Mixing of charmed mesons: theoretical overview

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• Charmed mixing: short and long distance
  ➡ exclusive approach: no experiment
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• Conclusions: things to take home
Introduction

★ Experimental fact: charm mixing parameters are non-zero

★ ... and rather large

- if CP-violation is neglected...

\[
x = (0.46^{+0.14}_{-0.15}) \%
\]
\[
y = (0.62 \pm 0.08) \%
\]

- if CP-violation is allowed

\[
x = (0.32 \pm 0.14) \%
\]
\[
y = (0.69^{+0.06}_{-0.07}) \%
\]
Main goal of the exercise: understand physics at the most fundamental scale

It is important to understand relevant energy scales for the problem at hand

New Physics

\[
\Lambda
\]

physics of beauty

c, u
t
small
dominant

physics of charm

\[m_t\]
\[m_W\]
\[m_b\]
\[m_c\]

\[b,s,d\]
\[s,d\]
\[b\]
dominant
small
Quark-hadron duality: lifetimes

★ New Physics couples to quark degrees of freedom, we observe hadrons!
  ➔ need to know how to compute non-perturbative matrix elements
  ➔ need to understand how quark-hadron duality works

★ Observables computed in terms of hadronic degrees of freedom...

$$\Gamma_{\text{hadron}}(H_b) = \sum_{\text{all final state hadrons}} \Gamma(H_b \rightarrow h_i)$$

★ ... must match observables computed in terms of quark degrees of freedom

$$\Gamma(H_b) = \frac{1}{2M_b} \langle H_b | T | H_b \rangle = \frac{1}{2M_b} \langle H_b | \text{Im} \int d^4x \, T \{ H_{\text{eff}}^{AB=1}(x) H_{\text{eff}}^{AB=1}(0) \} | H_b \rangle$$

$$\Gamma(H_b) = \frac{G_F^2 m_Q^5}{192\pi^3} \left[ A_0 + \frac{A_2}{m_Q^2} + \frac{A_2}{m_Q^3} + \ldots \right]$$

HQ expansion converges reasonably well...
How to define quark-hadron duality and quantify its violations?

- Compute quark correlator in Euclidean space and analytically continue to Minkowski space \([\text{exact calculation in ES} = \text{exact result in MS}]\)
- Expand it in \(a_5\) and \(1/Q \sim 1/m_Q\): series truncation
- Any deviation beyond “natural uncertainty” is treated as violation of quark-hadron duality [resonances, instantons,…]

This definition is due to M. Shifman
Quark-hadron duality: lifetimes

★ In case of b-flavored hadrons can compare directly to experiment

<table>
<thead>
<tr>
<th>Lifetime ratio</th>
<th>Experimental average</th>
<th>HQE prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau(B^+)/\tau(B^0)$</td>
<td>$1.076 \pm 0.004$</td>
<td>$1.04^{+0.05}_{-0.01} \pm 0.02 \pm 0.01$</td>
</tr>
<tr>
<td>$\tau(B^0_s)/\tau(B^0)$</td>
<td>$0.990 \pm 0.004$</td>
<td>$1.001 \pm 0.002$</td>
</tr>
<tr>
<td>$\tau(A^0_b)/\tau(B^0)$</td>
<td>$0.967 \pm 0.007$</td>
<td>$0.935 \pm 0.054$</td>
</tr>
<tr>
<td>$\tau(\Xi^0_b)/\tau(\Xi_b^-)$</td>
<td>$0.929 \pm 0.028$</td>
<td>$0.95 \pm 0.06$</td>
</tr>
</tbody>
</table>

★ ... works surprisingly well...

<table>
<thead>
<tr>
<th>Channel</th>
<th>Expansion parameter $x$</th>
<th>Numerical value</th>
<th>$\exp[-1/x]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \rightarrow c \bar{c}s$</td>
<td>$\frac{\Lambda}{\sqrt{m_b^2-4m_c^2}} \approx \frac{\Lambda}{m_b} \left(1 + \frac{2m_c^2}{m_b^3}\right)$</td>
<td>$0.054 - 0.58$</td>
<td>$9.4 \cdot 10^{-9} - 0.18$</td>
</tr>
<tr>
<td>$b \rightarrow c \bar{u}s$</td>
<td>$\frac{\Lambda}{\sqrt{m_b^2-m_c^2}} \approx \frac{\Lambda}{m_b} \left(1 + \frac{1}{2} \frac{m_c^2}{m_b^3}\right)$</td>
<td>$0.045 - 0.49$</td>
<td>$1.9 \cdot 10^{-10} - 0.13$</td>
</tr>
<tr>
<td>$b \rightarrow u \bar{u}s$</td>
<td>$\frac{\Lambda}{\sqrt{m_b^2-4m_u^2}} = \frac{\Lambda}{m_b}$</td>
<td>$0.042 - 0.48$</td>
<td>$4.2 \cdot 10^{-11} - 0.12$</td>
</tr>
</tbody>
</table>

Jubb, Kirk, Lenz, Tetlalmatzi-Xolocotzi, 2017

★ How does it work for charmed hadrons?

