Biologically motivated 3rd universal dynamical framework in sciences and technologies:

Issues on Evolution Mechanics

University of Science and Technology of China, Hefei, Anhui Province, China
2023.03.22
Ping Ao aoping@sjtu.edu.cn
College of Biomedical Engineering, Sichuan University, Chengdu, Sichuan, China

生命科学启发的第三个普适动力学框架：

演化力学的几个科学和数学问题

中国科学技术大学　粒子物理与原子核物理学科　安徽省基础学科（理论物理）研究中心
物质科研楼 C1403　合肥，安徽
2024.03.22

敖平，　四川大生物医学工程学院，成都，四川

(生命现象的量化；生命现象的机理；生命现象的运用)

I 量化达尔文演化动力学 Quantification of Darwin’s evolutionary process

II 新数学结构 **Needed mathematical structure**

III 对物理和其它科学领域的意义 Implications for physics and other scientific and technological fields

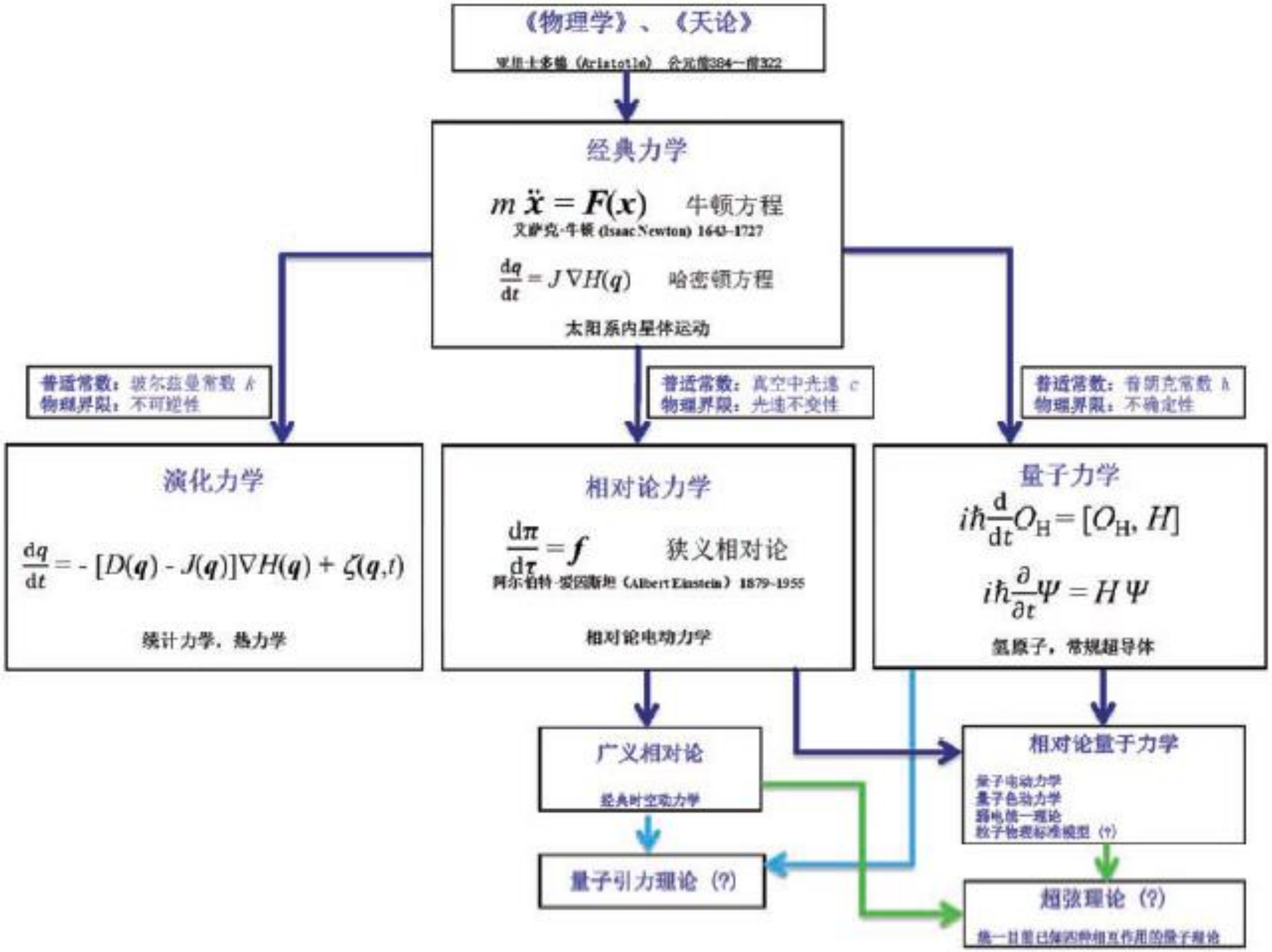


图 1 (网络版彩色)运动方程理论的演化: 时间的特殊地位

Charles Robert Darwin, 1809-1882

Modern word equation:

Evolution by Variation and Selection

Classical statement: principle of natural selection

Ludwig Eduard Boltzmann, 1844-1906

Modern expression:

$\rho \propto \exp(-H/kT)$

Classical formula: $S = k \ln(W)$

Evolution by Variation and Selection

演化由确定性的选择与随机性的变异共同地产生

（文字表述：论文- 达尔文、华莱斯，1858；　书- 《物种起源》，达尔文，1859）

- "… it requires interpretation to bring out the originality and force of the argument, more so than if Darwin were a Newton or an Einstein abstracting far beyond everyman's power to follow and to un*derstand.* ***A hero of science should be less accessible."***

C.C. Gillispie, The edge of objectivity, 1990

Consistent theoretical formulation was established in 2005:

- Laws of Darwinian evolutionary theory. P. Ao. Physics of Life Reviews 2 (**2005**) 117-156

  演化由确定性的选择与随机性的变异共同地产生 ⇨ $dx/dt = f(\mathbf{x}) + \zeta(\mathbf{x},t)$
- Emerging of Stochastic Dynamical Equalities and Steady State Thermodynamics from Darwinian Dynamics, P. Ao, Communications in Theoretical Physics 49 (**2008**) 1073-1090
- Darwinian dynamics implies developmental ascendency. P. Ao, Biological Theory 2 (**2007**) 113-115
- Equivalent formulations of "the equation of life". P. Ao. Chinese Physics B23 (**2014**) 070513

Selection makes complete disorder impossible; and variation makes complete order impossible.

选择让完全无序不能实现；变异让完全有序也不能实现。

RA Fisher and S Wright

3 main characteristics of evolutionary processes: stochasticity; adaptation; optimization

- RA Fisher: Fundamental theorem of natural selection(FTNS), 1930

  ‘The rate of increase in fitness of any orgainism at any time is **equal** to its genetic variance in fitness at that time.’

