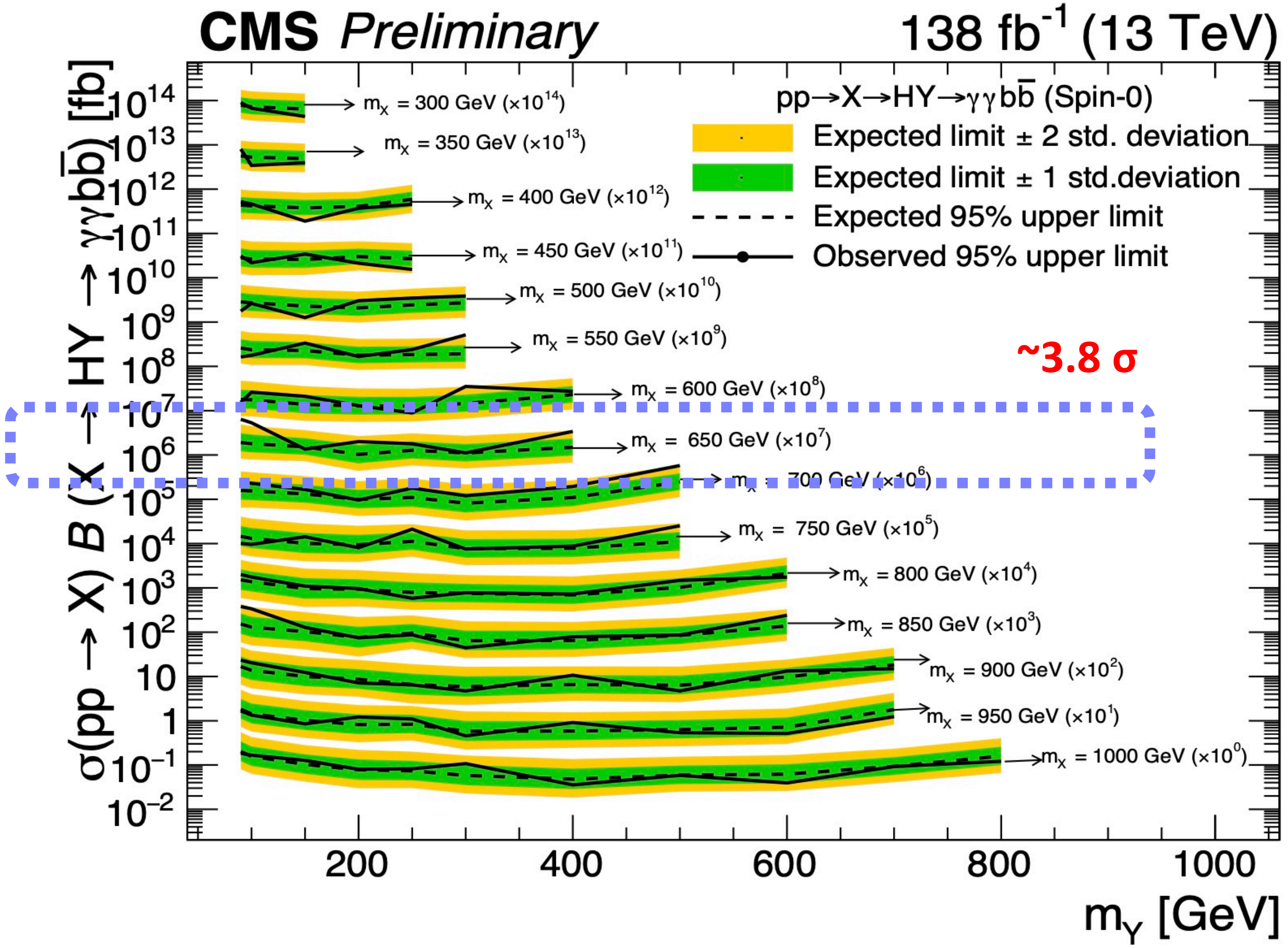


2. Beyond Higgs

“ A new resonance? ”

CMS-PAS-HIG-21-011





2. Beyond Higgs

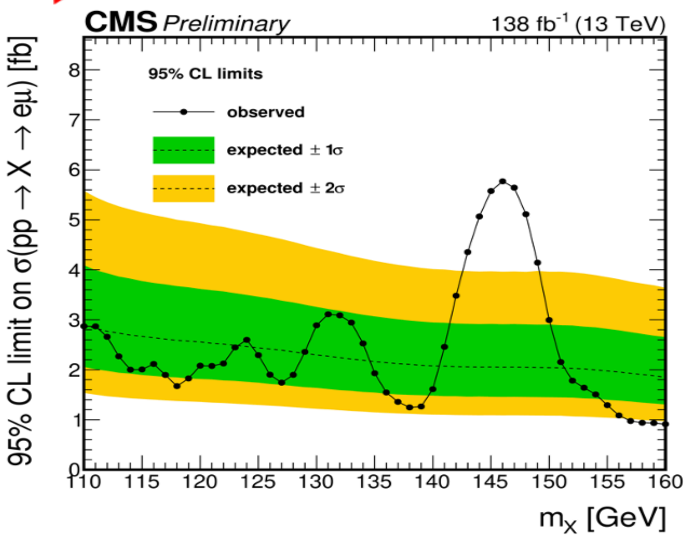
“ A new resonance? ”

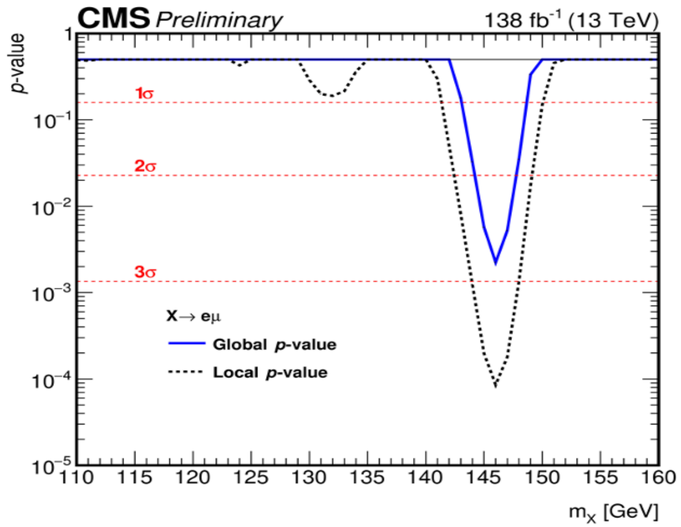
LFV SM (H_{125}) or SM-like Exotic X→eμ (110 GeV<m_H<160 GeV)

New!

CMS-PAS-HIG-22-002

http://cms-results.web.cern.ch/cms-results/public-results/preliminary-results/HIG-22-002/index.html





- Observed & expected 95% CL UL on σ (pp→X→eμ)

- Excess observed for $m_{e\mu}$~146 GeV with global (2.8 σ) and local (3.8σ) significances
- Not visible in ATLAS $m_{e\mu}$ spectrum (PLB 801 (2019) 135148), more data needed to conclude!

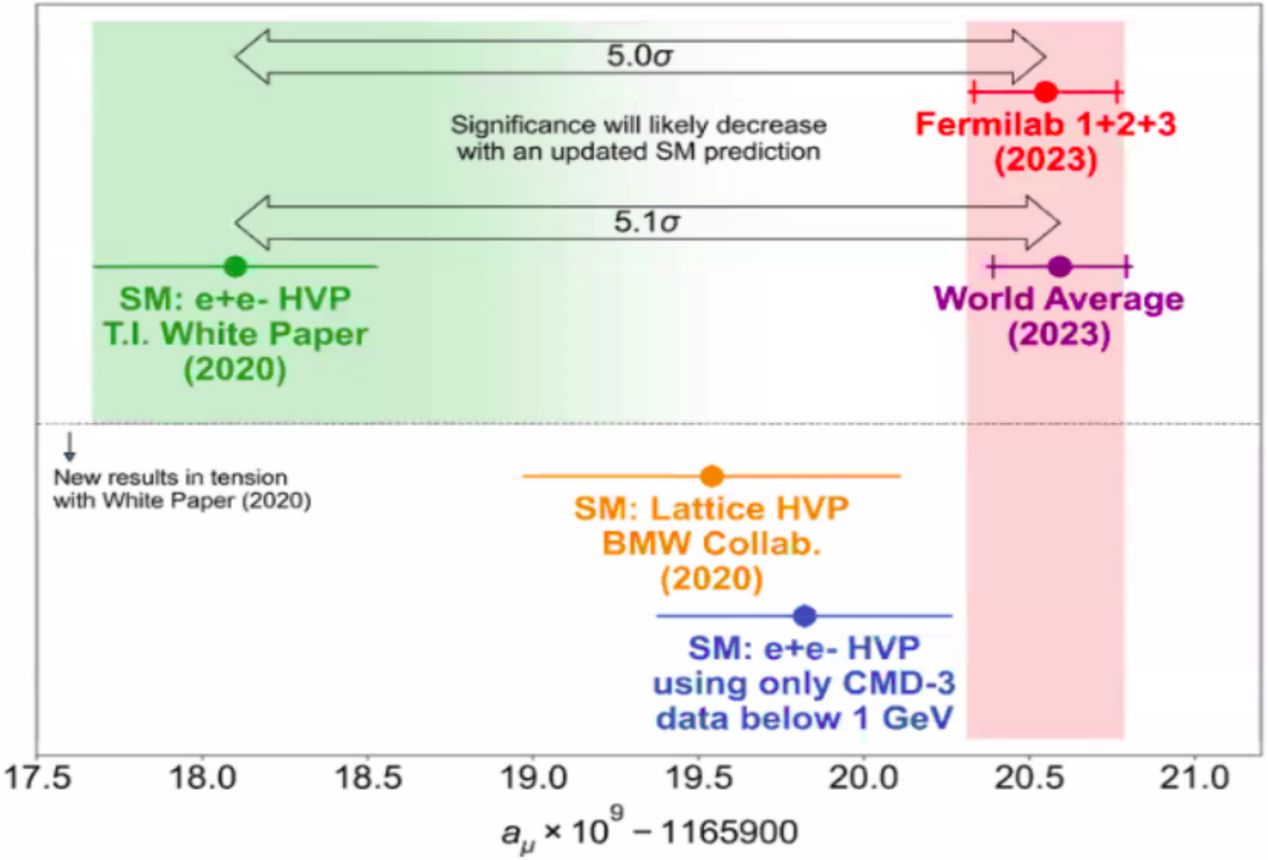
S. Gascon-Shotkin Rencontres de Moriond EW 2023, La Thuile, IT Mar 23 2023 13

2. Beyond Higgs

" Muon g-2? "

10th of August, 2023

https://muon-g-2.fnal.gov/



2. Beyond Higgs

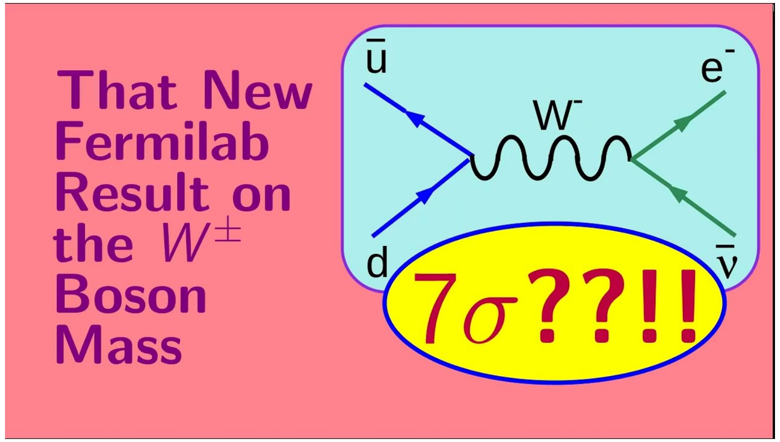
“ Overweight W mass ?”

Science, 2022, CDF-II

$$m_W^{\mathrm{CDF}} = 80.4335 \pm 0.0094\ \mathrm{GeV}$$

$$m_W^{\mathrm{SM}} = 80.357 \pm 0.006\ \mathrm{GeV}$$

April 8, 2022



Questions:

Higher order ✓
PDF ✓
Width ✓
Generator
Detector
New Physics
…

May 6, 2022

A key parameter in the EW fits

[27] arXiv:2205.02788 [pdf, other]

ResBos2 and the CDF W Mass Measurement

Joshua Isaacson, Yao Fu, C.-P. Yuan

Comments: 11 pages, 13 figures

Subjects: **High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Experiment (hep-ex)

The recent CDF W mass measurement of 80,433 ± 9 MeV is the most precise direct measurement. However, this result deviates from the Standard Model predicted mass of 80,359.1 ± 5.2 MeV by 7σ. The CDF experiment used an older version of the ResBos code that was only accurate at NNLL+NLO, while the state-of-the-art ResBos2 code is able to make predictions at N^3LL+NNLO accuracy. We determine that the data-driven techniques used by CDF capture most of the higher order corrections, and using higher order corrections would result in a decrease in the value reported by CDF by at most 10 MeV.

3. W mass

Weak theory

…

1961 ： Glashow, IVB Z boson

1964 ： Salam and Ward, EW and W boson mass

1967 ： Weinberg, EW and W/Z boson masses

…

Properties of IVBs become better specified within the theoretical frame of the unified weak and electro magnetic theory and of the Weinberg–Salam model [2]. The mass of the IVB is precisely predicted [3]:

$M_{W^\pm} = (82 \pm 2.4)\ \mathrm{GeV}/c^2$

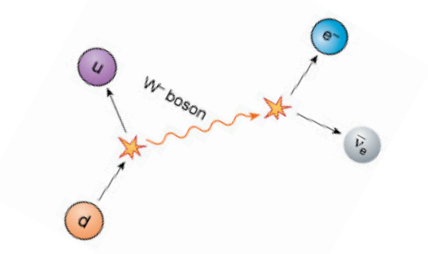
1979 Nobel Prize

Volume 122B, number 1 PHYSICS LETTERS 24 February 1983

EXPERIMENTAL OBSERVATION OF ISOLATED LARGE TRANSVERSE ENERGY ELECTRONS WITH ASSOCIATED MISSING ENERGY AT $\sqrt{s}$ = 540 GeV

UA1 Collaboration, CERN, Geneva, Switzerland

1983, UA1+2 Collab：



The result of a fit on electron angle and energy and neutrino transverse energy with allowance for systematic errors, is

$m_W = (81^{+5}_{-5})\ \mathrm{GeV}/c^2$,

in excellent agreement with the expectation of the Weinberg–Salam model [2].

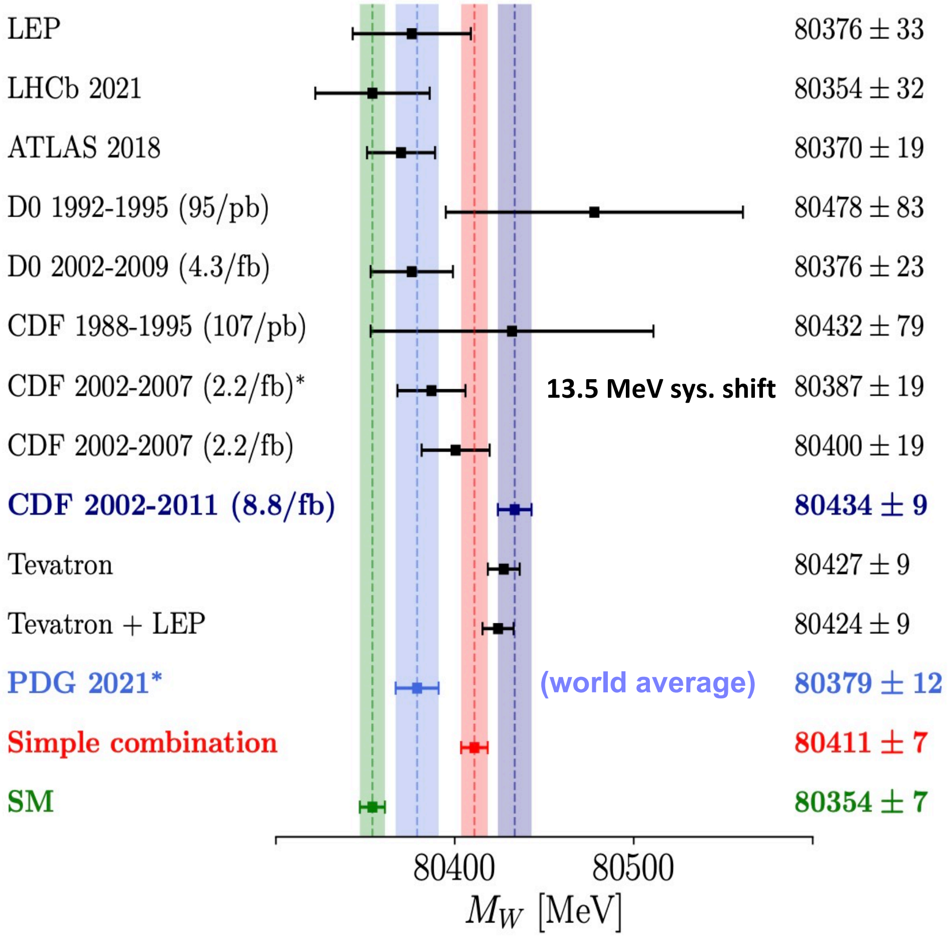
W boson turns 40

Forty years ago today, physicists at CERN announced to the world that they had discovered the electrically charged carrier of the weak force, one of nature's four fundamental forces

25 JANUARY, 2023 | By Ana Lopes

CERN Accelerating science

3. W mass



$$M_W^2 = M_Z^2 \left\{ \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2} G_\mu M_Z^2}(1+\Delta r)} \right\}$$

m_z ; Pure weak Corr.