For the lifetimes, see Prof. H.Y. Cheng’s talk from yesterday
Quark-hadron duality: mixing

How can one tell that a process is dominated by long-distance or short-distance?

To start thing off, mass and lifetime differences of mass eigenstates...

\[ x_D = \frac{M_2 - M_1}{\Gamma_D}, \quad y_D = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D} \]

...can be calculated as real and imaginary parts of a correlation function

\[
y_D = \frac{1}{2M_D\Gamma_D} \text{Im} \langle \overline{D^0} | i \int d^4 x T \left\{ \mathcal{H}_w^{\Delta C=1}(x) \mathcal{H}_w^{\Delta C=1}(0) \right\} | D^0 \rangle
\]

\[
x_D = \frac{1}{2M_D\Gamma_D} \text{Re} \left[ 2\langle \overline{D^0} | H^{\Delta C=2} | D^0 \rangle + \langle \overline{D^0} | i \int d^4 x T \left\{ \mathcal{H}_w^{\Delta C=1}(x) \mathcal{H}_w^{\Delta C=1}(0) \right\} | D^0 \rangle \right]
\]

... or can be written in terms of hadronic degrees of freedom...

\[
y = \frac{1}{2\Gamma} \sum_n \rho_n \left[ \langle D^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | \overline{D^0} \rangle + \langle \overline{D^0} | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | D^0 \rangle \right]
\]
How can one tell that a process is dominated by long-distance or short-distance?

\[ y = \frac{1}{2\Gamma} \sum_n \rho_n \left[ \langle D^0 | H_{\Delta}^{C=1} | n \rangle \langle n | H_{\Delta}^{C=1} | \bar{D}^0 \rangle + \langle \bar{D}^0 | H_{\Delta}^{C=1} | n \rangle \langle n | H_{\Delta}^{C=1} | D^0 \rangle \right] \]

It is important to remember that the expansion parameter is \( 1/E_{\text{released}} \)

\[ y_D = \frac{1}{2 M_D \Gamma_D} \text{Im} \langle \bar{D}^0 | i \int d^4 x \ T \left\{ H_{\Delta}^{C=1} (x) \ H_{\Delta}^{C=1} (0) \right\} | D^0 \rangle \]

**OPE-leading contribution:**

In the heavy-quark limit \( m_c \rightarrow \infty \) we have \( m_c \gg \sum m_{\text{intermediate quarks}} \), so \( E_{\text{released}} \sim m_c \)

- the situation is similar to B-physics, where it is "short-distance" dominated
- one can consistently compute pQCD and 1/m corrections

But wait, \( m_c \) is NOT infinitely large! What happens for finite \( m_c \)???

- how is large momentum routed in the diagrams?
- are there important hadronization (threshold) effects?
Threshold (and related) effects in OPE

★ How can one tell that a process is dominated by long-distance or short-distance?

★ Let's look at how the momentum is routed in a leading-order diagram
  - injected momentum is $p_c \sim m_c$
  - thus, $p_1 \sim p_2 \sim m_c/2 \sim O(\Lambda_{QCD})$?

Still OK with OPE, signals large nonperturbative contributions

★ For a particular example of the lifetime difference, have hadronic intermediate states
  - let's use an example of KKK intermediate state
  - in this example, $E_{\text{released}} \sim m_D - 3 m_K \sim O(\Lambda_{QCD})$

★ Similar threshold effects exist in B-mixing calculations
  - but $m_b \gg \sum m_{\text{intermediate quarks}}$, so $E_{\text{released}} \sim m_b$ (almost) always
  - quark-hadron duality takes care of the rest!

Thus, two approaches: 1. insist on $1/m_c$ expansion, hope for quark-hadron duality
2. saturate correlators by hadronic states
Mixing: Standard Model predictions

★ Predictions of $x$ and $y$ in the SM are complicated
- second order in flavor SU(3) breaking
- $m_c$ is not quite large enough for OPE
  - $x, y \ll 10^{-3}$ (“short-distance”)
  - $x, y \sim 10^{-2}$ (“long-distance”)

★ Short distance:
- assume $m_c$ is large
- combined $m_s$, $1/m_c$, $a_s$ expansions
- leading order: $m_s^2$, $1/m_c^6$!
- threshold effects?

★ Long distance:
- assume $m_c$ is NOT large
- sum of large numbers with alternating signs, SU(3) forces zero!
- multiparticle intermediate states dominate

* Not an actual representation of theoretical uncertainties. Objects might be bigger then what they appear to be...