  ~ fluctuation-dissipation theorem(FDT) in physics, 1950’s

- S Wright: adaptive landscape, 1932

  ~ energy function in physics

434 SEWALL WRIGHT

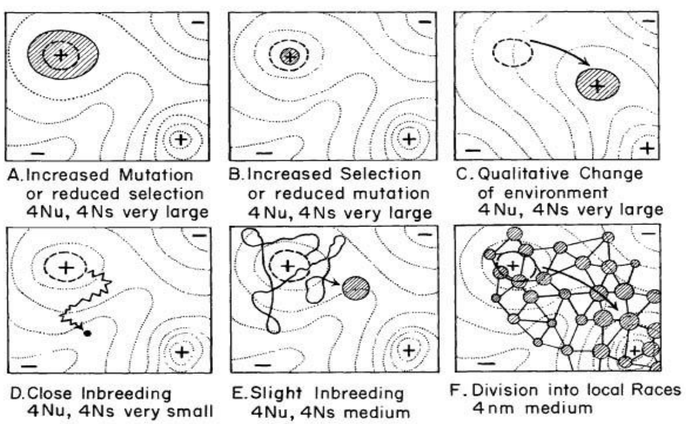
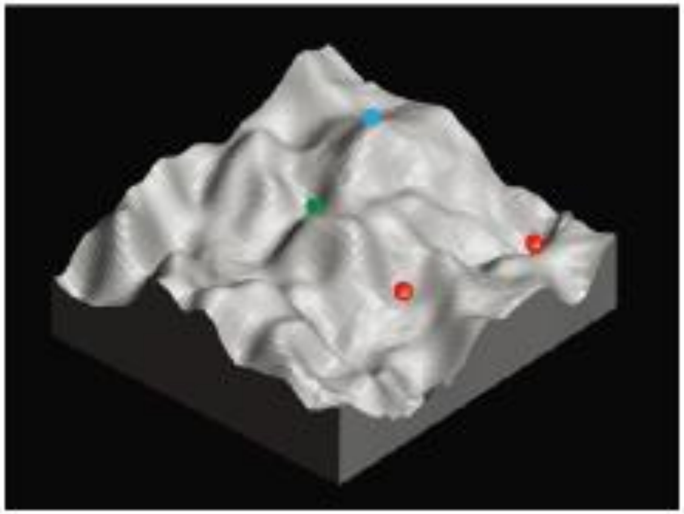


FIG. 7. Token representation of a portion of the multidimensional array of genotypes of a population with fitness contours. Field initially occupied indicated by heavy broken contour. Field occupied later indicated by crosshatched area (multiple subpopulations in F). Courses indicated in C, D, E and F by arrows. Effective population numbers, N (total), n (local); v (mutation), s (selection), m (migration) (from Wright, 1932, Fig. 4).

- “The principle of natural selection” is outdated.

Energy Function or Lyapunov Function

•Given a dynamical process described by an n-dimensional ordinary differential equation (ODE):



$$d\mathbf{x}/dt = \mathbf{f}(\mathbf{x})$$

•Does a Lyapunov function $H(\mathbf{x})$ always exist? $dH(\mathbf{x}(\mathbf{t}))/dt \leq 0$

•If yes, can we find a construction of $H(\mathbf{x})$?

Difficult but Important Question, I

encountered in many fields

- **Biology:**
  " … the idea that there is such a quantity (adaptive landscape—P.A.) remains one of the most widely held popular misconceptions about evolution".
  S.H. Rice, in *Evolutionary Theory: mathematical and conceptual foundations* (2004)
- **Chemistry**:
  "The search for a generalized thermodynamic potential in the nonlinear range has attracted a great deal of attention, but these efforts finally failed."
  G. Nicolis in *New Physics*, pp332 (1989)
- **Physics**:
  "Statistical physicists have tried to find such a variational formulation for many years because, if it existed in a useful form, it might be a powerful tool for the solution of many kinds of problems. My guess … is that no such general principle exists."
  J. Langer in *Critical Problems in Physics*, pp26 (1997)
  and, check recent issues of Physics Today, Physical Review Letters, …
- **Mathematics**:
  gradient *vs* vector systems, unsolved (Holmes, 2006; Strogatz, 1st ed., 2nd ed. 2015)
  dissipative, $\nabla \bullet f \neq 0$; asymmetric, $\nabla \times f \neq 0$ (absence of detailed balance); nonlinear f; stochastic, multiplicative noise
- **Economy (econophysics), finance, engineering, …**

Experts have been generally negative on its existence.
for example, Steve Smale, ~1970's; Strogatz, 2015
Udo Seifert, Reports on Progress in Physics (2012)
"… 甚至在很多动力学系统中并不存在这样的势函数" 《从动力学到统计物理学》，2016，63页
"达尔文进化论是赝科学" 未来论坛 理解未来系列讲座第18期 一北京教授 2016年7月

Nevertheless, a positive answer will provide not only an additional powerful tool to study nonlinear dynamics, also a unifying framework for statistical mechanics, complex systems and nonequilibrium processes.

Difficult but Important Question, II

a few recent confusions

- **Viscoelastic and Elastomeric Active Matter: Linear Instability and Nonlinear Dynamics**
  E. J. Hemingway, M. E. Cates, and S. M. Fielding, Phys. Rev. E93 (**2016**) 032702
  https://journals.aps.org/pre/abstract/10.1103/PhysRevE.93.032702

- **Machine learning the thermodynamic arrow of time**
  Alireza Seif, Mohammad Hafezi & Christopher Jarzynski. Nature Physics 17 (**2021**) 105–113
  https://www.nature.com/articles/s41567-020-1018-2

- **Non-reciprocal phase transitions**
  Michel Fruchart, Ryo Hanai, Peter B. Littlewood & Vincenzo Vitelli. Nature 592 (**2021**) 363
  https://www.nature.com/articles/s41586-021-03375-9.pdf

  奇异点——描述非平衡系统相变的数学语言 https://zhuanlan.zhihu.com/p/435419801
  集智科学家 2021-11-19 21:27
  Nature：复杂宏观系统中的非互易相变 (qq.com)
  集智俱乐部 2022年1月26日22:38

- **Symmetry, Thermodynamics, and Topology in Active Matter**
  Mark J. Bowick, Nikta Fakhri, M. Cristina Marchetti, and Sriram Ramaswamy. Phys. Rev. X12 (**2022**) 010501
  https://journals.aps.org/prx/abstract/10.1103/PhysRevX.12.010501 .

- **Mathematics**:
  gradient *vs* vector systems, unsolved (Holmes, 2006; Strogatz, 1st ed., 2nd ed. 2015)
  dissipative, $\nabla \bullet f \neq 0$; asymmetric, $\nabla \times f \neq 0$ (absence of detailed balance); nonlinear f; stochastic, multiplicative noise

Nevertheless, a positive answer will provide not only an additional powerful tool to study nonlinear dynamics, also a unifying framework for statistical mechanics, complex systems and non-equilibrium processes.

Construction: math perspective, m1

General dynamical system (ODE form) in n-dimension:

$d\mathbf{x}/dt = \mathbf{f}(\mathbf{x})$ For simplicity, **f** will be assumed to be smooth enough.

"Theorem":

There exists a decomposition such that

$$\mathbf{f}(\mathbf{x}) = - [\, D(\mathbf{x}) + Q(\mathbf{x}) \,] \nabla H(\mathbf{x}) \quad ,$$

with $H(\mathbf{x})$ single-valued scalar function

$D(\mathbf{x})$ symmetric and non-negative matrix $D = D^{\tau}$

$Q(\mathbf{x})$ anti-symmetric matrix $Q = - Q^{\tau}$

Two limiting situations are well known:

1) $D(\mathbf{x}) = 0$: (generalized) Hamilton equation: "energy $H(\mathbf{x})$" conserved

$$d\mathbf{x}/dt = - Q(\mathbf{x}) \nabla H(\mathbf{x})$$

2) $Q(\mathbf{x}) = 0$: (generalized) gradient system: purely dissipative dynamics

$$d\mathbf{x}/dt = - D(\mathbf{x}) \nabla H(\mathbf{x})$$

Construction: math perspective, m2

Plausibility 1: Hamilton-Jacobi equation

$$\mathbf{f}(\mathbf{x}) = -[\,D(\mathbf{x}) + Q(\mathbf{x})\,]\nabla H(\mathbf{x}) \quad \Rightarrow$$

$$\mathbf{f}(\mathbf{x}) \cdot \nabla H(\mathbf{x}) = -\nabla H(\mathbf{x}) \cdot [\,D(\mathbf{x}) + Q(\mathbf{x})\,]\nabla H(\mathbf{x})$$

$$= -\nabla H(\mathbf{x}) \cdot D(\mathbf{x})\,\nabla H(\mathbf{x})$$

For simplicity, set $D(\mathbf{x}) = 1$, we have

$$\mathbf{f}(\mathbf{x}) \cdot \nabla H(\mathbf{x}) + \nabla H(\mathbf{x}) \cdot \nabla H(\mathbf{x}) = 0,$$ Hamilton-Jacobi equation

with H and **f** known, Q can be found but generally not unique

Plausibility 2: 1st order quasi-linear partial differential equation (PDE)

Again, with the simplicity assumption of $D(\mathbf{x}) = 1$, $[1 + Q(\mathbf{x})\,]^{-1}$ exists.