* iterative calculation

1-loop [1980],

2-loop level [1987-2002, QCD, EW],

leading 3- and 4-loop corrections [1994–2005],

unknown higher-order corrections [2003, 2021 updated],

$$\delta M_W^{\rm theo} \approx 4 \text{ MeV}$$

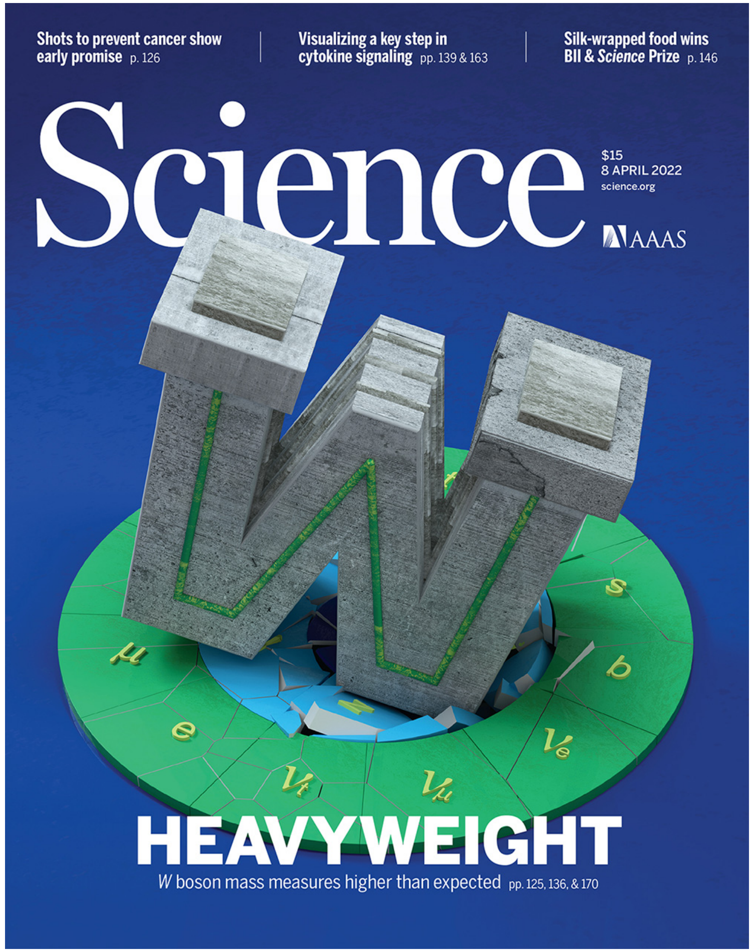
2204.03996, Athron, Fowlie, Lu, Wu, Wu, Zhu

0311148, Awramik, Czakon, Freitas, Weiglein

2. EW Fits and NP Hints

2204.03796, Lu, Wu, Wu, Zhu

2204.03996, Athron, Fowlie, Lu, Wu, Wu, Zhu



Three important questions:

Internal Consistency of the SM？

New Physics Scale？

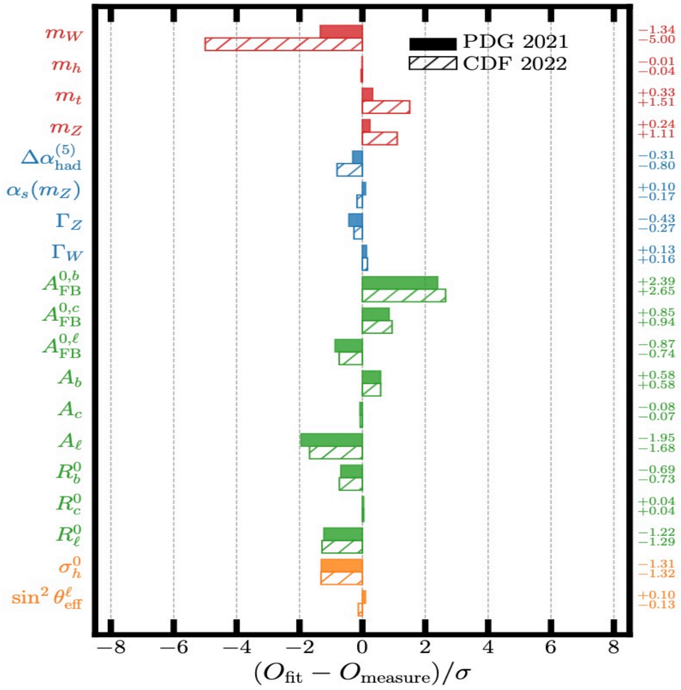
Possible New Physics Models？

2. EW Fits and NP Hints

Internal Consistency of the SM :

Parameter	Input Value	PDG 2021 $\chi^2_{\rm min}(\text{dof}) = 18.74(16)$ Fit Result	Pull	Fit w/o Input	Pull	CDF 2022 $\chi^2_{\rm min}(\text{dof}) = 62.58(16)$ Fit Result	Pull	Fit w/o Input	Pull
m_W [GeV]	80.379(12)	80.361(6)	−1.47	80.357(6)	−1.86	–	–	–	–
	80.4335(94)	–	–	–	–	80.380(5)	−5.71	80.357(6)	−8.17

$\Delta\alpha^{(5)}_{\rm had}$ [a]
m_h [GeV]
m_t [GeV] [b]
$\alpha_s(m_Z)$
Γ_W [GeV]
Γ_Z [GeV]
m_Z [GeV]
$A^{0,b}_{\rm FB}$
$A^{0,c}_{\rm FB}$
$A^{0,\ell}_{\rm FB}$
A_b
A_c
A_ℓ(SLD)
A_ℓ(LEP)
R^0_b
R^0_c
R^0_ℓ
σ^0_h [nb]
$\sin^2\theta^\ell_{\rm eff}(Q_{FB})$
$\sin^2\theta^\ell_{\rm eff}$(Teva)
$\overline{m}_c$ [GeV]
$\overline{m}_b$ [GeV]



$$M_W = \left[M_W^0 + c_t\Delta_t + c_t'\Delta_t^2 + c_Z\Delta_Z + c_\alpha\Delta_\alpha + c_{\alpha_s}\Delta_{\alpha_s}\right]\ \text{MeV}$$

0311148, Awramik, Czakon, Freitas, Weiglein

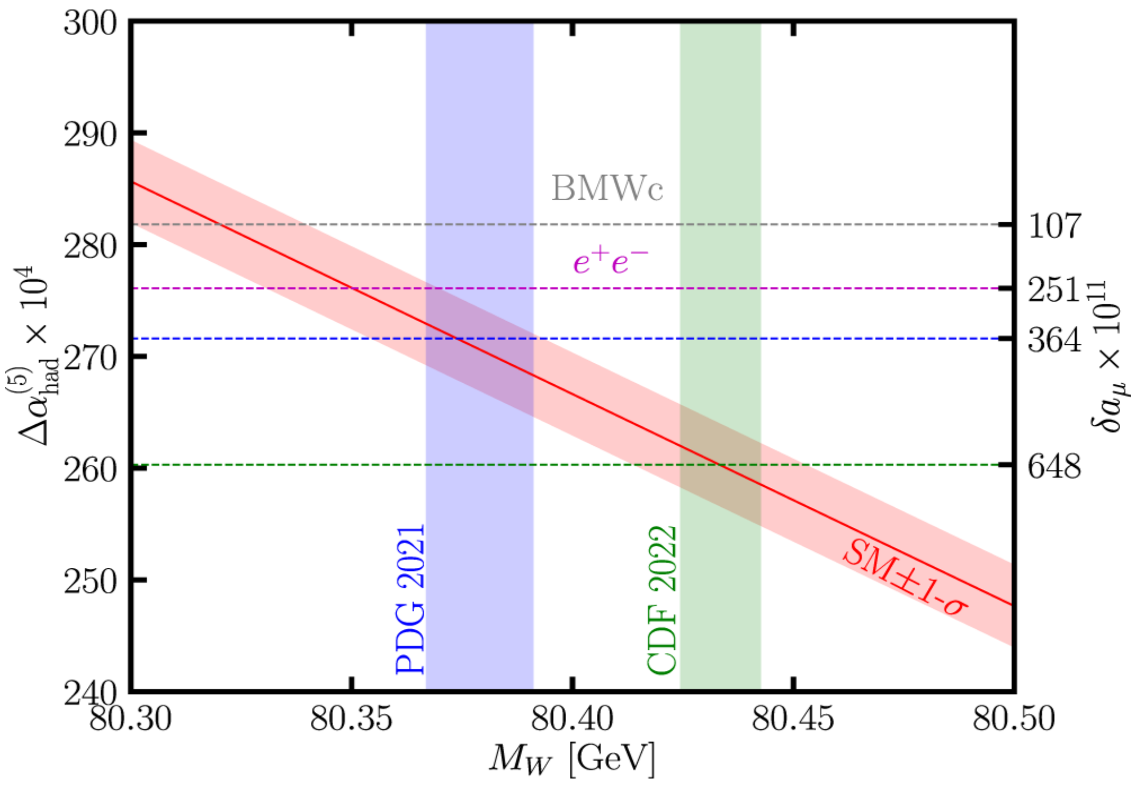
$M_W^0 = 80359.5$ $\quad c_t = 520.5$ $\quad c_t' = -67.7$

$c_Z = 115000.$ $\quad c_\alpha = -503.$ $\quad c_{\alpha_s} = -71.6$

M_W correlates
- **positively** with m_t and M_Z
- **negatively** with $\alpha(M_Z)$ and $\alpha_s(M_Z)$

2. EW Fits and NP Hints

Internal Consistency of the SM :



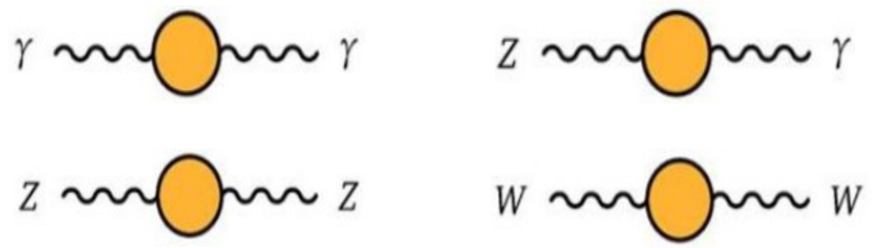
Muon g-2 and W mass

pull the physics in opposite directions

2. EW Fits and NP Hints

Oblique Parameters:

$$(M_W^2)_{\text{phys}} = (M_W^{SM})^2 \left[1 - \frac{\alpha S}{2(c_w^2 - s_w^2)} + \frac{c_w^2\, \alpha T}{c_w^2 - s_w^2} + \frac{\alpha U}{4 s_w^2}\right]$$



$$\begin{aligned}
\alpha S &= 4 s_W^2 c_W^2 \left[\Pi_{ZZ}(0) - (c_W^2 - s_W^2)/(s_W c_W)\cdot \Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right] \\
\alpha T &= \Pi_{WW}(0)/M_W^2 - \Pi_{ZZ}(0)/M_Z^2 \\
\alpha U &= 4 s_W^2 \left[\Pi'_{WW}(0) - c_W^2 \Pi'_{ZZ}(0) - 2 s_W c_W \Pi'_{Z\gamma}(0) - s_W^2 \Pi'_{\gamma\gamma}(0)\right]
\end{aligned}$$

PRD 46 (1992) 381-409, Peskin and Takeuchi

(1): The electroweak gauge group must be $SU_L(2)\times U_Y(1)$, with no new electroweal
;auge bosons apart from the photon, the $W^{\pm}$ and the Z.

(2): The couplings of the new physics to light fermions are suppressed compared to it
:ouplings to the gauge bosons.

(3): The intrinsic scale, M, of the new physics is large in comparison with M_w and M_z

- **Exception-1: NP is comparatively light.**
- **Exception-2: NP is comparatively large, but the low-energy measurements become sufficiently accurate.**

9306267, Maksymyk, Burgess, London

2. EW Fits and NP Hints

S, T and U:

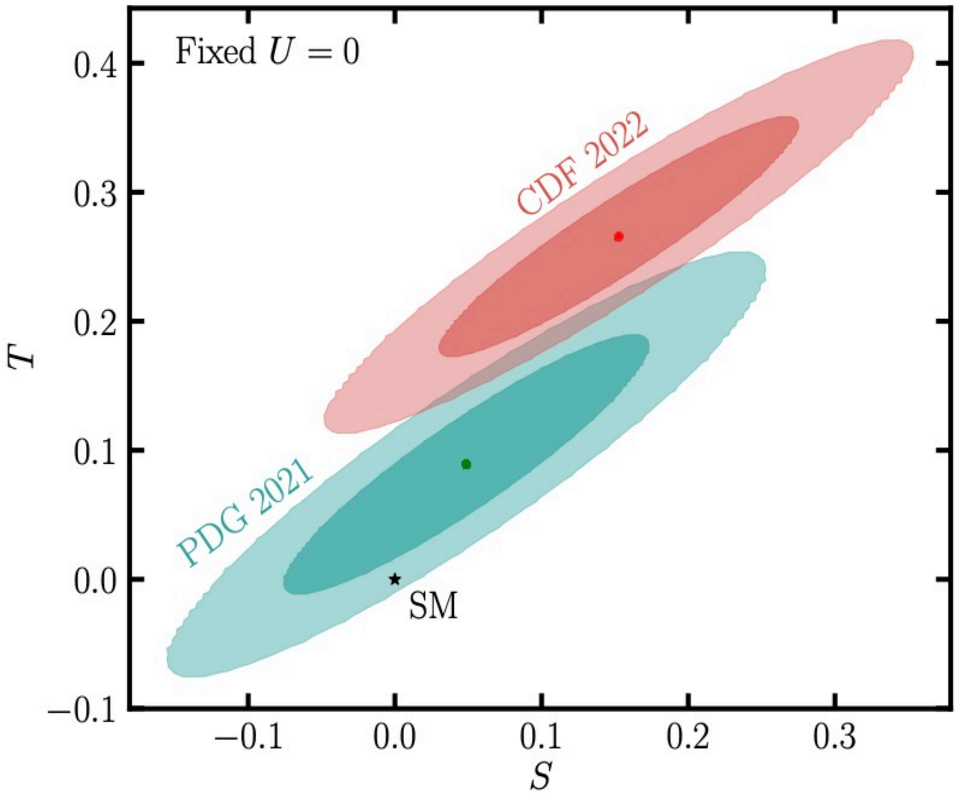
$U=0$ 14 dof	PDG 2021 Result $\chi^2_{min}=15.48$	Correlation S	Correlation T	CDF 2022 Result $\chi^2_{min}=17.82$	Correlation S	Correlation T
S	0.05 ± 0.08	1.00	0.92	0.15 ± 0.08	1.00	0.93
T	0.09 ± 0.07		1.00	0.27 ± 0.06		1.00

favor positive

13 dof	PDG 2021 Result $\chi^2_{min}=15.42$	Correlation S	Correlation T	Correlation U	CDF 2022 Result $\chi^2_{min}=15.44$	Correlation S	Correlation T	Correlation U
S	0.06 ± 0.10	1.00	0.90	-0.57	0.06 ± 0.10	1.00	0.90	-0.59
T	0.11 ± 0.12		1.00	-0.82	0.11 ± 0.12		1.00	-0.85
U	-0.02 ± 0.09			1.00	0.14 ± 0.09			1.00

due to W width in fits

2204.03796, Lu, Wu, Wu, Zhu



2. EW Fits and NP Hints

New Physics scale:

$$\left|H^\dagger D_\mu H\right|^2/\Lambda^2 \qquad H^\dagger W^{\mu\nu} B_{\mu\nu} H/\Lambda^2 \qquad \left(H^\dagger W^{\mu\nu} H\right)\left(H^\dagger W_{\mu\nu} H\right)/\Lambda^4$$

T parameter **S parameter** **U parameter**

$\mathcal{O}(0.1)$ —pert. couplings→ **Λ ~ 10 TeV**

$\mathcal{O}(0.1)$ —pert. couplings→ **Λ ~ 0.1 TeV**

2. EW Fits and NP Hints

How to make T larger: $\alpha T = \rho$

$$\rho = \frac{M_W^2}{c_W^2 M_Z^2}$$

= 1

(tree level)

NP @ tree-level

$$\rho = \frac{\sum_i \left[I_i(I_i+1) - (I_i^3)^2\right] v_i^2}{2\sum_i (I_i^3)^2 v_i^2}$$ (higher Higgs representations: Triplet etc)

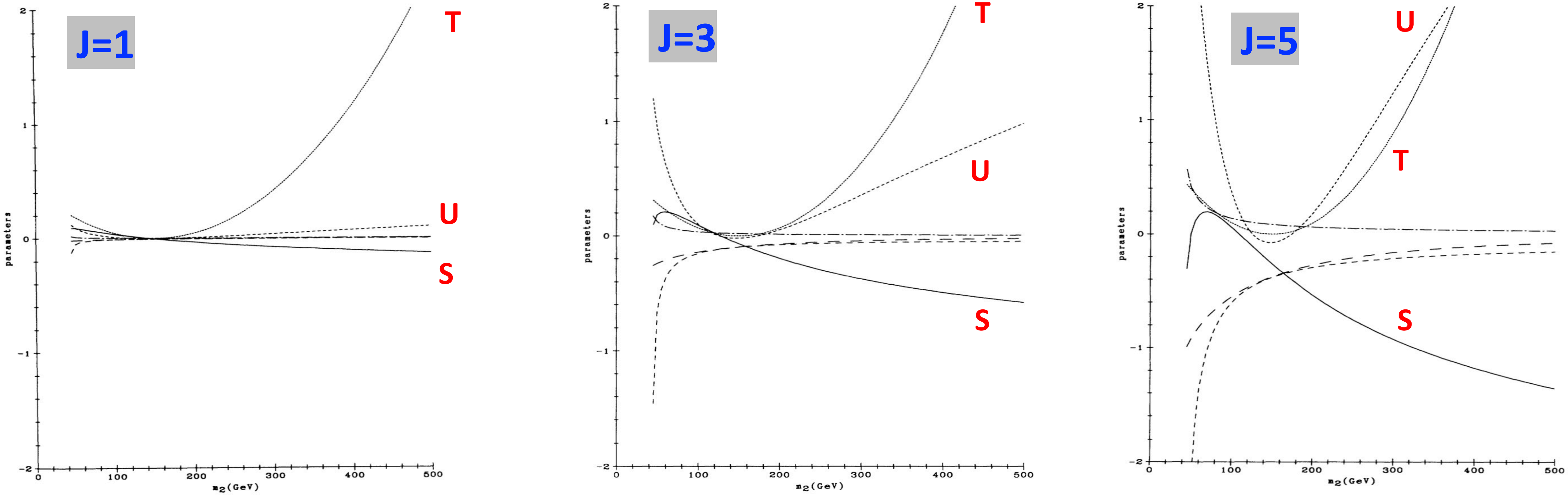
NP @ loop-level

$$\rho = 1 + \frac{3g^2}{64\pi^2}\frac{m_t^2 - m_b^2}{M_W^2} - \frac{11}{192\pi^2}g'^2 \log\frac{M_h^2}{M_Z^2} + \cdots$$ (new particles: 2HDM etc)

2. EW Fits and NP Hints

How to make S and U larger:

PRD 49 (1994) 1409-1416, Lavoura and Li   e.g. multiplet scalars   high isospin, light states



3. Inert 2HDM *2204.03693, Fan, Tang, Sming, Wu*

i2HDM:

Discrete Z_2 symmetry ($H_1 \to H_1$ and $H_2 \to -H_2$)

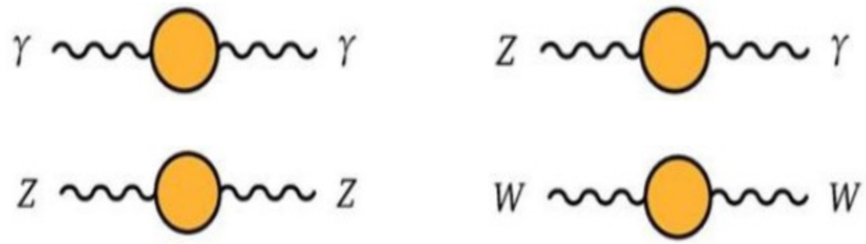
$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h + iG^0\right) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(S + iA) \end{pmatrix}$$

$$V = \mu_1^2|H_1|^2 + \lambda_1|H_1|^4 + \mu_2^2|H_2|^2 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1^\dagger H_2|^2 + \frac{\lambda_5}{2}\left\{(H_1^\dagger H_2)^2 + h.c.\right\}$$

PRD 18 (1978) 2574, Deshpande, Ma

3. Inert 2HDM

Oblique Parameter

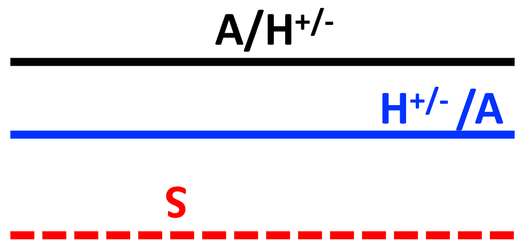


$$S = \frac{1}{2\pi}\left[\frac{1}{6}\log\left(\frac{m_S^2}{m_{H^\pm}^2}\right) - \frac{5}{36} + \frac{m_S^2 m_A^2}{3(m_A^2 - m_S^2)^2} + \frac{m_A^4(m_A^2 - 3m_S^2)}{6(m_A^2 - m_S^2)^3}\log\left(\frac{m_A^2}{m_S^2}\right)\right]$$

$$T = \frac{1}{32\pi^2\alpha v^2}\left[F(m_{H^\pm}^2, m_A^2) + F(m_{H^\pm}^2, m_S^2) - F(m_A^2, m_S^2)\right]$$

$$= 32 i\pi^2 \int \frac{d^4k}{(2\pi)^4} k^2 \frac{(m_+^2 - m_1^2)(m_+^2 - m_2^2)}{(k^2 - m_+^2)^2(k^2 - m_1^2)(k^2 - m_2^2)}$$

