Research of the production behavior of $\Lambda_c^+ \overline{\Lambda}_c^$ near the threshold at BESIII

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Briefly, we want to know how much does the Coulomb interaction and strong interaction affect the production behavior of $\Lambda_c^+ \bar{\Lambda}_c^-$ from the annihilation of electron and positron pair.

Study for production near threshold of $p\bar{p}$:



The cross section of $e^+e^- \rightarrow p\bar{p}$ near threshold is about 850 pb.

Study for production near threshold of $p\bar{p}$:

$$\sigma_{p\bar{p}} = \frac{4\pi\alpha^2\beta C}{3q^2} \left[\left| G_M^p(q^2) \right|^2 + \frac{2m_p^2}{q^2} \left| G_E^p(q^2) \right|^2 \right]$$

$$C = \frac{\pi\alpha}{\beta} \times R = \frac{\pi\alpha}{\beta} \frac{\sqrt{1-\beta^2}}{1-\exp(-\pi\alpha/\beta)}$$
(a)

or

$$C = \frac{\pi\alpha}{\beta} \times R_s = \frac{\pi\alpha}{\beta} \frac{\sqrt{1-\beta^2}}{1-\exp(-\pi\alpha_s/\beta)}$$
(b)

where $\alpha = 1/137$, $\alpha_s = 0.5$.

At threshold, the formula of production cross section can be written as follows:



Picture (a) for no strong interaction correction, (b) for taking appropriate strong interaction correction into consideration.

The result indicates:

- * appropriate Coulomb interaction and strong interaction should be considered when we explain the production behavior of $e^+e^- \rightarrow p\bar{p}$ near threshold.
- * form factor $|G| \approx 1$ at threshold for the process $e^+e^- \rightarrow p\bar{p}$.

So, what if we study the process $e^+e^- \rightarrow \Lambda_c^+ \overline{\Lambda_c^-}$, and whether we can obtain further information about the form factors of the particle Λ_c^+ ?

Data sets

- Boss version: 6.6.4.p01
- Data sample(online Luminosity)
 - * 4.575 GeV, 42.0 pb⁻¹
 - * 4.580 GeV, 7.94 pb⁻¹
 - * 4.590 GeV, 7.65 pb⁻¹
 - * 4.600 GeV, 506 pb⁻¹
- MC sample

* MC Generator: KKMC + BesEvtGen * inclusive Λ_c^+ channel, in PHSP * inclusive Λ_c^- channel, in PHSP

Decay mode

Mode	Decay modes	Br(modeN)/Br(mode1)	Branching fraction
1	$\Lambda_c^+ \rightarrow p^+ \pi^+ K^-$	1	(6.84±0.36)%
2	$\Lambda_c^+\!\!\rightarrow p^+K_s^{\ 0}\text{, } K_s^{\ 0}\!\rightarrow\pi^+\pi^-$	$(0.47 \pm 0.04) \times 50\% \times 69.2\%$	(1.11±0.11)%
3	$\Lambda_c^+\!\to\Lambda\pi^+$, $\Lambda\to p^+\pi^-$	$(0.20 \pm 0.02) \times 63.9\%$	$(0.87 \pm 0.10)\%$
4	$\Lambda_c^+\!\!\rightarrow p^+\pi^+K^{\!-}\!\pi^0\!\!,\ \pi^0\!\rightarrow\gamma\gamma$	$(0.67 \pm 0.12) \times 98.8\%$	(4.53±0.84)%
5	$\Lambda_c^+\!\!\rightarrow p^+K_s^{\ 0}\pi^0\!\!,\ K_s^{\ 0}\!\rightarrow\pi^+\pi^-\!\!,\ \pi^0\!\rightarrow\gamma\gamma$	$(0.66 \pm 0.09) \times 50\% \times 69.2\% \times 98.8\%$	$(1.54 \pm 0.23)\%$
6	$\Lambda_c^+\!\to\Lambda\pi^+\pi^0$, $\Lambda\to p^+\pi^-\!\!,\ \pi^0\to\gamma\gamma$	$(0.73 \pm 0.18) \times 63.9\% \times 98.8\%$	$(3.15 \pm 0.79)\%$
7	$\Lambda_c^+ \rightarrow p^+ K_s^{~0} \pi^+ \pi^\text{-}\text{, } K_s^{~0} \rightarrow \pi^+ \pi^\text{-}$	$(0.51 \pm 0.06) \times 50\% \times 69.2\%$	$(1.21 \pm 0.16)\%$
8	$\Lambda_c^+ \to \Lambda \pi^+ \pi^+ \pi^\text{-}$, $\Lambda \to p^+ \pi^\text{-}$	$(0.52 \pm 0.03) \times 63.9\%$	$(2.27 \pm 0.18)\%$
9	$\Lambda_c^+\to\Sigma^0\pi^+$, $\Sigma^0\to\Lambda\gamma$, $\Lambda\to p^+\pi^-$	$(0.20 \pm 0.04) \times 63.9\%$	$(0.87 \pm 0.18)\%$
10	$\Lambda_c^+ \to \Sigma^+ \pi^+ \pi^\text{-}, \Sigma^+ \to p \pi^0$, $\pi^0 \to \gamma \gamma$	$(0.69\pm0.08) \times 51.6\% \times 98.8\%$	(2.41±0.31)%