Hence, we have

$$[\,1 + Q(\mathbf{x})\,]^{-1}\,\mathbf{f}(\mathbf{x}) = -\nabla H(\mathbf{x})$$

$$\nabla \times \nabla H = 0 \;\Rightarrow$$

$$\nabla \times (\,[\,1 + Q(\mathbf{x})\,]^{-1}\,\mathbf{f}(\mathbf{x})\,) = 0,$$ anti-symmetric, **n(n-1)/2** equations

first order quasi-linear PDE

Q can be uniquely obtained, hence H upon to a constant.

H(x): Lyapunov Function

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}) = -[\,D(\mathbf{x}) + Q(\mathbf{x})\,]\nabla H(\mathbf{x}) \quad ,$$

$$\begin{aligned} dH(\mathbf{x}(t))/dt &= \nabla H(\mathbf{x}) \cdot d\mathbf{x}/dt \\ &= -\nabla H(\mathbf{x})\,[\,D(\mathbf{x}) + Q(\mathbf{x})\,]\nabla H(\mathbf{x}) \\ &= -\nabla H(\mathbf{x})\, D(\mathbf{x})\nabla H(\mathbf{x}) \\ &\leq 0 \,, \end{aligned}$$

i.e., $H(\mathbf{x})$ is a Lyapunov function.

- Remark 1: The Lyapunov function is generally unique when *D* known.
- Remark 2: The fixed points of $\mathbf{f}(\mathbf{x})$ are all preserved in $\nabla H(\mathbf{x})$.

$D(\mathbf{x}) = 0$: (generalized) Hamilton equation: "energy $H(\mathbf{x})$" conserved

$$d\mathbf{x}/dt = -Q(\mathbf{x})\nabla H(\mathbf{x})$$

System + Environment (origin of noise and friction, etc.)

Vortex Dynamics

大卫·邵乐思的治学和建树：纯粹探索凌绝顶 （上海科普网 https://www2.shkp.org.cn/content.html?type=if&id=190785 ）

$$m_V \frac{d^2 \boldsymbol{r}(t)}{dt^2} = -\nabla U(\boldsymbol{r}(t)) + q_V \boldsymbol{B} \times \frac{d\boldsymbol{r}(t)}{dt} - \int dt' \boldsymbol{S}(t-t') \frac{d\boldsymbol{r}(t')}{dt} + \xi(t), \quad (3)$$

其中r是涡旋位置矢量，q_V=+1或者-1（类似于电子电荷），▿是梯度算符，其他量在下面解释。

量子涡旋作为最简单的一类拓扑缺陷在很多系统中都出现。我们需要一个坚实可靠的理论出发点来描述量子涡旋。不幸的是，有的系统（如超流态氦）的微观理论至今仍未完善。邵乐思建议从多体波函数出发，并提示我只要对称性和很少几个一般性质在多体波函数里，结论很少会错，况且，我们已有两个非常成功的多体波函数：BCS波函数和Laughlin波函数。此外，Feynman用多体波函数处理超流态氦也很成功。从这里开始借助几何相位或贝利相位[10]我们很容易地计算出量子涡旋动力学最有争议一项——涡旋运动的横向力。我们发现它就是经典的Magnus力，和带电粒子在磁场中的洛仑兹力类似[11]，$B=h\rho_s/2$，其中h是普朗克常数、ρ_s是超导电子密度。我们很快认识到这个想法在超流现象中已有应用[12]，换了角度得到同样结果，并证明Magnus力在有限温度下也严格成立[13]。相互作用势能U理论上没有歧义，涡旋-反涡旋对的相互作用势能是$U=\rho_s h^2/8\pi m_c \ln(r/r_0)$，这里$m_c$是Cooper对有效质量，在外加超流$J_s$的势能是$U=hJ_s\cdot r$。

Construction: physics perspective, p1

Two equivalent forms of stochastic differential equations

- **Standard stochastic differential equation (SDE):**

$$d\mathbf{x}/dt = \mathbf{f}(\mathbf{x}) + \zeta(\mathbf{x},t) \qquad \text{(S)}$$

Gaussian and white, Wiener noise: $\langle \zeta \rangle = 0$, $\langle \zeta(\mathbf{x},t)\, \zeta^{\tau}(\mathbf{x},t') \rangle = 2\, \varepsilon\, D(\mathbf{x})\, \delta(t-t')$

- The desired equation (**local and trajectory view**):

$$[S(\mathbf{x}) + A(\mathbf{x})]\, d\mathbf{x}/dt = -\nabla H(\mathbf{x}) + \xi(\mathbf{x},t) \qquad \text{(N)}$$

Gaussian and white, Wiener noise: $\langle \xi \rangle = 0$, $\langle \xi(\mathbf{x},t)\, \xi^{\tau}(\mathbf{x},t') \rangle = 2\, \varepsilon\, S(\mathbf{x})\, \delta(t-t')$

which has **4** dynamical elements:
dissipative ($S=S^{\tau}$), transverse($A=-A^{\tau}$), driving(H), and stochastic forces(ξ).

- The desired distribution (**global and ensemble view**):

steady state **(Boltzmann-Gibbs)** distribution $\rho_{ss}(\mathbf{x}) \sim \exp(-H(\mathbf{x}) / \varepsilon)$.

From (N) to (S) is easy: $S+A$ nonsingular, i.e., n independent dynamics

Construction: physics perspective, p2

From (S) to (N): knowing f and D to find out S, A and H

- Describing **same dynamics** ⇒

  $[S(\mathbf{x}) + A(\mathbf{x})]\,[\mathbf{f}(\mathbf{x}) + \zeta(\mathbf{x},t)] = -\nabla H(\mathbf{x}) + \xi(\mathbf{x},t)$

- Noise and deterministic "force" have **independent origins** ⇒

  $[S(\mathbf{x}) + A(\mathbf{x})]\,\mathbf{f}(\mathbf{x}) = -\nabla H(\mathbf{x})$

  $[S(\mathbf{x}) + A(\mathbf{x})]\,\zeta(\mathbf{x},t) = \xi(\mathbf{x},t)$

- Potential condition: $\nabla\times\nabla H = 0$ ⇒

  $\nabla\times[S(\mathbf{x}) + A(\mathbf{x})]\,\mathbf{f}(\mathbf{x}) = 0$, anti-symmetric, **n(n-1)/2** equations

- Generalized Einstein relation

  $[S(\mathbf{x}) + A(\mathbf{x})]\,D(\mathbf{x})\,[S(\mathbf{x}) - A(\mathbf{x})] = S(\mathbf{x})$ (Einstein,1905: $S\,D = 1$)

  symmetric, **n(n+1)/2** equations

  $D(\mathbf{x}) + Q(\mathbf{x}) = 1/[\,S(\mathbf{x}) + A(\mathbf{x})\,]$

- Total **n**^{2} conditions to determine matrix $[S(\mathbf{x},t) + A(\mathbf{x},t)]$!

