Deep Inelastic Scattering

Collider Physics
Toni Baroncelli
Haiping Peng
USTC

Toni Baroncelli: Deep Inelastic Scattering

Toni Baroncelli: Deep Inelastic Scattering

This course is titled "Collider Physics"

- Colliders are today the most powerful instrument to study the innermost structure of matter
- Proton-proton colliders are the accelerators that can reach the highest energies, for reasons that will be clear when discussing about accelerators
- Proton are very complex objects, with a complex internal structure
- The interpretation of scattering experiments need to be based on the understanding of the proton structure
- The scattering lepton-nucleon allows us to study the structure of the proton
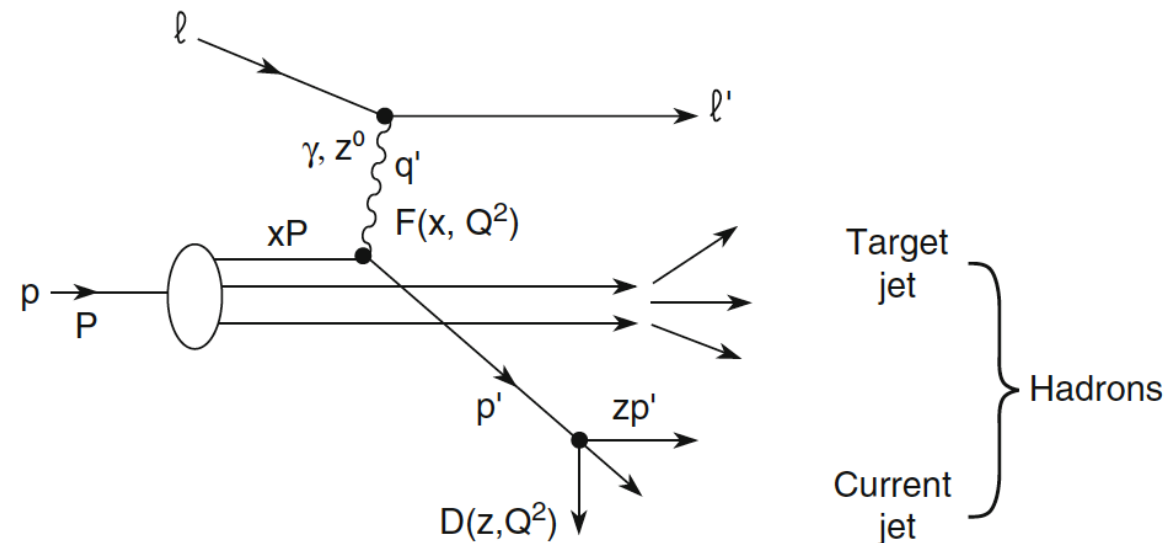
Deep Inelastic Scattering
→ *DIS*

Basis of QCD, the theory of hadronic interactions

Many generation of scattering experiments.
- Initially they used leptons (mostly electrons) produced in accelerators and sent on a target
- The last generation was the HERA collider at Desy, Germany

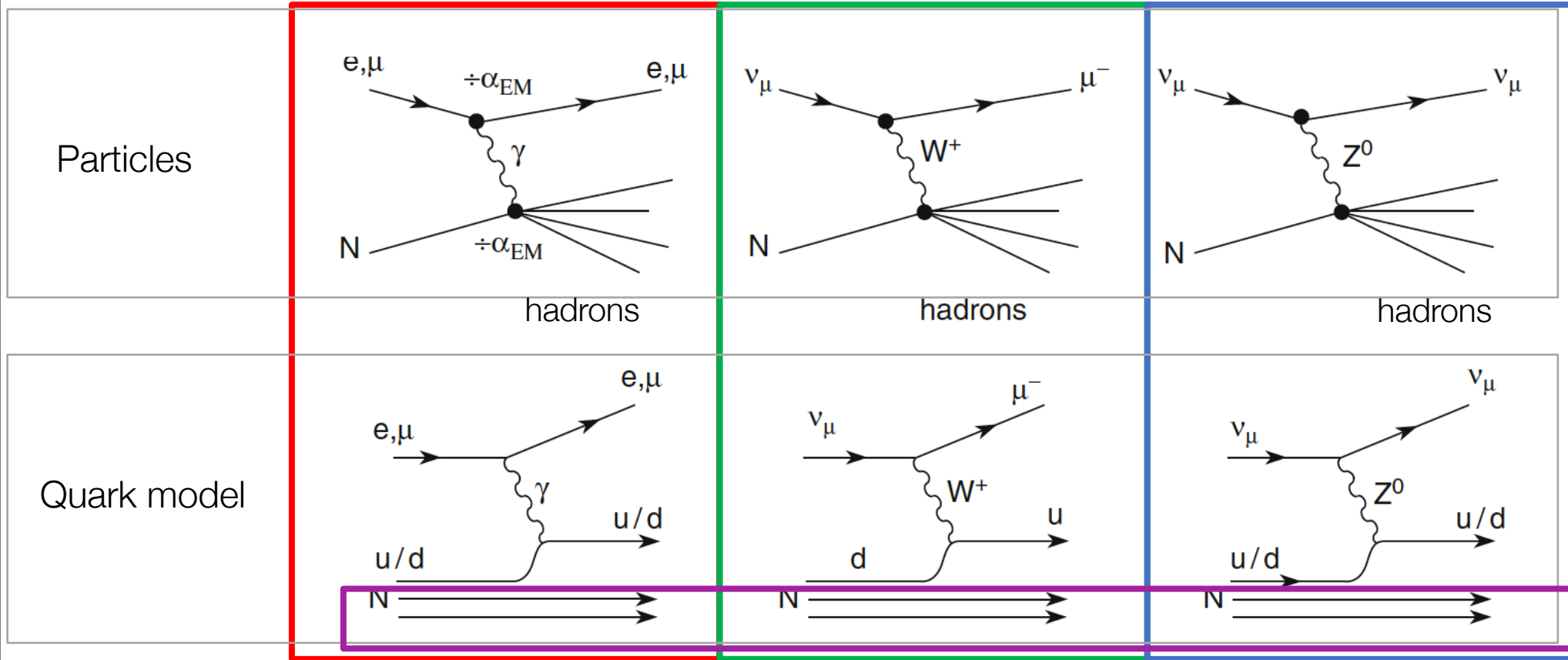30 GeV electrons against 900 GeV protons

$$ep: \quad e^{\pm} + p \to e^{\pm} + X^{+}$$

$$\mu p: \quad \mu^{\pm} + p \to \mu^{\pm} + X^{+}$$

$$\nu_{\mu} p(CC): \nu_{\mu} + p \to \mu^{-} + X^{++}, \quad \overline{\nu}_{\mu} + p \to \mu^{+} + X^{0}$$

$$\nu_{\mu} p(NC): \quad \nu_{\mu} + p \to \nu_{\mu} + X^{+}, \quad \overline{\nu}_{\mu} + p \to \overline{\nu}_{\mu} + X^{+}.$$

$$\Delta x \Delta Q c = \hbar c$$
$$\approx 197 MeV \cdot fm$$

$$Q^2 = 4 \cdot 10^2 GeV^2/c^2$$
$$\to \Delta x \approx 10^{-17} m$$

$$Q^2 = 4 \cdot 10^4 GeV^2/c^2$$
$$\to \Delta x \approx 10^{-18} m$$

Particles

Quark model

N = p / n

Electromagnetic   Weak Charged Current   Weak Neutral Current

Spectators

Toni Baroncelli: Deep Inelastic Scattering

Toni Baroncelli: Deep Inelastic Scattering

First stage:

~ elastic scattering of the virtual particle (boson) with one quark of the N carrying a fraction "x" of the N

First stage
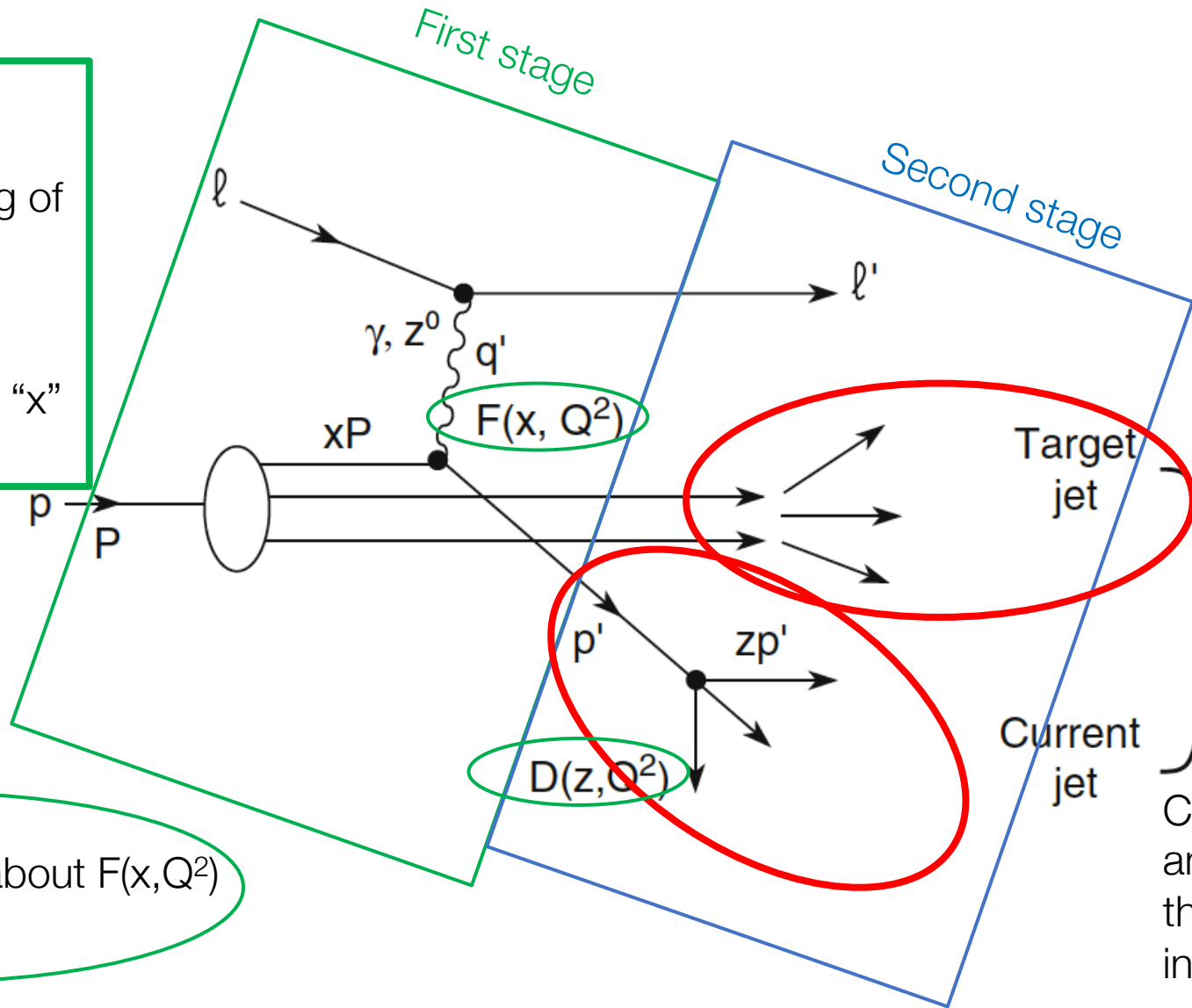
Second stage

Second stage:

Fragmentation → quarks cannot exist 'alone' → 'dress' into two jets

ℓ

ℓ'

γ, z^0 q'

$F(x, Q^2)$

xP

p

P

Target jet

p' zp'

$D(z, Q^2)$

Current jet

Target jet ~ in the same direction of the incoming ~~lepton~~ hadron

Hadrons

Current jet ~ at large angle with respect to the direction of the incoming lepton

Later we will talk about $F(x,Q^2)$ and $D(z,Q^2)$

Toni Baroncelli: Deep Inelastic Scattering

Excited States not observed
→ many hadrons

$E = 4.879\,\text{GeV}$

$\theta = 10^\circ$

inelastic

Elastic Peak

Excited States of the Proton
(Resonances)

Elastic scattering
(divided by 15)

$\dfrac{d^2\sigma}{d\Omega dE'}$ [nb/GeV sr]

E' [GeV]

W [GeV/c^2]
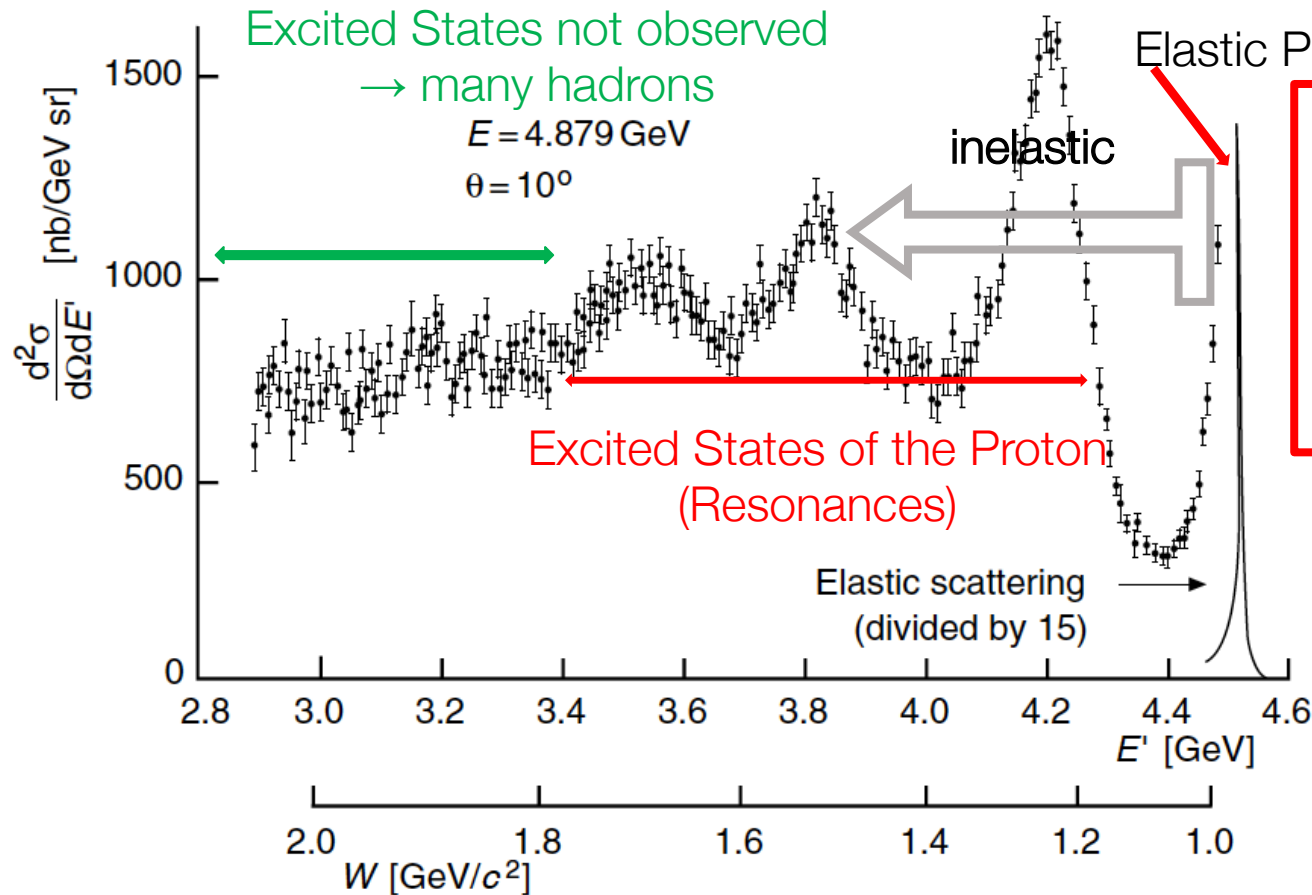
Figure ← electron-proton scattering.

- the incoming electron energy was E = 4. 9 GeV
- the scattering electron angle was fixed to θ = 10°
- The electron scattering energy is shown (part of the energy to the proton!)
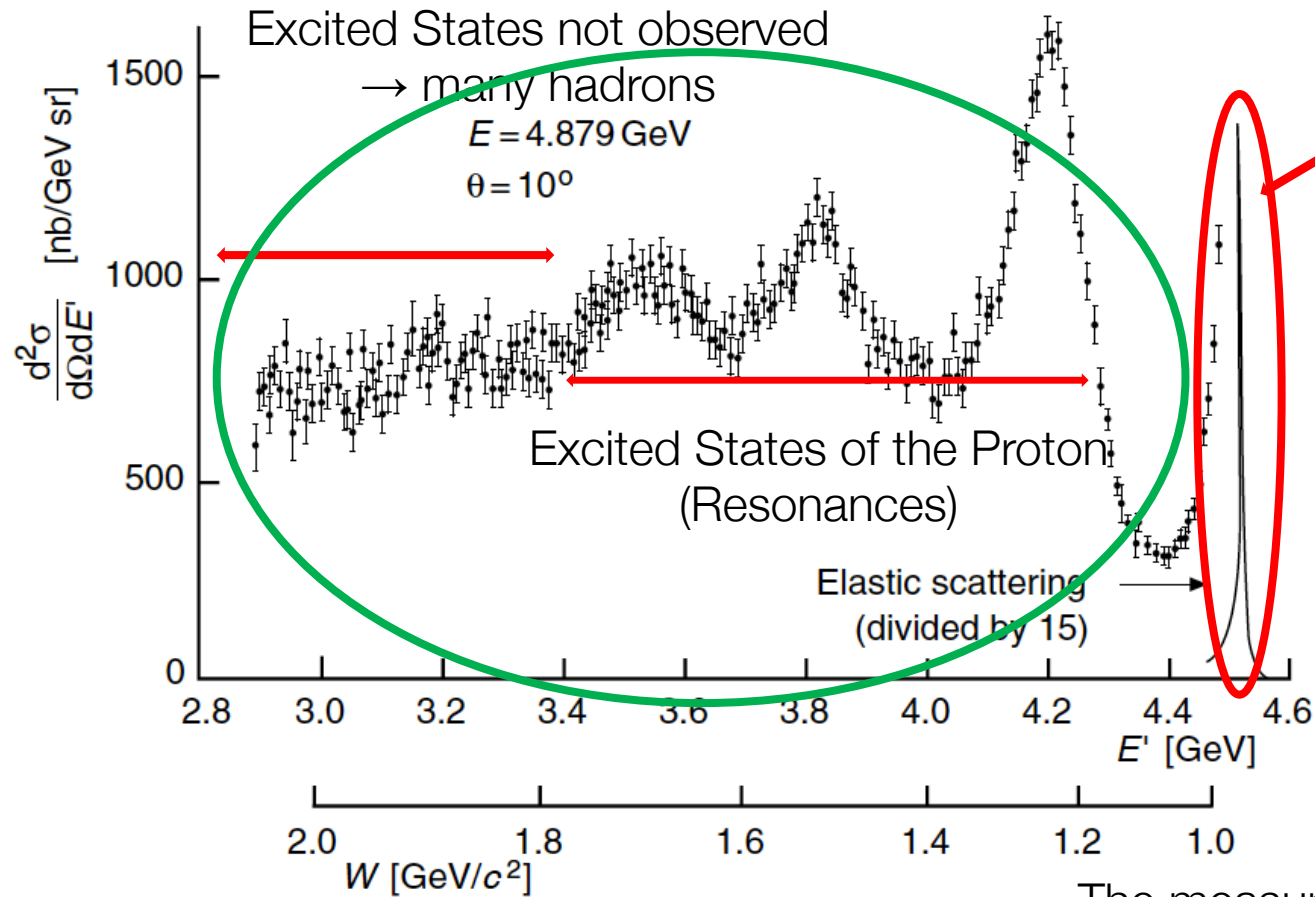
We see
- sharp elastic scattering peak (scaled down by 15)
- peaks at lower electron energies associated with inelastic excitations (excited states of the nucleon which we call nucleon resonances.
- Further down in energy, states with many hadrons

These excited states of the proton are an indication that **the proton is a composite system**. → quark model.

Toni Baroncelli: Deep Inelastic Scattering

Excited States not observed
→ many hadrons

$E = 4.879$ GeV

$\theta = 10°$

$\frac{d^2\sigma}{d\Omega dE'}$ [nb/GeV sr]

Excited States of the Proton
(Resonances)

Elastic scattering
(divided by 15)

E' [GeV]

W [GeV/c^2]

Elastic Scattering

Form Factors

$G_E(Q^2)$, $G_M(Q^2)$

Inelastic Scattering

Structure Functions

$W_2(Q^2,\nu)$, $W_1(Q^2,\nu)$

Elastic Peak → 2 particles in the final state./
The measurement of the electron scattering angle determines the kinematics of the event

electron

electron

P

P'

proton

proton

q

P_0

W
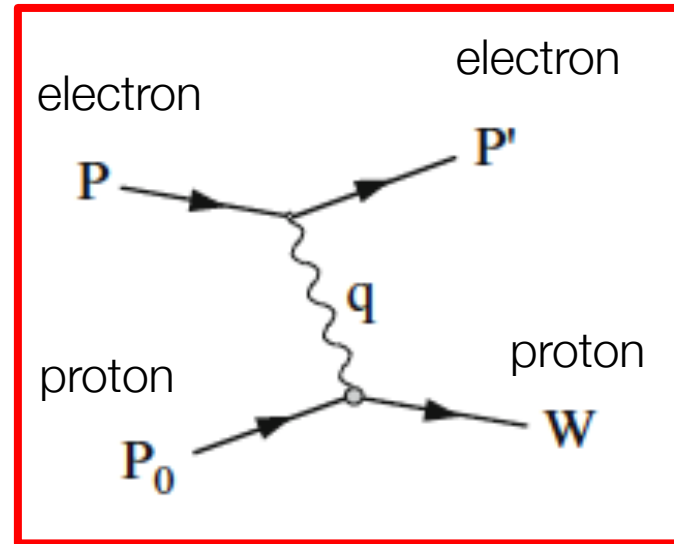
Inelastic scattering → many particles in the final state)
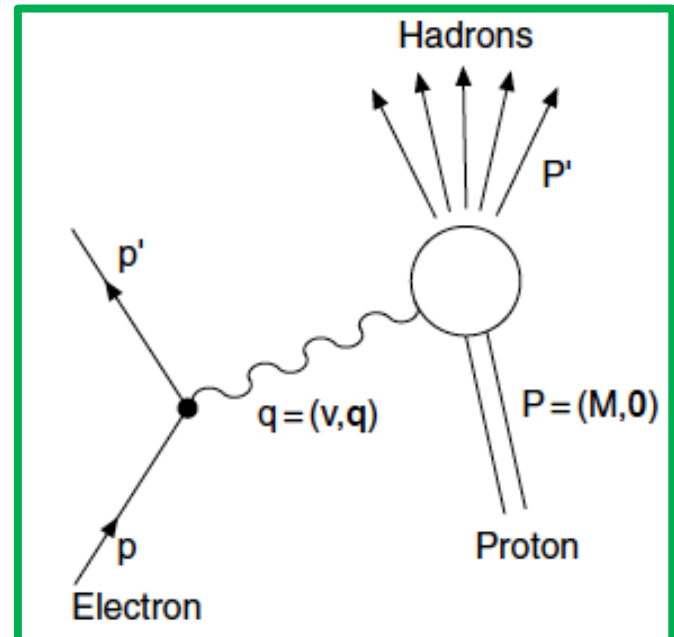
The measurement of the electron scattering angle does not determine the kinematics of the event → 2 variables needed (E',Θ) or (Q^2,υ)

Hadrons

p'

$P' = (M,0)$

$q = (\nu,\mathbf{q})$

p

Proton

Electron

Hadrons $\rightarrow$ W

W is the invariant mass of these hadrons

p'

q = (v, **q**)

P = (M,**0**)

p

Electron

Proton

Electron-proton inelastic scattering: more than the two incoming particles in the final state.

The scattering occurs between a proton at rest and an exchanged photon. In this representation the kinematics is defined as follows (*use quadri-momenta*):

W is defined as the invariant mass of all hadrons of the final state (*W>M*)

$$W^2 = P'^2 = (P + q)^2 = M^2 + 2Pq + q^2 =$$
$$= M^2 + 2Mv - Q^2 \qquad (Q^2 = -q^2)$$

And where

$$v = \frac{Pq}{M}$$

Quadri-momenta of particles are as follows: the target proton is at rest P=(Mc,**0**), the exchanged photon is

$$q=((E-E')/c,\mathbf{q}) \rightarrow \frac{Pq}{M} = v = \frac{Mc \cdot \frac{E-E'}{c} - q \cdot 0}{M} = E - E'$$

Therefore the energy transferred by the virtual photon from the electron to the proton in the laboratory frame is: $v = E - E'$

Toni Baroncelli: Deep Inelastic Scattering

Calculation	electron		Target, charge Ze (Z=1 proton)					Expression
	electron	Electron with spin	Point-like target, infinite Mass	Point-like target with mass M	Point-like proton	Point-like proton with spin	Finite size proton with spin	
Rutherford	✅		✅					$(\frac{d\sigma}{d\Omega})_R = \frac{Z^2 e^4}{4E_0^2 \, (\sin\theta/2)^4}$
Mott		✅		✅				$(\frac{d\sigma}{d\Omega})_M = (\frac{d\sigma}{d\Omega})_R \cdot (\cos\frac{\theta}{2})^2$
σ_{NS}		✅			✅			$(\frac{d\sigma}{d\Omega})_{NS} = (\frac{d\sigma}{d\Omega})_M \cdot 1/(1 - \frac{2E_0}{M}\sin\theta/2^2)$
σ		✅				✅		$(\frac{d\sigma}{d\Omega}) = (\frac{d\sigma}{d\Omega})_M \cdot (1 + \frac{q^2}{2M^2}\tan\theta/2^2)$
Rosenbluth		✅					✅	$(\frac{d\sigma}{d\Omega}) = (\frac{d\sigma}{d\Omega})_M \cdot \left| \frac{G_E^2(Q^2) + \tau \cdot G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan\theta/2^2 \right|$

$\longleftrightarrow$ e nucleus $\longleftrightarrow$ e proton

- Elastic scattering: G_E (Q^2) and G_M (Q^2) form factors.

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left| \frac{G_E^2(Q^2)+\tau \cdot G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan\frac{\theta^2}{2} \right| \text{ where } \tau = \frac{Q^2}{M^2c^2}$$

- The Q^2 dependence of the form factors gives us information about the radial charge distributions and the magnetic moments.

In elastic scattering, one parameter only fixes the kinematics of the event.
Example: the scattering angle θ is fixed, → squared four-momentum transfer Q^2, the energy transfer ν, the energy of the scattered electron E are also fixed. Since

$$W = M ,$$

(and remembering that $W^2 = P'^2 = (P+q)^2 = M^2 + 2Pq + q^2 = M^2 + 2M\nu - Q^2$ (inelastic scattering)
We get

$$M^2 = M^2 + 2M\nu - Q^2 \qquad \rightarrow \quad 2M\nu - Q^2 = 0. \qquad \text{(elastic scattering)}$$

Inelastic scattering: W_1 and W_2 structure functions.

Toni Baroncelli: Deep Inelastic Scattering

Toni Baroncelli: Deep Inelastic Scattering

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{Mott}\cdot\left[\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau}+2\tau G_M^2(Q^2)\tan\frac{\theta^2}{2}\right]\qquad\text{where }\tau=\frac{Q^2}{M^2c^2}$$
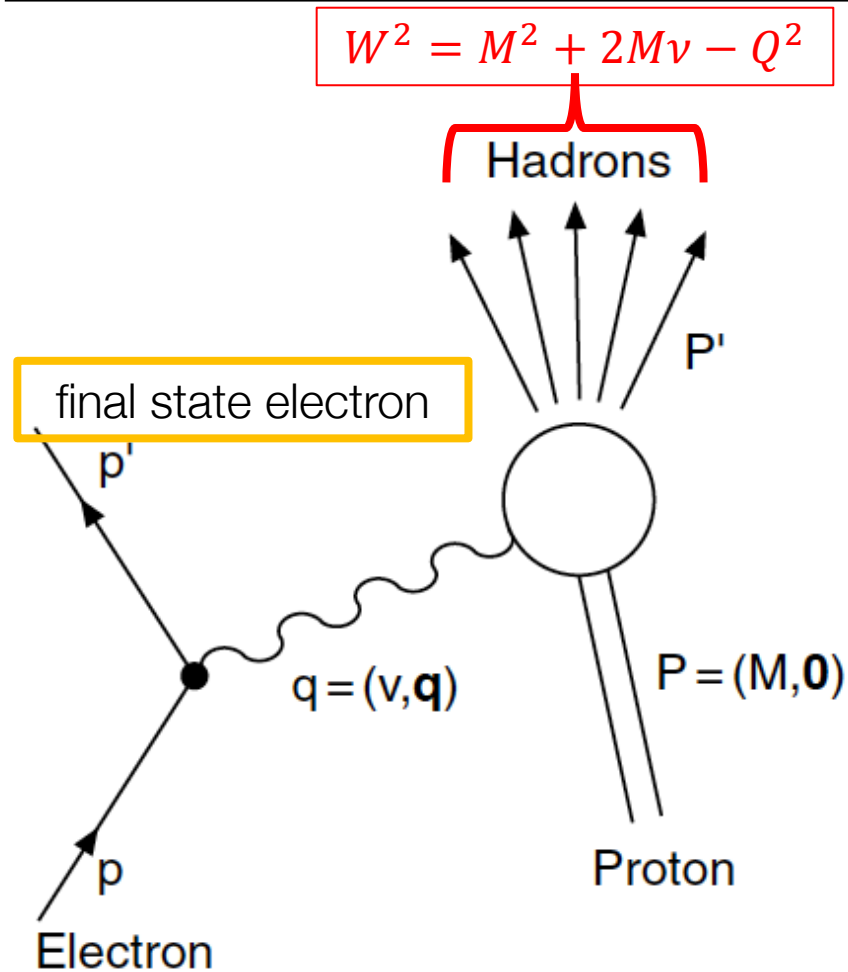
The modified cross section is generally rewritten now as

$$\left(\frac{d\sigma}{d\Omega dE'}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{Mott}\cdot\left[W_2(Q^2,\nu)+2\cdot W_1(Q^2,\nu)\tan\frac{\theta^2}{2}\right]\qquad\text{where }\nu=\frac{Pq}{M}$$

1 variable in elastic scattering → 2 variables in inelastic events
$$Q^2 \rightarrow Q^2, \nu$$

Here G_E (Q^2) and G_M (Q^2) are the electric and magnetic form factors both of which depend on Q^2.
$W_2(Q^2,n)$ and $W_2(Q^2,n)$ are the electric and magnetic structure functions both of which depend on Q^2 and ν

$$W^2 = M^2 + 2M\nu - Q^2$$

Hadrons

P'

final state electron

p'

$q = (\nu, \mathbf{q})$

$P = (M, \mathbf{0})$

p

Proton

Electron

The structure of the proton is best studied introducing a new Lorentz-invariant variable x defined as

$$x = \frac{Q^2}{2Pq} = \frac{Q^2}{2M\nu} = Q^2/2m_p(E_p - E_{p'})$$

This variable is generally known as "Bjorken scaling variable" and gives an indication of the inelasticity of the process.

$$Q^2 = 4E_e E_{e'} sin^2(\theta_e/2)$$
$$x = Q^2/2m_p(E_p - E_{p'})$$

In the elastic scattering W=M and the relation $2M\nu - Q^2 = 0$ gives x=1.

