Line shape Fitting for $\omega\pi^0$

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Fitting Function from SND

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[18]. The Born cross section is described by the following formula [15]:

$$\sigma(E) = \frac{4\pi\alpha^2}{3E^3}|F_{\omega\pi\gamma}(E^2)|^2 P_f(E) B(\omega \to \pi^0\gamma), \qquad (3)$$

where α is the fine-structure constant, $F_{\omega\pi\gamma}(E^2)$ is the $\gamma^* \to \omega\pi^0$ transition form factor, $B(\omega \to \pi^0\gamma)$ is the branching fraction for the $\omega \to \pi^0\gamma$ decay, and $P_f(E)$ is the phase-space factor. In the narrow ω-resonance approximation $P_f(E) = q_\omega^3$, where q_ω is the ω-meson momentum. The transition form factor is parametrized in the vector meson dominance (VMD) model as a sum of the $\rho(770)$, $\rho(1450)$, and $\rho(1700)$ resonance contributions,

$$F_{\omega\pi\gamma}(E^2) = \frac{g_{\rho\omega\pi}}{f_\rho}\left(\frac{m_\rho^2}{D_\rho} + A_1 e^{i\varphi_1}\frac{m^2_{\rho(1450)}}{D_{\rho(1450)}} + A_2 e^{i\varphi_2}\frac{m^2_{\rho(1700)}}{D_{\rho(1700)}}\right), \qquad (4)$$

where $g_{\rho\omega\pi}$ is the $\rho \to \omega\pi$ coupling constant, f_ρ is the $\gamma^* \to \rho$ coupling constant calculated from the $\rho \to e^+e^-$ decay width, $D_{\rho_i}(E) = m^2_{\rho_i} - E^2 - \imath E\Gamma_{\rho_i}(E)$, and m_{ρ_i} and $\Gamma_{\rho_i}(E)$ are the mass and width of the resonance ρ_i. The formula for $\Gamma_{\rho(770)}(E)$ is given in Ref. [15]. For the $\rho(1450)$ and $\rho(1700)$ resonances, the energy-independent widths are used.