- Once S and A are obtained, $H = -\int_c [S(\mathbf{x}') + A(\mathbf{x}')]\,\mathbf{f}(\mathbf{x}')\cdot d\mathbf{x}'$

- Hence, S, T, H can be constructed from D and **f** .

Lyapunov Function with Stochasticity

relative entropy or free energy

- Free energy — monotonically decreasing

  $$F(t) = \langle H \rangle(t) - \varepsilon\, S(t)$$

  with average energy $\langle H \rangle(t) = \int dx\, H(x)\, \rho(x,t)$, temperature ε,

  entropy $S = -\int dx\, \rho(x,t) \ln(\rho(x,t))$

- Relative entropy — monotonically increasing

  $$S_r(t) = -\int dx\, \rho(x,t) \ln(\rho(x,t)/\rho_{ss}(x))$$

  with stationary state distribution $\rho_{ss}(x) = \exp(-H(x)/\varepsilon)/Z$

- $$F(t) - F(\infty) = -\varepsilon\, S_r(t)$$

Three Typical Types of Dynamical Systems

- **Linear dynamics**: near stable or unstable fixed points

  to show that it is possible to construct a global Lyapunov function

  Structure of Stochastic Dynamics near Fixed Points,

  Kwon, Ao, Thouless. *Proc. Nat'l Acad. Sci. (USA)* **102** (2005) 13029

- **Limit cycles**: Many theorists, such as Prigogine and his school, once or even now, would not believe the existence of Lyapunov function for such dynamics.

  simple limit cycle: Limit Cycle and Conserved Dynamics,

  Zhu, Yin, Ao. *Int. J. Mod. Phy*. **B20** (2006) 817

  van der Pol like: Exploring a Noisy van der Pol Type Oscillator with a Stochastic Approach,

  Yuan, Wang, Ma, Yuan, Ao. *Phys Rev* **E87** (2013) 062109

- **Chaotic dynamics**: Many mathematicians would not think this is possible.

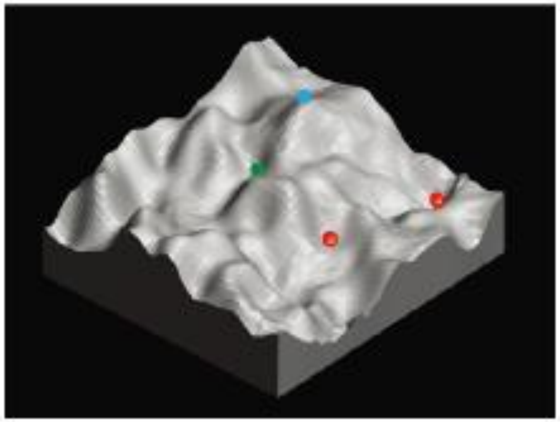
  Lorenz like systems Potential Function in a Continuous Dissipative Chaotic System: … ,

  Ma, Tan, Yuan, Yuan, Ao. *Intl J Bifurc Chaos* **24** (2014) 1450015

Linear Dynamics

- Linear dynamics, fixed point is either stable or unstable

$D + Q = 1/[S + A]$ Kwon, Ao, and Thouless, PNAS (2005)



$$\dot{x} = f(x) + \zeta(x, t)$$

$$\langle \zeta(x, t)\zeta^{\tau}(x, t') \rangle = 2D(x)\delta(t - t')$$

$$f_i(x) = F_{ij}x_j$$

$$(S + A)\dot{x} = -Ux + \xi(t)$$

$$\langle \xi(x, t)\xi^{\tau}(x, t') \rangle = 2S\delta(t - t')$$

$$F = -(D + Q)U = -(S + A)^{-1}U$$

$$FQ + QF^{\tau} = FD - DF^{\tau}$$

Analytically Explicit Solutions

1-d: easy; n-d linear: Kwon, Ao, Thouless, PNAS, 2005

- One dimension, arbitrary function f(x)

$$H(\mathbf{x}) = -\int_c^{\mathbf{x}} d\mathbf{x}' \, \mathbf{f}(\mathbf{x}')$$

$$Q(x) = 0$$

- n-dimensions, linear $f(x) = Fx$

$$H(\mathbf{x}) = -\int_c^{\mathbf{x}} d\mathbf{x}' \cdot (1 + Q)^{-1} \mathbf{f}(\mathbf{x}')$$

$$FQ + QF^{\tau} = F - F^{\tau}$$

Limit Cycles

- Limit cycles, $\nabla \cdot f \neq 0$ (dissipative?); at the limit cycle, Φ = constant, $S = 0$ (no dissipation)

$$\dot{\mathbf{q}} = \mathbf{f}(\mathbf{q}) + N(\mathbf{q})\xi(t) \qquad \langle \xi(t)\xi^\tau(t') \rangle = \delta(t-t')I_k$$

$$N(\mathbf{q})N^\tau(\mathbf{q}) = 2\epsilon D(\mathbf{q})$$

$$[S(\mathbf{q}) + A(\mathbf{q})]\dot{\mathbf{q}} = -\nabla\phi(\mathbf{q}) + \hat{N}(\mathbf{q})\xi(t)$$

$$\hat{N}(\mathbf{q})\hat{N}^\tau(\mathbf{q}) = 2\epsilon S(\mathbf{q})$$

simple limit cycle: Zhu, Yin, Ao, Intl J. Mod. Phys. B (2007)

$$\begin{cases} \dot{q}_1 = -q_2 + q_1(1 - q_1^2 - q_2^2) + \xi_1(t) \\ \dot{q}_2 = q_1 + q_2(1 - q_1^2 - q_2^2) + \xi_2(t) \end{cases}$$

$$S(\mathbf{q}) = \frac{(1-q_1^2-q_2^2)^2}{(1-q_1^2-q_2^2)^2+1}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

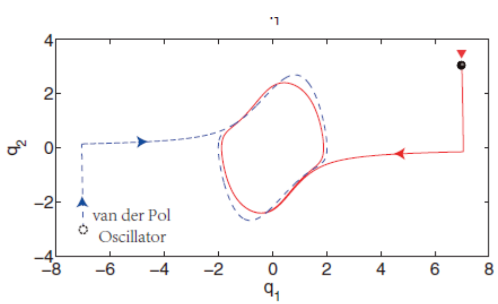
$$A(\mathbf{q}) = \frac{(1-q_1^2-q_2^2)}{(1-q_1^2-q_2^2)^2+1}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

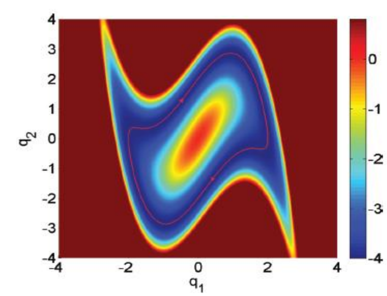
$$\phi(\mathbf{q}) = \frac{1}{4}(q_1^2+q_2^2)(q_1^2+q_2^2-2)$$

van der Pol like: Yuan, Wang, Ma, Yuan, Ao. Phys Rev E (2013)

$$\begin{cases} \dot{q}_1 = q_2 + \zeta_1(\mathbf{q}, t) \\ \dot{q}_2 = -\mu(q_1^2 - 1)q_2 - q_1 + h(q_1) + \zeta_2(\mathbf{q}, t)t \end{cases}$$

$$\text{with } h(q_1) = \mu^2 q_1^3/4 - \mu^2 q_1^5/16$$





$$\phi(\mathbf{q}) = \frac{1}{4}\left[q_1^2 + \left(q_2 - \mu q_1 + \frac{\mu}{4}q_1^3\right)^2\right]\left[q_1^2 + \left(q_2 - \mu q_1 + \frac{\mu}{4}q_1^3\right)^2 - 8\right]$$

