Research of the production behavior of $\Lambda_{c}^{+} \bar{\Lambda}_{c}^{-}$ near the threshold at BESIII

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## Outline

- Motivation
- Event selection
- Fit results
- Systematical uncertainty
- Calculations of cross section
- Summary


## Why the cross section near threshold?

Briefly, we want to know how much does the Coulomb interaction and strong interaction affect the production behavior of $\Lambda_{c}^{+} \bar{\Lambda}_{c}^{-}$from the annihilation of electron and positron pair.

## Why the cross section near threshold?

Study for production near threshold of $p \bar{p}$ :


The cross section of $e^{+} e^{-} \rightarrow p \bar{p}$ near threshold is about 850 pb .

## Why the cross section near threshold?

Study for production near threshold of $p \bar{p}$ :

$$
\begin{gather*}
\sigma_{p \bar{p}}=\frac{4 \pi \alpha^{2} \beta C}{3 q^{2}}\left[\left|G_{M}^{p}\left(q^{2}\right)\right|^{2}+\frac{2 m_{p}^{2}}{q^{2}}\left|G_{E}^{p}\left(q^{2}\right)\right|^{2}\right] \\
C=\frac{\pi \alpha}{\beta} \times R=\frac{\pi \alpha}{\beta} \frac{\sqrt{1-\beta^{2}}}{1-\exp (-\pi \alpha / \beta)} \tag{a}
\end{gather*}
$$

or

$$
\begin{equation*}
C=\frac{\pi \alpha}{\beta} \times R_{s}=\frac{\pi \alpha}{\beta} \frac{\sqrt{1-\beta^{2}}}{1-\exp \left(-\pi \alpha_{s} / \beta\right)} \tag{b}
\end{equation*}
$$

where $\alpha=1 / 137, \alpha_{s}=0.5$.

## Why the cross section near threshold?

At threshold, the formula of production cross section can be written as follows:

$$
\begin{equation*}
\sigma_{p \bar{p}}\left(q^{2}\right)=850 \times \frac{4 m_{p}^{2}}{q^{2}} \times R \quad(\mathrm{pb}) \text { or } \sigma_{p \bar{p}}\left(q^{2}\right)=850 \times \frac{4 m_{p}^{2}}{q^{2}} \times R_{S} \tag{pb}
\end{equation*}
$$




Picture (a) for no strong interaction correction, (b) for taking appropriate strong interaction correction into consideration.

## Why the cross section near threshold?

The result indicates:

* appropriate Coulomb interaction and strong interaction should be considered when we explain the production behavior of $e^{+} e^{-} \rightarrow p \bar{p}$ near threshold.
* form factor $|\mathrm{G}| \approx 1$ at threshold for the process $e^{+} e^{-} \longrightarrow p \bar{p}$.

So, what if we study the process $e^{+} e^{-} \rightarrow \Lambda_{c}^{+} \bar{\Lambda}_{c}^{-}$, and whether we can obtain further information about the form factors of the particle $\Lambda_{c}^{+}$?

## Data sets

- Boss version: 6.6.4.p01
- Data sample(online Luminosity)

$$
\begin{aligned}
& * 4.575 \mathrm{GeV}, 42.0 \mathrm{pb}^{-1} \\
& \text { * } 4.580 \mathrm{GeV}, 7.94 \mathrm{pb}^{-1} \\
& \text { * } 4.590 \mathrm{GeV}, 7.65 \mathrm{pb}^{-1} \\
& * 4.600 \mathrm{GeV}, 506 \mathrm{pb}^{-1}
\end{aligned}
$$

- MC sample
* MC Generator: KKMC + BesEvtGen
* inclusive $\Lambda_{c}^{+}$channel, in PHSP
* inclusive $\Lambda_{c}^{-}$channel, in PHSP