References:

H. Georgi; T. Ohl, ...
I. Bigi, N. Uraltsev;
M. Bobrowski et al

J. Donoghue et. al.
P. Colangelo et. al.

Falk, Grossman, Ligeti, Nir. A.A.P.
Phys.Rev. D69, 114021, 2004
Falk, Grossman, Ligeti, and A.A.P.
Phys.Rev. D65, 054034, 2002
Aside: classification of charm decays

★ Can be classified by SM CKM suppression

★ Cabibbo-favored (CF) decay
- originates from $c \to s \, ud$
- examples: $D^0 \to K\pi^+$

★ Singly Cabibbo-suppressed (SCS) decay
- originates from $c \to q \, u\bar{q}$
- examples: $D^0 \to \pi\pi$ and $D^0 \to KK$

★ Doubly Cabibbo-suppressed (DCS) decay
- originates from $c \to d \, u\bar{s}$
- examples: $D^0 \to K^+\pi^-$

★ “Common final states” for $D$ and $\bar{D}$ generate mixing in exclusive approach
Exclusive approach to mixing: use data?

★ LD calculation: saturate the correlator by hadronic states, e.g.

\[
y = \frac{1}{2\Gamma} \sum_n \rho_n \left[ \langle D^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | \bar{D}^0 \rangle + \langle \bar{D}^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | D^0 \rangle \right]
\]

... with \( n \) being all states to which \( D^0 \) and \( \bar{D}^0 \) can decay. Consider \( \pi\pi, \pi K, KK \) intermediate states as an example...

\[
y_2 = Br(D^0 \rightarrow K^+K^-) + Br(D^0 \rightarrow \pi^+\pi^-) - 2 \cos \delta \sqrt{Br(D^0 \rightarrow K^+\pi^-)Br(D^0 \rightarrow \pi^+K^-)}
\]

If every \( Br \) is known up to \( O(1\%) \) the result is expected to be \( O(1\%) \)!

The result here is a series of large numbers with alternating signs, \( SU(3) \) forces 0

If experimental data on \( Br \) is used, are we only sensitive to exit. uncertainties?

★ Need to “repackage” the analysis: look at complete multiplet contribution

\[
y = \sum_{F_R} y_{F,R} Br(D^0 \rightarrow F_R) = \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma(D^0 \rightarrow n)
\]
Exclusive approach to mixing: no data

★ LD calculation: consider the correlation

\[ \Sigma_{p_D}(q) = i \int d^4z \left\langle \overline{D}(p_D) | T \left[H_w(z)H_w(0)\right] | D(p_D) \right\rangle e^{i(q-p_D)z} \]

\[ \frac{1}{2m_D} \Sigma_{p_D}(p_D) = \left( \Delta m - \frac{i}{2} \Delta \Gamma \right) \]

★ \( \Sigma_{p_D}(q) \) is an analytic function of q. To write a disp. relation, go to to HQET:

\[ H_w = \frac{4G_F}{\sqrt{2}} V_{cq_1} V^*_{uq_2} \sum_i C_i O_i = \hat{H}_w \left[ e^{-im_v z} h^{(c)}_v + e^{im_v z} \tilde{h}^{(c)}_v \right] + \ldots \]

\[ |D(p = mv)\rangle = \sqrt{m} |H(v)\rangle + \ldots \]

Now we can interpret \( \Sigma_{p_D}(q) \) for all q.
Dispersion relations for mixing

★ ...this implies for the correlator

\[ \Sigma_{PD}(q) = i \int d^4z \left\langle H(v) \right| T e^{i(q_{PD} - m_c)z} \left[ \hat{H}_w h_v^{(c)}(z), \hat{H}_w \tilde{h}_v^{(c)}(0) \right]\left| H(v) \right\rangle + \]

\[ + i \int d^4z \left\langle H(v) \right| T e^{i(q_{PD} + m_c)z} \left[ \hat{H}_w \tilde{h}_v^{(c)}(z), \hat{H}_w h_v^{(c)}(0) \right]\left| H(v) \right\rangle + ... \]

★ HQ mass dependence drops out for the second term, so for \( \Sigma_v(q) = \Sigma_{PD}(q)/m_D \)

\[ \Sigma_v(q) = -2\Delta m(E) + i\Delta\Gamma(E) \]

★ Rapidly oscillates for large \( m_c \)

★ Thus, a dispersion relation

\[ \Delta m = -\frac{1}{2\pi} P \int_{2m_{\pi}}^{\infty} dE \left[ \frac{\Delta\Gamma(E)}{E - m_D} + O\left(\frac{\Lambda_{QCD}}{E}\right) \right] \]

mass and width difference of a heavy meson with mass \( E \)

Compute \( \Delta\Gamma \), then find \( \Delta m \)!
No data: SU(3)$_F$ and phase space

★ “Repackage” the analysis: look at the complete multiplet contribution

$$y = \sum_{F_R} y_{F,R} \ Br(D^0 \rightarrow F_R) = \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma(D^0 \rightarrow n)$$

y for each SU(3) multiplet

Each is 0 in SU(3)

★ Does it help? If only phase space is taken into account: mild model dependence

$$y_{F,R} = \frac{\sum_{n \in F_R} \langle D^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}{\sum_{n \in F_R} \Gamma(D^0 \rightarrow n)} = \frac{\sum_{n \in F_R} \langle D^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}{\sum_{n \in F_R} \langle D^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}$$