For instance,
$$Br(mode 2) = \left(\frac{\Gamma_{mode 2}}{\Gamma_{mode 1}}\right) \times \left(\frac{K_s^0}{\overline{K}^0}\right) \times Br.(K_s^0 \to \pi^+\pi^-)$$

The errors was obtained according to error transfer formula.

- Charged tracks: $|\cos\theta| \le 0.93$, $|V_r| \le 1 \text{ cm}$, $|V_z| \le 10 \text{ cm}$
- Neutral track: 0 < T < 14, $E_{barrel} > 25$ MeV, $E_{endcap} > 50$ MeV
- PID identification: proton, kaon, pion
- π^0 candidates: $|M_{\gamma\gamma} M_{\pi^0}| < 0.06$ GeV, $\chi^2_{1c} < 50$
- K_{s}^{0} candidates: L/Lerr > 2, $|M_{\pi^{+}\pi^{-}} M_{K_{s}^{0}}| \le 5\sigma$
- Λ candidates: L/Lerr > 2, $|M_{p\pi^-} M_{\Lambda}| \le 5\sigma$

The variables beam-constrained mass M_{BC} and energy difference ΔE are used to identify the signals, which defined as follows:

$$M_{BC} = \sqrt{E_{beam}^2 - |\vec{p}_{\Lambda_c^+}|^2}$$

$$\Delta E = E_{\Lambda_c^+} - E_{beam}$$

Here $\vec{p}_{\Lambda_c^+}$ and $E_{\Lambda_c^+}$ are the total momentum and energy of Λ_c^+ candidate, and E_{beam} is the beam energy. Only the Λ_c^+ candidates with the least $|\Delta E|$ will be kept.

ΔE cuts(MeV) for each mode of each energy points:

mode	4.575	4.580	4.590	4.600
1	4.0	4.1	4.7	4.6
2	5.2	5.4	5.2	5.4
3	5.5	4.9	5.1	5.1
4	7.5	8.1	8.1	7.4
5	6.8	6.6	6.6	6.0
6	5.5	5.8	6.1	5.4
7	2.6	3.0	3.3	7.9
8	3.1	3.7	3.8	4.0
9	5.8	5.0	6.0	6.5
10	4.8	5.5	6.5	6.7

We used the criteria fabs(ΔE) $\leq 5\sigma$ to constrain the Λ_c^+ candidates.

Fit result

We fitted the invariance mass of these intermediate states of MC data respectively to obtain resolutions of them.



The intermediate states mass window for each mode (weighted average for K_s and Λ):

mode	intermediate state	σ(MeV)
1, 4	None	
2, 5, 7	K _s	4.1
3, 6, 8	Λ	1.8
9	Σ^0	6.7
10	Σ^+	8.6

We used the criteria fabs($M_{intermediate} - M_{PDG}$) $\leq 5\sigma$ to constrain the Λ_c^+ candidates, and the result of 4.600 was applied to other energy points directly since there is no remarkable difference between them.