## Decay mode

| Mode | Decay modes | $\operatorname{Br}($ modeN $) / \operatorname{Br}($ mode1) | Branching fraction |
| :---: | :--- | :--- | :--- |
| 1 | $\Lambda_{\mathrm{c}}^{+} \rightarrow \mathrm{p}^{+} \pi^{+} \mathrm{K}^{-}$ | 1 | $(6.84 \pm 0.36) \%$ |
| 2 | $\Lambda_{\mathrm{c}}^{+} \rightarrow \mathrm{p}^{+} \mathrm{K}_{\mathrm{s}}{ }^{0}, \mathrm{~K}_{\mathrm{s}}{ }^{0} \rightarrow \pi^{+} \pi^{-}$ | $(0.47 \pm 0.04) \times 50 \% \times 69.2 \%$ | $(1.11 \pm 0.11) \%$ |
| 3 | $\Lambda_{\mathrm{c}}^{+} \rightarrow \Lambda \pi^{+}, \Lambda \rightarrow \mathrm{p}^{+} \pi^{-}$ | $(0.20 \pm 0.02) \times 63.9 \%$ | $(0.87 \pm 0.10) \%$ |
| 4 | $\Lambda_{\mathrm{c}}^{+} \rightarrow \mathrm{p}^{+} \pi^{+} \mathrm{K}^{-} \pi^{0}, \pi^{0} \rightarrow \gamma \gamma$ | $(0.67 \pm 0.12) \times 98.8 \%$ | $(4.53 \pm 0.84) \%$ |
| 5 | $\Lambda_{\mathrm{c}}^{+} \rightarrow \mathrm{p}^{+} \mathrm{K}_{\mathrm{s}}{ }^{0} \pi^{0}, \mathrm{~K}_{\mathrm{s}}{ }^{0} \rightarrow \pi^{+} \pi^{-}, \pi^{0} \rightarrow \gamma \gamma$ | $(0.66 \pm 0.09) \times 50 \% \times 69.2 \% \times 98.8 \%$ | $(1.54 \pm 0.23) \%$ |
| 6 | $\Lambda_{\mathrm{c}}^{+} \rightarrow \Lambda \pi^{+} \pi^{0}, \Lambda \rightarrow \mathrm{p}^{+} \pi^{-}, \pi^{0} \rightarrow \gamma \gamma$ | $(0.73 \pm 0.18) \times 63.9 \% \times 98.8 \%$ | $(3.15 \pm 0.79) \%$ |
| 7 | $\Lambda_{\mathrm{c}}^{+} \rightarrow \mathrm{p}^{+} \mathrm{K}_{\mathrm{s}}{ }^{0} \pi^{+} \pi^{-}, \mathrm{K}_{\mathrm{s}}{ }^{0} \rightarrow \pi^{+} \pi^{-}$ | $(0.51 \pm 0.06) \times 50 \% \times 69.2 \%$ | $(1.21 \pm 0.16) \%$ |
| 8 | $\Lambda_{\mathrm{c}}^{+} \rightarrow \Lambda \pi^{+} \pi^{+} \pi^{-}, \Lambda \rightarrow \mathrm{p}^{+} \pi^{-}$ | $(0.52 \pm 0.03) \times 63.9 \%$ | $(2.27 \pm 0.18) \%$ |
| 9 | $\Lambda_{\mathrm{c}}^{+} \rightarrow \Sigma^{0} \pi^{+}, \Sigma^{0} \rightarrow \Lambda \gamma, \Lambda \rightarrow \mathrm{p}^{+} \pi^{-}$ | $(0.20 \pm 0.04) \times 63.9 \%$ | $(0.87 \pm 0.18) \%$ |
| 10 | $\Lambda_{\mathrm{c}}^{+} \rightarrow \Sigma^{+} \pi^{+} \pi^{-}, \Sigma^{+} \rightarrow \mathrm{p} \pi^{0}, \pi^{0} \rightarrow \gamma \gamma$ | $(0.69 \pm 0.08) \times 51.6 \% \times 98.8 \%$ | $(2.41 \pm 0.31) \%$ |

For instance, $\quad \operatorname{Br}($ mode 2$)=\left(\frac{\Gamma_{\text {mode } 2}}{\Gamma_{\text {mode } 1}}\right) \times\left(\frac{K_{s}^{0}}{\bar{K}^{0}}\right) \times B r .\left(K_{s}^{0} \rightarrow \pi^{+} \pi^{-}\right)$
The errors was obtained according to error transfer formula.