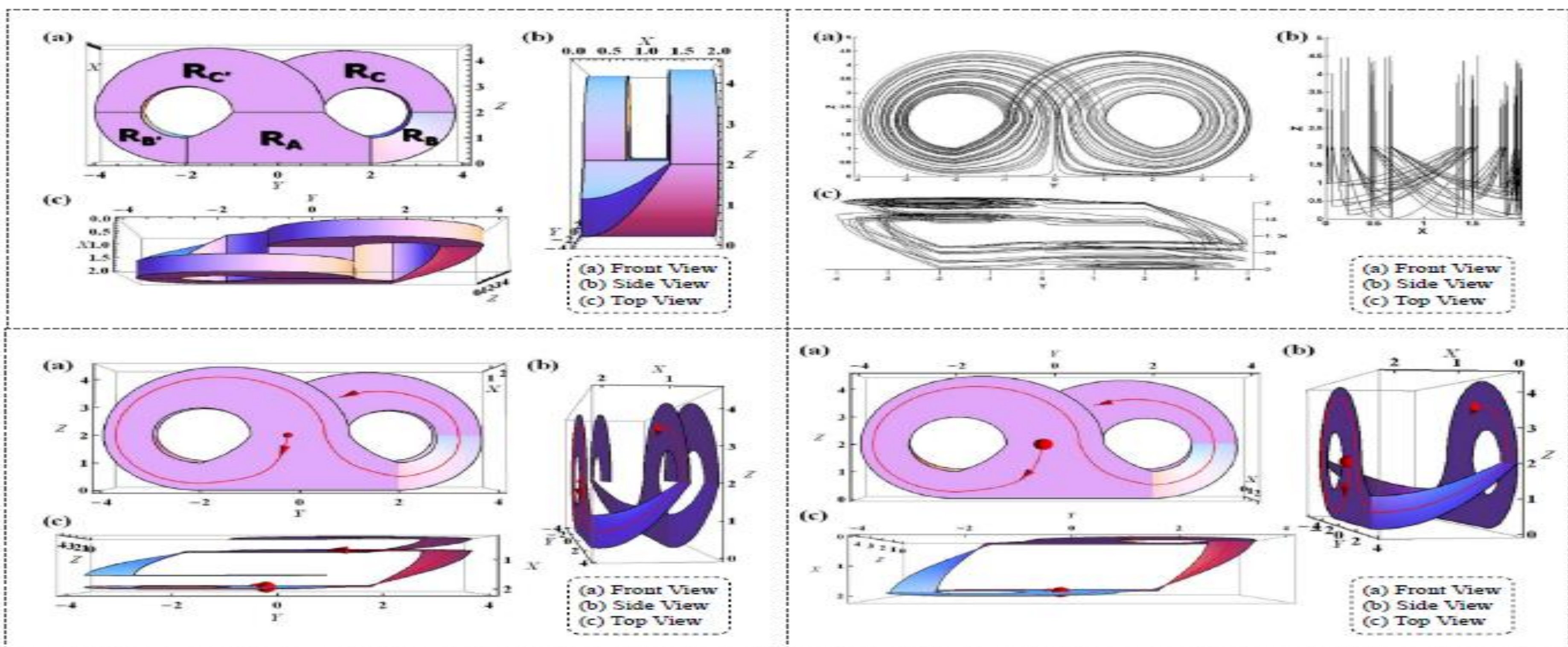
Dissipative Chaotic Dynamics

- Chaotic dynamics

Ma, Tan, Yuan, Yuan, Ao. Int'l J Bifurcation and Chaos (2014)

$$\dot{\mathrm{x}} = \mathrm{f}(\mathrm{x})$$
$$= -D(\mathrm{x})\nabla\Psi(\mathrm{x}) + Q(\mathrm{x})\nabla\Psi(\mathrm{x})$$

$$\dot{x}_i = -\{x_i, \Psi\} + [x_i, \Psi]$$ Bracket dynamics



Ito, Stratonovich, and Present Method on stochastic processes

1- d stochastic process (Ao, Kwon, Qian, Complexity, 2007)

Stochastic differential equation (pre-equation according to van Kampen, 1980):

$d_t x = f(x) + \zeta(x, t) = - D(x)H(x) + \zeta(x, t)$, Guassian-white noise, multiplicative

Ito Process:

- $\partial_t \rho(x, t) = [\varepsilon \partial_x \partial_x D(x) + \partial_x D(x) H_x(x)]\, \rho(x, t)$

  $\rho(x, t = \infty) \sim \exp\{ - H(x)/\varepsilon \} \,/\, D(x)$; $H(x) = - \int dx' \, f(x') / D(x')$

Stratonovich process:

- $\partial_t \rho(x, t) = [\varepsilon \partial_x D^{1/2}(x) \partial_x D^{1/2}(x) + \partial_x D(x) H_x(x)]\, \rho(x, t)$

  $\rho(x, t = \infty) \sim \exp\{ - H(x)/\varepsilon \} \,/\, D^{1/2}(x)$

Present process:

- $\partial_t \rho(x, t) = [\varepsilon \partial_x D(x) \partial_x + \partial_x D(x) H_x(x)]\, \rho(x, t)$

  $\rho(x, t = \infty) \sim \exp\{ - H(x)/\varepsilon \}$

Gradient Expansion
and Lyapunov Equation

Gradient expansion (Ao, 2004; 2005)

Exact equations: $\nabla\times[G^{-1}\mathbf{f}(\mathbf{x})] = 0$; $G + G^{T} = 2D$.

$$G \equiv [S + A]^{-1} = D + Q$$

Defining F matrix from the "force" **f** in n-d: $F_{ij} = \partial_j f_i$, $i,j = 1, \ldots, n$

Treating derivative $\partial_i G$ as a higher order term

Lowest order gradient expansion—linear matrix equation:

- $GF^{T} - FG^{T} = 0$
- $G + G^{T} = 2D$

$$Q = (FD - DF^{T}) / \mathrm{tr}(F) \ , \quad \text{2-d}$$

- $H(\mathbf{x}) = -\int_c d\mathbf{x}' \cdot [G^{-1}(\mathbf{x}')\,\mathbf{f}(\mathbf{x}')]$

Where We Are

- **General construction**
  Ao, physics/0302081; Ao, J. Phys. A (2004); Yuan and Ao, J Stat Mech (2012)

- **Linear dynamics, stable or unstable**
  Kwon, Ao, and Thouless, PNAS (2005)
- **Equivalence to other methods**
  Yin and Ao, J. Phys. A (2006); Ao, Kwon, Qian, Complexity (2007); Yuan et al. Chin Phys B (2013)
- **Limit cycles**
  Zhu, Yin, Ao, Intl J. Mod. Phys. B (2007); Yuan et al, Phys Rev E (2013)
- **Chaotic dynamics**
  Ma, Tan, Yuan, Yuan, Ao, Int'l J Bifurcation and Chaos (2014)
- **Partial differential equations**
  Chen, Shi, Kosterlitz, Zhu, Ao. Proc. Nat'l Acad Sci (USA) (2022)

- **Applications in biology**:
  Phage lambda gene switch:
  Zhu et al, Funct Integr Geno (2004); Lei et al, Sci Rep (2015)
  Cancer genesis and progression:
  Ao et al Med Hyp (2008); Yuan, Zhu, Wang, Li, Ao. Rep Prog Phys (2017)

- **Gradient expansion**: linear problem at each order
  Ao, J. Phys. A (2004)
- **Markov Processes**: same three parts--energy function, diffusion, no-detailed balance
  Ao, Chen, Shi. Chin Phys Lett (2013)
- **Dynamical structure decomposition** and A-type stochastic interpretation in nonequilibrium processes:
  Yuan, Tang, Ao. Frontiers of Physics. 12: 120201 (2017)

New Predictions

- Generalized Einstein relation (Ao, J Phys A, 2004)

$$[S(\mathbf{x}) + A(\mathbf{x})]\, D(\mathbf{x})\, [S(\mathbf{x}) - A(\mathbf{x})] = S(\mathbf{x})$$

$S\,D = 1$ (Einstein, 1905) $A = 0$, detailed balance

No**:** Luposchainsky and Hinrichsen, J Stat Phys, 2013

- Beyond Ito vs Stratonovich (Ao, Kwon, Qian, 2007; Yuan, Ao, 2012)

No: Udo Seifert, Rep Prog Phys 2012

- “Free energy” equality (Ao, Comm Theor Phys 2008; Tang et al, 2015)

$$< \exp[- \int_c d\mathbf{x} \cdot \nabla_x H(\mathbf{x}, \mathbf{q})] > = \exp[- (F(end) - F(initial))]$$

Jarzynski, 1997, when Hamiltonian is known *a priori*

Ao, 2008, when Hamiltonian is unknown, *but emerging as shown*

How Would **Nature** Decide?