Can consistently compute
Example: PP intermediate states

★ Consider PP intermediate state. Note that \((8 \times 8)_s = 27+8+1\). Look at 8 as an example

Numerator:

\[
A_{N,8} = |A_0|^2 s_1^2 \left[ \frac{1}{2} \Phi(\eta,\eta) + \frac{1}{2} \Phi(\pi^0,\pi^0) + \frac{1}{3} \Phi(\eta,\pi^0) + \Phi(\pi^+,\pi^-) - \Phi(K^0,\pi^0) \\
+ \Phi(K^+,K^-) - \frac{1}{6} \Phi(\eta,K^0) - \frac{1}{6} \Phi(\eta,\bar{K}^0) - \Phi(K^+,\pi^-) - \Phi(K^-,\pi^+) \right]
\]

Denominator:

\[
A_{D,8} = |A_0|^2 \left[ \frac{1}{6} \Phi(\eta,K^0) + \Phi(K^+,\pi^-) + \frac{1}{2} \Phi(K^0,\pi^0) + O(s_1^2) \right]
\]

★ This contribution is calculable....

\[
y_{2,8} = \frac{A_{N,8}}{A_{D,8}} = -0.038 s_1^2 = -1.8 \times 10^{-4}
\]

.... but completely negligible!

1. Repeat for other states
2. Multiply by Br_{Fr} to get y
Old results

★ Repeat for other intermediate states:

<table>
<thead>
<tr>
<th>Final state representation</th>
<th>$y_{F,R}/s_1^2$</th>
<th>$y_{F,R}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PP$</td>
<td>8</td>
<td>-0.0038</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>-0.00071</td>
</tr>
<tr>
<td>$PV$</td>
<td>$8_s$</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>$8_A$</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>0.016</td>
</tr>
<tr>
<td>$(VV)_s$-wave</td>
<td>8</td>
<td>-0.081</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>-0.061</td>
</tr>
<tr>
<td>$(VV)_p$-wave</td>
<td>8</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>-0.10</td>
</tr>
<tr>
<td>$(VV)_d$-wave</td>
<td>8</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>0.57</td>
</tr>
<tr>
<td>$(3P)_s$-wave</td>
<td>8</td>
<td>-0.48</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>-0.11</td>
</tr>
<tr>
<td>$(3P)_p$-wave</td>
<td>8</td>
<td>-1.13</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>-0.07</td>
</tr>
<tr>
<td>$(3P)_{form}$-factor</td>
<td>8</td>
<td>-0.44</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>-0.13</td>
</tr>
<tr>
<td>$4P$</td>
<td>8</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>$27'$</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
</tr>
</tbody>
</table>

- Product is naturally $O(1\%)$
- No (symmetry-enforced) cancellations
- Disp relation: compute $x$ (model-dependence)

naturally implies that $x,y \sim 1\%$ is expected in the Standard Model

Note dominance of near-threshold states!
**Exclusive approach to mixing: use data!**

★ What if we insist on using experimental data anyway?