## Event selection

- Charged tracks: $|\cos \theta|<=0.93,\left|V_{r}\right|<1 \mathrm{~cm},\left|V_{z}\right|<10 \mathrm{~cm}$
- Neutral track: $0<\mathrm{T}<14, E_{\text {barrel }}>25 \mathrm{MeV}, E_{\text {endcap }}>50 \mathrm{MeV}$
- PID identification: proton, kaon, pion
- $\pi^{0}$ candidates: $\left|M_{\gamma \gamma}-M_{\pi^{0}}\right|<0.06 \mathrm{GeV}, \chi_{1 c}^{2}<50$
- $\mathrm{K}_{\mathrm{s}}{ }^{0}$ candidates: L/Lerr $>2,\left|M_{\pi^{+} \pi^{-}}-M_{K_{s}^{0}}\right|<=5 \sigma$
- $\Lambda$ candidates: $\mathrm{L} / \mathrm{Lerr}>2,\left|M_{p \pi^{-}}-M_{\Lambda}\right|<=5 \sigma$


## Event selection

The variables beam-constrained mass $M_{B C}$ and energy difference $\Delta E$ are used to identify the signals, which defined as follows:

$$
\begin{gathered}
M_{B C}=\sqrt{E_{\text {beam }}^{2}-\left|\vec{p}_{\Lambda_{c}^{+}}\right|^{2}} \\
\Delta E=E_{\Lambda_{c}^{+}}-E_{\text {beam }}
\end{gathered}
$$

Here $\vec{p}_{\Lambda_{c}^{+}}$and $E_{\Lambda_{c}^{+}}$are the total momentum and energy of $\Lambda_{c}^{+}$candidate, and $E_{\text {beam }}$ is the beam energy. Only the $\Lambda_{c}^{+}$candidates with the least $|\Delta \mathrm{E}|$ will be kept.

## Event selection

$\Delta \mathrm{E}$ cuts $(\mathrm{MeV})$ for each mode of each energy points:

| mode | 4.575 | 4.580 | 4.590 | 4.600 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4.0 | 4.1 | 4.7 | 4.6 |
| 2 | 5.2 | 5.4 | 5.2 | 5.4 |
| 3 | 5.5 | 4.9 | 5.1 | 5.1 |
| 4 | 7.5 | 8.1 | 8.1 | 7.4 |
| 5 | 6.8 | 6.6 | 6.6 | 6.0 |
| 6 | 5.5 | 5.8 | 6.1 | 5.4 |
| 7 | 2.6 | 3.0 | 3.3 | 7.9 |
| 8 | 3.1 | 3.7 | 3.8 | 4.0 |
| 9 | 5.8 | 5.0 | 6.0 | 6.5 |
| 10 | 4.8 | 5.5 | 6.5 | 6.7 |

We used the criteria fabs $(\Delta \mathrm{E}) \leq 5 \sigma$ to constrain the $\Lambda_{c}^{+}$candidates.

## Fit result

We fitted the invariance mass of these intermediate states of MC data respectively to obtain resolutions of them.


## Event selection

The intermediate states mass window for each mode (weighted average for $K_{s}$ and $\Lambda$ ):

| mode | intermediate state | $\sigma(\mathrm{MeV})$ |
| :---: | :---: | :---: |
| 1,4 | None | ---- |
| $2,5,7$ | $K_{S}$ | 4.1 |
| $3,6,8$ | $\Lambda$ | 1.8 |
| 9 | $\Sigma^{0}$ | 6.7 |
| 10 | $\Sigma^{+}$ | 8.6 |

We used the criteria fabs $\left(M_{\text {intermediate }}-M_{P D G}\right) \leq 5 \sigma$ to constrain the $\Lambda_{c}^{+}$candidates, and the result of 4.600 was applied to other energy points directly since there is no remarkable difference between them.

## Event selection

Mass window implemented to intermediate states of real data:


## background study

The contribution of $q \bar{q}$ and DD events to $M_{b c}$ makes up main part of the background of $M_{b c}$. But no enhancement around $\Lambda_{c}^{+}$is observed.


## background study

The sideband of the intermediate states are also used to estimate background. The Sideband region for this intermediate states are:

$$
\begin{aligned}
& * K_{s}^{0}(\operatorname{mode} 2,5,7): 0.45645<M_{K_{S}^{0}}<0.47700 \& \& 0.52000<M_{K_{S}^{0}}<0.54055 \\
& * \Lambda(\operatorname{mode} 3,6,8): 1.09112<M_{\Lambda}<1.10000 \& \& 1.13000<M_{\Lambda}<1.13888 \\
& * \Sigma^{0}(\operatorname{mode} 9): 1.12135<M_{\Sigma^{0}}<1.14500 \& \& 1.23000<M_{\Sigma^{0}}<1.26365 \\
& * \Sigma^{+}(\text {mode10 }): 1.1018<M_{\Sigma^{+}}<1.1450 \& \& 1.2500<M_{\Sigma^{+}}<1.2932
\end{aligned}
$$

The range of each mass interval covers 5 resolution.