Mass window implemented to intermediate states of real data:



background study

The contribution of $q\bar{q}$ and DD events to M_{bc} makes up main part of the background of M_{bc} . But no enhancement around Λ_c^+ is observed.



background study

The sideband of the intermediate states are also used to estimate background. The Sideband region for this intermediate states are:

- * K_s^0 (mode2, 5, 7): 0.45645 < $M_{K_s^0}$ < 0.47700 && 0.52000 < $M_{K_s^0}$ < 0.54055
- * Λ (mode3, 6, 8) : 1.09112 < M_{Λ} < 1.10000 & 1.13000 < M_{Λ} < 1.13888
- * Σ^0 (mode9): 1.12135 < M_{Σ^0} < 1.14500 & 1.23000 < M_{Σ^0} < 1.26365
- * Σ^+ (mode10): 1.1018 < M_{Σ^+} < 1.1450 && 1.2500 < M_{Σ^+} < 1.2932

The range of each mass interval covers 5 resolution.

background study

The distribution of M_{bc} in data at 4.60 GeV, where the hatched histograms are from the sideband of intermediate states.



From the background study above, we can conclude that there is no peaking background in M_{bc} distribution.

fit method

- * The signal is fitted by MC shape convoluted with a Gaussian function, and the mean value and sigma is float in the fitting of data at energy point 4.60
- * The background is described by a third(or second)-order polynomial, and for energy point 4.60, the parameters of the polynomial are float.
- * For other energy points, the parameters of Gaussian function and polynomial is fixed by that of energy point 4.60, since the low statistic.

Ecm = 4.575 GeVA RooPlot of "x" mean = 0.001466 ± 0.000095 nts / (0.0012 80 70 $nbkg = 5.8 \pm 3.6$ nsig = 156 ± 13 sigma = 0.00043 ± 0.00014 60 50 mode1 30 10 2 276 2.28 2.284 2.286 2.288 2.278 2.282 M_{BC}(GeV/c²)













A RooPlot of "x"

mean = 0.00152 ± 0.00034

mode

2.28 2.282

2.284 2.286

nbkg = 10.6 ± 4.1

 $nsig = 10.4 \pm 4.0$

2)

(0.001

tts

2.276 2.278





Ecm = 4.580 GeV













A RooPlot of "x"



M_{BC}(GeV/c²)

A RooPlot of "x"





Ecm = 4.590 GeV



















A RooPlot of "x"





Ecm = 4.575 GeV



M_{RC}(GeV/c²)

Ecm = 4.580 GeV





















Ecm = 4.590 GeV























Ecm = 4.600 GeV





















Systematic error

The systematic uncertainty includes the uncertainties from tracking, PID, ΔE requirement, mass window for intermediate states, fitting method of M_{bc} , ISR correction and luminosity.

The uncertainty of ΔE is estimated by varying the requirement on ΔE from 5σ to 8σ .

The uncertainty of the mass window of the intermediate states is estimated by varying the mass window from 5σ to 8σ .

The uncertainty of the fit of M_{bc} is studied from two aspects, one is changing the fitting range of the M_{bc} , and the second is changing the background shape to 2-order or 3-order polynomial and the largest error is taken as the systematic uncertainty.

Systematic error

This table shows the systematic uncertainty at the energy point 4.60 by tagging Λ_c^+ :

mode	tracking	PID	Ks	Λ	π^0	ΔE	mass win.	fit range	bkg. shape	lum.	total
1	3.0	3.0	0	0	0	0.36	0	6.43	1.54	1.0	7.93
2	1.0	1.0	3.5	0	0	2.44	1.29	6.85	2.22	1.0	8.64
3	1.0	1.0	0	2.5	0	5.86	4.20	1.13	5.30	1.0	9.52
4	3.0	3.0	0	0	1.0	3.18	0	1.81	5.04	1.0	7.67
5	1.0	1.0	3.5	0	1.0	2.70	5.91	5.39	5.29	1.0	10.75
6	1.0	1.0	0	2.5	1.0	3.41	7.80	16.84	3.94	1.0	19.51
7	3.0	3.0	3.5	0	0	3.14	7.85	11.78	2.44	1.0	15.75
8	3.0	3.0	0	2.5	0	7.54	8.80	3.64	1.45	1.0	13.22
9	1.0	1.0	0	2.5	0	0.56	2.84	5.17	0	1.0	6.17
10	3.0	3.0	0	0	1.0	1.45	1.77	9.77	4.96	1.0	12.05

The systematic errors of the reconstruction of Ks and Λ was chosen as 3.5% and 2.5% for the sake of conservative, and it will be analyzed in detail soon.