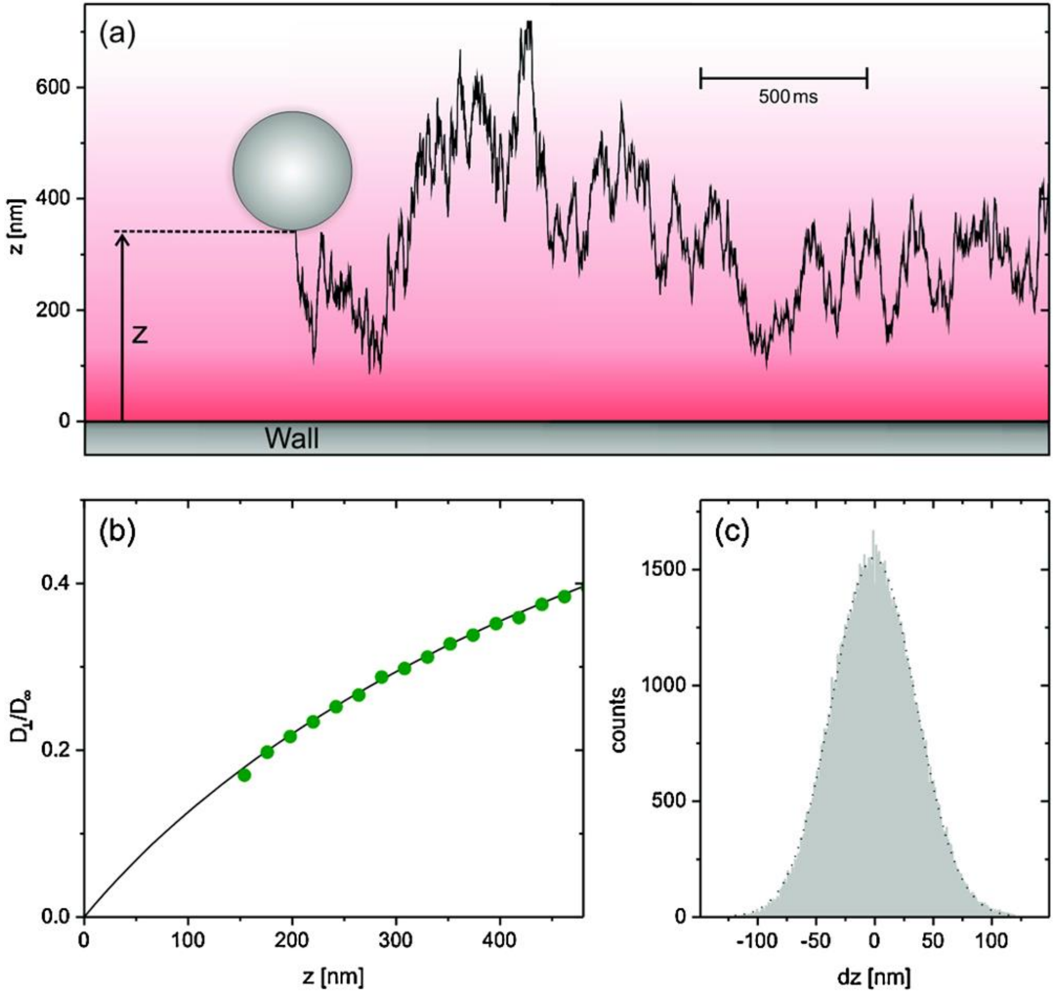
“ I assign more value to discovering a fact, even about the minute thing, than to lengthy disputations on the Grand Questions that fail to lead to true understanding whatever. ”

Galileo Galilei (1564-1642)

First new experimental evidence:

Influence of Noise on Force Measurements.

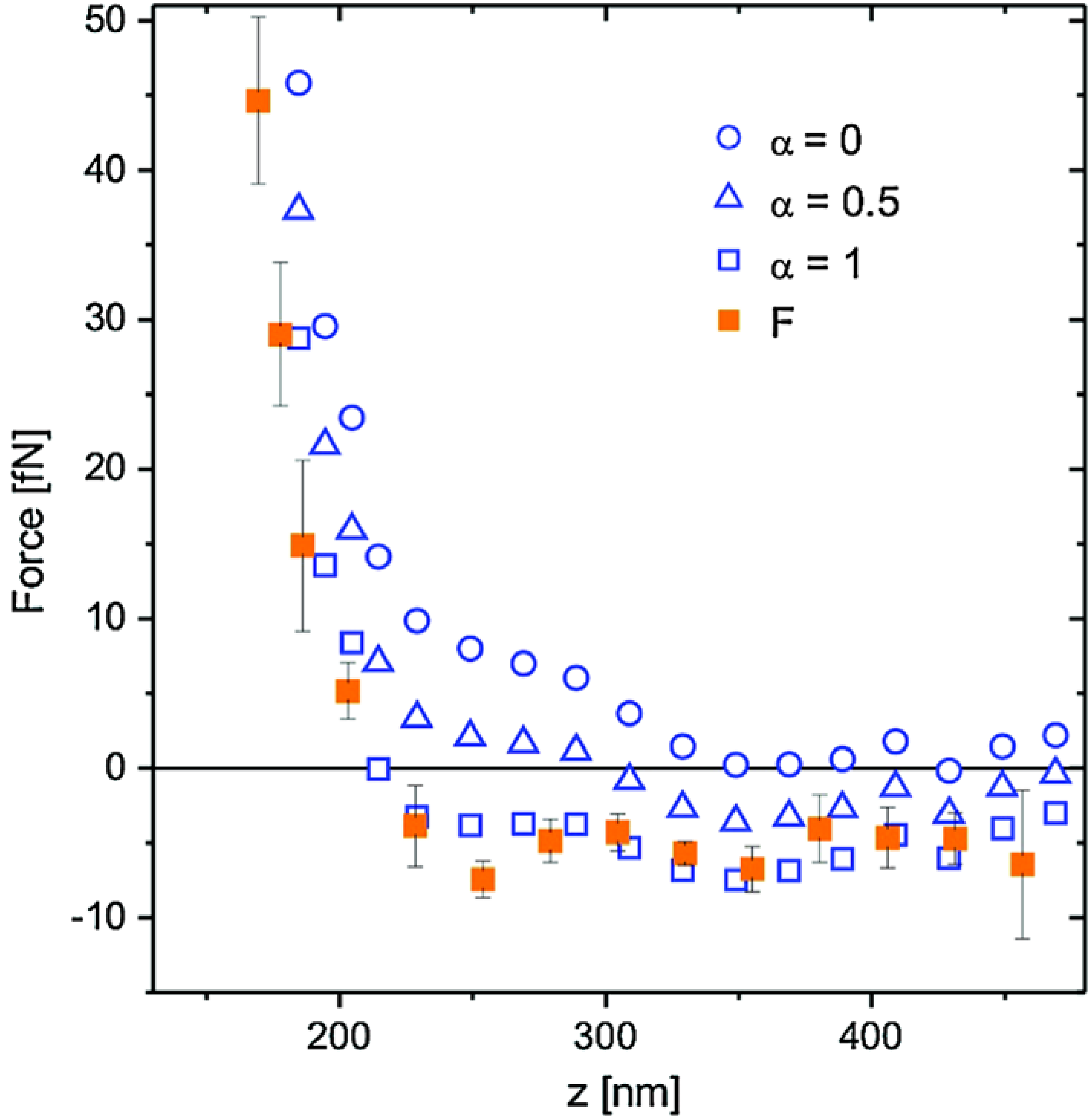
Giovanni Volpe, Laurent Helden, Thomas Brettschneider, Jan Wehr, and Clemens Bechinger. Phy Rev Lett **104** (2010) 170602