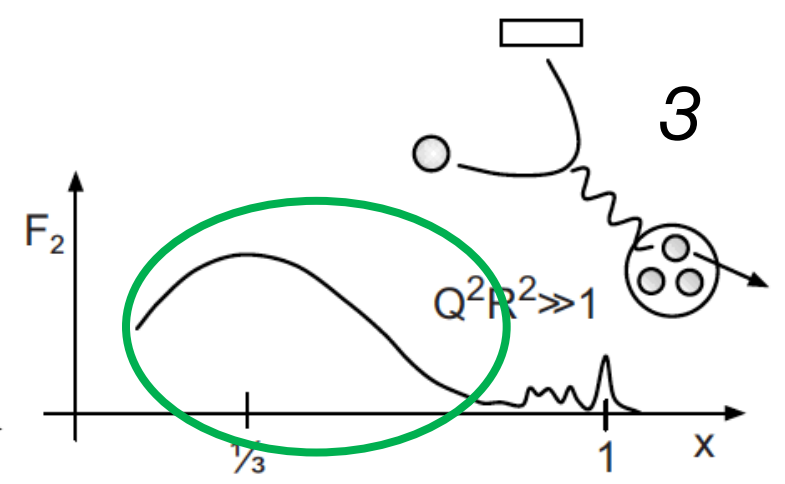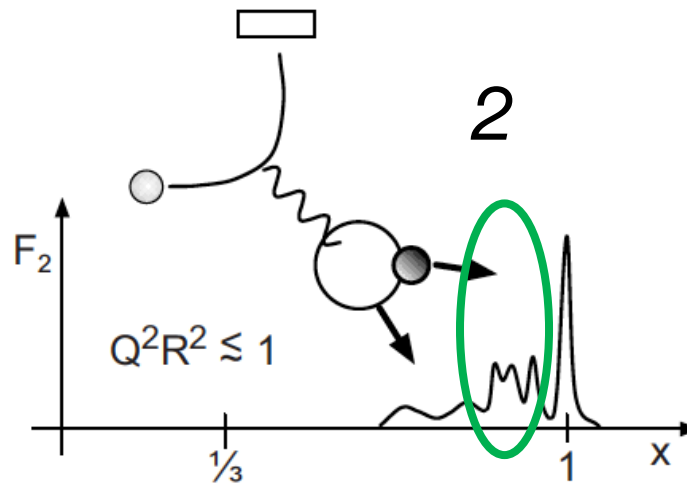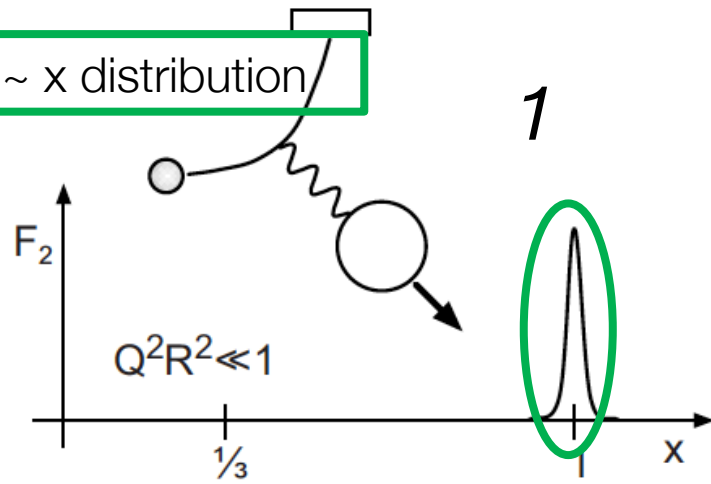In the case of inelastic processes however W>M and $0 \leq x \leq 1$.

To deduce the momentum transfer Q² and the energy loss ν , *the energy and the scattering angle of the electron have to be determined in the experiment*

$W = M$	$W > M$
x = 1	$0 \leq x \leq 1$

Toni Baroncelli: Deep Inelastic Scattering

F$_2$ ~ x distribution

1

2

3

F$_2$

$Q^2R^2 \ll 1$

⅓   X

F$_2$

$Q^2R^2 \lesssim 1$

⅓   1   X

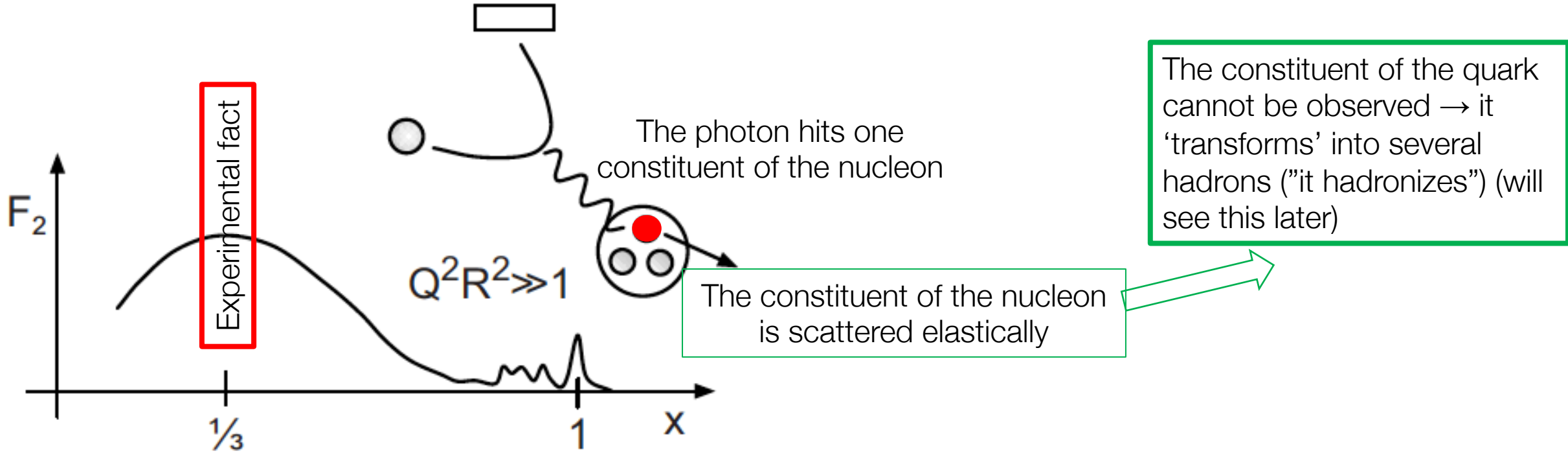F$_2$

$Q^2R^2 \gg 1$

⅓   1   X

What do we see with increasing Q^2?

See above 3 different cases

Q2 ↑ wave length of the probe particle ↓

1. The Q^2 of the reaction is ~low, the **nucleon** is seen by the exchanged photon as **a unique obje**ct. We have elastic scattering
2. The Q^2 of the reaction is not as ~low as in 1, not enough to probe the inner structure but enough to **excite the nucleon**
3. The Q^2 of the reaction is ~large enough to see the internal structure of the proton and the photon scatters elastically on one of the **internal constituents of the nucleon**

Experimental fact

F_2

$Q^2R^2 \gg 1$

⅓   1   X

The photon hits one constituent of the nucleon

The constituent of the nucleon is scattered elastically

The constituent of the quark cannot be observed → it 'transforms' into several hadrons ("it hadronizes") (will see this later)

The peak at ~1/3 can be understood as the "most probable" x value corresponding to the *elastic scattering of the photon and one of the nucleon constituents.*

If we assume that the 'x' budget is equally shared by '**n**' nucleon constituents then

$$x = \frac{1}{n}\frac{Q^2}{2Pq} = \frac{1}{n}\frac{Q^2}{2M\nu}$$

⟶   This term is equal to 1 in case of elastic scattering

$$\frac{1}{3} = \frac{1}{n} \rightarrow \text{there are 3 components in the nucleon}$$

For elastic scattering, two form factors G_E^2, G_M^2 are necessary to describe the electric and magnetic distributions. The cross-section for the scattering of an electron off a nucleon is described by the Rosenbluth formula,.

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[\frac{G_E^2(Q^2)+\tau \cdot G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\, tan\frac{\theta^2}{2}\right] \quad \leftarrow \text{Elastic Scattering, Q}^2 \qquad \text{where } \tau = \frac{Q^2}{M^2 c^2}$$

In the e-p inelastic scattering, it transforms into

$$\left(\frac{d\sigma}{d\Omega dE\prime}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[W_2(Q^2,\nu) + 2 \cdot W_1(Q^2,\nu)\, tan\frac{\theta^2}{2}\right] \quad \leftarrow \text{Inelastic Scattering, Q}^2,\nu$$

where the first term describes electrical interactions and the second term represents the magnetic interaction.

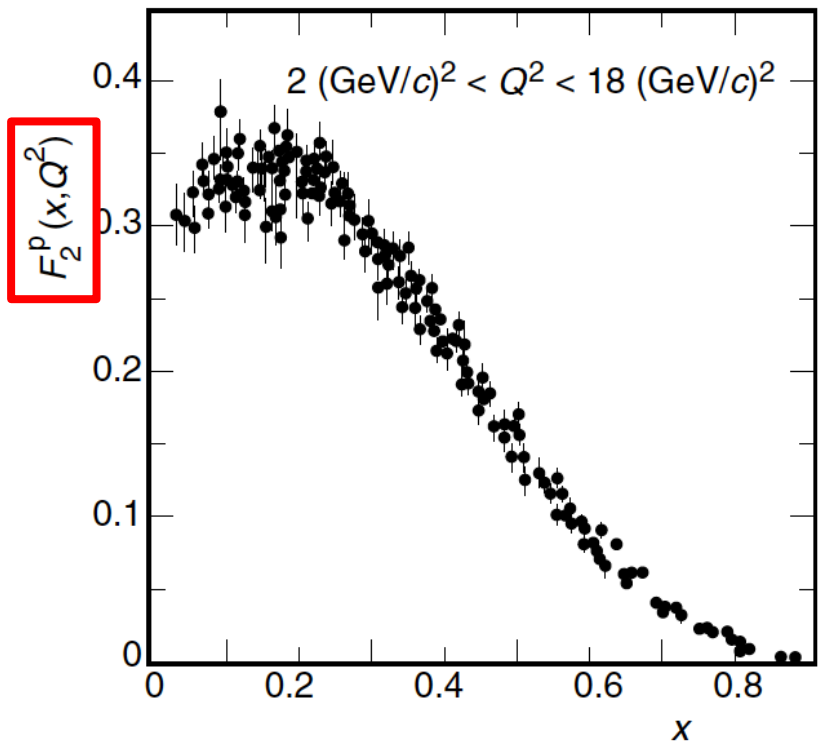One variable, Q², in the elastic case → two variables, Q² and ν, in the inelastic case

The two structure functions $W_1(Q^2, \nu)$ and $W_2(Q^2, \nu)$ in

$$\left(\frac{d\sigma}{d\Omega dE'}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[W_2(Q^2, \nu) + 2 \cdot W_1(Q^2, \nu) \tan\frac{\theta^2}{2} \right]$$

Can be replaced by two dimensionless functions

$$F_1(x, Q^2) = Mc^2 W_1(Q^2, \nu)$$
$$F_2(x, Q^2) = \nu W_2(Q^2, \nu)$$

Magnetic interaction term: W_1 & F_1 vanishes for scattering off spin 0 particles

$$G_E^2, G_M^2 \rightarrow W_2, W_1 \rightarrow F_1, F_2$$

different functions correspond better to inner structure. *Weak x-dependence*

The measured structure function $F_2(x, Q^2)$ is shown in the figure ← for a Q^2 interval between 2 and 18 (GeV/c)².

$F_2^p(x, Q^2)$ → measured in protons, you cannot choose a single quark! Experimental points taken at different Q^2 are seen to be superimposed.

It can be shown that this implies the scattering off point like objects.

Toni Baroncelli: Deep Inelastic Scattering
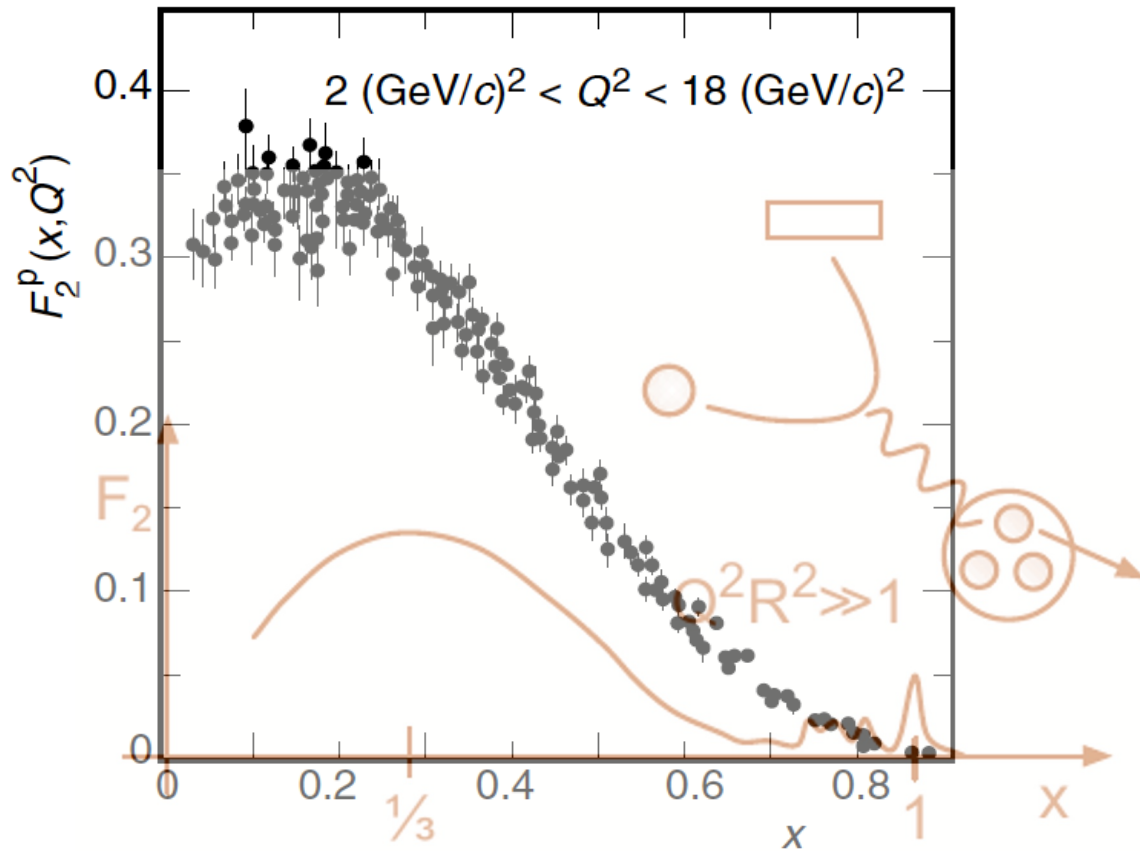
The peak of the experimental distribution is seen at a value of about ~ 0.2, lower than the 1/3 shown in the qualitative distribution. The shift is due to understood effects that will be discussed later

Studying the nucleon's constituents the wave length of the probe particle λ has to be small compared to the nucleon's radius, R

$$\lambda \ll R \;\; \rightarrow \;\; Q^2 \gg \hbar^2/R^2$$

Large Q^2 values are needed → high energies are required.

- The first generation ~1960 @ **SLAC** 25 GeV electrons on a target
- The second generation ~ 1980 @ **CERN** using beams of *muons* of up to 300 GeV (*). Muons on a target
- The last generation ~1990 → 2007 @ **DESY** Collider **HERA:** 30 GeV electrons against 900 GeV protons (see next slides).
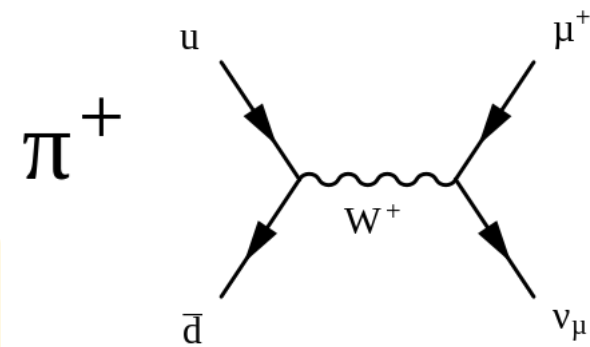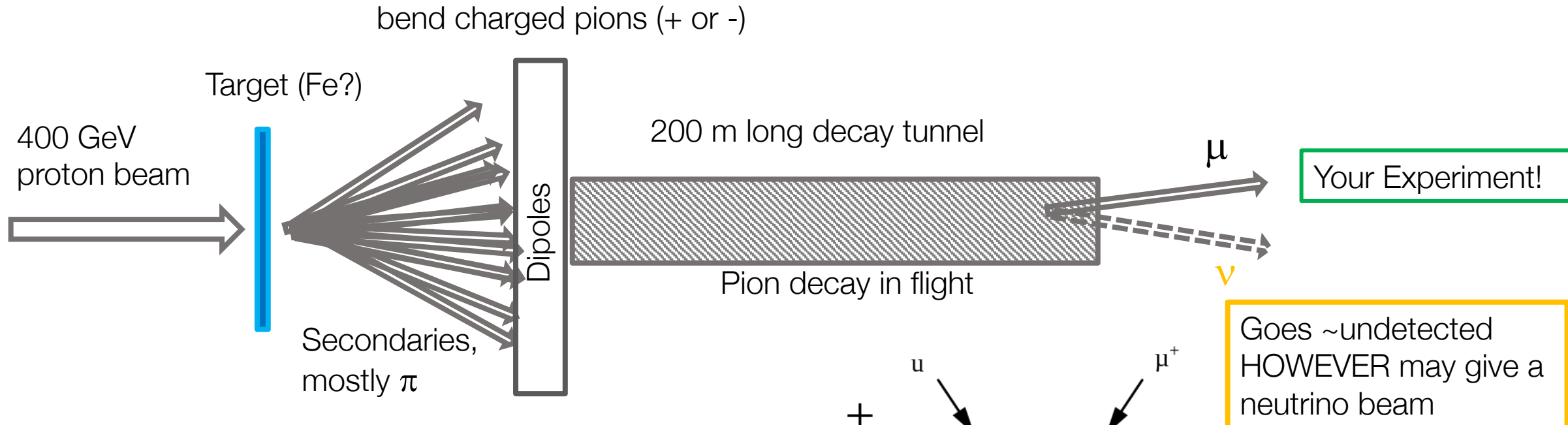
Photon exchange:
You need to use charged particles!

➤ In the SLAC experiments, the basic properties of the quark and gluon structure of the hadrons were established.
➤ The second and the third generations of experiments are at the basis of the

Quantum Chromodynamics,

the theory of the strong interaction.

(*) Protons of 400 GeV on a target produced pions which were kept confined in a 200 meters tunnel. During the flight part of the pions decayed into muons which were collected into a beam with energies up to 300 GeV.

Toni Baroncelli: Deep Inelastic Scattering

Circular $e + p$ accelerator @ Desy, Hamburg-DE.

Toni Baroncelli: Deep Inelastic Scattering

- 15 to 30 m underground and circumference of 6.3 km. Leptons and protons → two independent rings

- At HERA, 27.5 GeV electrons (or positrons) collided with 920 GeV protons, cms energy of 318 GeV (*).
- electrons or positrons: 450 MeV, 7.5 GeV, 14 GeV, 27.5 GeV.
- Protons: 50 MeV, 7 GeV, 40 GeV, 920 GeV.

- 4 interaction regions, 4 experiments H1, ZEUS, HERMES and Hera-B.
- About 40 minutes to fill the machine

- Operated between 1992 and 2007.

$$(*) \; E_{cm}(or \; cms) = \sqrt{m_p^2 + m_e^2 + 2E_pE_e(1 - \beta_1\beta_2\cos(\theta))} \approx \sqrt{2E_pE_e \cdot 2}$$

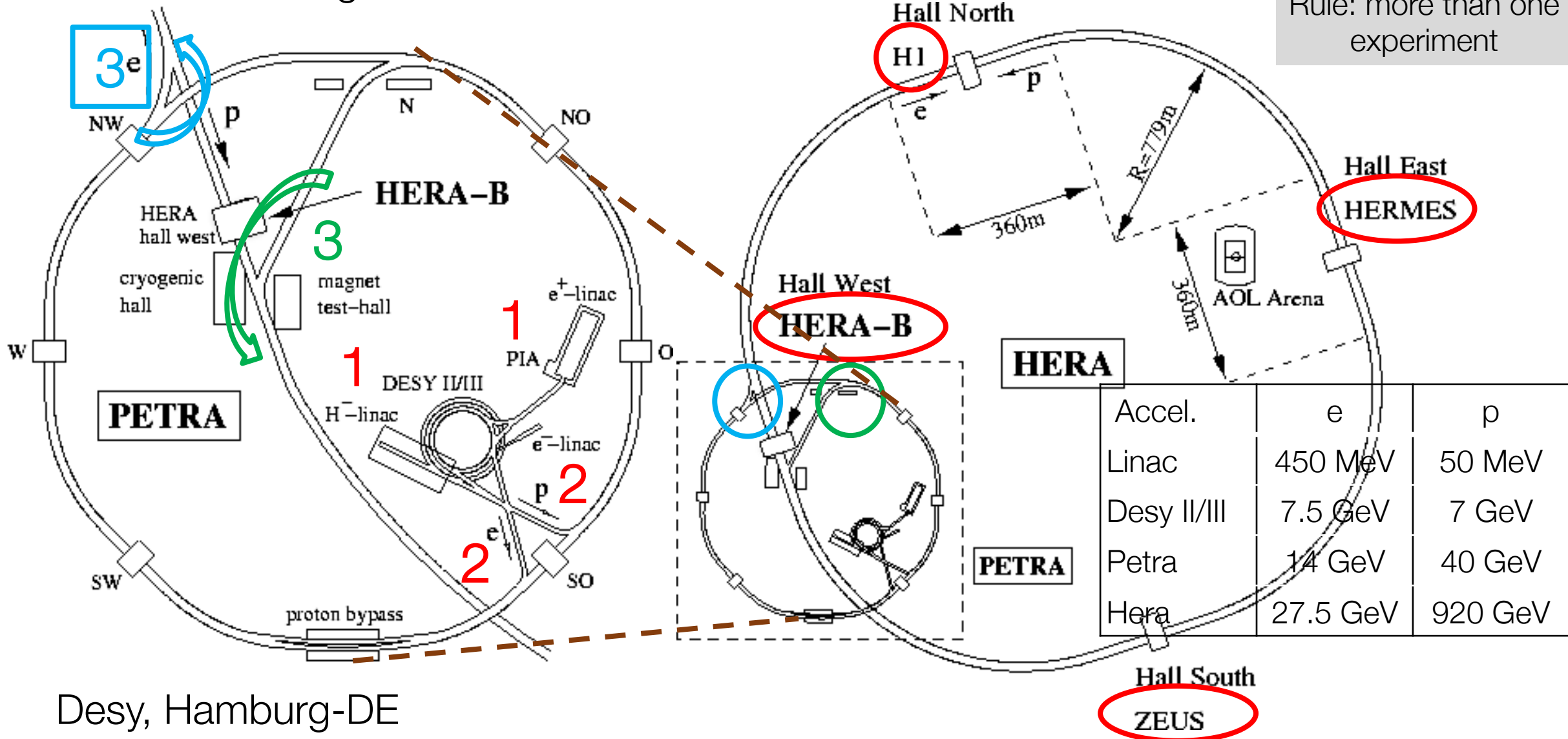At high energy
$\beta_1 \cdot \beta_2 = 1 \cdot -1$

Three stage acceleration: Linac and *Petra and then Hera*

Desy, Hamburg-DE

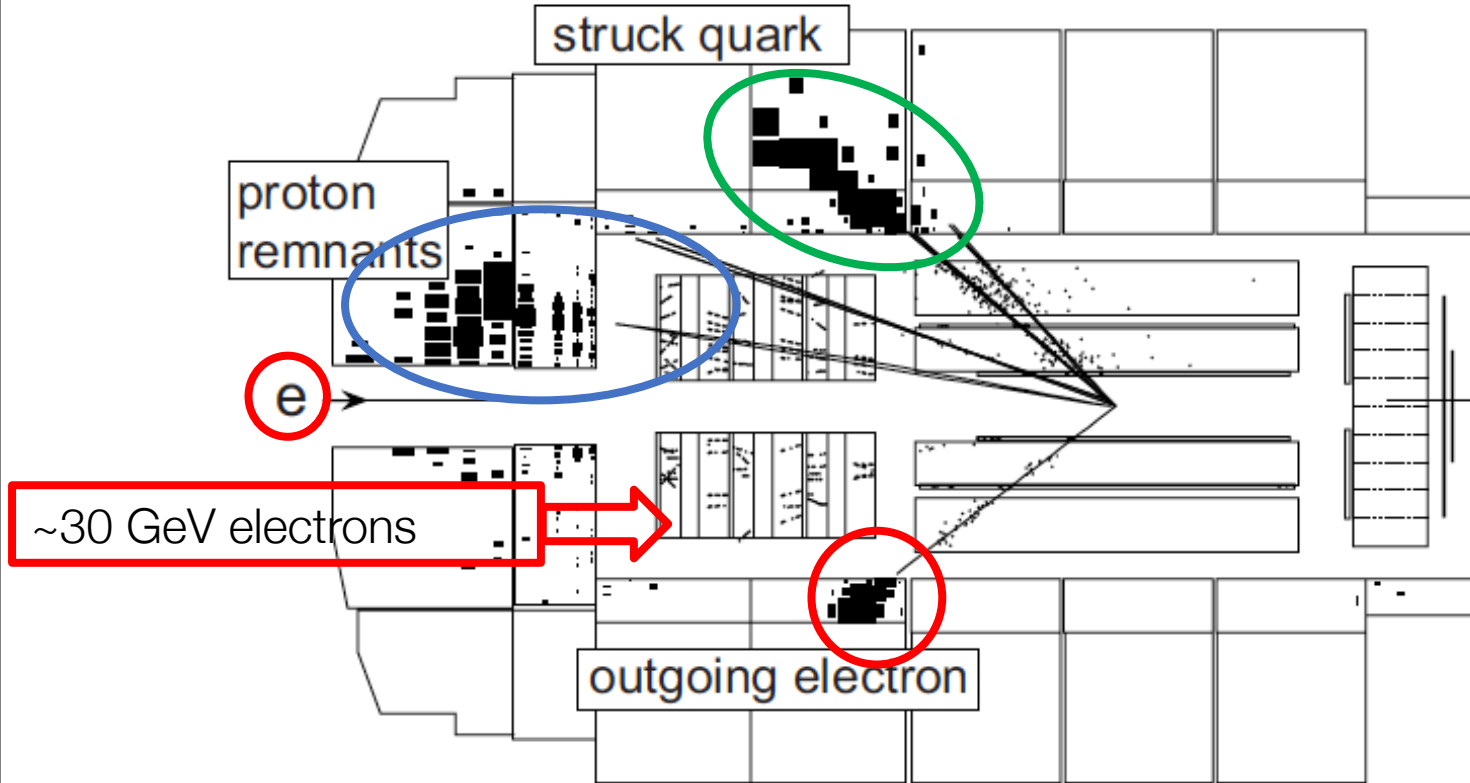| Accel.	e	p
Linac	450 MeV	50 MeV
Desy II/III	7.5 GeV	7 GeV
Petra	14 GeV	40 GeV
Hera	27.5 GeV	920 GeV

Toni Baroncelli: Deep Inelastic Scattering

struck quark

proton remnants

e

~30 GeV electrons

outgoing electron

To deduce the momentum transfer Q^2 and the **energy loss** $\nu = E' - E$, the **energy** and the **scattering angle** of the electron have to be determined in the experiment.
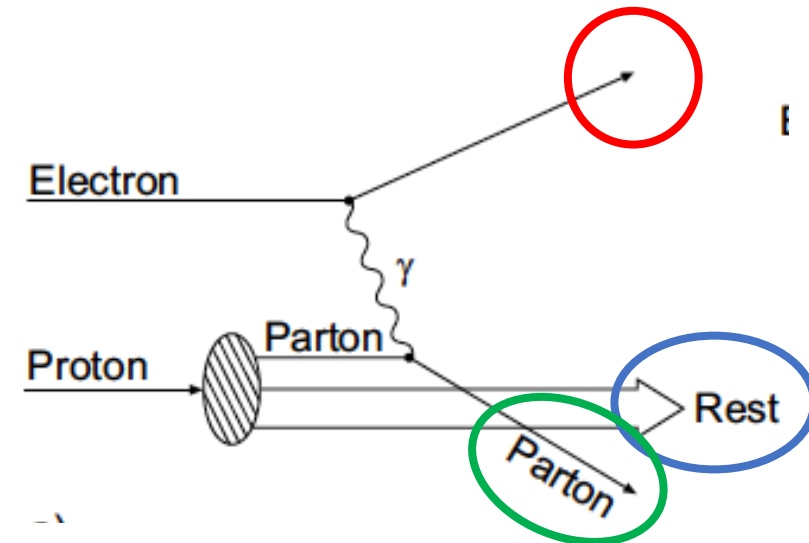
Very asymmetric event topology!