★ Ex., one can employ Factorizaton-Assisted Topological Amplitudes

<table>
<thead>
<tr>
<th>Modes</th>
<th>$\mathcal{B}(\text{exp})$</th>
<th>$\mathcal{B}(\text{FAT})$</th>
<th>Modes</th>
<th>$\mathcal{B}(\text{exp})$</th>
<th>$\mathcal{B}(\text{FAT})$</th>
<th>Modes</th>
<th>$\mathcal{B}(\text{exp})$</th>
<th>$\mathcal{B}(\text{FAT})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0\overline{K}^0$</td>
<td>24.0 ± 0.8</td>
<td>24.2 ± 0.8</td>
<td>$\pi^0\overline{K}^0$</td>
<td>37.5 ± 2.9</td>
<td>35.9 ± 2.2</td>
<td>$\overline{K}^0\rho^0$</td>
<td>12.8 ± 1.4</td>
<td>13.5 ± 1.4</td>
</tr>
<tr>
<td>$\pi^+K^-$</td>
<td>39.3 ± 0.4</td>
<td>39.2 ± 0.4</td>
<td>$\pi^+K^-$</td>
<td>54.3 ± 4.4</td>
<td>62.5 ± 2.7</td>
<td>$K^-\rho^+$</td>
<td>111.0 ± 9.0</td>
<td>105.0 ± 5.2</td>
</tr>
<tr>
<td>$\eta\overline{K}^0$</td>
<td>9.70 ± 0.6</td>
<td>9.6 ± 0.6</td>
<td>$\eta\overline{K}^0$</td>
<td>9.6 ± 3.0</td>
<td>6.1 ± 1.0</td>
<td>$\overline{K}^0\omega$</td>
<td>22.2 ± 1.2</td>
<td>22.3 ± 1.1</td>
</tr>
<tr>
<td>$\eta'\overline{K}^0$</td>
<td>19.0 ± 1.0</td>
<td>19.5 ± 1.0</td>
<td>$\eta'\overline{K}^0$</td>
<td>&lt; 1.10</td>
<td>0.19 ± 0.01</td>
<td>$\overline{K}^0\phi$</td>
<td>8.47 ± 0.66</td>
<td>8.2 ± 0.6</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>1.421 ± 0.025</td>
<td>1.44 ± 0.02</td>
<td>$\pi^+\rho^-$</td>
<td>5.09 ± 0.34</td>
<td>4.5 ± 0.2</td>
<td>$\pi^-\rho^+$</td>
<td>10.0 ± 0.6</td>
<td>9.2 ± 0.3</td>
</tr>
<tr>
<td>$K^+K^-$</td>
<td>4.01 ± 0.07</td>
<td>4.05 ± 0.07</td>
<td>$K^+K^-$</td>
<td>1.62 ± 0.15</td>
<td>1.8 ± 0.1</td>
<td>$K^-K^+$</td>
<td>4.50 ± 0.30</td>
<td>4.3 ± 0.2</td>
</tr>
<tr>
<td>$K^0\overline{K}^0$</td>
<td>0.36 ± 0.08</td>
<td>0.29 ± 0.07</td>
<td>$K^0\overline{K}^0$</td>
<td>0.18 ± 0.04</td>
<td>0.19 ± 0.03</td>
<td>$\overline{K}^0\overline{K}^0$</td>
<td>0.21 ± 0.04</td>
<td>0.19 ± 0.03</td>
</tr>
<tr>
<td>$\pi^0\eta$</td>
<td>0.69 ± 0.07</td>
<td>0.74 ± 0.03</td>
<td>$\eta\rho^0$</td>
<td>1.4 ± 0.2</td>
<td></td>
<td>$\pi^0\omega$</td>
<td>0.117 ± 0.035</td>
<td>0.10 ± 0.03</td>
</tr>
<tr>
<td>$\pi^0\eta'$</td>
<td>0.91 ± 0.14</td>
<td>1.08 ± 0.05</td>
<td>$\eta'\rho^0$</td>
<td>0.25 ± 0.01</td>
<td></td>
<td>$\pi^0\phi$</td>
<td>1.35 ± 0.10</td>
<td>1.4 ± 0.1</td>
</tr>
<tr>
<td>$\eta\eta$</td>
<td>1.70 ± 0.20</td>
<td>1.86 ± 0.06</td>
<td>$\eta\omega$</td>
<td>2.21 ± 0.23</td>
<td>2.0 ± 0.1</td>
<td>$\eta\phi$</td>
<td>0.14 ± 0.05</td>
<td>0.18 ± 0.04</td>
</tr>
<tr>
<td>$\eta\eta'$</td>
<td>1.07 ± 0.26</td>
<td>1.05 ± 0.08</td>
<td>$\eta'\omega$</td>
<td>0.044 ± 0.004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^0\pi^0$</td>
<td>0.826 ± 0.035</td>
<td>0.78 ± 0.03</td>
<td>$\pi^0\rho^0$</td>
<td>3.82 ± 0.29</td>
<td>4.1 ± 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^-K^+$</td>
<td>0.133 ± 0.009</td>
<td>0.133 ± 0.001</td>
<td>$\pi^-K^+$</td>
<td>0.345 ± 0.180</td>
<td>0.40 ± 0.02</td>
<td>$K^+\rho^-$</td>
<td>0.144 ± 0.009</td>
<td></td>
</tr>
<tr>
<td>$\eta K^0$</td>
<td>0.027 ± 0.002</td>
<td></td>
<td>$\eta K^0$</td>
<td>0.017 ± 0.003</td>
<td></td>
<td>$K^0\omega$</td>
<td>0.064 ± 0.003</td>
<td></td>
</tr>
<tr>
<td>$\eta' K^0$</td>
<td>0.056 ± 0.003</td>
<td></td>
<td>$\eta' K^0$</td>
<td>0.00055 ± 0.00004</td>
<td></td>
<td>$K^0\phi$</td>
<td>0.024 ± 0.002</td>
<td></td>
</tr>
</tbody>
</table>

Jiang, Yu, Qin, Li, and Lu, 2017

★ ... but it appears to yield a smaller result, $y_{PP+PV} = (0.21 ± 0.07)\%$. 
Exclusive approach to mixing: use data!

★ What if we insist on using experimental data anyway?

★ Possible additional contributions?
   - each intermediate state has a finite width, i.e. is not a proper asymptotic state
   - within each multiplet widths experience (incomplete) SU(3) cancelations
   - this effect already happens for the simplest intermediate states!