(a) A Brownian particle (drawn not to scale) diffuses near a wall in the presence of gravitational and electrostatic forces.

(b) Comparison of measured (bullets) and calculated (line) vertical diffusion coefficient as a function of the particle-wall distance.

(c) Experimentally determined probability distribution of the local drift dz for dt = 5 ms at z = 380 nm (grey). The dashed line is a Gaussian in excellent agreement with the experimental data.



Forces obtained from a drift-velocity experiment with added noise-induced drift
[alpha = 1 (open squares) (A-type), alpha = 0:5 (open triangles) (S-type), and alpha = 0 (open dots) (I-type)].
The solid squares represent the forces obtained from an equilibrium measurement.

Fundamental Theorem of (Stochastic) Dynamics

$$[S(\mathbf{x}) + A(\mathbf{x})]\, d\mathbf{x}/dt = - \nabla H(\mathbf{x}) + \xi\,(\mathbf{x},t) \qquad (N)$$

Three independent dynamical elements, the representation:

- *S*: Semi-positive definite symmetric matrix: dissipation; degradation
- *A*: Anti-symmetric matrix, transverse: oscillation, conservative
- H: Potential function or Lyapunov function: capacity, cost function, landscape
- ξ: Stochastic force: noise, related to *S* by "FDT".

Steady state distribution is determined by H(**X**), when independent of time, according to Boltzmann-Gibbs distribution, $\rho \sim \exp(-H/\varepsilon)$!
Direct and quantitative measure for robustness and stability

Global view of bionetwork dynamics: adaptive landscape. Ao. J Genetics and Genomics **36** (2009) 63

Unifying dynamical framework for nonequilibrium processes

Unity of Biology and Physics

Evolutionary processes expressed by Stochastic Differential Equation

Fundamental dynamical equation in biology--Evolution by Variation and Selection (Darwin and Wallace, 1858):

$$d\mathbf{x}/dt = \mathbf{f}(\mathbf{x}) + \zeta(\mathbf{x},t) \qquad \text{(S)}$$

Gaussian and white, Wiener noise: $< \zeta > \mathbf{= 0,}$ $< \zeta(\mathbf{x},t)\, \zeta^{\tau} (\mathbf{x},t') > = 2\, \varepsilon\, D\, (\mathbf{x})\, \delta(t-t')$

Dynamical structure decomposition connects biology and physics (Ao, 2005):

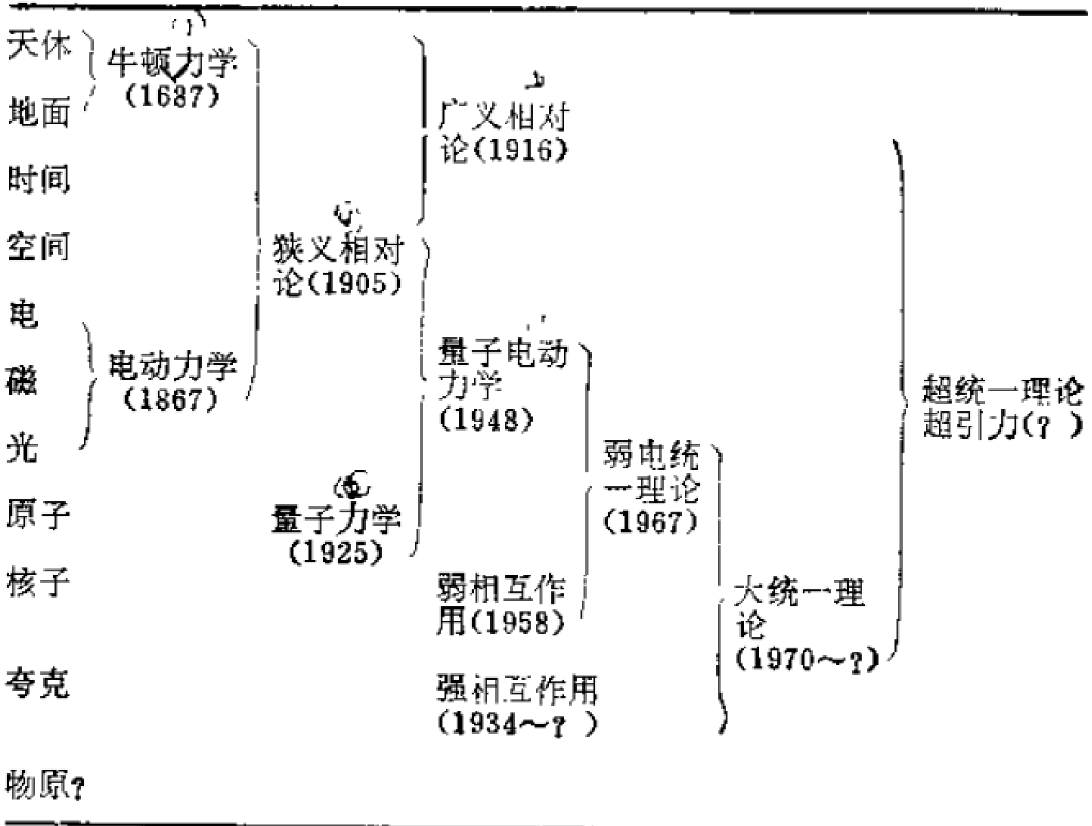
$$[S(\mathbf{x}) + A(\mathbf{x})]\, d\mathbf{x}/dt = - \nabla H(\mathbf{x}) + \xi\, (\mathbf{x},t) \qquad \text{(N)}$$

Gaussian and white, Wiener noise: $< \xi > \mathbf{= 0,}$ $< \xi(\mathbf{x},t)\;\; \xi^{\tau} (\mathbf{x},t') > = 2\, \varepsilon\, S\, (\mathbf{x})\, \delta(t-t')$