p

Up to 920 GeV protons

The direction of all charged particles is measured in the inner tracking detector. The energy of the scattered electron is measured in the electromagnetic calorimeter, that of the hadrons in the hadron calorimeter.
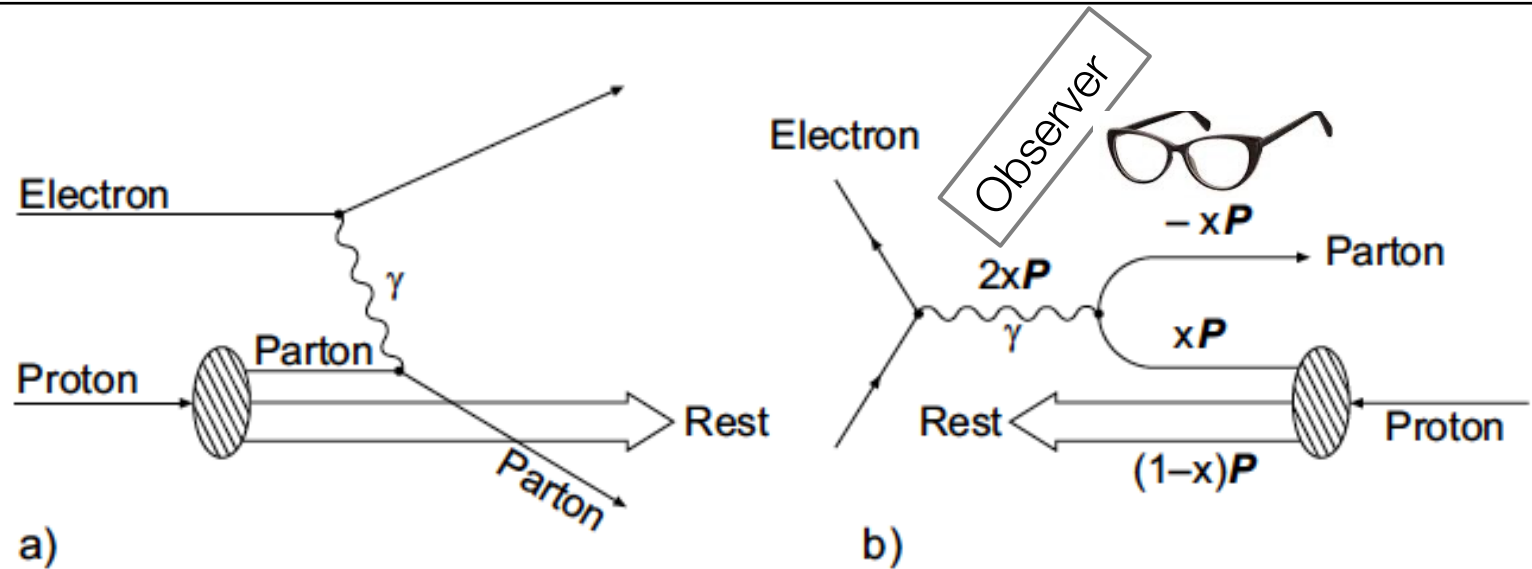
Electron

γ

Proton

Parton

Parton

Rest

$$Q^2 = 4E_e E_{e'} sin^2(\theta_e / 2)$$
$$x = Q^2 / 2m_p (E_p - E_{p'})$$

- Physics is independent of reference frame
- Proton observed in a reference system where it appears to be very fast → only longitudinal components, neglect p_T
- Masses can be neglected

Partons =
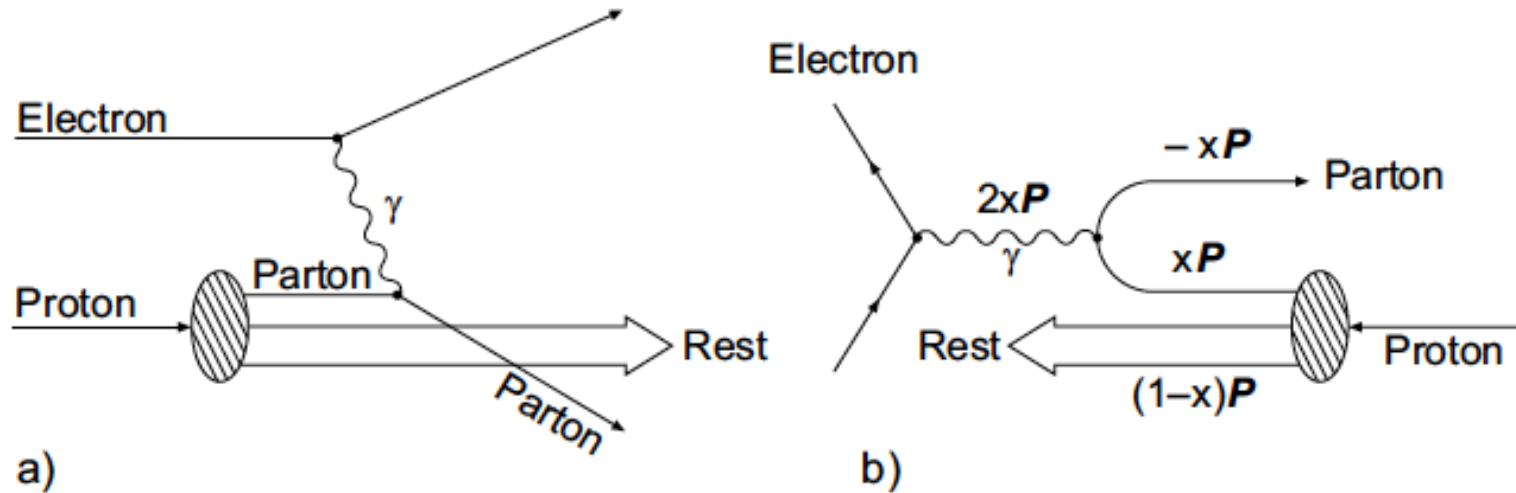quarks (charged) and neutral (gluons)

parton point of view of deep inelastic e-p scattering:
- (a) in the laboratory system
- (b) in a fast moving system (the Breit frame) in which the momentum transferred by the virtual photon is zero. Hence the momentum of the parton hit by the electron is turned around but its magnitude is unchanged.

Decomposing the proton into a sum of independent components allows us to see the
Interaction electron proton = sum of elastic interactions between the electron (via photon exchange) and partons

Toni Baroncelli: Deep Inelastic Scattering

Electron

Electron

Proton

Parton

Rest

Parton

$-x\boldsymbol{P}$

Parton

$2x\boldsymbol{P}$

γ

$x\boldsymbol{P}$

Rest

Proton

$(1-x)\boldsymbol{P}$

a)

b)

remember: ν = energy transferred by the virtual γ from the e to the p

It is assumed that
- the duration of the interaction photon – parton is so short that partons do not have time to interact between themselves →

Impulse Approximation

- Masses can be neglected →
$$Q^2 \gg M^2 c^2$$

In the laboratory system the photon which has four-momentum q=(ν/c, **q**) interacts with a parton carrying the four-momentum xP

The reduced wave-length λ– of the virtual photon is given by

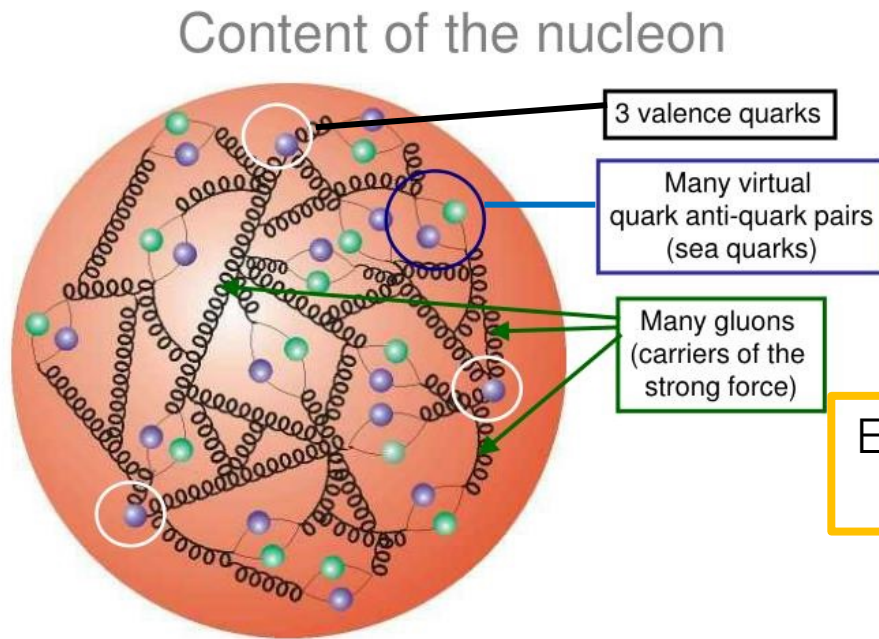$$\lambda = \frac{\hbar}{|\boldsymbol{q}|} = \frac{\hbar}{\sqrt{Q^2}}.$$

This gives the size of the structures of the proton we can study using a photon with momentum transfer Q^2
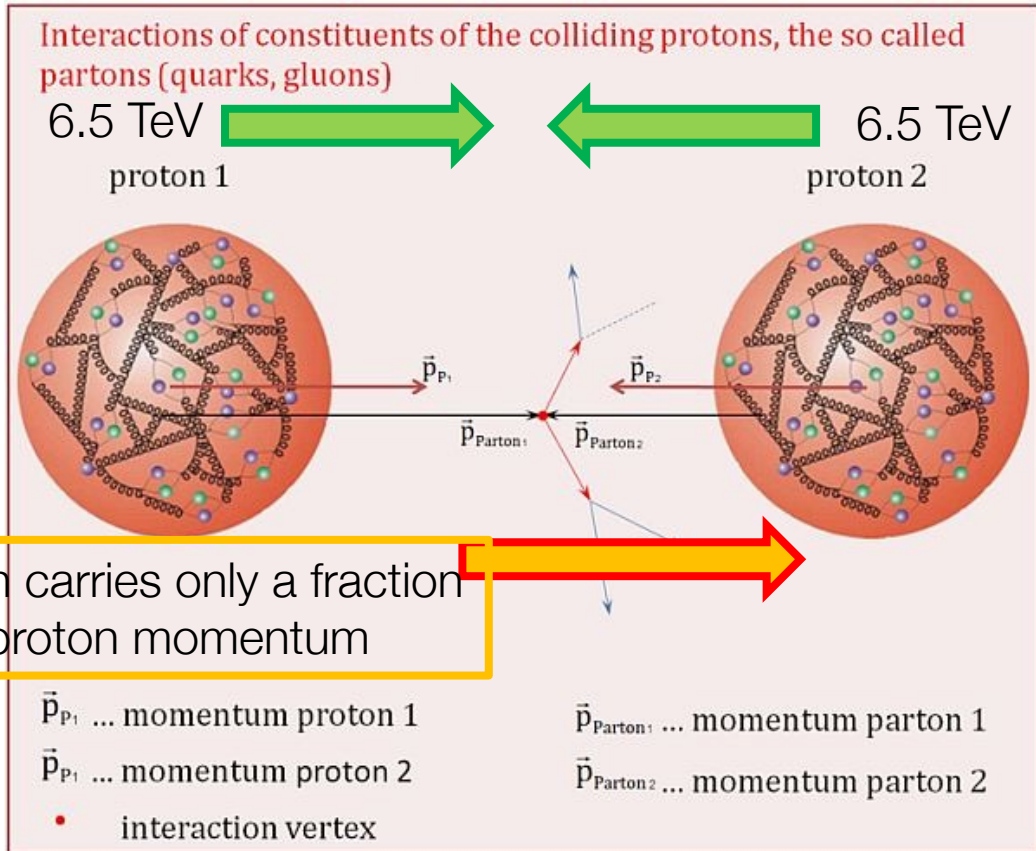
LHC: the largest accelerator in the world: proton beam against proton protons at a cms energy of 6.5 TeV + 6.5 TeV. collisions between two very complex objects!

$F_2(x, Q^2)$ tells us how quarks are distributed in x

→ To interpret these collisions you need to know the structure of the proton

Content of the nucleon

3 valence quarks

Many virtual quark anti-quark pairs (sea quarks)

Many gluons (carriers of the strong force)

… only quarks and anti-quarks interact with neutrinos

Interactions of constituents of the colliding protons, the so called partons (quarks, gluons)

6.5 TeV

proton 1

6.5 TeV

proton 2

$\vec{P}_{P_1}$ $\vec{P}_{P_2}$

$\vec{P}_{Parton_1}$ $\vec{P}_{Parton_2}$

Each parton carries only a fraction of the proton momentum

$\vec{P}_{P_1}$ … momentum proton 1

$\vec{P}_{P_1}$ … momentum proton 2

• interaction vertex

$\vec{P}_{Parton_1}$ … momentum parton 1

$\vec{P}_{Parton_2}$ … momentum parton 2

Toni Baroncelli: Deep Inelastic Scattering

- Scattering *ep* and *μp* (at accelerators)

$$ep : \quad e^{\pm} + p \to e^{\pm} + X^+$$

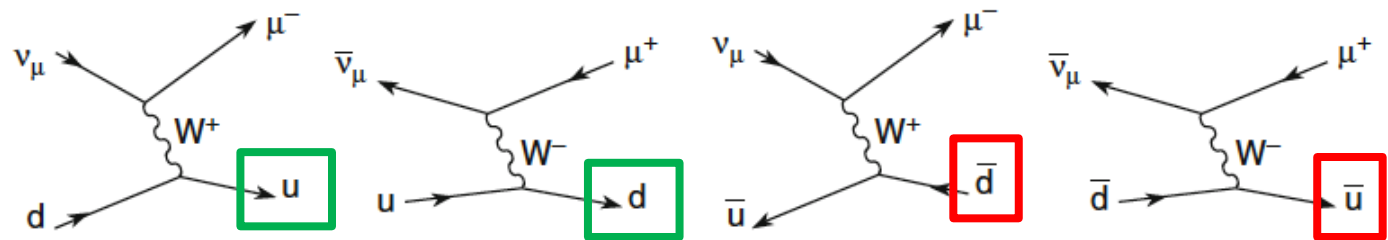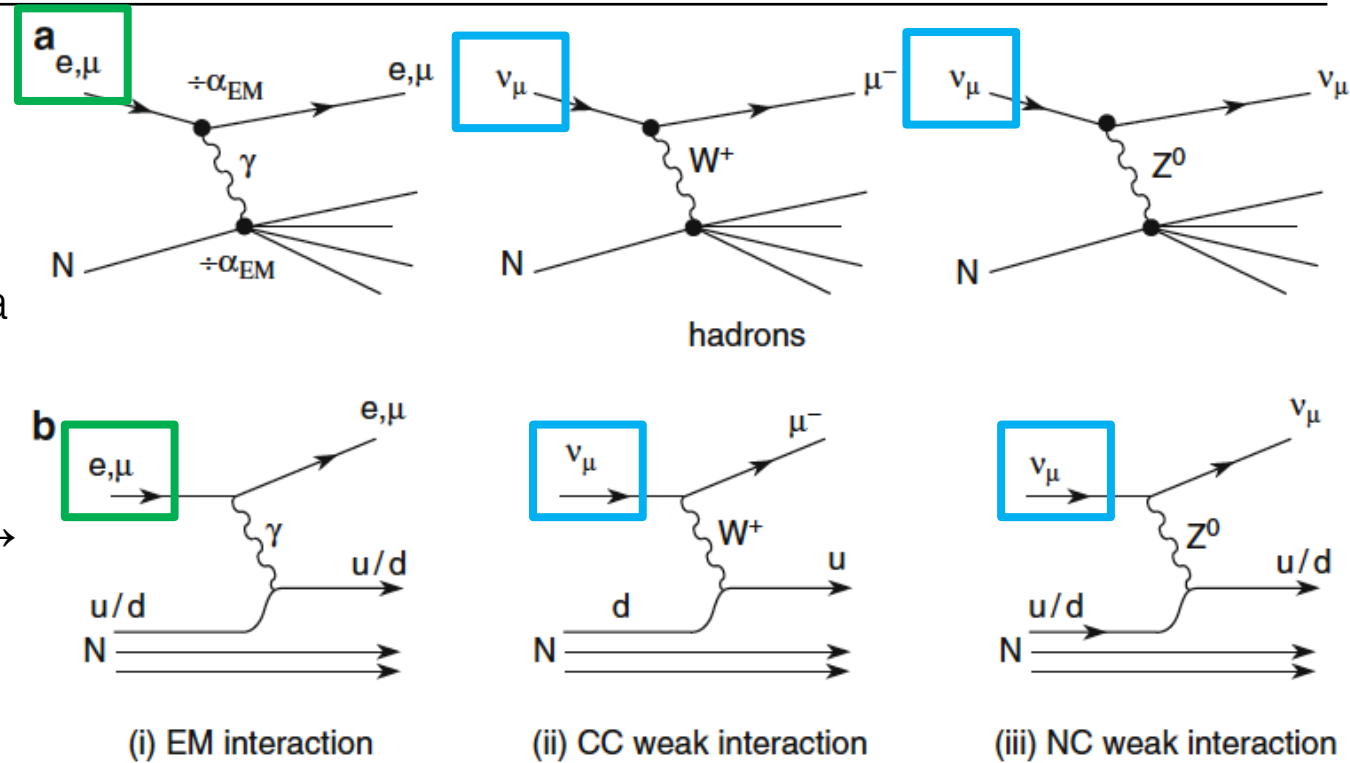$$\mu p : \quad \mu^{\pm} + p \to \mu^{\pm} + X^+$$

- Scattering of leptons (electrons and neutrinos) on a hydrogen (1p), deuterium (1p+1n) and heavier nuclei target (targets with #protons=#neutrons).
- F_2^d : Scattering on nuclei the structure function is always given per nucleon (protons and neutrons) → How to distinguish F_2^p *from* F_2^n? **Compare targets!**
- The structure function of the deuteron F^d_2 is equal to the average structure function of the nucleons

$$F^N_2 \quad F_2^d \approx \frac{F_2^p + F_2^n}{2} = F_2^N$$

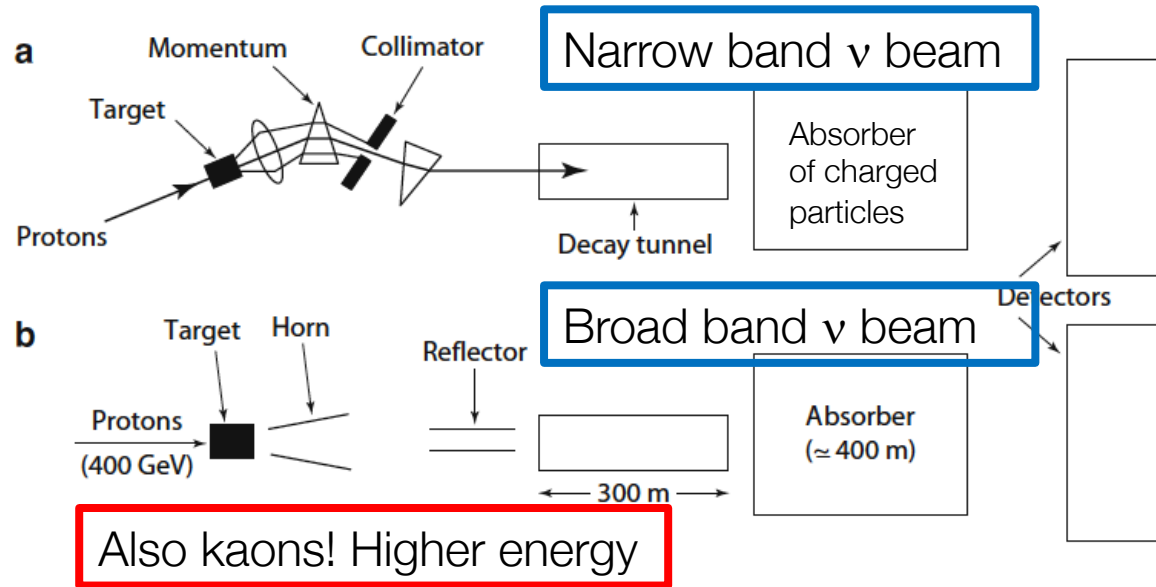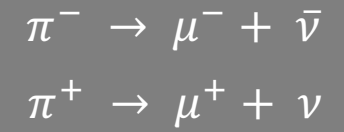- Neutrinos on a target → it is (im)possible to distinguish between 'valence' and 'sea' quarks

a

(i) EM interaction   (ii) CC weak interaction   (iii) NC weak interaction

$$\nu_\mu p(CC) : \nu_\mu + p \to \mu^- + X^{++}, \; \bar{\nu}_\mu + p \to \mu^+ + X^0$$

$$\nu_\mu p(NC) : \; \nu_\mu + p \to \nu_\mu + X^+, \; \bar{\nu}_\mu + p \to \bar{\nu}_\mu + X^+$$

Toni Baroncelli: Deep Inelastic Scattering

Narrow band ν beam

Broad band ν beam

Also kaons! Higher energy
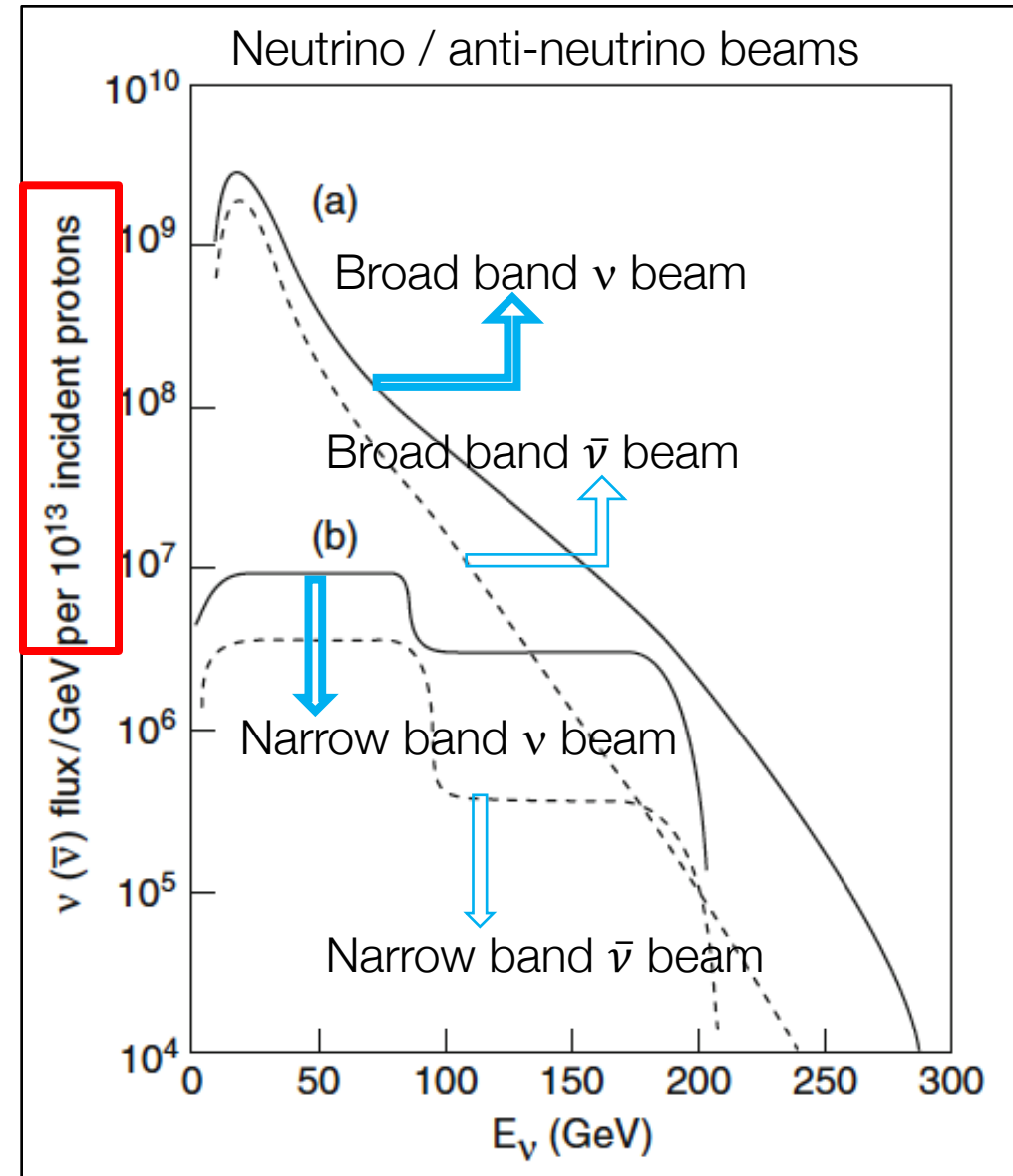
Narrow band ν beam: ~ π selected in momentum ~ low intensity

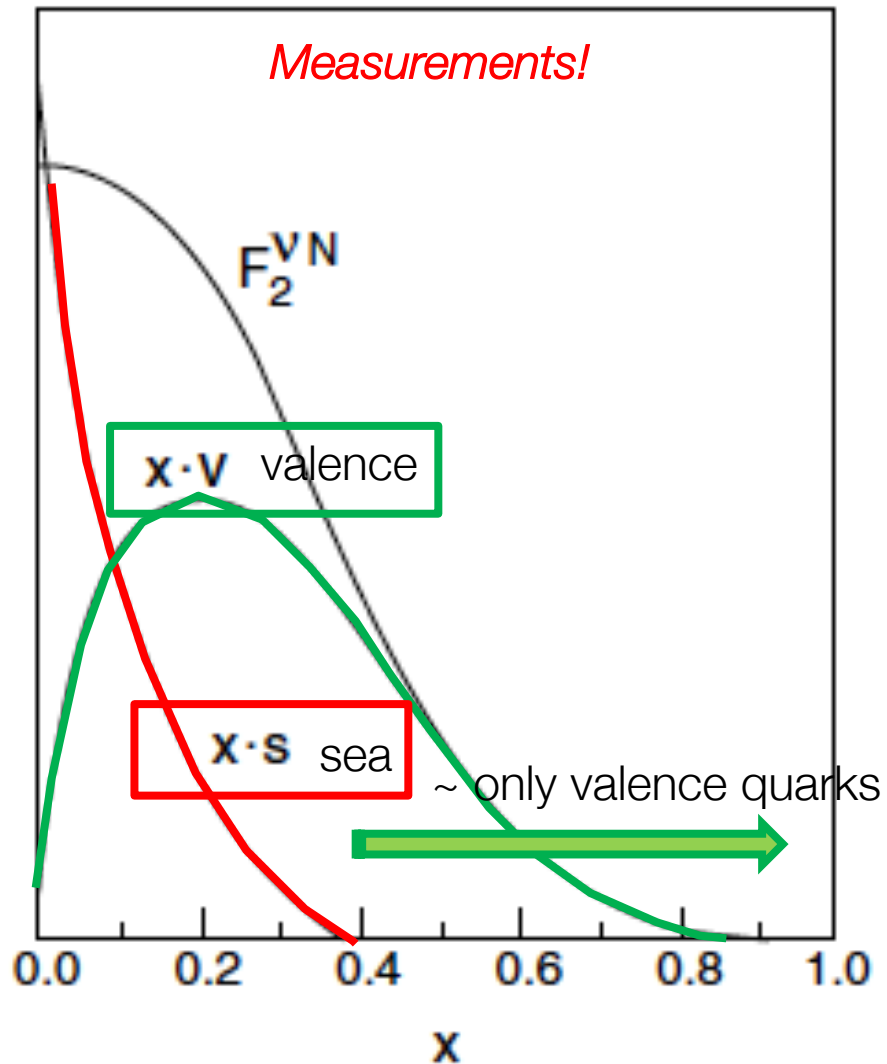Broad band ν beam: ~ π not selected in momentum ~ high intensity

Experiments:

The mean free path in iron of 10GeV neutrinos is $\lambda \approx 2.6 \cdot 10^9 Km$ (~ 20 cm for hadrons!). This means that only a very small fraction $3 \cdot 10^{-13}$ of 10 GeV neutrinos interact in a meter of iron. With a flux of 10^{12} neutrinos (for 10^{13} accelerated protons incident on the target), there are only 0.3 interactions in one meter of iron.

→ very long and massive detectors

Neutrino / anti-neutrino beams

Broad band ν beam

Broad band $\bar{\nu}$ beam

Narrow band ν beam

Narrow band $\bar{\nu}$ beam

Measurements!

$F_2^{\nu N}$

$x \cdot V$ valence

$x \cdot S$ sea

~ only valence quarks

- Neutrino scattering gives complementary information about the quark distribution.
- Neutrinos couple to the weak charge of the quarks via the weak interaction.
- In neutrino scattering you distinguish between (spin → helicity conservation → different angular distributions)

  - types of quarks,
  - quarks and antiquarks.

- sea quarks contribute to F_2 only at small values of x; ~0 above x ≈ 0.35.
- valence quarks F_2 maximum at x≈0.2 and ~0 for x → 1 and x → 0.

one quark alone ~never carries the major part of the nucleon momentum.

Structure Functions describe the internal structure of a nucleon. Let's say that

Definitions

- A nucleon is made of quarks of type f;
- Each quark carries a charge $z_f \cdot e$;
- The **electro-magnetic** cross section for a scattering on a quark is $|z_f \cdot e|^2$
- $q_f(x)$ is the probability f-quark carries a fraction of the nucleon momentum in the interval $(x, x + dx)$ (similarly $\overline{q_f}(x)$ for anti-quarks)
- There are two types of quarks:
  - *valence* quarks: they determine the quantum numbers of the nucleon
  - *sea quarks*, they exist in **pairs**, quark + anti-quark. They are produced and annihilated as virtual particles in the field of the strong interaction (as in the production of virtual electron–positron pairs in the Coulomb field)
- The nucleon also contains neutral components, *gluons*, with *NO CHARGE* and momentum distribution $g(x)$

The Structure Function $F_2(x)$ is the superposition of the momentum distributions carried by the quarks and weighted by x and $z_f{}^2$

$$F_{2(x)} = x \cdot \sum z_f^2 \cdot (q_f(x) + \overline{q_f}(x))$$

DIS is not sensitive to gluons (gq interaction)

Toni Baroncelli: Deep Inelastic Scattering

Toni Baroncelli: Deep Inelastic Scattering

- deep inelastic scattering (DIS) → nucleon structure and information about the structure of the hadrons and the forces acting between them.
- By the mid-sixties a large number of apparently different hadrons were known.
- The quark model was invented to accommodate the 'zoo' of hadrons which had been discovered

		u	d	p (uud)	n (udd)
Charge	z	+2/3	−1/3	1	0
Isospin	I	1/2		1/2	
	I_3	+1/2	−1/2	+1/2	−1/2
Spin	s	1/2	1/2	1/2	1/2

Quantum numbers of u, d quarks and of protons and neutrons

Use information from both

- deep inelastic scattering and
- spectroscopy to

extract the properties of the quarks.