## background study

The distribution of $M_{b c}$ in data at 4.60 GeV , where the hatched histograms are from the sideband of intermediate states.


From the background study above, we can conclude that there is no peaking background in $M_{b c}$ distribution.

## fit method

* The signal is fitted by MC shape convoluted with a Gaussian function, and the mean value and sigma is float in the fitting of data at energy point 4.60
* The background is described by a third(or second)-order polynomial, and for energy point 4.60 , the parameters of the polynomial are float.
* For other energy points, the parameters of Gaussian function and polynomial is fixed by that of energy point 4.60 , since the low statistic.


## Fit result (tagging $\Lambda_{c}^{+}$)

## $\mathrm{Ecm}=4.575 \mathrm{GeV}$






A Rooplot of " x "


A Rooplot of " x "




## Fit result (tagging $\Lambda_{c}^{+}$)

## $\mathrm{Ecm}=4.580 \mathrm{GeV}$





A Rooplot of " x "





## Fit result (tagging $\Lambda_{c}^{+}$)

## $\mathrm{Ecm}=4.590 \mathrm{GeV}$






## Fit result (tagging $\Lambda_{c}^{+}$)

## $\mathrm{Ecm}=4.600 \mathrm{GeV}$












## Fit result (tagging $\Lambda_{c}^{-}$)

## $\mathrm{Ecm}=4.575 \mathrm{GeV}$




A Rooplot of " x "

## A Rooplot of " x "




A Rooplot of " $\mathrm{xn}^{\prime \prime}$


## Fit result (tagging $\Lambda_{c}^{-}$)

## $\mathrm{Ecm}=4.580 \mathrm{GeV}$



## Fit result (tagging $\Lambda_{c}^{-}$)

## $\mathrm{Ecm}=4.590 \mathrm{GeV}$



## Fit result (tagging $\Lambda_{c}^{-}$)

## $\mathrm{Ecm}=4.600 \mathrm{GeV}$











## Systematic error

The systematic uncertainty includes the uncertainties from tracking, PID, $\Delta E$ requirement, mass window for intermediate states, fitting method of $M_{b c}$, ISR correction and luminosity.

The uncertainty of $\Delta E$ is estimated by varying the requirement on $\Delta E$ from $5 \sigma$ to $8 \sigma$.

The uncertainty of the mass window of the intermediate states is estimated by varying the mass window from $5 \sigma$ to $8 \sigma$.

The uncertainty of the fit of $M_{b c}$ is studied from two aspects, one is changing the fitting range of the $M_{b c}$, and the second is changing the background shape to 2 -order or 3 -order polynomial and the largest error is taken as the systematic uncertainty.

## Systematic error

This table shows the systematic uncertainty at the energy point 4.60 by tagging $\Lambda_{c}^{+}$:

| mode | tracking | PID | Ks | $\Lambda$ | $\pi^{0}$ | $\Delta \mathrm{E}$ | mass <br> win. | fit <br> range | bkg. <br> shape | lum. | total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 3.0 | 3.0 | 0 | 0 | 0 | 0.36 | 0 | 6.43 | 1.54 | 1.0 | 7.93 |
| 2 | 1.0 | 1.0 | 3.5 | 0 | 0 | 2.44 | 1.29 | 6.85 | 2.22 | 1.0 | 8.64 |
| 3 | 1.0 | 1.0 | 0 | 2.5 | 0 | 5.86 | 4.20 | 1.13 | 5.30 | 1.0 | 9.52 |
| 4 | 3.0 | 3.0 | 0 | 0 | 1.0 | 3.18 | 0 | 1.81 | 5.04 | 1.0 | 7.67 |
| 5 | 1.0 | 1.0 | 3.5 | 0 | 1.0 | 2.70 | 5.91 | 5.39 | 5.29 | 1.0 | 10.75 |
| 6 | 1.0 | 1.0 | 0 | 2.5 | 1.0 | 3.41 | 7.80 | 16.84 | 3.94 | 1.0 | 19.51 |
| 7 | 3.0 | 3.0 | 3.5 | 0 | 0 | 3.14 | 7.85 | 11.78 | 2.44 | 1.0 | 15.75 |
| 8 | 3.0 | 3.0 | 0 | 2.5 | 0 | 7.54 | 8.80 | 3.64 | 1.45 | 1.0 | 13.22 |
| 9 | 1.0 | 1.0 | 0 | 2.5 | 0 | 0.56 | 2.84 | 5.17 | 0 | 1.0 | 6.17 |
| 10 | 3.0 | 3.0 | 0 | 0 | 1.0 | 1.45 | 1.77 | 9.77 | 4.96 | 1.0 | 12.05 |