Systematic error

Two notes :

* the uncertainty of MC model was not given since there are some additional complication.

* Further research on background at mode6 and mode7 will reduce the uncertainty of their fit range.

cross section

The calculation of cross section of each mode:

$$X_{sec}^{i} = \frac{N_{data}^{i}}{Lum \times f_{VP} \times f_{cor} \times Eff.^{i} \times Br.^{i}}$$
(a)

A data set (x_i, y_i) contains *n* elements, where $i = 1, 2, \dots, n$, and a linear function $f(\vec{x}, \vec{\beta}) = \sum_{j=1}^{m} \beta_j \phi_j(\vec{x})$, which contains *m* parameters was used to fit the data. The least square method told us that when

$$\frac{\partial S}{\partial \beta_j} = 0 \qquad \text{for } j = 1, 2, \cdots, m$$

we
$$S = \sum_{i=1}^n r_i^2 = \sum_{i=1}^n [y_i - f(x_i, \vec{\beta})]^2$$

where

will gets its minimum value, and we can obtain that

$$X^T X \hat{\beta} = X^T \hat{y}$$
 or $\hat{\beta} = (X^T X)^{-1} X^T \hat{y}$

and the error of $\vec{\beta}$ can be obtained according the error transfer formula

$$M_{\beta} = (X^{T}X)^{-1} X^{T}M_{y}X(X^{T}X)^{-1}$$

where $M_{\beta} \in \mathbb{R}^{m \times m}$, $M_{y} \in \mathbb{R}^{n \times n}$, is the variance-covariance matrix of $\vec{\beta}$ and \vec{y} respectively, and $X_{ij} = \frac{\partial f(x_i, \vec{\beta})}{\partial \beta_j} = \phi_j(x_i), X \in \mathbb{R}^{n \times m}$, and $\hat{\beta} \in \mathbb{R}^{m \times 1}$, $\hat{y} \in \mathbb{R}^{n \times 1}$ is the parameter set and data set respectively.

However, in the weighted case, we written *S* as

$$S = \sum_{i=1}^{n} W_{ii} r_i^2 = \sum_{i=1}^{n} W_{ii} [y_i - f(x_i, \vec{\beta})]^2$$

where the matrix W denotes the weights matrix, then the least square method told us that

$$\hat{\beta} = (X^T W X)^{-1} X^T W \hat{y}$$
(b)
$$M_{\beta} = (X^T W X)^{-1} X^T W M_v W^T X (X^T W^T X)^{-1}$$
(c)

are the **best liner unbiased estimator** of parameter set $\hat{\beta}$ and its error respectively, when $W = M_{\gamma}^{-1}$ is the case.

As for the case of my analysis, m = 1, and n = 10. The function which we used to fit the data set (x_i, y_i) is $f(\vec{x}, \vec{\beta}) = \sum_{j=1}^{1} \beta_j \phi_j(\vec{x}) = \beta$, so $X_{ij} = \frac{\partial f(x_i, \vec{\beta})}{\partial \beta_j} = 1$, and $X^T = (1, 1, \dots, 1)$. $M_\beta \in \mathbf{R}$, and $M_y \in \mathbf{R}^{10 \times 10}$.

When we choose the weights matrix as the inverse of variance-covariance matrix M_y , that is, $W = M_y^{-1}$, we can obtain the weighted average of cross section and its total error as follow

$$\overline{\beta} \pm \overline{\delta\beta} = \frac{\sum_{j} y_{j} \cdot \sum_{i} \omega_{ij}}{\sum_{j} \sum_{i} \omega_{ij}} \pm \sqrt{\frac{1}{\sum_{j} \sum_{i} \omega_{ij}}}$$
(d)

where ω_{ij} is element of (i, j) position of weights matrix, which will be defined later.