Idea: reconstruct the properties of the nucleons (charge, mass, magnetic moment, isospin, etc.) by combining the quantum numbers of these constituents.

Toni Baroncelli: Deep Inelastic Scattering

		u	d	p (uud)	n (udd)
Charge	z	+2/3	−1/3	1	0
Isospin	I	1/2		1/2	
	I_3	+1/2	−1/2	+1/2	−1/2
Spin	s	1/2	1/2	1/2	1/2

- The quarks have spin ½
- in the quark model, their spins must combine to give the total spin 1/2 of the nucleon → nucleons are built up out of at least 3 quarks.
- The proton has two u-quarks and one d-quark
- The neutron has two d-quarks and one u-quark.

- u and d quarks form an isospin doublet, it is natural to assume that also the proton and the neutron form an isospin doublet (I = 1/ 2) u-quark and d-quark can be exchanged (isospin symmetry) → *proton ↔ neutron*.

- The fact that the charges of the quarks *are multiples of 1/3 is derived by* the fact that
  - the maximum positive charge in hadrons is two (e. g., Δ^{++}). Generated by 3 u quarks → charge +2e/3
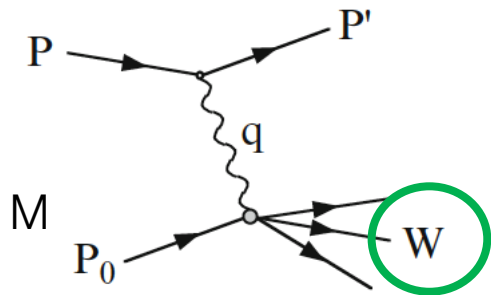 - the maximum negative charge is one (e. g., Δ^-). Generated by 3 d quarks -1e/3

Toni Baroncelli: Deep Inelastic Scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{point} \\ \text{spin } 1/2}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[1 + 2\tau \tan^2 \frac{\theta}{2}\right] \qquad \tau = \frac{Q^2}{4M^2c^2}$$

Elastic scattering, one variable
Spin ½ e on spin ½ point-like p

If the reaction $\ell + N \to \ell' + X$ is inelastic $\to$ the lepton scattering angle and energy are independent

$$W^2 = (P_0 + q)^2 = M^2 + q^2 + 2M\nu = M^2 - Q^2 + 2M\nu > M^2$$

(while in elastic scattering $W^2 = M^2 \to Q^2 = 2M\nu \to \nu - \frac{Q^2}{2M} = 0$)

$P = (E, \mathbf{p}); \ P' = (E', \mathbf{p}')$ for the incident and scattered electron
$P_0 = (M, 0) \ ; \ W = (E_0', \mathbf{p}_0')$ for the proton before and after impact.

p at rest, M =proton mass;
$P_0^2 = M^2; P^2 = m_e^2;$

$$q = P - P' = (E - E', \mathbf{p} - \mathbf{p}') = (\nu, \mathbf{q});$$
$$q^2 = 2m_e^2 - 2E'E + 2pp' \cos(\theta) \to m_e \sim 0; p \sim E \to$$
$$q^2 = -2E'E(1 - \cos\theta) = -4EE' \sin^2\left(\frac{\theta}{2}\right)$$
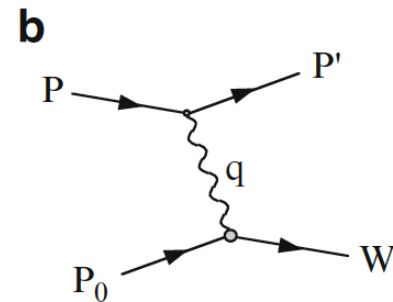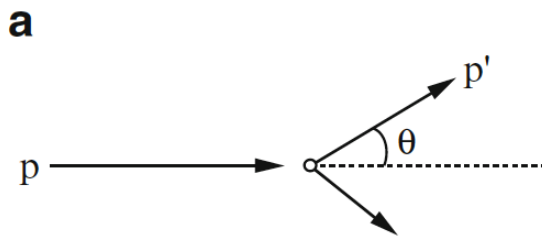
$$q = P - P' = (E - E', \mathbf{p} - \mathbf{p}') = (\nu, \mathbf{q}) \tag{10.7}$$

and its square $t = q^2$ is

$$t = q^2 = (P' - P)^2 = (E'/c - E/c)^2 - (\mathbf{p}' - \mathbf{p})^2$$

$$\xrightarrow{c=1} 2m_e^2 - 2E'E + 2p'p\cos\theta. \tag{10.8}$$

At high energy, the electron mass can be neglected ($m_e = 0$, $p \simeq E$), that is,

$$\boxed{q^2 = -Q^2 \simeq -2EE'(1 - \cos\theta) = -4EE'\sin^2(\theta/2).} \tag{10.9}$$

a

b

Toni Baroncelli: Deep Inelastic Scattering

Calculation	electron	Electron with spin	Point-like target, infinite Mass	Point-like target with mass M	Point-like proton	Point-like proton with spin	Finite size proton with spin	Expression
Rutherford	✅		✅					$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{Z^2 e^4}{4 E_0^2 \,(\sin\theta/2)^4}$
Mott		✅		✅				$\left(\frac{d\sigma}{d\Omega}\right)_M = \left(\frac{d\sigma}{d\Omega}\right)_R \cdot \left(\cos\frac{\theta}{2}\right)^2$
σ_{NS}		✅			✅			$\left(\frac{d\sigma}{d\Omega}\right)_{NS} = \left(\frac{d\sigma}{d\Omega}\right)_M \cdot 1/\left(1 - \frac{2E_0}{M}\sin\theta/2^2\right)$
σ		✅				✅		$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_M \cdot \left(1 + \frac{q^2}{2M^2} tan\,\theta/2^2\right)$
Rosenbluth		✅					✅	$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_M \cdot \left|\frac{G_E^2(Q^2) + \tau \cdot G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\,tan\,\theta/2^2\right|$

The "Target, charge Ze (Z=1 proton)" spans the columns from "Point-like target, infinite Mass" through "Finite size proton with spin".

e nucleus ↔ (Point-like target, infinite Mass — Point-like target with mass M)

e proton ↔ (Point-like proton — Finite size proton with spin)

Toni Baroncelli: Deep Inelastic Scattering

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\substack{\text{point} \\ \text{spin } 1/2}} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} \cdot \left[1 + 2\tau\tan^2\frac{\theta}{2}\right] \qquad \tau = \frac{Q^2}{4M^2c^2}$$

Elastic scattering, one variable
Spin ½ e on spin ½ proton

Condition for DIS: $Q^2 \gg M^2;\quad \nu = E - E' \gg M.$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^*_{\text{Mott}}\left[W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right]$$

Inelastic scattering, two variables

$$\frac{d^2\sigma}{dQ^2\,d\nu} = \underbrace{\frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cos^2\frac{\theta}{2}}_{\text{Mott*}}\left(W_2(Q^2,\nu) + W_1(Q^2,\nu)2\tan^2\frac{\theta}{2}\right)$$

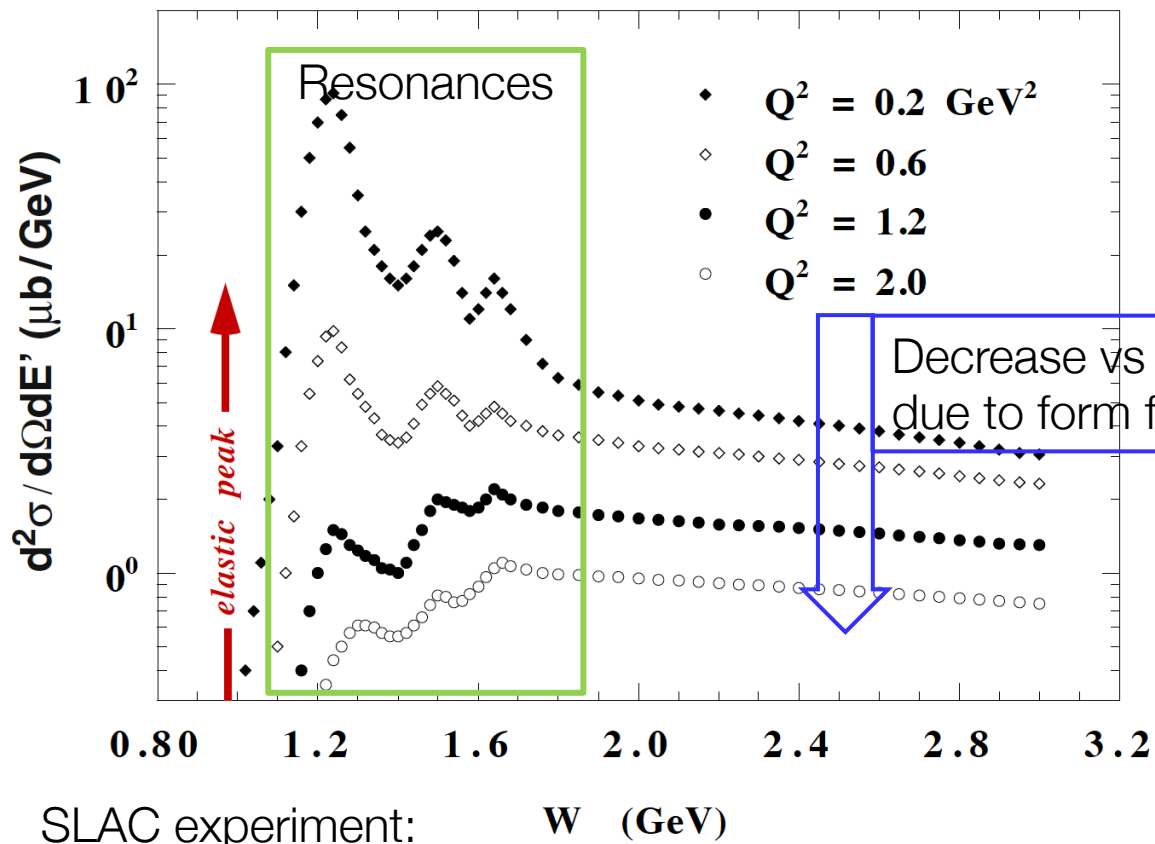Mott* includes also the (small) recoil of the proton → E'/E

Mott*

$$\frac{d^2\sigma}{dQ^2 dv} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cos^2\frac{\theta}{2}\left(W_2(Q^2,v) + W_1(Q^2,v)2\tan^2\frac{\theta}{2}\right)$$

Mott* includes also the (small) recoil of the proton $\rightarrow E'/E$

What happens if we, taking into account that nucleon is made of quarks, interpret the DIS scattering as interaction of

Virtual photon with one quark?

$\rightarrow$

Elastic scattering electron – quark (via photon exchange)?

Have to request that

the scattering is elastic: $\rightarrow$ $2Mv - Q^2 = 0$.
Use the "quark mass" m instead of proton mass

DIS of point-like particles with nucleons (p or n) $\rightarrow$ sum of elastic scattering on components (with mass m) of nucleons

Mott*

$$\left(\frac{d^2\sigma}{dQ^2 dv}\right)_{ela} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cos^2\frac{\theta}{2}\left(1 + \frac{Q^2}{4m^2}2\tan^2\frac{\theta}{2}\right)\delta\left(v - \frac{Q^2}{2m}\right)$$

$\delta \rightarrow$ condition for elastic scattering, one variable

If we compare the two expressions

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{point \\ spin\ 1/2}} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[1 + 2\tau \tan^2 \frac{\theta}{2}\right] \qquad \tau = \frac{Q^2}{4M^2c^2}$$

Elastic scattering, one variable
Spin ½ e on spin ½ point-like proton
with finite mass

$$\left(\frac{d^2\sigma}{dQ^2 d\nu}\right)_{ela} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cos^2\frac{\theta}{2}\left(1 + \frac{Q^2}{4m^2}2\tan^2\frac{\theta}{2}\right)\delta\left(\nu - \frac{Q^2}{2m}\right)$$

Replace M with m !
Which physical meaning?

$$\frac{d^2\sigma}{dQ^2 d\nu} = \underbrace{\frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cos^2\frac{\theta}{2}}_{\text{Mott*}}\left(W_2(Q^2,\nu) + W_1(Q^2,\nu)2\tan^2\frac{\theta}{2}\right)$$

$$W_2(Q^2,\nu) \to \delta\left(\nu - \frac{Q^2}{2m}\right); \qquad W_1(Q^2,\nu) \to \frac{Q^2}{4m^2}\delta\left(\nu - \frac{Q^2}{2m}\right)$$
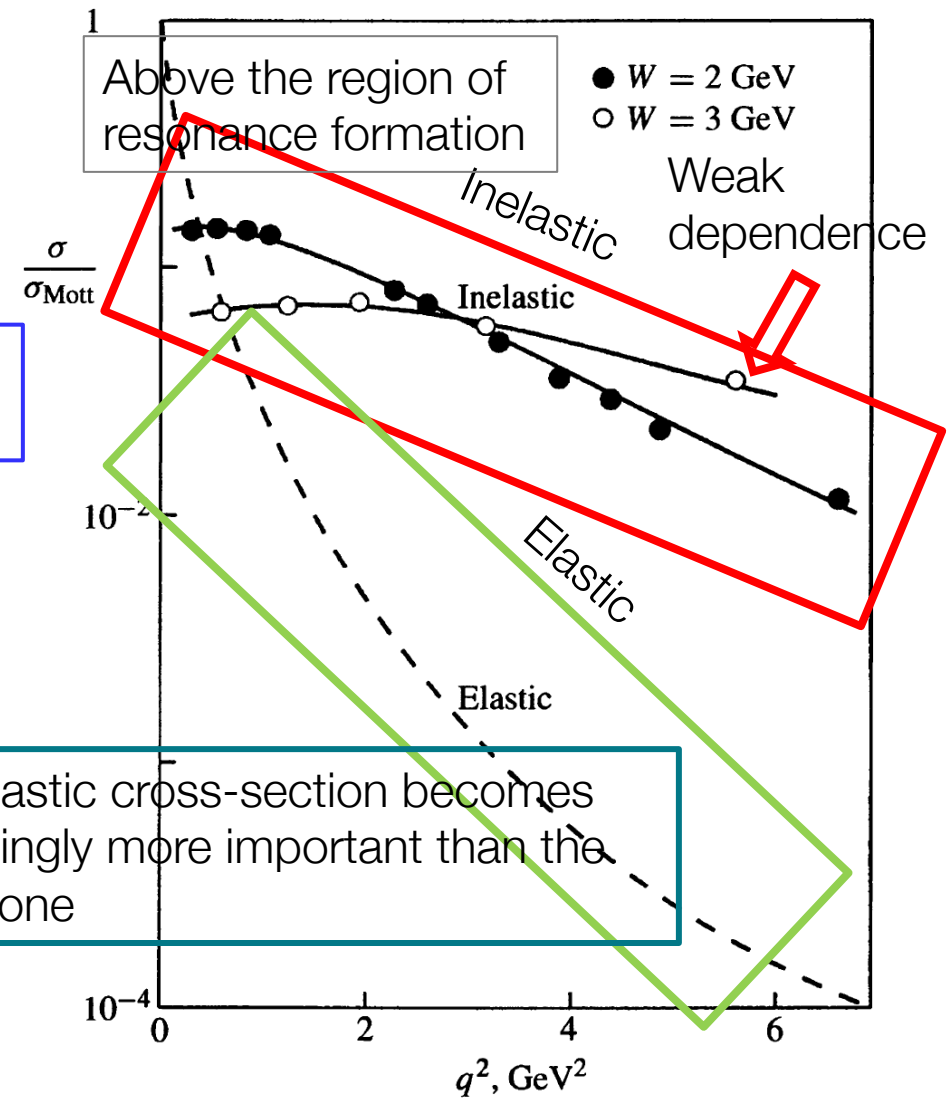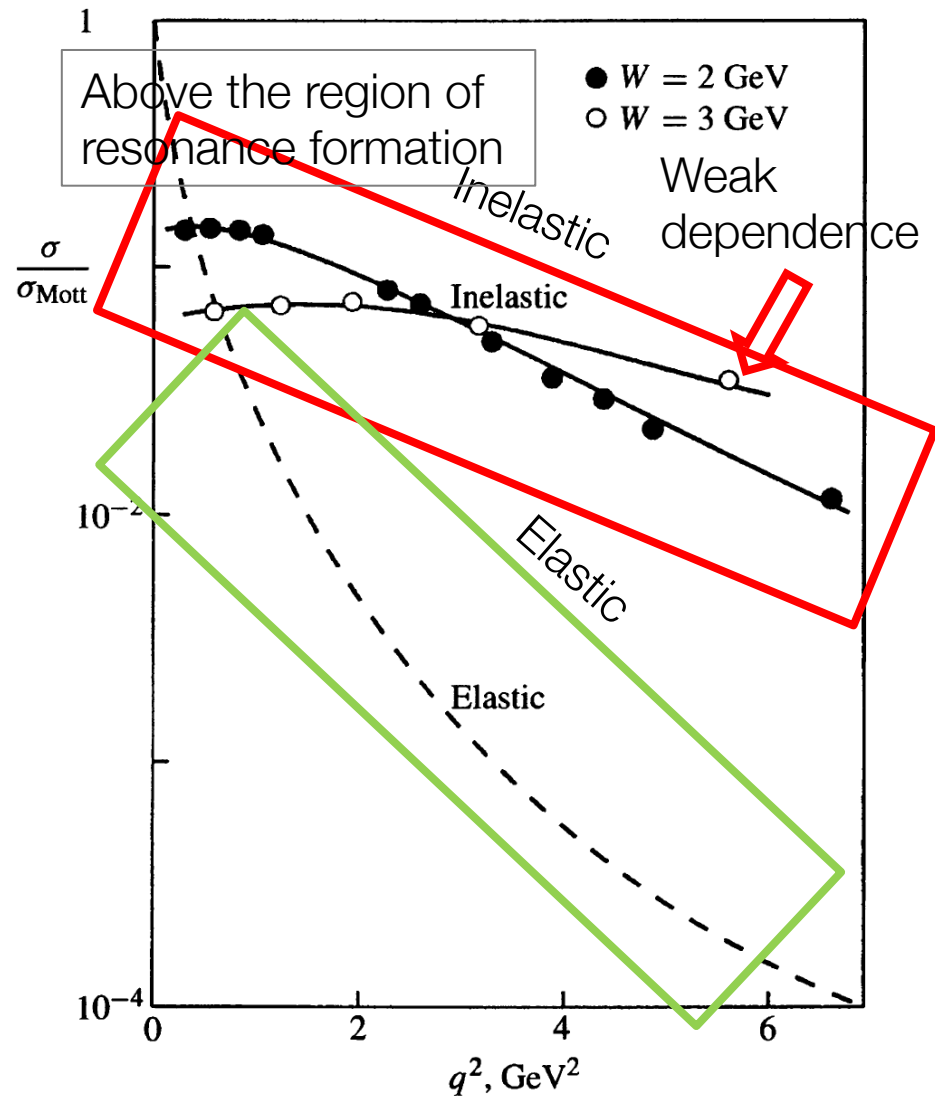
Resonances

$Q^2 = 0.2$ GeV^2
$Q^2 = 0.6$
$Q^2 = 1.2$
$Q^2 = 2.0$

elastic peak

d$^2\sigma$ / dΩdE' (μb / GeV)

W (GeV)

Decrease vs Q^2 at a fixed W due to form factor F(Q^2)

Above the region of resonance formation

$\bullet \ W = 2$ GeV
$\circ \ W = 3$ GeV

Weak dependence

Inelastic

$\dfrac{\sigma}{\sigma_{\text{Mott}}}$

Inelastic

Elastic

Elastic

the inelastic cross-section becomes increasingly more important than the elastic one

q^2, GeV^2

SLAC experiment:
- 20 GeV electron beam on hydrogen and deuterium
- The energy and direction of the scattered electron was measured
- $\rightarrow Q^2, \nu, W$ were measured

Toni Baroncelli: Deep Inelastic Scattering

- ● $W = 2$ GeV
- ○ $W = 3$ GeV

$\dfrac{\sigma}{\sigma_{\text{Mott}}}$

Inelastic

Inelastic

Elastic

Elastic

Above the region of resonance formation

Weak dependence

q^2, GeV^2

The weak dependence indicates that the "quark" is a point-like object!

In the electron Nucleon scattering $Q^2\sim0$ gives the largest cross-section (the electron sees all the charge)

In the parton picture, increasing Q^2 doesn't change the cross-section: the quark is point-like!
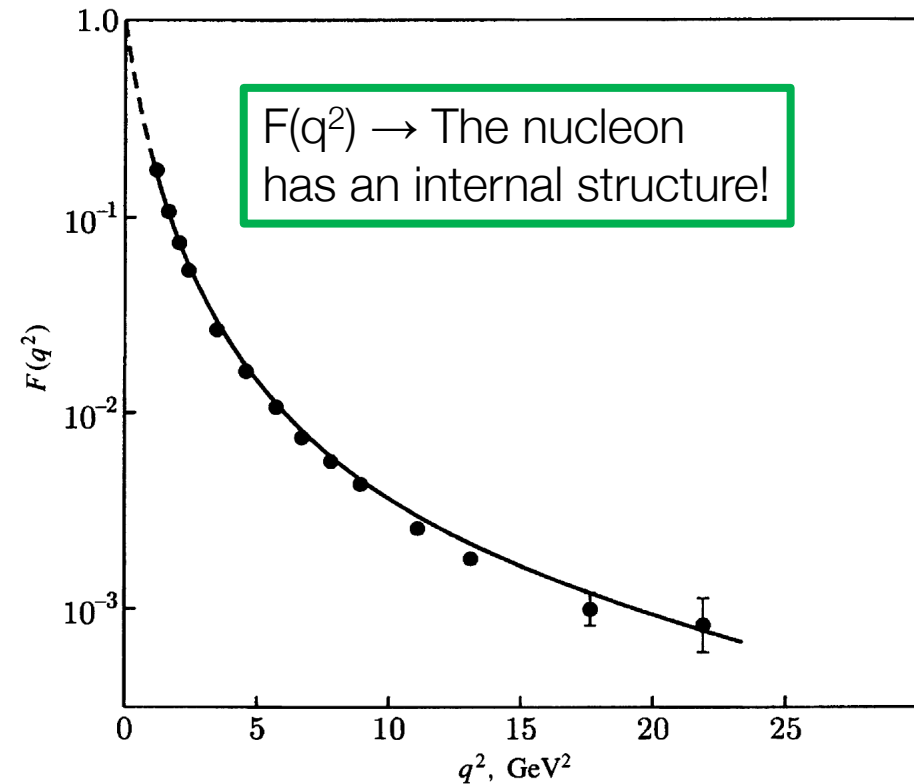
$F(q^2)$

q^2, GeV^2

$F(q^2) \rightarrow$ The nucleon has an internal structure!

the inelastic cross-section becomes increasingly more important than the elastic one

In late 60' Bjorken showed that in the 'DIS' region (in elastic region: $\nu - \frac{Q^2}{2M} = 0$)

$$Q^2 \gg M^2$$
$$\nu \gg M$$

the ratio

$$x = \frac{Q^2}{2M\nu}$$

remains finite for

$$Q^2 \to \infty \; and \; \nu \to \infty$$

- Two new functions: $F_2 = \nu \cdot W_2$ and $F_1 = M \cdot W_1$
- depend on a dimensionless variable x and not on $Q^2 \; and \; \nu$

$$\lim_{Q^2, \nu \to \infty} \nu W_2(Q^2, \nu) = F_2(x) \qquad \lim_{Q^2, \nu \to \infty} M W_1(Q^2, \nu) = F_1(x)$$

Bjorken scaling
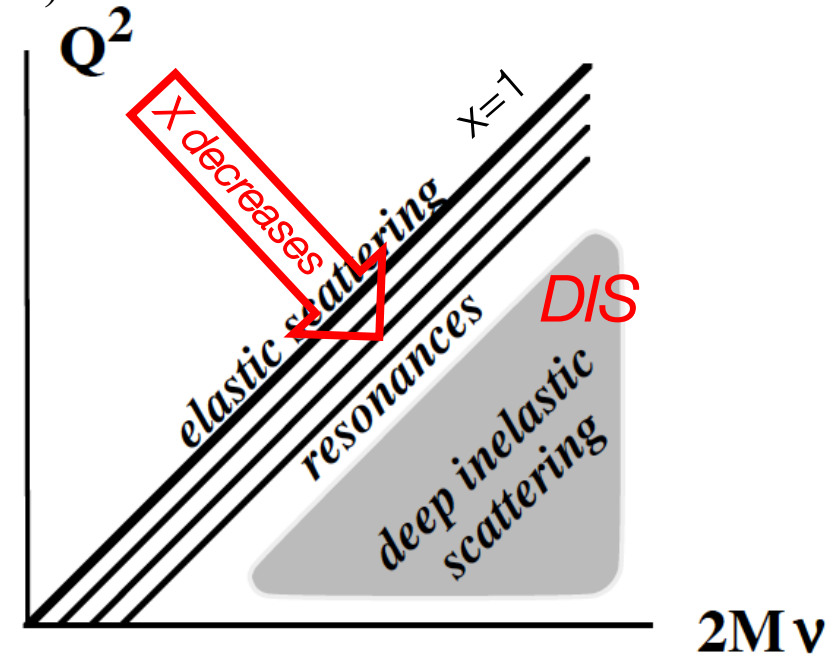
This hypothesis was

- derived in the assumption DIS consists of elastic lepton scattering on proton constituents $\to Q^2 = 2m\nu \to$

  $x = m/M$ can be seen as the fraction of nucleon mass carried by the parton

- experimentally tested in the years after using a 20 GeV electron beam on hydrogen and deuterium

(figure: Q^2 vs $2M\nu$ plot showing regions — elastic scattering ($x=1$ line), resonances, deep inelastic scattering (DIS); arrow indicating X decreases)

Toni Baroncelli: Deep Inelastic Scattering

We understand more if we use $d^2\sigma/dQ^2dx$

$$\frac{d^2\sigma}{dQ^2dx} = \frac{\nu}{x}\frac{d^2\sigma}{dQ^2d\nu} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\frac{1}{x}\cos^2\frac{\theta}{2}\left(\nu W_2(Q^2,\nu) + \nu W_1(Q^2,\nu)2\tan^2\frac{\theta}{2}\right)$$

$$= \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\frac{1}{x}\cos^2\frac{\theta}{2}\left(F_2(x) + \frac{\nu F_1(x)}{M}2\tan^2\frac{\theta}{2}\right)$$

$$x = Q^2/2M\nu \rightarrow dx/d\nu = Q^2/\nu^2 2M = x/\nu$$

We also passed from $F_1(x,Q^2)$ to $F_1(x) \rightarrow$ scaling assumption

$$F_1(x,Q^2) = Mc^2\,W_1(Q^2,\nu)$$
$$F_2(x,Q^2) = \nu\,W_2(Q^2,\nu).$$

multiply and divide F_1 by $\quad 2x \rightarrow \dfrac{2x\,\nu\,F_1}{2x\,M}\quad \nu = \dfrac{Q^2}{2Mx}\quad\quad \dfrac{2x\,\nu\,F_1(x)}{2x\,M} = \dfrac{2x\,Q^2\,F_1}{2x\,2Mx\,M} = \dfrac{2x\,Q^2\,F_1}{4M^2x^2}$

$$= \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\frac{1}{x}\cos^2\frac{\theta}{2}\left(F_2(x) + 2xF_1(x)\frac{Q^2}{4M^2x^2}2\tan^2\frac{\theta}{2}\right).$$

Toni Baroncelli: Deep Inelastic Scattering

$$= \frac{4\pi\alpha^2}{Q^4} \frac{E'}{E} \frac{1}{x} \cos^2\frac{\theta}{2} \left(F_2(x) + 2xF_1(x) \frac{Q^2}{4M^2x^2} 2\tan^2\frac{\theta}{2} \right).$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{NS} \left[1 + \frac{q^2}{2M^2}\tan^2(\theta/2) \right] \quad \text{Spin 1/2 particle}$$

M²x² → m²

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \left(\frac{d\sigma}{d\Omega}\right)_R (1 - \beta^2\sin^2\theta/2) \simeq \left(\frac{d\sigma}{d\Omega}\right)_R \cos^2(\theta/2). \quad \text{Spin 0 particle}$$

$$\tau = \frac{Q^2}{4M^2c^2}$$

$$x = \frac{Q^2}{2M\nu}$$

If we compare the expression with F_1 and F_2 with the elastic expression for *e q* scattering (2 point-like objects) spin 0 and spin 1/2 (with mass xM) expression we conclude that

If we want to have the same expression for elastic scattering of a point-like electron on a point-like spin ½ quark

- $F_1 = 0$ for spin 0 particles and

- $F_2 = 2x F_1$ for spin ½ particles (Callan-Gross relation)