The systematic errors of the reconstruction of Ks and $\Lambda$ was chosen as $3.5 \%$ and $2.5 \%$ for the sake of conservative, and it will be analyzed in detail soon.

## Systematic error

Two notes:

* the uncertainty of MC model was not given since there are some additional complication.
* Further research on background at mode6 and mode7 will reduce the uncertainty of their fit range.


## cross section

The calculation of cross section of each mode:

$$
\begin{equation*}
X_{\text {sec }}^{i}=\frac{N_{\text {data }}^{i}}{L u m . \times f_{V P} \times f_{\text {cor }} \times E f f .^{i} \times B r . .^{i}} \tag{a}
\end{equation*}
$$

## The weighted least squares methods

A data set $\left(x_{i}, y_{i}\right)$ contains $n$ elements, where $i=1,2, \cdots, n$, and a linear function $f(\vec{x}, \vec{\beta})=\sum_{j=1}^{m} \beta_{j} \phi_{j}(\vec{x})$, which contains $m$ parameters was used to fit the data. The least square method told us that when
where

$$
\begin{aligned}
& \frac{\partial S}{\partial \beta_{j}}=0 \quad \text { for } j=1,2, \cdots, m \\
& S=\sum_{i=1}^{n} r_{i}^{2}=\sum_{i=1}^{n}\left[y_{i}-f\left(x_{i}, \vec{\beta}\right)\right]^{2}
\end{aligned}
$$

will gets its minimum value, and we can obtain that

$$
X^{T} X \widehat{\beta}=X^{T} \hat{y} \quad \text { or } \quad \widehat{\beta}=\left(X^{T} X\right)^{-1} X^{T} \hat{y}
$$

and the error of $\vec{\beta}$ can be obtained according the error transfer formula

$$
M_{\beta}=\left(X^{T} X\right)^{-1} X^{T} M_{y} X\left(X^{T} X\right)^{-1}
$$

## The weighted least squares methods

where $\mathrm{M}_{\beta} \in \boldsymbol{R}^{m \times m}, \mathrm{M}_{\mathrm{y}} \in \boldsymbol{R}^{n \times n}$, is the variance-covariance matrix of $\vec{\beta}$ and $\vec{y}$ respectively, and $X_{i j}=\frac{\partial f\left(x_{i}, \vec{\beta}\right)}{\partial \beta_{j}}=\phi_{j}\left(x_{i}\right), X \in \boldsymbol{R}^{n \times m}$, and $\hat{\beta} \in \boldsymbol{R}^{m \times 1}, \hat{\mathrm{y}} \in \boldsymbol{R}^{n \times 1}$ is the parameter set and data set respectively.

However, in the weighted case, we written $S$ as

$$
S=\sum_{i=1}^{n} W_{i i} r_{i}^{2}=\sum_{i=1}^{n} W_{i i}\left[y_{i}-f\left(x_{i}, \vec{\beta}\right)\right]^{2}
$$

where the matrix W denotes the weights matrix, then the least square method told us that

$$
\begin{gather*}
\hat{\beta}=\left(X^{T} W X\right)^{-1} X^{T} W \hat{y}  \tag{b}\\
M_{\beta}=\left(X^{T} W X\right)^{-1} X^{T} W M_{y} W^{T} X\left(X^{T} W^{T} X\right)^{-1} \tag{c}
\end{gather*}
$$

are the best liner unbiased estimator of parameter set $\widehat{\beta}$ and its error respectively, when $\mathrm{W}=M_{y}^{-1}$ is the case.