现代技术的科学理论基础

- 20世纪，两大理论基础：相对论、量子力学

表1　物理学发展中的统一*



* 括号中的数字表示相应的理论建立的年代，有问号的表示尚未完成。

《力学概论》，中国科学技术大学/北京大学，1988

当时尚无演化力学，作者们未提及热力学、统计力学—奇怪的是这两领域直到现在也总是有意或无意地被大家忽视

- 21世纪，三大理论基础：相对论、量子力学、演化力学

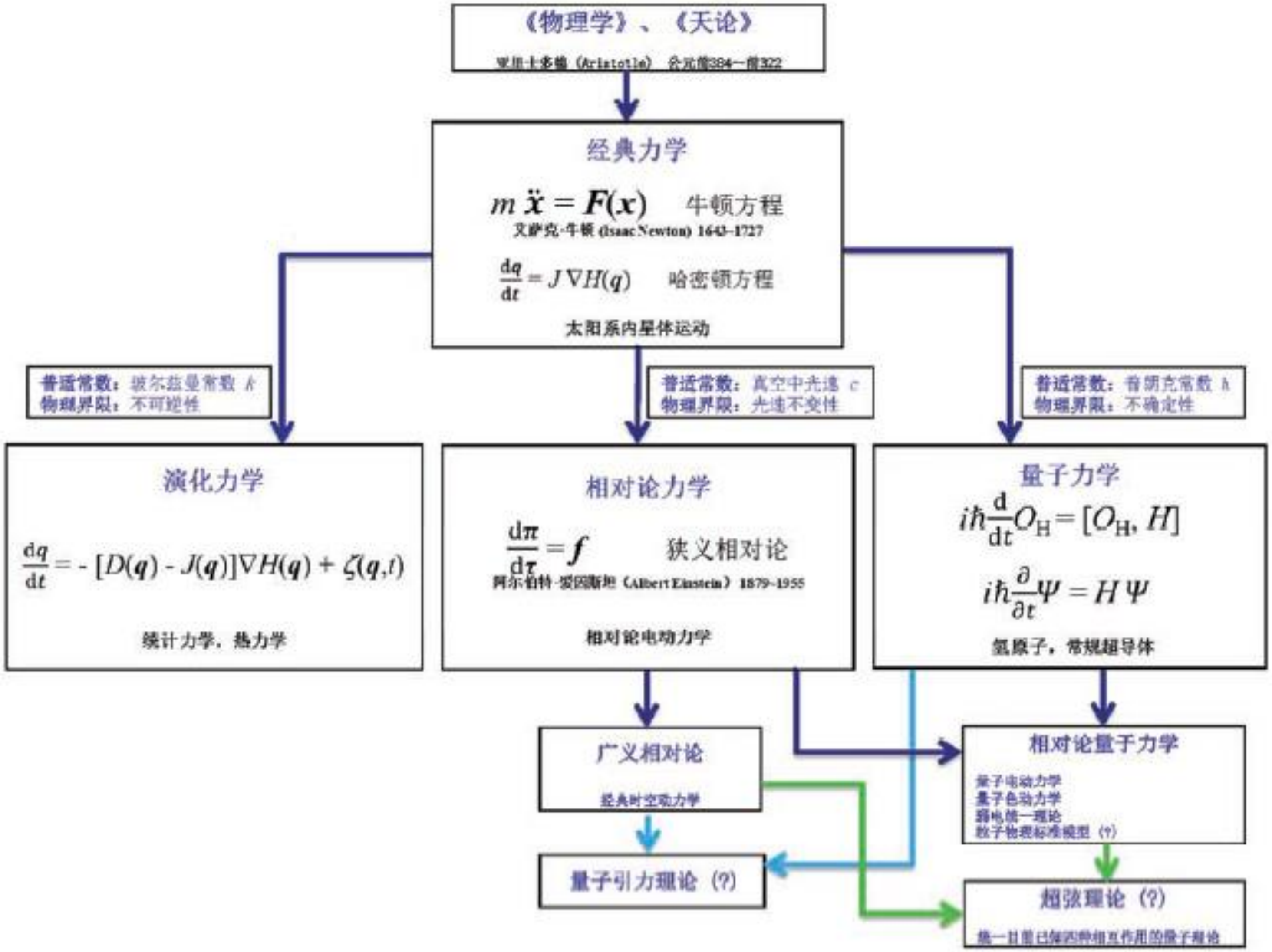


图 1 （网络版彩色）运动方程理论的演化：时间的特殊地位

在所有层次上，宇宙、地质、人类社会、生物、分子、原子、…，都有演化过程。存在统一的演化方程

1939年 福勒提出**热平衡定律** – 热力学第零定律

19世纪中期 迈耶、焦耳、亥姆霍兹提出热力学第一定律：**能量守恒**

克劳修斯、开尔文提出热力学第二定律：**熵原理**

1912年 能斯特：**绝对零度不可达到** – 热力学第三定律

1931年 昂萨格提出倒易关系，于1968年获得诺贝尔化学奖

1945年 普利高津提出最小熵产生原理

1969年 普利高津提出热力学中的耗散结构定理，为认识自然界中（特别是生命体系中）发生的各种自组织现象开辟了一条新路，奠定了远离平衡的非平衡热力学的初步理论，于1977年获诺贝尔化学奖，非平衡可能是秩序的根源

1858年 1859年 达尔文、华莱斯提出生物演化基本理论 达尔文出版《物种起源》

1930年 RA Fisher 提出自然选择基本定理(FTNS)，(不正确地)认为等同于热力学第二定律

1932年 S Wright 提出演化的适应性景观(fitness landscape)

1940年 沃丁顿提出基因调控的表观遗传学(epigenetic landscape)

⇩

2004年 敖平提出动力过程的能量函数构造一般方法，证明在非平衡状态下，稳态分布函数是玻尔兹曼-吉布斯分布

2005年 该变换由 Kwon、Ao（敖平）、Thouless 在线性情况下给出了解析构造

2005年 敖平成功把演化过程表达为一组的简洁数学方程，证明FTNS是涨落-耗散关系

1997年，普利高津：“能量函数”在远离热平衡态情况下不适用 （*wrong!* ）

演化力学的信息论应用-人工智能

- 智能是生物演化过程中出现的一种性能。（**演化 I**）
- 智能作为一种现象，就表现在个体与自然、社会群体的相互作用和行为过程中。这些主体-客体的行为和现象必然有统一的力、相互作用、基本元素来描述：**有规律可循 - 本构性机理存在--科学/物理学的基本要素。**
- 机器在演化过程中出现智能是逻辑的必然，有两个演化阶段：

  人类干预和参与下的演化（**演化 II**）；

  机器自主的演化（**演化 III**）

- **演化力学贯穿于所有演化过程中。**

- 国际、国内人工智能的研究极少关注这类重要的基础科学问题。大学里通常也不教这个事情，大家忙着学一些技能。会解决一些小问题，日子就能过得红火滋润。部分知名学者认为智能现象那么复杂，根本不可能有统一的理论，只能是“a bag of tricks”（一麻袋的花招）。开发一些“兵来将挡、水来土掩”的实用法则就行了。
- 我们是着重于技术热点上的跟踪追赶，还是要有基础理论上的深入研究突破创新？

- 机器智能与生物智能遵从相同的演化力学，也会有相同的本构性机理吗？会有类似的实现方式吗？

  游泳：　鲤鱼、鲸鱼、企鹅、潜水艇；　　**飞行**：　蜻蜓、麻雀、蝙蝠、波音787

- On the Stochastic Gradient Descent and Inverse Variance-flatness Relation in Artificial Neural Networks

  Xiong, Chen, Shi, Ao　Chinese Physics Letters (2023)　　https://arxiv.org/abs/2207.04932

演化由确定性的选择与随机性的变异共同地产生

诗词的特征之一是不确定，而不确定性也是生命演化的基本特质之一，演化力学不缺乏诗词的意境

咏梅诗词--不同的诗人用同一格式对同一现象作出完全不同的理解和表达

•被动、自怜：

驿外断桥边，寂寞开无主。已是黄昏独自愁，更著风和雨。
无意苦争春，一任群芳妒。零落成泥碾作尘，只有香如故。

•乐观、开放：

风雨送春归，飞雪迎春到，已是悬崖百丈冰，犹有花枝俏。
俏也不争春，只把春来报。待到山花烂漫时，她在丛中笑。

演化力学也不缺乏哲学的品质：偶然与必然的关系

Thank you very much for your attention!

谢谢倾听，期待讨论

应用演化力学研究院（拟建）

追求卓越：树一流的目标；创活跃的气氛；育健全的人才