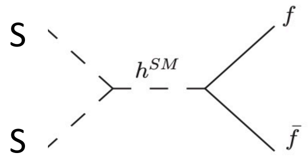
"Spectrum"



3. Inert 2HDM

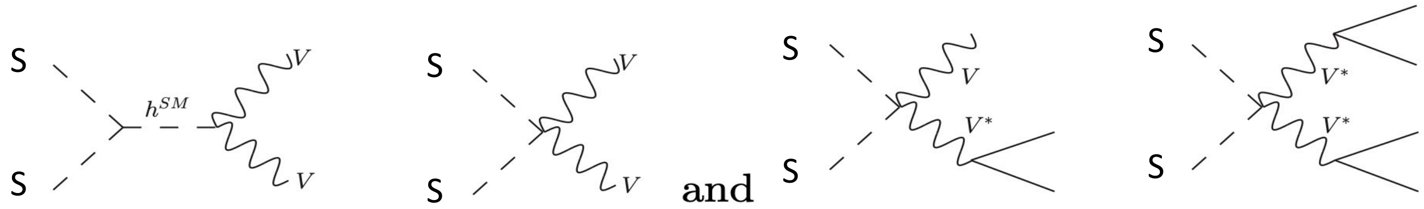
Relic Density of DM

- annihilation through Higgs into fermions



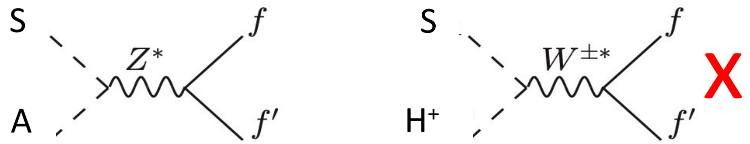
depends on g_{DMh} coupling; dominant channel for $M_{DM} < M_h/2$

- annihilation into gauge bosons (also into virtual states)



crucial for heavy masses; non-negligible for $M_h/2 < M_{DM} < M_W$

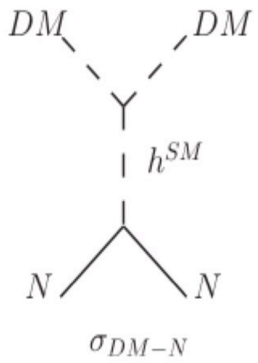
- coannihilation



very important when particles have similar masses

3. Inert 2HDM

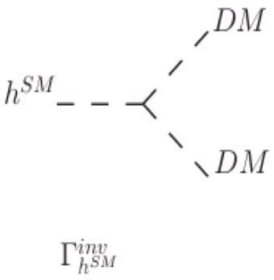
Direct Detection



DM-scattering on nucleus through the Higgs exchange

$$\sigma_{DM-N} \propto g^2_{DMh}/(M_{H_1}+M_N)^2$$

Higgs invisible decay



Higgs invisible decays for $M_{DM} < M_h/2$

$$\Gamma(h \to H_1 H_1) = \frac{g^2_{DMh} v^2}{32\pi M_h}\sqrt{1-\frac{4M^2_{H_1}}{M^2_h}}$$

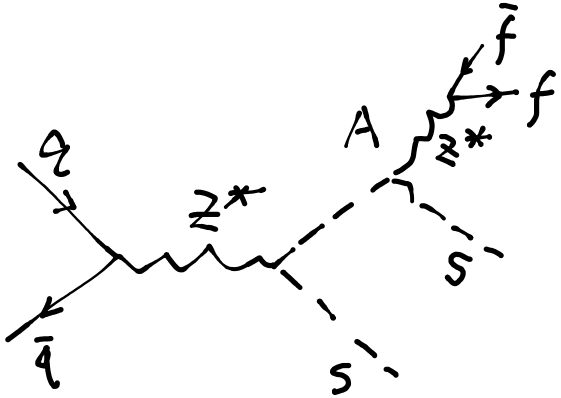
3. Inert 2HDM

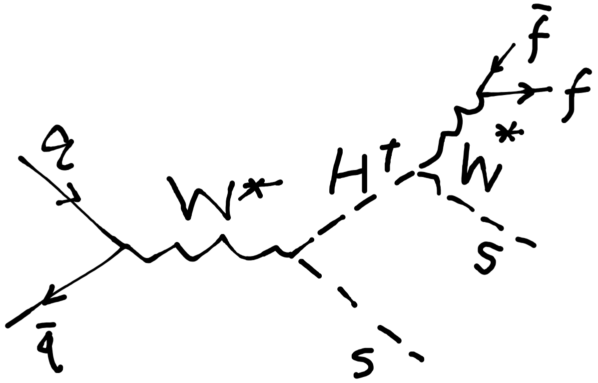
LHC signatures:

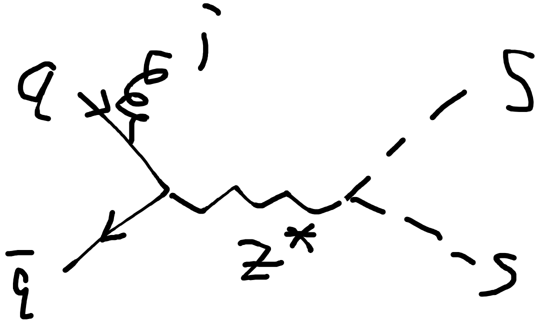
2 leptons+missing E_T

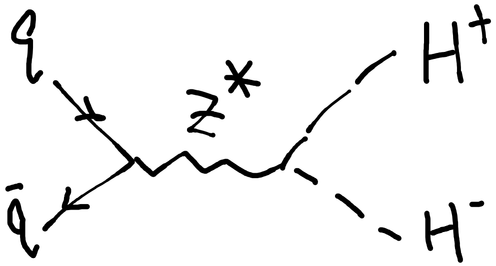
mono-jet

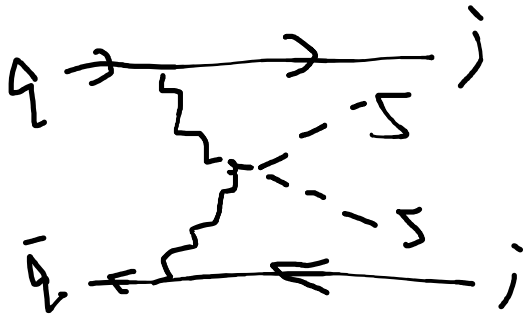
multi-jet+missing E_T



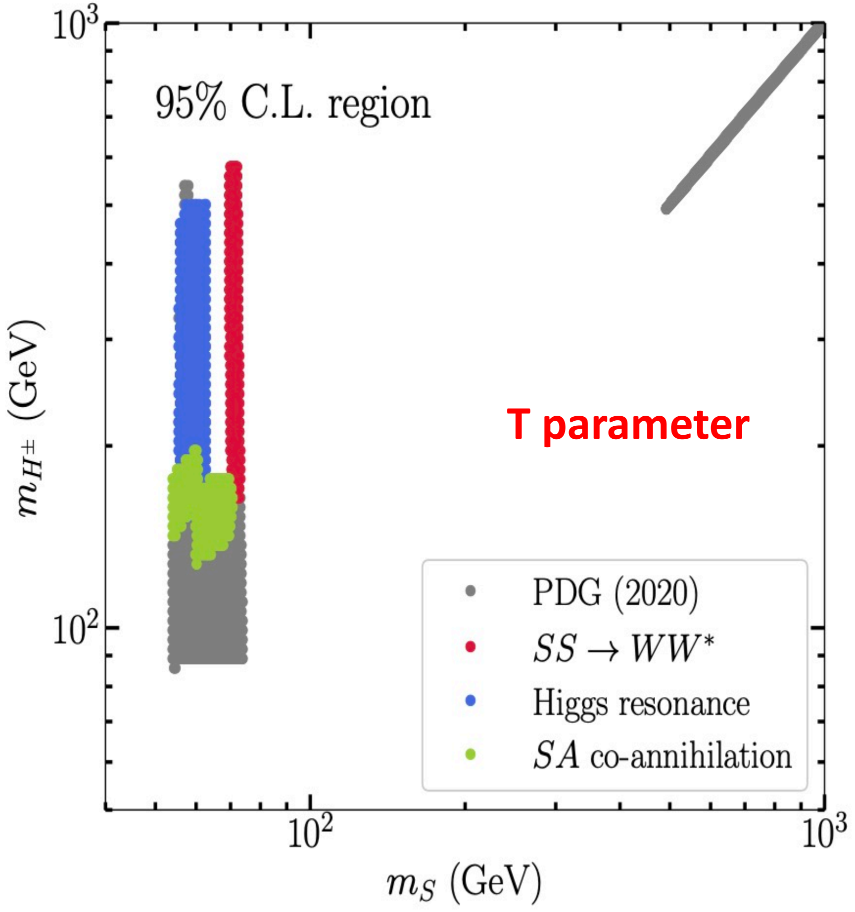




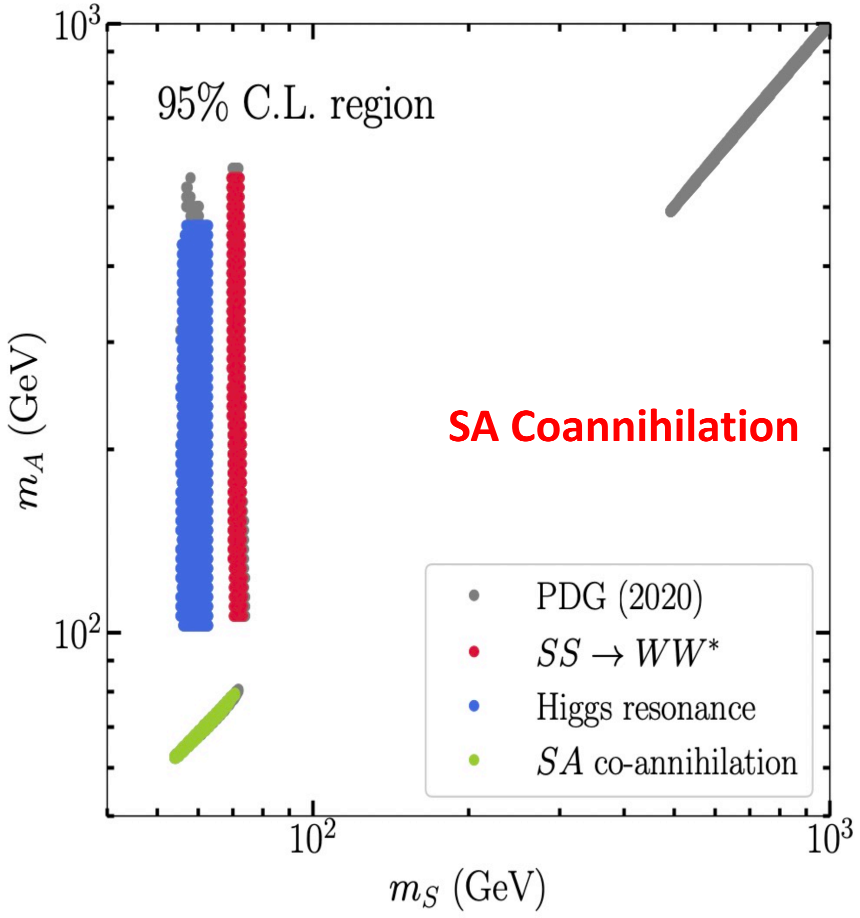




3. Inert 2HDM



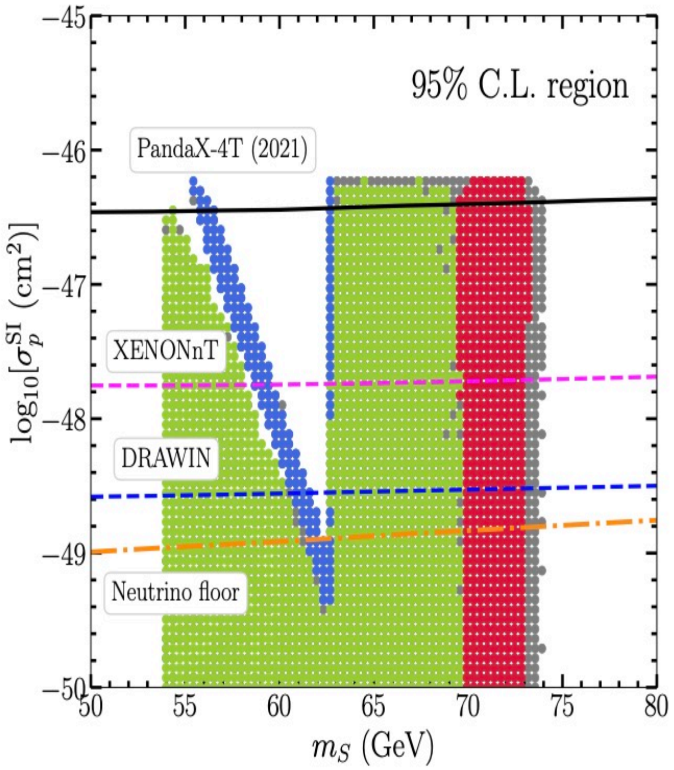
95% C.L. region
T parameter
PDG (2020)
$SS \to WW^*$
Higgs resonance
SA co-annihilation
$m_{H^\pm}$ (GeV)
m_S (GeV)



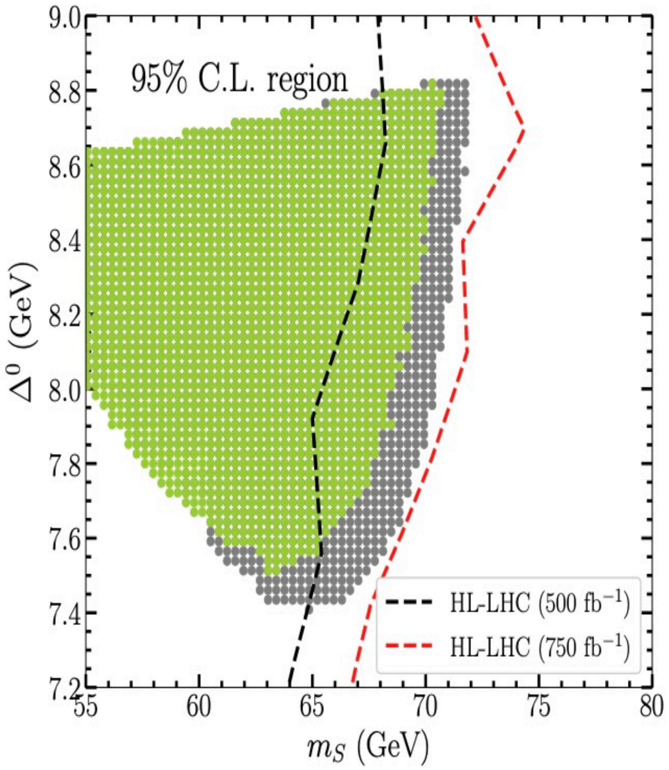
95% C.L. region
SA Coannihilation
PDG (2020)
$SS \to WW^*$
Higgs resonance
SA co-annihilation
m_A (GeV)
m_S (GeV)

3. Inert 2HDM

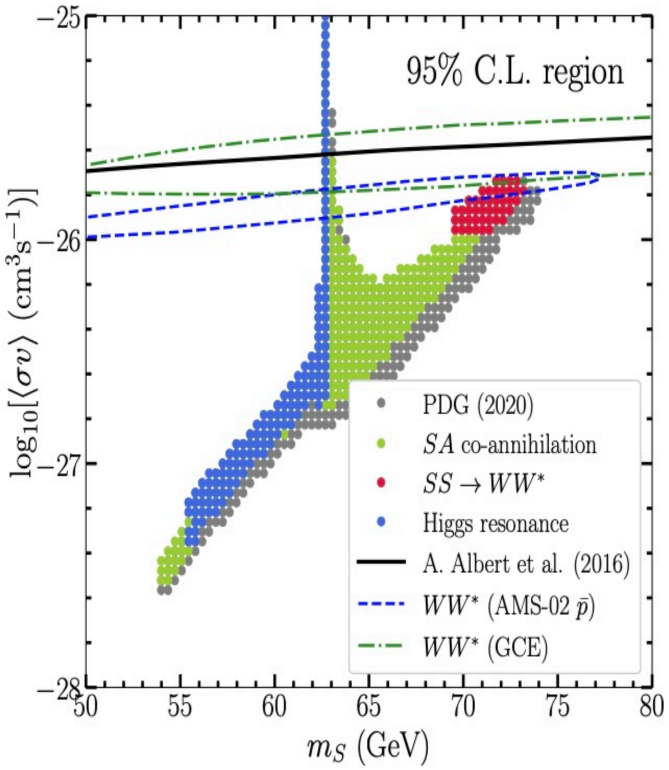
Direct Detection



LHC



Indirect Detection

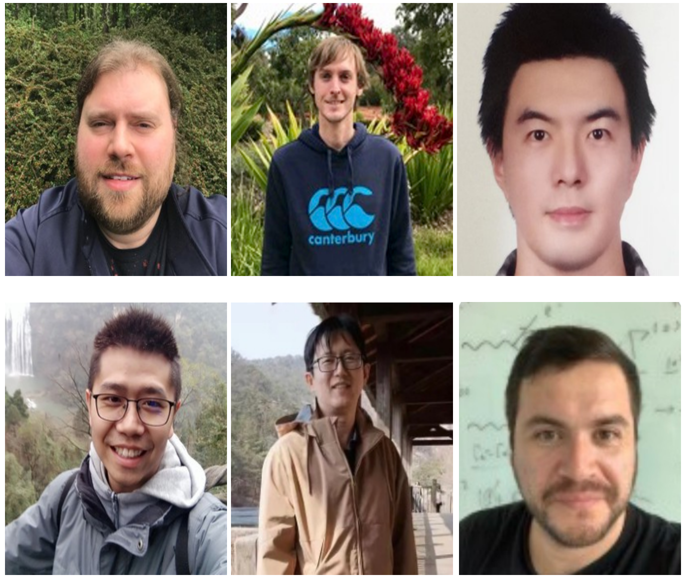


It is very predictive, and can be tested soon!

4. Summary

Thank you!