When we want to calculate the error comes from different sources, we can use the original formula (c) by using the variance-covariance matrix of the error comes from different sources, that is

$$\mathbf{M} = (\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X})^{-1} \mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{M}_{\mathbf{i}}\mathbf{W}^{\mathrm{T}}\mathbf{X}(\mathbf{X}^{\mathrm{T}}\mathbf{W}^{\mathrm{T}}\mathbf{X})^{-1} \qquad (\mathbf{e})$$

where M_i is the variance-covariance matrix of i-th error which shares the same definition with that of total error.

In my analysis, this formula can be simplified as

$$\overline{\delta\beta}_{i} = M = \frac{\sum_{i=1}^{i=10} \sum_{j=1}^{j=10} \omega_{i} \omega_{j} m_{ij}}{(\sum_{i=1}^{i-10} \omega_{i})^{2}}$$
(e)

where m_{ij} is the (i, j) position element of M_i , and $\omega_i = \sum_{k=1}^{k=10} \omega_{ik}$, and ω_{ik} is the (i, k) position element of weight matrix W.

Error Matrix

The covariance error matrix M_y is:

$$\begin{pmatrix} \sigma_{T1}^2 & Cov(x_1, x_2) & \cdots & Cov(x_1, x_n) \\ Cov(x_2, x_1) & \sigma_{T2}^2 & \cdots & Cov(x_2, x_n) \\ \cdots & \cdots & \cdots & \cdots \\ Cov(x_n, x_1) & Cov(x_n, x_2) & \cdots & \sigma_{Tn}^2 \end{pmatrix}$$

$$\sigma_{Ti}^{2} = \sigma_{i}^{2}(stat.) + \sigma_{i}^{2}(sys1.) + \sigma_{i}^{2}(sys2.) + \cdots$$
$$Cov(x_{i}, x_{j}) = x_{i} \cdot \varepsilon_{ij} \cdot x_{j} \cdot \varepsilon_{ji}$$

Where σ_{Ti} stands for the total uncertainty in the measurement of mode i, and $\sigma_i(stat.)$ and $\sigma_i(sysj.)$ are the statistical error and the systematic error for source j in mode i respectively. $Cov(x_i, x_j)$ is the covariance systematic error between mode i and j. x_i is The measured value of mode i, and $\varepsilon_{ij} = \varepsilon_{ji}$ is the common relative systematic error (in percentage) between mode i and j.

Weighted average

As for the variance-covariance matrix of i-th error source(i.e. M_i), the only difference is that $\varepsilon_{ij} = \varepsilon_{ji}$ just contains the common relative error which comes from source i only. We can find that

$$M_{y} = \sum_{i} M_{i}$$
 (f)

The inverse of the covariance matrix M_y^{-1} is:

$/\omega_{11}$	ω_{12}	•••	ω_{1n}
ω_{21}	ω_{22}	•••	ω_{2n}
	•••	•••]
$\setminus \omega_{n1}$	ω_{n2}	•••	$\omega_{nn}/$

The weighted average cross section and the corresponding total uncertainty can be calculated according to formula(d).

According this formula, the production cross section of $\Lambda_c^+ \overline{\Lambda_c^-}$ is 232.22 pb, with the total error 18.01 pb.

Updated cross section(@4600)

Mode	N ⁺ _{total}	Eff. ⁺ (%)	Br.(%)	<i>X</i> ⁺ _{sec} (pb)
1	2662±57	46.95	6.84±0.36	$211.71 \pm 4.53 \pm 16.78 \pm 11.14 \pm 0.00$
2	540 ± 26	47.40	1.11±0.11	$262.13 \pm 12.62 \pm 22.66 \pm 13.80 \pm 22.31$
3	286±21	32.76	0.87 ± 0.10	$256.28 \pm 18.82 \pm 24.39 \pm 13.49 \pm 25.63$
4	813±71	18.30	4.53±0.84	$250.47 \pm 21.87 \pm 19.20 \pm 13.18 \pm 44.86$
5	196±22	13.34	1.54±0.23	$242.04 \pm 27.17 \pm 26.01 \pm 12.74 \pm 33.00$
6	330 ± 45	6.48	3.15 <u>+</u> 0.79	$412.90\pm56.30\pm80.54\pm21.73\pm101.81$
7	287±31	21.90	1.21±0.16	$276.61 \pm 29.88 \pm 43.55 \pm 14.56 \pm 32.54$
8	275 <u>+</u> 52	12.69	2.27 ± 0.18	$243.82 \pm 46.10 \pm 32.24 \pm 12.83 \pm 14.07$
9	205 ± 20	20.41	0.87 ± 0.18	$294.86 \pm 28.77 \pm 18.20 \pm 15.52 \pm 58.97$
10	587±65	19.53	2.41±0.31	$318.52 \pm 35.27 \pm 38.39 \pm 16.76 \pm 36.93$
summary				$232.22 \pm 4.56 \pm 11.53 \pm 12.22 \pm 4.63$