Toni Baroncelli: Deep Inelastic Scattering

$(\blacktriangledown)$ $1.5 < Q^2 < 4\,GeV^2$; $(\bullet)$ $5 < Q^2 < 11\,GeV^2$

$(\blacktriangle)$ $12 < Q^2 < 16\,GeV^2$

$2xF_1(x)\,/\,F_2(x)$

x

Spin 1/2 partons

Spin 0 partons
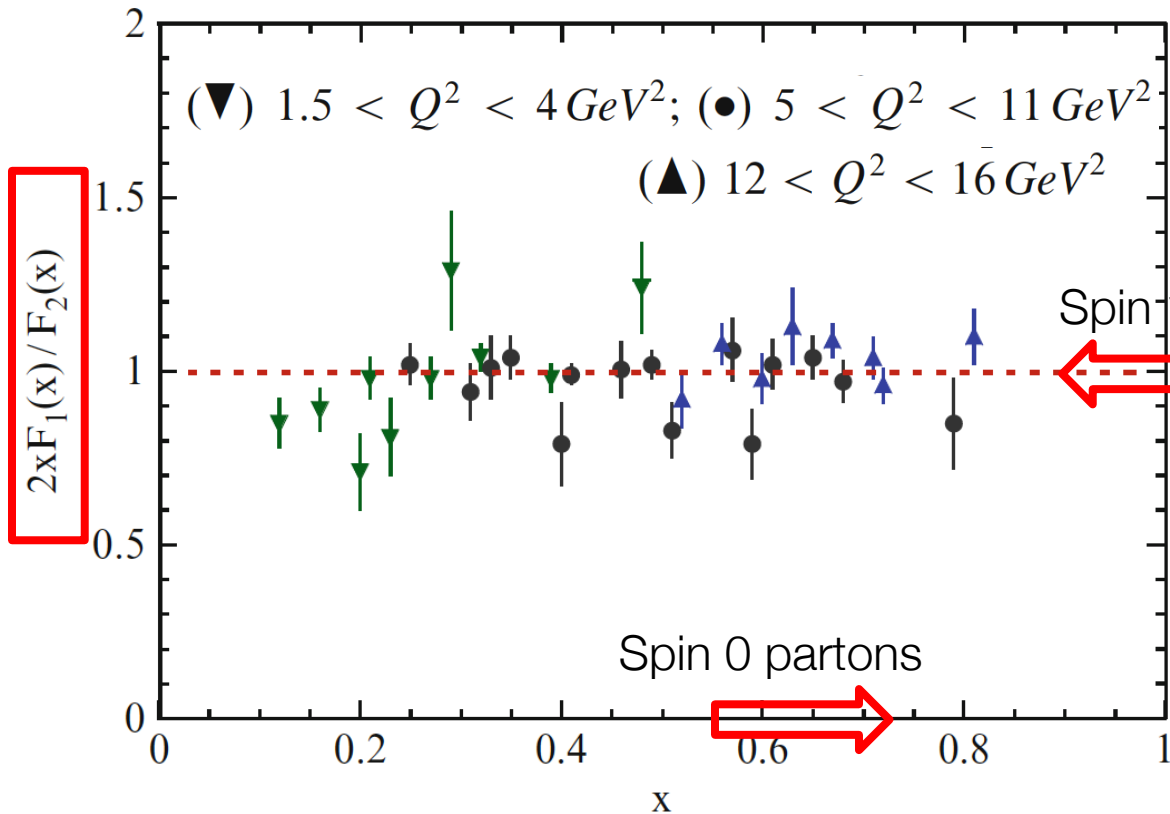
The Callan-Gross relation is found to be valid for different Q² intervals

→ partons are spin ½ particles

Indeed F_2 seems to depend on x and not on Q²

$F_2(x)$

x
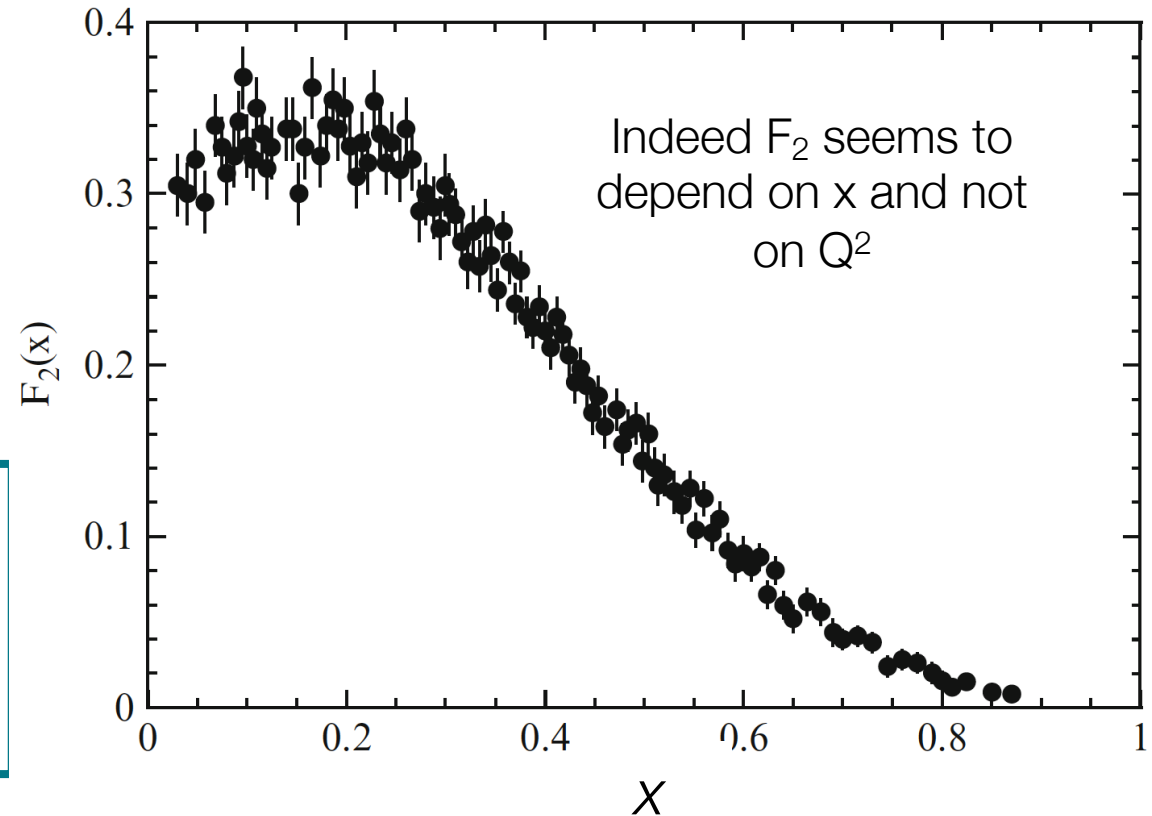
- The hadron is made of point-like charged partons
- Partons are spin ½ objects
- Partons share a fraction x of the hadron 4-momentum
- The function $F_2(x)$ represents the x-distribution of partons inside the hadron

Toni Baroncelli: Deep Inelastic Scattering

- Nucleons: three *valence quarks* determine the *quantum numbers*
- Virtual quark-antiquark pairs, (*sea quarks)* exist in the nucleon. Their *quantum numbers sum out to zero* and do not change those of the nucleon by three *valence quarks*
- *Sea quarks* carry a very small fractions x of the nucleon's momentum.
- There are not only "u" and "d" quarks but also s (strange), c (charm), b (bottom) and t (top). These heavy quarks contribute very little to the 'sea'.
- Because of their *electrical charge*, *sea quarks* are "*visible*" in deep inelastic scattering.

> The cross-section for electro-magnetic interactions is proportional charge[2], e_k^2
>
> $\Longrightarrow$ $F_2(x) = \sum_k e_k^2 \cdot x \cdot f_k(x).$

- The six quark types can be arranged in doublets (called families or generations), according to their increasing mass :

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

Charge 2/3 e
Charge -1/3 e
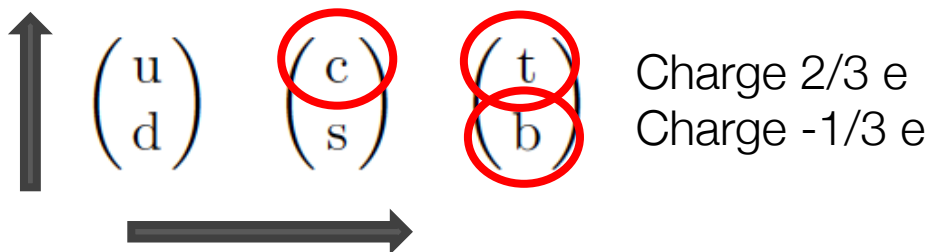
> Pairs of $q\bar{q}$ are continuously created and exist for a time $\Delta t \cdot 2m_q < \hbar \rightarrow$ heavy quarks have 'less time' for creation $\rightarrow$ contribute very little to Deep Inelastic Scattering at ~low or moderate Q^2 . They can be neglected

Call $F_2^{e,p}$ $and F_2^{e,n}$ the structure functions of of protons and neutrons respectively. $d_s, \overline{d_s}$ the x-distribution of d-valence quarks and of anti-d x-distribution of sea quarks (similarly for other quarks)

$$F_2^{e,P}(x) = x \cdot \left[\frac{1}{9}\left(d_v^P + d_s + \bar{d}_s\right) + \frac{4}{9}\left(u_v^P + u_s + \bar{u}_s\right) + \frac{1}{9}\left(s_s + \bar{s}_s\right) \right]$$

$$F_2^{e,n}(x) = x \cdot \left[\frac{1}{9}\left(d_v^n + d_s + \bar{d}_s\right) + \frac{4}{9}\left(u_v^n + u_s + \bar{u}_s\right) + \frac{1}{9}\left(s_s + \bar{s}_s\right) \right]$$

Valence quarks

Sea quarks

F$_2$ between x and x+dx

F_2^{vN}

Valence

Sea
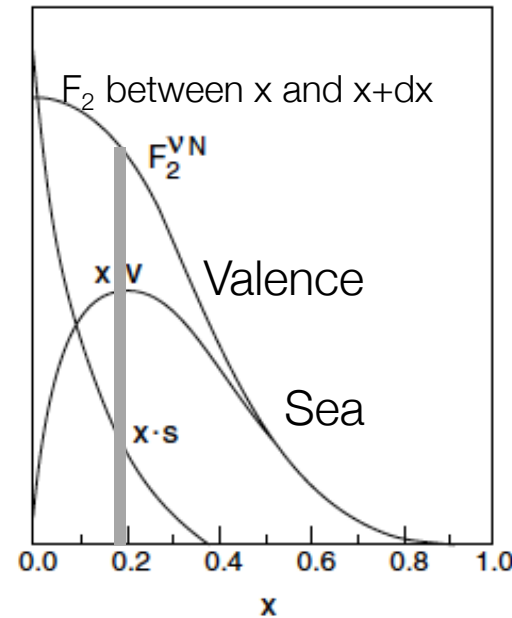
The proton and the neutron can be interchanged by exchanging d and u quarks (isospin symmetry)

The proton has two u-quarks and one d-quark, the neutron has two d-quarks and one u-quark.

$$u_v^P(x) = d_v^n(x) \,,$$
$$d_v^P(x) = u_v^n(x) \,,$$
$$u_s^P(x) = d_s^P(x) = d_s^n(x) = u_s^n(x)$$

And the 'average' Nucleon structure function can be written as

Term with sea quarks only → negligible

$$F_2^{e,N}(x) = \frac{F_2^{e,P}(x) + F_2^{e,n}(x)}{2}$$

$$= \frac{5}{18} x \cdot \sum_{q=d,u} (q(x) + \bar{q}(x)) + \frac{1}{9} x \cdot \left[s_s(x) + \bar{s}_s(x) \right]$$

5/18 is ~ the mean square charge of u + d quarks

Toni Baroncelli: Deep Inelastic Scattering

- In deep inelastic *neutrino* scattering, the charge factors z_f^2 are not present, as the *weak charge is the same for all quarks*.
- Because of charge conservation and helicity, neutrinos and antineutrinos couple differently to the different types of quarks and antiquarks. These differences, however, cancel out when the structure function of an average nucleon is considered. One then obtains:

e,N

νN

$$F_2^{e,N}(x) = \frac{5}{18} x \cdot \sum_{q=d,u} (q(x) + \bar{q}(x))$$

$$F_2^{\nu,N}(x) = x \cdot \sum_{f} (q_f(x) + \bar{q}_f(x))$$

Experiments show that $F_2^{v,N}$ and $F_2^{e,N}$ are identical ((but for the factor 5/18 due to charge) → This means that the charge numbers +2/3 and −1/3 have been correctly attributed to the u- and d-quarks.

- Valence quarks peak at $x \approx 0.17$ and an average value of $\langle x_v \rangle \approx 0.12$
- Sea quark →low x values with an average value of $\langle x_s \rangle \approx 0.04$

DIS with v

	CCFRR
●,○	CCFRR
▲,△	CDHSW
■,□	CHARM

F_2

x 18/5

▼	BCDMS x 18/5
+	BFP x 18/5
★	EMC x 18/5

$10\ (\mathrm{GeV}/c)^2 < Q^2 < 100\ (\mathrm{GeV}/c)^2$

DIS with e

$x F_3$

x F_3 ~ valence distribution

$\bar{q}$

$\bar{q}$ ~ sea distribution

Toni Baroncelli: Deep Inelastic Scattering
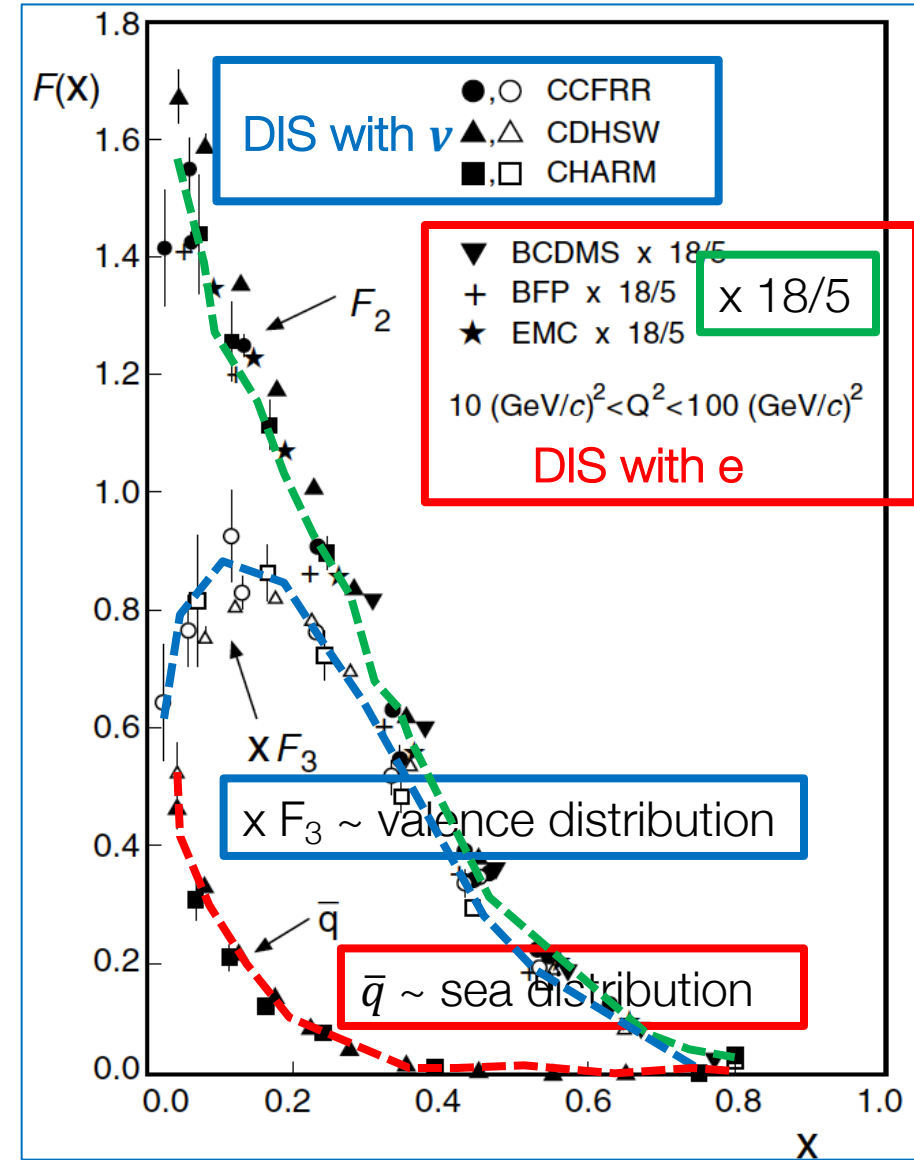
WARNING!

The integral of $F_2^{v,N}$ and $F_2^{e,N}$ gives about 0.5 → IMPORTANT INFORMATION: half of the momentum of a nucleon is carried by components that are NOT quarks

$$\int_0^1 F_2^{\nu,N}(x)\,\mathrm{d}x \approx \frac{18}{5}\int_0^1 F_2^{e,N}(x)\,\mathrm{d}x \approx 0.5$$

This component is not detected in $F_2^{v,N}$ or $F_2^{e,N}$. This means it is sensible neither to electromagnetic interactions nor to weak interactions → *gluons*

DIS with v

●,○ CCFRR
▲,△ CDHSW
■,□ CHARM

▼ BCDMS x 18/5
+ BFP x 18/5
★ EMC x 18/5

x 18/5

$10\ (\mathrm{GeV}/c)^2 < Q^2 < 100\ (\mathrm{GeV}/c)^2$

DIS with e

F_2

xF_3

x F_3 ~ valence distribution

$\bar{q}$

$\bar{q}$ ~ sea distribution
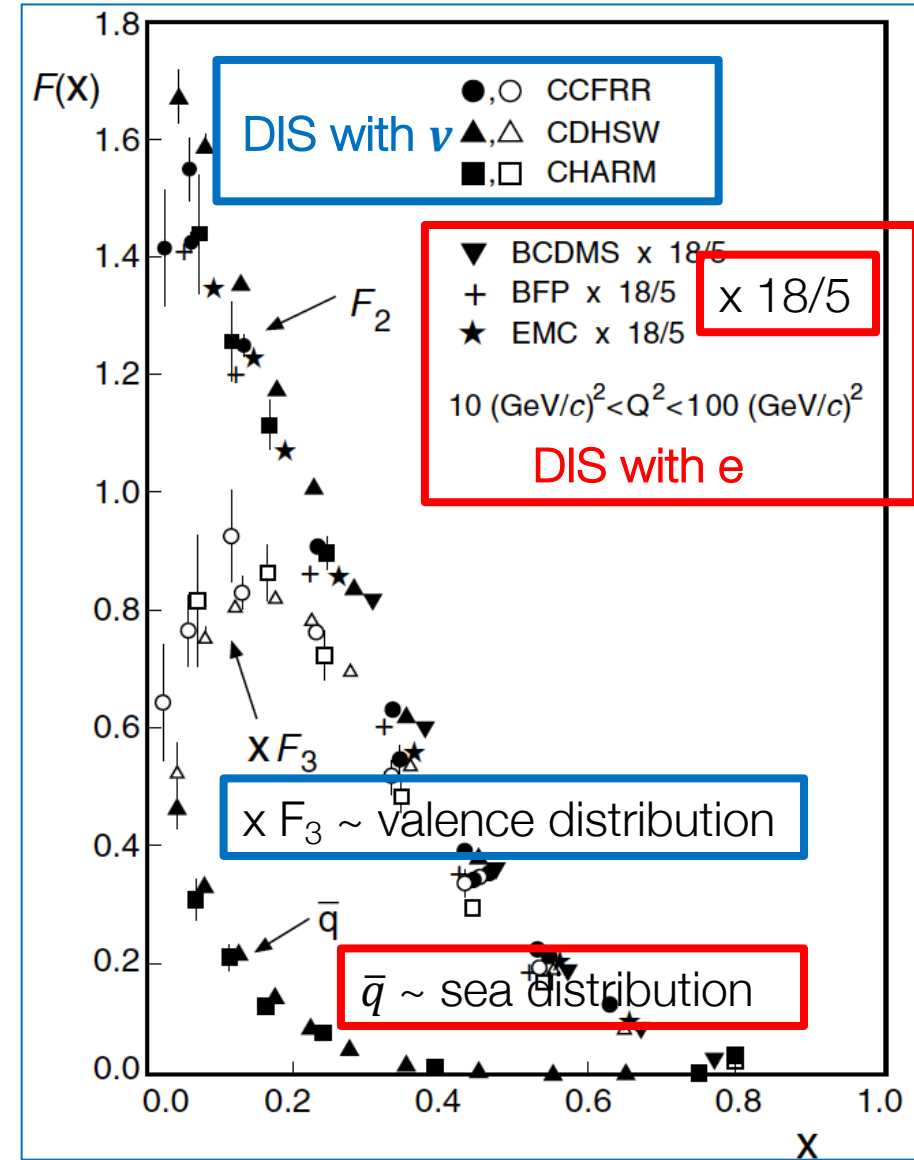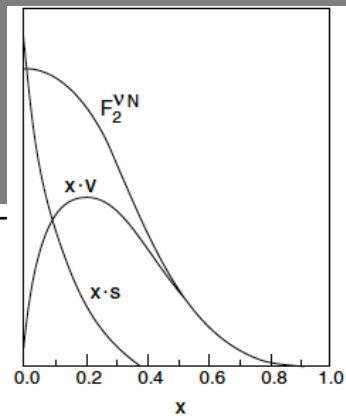
$$F_2^{e,N}(x) = \frac{F_2^{e,P}(x) + F_2^{e,n}(x)}{2} = \frac{5}{18}\, x \cdot \sum_{q=d,u} (q(x) + \bar{q}(x)) + \frac{1}{9}\, x \cdot \left[s_s(x) + \bar{s}_s(x) \right]$$

- $F_2^n / F_2^p \sim 1$ at very low x: few valence quarks. The ratio is sensitive to sea quarks expected to be equally present in protons and neutrons

- F_2^n / F_2^p at x ~ 1 (mostly valence quarks) should be about $(2z_d^2 + z_u^2)/(2z_u^2 + z_d^2) \approx \frac{2}{3}$ (neutron / proton), ratio of the square charges of the valence quarks of the neutron and proton

- It is found to be ~ ¼ ~ $(2/3)^2/(1/3)^2$ z_d, z_u charges

  - $(2z_d^2 + \cancel{z_u^2})/(2z_u^2 + \cancel{z_d^2})$
     neutron   proton

- → large momentum fractions in the proton are carried by u-quarks, and, in the neutron, by d-quarks.

F_2^n / F_2^p

Mostly sea quarks — Mostly valence quarks

~0.25

○ 90 GeV
● 280 GeV
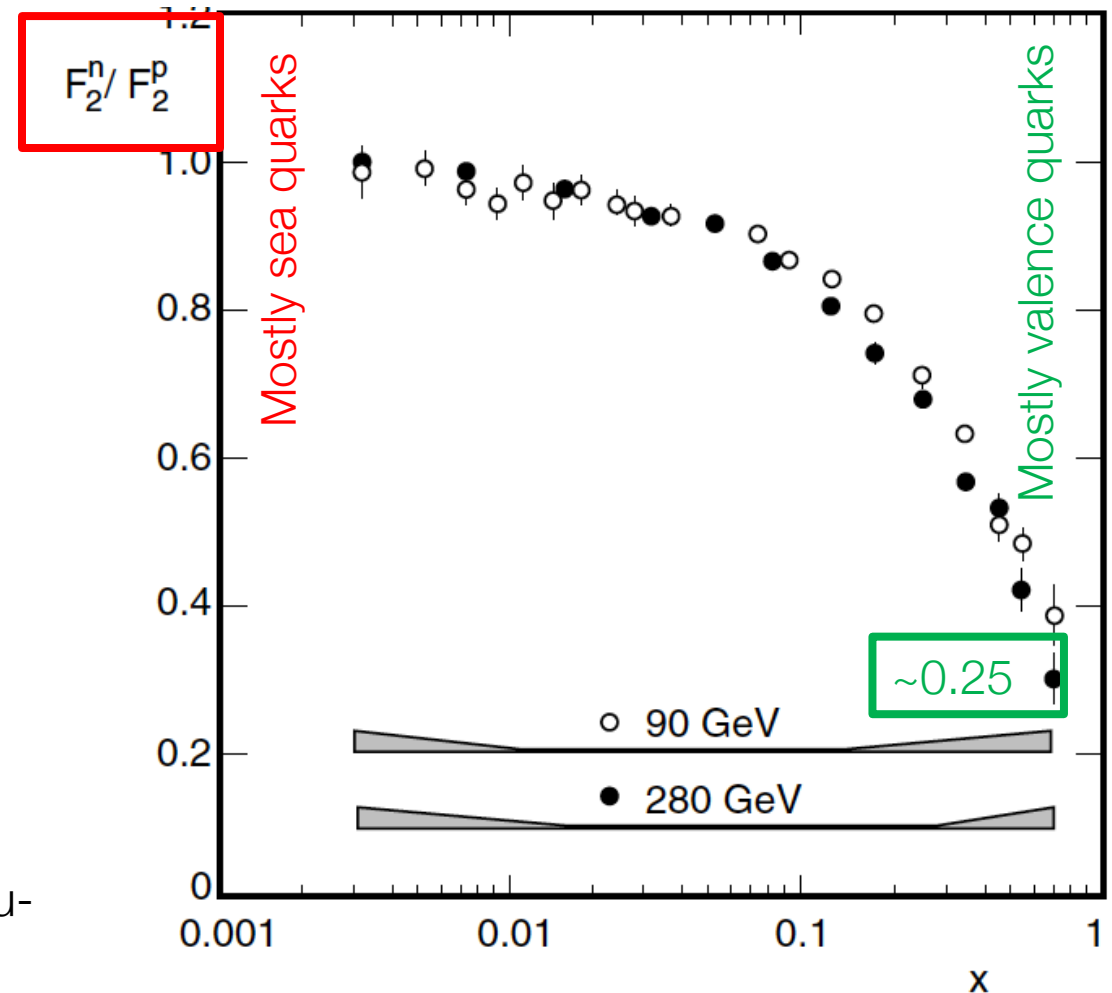
- valence and sea quarks carry ~1/2 of the momentum of a nucleon.
- Nucleons can be constructed using only the valence quarks.
- *quarks are never free → Quark masses cannot be measured.*
- Masses of 'bare' u and d quarks are (expected to be) small: m_u =1.5 – 5 MeV/c² , m_d =3 – 9 MeV/c² . These masses are commonly called *current quark masses*.

- "*constituent quarks*" *masses:* enlarged masses (~"incorporating sea & gluons") but unchanged quantum numbers.
- The *constituent quark masses* are much larger (300 MeV/c²). The *constituent masses* must be mainly due
  - the electromagnetic interaction → mass differences of a few MeV;
  - Additional effects must be due to differences between quark–quark interaction.

- It is often assumed that m_u ~ m_d ~ few MeV and m_s ~ m_u + 150MeV.
- The masses of heavier quarks are m_c ~1.550 MeV and m_b ~ 4.300 MeV.
- Hadrons and mesons made of the t quarks cannot be formed because the quark t is free for a very short time.

Quark	Colour	Electr. Charge	Mass [MeV/c^2]	
			Bare Quark	Const. Quark
down	b, g, r	$-1/3$	3 – 9	≈ 300
up	b, g, r	$+2/3$	1.5 – 5	≈ 300
strange	b, g, r	$-1/3$	60 – 170	≈ 450
charm	b, g, r	$+2/3$	1 100 – 1 400	
bottom	b, g, r	$-1/3$	4 100 – 4 400	
top	b, g, r	$+2/3$	$168 \cdot 10^3 – 179 \cdot 10^3$	

Toni Baroncelli: Deep Inelastic Scattering

Hadrons can be classified in two groups:

1. the baryons , fermions with half-integral spin
2. the mesons, bosons with integral spin.

Baryons.

.
- Like the proton and neutron, other baryons are also composed of three quarks.
- Since quarks have spin 1/2, baryons have half-integral spin.
- # baryons = # antibaryons are produced in particle interactions.
- baryon number B, = 1 for baryons and B = − 1 for antibaryons. ($\rightarrow$ B = +1/ 3 for quarks, B = − 1/ 3 for antiquarks.
- Experiments indicate that baryon number is conserved in all particle reactions and decays.
- The quark minus antiquark number is conserved.
- This would be violated by, e. g., the hypothetical **decay of the proton: p $\rightarrow$ π^0 + e$^+$** . Without baryon number conservation this decay mode would be energetically favoured. Yet, it has not been observed.

Mesons.

- Pions are the lightest hadrons ~ 140 MeV/c^2.
- They are found in three different charge states: π^-, π^0 and π^+ .
- Pions have spin 0. It is, therefore, natural to assume that they are composed of a quark and an antiquark: this is the only way to build the three charge states out of quarks.

$$|\pi^+\rangle = |u\bar{d}\rangle \quad |\pi^-\rangle = |d\bar{u}\rangle \quad |\pi^0\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle$$

- The pions are the lightest systems of quarks. Hence, they can only decay into the even lighter leptons or into photons.

- The pion mass is considerably smaller than the constituent quark mass → the interquark interaction energy has a substantial effect on hadron masses.
- The total angular momentum = vector sum of the quark, antiquark spins, integer orbital angular momentum contribution.

- Mesons eventually decay into electrons, neutrinos and/or photons; **there is no "meson number conservation** (the number of quarks minus the number of antiquarks is zero) → any number of mesons may be produced or annihilated.

Toni Baroncelli: Deep Inelastic Scattering

We MUST introduce another important property called colour: needed to satisfy the Pauli principle.

Δ^{++} resonance (baryon!)

- It is made of three u-quarks, has spin J = 3/2 and positive parity; it is the lightest baryon with J^P = $3/2^+$ → we therefore can assume that its orbital angular momentum is = 0;
- it has a symmetric spatial wave function. In order to yield total angular momentum 3/2, the spins of all three quarks have to be parallel:

$$\Delta^{++} = |u^{\uparrow}u^{\uparrow}u^{\uparrow}\rangle$$

- Thus, the spin wave function is also symmetric.
- The wave function of this system is furthermore symmetric under the interchange of any two quarks, as only quarks of the same flavour are present.
- The total wave function is symmetric, in violation of the Pauli principle.

To fulfil the Pauli principle the colour, a kind of quark charge, has to be introduced → distinguish quarks!

HP: Colour can assume three values: red, blue and green. (confirmed by data)
antiquarks carry the anti-colours anti-red, anti-blue , and anti-green.

The strong interaction binds quarks into a hadron → mediated by force carriers → gluons.
... And gluons? Do they carry colour?