Welcome to NNU



- **Theory (18+2)**

Heavy Flavor: 肖振军、郭立波、朱瑞林、邢志鹏
Hadron Physics：平加伦、何军、黄虹霞、何秉然、黄琦
New Physics： 武雷、Peter Arthon、吴永成、卢致廷、Ariel Arza
+ Andrew Fowlie、祝斌
QCD Physics： 王琦、黄日俊、刘宁、金立刚

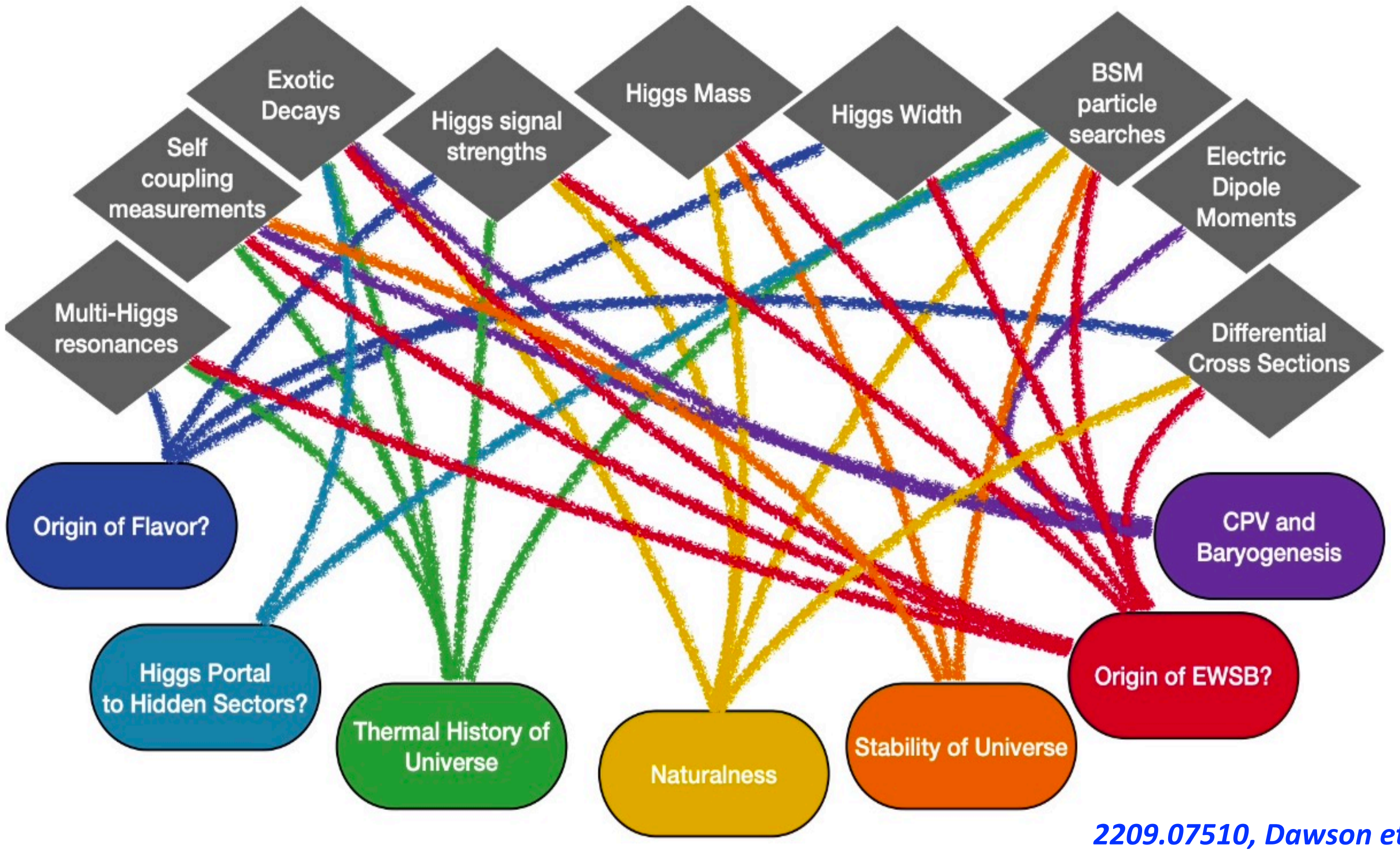
- **Experiment (4)**

BESIII： 钟彬、张敬庆
CMS： 易凯、张敬庆、Gary Baur
Belle II： 易凯、钟彬

Backup

1. Higgs Era

Higgs Era: Precision Measurements and New Physics



2209.07510, Dawson et al.

B1. W mass status and future

Questions:

Higher order ✓

PDF ✓

Width ✓

Generator

Detector

New Physics

…

May 6, 2022

$\sim 6\sigma$

[27] arXiv:2205.02788 [pdf, other]

ResBos2 and the CDF W Mass Measurement

Joshua Isaacson, Yao Fu, C.-P. Yuan

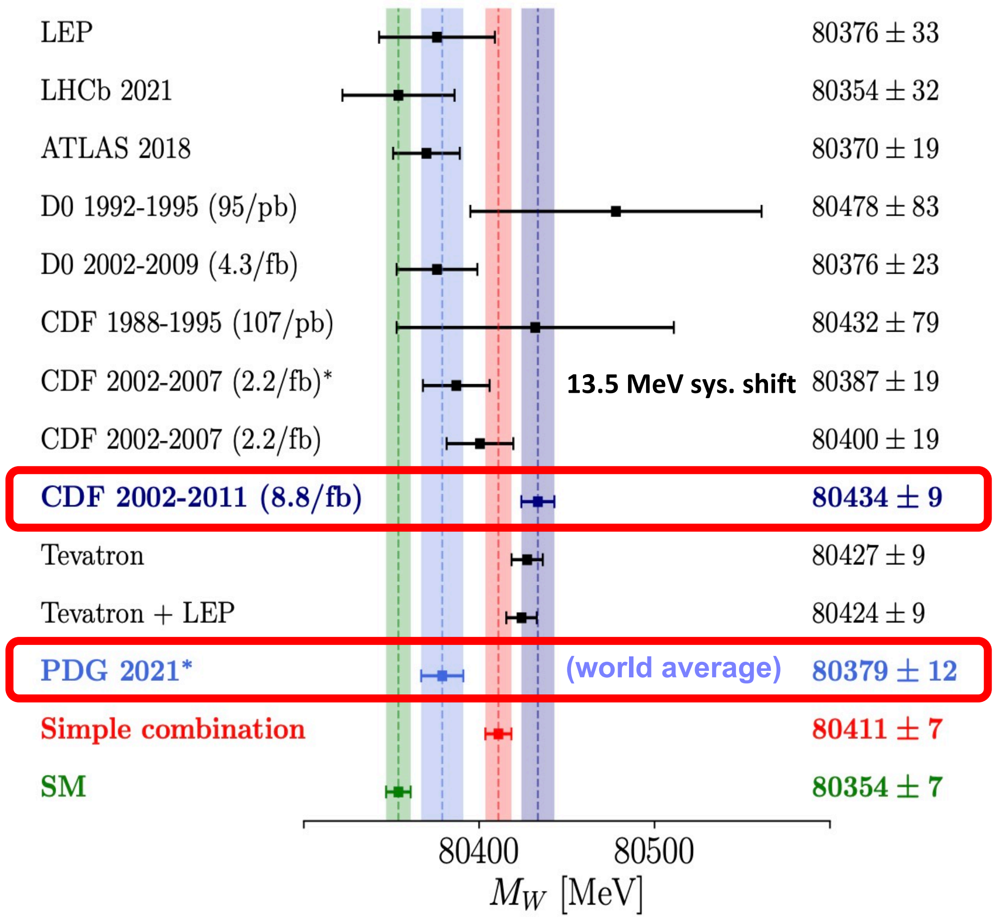
Comments: 11 pages, 13 figures

Subjects: **High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Experiment (hep-ex)

The recent CDF W mass measurement of $80{,}433 \pm 9$ MeV is the most precise direct measurement. However, this result deviates from the Standard Model predicted mass of $80{,}359.1 \pm 5.2$ MeV by 7σ. The CDF experiment used an older version of the ResBos code that was only accurate at NNLL+NLO, while the state-of-the-art ResBos2 code is able to make predictions at N^3LL+NNLO accuracy. We determine that the data-driven techniques used by CDF capture most of the higher order corrections, and using higher order corrections would result in a decrease in the value reported by CDF by at most 10 MeV.

HL-LHC: 5 MeV CEPC: 1 MeV

2204.03996, Athron, Fowlie, Lu, Wu, Wu, Zhu



What is experimentally measured?

- **LEP: $e^+e^- \to W^+ W^-$ in the continuum and at threshold (small amount of data);**
- **Tevatron/LHC: transverse mass distribution**

m_Z **Pure weak Corr.**

$$M_W^2 = M_Z^2 \left\{ \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi \alpha}{\sqrt{2} G_\mu M_Z^2} (1 + \Delta r)} \right\}$$

* iterative calculation

α is extracted from low-energy experiments,
G_F is extracted from the muon lifetime,
m_Z is measured from e^+e^- annihilation near the Z mass,
Δr is radiative corrections.

$$\delta M_W^{\text{theo}} \approx 4 \text{ MeV}$$

1-loop [1980],
2-loop level [1987-2002, QCD, EW],
leading 3- and 4-loop corrections [1994–2005],
unknown higher-order corrections [2003, 2021 updated],

$$\Delta r = \Delta\alpha - \cot^2\theta_W \Delta\rho + \Delta_r^{\text{rem}}$$

scale dependence of α

$$\alpha(M_Z^2) = \frac{\alpha(0)}{1-\Delta\alpha}$$

$$\Delta\alpha(M_Z^2) = \Pi_{\gamma\gamma}(0) - \Pi_{\gamma\gamma}(M_Z^2)$$

$$\Delta\alpha^{\text{lept}}(M_Z^2) = \sum_{\ell=e,\mu,\tau} \frac{\alpha}{3\pi}\left[\log\frac{M_Z^2}{m_\ell^2} - \frac{5}{3}\right] + \mathcal{O}\left(\frac{m_\ell^2}{M_Z^2}\right) + \mathcal{O}(\alpha^2) + \mathcal{O}(\alpha^3) \simeq 0.0315$$

$$\Delta\alpha^{\text{had}}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\text{Re}\left(\int_{4m_\pi^2}^{\infty} \mathrm{d}s' \frac{R_{\gamma\gamma}(s')}{s'(s'-M_Z^2)}\right)$$

$$R_{\gamma\gamma}(s) = \frac{\sigma(e^+e^- \to \gamma^* \to \text{had.})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

$$\Delta\alpha^{\text{had}}(M_Z^2) = 0.02761 \pm 0.00036$$

New Physics

(1) parity violation effects in electron scattering;

(2) atomic parity violation (weak charge), Lamb shift

(3) electron and muon magnetic moments

$$\Delta r = \Delta\alpha - \cot^2\theta_W \Delta\rho + \Delta_r^{\text{rem}}$$

Strength of the ratio of NC to CC at zero–momentum transfer in DIS

$$\rho = \frac{1}{1-\Delta\rho}$$

$$\rho = \frac{M_W^2}{c_W^2 M_Z^2}$$ (=1, Custorial symmetry)

$$\Delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$

$$\rho = 1 + \frac{3g^2}{64\pi^2}\frac{m_t^2 - m_b^2}{M_W^2} - \frac{11}{192\pi^2}g'^2 \log\frac{M_h^2}{M_Z^2} + \cdots$$

$$\alpha T = \rho$$

$$\rho = \frac{\sum_i \left[I_i(I_i+1) - (I_i^3)^2\right] v_i^2}{2\sum_i (I_i^3)^2 v_i^2}$$ (other Higgs representations)

New Physics

(1) non-degenerate SU(2) doublet :

2HDM, SUSY, vector-like fermion…

(2) heavy Z′ bosons

(3) Other higher representations

EWPO Uncertainties	Current	HL-LHC
Δm_W (MeV)	12 / $9.4^{\dagger}$	5
Δm_Z (MeV)	2.1	
$\Delta \Gamma_Z$ (MeV)	2.3	
Δm_t (GeV)	0.6*	0.2
$\Delta \sin^2 \theta^{\ell}_{\text{eff}}$ $(\times 10^5)$	13	< 10
δR_{μ} $(\times 10^3)$	1.6	
δR_b $(\times 10^3)$	3.1	

Quantity	current	ILC250	ILC-GigaZ	FCC-ee	CEPC	CLIC380
$\Delta\alpha(m_Z)^{-1}$ ($\times 10^3$)	17.8*	17.8*		3.8 (1.2)	17.8*	
Δm_W (MeV)	12*	0.5 (2.4)		0.25 (0.3)	0.35 (0.3)	
Δm_Z (MeV)	2.1*	0.7 (0.2)	0.2	0.004 (0.1)	0.005 (0.1)	2.1*
Δm_H (MeV)	170*	14		2.5 (2)	5.9	78
$\Delta\Gamma_W$ (MeV)	42*	2		1.2 (0.3)	1.8 (0.9)	
$\Delta\Gamma_Z$ (MeV)	2.3*	1.5 (0.2)	0.12	0.004 (0.025)	0.005 (0.025)	2.3*
ΔA_e ($\times 10^5$)	190*	14 (4.5)	1.5 (8)	0.7 (2)	1.5 (2)	60 (15)
ΔA_μ ($\times 10^5$)	1500*	82 (4.5)	3 (8)	2.3 (2.2)	3.0 (1.8)	390 (14)
ΔA_τ ($\times 10^5$)	400*	86 (4.5)	3 (8)	0.5 (20)	1.2 (20)	550 (14)
ΔA_b ($\times 10^5$)	2000*	53 (35)	9 (50)	2.4 (21)	3 (21)	360 (92)
ΔA_c ($\times 10^5$)	2700*	140 (25)	20 (37)	20 (15)	6 (30)	190 (67)
$\Delta\sigma^0_{\text{had}}$ (pb)	37*			0.035 (4)	0.05 (2)	37*
δR_e ($\times 10^3$)	2.4*	0.5 (1.0)	0.2 (0.5)	0.004 (0.3)	0.003 (0.2)	2.5 (1.0)
δR_μ ($\times 10^3$)	1.6*	0.5 (1.0)	0.2 (0.2)	0.003 (0.05)	0.003 (0.1)	2.5 (1.0)
δR_τ ($\times 10^3$)	2.2*	0.6 (1.0)	0.2 (0.4)	0.003 (0.1)	0.003 (0.1)	3.3 (5.0)
δR_b ($\times 10^3$)	3.1*	0.4 (1.0)	0.04 (0.7)	0.0014 ($<$ 0.3)	0.005 (0.2)	1.5 (1.0)
δR_c($\times 10^3$)	17*	0.6 (5.0)	0.2 (3.0)	0.015 (1.5)	0.02 (1)	2.4 (5.0)

B2. EW fit

9306267, Maksymyk, Burgess, London

$$
\begin{aligned}
\Gamma_Z &= 2.4950 - 0.0092S + 0.026T + 0.019V - 0.020X \\
\sigma^0_{had} &= 41.484 + 0.014S - 0.0098T + 0.031X \\
R_\ell &= 20.743 - 0.062S + 0.042T - 0.14X \\
A^\ell_{FB} &= 0.01626 - 0.0061S + 0.0042T - 0.013X \\
A_\ell &= 0.1472 - 0.028S + 0.019T - 0.061X \\
A_c &= 0.6680 - 0.012S + 0.0084T - 0.027X \\
A_b &= 0.93463 - 0.0023S + 0.0016T - 0.0050X \\
A^c_{FB} &= 0.0738 - 0.015S + 0.010T - 0.033X \\
A^b_{FB} &= 0.1032 - 0.020S + 0.014T - 0.043X \\
R_c &= 0.17226 - 0.00021S + 0.00015T - 0.00046X \\
R_b &= 0.21578 + 0.00013S - 0.000091T + 0.00030X \\
s^2_{\theta_{\rm eff}} &= 0.23150 + 0.0035S - 0.0024T + 0.0078X \\
m_W &= 80.364 - 0.28S + 0.43T + 0.35U \\
\Gamma_W &= 2.091 - 0.015S + 0.023T + 0.018U + 0.016W
\end{aligned}
$$

U only contributes to W mass and width

EW Global Fitting

- SM is well-known to explain our nature extremely well
- Global Fitting is a way to quantitatively show above statement
- In the fitting
  - Varying the independent input parameters of a model
  - Calculating observables
  - Comparing with measurements

EW Global Fitting

- Input Parameters for SM
  - Fixed:
    - $G_F = 1.1663787 \times 10^{-5}$
    - $m_u,\ m_d,\ m_s$ (MSbar)
  - Free in Fitting
    - $m_h,\ m_Z,\ m_t,\ \overline{m}_c,\ \overline{m}_b$
    - $\Delta\alpha_{had}^{(5)},\ \alpha_s$
- All other parameters/observable in terms of above parameters.
  - $M_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha(1-\Delta r)}{G_F M_Z^2}}\right)$
  - Etc.

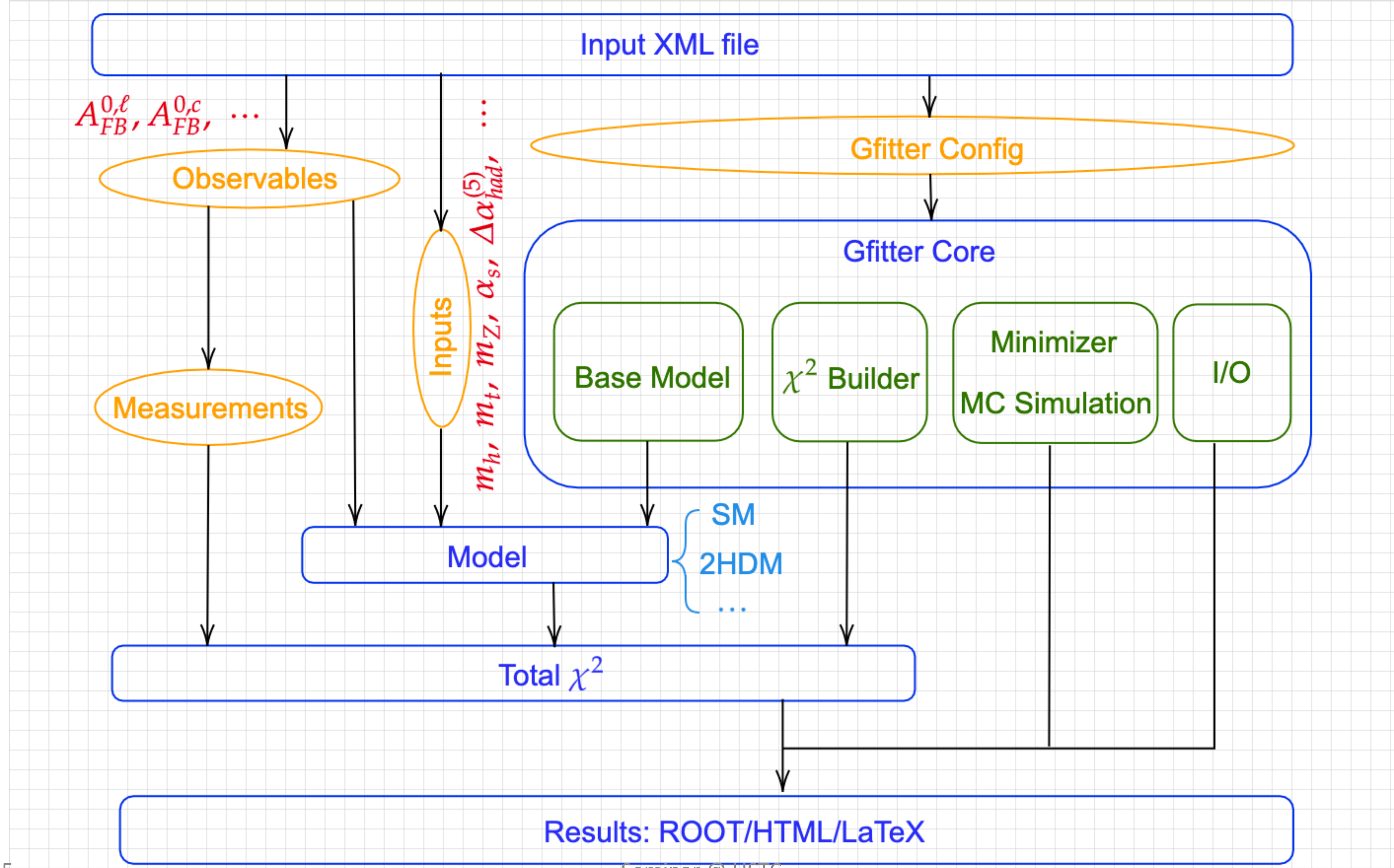
Table I. The input parameters and the best points in the global EW fit. The Fermi constant $G_F = 1.1663787(6) \times 10^{-5}$ [GeV^{-2}] and fine-structure constant $\alpha = 1/137.035999074(44)$ [36] are fixed in our calculation. Correlations among $(m_Z, \Gamma_Z, \sigma_h^0, R_\ell^0, A_{FB}^{0,\ell})$ and among $(A_{FB}^{0,c}, A_{FB}^{0,b}, A_c, A_b, R_c^0, R_b^0)$ are also taken into account [4]. The value of "Pull" is defined as $(O_{fit} - O_{measure})/\sigma_{measure}$, where $\sigma_{measure}$ is the error of each input observable.