★ Consider, for illustration, a set of single-particle intermediate states:

\[
-\sum_{p_D} \left( p_D \right)_{\text{res}}^{\text{tot}} = \frac{1}{2m_D} \sum_R \text{Re} \left\langle D_L | \mathcal{H}_W | R \right\rangle \left\langle R | \mathcal{H}_W^\dagger | D_L \right\rangle \frac{m_D^2 - m_R^2 + i\Gamma_R m_D}{m_D^2 - m_R^2 + i\Gamma_R m_D} - (D_L \rightarrow D_S)
\]

\[\Delta m_D^{\text{res}}_R \propto \frac{m_D^2 - m_R^2}{(m_D^2 - m_R^2)^2 + \Gamma_R^2 m_D^2}\]

\[\Delta \Gamma_D^{\text{res}}_R \propto -\frac{\Gamma_R m_D}{(m_D^2 - m_R^2)^2 + \Gamma_R^2 m_D^2}\]

★ Each resonance contributes to $\Delta \Gamma$ only because of its finite width!
Finite width effects and exclusive approach

★ Multiplet effects for (single-particle) intermediate states
- in this simple example: heavy pion, kaon and eta/eta’
- each single-particle intermediate state has a rather large width

\[ \Delta \Gamma_D|_{\text{octet}}^{\text{res}} = \Delta \Gamma_D^{(K_H)} - \frac{1}{4} \Delta \Gamma_D^{(\pi_H)} - \frac{3}{4} \cos^2 \theta_H \Delta \Gamma_D^{(\eta_H)} - \frac{1}{4} \sin^2 \theta_H \Delta \Gamma_D^{(\eta_H')} \]

- where for each state \( \Delta \Gamma_D^{\text{res}} = -C f_R^2 \frac{\mu_R \gamma_R}{(1 - \mu_R)^2 + \gamma_R^2} \)
- ... and a model calculation gives \( C \equiv 2m_D (G_F a_2 f_D \xi_d / \sqrt{2})^2 \)
- SU(3) forces cancellations between members: a new SU(3) breaking effect!

| Resonance   | \( |\Delta m_D| \times 10^{-16} \) (GeV) | \( |\Delta \Gamma_D| \times 10^{-16} \) (GeV) |
|-------------|----------------------------------------|-----------------------------------------|
| \( K(1460) \) | \( \sim 1.24 (f_K(1460)/0.025)^2 \) | \( \sim 0.88 (f_K(1460)/0.025)^2 \) |
| \( \eta(1760) \) | \( 0.77 \pm 0.27 \) \( (f_\eta(1760)/0.01)^2 \) | \( 0.43 \pm 0.53 \) \( (f_\eta(1760)/0.01)^2 \) |
| \( \pi(1800) \) | \( 0.13 \pm 0.06 \) \( (f_\pi(1800)/0.01)^2 \) | \( 0.41 \pm 0.11 \) \( (f_\pi(1800)/0.01)^2 \) |
| \( K(1830) \) | \( \sim 0.29 (f_K(1830)/0.01)^2 \) | \( \sim 1.86 (f_K(1830)/0.01)^2 \) |

★ Similar effect for PP’, PV, PA, ... intermediate states!

A.A.P. and R. Briere
arXiv:1804.xxxx
To counteract the effects of finite widths and avoid double counting, work directly with Dalitz plot decays of D-mesons

A.A.P. and R. Briere
arXiv:1804.xxxx
★ Multitude of various models of New Physics can affect $\chi$
How would New Physics affect charm mixing?

- Local $\Delta C=2$ piece of the mass matrix affects $x$:

$$\left(M - \frac{i}{2} \Gamma\right)_{ij} = m_D^{(0)} \delta_{ij} + \frac{1}{2m_D} \langle D_i^0 | H_{W}^{\Delta C=2} | D_j^0 \rangle + \frac{1}{2m_D} \sum_I \langle D_i^0 | H_{W}^{\Delta C=1} | I \rangle \langle I | H_{W}^{\Delta C=1} | D_j^0 \rangle \frac{m^2_D - m_i^2 + i\epsilon}{m^2_D - m_j^2 + i\epsilon}$$

- Double insertion of $\Delta C=1$ affects $x$ and $y$:

Amplitude $A_n = \langle D^0 | (H_{SM}^{\Delta C=1} + H_{NP}^{\Delta C=1}) | n \rangle = A_n^{SM} + A_n^{NP}$

Suppose $|A_n^{NP}|/|A_n^{SM}| \sim O(\text{exp. uncertainty}) \leq 10\%$

Example: $y = \frac{1}{2\Gamma} \sum_n \rho_n \left( \overline{A}_n^{SM} + \overline{A}_n^{NP} \right) \left( A_n^{SM} + A_n^{NP} \right) \approx \frac{1}{2\Gamma} \sum_n \rho_n \overline{A}_n^{SM} A_n^{SM} + \frac{1}{2\Gamma} \sum_n \rho_n \left( \overline{A}_n^{SM} A_n^{NP} + \overline{A}_n^{NP} A_n^{SM} \right)$

- Zero in the SU(3) limit
- Can be significant!!!