The combined error comes from absolute branch fraction of these multiple modes is $\sqrt{4.63^2 + 12.22^2}/232.22 = 13.07/232.22 = 5.63\%$

$|G_E/G_M|$ ratio measurement at 4.6 GeV

Angular distribution:

Data of mode $pK^{-}\pi^{+}$ only.

The angular distribution is then fitted by $1 + \alpha \cos^2 \theta$.



$|G_E/G_M|$ ratio measurement at 4.6 GeV

The differential cross section can be expressed as:

$$\frac{d\sigma_{Born}(s)}{d\Omega} = \frac{\alpha^2 \beta C}{4s} \left[|G_M(s)|^2 \left(1 + \cos^2 \theta_{\Lambda_c} \right) + \frac{4m_{\Lambda_c}^2}{s} |G_E(s)|^2 (\sin^2 \theta_{\Lambda_c}) \right]$$

 $|G_E/G_M|$ ratio can be described by the following:

$$\left|\frac{G_E}{G_M}\right|^2 = (1-\alpha)/(\frac{4m_{A_c}^2}{s}\alpha + \frac{4m_{A_c}^2}{s})$$

 $|G_E/G_M|$ ratio is calculated to be 1.40±0.13 at 4.60 GeV.

$|G_E/G_M|$ ratio measurement at 4.6 GeV



- * At threshold (4.575 GeV), we assume $|G_E/G_M|=1$.
- * At 4.58, 4.59 GeV, $|G_E/G_M|$ is obtained by interpolation.
- * The result of $|G_E/G_M|$ ratio is needed in the line-shape fitting.

fit the line-shape

The function of non-resonant (NonR) contribution can be parameterized as:

$$NonR = \frac{4\pi\alpha^2\beta C}{3q^2} [|G_M(q^2)|^2 + \frac{1}{2\tau}|G_E(q^2)|^2]$$

where $\beta{=}\sqrt{(1{-}4m^2{}_{\Lambda c}\!/q^2)}$, $\tau{=}q^2/4m^2{}_{\Lambda c}\!.$

The Coulomb factor $C = \epsilon \times R$, $\epsilon = \pi \alpha / \beta$ is the enhancement factor, and R is the resummation factor. In traditional prediction: $R = \sqrt{1 - \beta^2} / (1 - e^{-\pi \alpha / \beta})$

From the prediction by R. Baldini Ferroli, S. Pacetti

$$C = \frac{\pi\alpha}{\beta} \times \frac{R_s}{R_s} = \frac{\pi\alpha}{\beta} \frac{\sqrt{1-\beta^2}}{1-\exp(-\pi\alpha_s/\beta)}$$

where the coupling constant $\alpha = 1/137$, $\alpha_s = 0.5$.

fit the line-shape

The fitting results with previous formula, $|G_E/G_M|$ ratio is inputted in the fitting, G_M is the only fitting parameter :



- * Using the updated R_s to fit line-shape, the fit status is good, and $|G_M|$ is 1.137 ±0.039.
- * Using the traditional *R* to fit line-shape, the fit status is bad, and $|G_M|$ is 0.5436 ±0.0199.

Summary

- We present the measurement of $\Lambda_c^+ \overline{\Lambda}_c^-$ production cross section at threshold by tagging 10 decay modes, the systematic uncertainty at 4.6 GeV is studied by tagging Λ_c^+ , and it is about 7.5%.
- The $|G_E/G_M|$ ratio at 4.60 GeV is obtained, to be 1.40 ± 0.13 , which is significantly larger than 1.
- The line-shape of $\Lambda_c^+ \overline{\Lambda}_c^-$ production is fitted, and the result favors the prediction with $R \to R_s$ in the fitting. The fitted form factor $|G_M|$ is 1.14 ± 0.04 .