Parameter	Input Value	PDG 2021				CDF 2022				Refs
		$\chi^2_{min}(dof) = 18.74(16)$				$\chi^2_{min}(dof) = 62.58(16)$				
		Fit Result	Pull	Fit w/o Input	Pull	Fit Result	Pull	Fit w/o Input	Pull	
m_W [GeV]	80.379(12)	80.361(6)	−1.47	80.357(6)	−1.86	–	–	–	–	[36]
	80.4335(94)	–	–	–	–	80.380(5)	−5.71	80.357(6)	−8.17	[9]
$\Delta\alpha_{had}^{(5)}$ [a]	0.02761(11)	0.02756(11)	−0.44	0.02716(38)	−4.06	0.02747(10)	−1.11	0.02609(36)	−13.64	[37–39]
m_h [GeV]	125.25(17)	125.25(17)	−0.02	$92^{(21)}_{(18)}$	−193.26	125.24(17)	−0.05	$44^{(10)}_{(8)}$	−479.17	[36]
m_t [GeV] [b]	172.76(58)	173.02(56)	0.45	176.2(20)	5.83	173.97(55)	2.07	184.1(16)	19.39	[36]
$\alpha_s(m_Z)$	0.1179(9)	0.1180(9)	0.14	0.1193(9)	1.53	0.1177(9)	−0.23	0.1155(29)	−2.68	[36]
Γ_W [GeV]	2.085(42)	2.0905(5)	0.13	2.0905(5)	0.13	2.0918(5)	0.16	2.0918(5)	0.16	[36]
Γ_Z [GeV]	2.4952(23)	2.4942(6)	−0.45	2.4940(7)	−0.51	2.4946(6)	−0.28	2.4945(7)	−0.32	[4]
m_Z [GeV]	91.1875(21)	91.1882(21)	0.34	91.2037(90)	7.72	91.1907(20)	1.54	91.2386(79)	24.36	[4]
$A_{FB}^{0,b}$	0.0992(16)	0.1031(3)	2.44	0.1033(3)	2.54	0.1035(3)	2.70	0.1037(3)	2.80	[4]
$A_{FB}^{0,c}$	0.0707(35)	0.0737(3)	0.85	0.0737(3)	0.85	0.07401(25)	0.95	0.07402(25)	0.95	[4]
$A_{FB}^{0,\ell}$	0.0171(10)	0.01623(10)	−0.87	0.01622(10)	−0.88	0.01636(10)	−0.74	0.01635(10)	−0.75	[4]
A_b	0.923(20)	0.93462(4)	0.58	0.93462(4)	0.58	0.93464(4)	0.58	0.93464(4)	0.58	[4]
A_c	0.670(27)	0.6679(2)	−0.08	0.6679(2)	−0.08	0.6682(2)	−0.07	0.6682(2)	−0.07	[4]
A_ℓ(SLD)	0.1513(21)	0.1471(5)	−2.00	0.1469(5)	−2.10	0.1477(5)	−1.72	0.1475(5)	−1.81	[4]
A_ℓ(LEP)	0.1465(33)	0.1471(5)	0.18	0.1469(5)	0.12	0.1477(5)	0.37	0.1475(5)	0.32	[4]
R_b^0	0.21629(66)	0.21583(10)	−0.69	0.21582(10)	−0.71	0.21580(10)	−0.74	0.21579(10)	−0.76	[4]
R_c^0	0.1721(30)	0.17222(6)	0.04	0.17222(6)	0.04	0.17223(6)	0.04	0.17223(6)	0.04	[4]
R_ℓ^0	20.767(25)	20.735(8)	−1.28	20.732(8)	−1.40	20.733(8)	−1.34	20.730(8)	−1.48	[4]
σ_h^0 [nb]	41.540(37)	41.491(8)	−1.34	41.489(8)	−1.39	41.490(8)	−1.34	41.488(8)	−1.39	[4]
$\sin^2\theta_{eff}^\ell(Q_{FB})$	0.2324(12)	0.23151(6)	−0.74	0.23151(6)	−0.74	0.23144(6)	−0.81	0.23143(6)	−0.81	[4]
$\sin^2\theta_{eff}^\ell$(Teva)	0.23148(33)	0.23151(6)	0.10	0.23151(6)	0.10	0.23144(6)	−0.13	0.23144(6)	−0.13	[40]
$\overline{m}_c$ [GeV]	1.27(2)	1.27(2)	0.00	–	–	1.27(2)	0.00	–	–	[36]
$\overline{m}_b$ [GeV]	$4.18^{(3)}_{(2)}$	$4.18^{(3)}_{(2)}$	0.00	–	–	$4.18^{(3)}_{(2)}$	0.00	–	–	[36]

EW Observables

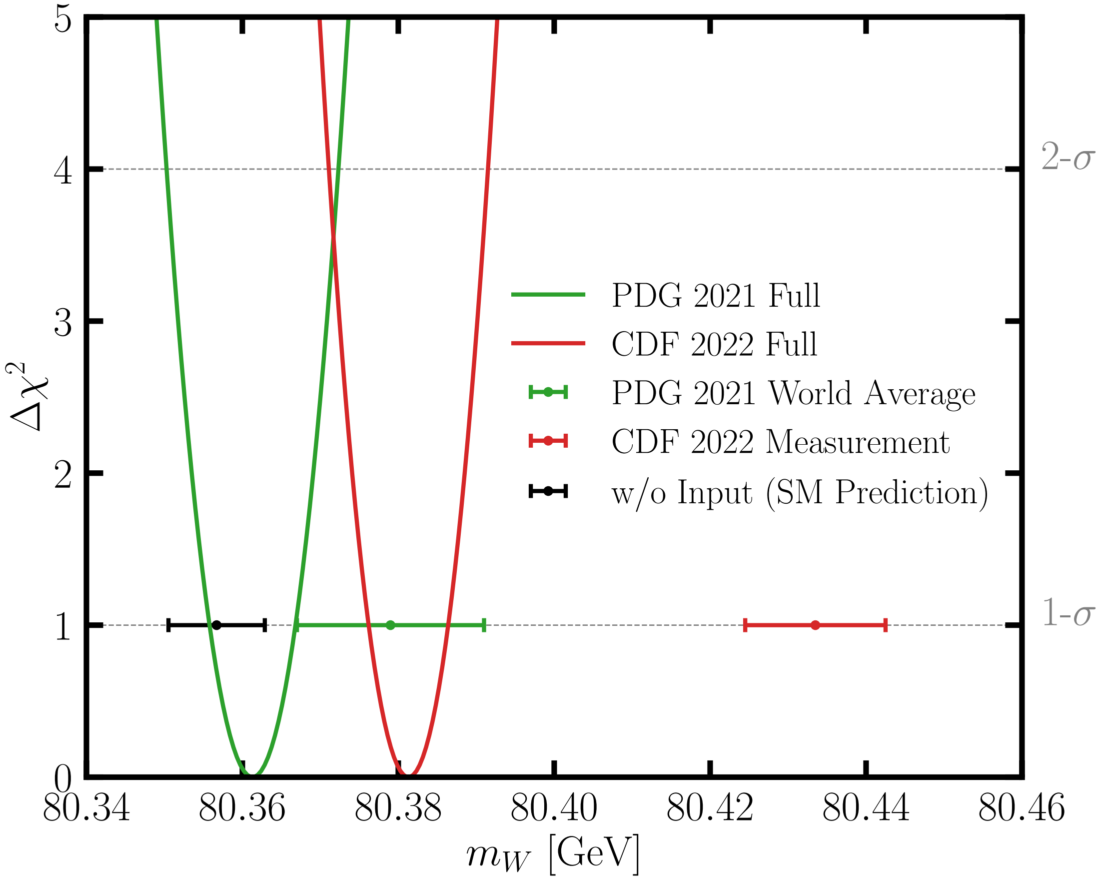
- Z resonance parameters:
  - M_Z, Γ_Z
  - $\sigma(e^+e^- \to Z \to hadron)$
- Partial Z cross section
  - R_ℓ^0, R_c^0, R_b^0
- Neutral current couplings
  - $\sin\theta_{eff}^\ell$
  - A_ℓ, A_c, A_b
  - $A_{FB}^{0,\ell},\ A_{FB}^{0,c},\ A_{FB}^{0,b}$
- W-boson parameters
  - M_W, Γ_W
- Higgs parameter
  - m_h
- Misc
  - m_t, m_b, m_c
  - $\alpha_s, \alpha\ \left(\Delta\alpha_{had}^{(5)}\right)$

EW Global Fitting in GFitter



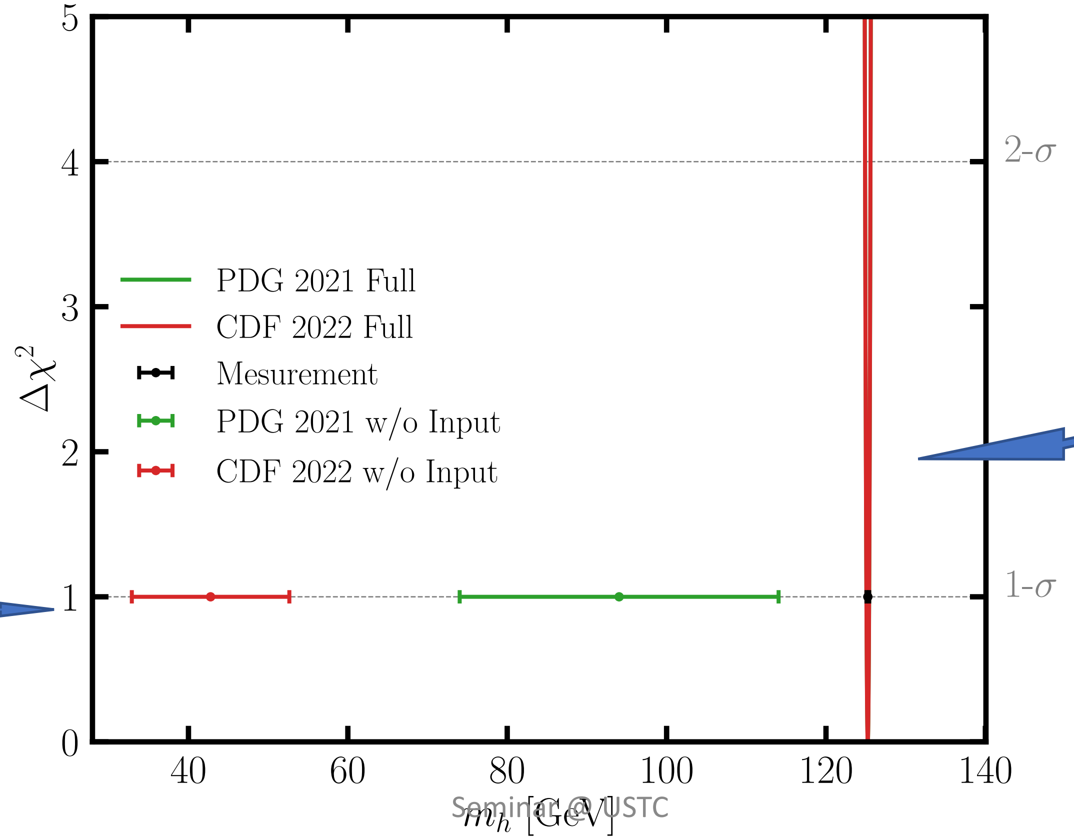
EW Global Fitting Individual Result

- m_W
  - There is huge deviation between
    - CDF measurement and SM prediction



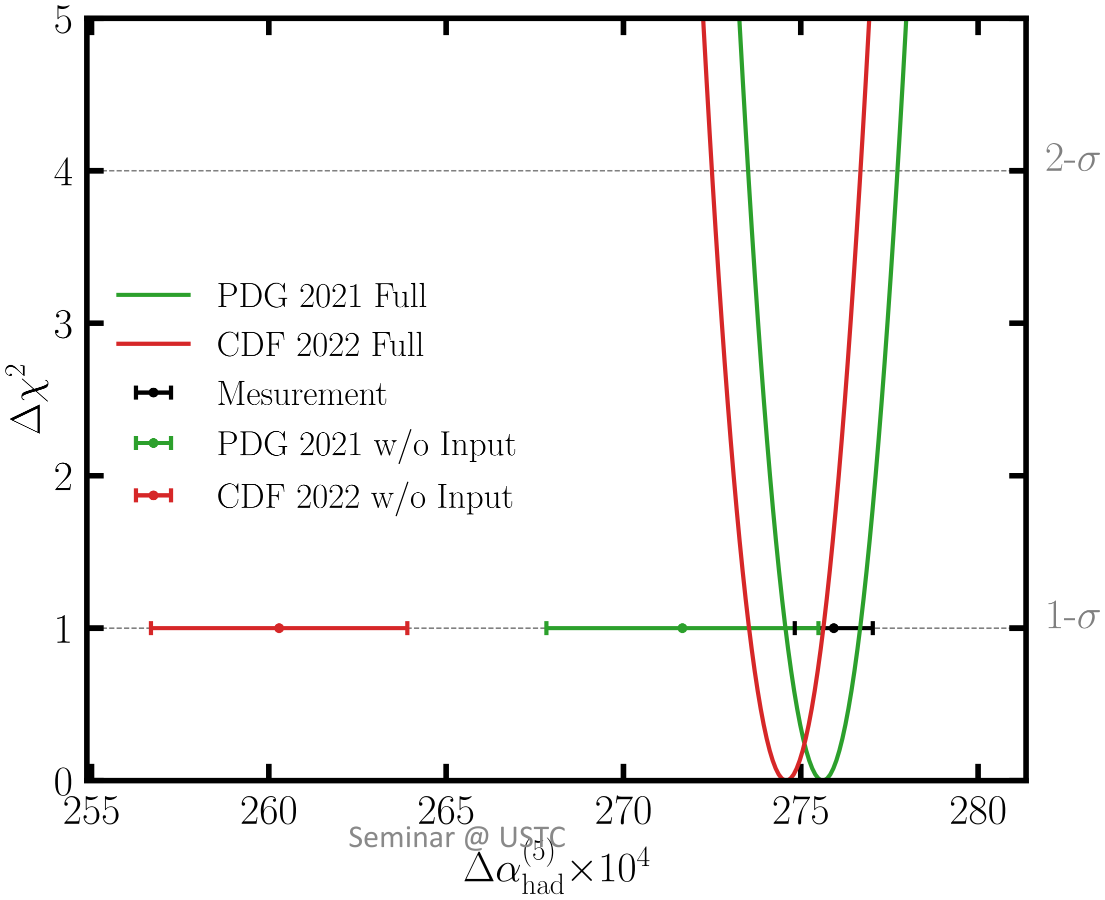
EW Global Fitting Individual Result

- **Higgs Mass**
  - Dominated by Current Measurement
  - CDF Measurement prefer much lighter Higgs



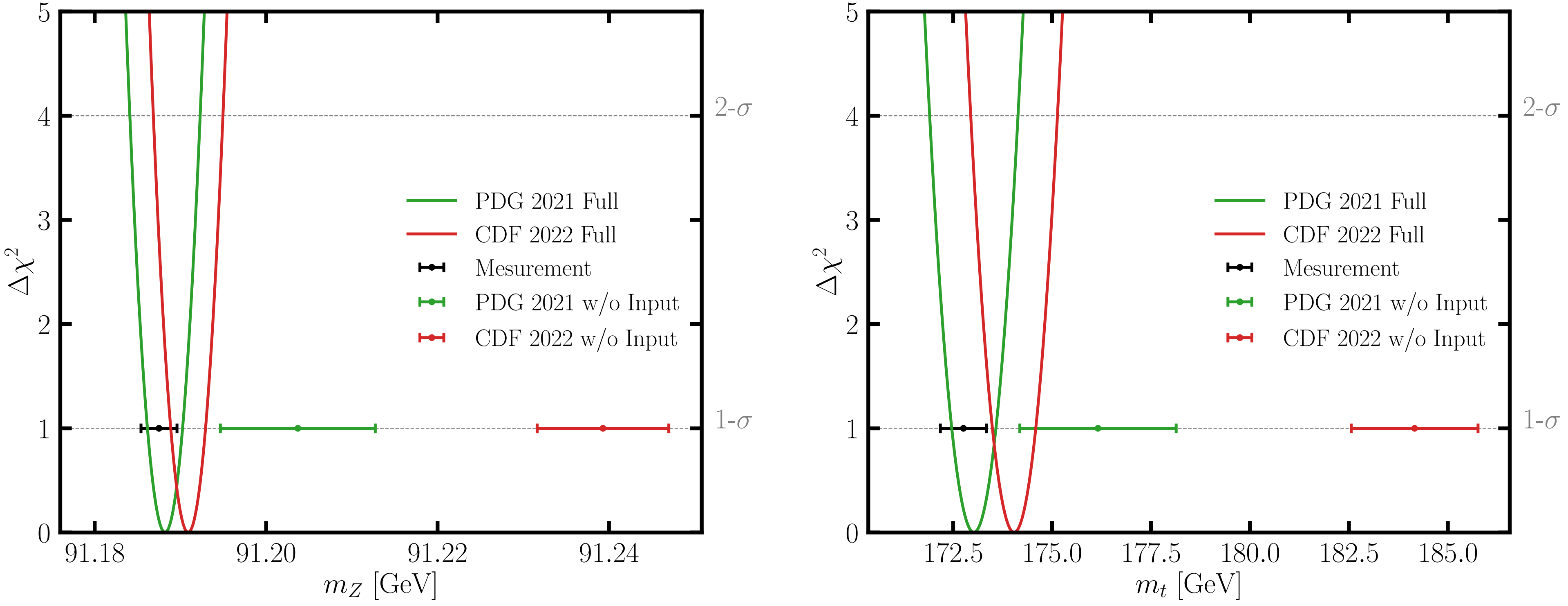
EW Global Fitting Individual Result

- $\Delta\alpha_{had}^{(5)}$
  - Is also important for g-2 interpretation
  - CDF prefer lower value which will enlarge the g-2 deviation



EW Global Fitting Individual Result

- $m_Z,\ m_t$
  - Fitting w/ CDF measurement prefer larger value



2. EW Precision Fits and NP Hints

~ 0.5 MeV precision wrt exact formulae

$$M_W = \left[M_W^0 + c_t\Delta_t + c_t'\Delta_t^2 + c_Z\Delta_Z + c_\alpha\Delta_\alpha + c_{\alpha_s}\Delta_{\alpha_s}\right]\ \text{MeV},$$

with the definitions,

0311148, Awramik, Czakon, Freitas, Weiglein

$$\Delta_t \equiv \left(\frac{m_t}{173\ \text{GeV}}\right)^2 - 1, \qquad \Delta_Z \equiv \frac{M_Z}{91.1876\ \text{GeV}} - 1, \qquad \Delta_\alpha \equiv \frac{\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)}{0.0276} - 1, \qquad \Delta_{\alpha_s} \equiv \frac{\alpha_s(M_Z^2)}{0.119} - 1,$$

and the numerical values,

$$M_W^0 = 80359.5 \quad c_t = 520.5 \quad c_t' = -67.7 \quad c_Z = 115000. \quad c_\alpha = -503. \quad c_{\alpha_s} = -71.6$$