Generic restrictions on NP from $D\bar{D}$-mixing

★ Comparing to experimental value of $x$, obtain constraints on NP models
  - assume $x$ is dominated by the New Physics model
  - assume no accidental strong cancellations b/w SM and NP

$$\mathcal{H}_{NP}^{\Delta C=2} = \frac{1}{\Lambda_{NP}^2} \sum_{i=1}^{8} \Lambda_{NP}^2 z_i(\mu) Q'_i$$

\[
\begin{align*}
Q_{cu}^{1} &= \bar{u}_L^\alpha \gamma_\mu c_L^\beta \bar{u}_L^\gamma c_L^- \mu c_L^- \\
Q_{cu}^{2} &= \bar{u}_R^\alpha c_L^\beta \bar{u}_R^\gamma c_L^- \\
Q_{cu}^{3} &= \bar{u}_R^\alpha c_L^\beta \bar{u}_R^\gamma c_L^- \\
Q_{cu}^{4} &= \bar{u}_R^\alpha c_L^\beta \bar{u}_L^\gamma c_R^- \\
Q_{cu}^{5} &= \bar{u}_R^\alpha c_L^\beta \bar{u}_L^\gamma c_R^- \\
\end{align*}
\]

★ ... which are

\[
\begin{align*}
|z_1| &\approx 5.7 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2 \\
|z_2| &\approx 1.6 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2 \\
|z_3| &\approx 5.8 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2 \\
|z_4| &\approx 5.6 \times 10^{-8} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2 \\
|z_5| &\approx 1.6 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2 \\
\end{align*}
\]

New Physics is either at a very high scales

- tree level: $\Lambda_{NP} \geq (4 - 10) \times 10^3 \text{ TeV}$
- loop level: $\Lambda_{NP} \geq (1 - 3) \times 10^2 \text{ TeV}$

or have highly suppressed couplings to charm!

★ Constraints on particular NP models available

Gedalia, Grossman, Nir, Perez
Phys. Rev. D80, 055024, 2009
E. Golowich, J. Hewett, S. Pakvasa and A.A.P.

Alexey A Petrov (WSU & MCTP)

HIEPA-II Workshop, UCAS, 18-21 March 2018
## New Physics in mixing: particular models

<table>
<thead>
<tr>
<th>Model</th>
<th>Approximate Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourth Generation (Fig. 2)</td>
<td>$</td>
</tr>
<tr>
<td>$Q = -1/3$ Singlet Quark (Fig. 4)</td>
<td>$m_2 - m_1 &lt; 0.27$ (GeV)</td>
</tr>
<tr>
<td>$Q = +2/3$ Singlet Quark (Fig. 5)</td>
<td>$</td>
</tr>
<tr>
<td>Little Higgs</td>
<td>Box: Region of parameter space can reach observed $x_D$</td>
</tr>
<tr>
<td>Generic $Z'$ (Fig. 7)</td>
<td>$M_{Z'}/C &gt; 2.2 \cdot 10^6$ TeV</td>
</tr>
<tr>
<td>Family Symmetries (Fig. 8)</td>
<td>$m_{Z''}/f &gt; 1.2 \cdot 10^3$ TeV (with $m_{Z''}/m_1 = 0.5$)</td>
</tr>
<tr>
<td>Left-Right Symmetric (Fig. 9)</td>
<td>No constraint</td>
</tr>
<tr>
<td>Alternate Left-Right Symmetric (Fig. 10)</td>
<td>$M_{LR} &gt; 1.2$ TeV ($m_{U_R} = 0.5$ TeV)</td>
</tr>
<tr>
<td>Vector Leptoquark Bosons (Fig. 11)</td>
<td>$(\Delta m/m_{D_1})/M_R &gt; 0.4$ TeV$^{-1}$</td>
</tr>
<tr>
<td>Flavor Conserving Two-Higgs-Doublets (Fig. 13)</td>
<td>$M_{Z'} &gt; 5\times\Lambda_{D_1}/0.1$ TeV</td>
</tr>
<tr>
<td>Flavor Changing Neutral Higgs (Fig. 15)</td>
<td>No constraint</td>
</tr>
<tr>
<td>FC Neutral Higgs (Cheng-Sher ansatz) (Fig. 16)</td>
<td>$m_H/C &gt; 2.4 \cdot 10^9$ TeV</td>
</tr>
<tr>
<td>Scalar Leptoquark Bosons</td>
<td>$m_{H}/</td>
</tr>
<tr>
<td>Higgsless (Fig. 17)</td>
<td>See entry for RPV SUSY</td>
</tr>
<tr>
<td>Universal Extra Dimensions</td>
<td>$M &gt; 100$ TeV</td>
</tr>
<tr>
<td>Split Fermion (Fig. 19)</td>
<td>No constraint</td>
</tr>
<tr>
<td>Warped Geometries (Fig. 21)</td>
<td>$M/</td>
</tr>
<tr>
<td>Minimal Supersymmetric Standard (Fig. 23)</td>
<td>$M_1 &gt; 3.5$ TeV</td>
</tr>
<tr>
<td>Supergravity Alignment</td>
<td>$</td>
</tr>
<tr>
<td>Supersymmetry with RPV (Fig. 27)</td>
<td>$</td>
</tr>
<tr>
<td>Split Supersymmetry</td>
<td>$\tilde{m} &gt; 2$ TeV</td>
</tr>
<tr>
<td></td>
<td>$\lambda_{13k} m_k/\tilde{m}^2_{3k} &lt; 1.8 \cdot 10^{-5}/100$ GeV</td>
</tr>
<tr>
<td></td>
<td>No constraint</td>
</tr>
</tbody>
</table>