The gluons carry simultaneously colour and anti-colour

→ *3 colors x 3 anti-colors → 9 combinations.*

Colour forms combinations that may be organised in multiplets of states: a singlet and an octet. One possible choice is (others exist):

Octet $r\bar{g}, \quad r\bar{b}, \quad g\bar{b}, \quad g\bar{r}, \quad b\bar{r}, \quad b\bar{g}, \quad \sqrt{1/2}\,(r\bar{r} - g\bar{g}), \quad \sqrt{1/6}\,(r\bar{r} + g\bar{g} - 2b\bar{b})$

Singlet $\sqrt{1/3}\,(r\bar{r} + g\bar{g} + b\bar{b})$ Net colour of singlet = 0 → does not mediate QCD

Exchange of the eight gluons mediate the interaction between particles carrying colour charge:
Between quarks but also between gluons.
→ This is an important difference to the electromagnetic interaction, where the photon has no charge,
→ cannot interact with each other.

(a)   (b)   (c)   (d)
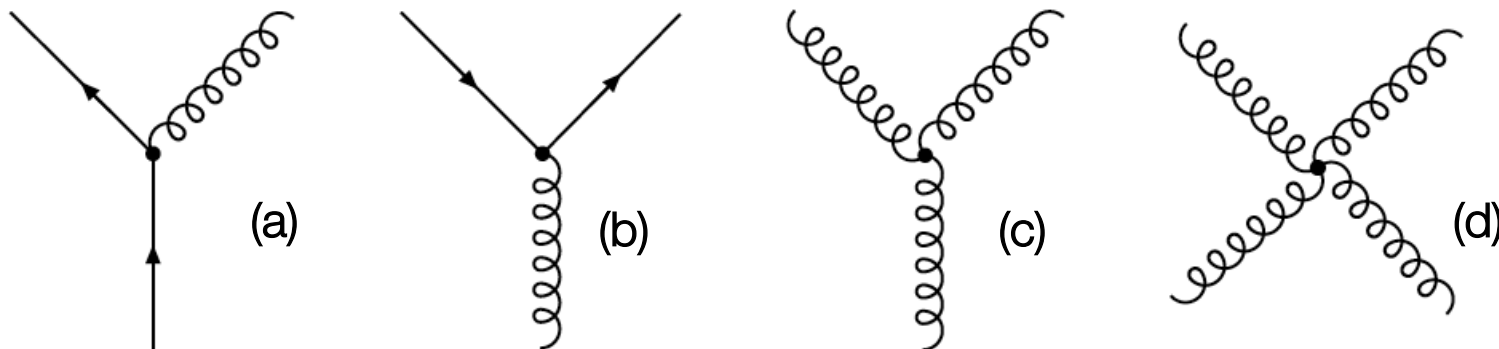
The fundamental interaction diagrams of the strong interaction: emission of a gluon by a quark **(a)**, splitting of a gluon into a quark–antiquark pair **(b)** and "self-coupling" of gluons (**c, d**).

Toni Baroncelli: Deep Inelastic Scattering

Colour Carriers

	Quarks	Anti-quarks	Gluon	Photon
Charge	✅	✅		
Colour	✅	✅	✅	

In principle each hadron might exist in many different colours (the colours of the constituent quarks involved), would
- have different total (net) colours
- but would be equal in all other respects.

In practice only one type of each hadron is observed (one π^-, p, Δ^0 etc.)

additional condition: only colourless particles can exist as free particles $\longrightarrow$

Hadrons as colour-neutral objects.
- colour + anti-colour = "white" = white objects!
- Three different colours = "white" as well.

➤ This is why quarks are not observed as free particles. Breaking one hadron into quarks would produce at least two objects carrying colour: the quark, and the rest of the hadron. This would be a violation of the hadron colour-neutrality.
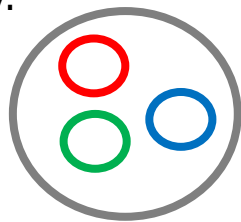
This phenomenon is called confinement.
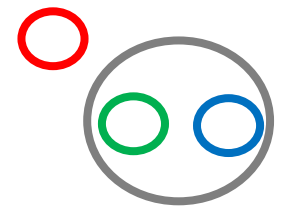
This implies that the potential acting on a quark increases with increasing separation $\rightarrow$ in sharp contrast to the Coulomb potential.

Toni Baroncelli: Deep Inelastic Scattering

Graphically: three vectors in a plane symbolising the three colours, rotated by 120°
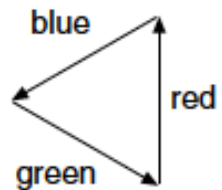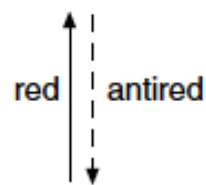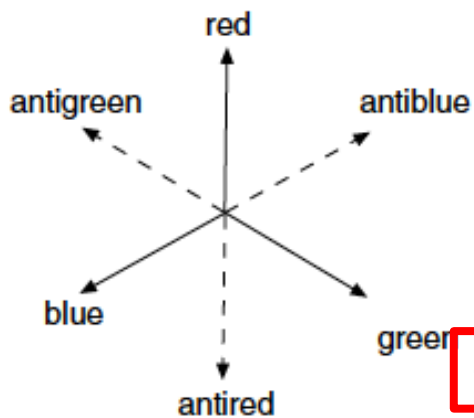
$$|\pi^+\rangle = \begin{cases} |u_r\bar{d_{\bar{r}}}\rangle \\ |u_b\bar{d_{\bar{b}}}\rangle \\ |u_g\bar{d_{\bar{g}}}\rangle \end{cases}$$

The pion π^+ Is a superposition of these states

Combination of 2/3 colours giving white: colour+anti-colour or r+b+g

Gluons are not white: they carry colour and anti-colour

➤ Due to exchange of gluons the colour combination of hadrons continuously changes; but the net-colour "white" remains.

➤ to obtain a colour neutral baryon, each quark must have a different colour. The proton is a mixture of such states:

$$|p\rangle = \begin{cases} |u_b u_r d_g\rangle \\ |u_r u_g d_b\rangle \\ \vdots \end{cases}$$

➤ From this argument, it also becomes clear why no hadrons exist which are $|qq\rangle$ or $|qq\bar{q}\rangle$ combinations, or similar combinations. These states would not be colour neutral
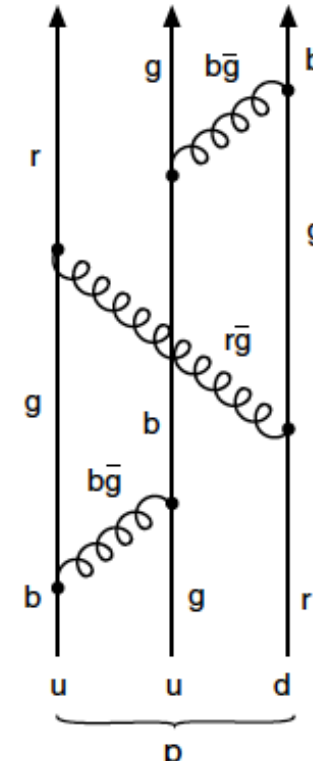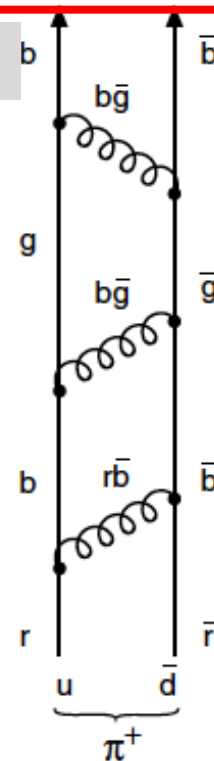
$$\lambda = \frac{\hbar}{|q|} = \frac{\hbar}{\sqrt{Q^2}}$$

Virtual pairs of e⁺e⁻ in *em* interactions have the effect of screening the real e⁻ charge.

At low Q^2 is, the the distances between the interacting particles are large →

➤ the virtual photon sees a cloud of charges
➤ the effective charge of the interacting particles decreases:
➤ the coupling constant is (a bit) smaller (than for a 'naked' electron).
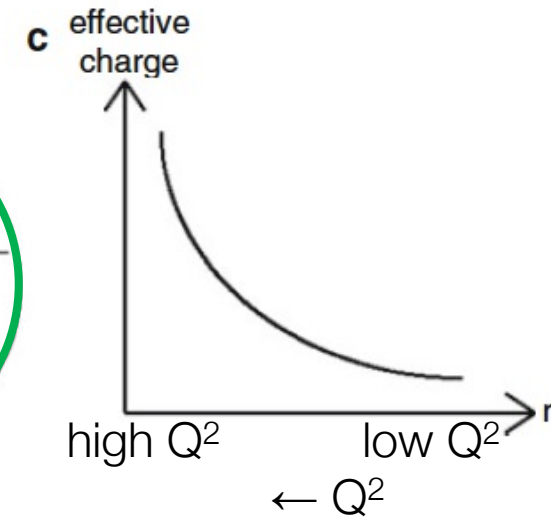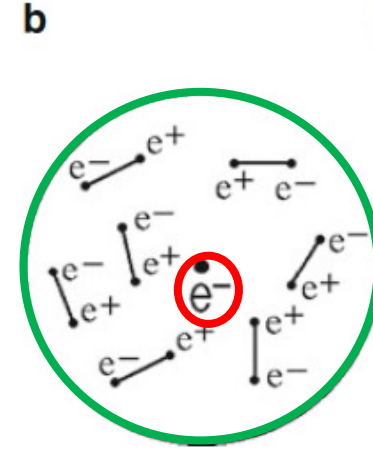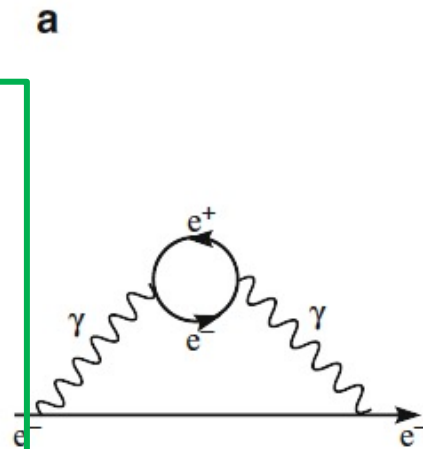
At high Q^2 is, the the distances between the interacting particles are small →

➤ the virtual photon sees the individual charge
➤ the effective charge of the interacting particles increases:
➤ the coupling constant is large.

A parametrization describing the variation of α with Q^2 is given here and it is defined at a given scale μ^2.

$\alpha(m_e) = 1/137 \quad \alpha(m_Z) = 1/128$

a **b** **c** effective charge

high Q^2 low Q^2
← Q^2

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\ln(Q^2/\mu^2)}$$

Toni Baroncelli: Deep Inelastic Scattering

- The coupling "constant" α_s describing the strength of the hadronic interaction between two particles depends on Q^2.

- While in the *em* interaction α_{em} depends weakly on Q^2, in the strong interaction, however, it is stronger.

Why?

The fluctuation of the photon into a electron-positron pair
and
The fluctuation of the gluon into the quark-antiquark pair
generate a

➤ repulsive force between two quarks of the same colour (same charge) and
➤ the attractive force between quarks with (opposite charge) colour and anticolour

Generates screening of the electric and strong charge.

QCD   QED
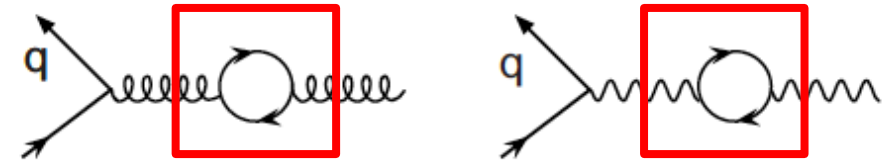
Screening of em or strong charge

Same for QED and QCD ?

coupling constant

quark confined in hadron

α_s

α_{em}

Q^2

Toni Baroncelli: Deep Inelastic Scattering
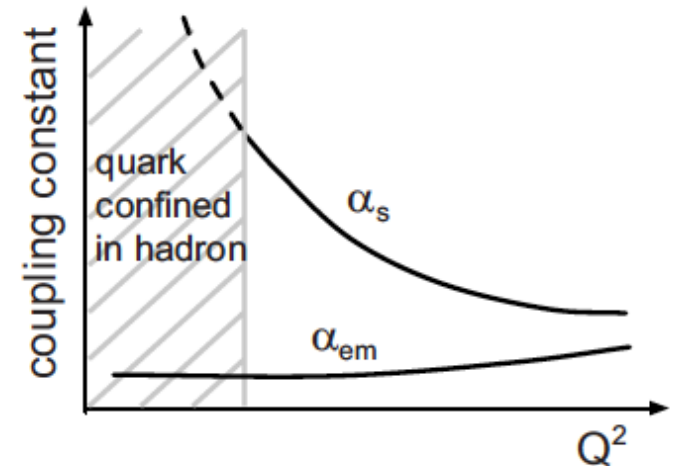
Gluons couple with gluons (photons do NOT couple to photons)!

Different colours may give rise to an attractive force if the quantum state is antisymmetric, and a repulsive force if it is symmetric under the interchange of quarks.

This means that the favourite state of three quarks is the state with three quarks of different colours, q_r q_b q_g, that is, the colourless state of baryons.

The higher Q^2 is, the smaller are the distances between the interacting particles; effective charge of the interacting particles increases: the coupling constant increases.

Gluons can fluctuate into gluons → this can be shown to give anti-screening. The closer the interacting particles are, the smaller is the charge they see.

a_s decreases with increasing Q^2.

QCD　　　　**QED**
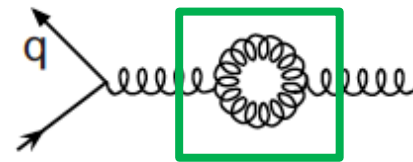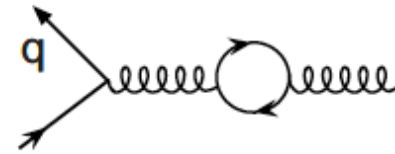
Anti-Screening of strong charge
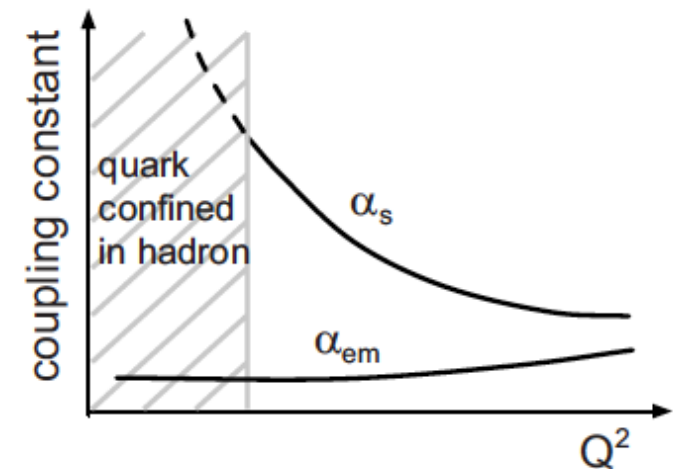
At low Q^2 is, the the distances between the interacting particles are large →

➤ the virtual photon sees a cloud of charges
➤ the effective charge of the interacting particles decreases:
➤ the coupling constant is small.
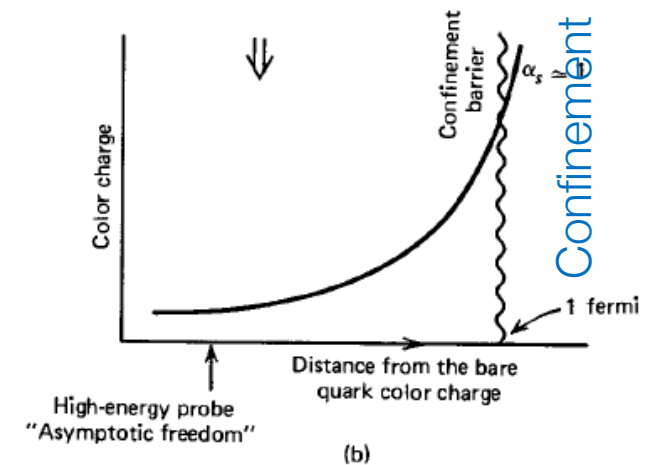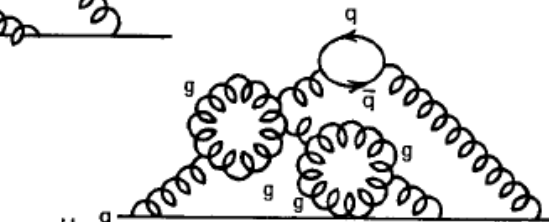
At high Q^2 is, the the distances between the interacting particles are small →
➤ the virtual photon sees the individual charge
➤ the effective charge of the interacting particles increases:
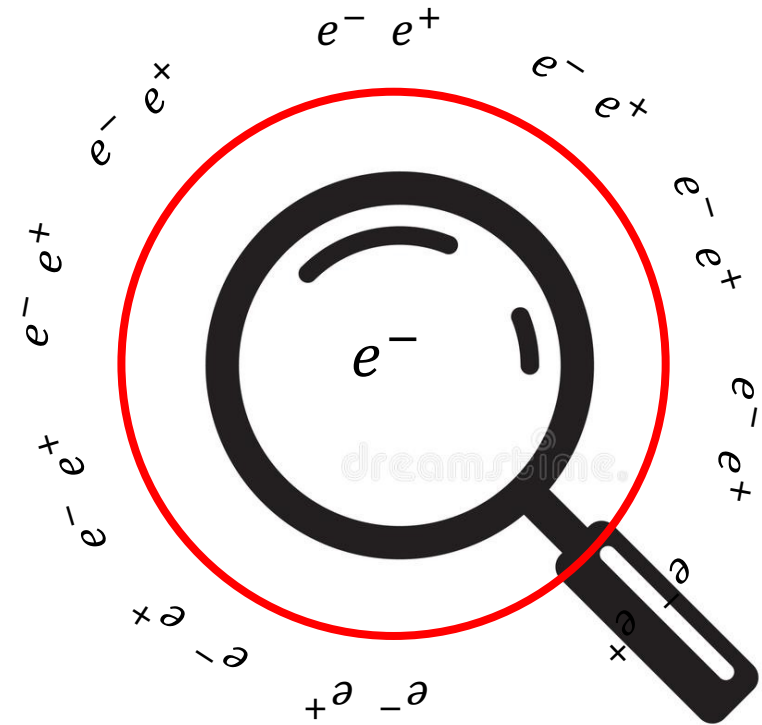➤ the coupling constant is large.

Quantum electrodynamics

Quantum chromodynamics
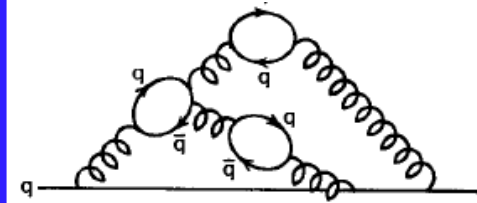
Asymptotic Freedom

Confinement

Low q^2 →
the bare charge is screened →
screening

High q^2 →
you see the bare charge

In the case of gluons the anti-screening is far stronger than the screening. A first-order perturbation calculation in QCD gives:

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2 \cdot n_f) \cdot \ln(Q^2/\Lambda^2)}$$
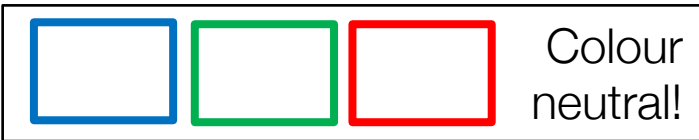
Colour neutral!

n_f number of flavours that contribute to the interaction

$Q^2 \rightarrow$ separation among different components

Λ parameter of the function determined from data

$33 = 11 \times N_c$

- A heavy $q\bar{q}$ pair has a very short lifetime → exists at very high Q^2. → n_f varies between $n_f \approx$ 3–6 → when Q^2 increases n_f increases too.
- The free parameter Λ (1 parameter!) is was found to be $\Lambda \approx$ 250 MeV/c by comparing the prediction and data.
- Perturbative expansion procedures in QCD are valid only if $\alpha_s \ll 1$. This is satisfied for $Q^2 \gg \Lambda^2 \approx 0.06$ (GeV/c)² .

The formula indicates two regions:

- For very small distances (high values of Q^2) " α_s decreases, vanishing asymptotically. In the limit $Q^2 \rightarrow \infty$, quarks can be considered "free", this is called asymptotic freedom.
- At large distances, (low values of Q^2) α_s increases so strongly that it is impossible to separate individual quarks inside hadrons (confinement).
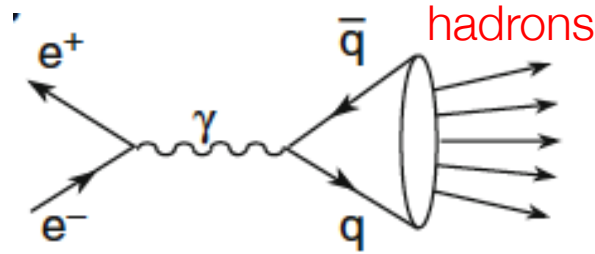
Study the production of
- $q\bar{q}$ pairs and of
- $\mu^+\mu^-$ pairs

in e^+e^- interactions

The two cross-sections are due to the exchange of one photon and are described by the two expressions below
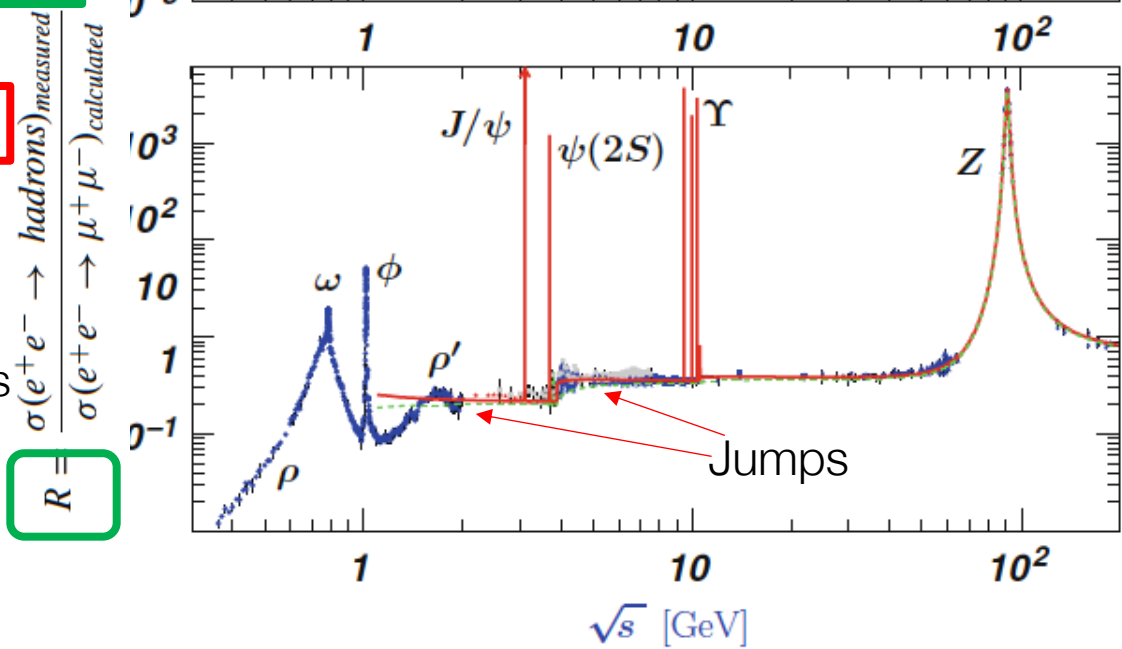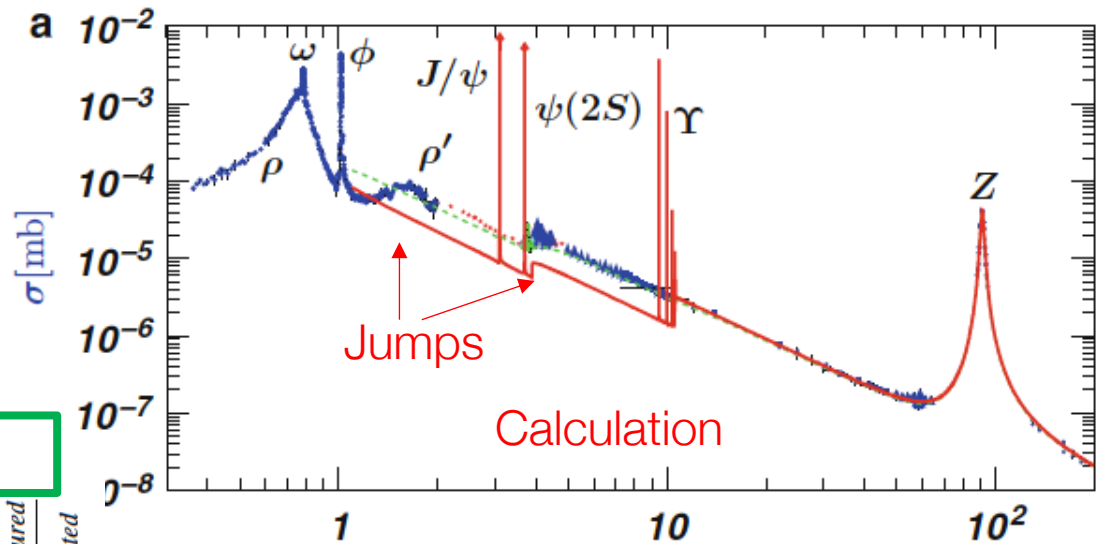
N_f # accessible quarks

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha_{EM}^2(\hbar c)^2}{3s}$$

$$\sigma(e^+e^- \to \gamma \to q\bar{q} \to hadrons) = N_C \frac{4\pi\alpha_{EM}^2}{3} \frac{(\hbar c)^2}{s} \sum_{n=1}^{N_f} Q_n^2$$

Q_q^2 *charge of quarks involved*

Jumps are understood with the opening of kinematical windows as soon as $\sqrt{s} > m_q + m_{\bar{q}}$

A factor $N_C = 3$ had to be introduced to account for the number of different coloured hadrons

$$R = \frac{\sigma(e^+e^- \to hadrons)_{measured}}{\sigma(e^+e^- \to \mu^+\mu^-)_{calculated}}$$

hadrons

e^+ $\bar{q}$

γ

e^- q

a

Jumps

Calculation

ω ϕ J/ψ $\psi(2S)$ Υ Z

ρ ρ'

σ[mb]

J/ψ $\psi(2S)$ Υ Z

ω ϕ ρ'

ρ

Jumps

$\sqrt{s}$ [GeV]

We showed that
initial measurements
of the structure function
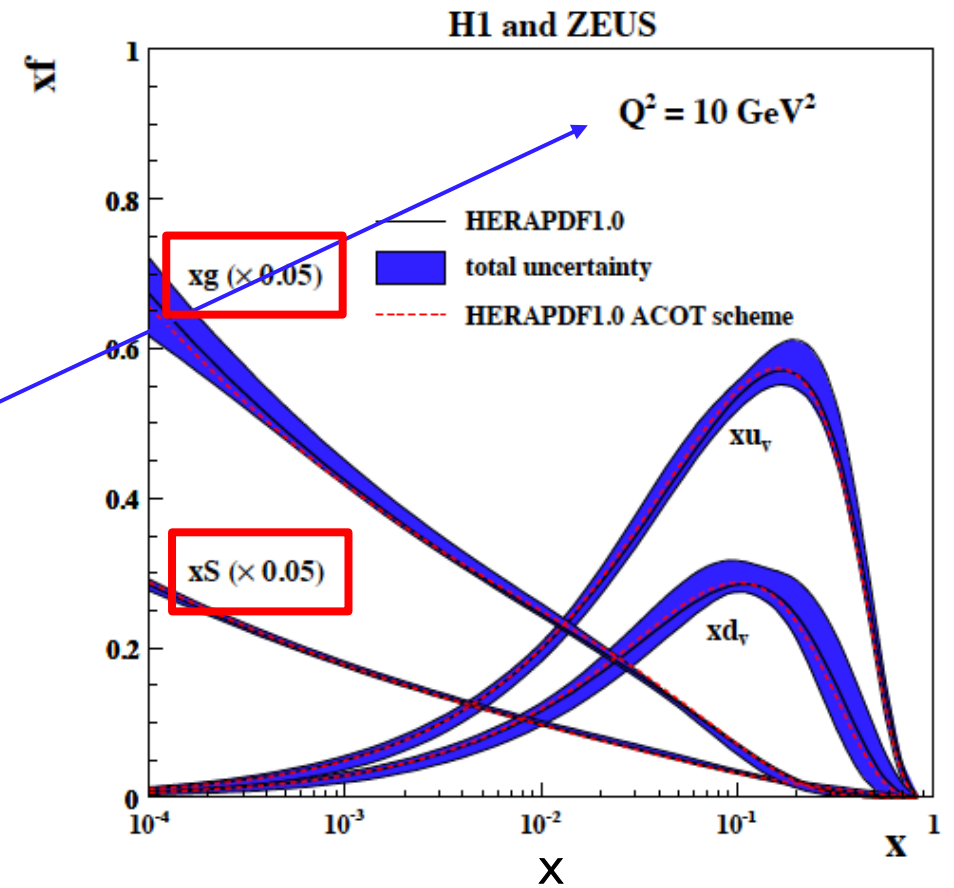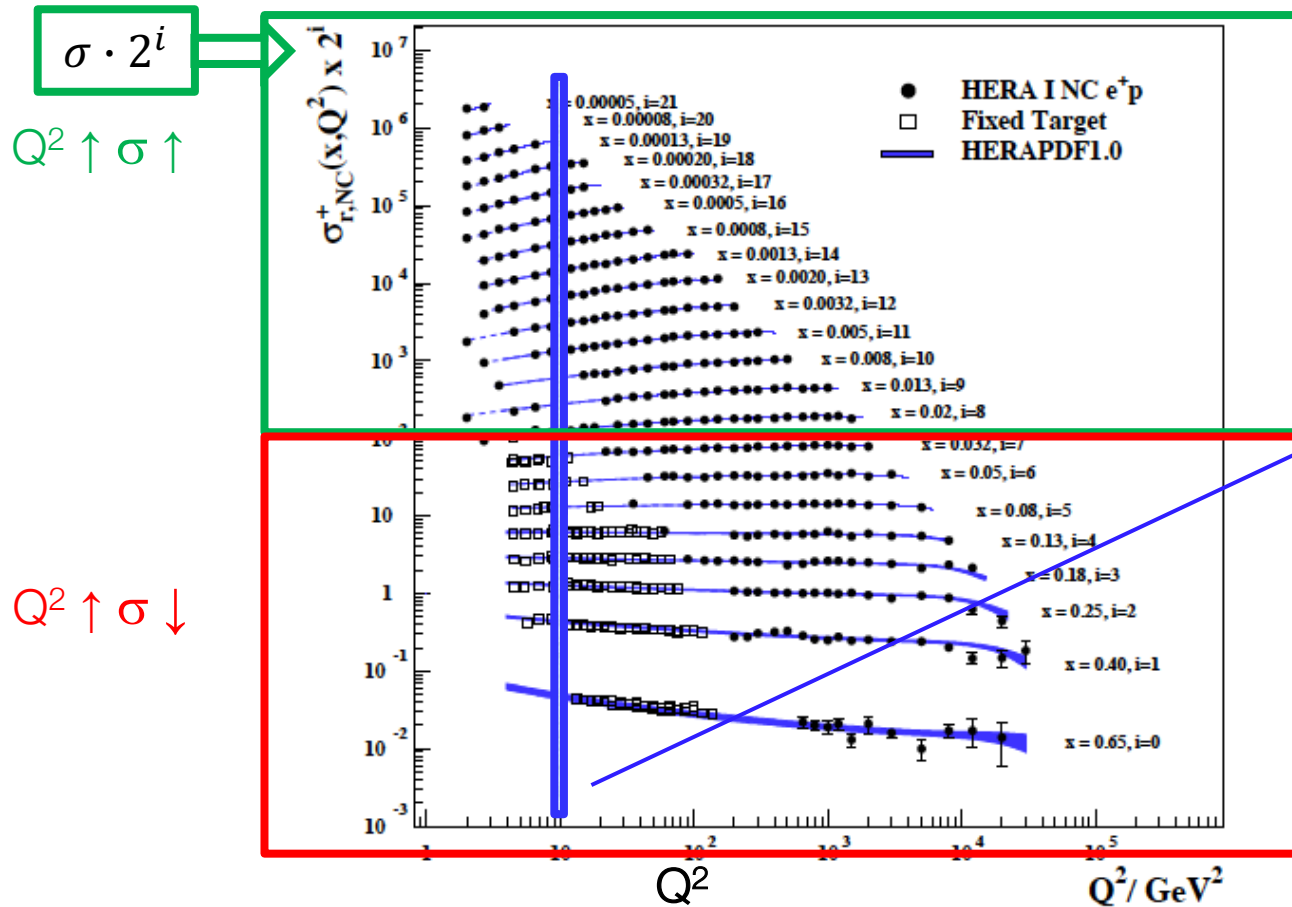F_2 depend only on the scaling
variable x *(Bjorken scaling)* .