M_W correlates
- **positively** with m_t and M_Z
- **negatively** with $\alpha(M_Z)$ and $\alpha_s(M_Z)$

Oblique Parameters

- New CDF measurement worse the SM global fitting
- New Physics can be parameterized through
  - its correction to the self-energy of gauge bosons

$$\frac{\alpha S}{4s_w^2c_w^2} = \left[\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}\right] - \frac{(c_w^2 - s_w^2)}{s_w c_w}\,\delta\Pi'_{Z\gamma}(0) - \delta\Pi'_{\gamma\gamma}(0),$$

$$\alpha T = \frac{\delta\Pi_{WW}(0)}{M_W^2} - \frac{\delta\Pi_{ZZ}(0)}{M_Z^2},$$

$$\frac{\alpha U}{4s_w^2} = \left[\frac{\delta\Pi_{WW}(M_W^2) - \delta\Pi_{WW}(0)}{M_W^2}\right] - c_w^2\left[\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}\right] - s_w^2\delta\Pi'_{\gamma\gamma}(0) - 2s_w c_w\delta\Pi'_{Z\gamma}(0).$$

$$m_W^2 = \left(m_W^{SM}\right)^2\left[1 - \frac{\alpha S}{2(c_w^2 - s_w^2)} + \frac{c_w^2\alpha T}{c_w^2 - s_w^2} + \frac{\alpha U}{4s_w^2}\right]$$

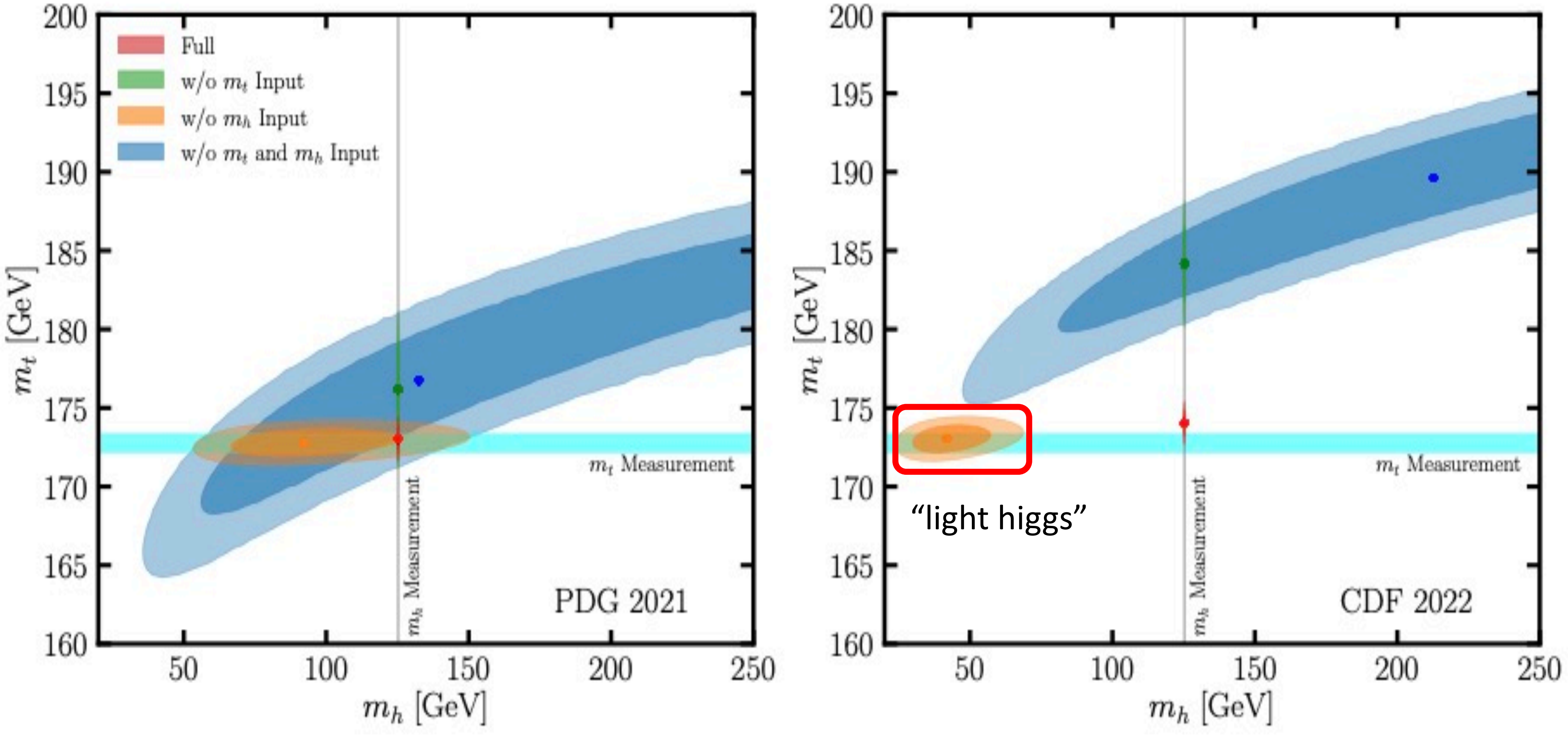
Fitting with S/T/U

- Fitting with S/T/U
  - Need larger U

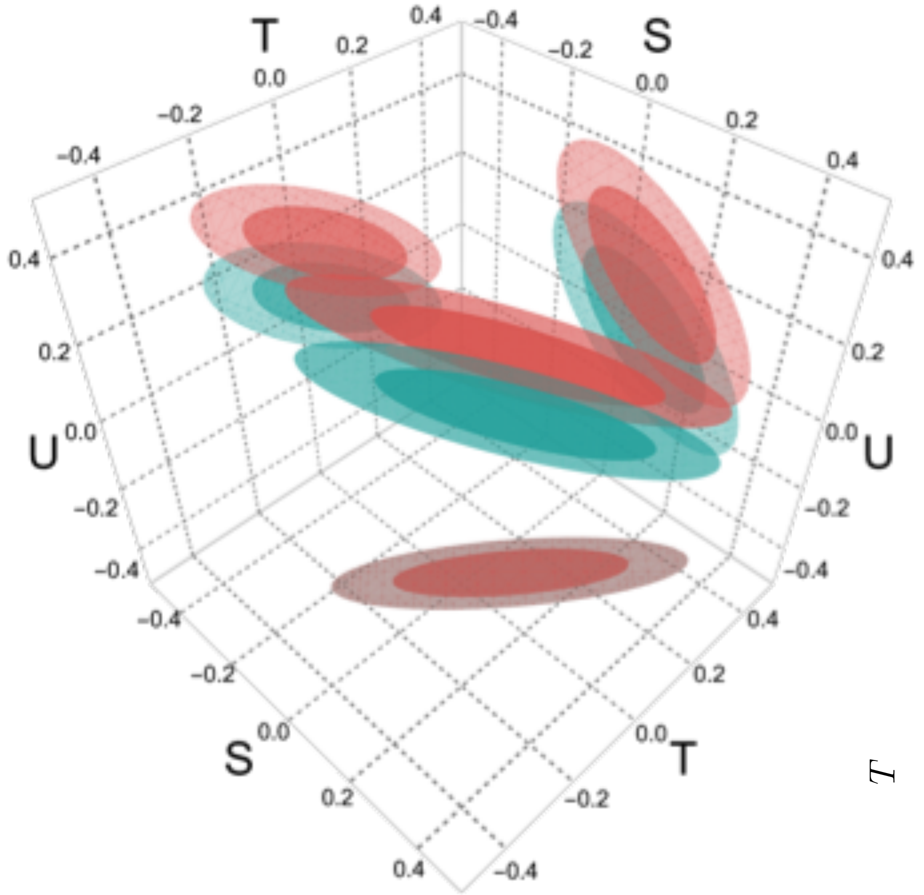
13 dof	PDG 2021				CDF 2022			
	Result	Correlation			Result	Correlation		
	$\chi^2_{\min} = 15.42$	S	T	U	$\chi^2_{\min} = 15.44$	S	T	U
S	0.06 ± 0.10	1.00	0.90	-0.57	0.06 ± 0.10	1.00	0.90	-0.59
T	0.11 ± 0.12		1.00	-0.82	0.11 ± 0.12		1.00	-0.85
U	-0.02 ± 0.09			1.00	0.14 ± 0.09			1.00

- Fix U = 0
  - S and T are significantly deviated from 0

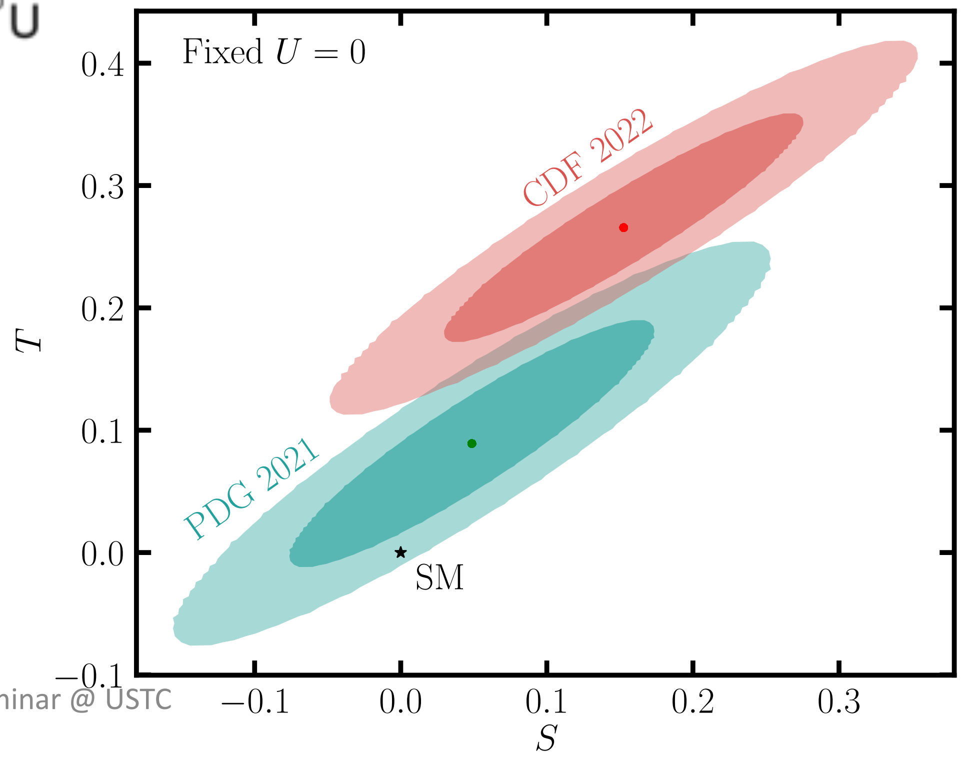
$U = 0$	PDG 2021			CDF 2022		
	Result	Correlation		Result	Correlation	
14 dof	$\chi^2_{\min} = 15.48$	S	T	$\chi^2_{\min} = 17.82$	S	T
S	0.05 ± 0.08	1.00	0.92	0.15 ± 0.08	1.00	0.93
T	0.09 ± 0.07		1.00	0.27 ± 0.06		1.00


Full
w/o m_t Input
w/o m_h Input
w/o m_t and m_h Input
m_t [GeV]
m_h [GeV]
m_t Measurement
m_h Measurement
PDG 2021
CDF 2022
"light higgs"

Fitting with S/T/U



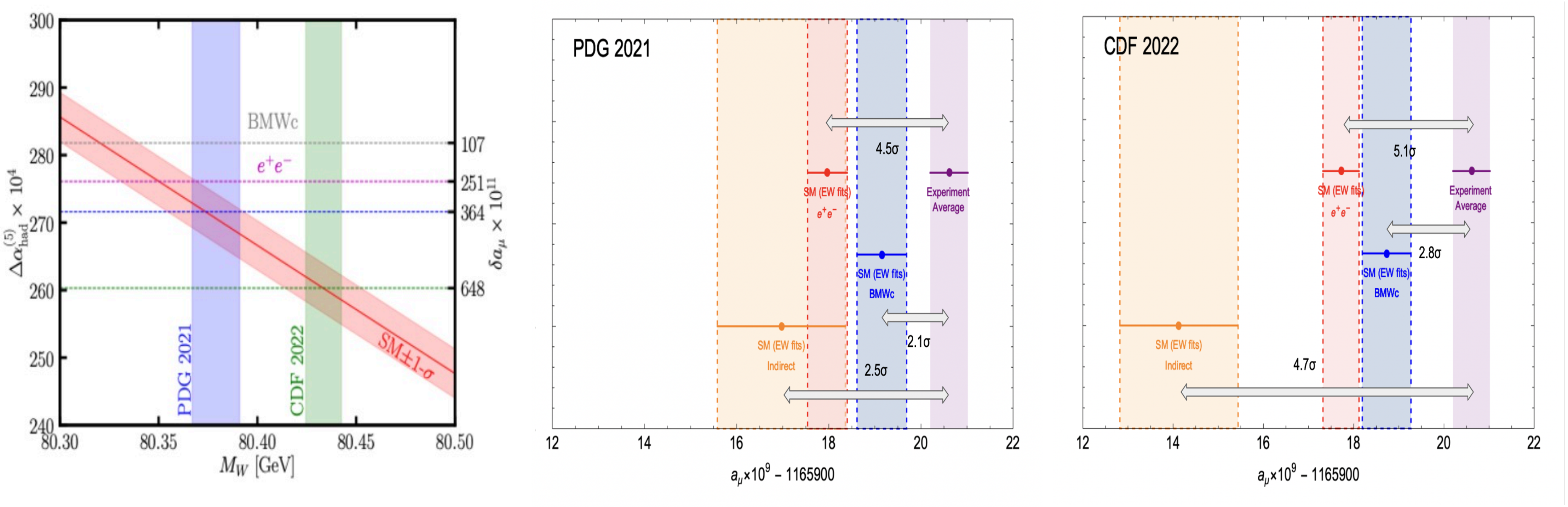
- Most BSM has $U \approx 0$
  - Large correction to S/T is needed



B3. EW Fit and Muon g-2

Muon g-2:

Relationship: $$\Delta\alpha_{had}^{(5)}(ew) = \Delta\alpha_{had}^{(5)}(e^+e^-) * \frac{\delta a_\mu^{HVP}(ew)}{\delta a_\mu^{HVP}(e^+e^-)}$$ *Nature* 593 (2021) 7857, 51-55



2204.03996, Athron, Fowlie, Lu, Wu, Wu, Zhu

- Definition of R value

$$R \equiv \frac{\sigma^0(e^+e^-\to \text{hadrons})}{\sigma^0(e^+e^-\to\mu^+\mu^-)} \equiv \frac{\sigma^0_{\text{had}}}{\sigma^0_{\mu\mu}}$$

EW fit

- Determination of running coupling constant of QED theory

$$\alpha \equiv \frac{\alpha_0}{1-\Delta\alpha},\ \Delta\alpha(s) = \Delta\alpha(s)_{\text{lep}} + \Delta\alpha(s)_{\text{had}}$$

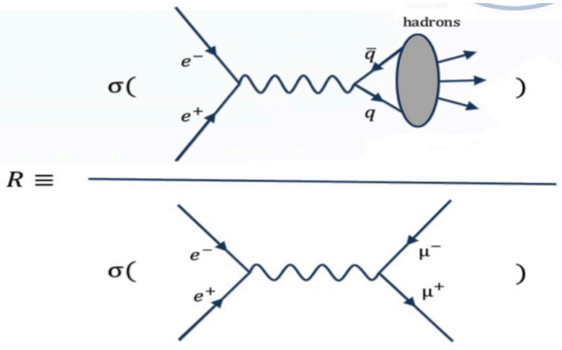
$$\Delta\alpha(M_Z^2) = -\frac{\alpha(0)M_Z^2}{3\pi} Re\int_{4M_\pi^2}^{\infty}\frac{ds\, R(s)}{s(s-M_Z^2)-i\epsilon}$$

- Anomalous magnetic moment of muon $g_\mu - 2$

$$a_\mu = \frac{g_\mu - 2}{2},\ a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{had}}$$

$$a_\mu^{\text{had}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s^2} R(s)$$

$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$: 4.2σ → new physics



Source	Contribution($\times 10^4$)
$\Delta\alpha_{\text{lepton}}(M_Z^2)$	314.979 ± 0.002
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$	276.0 ± 1.0
$\Delta\alpha_{\text{top}}(M_Z^2)$	-0.7180 ± 0.0054

Source	Contribution($\times 10^{11}$)
a_μ^{QED}	116 584 718.931(104)
a_μ^{Weak}	153.6(1.0)
a_μ^{had}[LO]	6931(40)
a_μ^{had}[NLO]	–98.3(7)
a_μ^{had}[NNLO]	12.4(1)
$a_\mu^{\text{had,l–l}}$	92(18)
a_μ^{SM}	116 591 810(43)
a_μ^{exp}	116 592 061(41)
Δa_μ	251(59)

From Dong Liu's Slides @LP2021

The time-like and space-like master formulae for $\Delta\alpha_{\rm had}$ and $a_\mu^{\rm HVP}$

1. **Time-like** : Using for $e^+e^- \to \rm hadrons$ data calculations

$$a_\mu^{\rm HVP,\,LO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\rm thr}}^{\infty} ds\, \frac{\hat{K}(s)}{s^2} R_{\rm had}(s)$$

$$R_{\rm had}(s) = \frac{3s}{4\pi\alpha^2}\sigma(e^+e^- \to \text{hadrons})$$

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\rm thr}}^{\infty} ds\, \frac{R_{\rm had}(s)}{s(M_Z^2 - s)}$$

where $s_{\rm thr} = m_{\pi^0}^2$

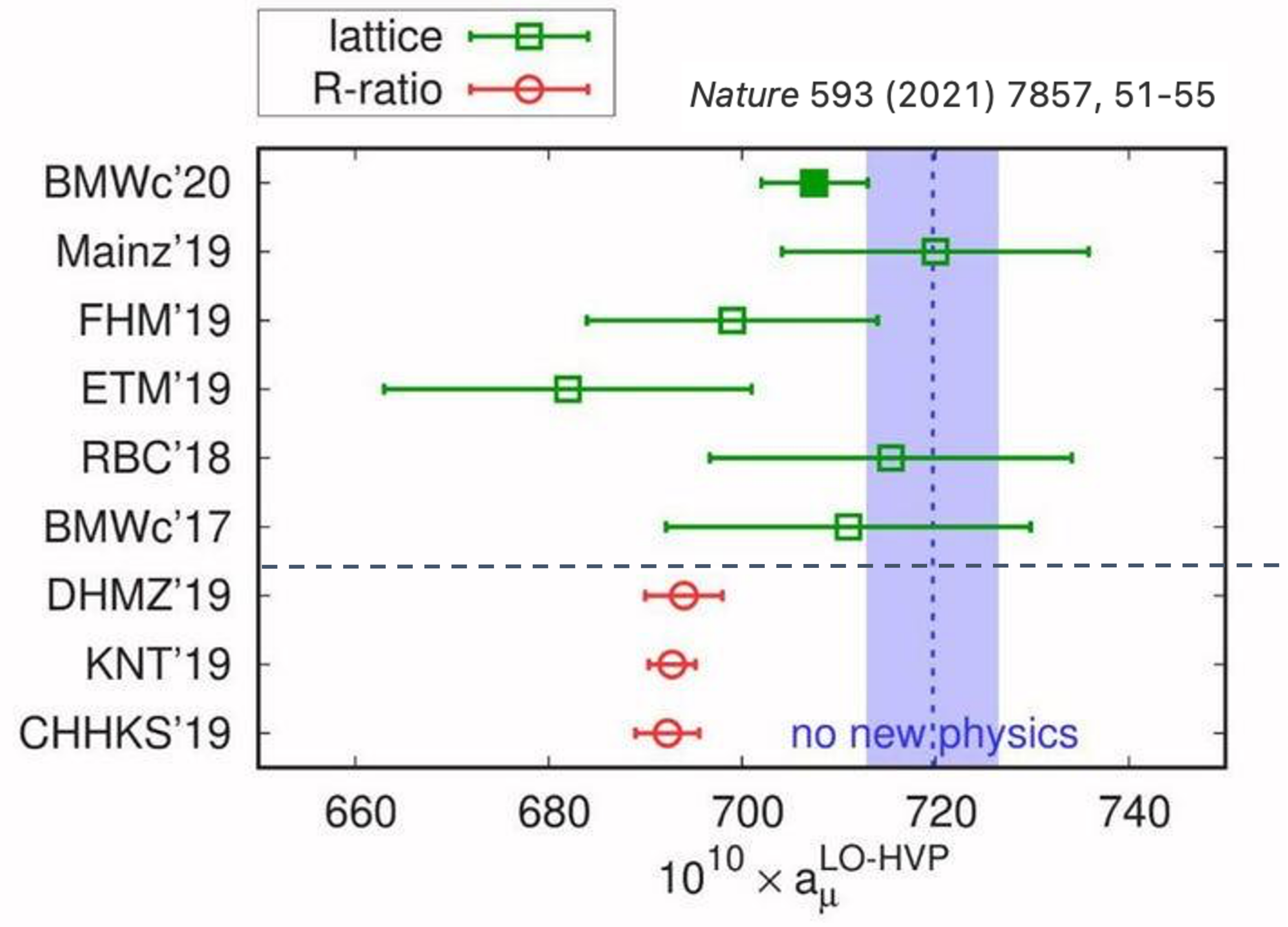
1. **Space-like** : Using for lattice QCD calculations

$$a_\mu^{\rm HVP,\,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} ds\, f(s)\hat{\Pi}(-s)$$

$$\hat{\Pi}(s) = 4\pi^2\left[\Pi(s) - \Pi(0)\right]$$

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi}\left(\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2)\right)$$