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Gedalia, Grossman, Nir, Perez

Bigi, Blanke, Buras, Recksiegel, JHEP 0907:097, 2009
Measuring charm mixing with HIEPA

★ If CP violation is neglected: mass eigenstates = CP eigenstates
★ CP eigenstates do NOT evolve with time, so can be used for “tagging”

\[ D_{CP(\pm)} \rightarrow f_1 f_2 \]

\[
\left| D^0 \bar{D}^0 \right>_L = \frac{1}{\sqrt{2}} \left[ D^0(k_1) \bar{D}^0(k_2) + (-1)^L \left| D^0(k_2) \bar{D}^0(k_1) \right| \right]
\]

★ τ-charm factories have good CP-tagging capabilities

- CP anti-correlated \( \psi(3770) \): \( CP(\text{tag}) \) \((-1)^L = [CP(K_S) CP(\pi^0)] \) \((-1) = +1 \)
- CP correlated \( \psi(4140) \)

Can measure \( y \cos \phi \):

\[
B^l_\pm = \frac{\Gamma(D_{CP\pm} \rightarrow X \ell \nu)}{\Gamma_{tot}}
\]

\[
y \cos \phi = \frac{1}{4} \left( \frac{B^l_+}{B^l_-} - \frac{B^l_-}{B^l_+} \right)
\]

No need for time dependence!

D. Atwood, A.A.P., hep-ph/0207165
D. Asner, W. Sun, hep-ph/0507238
4. Things to take home

- Computation of charm mixing amplitudes is a difficult task
  - no dominant heavy dof, as in beauty decays
  - light dofs give no contribution in the flavor SU(3) limit

- Charm quark is neither heavy nor light enough for a clean application of well-established techniques
  - “heavy-quark-expansion” techniques miss threshold effects
  - “heavy-quark” techniques give numerically leading contribution that is parametrically suppressed by $1/m^6$
  - “hadronic” techniques need to sum over large number of intermediate states, AND cannot use current experimental data on D-decays
  - “hadronic” techniques currently neglect some sources of SU(3) breaking

- Finite width effects complicate use of experimental data in exclusive analyses to obtain mass and lifetime differences
  - instead, direct use of Dalitz decays of D-mesons is desirable

- Quantum-coherent initial states allow for unique measurements
  - lifetime differences, hadronic and CP-violating observables
Mixing: short-distance estimates

★ SD calculation: expand the operator product in $1/m_c$, e.g.

$$\Gamma_{12} = -\lambda_s^2 \left( \Gamma^{ss}_{12} - 2\Gamma^{sd}_{12} + \Gamma^{dd}_{12} \right) + 2\lambda_s \lambda_b \left( \Gamma^{sd}_{12} - \Gamma^{dd}_{12} \right) - \lambda_b^2 \Gamma^{dd}_{12}$$

LO: $O(m_s^4)$
NLO: $O(m_s^3)$

- ... main contribution comes from dim-12 operators!!!
Correlate rare decays with D-mixing?

Let's write the most general $\Delta C=2$ Hamiltonian

$$\mathcal{H}_{NP}^{\Delta C=2} = \frac{1}{\Lambda_{NP}^2} \sum_{i=1}^{8} C_i(\mu)Q_i$$

...with the following set of 8 independent operators...

$$Q_1 = (\bar{u}_L \gamma_\mu c_L) (\bar{u}_L \gamma^\mu c_L)$$
$$Q_2 = (\bar{u}_L \gamma_\mu c_L) (\bar{u}_R \gamma^\mu c_R)$$
$$Q_3 = (\bar{u}_L c_R) (\bar{u}_R c_L)$$
$$Q_4 = (\bar{u}_R c_L) (\bar{u}_R c_L)$$
$$Q_5 = (\bar{u}_R \sigma_{\mu\nu} c_L) (\bar{u}_R \sigma^{\mu\nu} c_L)$$
$$Q_6 = (\bar{u}_R \gamma_\mu c_R) (\bar{u}_R \gamma^\mu c_R)$$
$$Q_7 = (\bar{u}_L c_R) (\bar{u}_L c_R)$$
$$Q_8 = (\bar{u}_L \sigma_{\mu\nu} c_R) (\bar{u}_L \sigma^{\mu\nu} c_R)$$

RG-running relate $C_i(m)$ at NP scale to the scale of $m \sim 1$ GeV, where ME are computed (on the lattice)

$$\frac{d}{d \log \mu} \tilde{C}(\mu) = \mathcal{T}(\mu) \tilde{C}(\mu)$$

Comparing to experimental value of $x$, obtain constraints on NP models
- assume $x$ is dominated by the New Physics model
- assume no accidental strong cancellations b/w SM and NP

E. Golowich, J. Hewett, S. Pakvasa and A.A.P. (07)
Gedalia, Grossman, Nir, Perez (09)

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