High precision measurements
(and higher energies) showed
that F_2 does depend also on
Q^2 (but weakly).

Figure → shows the
experimental measurements
of F_2 as a function of Q^2 at
several fixed values of x .

Toni Baroncelli: Deep Inelastic Scattering
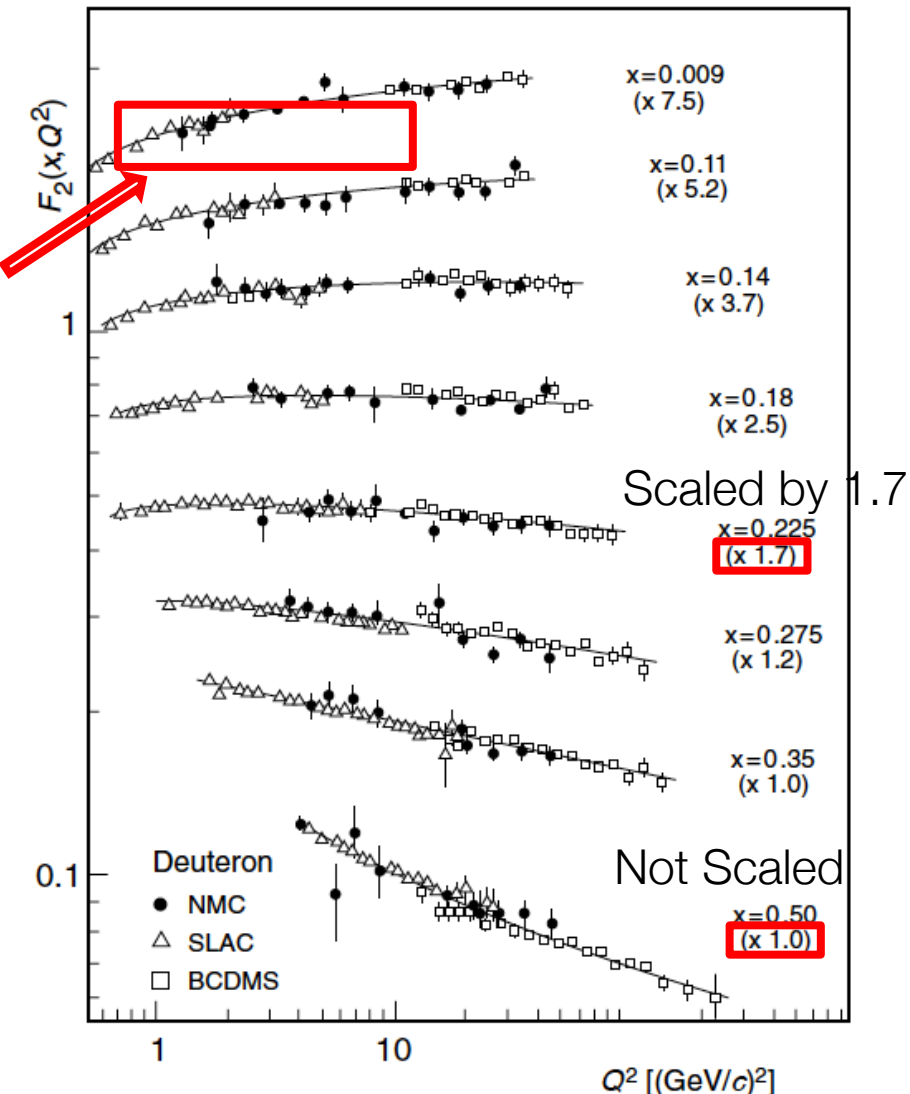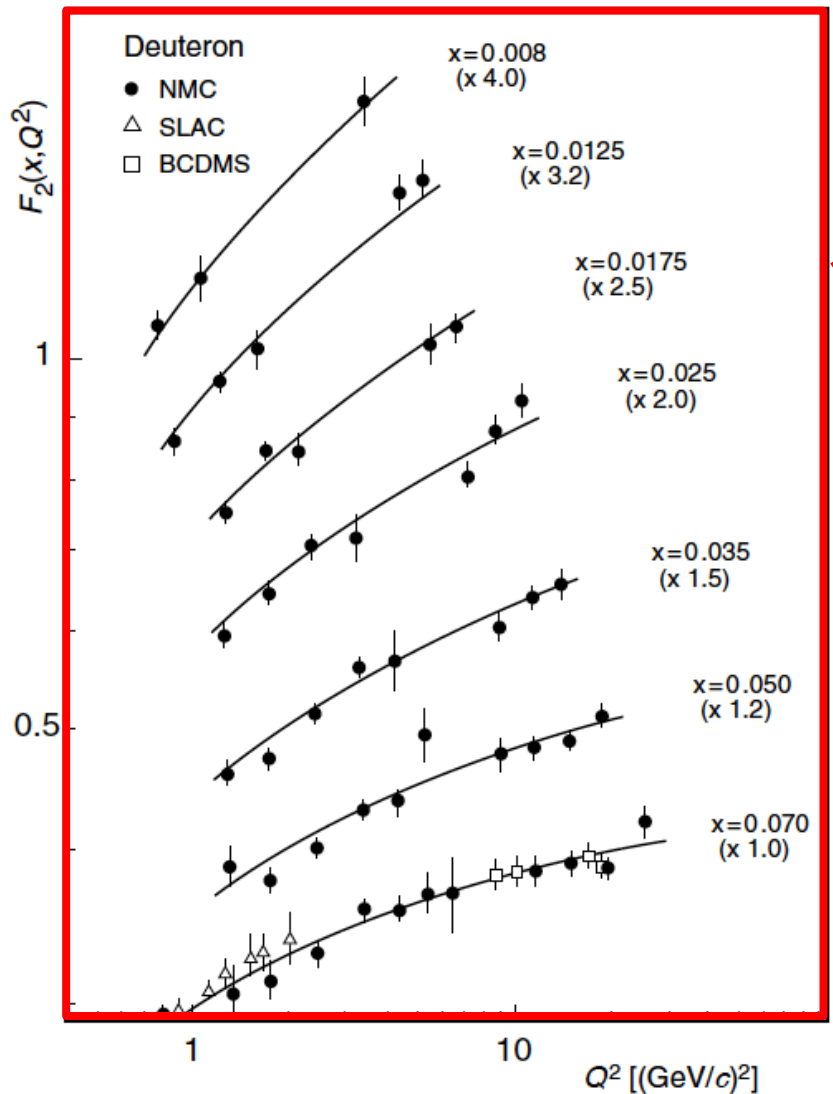
On the left the HERA combined NC e+p reduced cross section and fixed-target data as a function of Q^2. The error bars indicate the total experimental uncertainty. An analytic parametrisation is superimposed.
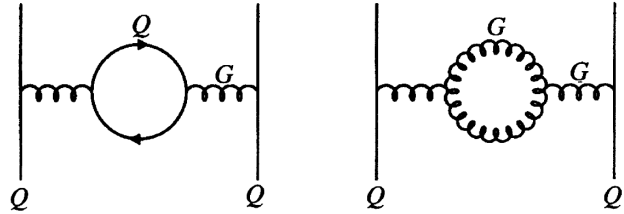
The data shows a large range of x and Q^2.

On the right $x \cdot u_v, x \cdot d_v, \ x \cdot g, \ x \cdot s$ (bands are the total uncertainty of the fit).
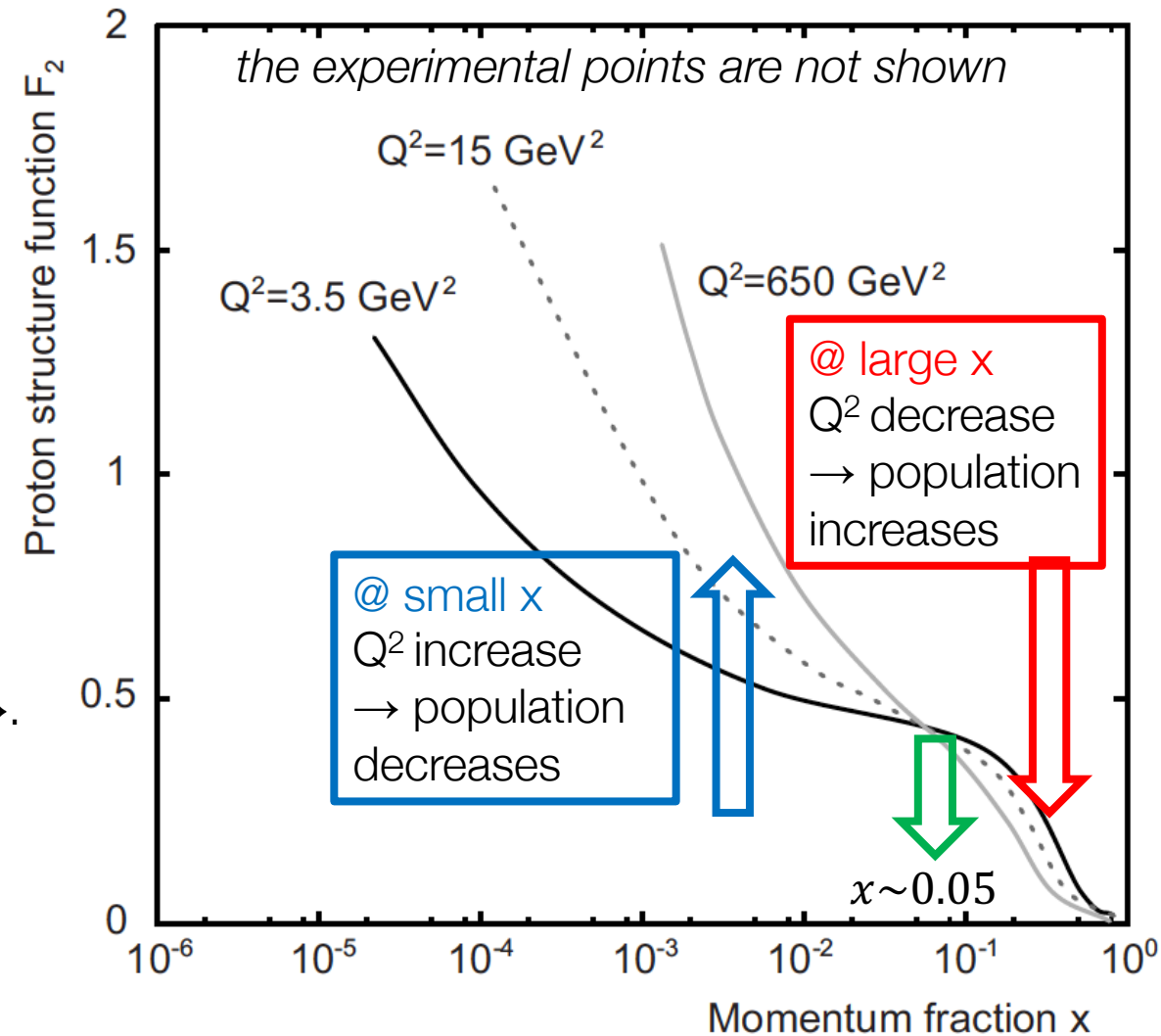
$$x = \frac{Q^2}{2M\nu}$$

- Quarks emit or absorb gluons
- gluons may split into $q\bar{q}$ pairs or emit gluons →
- The momentum distribution changes continuously.

The structure function

- increases with Q^2 at small values of x and
- decreases when Q^2 increases at large values of x .

This behaviour, called **scaling violation**, is sketched in Fig. →.

With increasing values of Q^2 many quarks seen → the momentum of the proton is shared among many partons → there are few quarks with large momentum fractions in the nucleon → quarks with small momentum fractions predominate.

the experimental points are not shown

Q^2=15 GeV^2

Q^2=3.5 GeV^2

Q^2=650 GeV^2

@ large x
Q^2 decrease
→ population
increases

@ small x
Q^2 increase
→ population
decreases

$x \sim 0.05$

Proton structure function F$_2$

Momentum fraction x

A virtual photon can resolve dimensions of the order of $\hbar/Q^2$. At small Q^2 quarks and emitted gluons cannot be distinguished and a quark distribution $q(x,Q^2)$ is measured.

At larger Q^2 and higher resolution, emission and splitting processes must be considered → the number of partons that share the momentum of the nucleon increases.

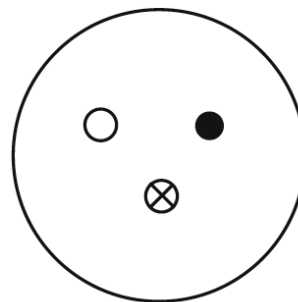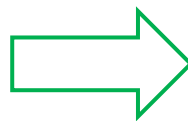- The quark distribution $q(x,Q^2)$ at small momentum fractions x, therefore, is larger than $q(x,Q^2)$ at high values of x;
- the effect is reversed for large x.
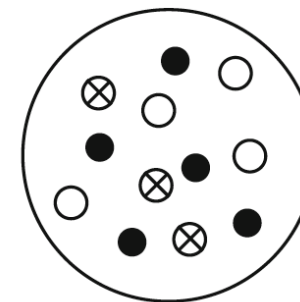
Large x, low Q^2
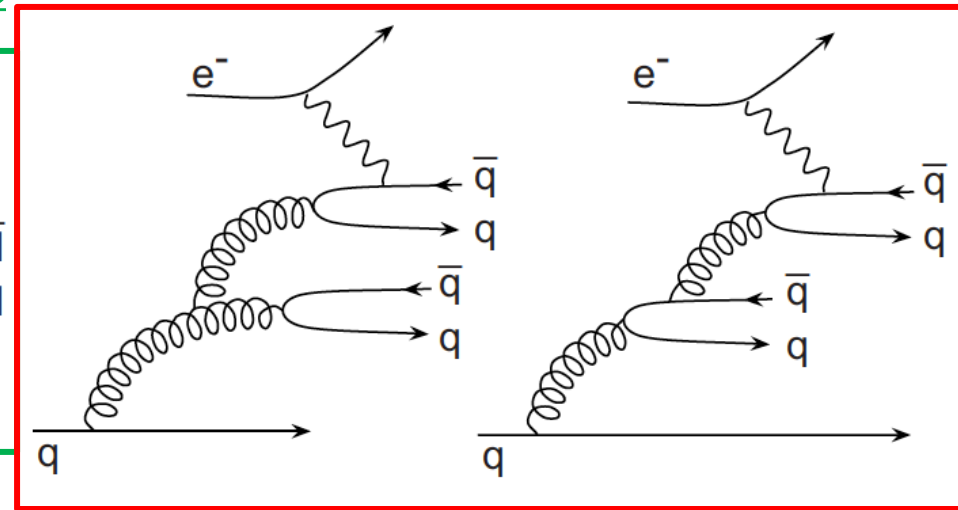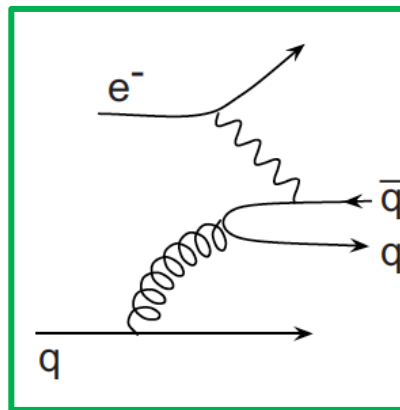
Small x , high Q^2

Evolution of the structure function with Q^2 at small values of x and its decrease at large x. The gluon distribution $g(x,Q^2)$ has a similar behaviour.

$x = 0.1$ $x = 0.001$ $x < x_c$
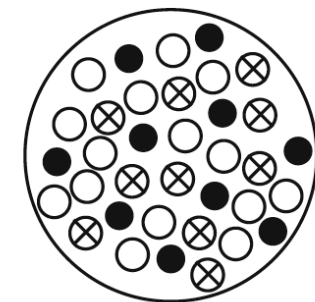
The photon exchanged in DIS has an equivalent length of $\frac{\hbar}{\sqrt{Q^2}}$ and cannot resolve any structure smaller than this

$\rightarrow$

- at low Q^2 the photon cannot see the effect of gluons and sees the x distribution of quarks
- at high Q^2 the photon starts to resolve inner structure of quarks and splitting processes must be accounted for.

Resolving power of an high Q^2 photon

Resolving power of a low Q^2 photon
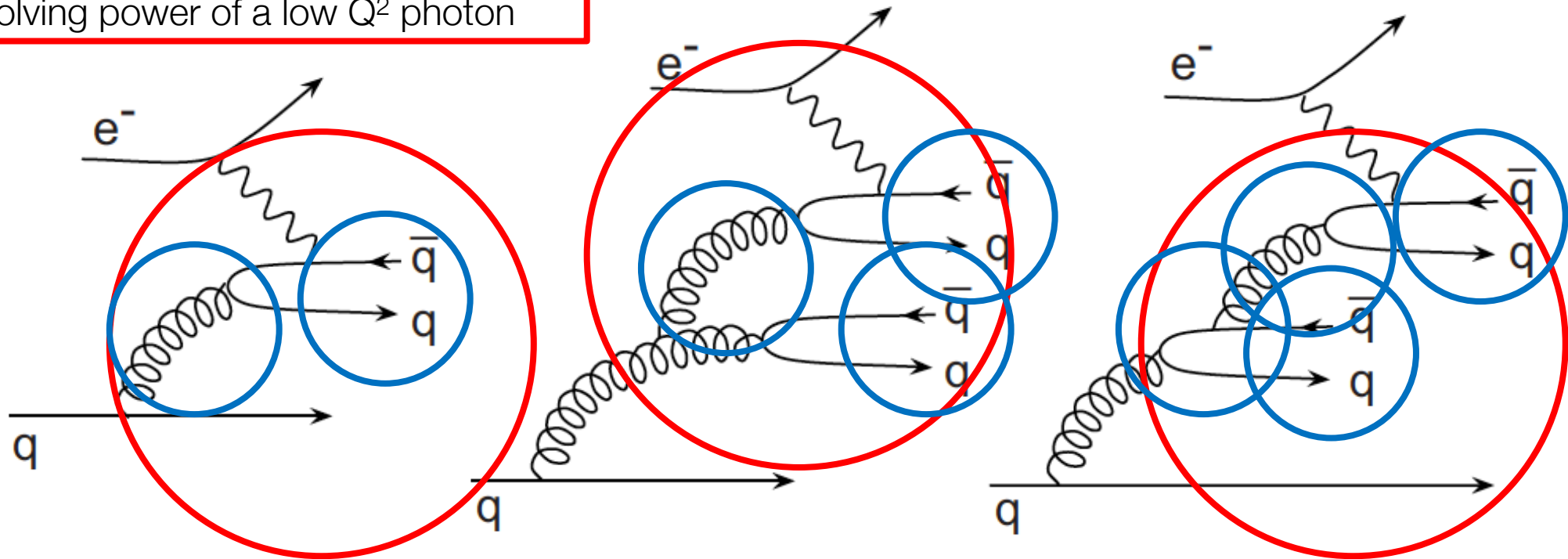
The number of partons seen to share the momentum of the nucleon increases when Q^2 increases.

The quark distribution $q(x,Q^2)$ at small momentum fractions x is larger than $q(x,Q^2)$ al large values of x; whereas the effect is reversed for large x .

This is the origin of the increase of the structure function with Q^2 at small values of x and its decrease at large x . The gluon distribution $g(x,Q^2)$ shows a Q^2-dependence as well which originates from processes of gluon emission by a quark or by another gluon.

The change in the quark distribution and in the gluon distribution with Q^2 at fixed values of x is proportional to the strong coupling constant $\alpha_s(Q^2)$ and depends upon the size of the quark and gluon distributions at all larger values of x.

The dependence of the quark and gluon distributions can be described by a system of coupled integral-differential equations [Altarelli Parisi equations].
- If $\alpha_s(Q^2)$ and the shape of $q(x,Q^2)$ and $g(x,Q^2)$ are known at a given value Q^2
- $\rightarrow$ $q(x,Q^2)$ and $g(x,Q^2)$ can be predicted from QCD for all other values of Q^2.
- The coupling $\alpha_s(Q^2)$ and the gluon distribution $g(x,Q^2)$, which cannot be directly measured, can be determined from the observed scaling violation of the structure function $F_2(x,Q^2)$.

In QCD, the above process is described in terms of scale-dependent parton distributions $f_a(x, \mu^2)$, where $a = g$ or q and, typically, μ is the scale of the probe Q. For $Q^2 \gg M^2$, the structure functions are of the form

$$F_i = \sum_a C_i^a \otimes f_a, \qquad (16.21)$$

where $\otimes$ denotes the convolution integral

$$C \otimes f = \int_x^1 \frac{dy}{y} C(y) f\left(\frac{x}{y}\right), \qquad (16.22)$$

and where the coefficient functions C_i^a are given as a power series in α_s. The parton distribution f_a corresponds, at a given x, to the density of parton a in the proton integrated over transverse momentum k_t up to μ. Its evolution in μ is described in QCD by a DGLAP equation (see Refs. 14–17) which has the schematic form

$$\frac{\partial f_a}{\partial \ln \mu^2} \sim \frac{\alpha_s(\mu^2)}{2\pi} \sum_b (P_{ab} \otimes f_b), \qquad (16.23)$$

where the P_{ab}, which describe the parton splitting $b \rightarrow a$, are also given as a power series in α_s. Although perturbative QCD can predict, via Eq. (16.23), the evolution of the parton distribution functions from a particular scale, μ_0, these DGLAP equations cannot predict them *a priori* at any particular μ_0. Thus they must be measured at a starting point μ_0 before the predictions of QCD can be compared to the data at other scales, μ. In general, all observables involving a hard hadronic interaction (such as structure functions) can be expressed as a convolution of calculable, process-dependent coefficient functions and these universal parton distributions, e.g. Eq. (16.21).

It is often convenient to write the evolution equations in terms of the gluon, non-singlet (q^{NS}) and singlet (q^S) quark distributions, such that

$$q^{NS} = q_i - \bar{q}_i \quad (\text{or } q_i - q_j), \qquad q^S = \sum_i (q_i + \bar{q}_i). \qquad (16.24)$$

The non-singlet distributions have non-zero values of flavor quantum numbers, such as

Nomenclature
$f_a(x, q^2)$ parton distributions
P_{ab} parton splitting $b \rightarrow a$
n_f number of active quark flavors

isospin and baryon number. The DGLAP evolution equations then take the form

$$\frac{\partial q^{NS}}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} P_{qq} \otimes q^{NS},$$

$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} q^S \\ g \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi} \begin{pmatrix} P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q^S \\ g \end{pmatrix}, \qquad (16.25)$$

where P are splitting functions that describe the probability of a given parton splitting into two others, and n_f is the number of (active) quark flavors. The leading-order

into two others, and n_f is the number of (active) quark flavors. The leading-order Altarelli-Parisi [16] splitting functions are

$$P_{qq} = \frac{4}{3}\left[\frac{1+x^2}{(1-x)}\right]_+ = \frac{4}{3}\left[\frac{1+x^2}{(1-x)_+}\right] + 2\delta(1-x) , \tag{16.26}$$

$$P_{qg} = \frac{1}{2}\left[x^2 + (1-x)^2\right] , \tag{16.27}$$

$$P_{gq} = \frac{4}{3}\left[\frac{1+(1-x)^2}{x}\right] , \tag{16.28}$$

$$P_{gg} = 6\left[\frac{1-x}{x} + x(1-x) + \frac{x}{(1-x)_+}\right]$$
$$+ \left[\frac{11}{2} - \frac{n_f}{3}\right]\delta(1-x), \tag{16.29}$$

where the notation $[F(x)]_+$ defines a distribution such that for any sufficiently regular test function, $f(x)$,

$$\int_0^1 dx\, f(x)[F(x)]_+ = \int_0^1 dx\, (f(x) - f(1))F(x) . \tag{16.30}$$

In general, the splitting functions can be expressed as a power series in α_s. The series contains both terms proportional to $\ln\mu^2$ and to $\ln\, 1/x$. The leading-order DGLAP evolution sums up the $(\alpha_s \ln\mu^2)^n$ contributions, while at next-to-leading order (NLO) the sum over the $\alpha_s(\alpha_s \ln\mu^2)^{n-1}$ terms is included [18,19]. In fact, the NNLO contributions to the splitting functions and the DIS coefficient functions are now also all known [20–22].

In the kinematic region of very small x, it is essential to sum leading terms in $\ln\, 1/x$, independent of the value of $\ln\mu^2$. At leading order, LLx, this is done by the BFKL equation for the unintegrated distributions (see Refs. [23,24]). The leading-order $(\alpha_s \ln(1/x))^n$ terms result in a power-like growth, $x^{-\omega}$ with $\omega = (12\alpha_s \ln2)/\pi$, at asymptotic values of $\ln\, 1/x$. More recently, the next-to-leading $\ln\, 1/x$ (NLLx) contributions have become available [25,26]. They are so large (and negative) that the result appears to be perturbatively unstable. Methods, based on a combination of collinear and small x resummations, have been developed which reorganize the perturbative series into a more stable hierarchy [27–30]. There are indications that small x resummations become necessary for real precision for $x \lesssim 10^{-3}$ at low scales. On the

Toni Baroncelli: Deep Inelastic Scattering

Proton contains both quarks and gluons — so DGLAP is a *matrix in flavour space:*

$$\frac{d}{d \ln Q^2} \begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q \leftarrow q} & P_{q \leftarrow g} \\ P_{g \leftarrow q} & P_{g \leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

[In general, matrix spanning all flavors, anti-flavors, $P_{qq'} = 0$ (LO), $P_{\bar{q}g} = P_{qg}$]

Splitting functions are:

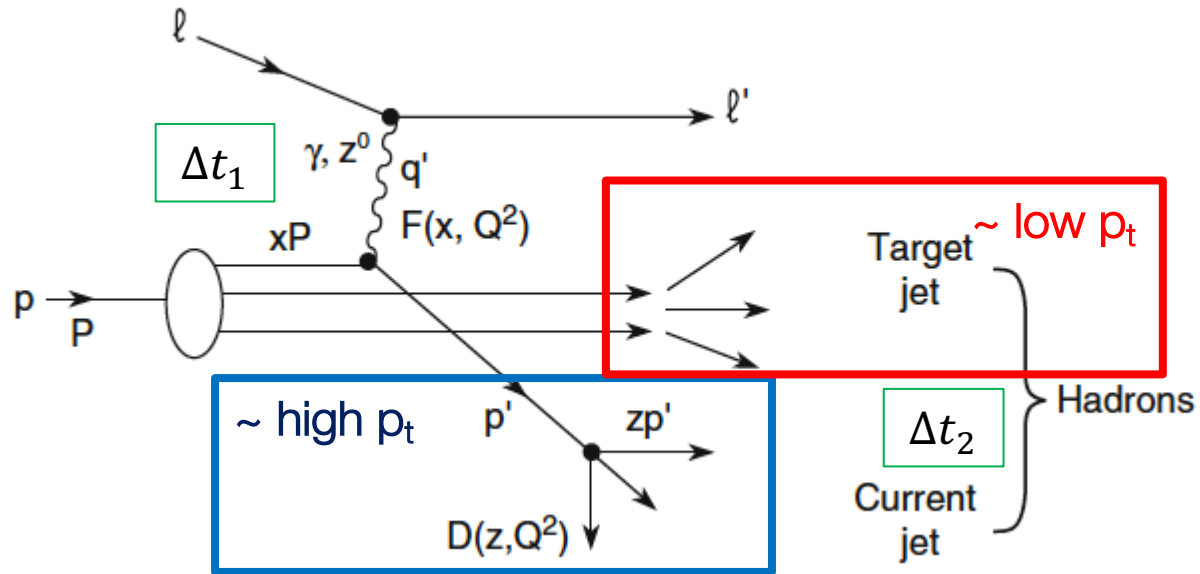$$P_{qg}(z) = T_R \left[z^2 + (1-z)^2 \right], \qquad P_{gq}(z) = C_F \left[\frac{1 + (1-z)^2}{z} \right],$$

$$P_{gg}(z) = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] + \delta(1-z) \frac{(11 C_A - 4 n_f T_R)}{6}.$$

Have various symmetries / significant properties, e.g.