A summary for a_μ^{HVP} form data-driven and lattice QCD calculations

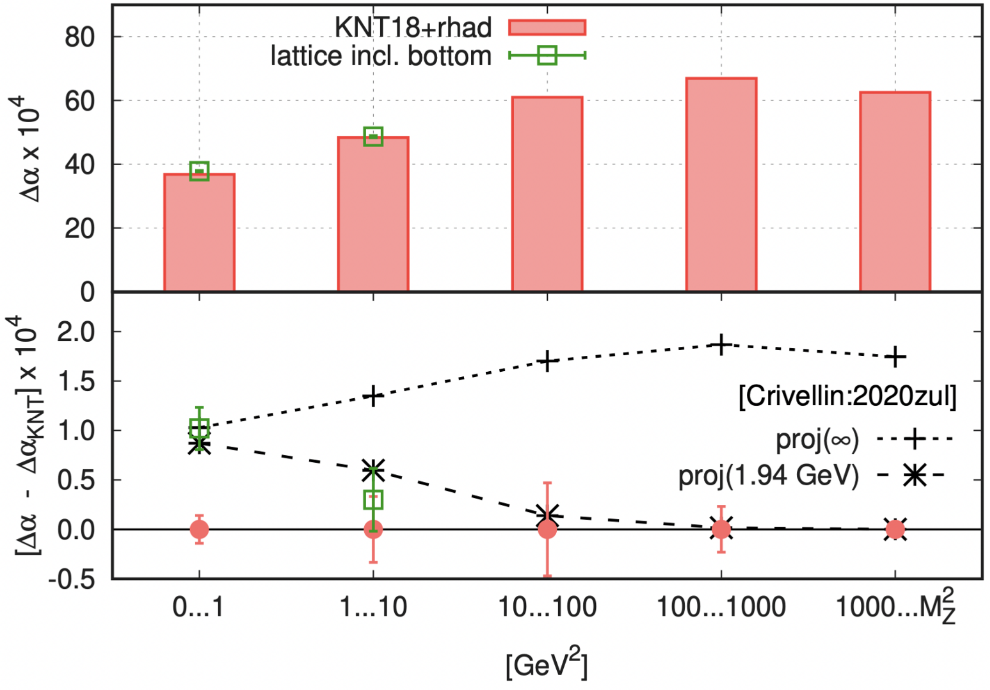


$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty ds\, f(s) \hat{\Pi}(-s)$$

$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\mathrm{thr}}}^\infty ds\, \frac{\hat{K}(s)}{s^2} R_{\mathrm{had}}(s)$$

The problem to compare $\Delta\alpha_{\rm had}$ form data-driven and lattice QCD

1. The $\Delta\alpha_{\rm had}$ is calculated at the scale MZ for five quark flavors from data-driven method with $\Delta\alpha^{(5)}_{\rm had}|_{e^+e^-} = 276.1(1.1) \times 10^{-4}$. KNT, DHMZ
2. However, we don't have enough informaiton for $\Delta\alpha_{\rm had}$ from lattice QCD side.



For example,
using the whole energy range project [proj(∞)] :

$$a_\mu^{\rm HVP}({\rm BMWc}) = 707.5(5.5) \times 10^{-10}$$

$$\Rightarrow \Delta\alpha_{\rm had}({\rm BMWc}) = 276.1(1.1) \times 10^{-4} \times \frac{707.5}{693.1} = 281.8(1.5) \times 10^{-4}$$

Nature 593 (2021) 7857, 51-55

The 3rd way to extract $\Delta\alpha_{\rm had}$ Global Electrowek Fits

GFitter

P.Athron, A.Fowlie, C.T.Lu., L.Wu, Y.C.Wu, B.Zhu

M_W		Indirect			PDG 2021			CDF 2022		
$\Delta\alpha_{\rm had}$		BMWc	e^+e^-	Indirect	BMWc	e^+e^-	Indirect	BMWc	e^+e^-	Indirect
Input	M_W [GeV]	-	-	-	80.379(12)	80.379(12)	80.379(12)	80.4335(94)	80.4335(94)	80.4335(94)
	$\Delta\alpha_{\rm had} \times 10^4$	281.8(1.5)	276.1(1.1)	-	281.8(1.5)	276.1(1.1)	-	281.8(1.5)	276.1(1.1)	-
Fitted	χ^2/dof	18.32/15	16.01/15	15.89/14	23.41/16	18.74/16	17.59/15	74.51/16	62.58/16	47.19/15
	M_W [GeV]	80.348(6)	80.357(6)	80.359(9)	80.355(6)	80.361(6)	80.367(7)	80.375(5)	80.380(5)	80.396(7)
	$\Delta\alpha_{\rm had} \times 10^4$	280.9(1.4)	275.9(1.1)	274.4(4.4)	280.3(1.4)	275.6(1.1)	271.7(3.8)	278.6(1.4)	274.7(1.0)	260.9(3.6)
	$\delta a_\mu \times 10^{11}$	-	-	294(166)	146(68)	264(59)	364(145)	188(68)	289(57)	648(137)
	Tension	-	-	1.8σ	2.1σ	4.5σ	2.5σ	2.8σ	5.1σ	4.7σ
	δM_W [MeV]	86(11)	77(11)	75(13)	79(11)	73(11)	67(12)	59(11)	54(11)	38(12)
	Tension	7.8σ	7.0σ	5.8σ	7.2σ	6.6σ	5.6σ	5.4σ	4.9σ	3.2σ

Then, how to transform the information between $\Delta\alpha_{\rm had}$ and $a_\mu^{\rm HVP}$?

Here we consider the whole energy range projection.

Using Global EW Fits to extract $\Delta\alpha_{\rm had}$: Low energy projection

P.Athron, A.Fowlie, C.T.Lu., L.Wu, Y.C.Wu, B.Zhu

GFitter

M_W		Indirect			PDG 2021			CDF 2022		
$\Delta\alpha_{\rm had}$		BMWc	e^+e^-	Indirect	BMWc	e^+e^-	Indirect	BMWc	e^+e^-	Indirect
Input	M_W [GeV]	-	-	-	80.379(12)	80.379(12)	80.379(12)	80.4335(94)	80.4335(94)	80.4335(94)
	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)\times 10^4$	277.4(1.2)	276.1(1.1)	-	277.4(1.2)	276.1(1.1)	-	277.4(1.2)	276.1(1.1)	-
Fitted	χ^2/dof	16.28/15	16.01/15	15.89/14	19.51/16	18.74/16	17.59/15	65.07/16	62.58/16	47.19/15
	M_W [GeV]	80.355(6)	80.357(6)	80.359(9)	80.360(6)	80.361(6)	80.367(7)	80.379(5)	80.380(5)	80.396(7)
	$\Delta\alpha_{\rm had}\times 10^4$	277.1(1.2)	275.9(1.1)	274.4(4.4)	276.8(1.1)	275.6(1.1)	271.7(3.8)	275.6(1.1)	274.7(1.0)	260.9(3.6)
	$\delta a_\mu \times 10^{11}$	-	-	438(396)	173(54)	306(54)	748(339)	306(54)	416(54)	1997(320)
	Tension	-	-	1.1σ	3.2σ	5.7σ	2.2σ	5.7σ	7.7σ	6.2σ
	δM_W [MeV]	79(11)	77(11)	75(13)	74(11)	73(11)	67(12)	55(11)	54(11)	38(12)
	Tension	7.2σ	7.0σ	5.8σ	6.7σ	6.6σ	5.6σ	5.0σ	4.9σ	3.2σ

For the case of low energy projection, $\Delta\alpha_{\rm had}$ is shrunk, but $a_\mu^{\rm HVP}$ is enlarged after the transformation compared with the whole energy range projection.

Three various projections between $\Delta\alpha_{\rm had}$ and $a_\mu^{\rm HVP}$

1. According to *Crivellin:2020zul*, there are three different hypotheses for the projection between

   $\Delta\alpha_{\rm had}$ and $a_\mu^{\rm HVP}$:

   (1) Low energy for the sum of exclusive channels : $m_{\pi_0} \leq \sqrt{s} \leq 1.937\,{\rm GeV}$,

   (2) Energy below the perturbative contributions : $m_{\pi_0} \leq \sqrt{s} \leq 11.199\,{\rm GeV}$ or

   (3) The whole energy range : $m_{\pi_0} \leq \sqrt{s} \leq \infty$,

(**Hypothesis** : The part above the upper energy threshold is the same as data driven one and the uniform scaling is applied.)

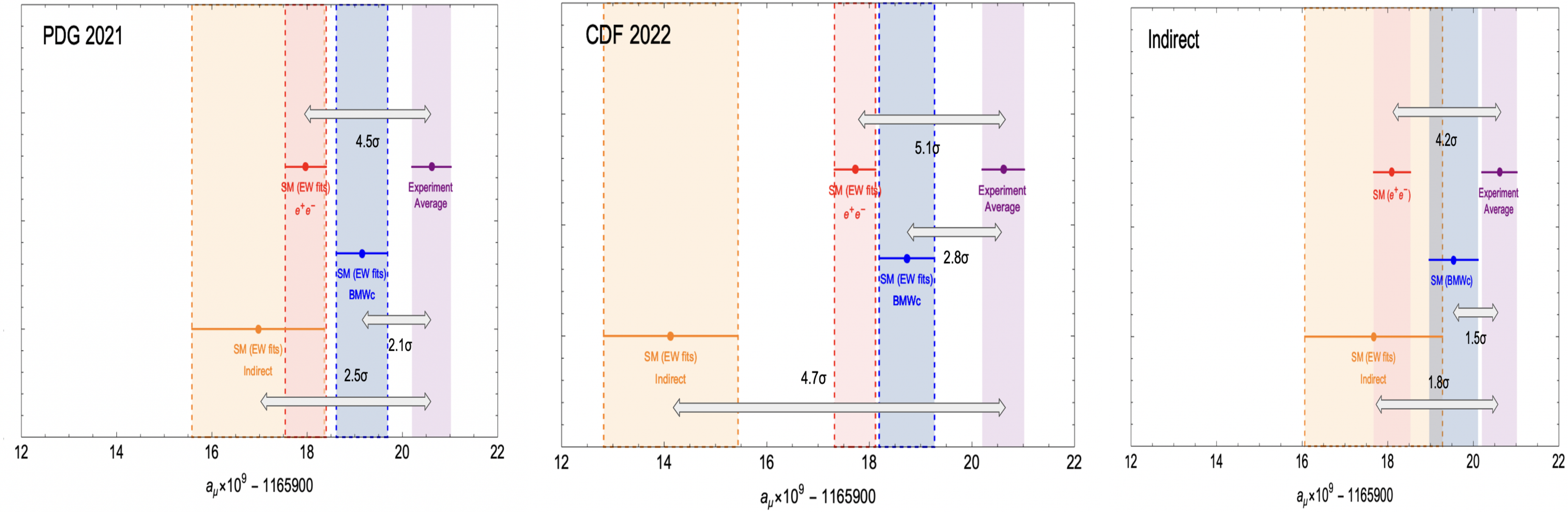
1. Open questions : (1) Which projection should be preferred ?

   (The low energy projection agrees better with BMWc restuls.)

   (2) Can we go beyond the uniform scaling (energy independent) hypothesis ?

The impact to muon g-2 from the global EW fits

P.Athron, A.Fowlie, C.T.Lu., L.Wu, Y.C.Wu, B.Zhu



The EW fits from indirect and BMWc inputs go into opposite direction compared with the one fromm e+e- input.

Channel	Energy range (GeV)	$a_\mu^{\text{had, LO VP}} \times 10^{10}$	$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4$	Our results
Chiral perturbation theory (ChPT) threshold contributions				
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01	0.00 ± 0.00	0.01 ± 0.00
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	0.01 ± 0.00
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	0.01 ± 0.00
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
Data based channels ($\sqrt{s} \le 1.937$ GeV)				
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	0.43 ± 0.01
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	26.60 ± 0.10
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	6.51 ± 0.12
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	1.85 ± 0.02
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	3.57 ± 0.14
$(2\pi^+2\pi^-\pi^0)_{\text{no }\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	0.22 ± 0.02
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	0.07 ± 0.00
$(2\pi^+2\pi^-2\pi^0)_{\text{no }\eta\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	0.41 ± 0.05
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22	3.37 ± 0.03	5.05 ± 0.05
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	2.91 ± 0.04
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	0.78 ± 0.03
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	0.60 ± 0.02
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	0.09 ± 0.02
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06	0.39 ± 0.02	0.32 ± 0.01
$(\eta\pi^+\pi^-\pi^0)_{\text{no }\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15	0.21 ± 0.05	0.18 ± 0.05
$\eta 2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01	0.03 ± 0.00	0.02 ± 0.00
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	0.09 ± 0.01
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	0.18 ± 0.00
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	0.15 ± 0.01
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04	0.01 ± 0.01	0.01 ± 0.01
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	0.13 ± 0.03
$\eta(\to \text{npp})K\bar{K}_{\text{no }\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	0.00 ± 0.01
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	0.01 ± 0.00
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01	0.01 ± 0.00	0.01 ± 0.00
Estimated contributions ($\sqrt{s} \le 1.937$ GeV)				
$(\pi^+\pi^-3\pi^0)_{\text{no }\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04	0.16 ± 0.01	0.11 ± 0.01
$(\pi^+\pi^-4\pi^0)_{\text{no }\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21	0.08 ± 0.08	0.06 ± 0.06
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02	0.02 ± 0.01	0.01 ± 0.01
$\omega(\to \text{npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02	0.03 ± 0.01	0.03 ± 0.01
$\omega(\to \text{npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.06 ± 0.01	0.05 ± 0.01
$\omega(\to \text{npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.03 ± 0.02	0.02 ± 0.01
Other contributions ($\sqrt{s} > 1.937$ GeV)				
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	69.94 ± 1.07
J/ψ	-	6.26 ± 0.19	7.07 ± 0.22	10.03 ± 0.30
ψ'	-	1.58 ± 0.04	2.51 ± 0.06	2.53 ± 0.06
$\Upsilon(1S-4S)$	-	0.09 ± 0.00	1.06 ± 0.02	0.14 ± 0.00
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	127.25 ± 0.00
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46	276.11 ± 1.11	260.39 ± 1.14

B4. 2HDM

2HDM (alignment):

0207010, Gunion and Haber

$$\Phi_i = \begin{pmatrix} w_i^+ \\ \dfrac{v_i + h_i + i\eta_i}{\sqrt{2}} \end{pmatrix}, \quad i = 1, 2,$$

making a global SU(2) transformation in the scalar space spanned by the two doublets, it is always possible to work in the so-called Higgs basis,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1\Phi_1 + v_2\Phi_2}{v}, \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2\Phi_1 + v_1\Phi_2}{v},$$

$$\mathcal{V} \ni \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 + \left\{\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + Z_6(H_1^\dagger H_1)(H_1^\dagger H_2) + \text{h.c.}\right\}$$

2HDM (alignment):

$$\mathcal{M}_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix}$$

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}\,H_1^0 - v \\ \sqrt{2}\,\mathrm{Re}\,H_2^0 \end{pmatrix}$$

$$Z_1 v^2 = m_h^2 s_{\beta-\alpha}^2 + m_H^2 c_{\beta-\alpha}^2\,,$$

$$Z_6 v^2 = (m_h^2 - m_H^2) s_{\beta-\alpha} c_{\beta-\alpha}\,,$$

$$Z_5 v^2 = m_H^2 s_{\beta-\alpha}^2 + m_h^2 c_{\beta-\alpha}^2 - m_A^2\,.$$

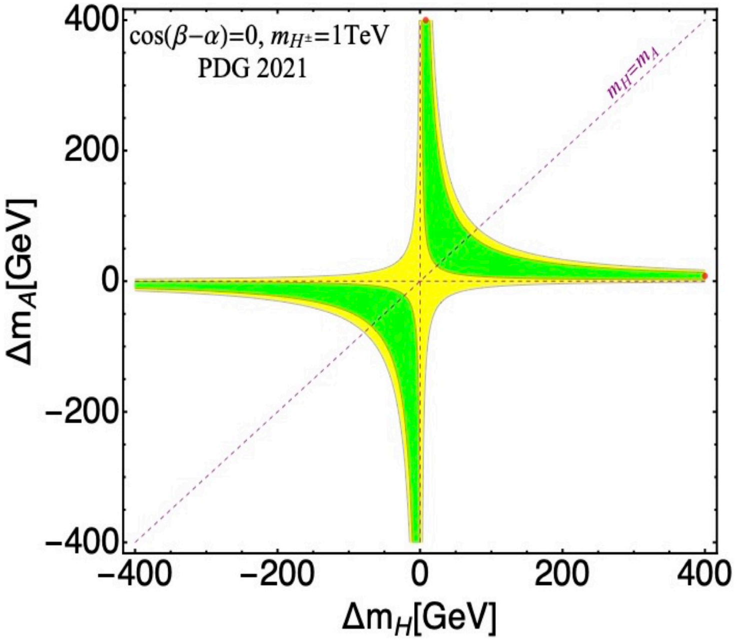
h is SM-like if $|c_{\beta-\alpha}| \ll 1$