## The weighted least squares methods

As for the case of my analysis, $m=1$, and $n=10$. The function which we used to fit the data set $\left(x_{i}, y_{i}\right)$ is $f(\vec{x}, \vec{\beta})=\sum_{j=1}^{1} \beta_{j} \phi_{j}(\vec{x})=\beta$, so $X_{i j}=\frac{\partial f\left(x_{i} \vec{\beta}\right)}{\partial \beta_{j}}=1$, and $X^{T}=(1,1, \cdots, 1) . \mathrm{M}_{\beta} \in \boldsymbol{R}$, and $\mathrm{M}_{\mathrm{y}} \in \boldsymbol{R}^{10 \times 10}$.

When we choose the weights matrix as the inverse of variance-covariance matrix $M_{y}$, that is, $W=M_{y}{ }^{-1}$, we can obtain the weighted average of cross section and its total error as follow

$$
\begin{equation*}
\bar{\beta} \pm \overline{\delta \beta}=\frac{\sum_{j} y_{j} \cdot \sum_{i} \omega_{i j}}{\sum_{j} \sum_{i} \omega_{i j}} \pm \sqrt{\frac{1}{\sum_{j} \sum_{i} \omega_{i j}}} \tag{d}
\end{equation*}
$$

where $\omega_{i j}$ is element of $(i, j)$ position of weights matrix, which will be defined later.

## The weighted least squares methods

When we want to calculate the error comes from different sources, we can use the original formula (c) by using the variance-covariance matrix of the error comes from different sources, that is

$$
\begin{equation*}
M=\left(X^{T} W X\right)^{-1} X^{T} W M_{i} W^{T} X\left(X^{T} W^{T} X\right)^{-1} \tag{e}
\end{equation*}
$$

where $M_{i}$ is the variance-covariance matrix of i-th error which shares the same definition with that of total error.

In my analysis, this formula can be simplified as

$$
\begin{equation*}
\overline{\delta \beta}_{i}=M=\frac{\sum_{i=1}^{i=10} \sum_{j=1}^{j=10} \omega_{i} \omega_{j} m_{i j}}{\left(\sum_{i=1}^{i-10} \omega_{i}\right)^{2}} \tag{e}
\end{equation*}
$$

where $m_{i j}$ is the $(i, j)$ position element of $\mathrm{M}_{\mathrm{i}}$, and $\omega_{i}=\sum_{k=1}^{k=10} \omega_{i k}$, and $\omega_{i k}$ is the $(i, k)$ position element of weight matrix W .

## Error Matrix

The covariance error matrix $\mathrm{M}_{\mathrm{y}}$ is:

$$
\begin{gathered}
\left(\begin{array}{cccc}
\sigma_{T 1}^{2} & \operatorname{Cov}\left(x_{1}, x_{2}\right) & \cdots & \operatorname{Cov}\left(x_{1}, x_{n}\right) \\
\operatorname{Cov}\left(x_{2}, x_{1}\right) & \sigma_{T 2}^{2} & \cdots & \operatorname{Cov}\left(x_{2}, x_{n}\right) \\
\cdots & \cdots & & \cdots \\
\operatorname{Cov}\left(x_{n}, x_{1}\right) & \operatorname{Cov}\left(x_{n}, x_{2}\right) & & \cdots \\
\sigma_{T n}^{2}
\end{array}\right) \\
\sigma_{T i}^{2}=\sigma_{i}^{2}(\text { stat. })+\sigma_{i}^{2}(\operatorname{sys} 1 .)+\sigma_{i}^{2}(\text { sys } 2 .)+\cdots \\
\operatorname{Cov}\left(x_{i}, x_{j}\right)=x_{i} \cdot \varepsilon_{i j} \cdot x_{j} \cdot \varepsilon_{j i}
\end{gathered}
$$

Where $\sigma_{T i}$ stands for the total uncertainty in the measurement of mode i , and $\sigma_{i}$ (stat.) and $\sigma_{i}$ (sysj.) are the statistical error and the systematic error for source j in mode i respectively. $\operatorname{Cov}\left(x_{i}, x_{j}\right)$ is the covariance systematic error between mode i and j. $x_{i}$ is The measured value of mode i , and $\varepsilon_{i j}=\varepsilon_{j i}$ is the common relative systematic error (in percentage) between mode $i$ and $j$.