- ▶ P_{qg}, P_{gg}: *symmetric* $z \leftrightarrow 1 - z$ (except virtuals)
- ▶ P_{qq}, P_{gg}: *diverge for* $z \to 1$ soft gluon emission
- ▶ P_{gg}, P_{gq}: *diverge for* $z \to 0$ Implies PDFs grow for $x \to 0$

1. γ-parton collision occurs within a time $\Delta t_1 \approx \dfrac{\hbar}{v}, v = E - E'$

2. The quark hadronization has a time $\Delta t_2 \approx \dfrac{\hbar}{m_p^2} \approx 10^{-24} s$

If $v \gg m_p$, one has $\Delta t_1 \gg \Delta t_2$ and the two subprocesses are distinct.

DIS second stage: the parton fragments into two jets of hadrons (hadronises).
naked quarks to hadrons in the final state.

The fragmentation function,

$$D(z;Q^2)$$

describes the hadrnisation.

D(z;Q²): probability that a given hadron carries a fraction z of the interacting parton energy, it must be estimated.

In this stage, the gluons play an important role and modify the structure function, making it dependent on Q².

1. Basic (EW) Interaction

2. The quark or the antiquark can radiate a gluon, which can radiate another gluon, or produce a $q\bar{q}$ pair.

3. The coloured partons (quarks and gluons) fragment (hadronize) in colourless hadrons. The process cannot be treated with perturbation methods; in the absence of an exact analysis, the fragmentation is described by *models*

4. In the fourth phase, the produced hadronic resonances decay rapidly into hadrons
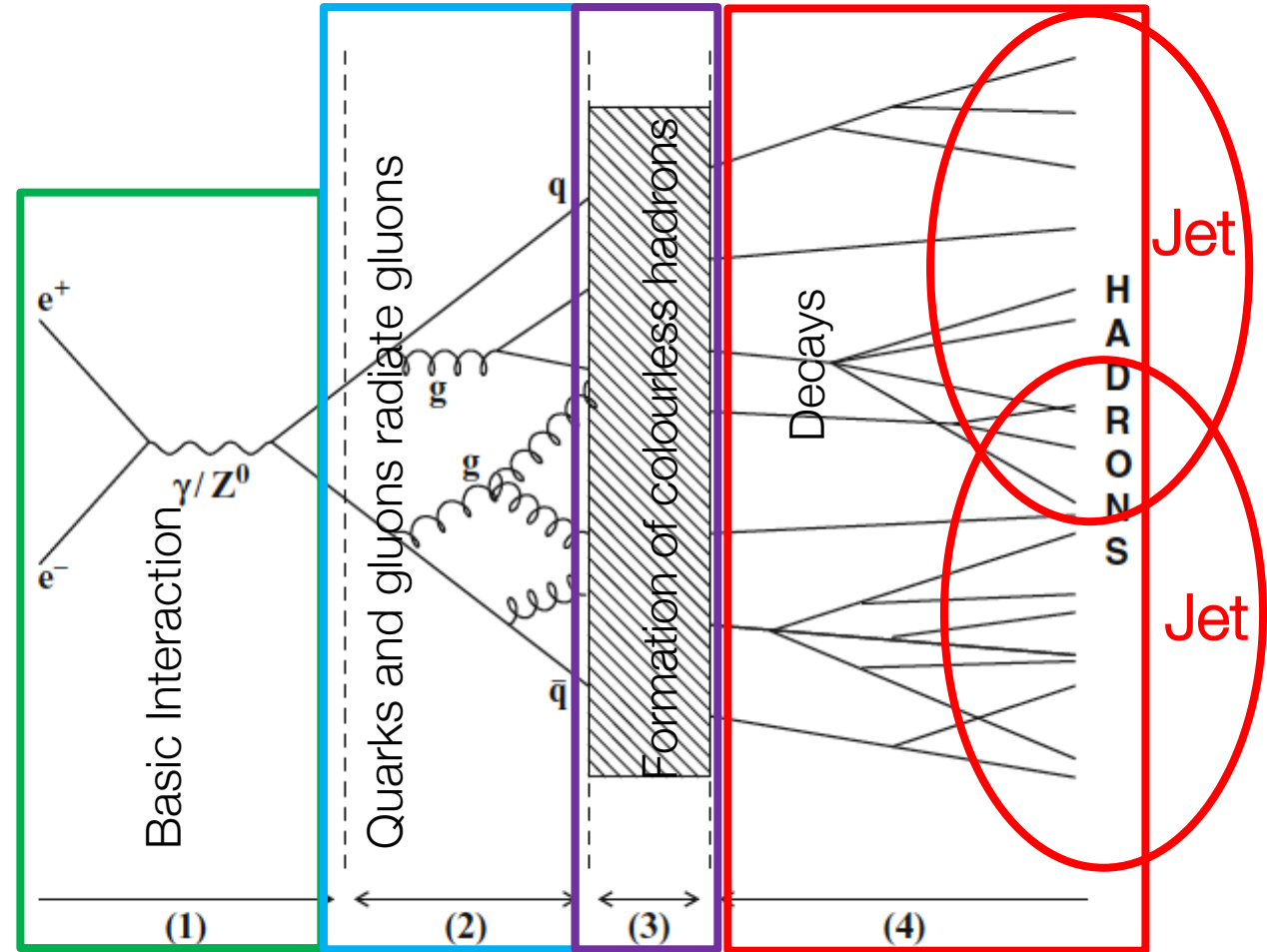
Jet production in $pp, p\bar{p}$ interactions

- At high energies, hadrons are typically produced in two jets, emitted in opposite directions.
- These jets are produced in the hadronization of the primary quarks and antiquarks.
- In addition to simple qq production, higher-order processes can happen. For example, a high-energy ("hard") gluon can be emitted, which can then manifest itself as a third jet of hadrons.
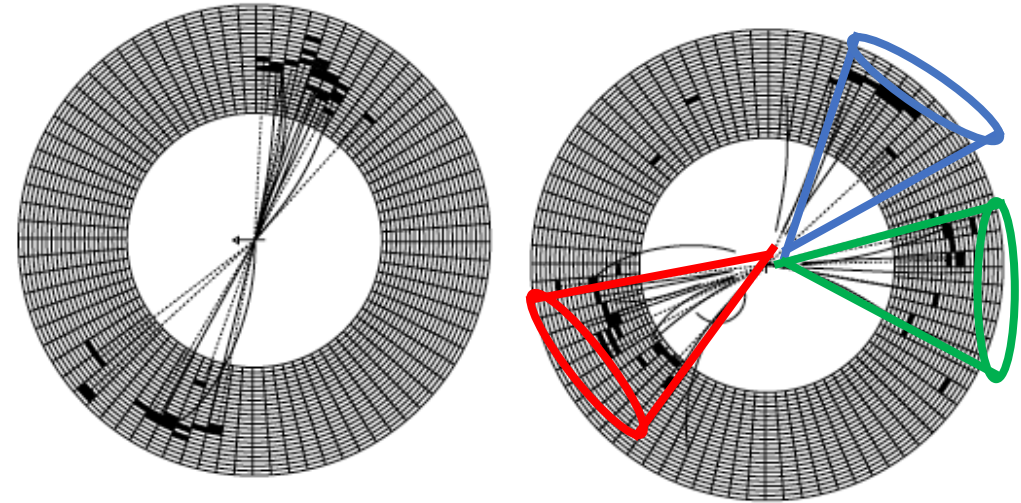
This is ~ to the emission of a γ in em bremsstrahlung. The em coupling constant α is small → emission of a hard photon is a relatively rare process.
The probability of gluon bremsstrahlung (right part of the Figure) is given by the coupling constant α_s .

A comparison of the 3- and 2-jet event rates → α_s.
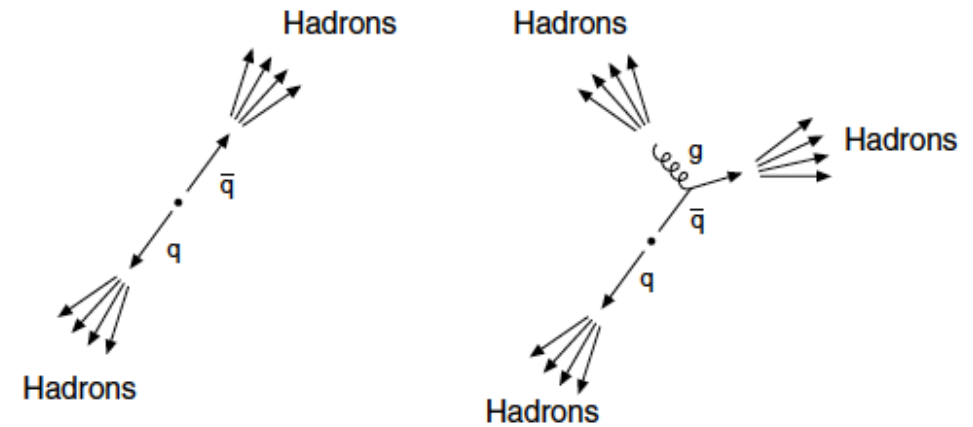
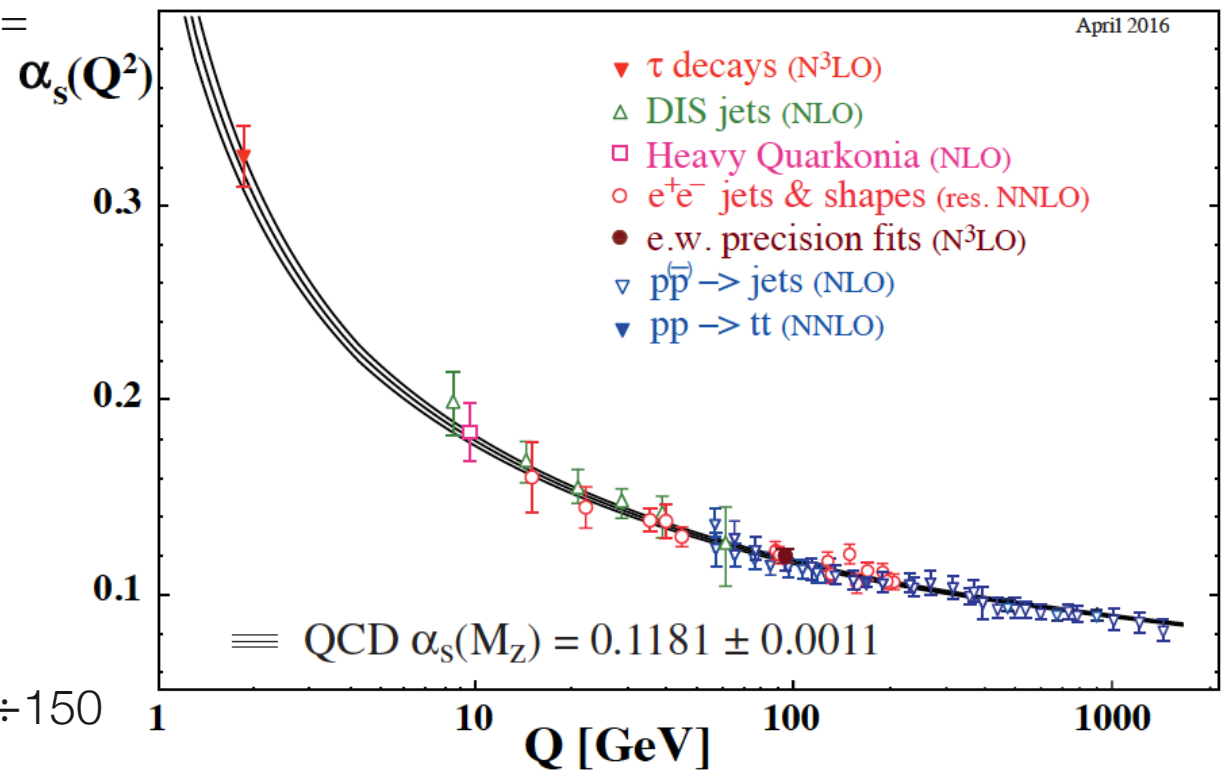Measurements at different energies show that α_s decreases with increasing Q^2 as predicted by

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \cdot \ln(Q^2/\Lambda^2)}$$

Toni Baroncelli: Deep Inelastic Scattering

Review of Particles Properties 2018 edition: http://pdg.lbl.gov/2018/reviews/rpp2018-rev-qcd.pdf

- Hadronic decays of the τ lepton: $\tau \rightarrow \nu_\tau + hadrons$ (Q=1.77GeV)
- Evolution of the nucleon structure functions measured in inelastic scattering of e,μ,ν on nucleons (Q=2 ÷ 50 GeV)
- Jet production in the inelastic scattering $ep \rightarrow eX$ (Q= 2 ÷ 50 GeV)
- Analyses of the energy levels of bound states $q\bar{q}$ (quarkonium) (Q = 1.5 ÷ 5 GeV)
- Decays of the vector mesons ϒ (Q = 5 GeV)
- Hadronic cross-section of the annihilation $e^+e^- \rightarrow hadrons$ (Q = 10 ÷ 200 GeV)
- Fragmentation function of jets produced in $e^+e^- \rightarrow hadrons$ (Q = 10 ÷ 200 GeV)
- *Hadronic decays of the Z^0 boson (Q = 91 GeV)*
- *Jet production in $pp, p\bar{p}$ interactions (Q = 50 ÷1000 GeV)*
- Photon production in in $pp, p\bar{p}$ interactions (Q = 30 ÷150 GeV)

https://pdg.lbl.gov/2021/reviews/contents_sports.html
Pg 32

Toni Baroncelli: Deep Inelastic Scattering

τ decay (N³LO)
low Q² cont. (N³LO)
HERA jets (NNLO)
Heavy Quarkonia (NNLO)
e⁺e⁻ jets/shapes (NNLO+res)
pp/pp̄ (jets NLO)
EW precision fit (N³LO)
pp (top, NNLO)

$\alpha_s(M_Z^2) = 0.1179 \pm 0.0009$

August 2021

Q [GeV]

$\alpha_s(Q^2)$
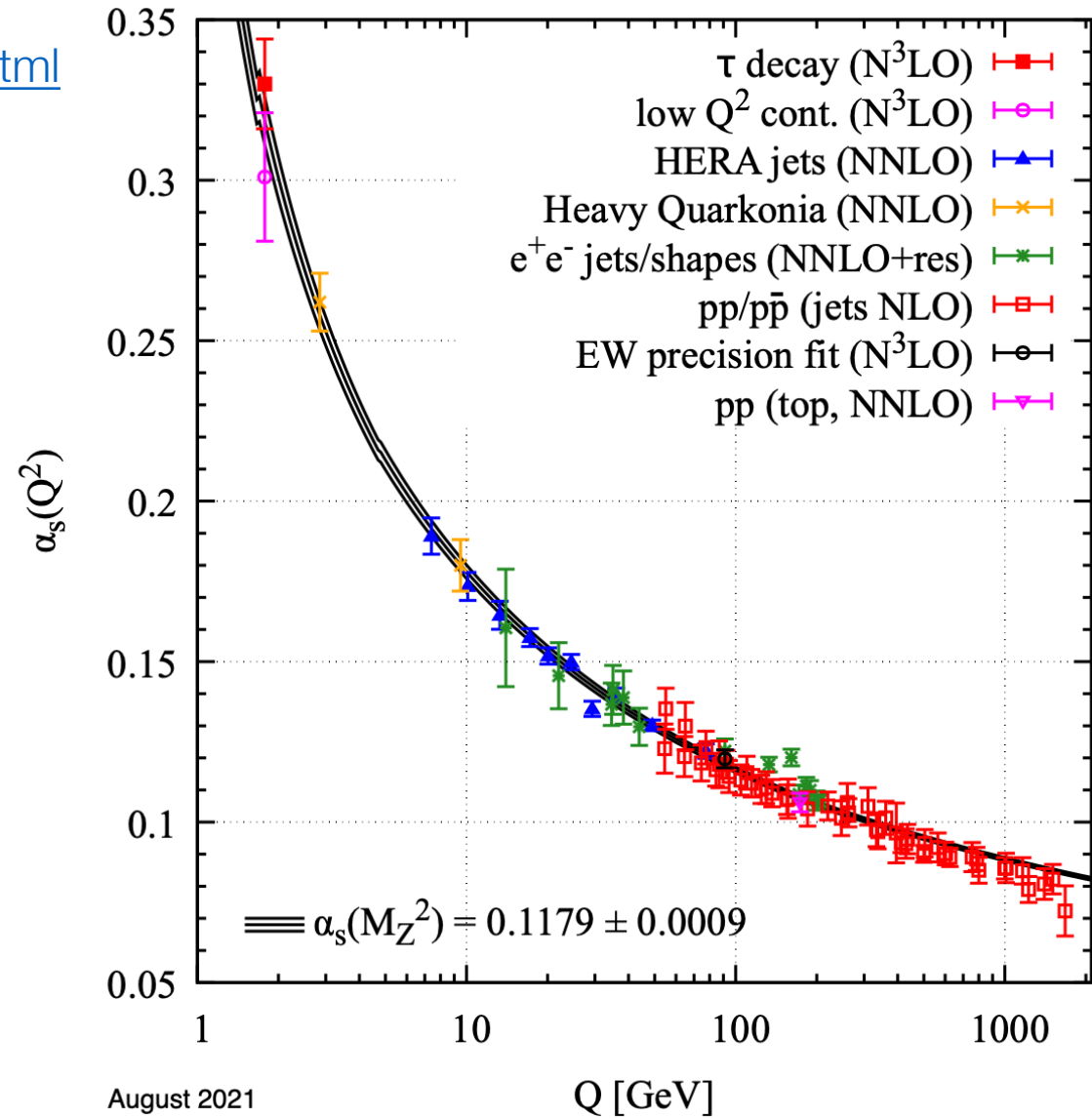
1. Povh, Rith, Scholz, Zetsche: Particles and Nuclei, An Introduction to the Physical Concepts. Springer
2. Braibant, Giacomelli, Spurio: Particles and Fundamental Interactions, An Introduction to Particle Physics, Springer
3. P.Bagnaia: Sapienza University, Particle Physics, Hadron Structure
4. M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018), Standard Model and Related Topics
5. http://th-www.if.uj.edu.pl/~erichter/dydaktyka/Dydaktyka2017/SpecFizCzast-2017/WyklSpec-4-theory-2017.pdf from (http://th-www.if.uj.edu.pl/~erichter/dydaktyka/Dydaktyka2017/SpecFizCzast-2017/)
6. Collider Physics at Hera, M.Klein and R.Yoshida

Particle Physics
Toni Baroncelli
Haiping Peng
USTC

End of Deep Inelastic Scattering

Toni Baroncelli: Deep Inelastic Scattering

$$ep: \quad e^{\pm} + p \rightarrow e^{\pm} + X^+$$
$$\mu p: \quad \mu^{\pm} + p \rightarrow \mu^{\pm} + X^+$$
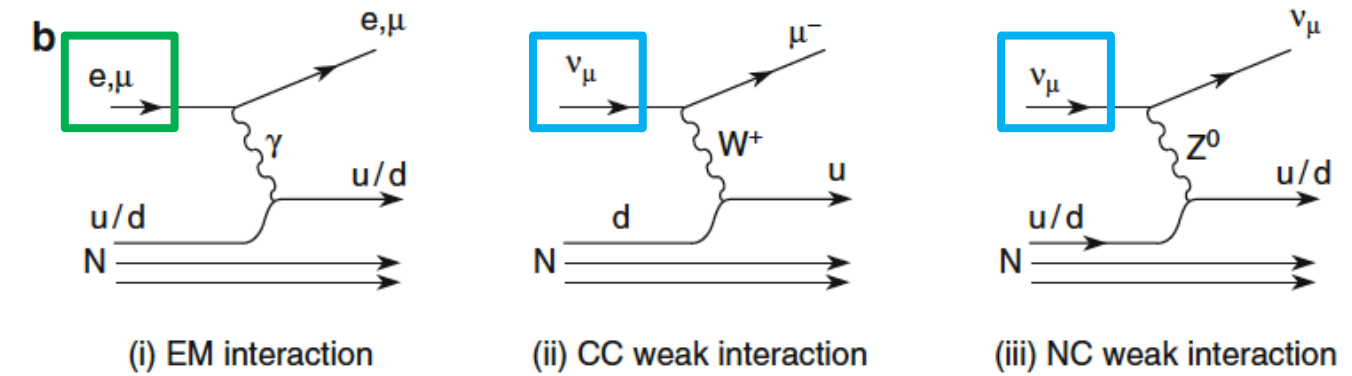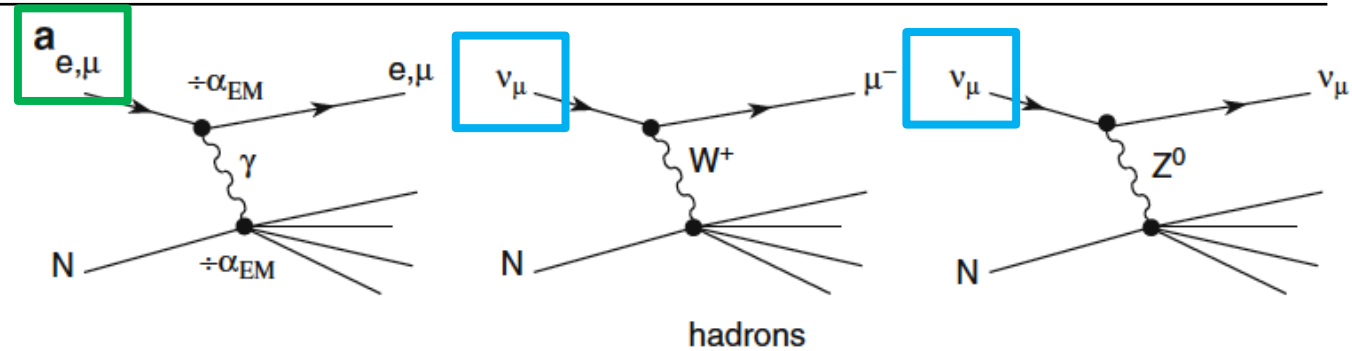
Incoming particle	Outgoing particle	Parton involved
$lepton^+$	$lepton^+$	Sea & valence
$lepton^-$	$lepton^-$	
ν_μ	μ^-	$d \rightarrow u \, (CC, W^+)$
ν_μ	μ^-	$\bar{u} \rightarrow \bar{d} \, (CC, W^-)$
ν_μ	ν_μ	ν_μ
ν_μ		
$\overline{\nu_\mu}$		

(i) EM interaction   (ii) CC weak interaction   (iii) NC weak interaction

$$\nu_\mu p(CC): \nu_\mu + p \rightarrow \mu^- + X^{++}, \quad \bar{\nu}_\mu + p \rightarrow \mu^+ + X^0$$
$$\nu_\mu p(NC): \nu_\mu + p \rightarrow \nu_\mu + X^+, \quad \bar{\nu}_\mu + p \rightarrow \bar{\nu}_\mu + X^+$$
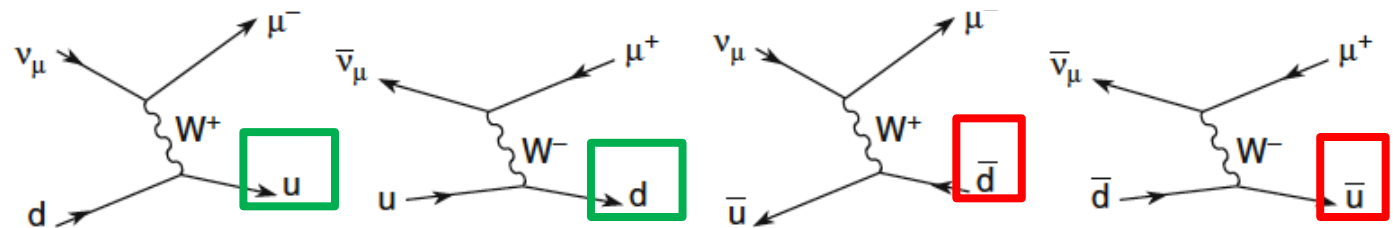
Bjorken scaling : Callan-Gross formula

a) the cross sections of pointlike spin ½ particle of mass m (à la Rosenbluth with $G_E=G_M=1$) :

$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{\text{point-like, spin1/2}} = \frac{12\alpha^2 E'^2}{EQ^4}\left[\cos^2\frac{\theta}{2}+2\tau\sin^2\frac{\theta}{2}\right];$$

$$\left[\frac{d^2\sigma}{d\Omega dE'}\right]_{DIS} = \frac{4\alpha^2 E'^2}{Q^4}\left[W_2\cos^2\frac{\theta}{2}+2W_1\sin^2\frac{\theta}{2}\right];$$

$$W_2\cos^2\frac{\theta}{2}+2W_1\sin^2\frac{\theta}{2} = \frac{3}{E}\left[\cos^2\frac{\theta}{2}+2\tau\sin^2\frac{\theta}{2}\right];$$

$$W_1 = \frac{3\tau}{E}; \qquad W_2 = \frac{3}{E}; \qquad \frac{W_1}{W_2} = \frac{F_1(x)}{F_2(x)}\frac{\nu}{M} = \tau = \frac{Q^2}{4m^2};$$

b) from the kinematics of elastic scattering of point-like constituents of mass m :

$$Q^2 = 2m\nu = 2M\nu x \rightarrow m = xM;$$

$$\frac{F_1(x)}{F_2(x)} = \frac{Q^2}{4m^2}\frac{M}{\nu} = \frac{2m\nu}{4m^2}\frac{M}{\nu} = \frac{M}{2m} = \frac{1}{2x}; \quad \rightarrow$$

$$\boxed{2xF_1(x) = F_2(x).}$$

Warnings :

- don't confuse M (the nucleon) with m (the constituent);

- don't confuse the inelastic scattering ep with the elastic scattering eq;

- x refers to the inelastic case;

- an hypothetical [*nobody uses it*] variable ξ, analogous to x but for the constituent scattering; in this case, $Q^2=2m\nu\xi$, $\xi = $ **1**;

- we learn that x = m/M [REMEMBER].
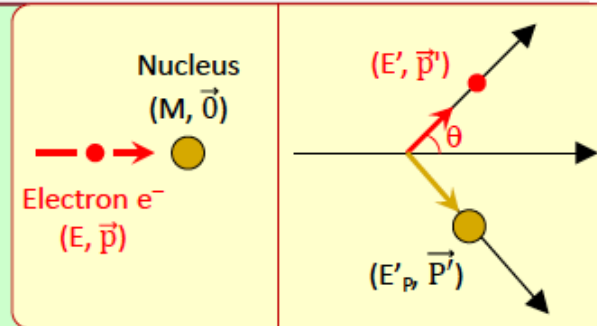
Prof. Paolo Bagnaia
University of Rome "La Sapienza"

Toni Baroncelli: Deep Inelastic Scattering

(In)elastic scattering e-N : kinematics

3/8

$$\begin{cases} e^-_{init} & (E, \vec{p}; m); \\ N_{init} & (M, \vec{0}; M); \end{cases} \qquad \begin{cases} e^-_{fin} & (E', \vec{p}'; m); \\ N_{fin} & (E'_P, \vec{P}'_P; M); \end{cases}$$

4-momentum conservation
$$\begin{cases} E + M = E' + E'_P \rightarrow E'_P = E + M - E'; \\ \vec{p} + \vec{0} = \vec{p}' + \vec{P}'_P \rightarrow \vec{P}'_P = \vec{p} - \vec{p}'; \end{cases}$$

Square and subtract
$$\begin{cases} \left(E'_P\right)^2 = E^2 + M^2 + E'^2 + 2EM - 2EE' - 2ME'; \\ \left(\vec{P}'_P\right)^2 = p^2 + p'^2 - 2pp'\cos\theta; \\ \left(E'_P\right)^2 - \left(\vec{P}'_P\right)^2 = M^2 = E^2 + M^2 + E'^2 + 2EM - 2EE' - 2ME' - p^2 - p'^2 + 2pp'\cos\theta; \end{cases}$$

Ultra-relativistic approx.
$(m_e \ll E, E') \rightarrow (p \approx E, p' \approx E')$
$$\begin{cases} \cancel{M^2} = \cancel{E^2} + \cancel{M^2} + \cancel{E'^2} + 2EM - 2EE' - 2ME' - \cancel{E^2} - \cancel{E'^2} + 2EE'\cos\theta; \\ 0 = 2EM - 2EE' - 2ME' + 2EE'\cos\theta; \end{cases}$$

$$E' = \frac{EM}{E + M - E\cos\theta} = \frac{EM}{E(1 - \cos\theta) + M} \qquad \text{q.e.d.}$$

NB – The reaction is planar (why?). The final state is defined by 6 variables. There are 3 (E, p conservations) and 2 ($m^2 = E^2 - p^2$) rules. Therefore : 6-5=1 independent variable.

Toni Baroncelli: Deep Inelastic Scattering

The comparison between these two formulas

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{point} \\ \text{spin } 1/2}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[1 + 2\tau \tan^2 \frac{\theta}{2}\right] \qquad \tau = \frac{Q^2}{4M^2c^2} \cdot$$

$$\frac{d^2\sigma}{d\Omega\, dE'} = \left(\frac{d\sigma}{d\Omega}\right)^*_{\text{Mott}} \left[W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right] \quad \begin{array}{l} F_1(x, Q^2) = Mc^2\, W_1(Q^2,\nu) \\ F_2(x, Q^2) = \nu\, W_2(Q^2,\nu)\,. \end{array}$$

Gives the Callan-Gross relation: $2xF_1(x) = F_2(x)\,.$
This relation is expected to be valid ONLY for spin ½ target objects. This is verified experimentally as shown in the figure to the right.

Experimental evidence tells us that
- Nucleons are made of point-like objects
- These point-like objects carry spin 1/2