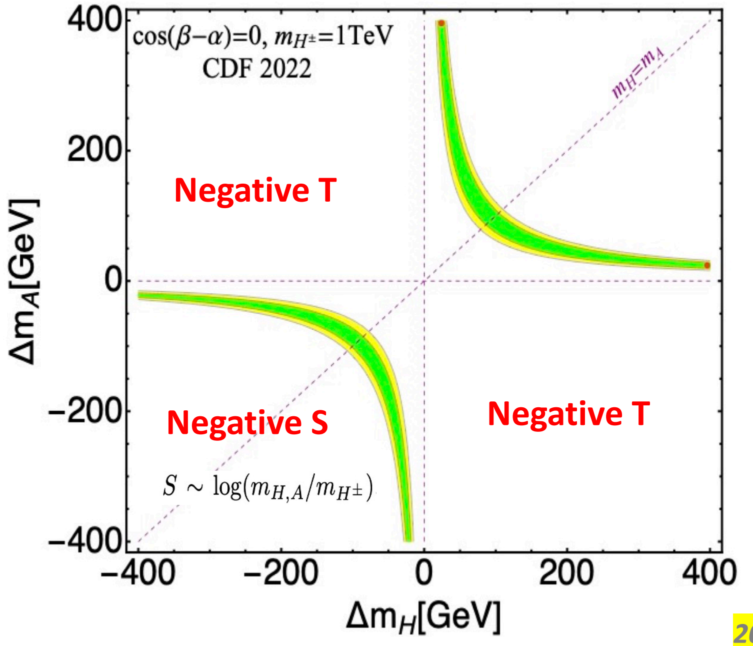
Alignment limit: Higgs base = Mass base

2HDM (alignment):

$$T = \frac{1}{32\pi \sin^2\theta_W m_W^2}\left[\theta_+(m_+, m_1) + \theta_+(m_+, m_2) - \theta_+(m_1, m_2)\right]$$

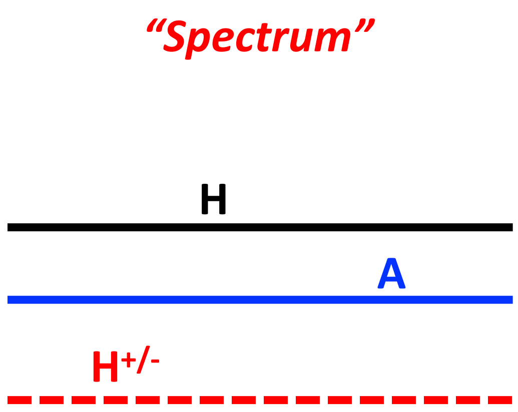
$$\theta_+(m_+, m_1) + \theta_+(m_+, m_2) - \theta_+(m_1, m_2)$$
$$= 32 i\pi^2 \int \frac{d^4k}{(2\pi)^4} k^2 \frac{(m_+^2 - m_1^2)(m_+^2 - m_2^2)}{(k^2 - m_+^2)^2 (k^2 - m_1^2)(k^2 - m_2^2)} .$$

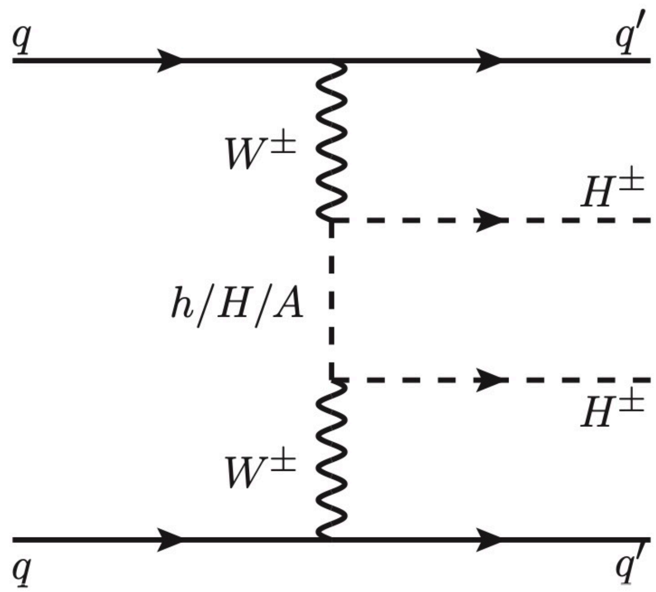




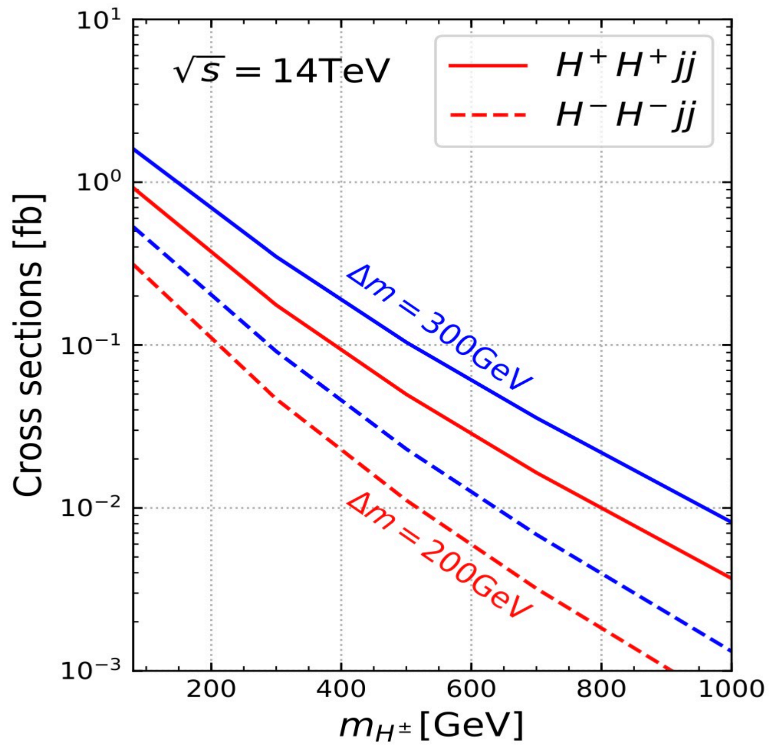
2022.03796, Lu, Wu, Wu, Zhu

2HDM (alignment):

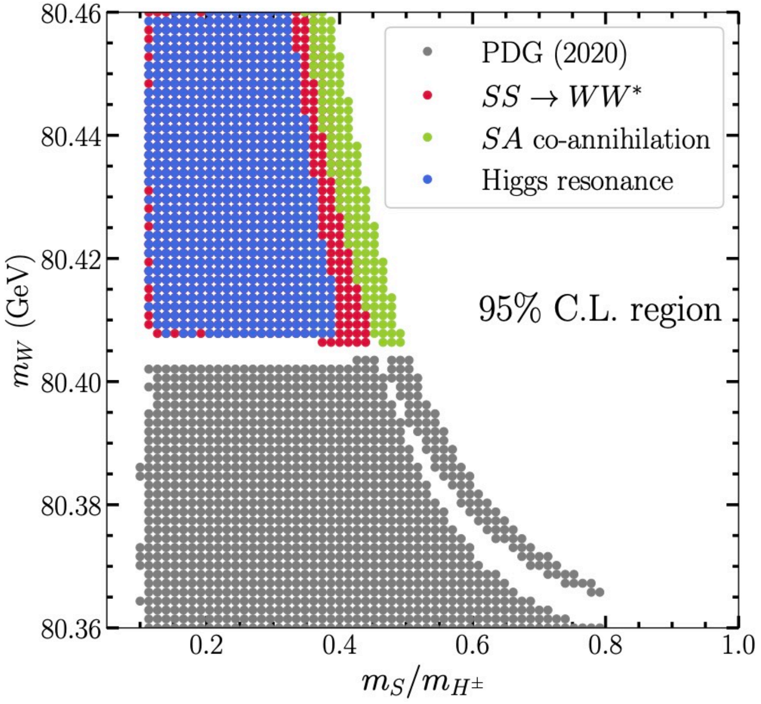




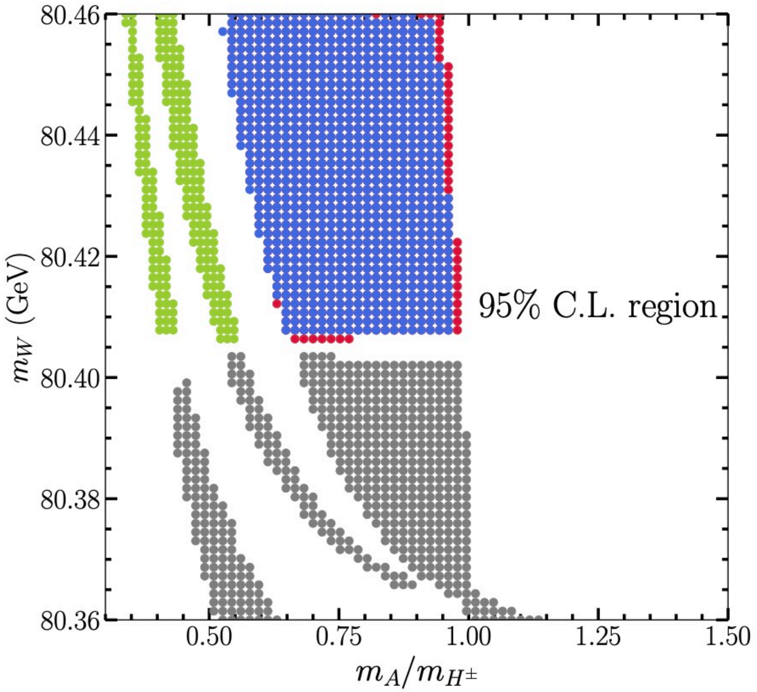
1906.09101, Aiko, Kanemura, Mawatari



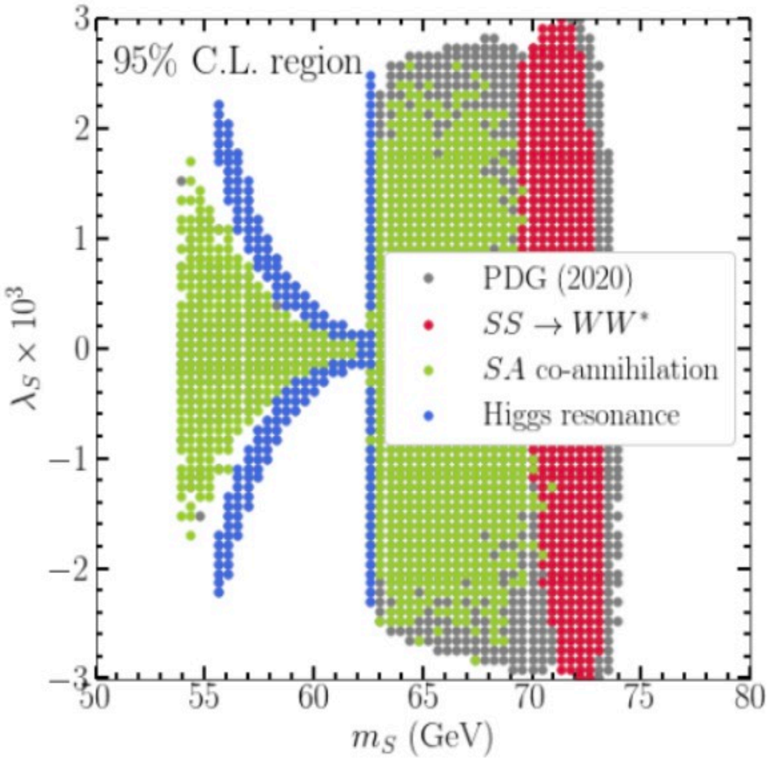
2HDM (inert):

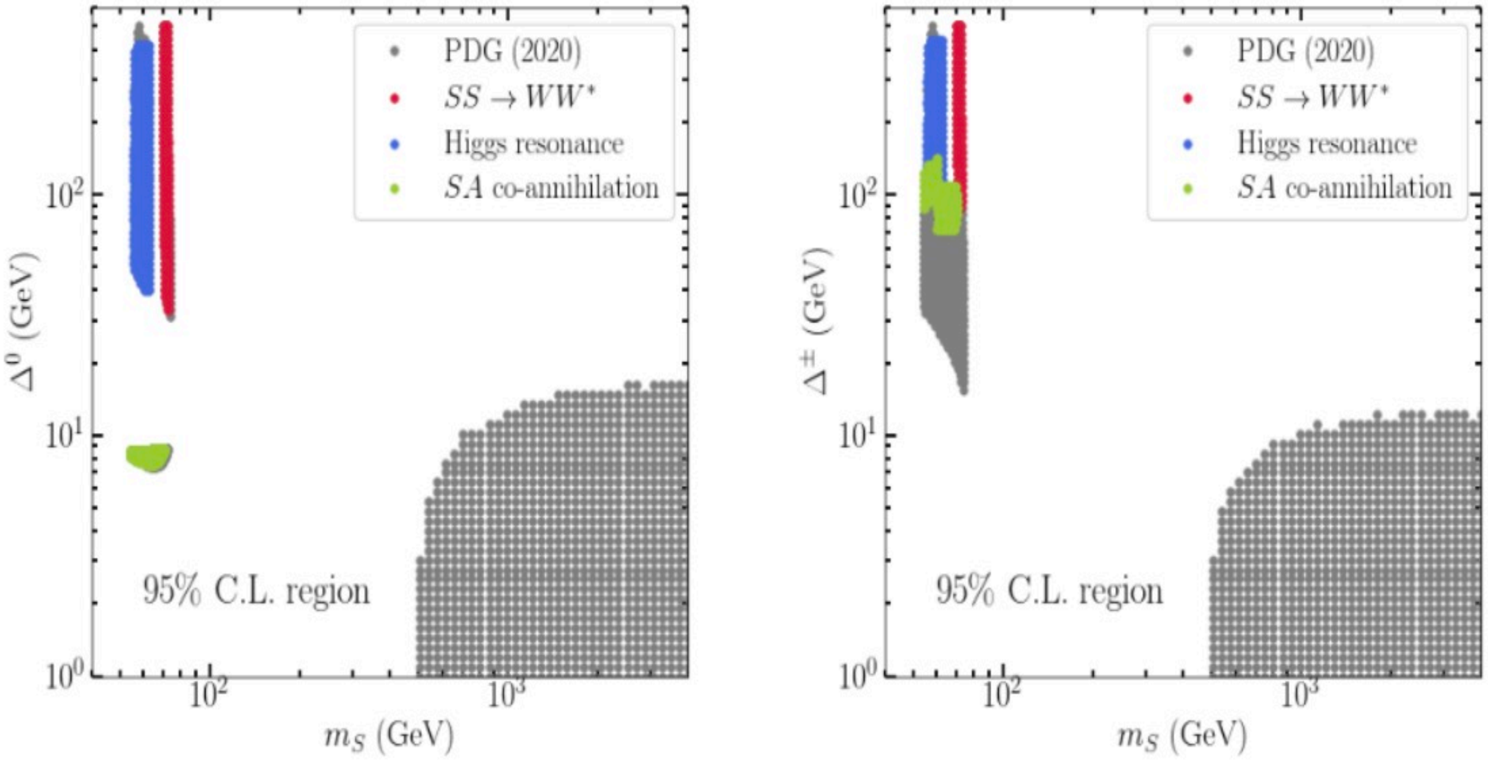


80.46
80.44
80.42
80.40
80.38
80.36
m_W (GeV)
0.2
0.4
0.6
0.8
1.0
$m_S/m_{H^\pm}$
PDG (2020)
$SS \to WW^*$
SA co-annihilation
Higgs resonance
95% C.L. region

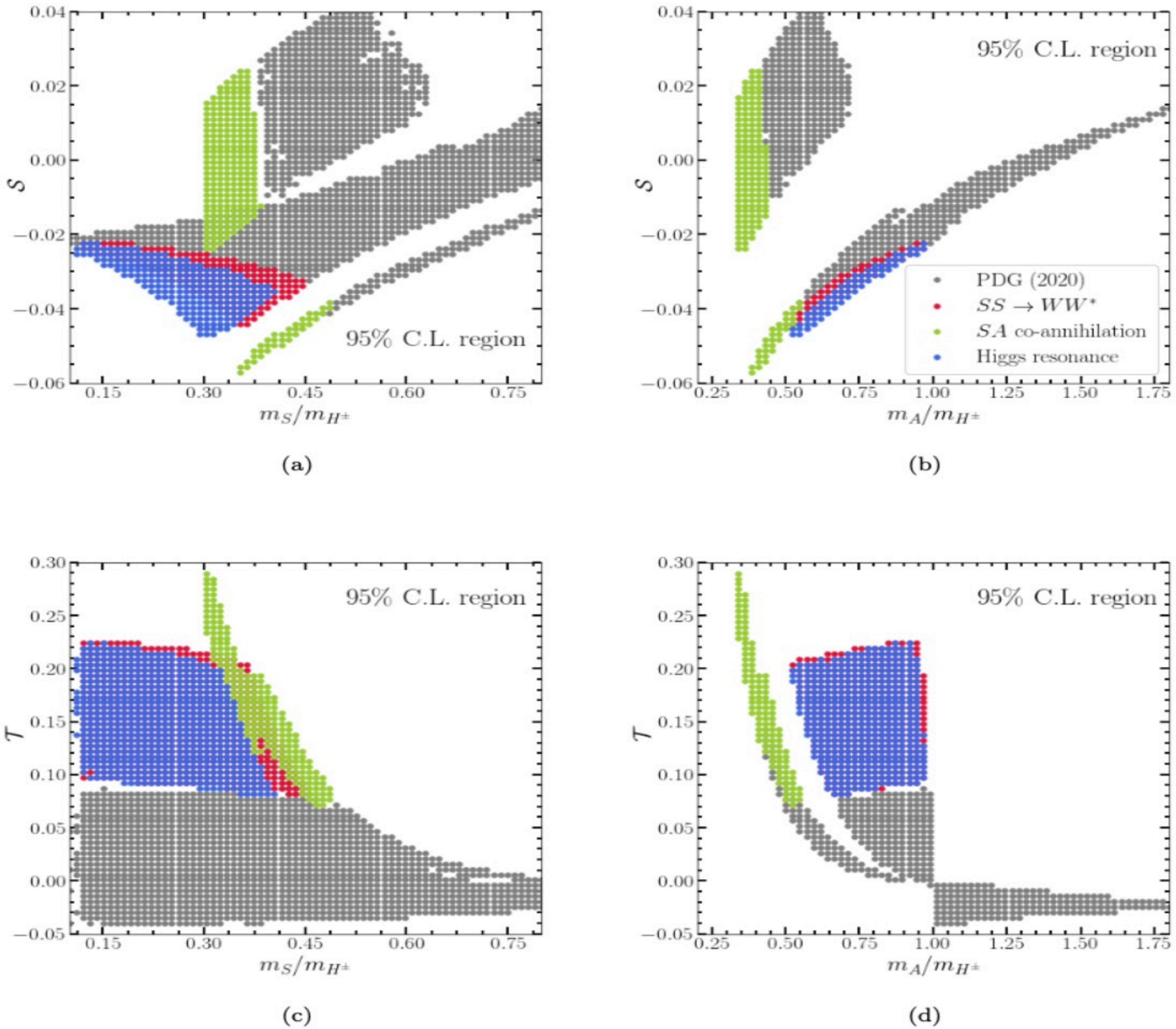


80.46
80.44
80.42
80.40
80.38
80.36
m_W (GeV)
0.50
0.75
1.00
1.25
1.50
$m_A/m_{H^\pm}$
95% C.L. region


95% C.L. region
PDG (2020)
$SS \to WW^*$
SA co-annihilation
Higgs resonance
$\lambda_S \times 10^3$
m_S (GeV)


PDG (2020)
$SS \to WW^*$
Higgs resonance
SA co-annihilation
95% C.L. region
Δ^0 (GeV)
$\Delta^\pm$ (GeV)
m_S (GeV)

(a) (b)


95% C.L. region
PDG (2020)
$SS \to WW^*$
SA co-annihilation
Higgs resonance
$\mathcal{S}$
$\mathcal{T}$
$m_S/m_{H^\pm}$
$m_A/m_{H^\pm}$
(a)
(b)
(c)
(d)

Monash U.

PMO