## Weighted average

As for the variance-covariance matrix of i-th error source(i.e. $\mathrm{M}_{\mathrm{i}}$ ), the only difference is that $\varepsilon_{i j}=\varepsilon_{j i}$ just contains the common relative error which comes from source i only. We can find that

$$
\begin{equation*}
\mathrm{M}_{\mathrm{y}}=\sum_{i} \mathrm{M}_{\mathrm{i}} \tag{f}
\end{equation*}
$$

The inverse of the covariance matrix $\mathrm{M}_{\mathrm{y}}{ }^{-1}$ is:

$$
\left(\begin{array}{cccc}
\omega_{11} & \omega_{12} & \cdots & \omega_{1 n} \\
\omega_{21} & \omega_{22} & \cdots & \omega_{2 n} \\
\cdots & \cdots & \cdots & \cdots \\
\omega_{n 1} & \omega_{n 2} & \cdots & \omega_{n n}
\end{array}\right)
$$

The weighted average cross section and the corresponding total uncertainty can be calculated according to formula(d).

According this formula, the production cross section of $\Lambda_{c}^{+}{\overline{\Lambda_{c}}}^{-}$is 232.22 pb , with the total error 18.01 pb .

## Updated cross section(@4600)

| Mode | $N_{\text {total }}^{+}$ | Eff. $+(\%)$ | Br. (\%) | $X_{\text {sec }}^{+}(\mathrm{pb})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $2662 \pm 57$ | 46.95 | $6.84 \pm 0.36$ | $211.71 \pm 4.53 \pm 16.78 \pm 11.14 \pm 0.00$ |
| 2 | $540 \pm 26$ | 47.40 | $1.11 \pm 0.11$ | $262.13 \pm 12.62 \pm 22.66 \pm 13.80 \pm 22.31$ |
| 3 | $286 \pm 21$ | 32.76 | $0.87 \pm 0.10$ | $256.28 \pm 18.82 \pm 24.39 \pm 13.49 \pm 25.63$ |
| 4 | $813 \pm 71$ | 18.30 | $4.53 \pm 0.84$ | $250.47 \pm 21.87 \pm 19.20 \pm 13.18 \pm 44.86$ |
| 5 | $196 \pm 22$ | 13.34 | $1.54 \pm 0.23$ | $242.04 \pm 27.17 \pm 26.01 \pm 12.74 \pm 33.00$ |
| 6 | $330 \pm 45$ | 6.48 | $3.15 \pm 0.79$ | $412.90 \pm 56.30 \pm 80.54 \pm 21.73 \pm 101.81$ |
| 7 | $287 \pm 31$ | 21.90 | $1.21 \pm 0.16$ | $276.61 \pm 29.88 \pm 43.55 \pm 14.56 \pm 32.54$ |
| 8 | $275 \pm 52$ | 12.69 | $2.27 \pm 0.18$ | $243.82 \pm 46.10 \pm 32.24 \pm 12.83 \pm 14.07$ |
| 9 | $205 \pm 20$ | 20.41 | $0.87 \pm 0.18$ | $294.86 \pm 28.77 \pm 18.20 \pm 15.52 \pm 58.97$ |
| 10 | $587 \pm 65$ | 19.53 | $2.41 \pm 0.31$ | $318.52 \pm 35.27 \pm 38.39 \pm 16.76 \pm 36.93$ |
| summary |  |  |  | $232.22 \pm 4.56 \pm 11.53 \pm 12.22 \pm 4.63$ |

The combined error comes from absolute branch fraction of these multiple modes is $\sqrt{4.63^{2}+12.22^{2}} / 232.22=13.07 / 232.22=5.63 \%$

## $\left|G_{E} / G_{M}\right|$ ratio measurement at 4.6 GeV

## Angular distribution:

Data of mode $\mathrm{pK}^{-} \pi^{+}$only.
The angular distribution is then fitted by $1+\alpha \cos ^{2} \theta$.



| $\operatorname{Ecm}(\mathrm{GeV})$ | $\alpha_{\Lambda_{c}^{+}}$ | $\alpha_{\Lambda_{\bar{c}}}$ | $\alpha$ |
| :---: | :---: | :---: | :---: |
| 4.60 | $-0.34 \pm 0.083$ | $-0.30 \pm 0.08$ | $-0.32 \pm 0.06$ |

## $\left|G_{E} / G_{M}\right|$ ratio measurement at 4.6 GeV

The differential cross section can be expressed as:

$$
\frac{d \sigma_{\text {Born }}(s)}{d \Omega}=\frac{\alpha^{2} \beta C}{4 s}\left[\left|G_{M}(s)\right|^{2}\left(1+\cos ^{2} \theta_{\Lambda_{c}}\right)+\frac{4 m_{\Lambda_{c}}^{2}}{s}\left|G_{E}(s)\right|^{2}\left(\sin ^{2} \theta_{\Lambda_{c}}\right)\right]
$$

$\left|\mathrm{G}_{\mathrm{E}} / \mathrm{G}_{\mathrm{M}}\right|$ ratio can be described by the following:

$$
\left|\frac{G_{E}}{G_{M}}\right|^{2}=(1-\alpha) /\left(\frac{4 m_{\Lambda_{c}}^{2}}{s} \alpha+\frac{4 m_{\Lambda_{c}}^{2}}{s}\right)
$$

$\left|\mathrm{G}_{\mathrm{E}} / \mathrm{G}_{\mathrm{M}}\right|$ ratio is calculated to be $1.40 \pm 0.13$ at 4.60 GeV .

## $\left|G_{E} / G_{M}\right|$ ratio measurement at 4.6 GeV



* At threshold (4.575 GeV), we assume $\left|\mathrm{G}_{\mathrm{E}} / \mathrm{G}_{\mathrm{M}}\right|=1$.
* At $4.58,4.59 \mathrm{GeV},\left|\mathrm{G}_{\mathrm{E}} / \mathrm{G}_{\mathrm{M}}\right|$ is obtained by interpolation.
* The result of $\left|\mathrm{G}_{\mathrm{E}} / \mathrm{G}_{\mathrm{M}}\right|$ ratio is needed in the line-shape fitting.


## fit the line-shape

The function of non-resonant (NonR) contribution can be parameterized as:

$$
N o n R=\frac{4 \pi \alpha^{2} \beta C}{3 q^{2}}\left[\left|G_{M}\left(q^{2}\right)\right|^{2}+\frac{1}{2 \tau}\left|G_{E}\left(q^{2}\right)\right|^{2}\right]
$$

where $\beta=\sqrt{\left(1-4 m^{2}{ }_{\Lambda c} / q^{2}\right)}, \tau=q^{2} / 4 m^{2}{ }_{\Lambda c}$.
The Coulomb factor $\mathrm{C}=\varepsilon \times \mathrm{R}, \varepsilon=\pi \alpha / \beta$ is the enhancement factor, and R is the resummation factor. In traditional prediction: $\mathrm{R}=\sqrt{1-\beta^{2}} /\left(1-\mathrm{e}^{-\pi \alpha / \beta}\right)$

From the prediction by R. Baldini Ferroli, S. Pacetti

$$
C=\frac{\pi \alpha}{\beta} \times R_{S}=\frac{\pi \alpha}{\beta} \frac{\sqrt{1-\beta^{2}}}{1-\exp \left(-\pi \alpha_{s} / \beta\right)}
$$

where the coupling constant $\alpha=1 / 137, \alpha_{s}=0.5$.

## fit the line-shape

The fitting results with previous formula, $\left|\mathrm{G}_{\mathrm{E}} / \mathrm{G}_{\mathrm{M}}\right|$ ratio is inputted in the fitting, $G_{M}$ is the only fitting parameter :



* Using the updated $R_{s}$ to fit line-shape, the fit status is good, and $\left|\mathrm{G}_{\mathrm{m}}\right|$ is $1.137 \pm 0.039$.
* Using the traditional $R$ to fit line-shape, the fit status is bad, and $\left|\mathrm{G}_{\mathrm{M}}\right|$ is $0.5436 \pm 0.0199$.


## Summary

- We present the measurement of $\Lambda_{c}^{+} \bar{\Lambda}_{c}^{-}$production cross section at threshold by tagging 10 decay modes, the systematic uncertainty at 4.6 GeV is studied by tagging $\Lambda_{c}^{+}$, and it is about $7.5 \%$.
- The $\left|G_{E} / G_{M}\right|$ ratio at 4.60 GeV is obtained, to be $1.40 \pm 0.13$, which is significantly larger than 1.
- The line-shape of $\Lambda_{c}^{+} \bar{\Lambda}_{c}^{-}$production is fitted, and the result favors the prediction with $R \rightarrow R_{S}$ in the fitting. The fitted form factor $\left|G_{M}\right|$ is $1.14 \pm 0.04$.