Fitting result of SND

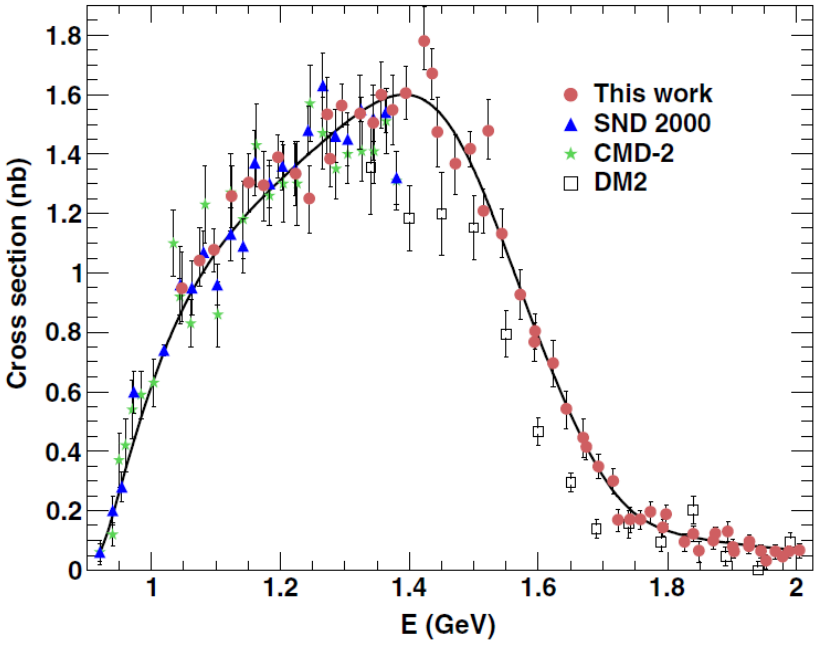


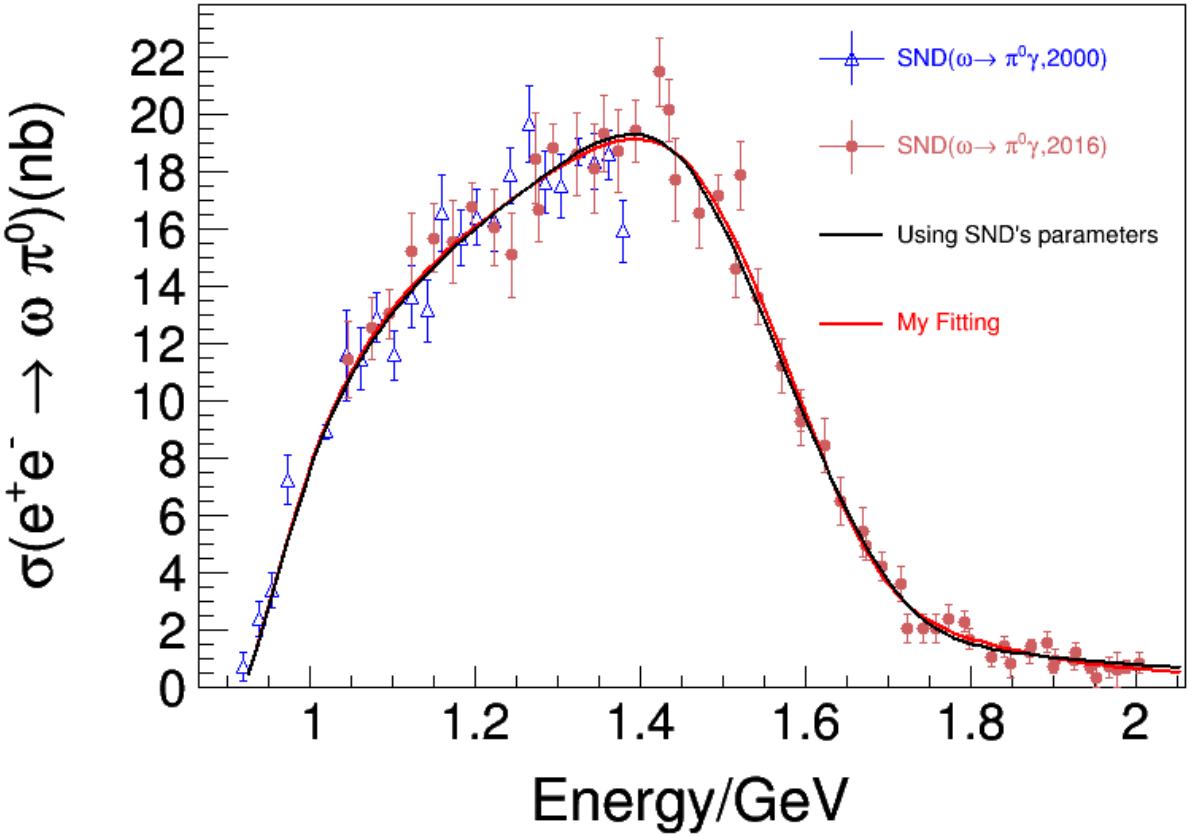
FIG. 2. The cross section for $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$ measured in this work (circles), and in the SND@VEPP-2M [3,4] (triangles), CMD-2 [7] (stars), and DM2 [9] (squares) experiments. Only statistical errors are shown. The curve is the result of the fit to SND 2000 and SND 2013 data described in the text (Model 1).

TABLE II. The fitted parameters of the $e^+e^- \to \omega\pi^0$ cross-section model.

lccc Parameter	Model 1	Model 2	Model 3
$g_{\rho\omega\pi}$, GeV^{-1}	15.9 ± 0.4	16.5 ± 0.2	$\cdots$
A_1	0.175 ± 0.016	0.137 ± 0.006	0.251 ± 0.006
A_2	0.014 ± 0.004	$\equiv 0$	0.027 ± 0.003
$M_{\rho(1450)}$, MeV	1510 ± 7	1499 ± 4	1516 ± 10
$\Gamma_{\rho(1450)}$, MeV	440 ± 40	367 ± 13	500 ± 30
$M_{\rho(1700)}$, MeV	$\equiv 1720$	$\cdots$	$\equiv 1720$
$\Gamma_{\rho(1700)}$, MeV	$\equiv 250$	$\cdots$	$\equiv 250$
φ_1, deg.	124 ± 17	122 ± 8	162 ± 6
φ_2, deg.	-63 ± 21	$\cdots$	-24 ± 10
χ^2/ν	71/73	85/75	83/74

Fitting results from Mode1

Following SND's result



Almost the same fitting results

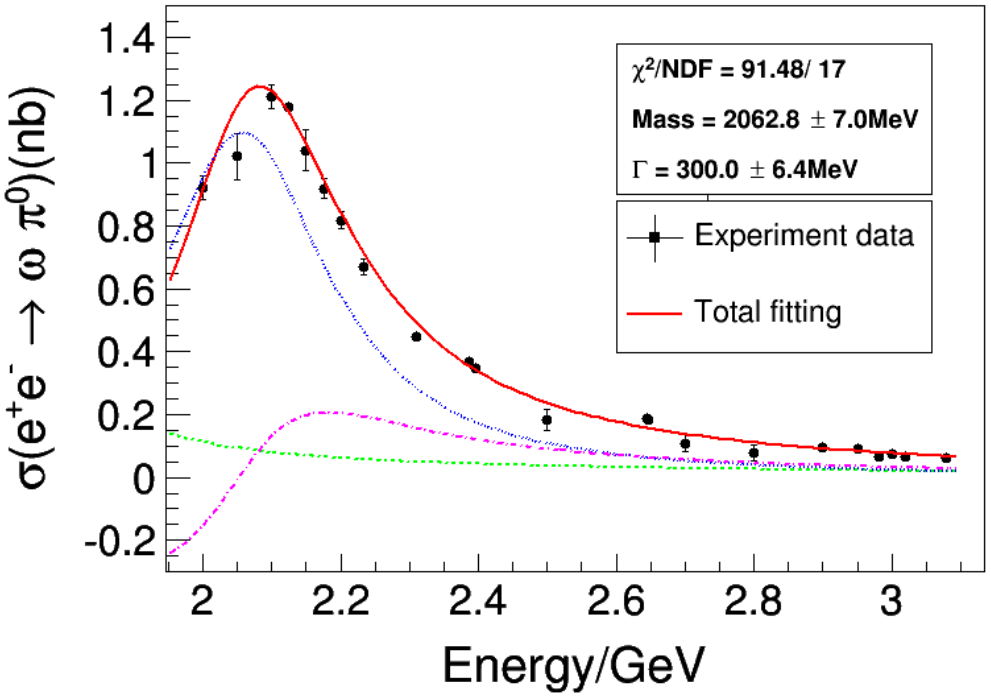
Following SND's method (only statistics included)

$$\sigma(E) = \frac{4\pi\alpha^2}{3E^3}\left|F_{\omega\pi\gamma}\left(E^2\right)\right|^2 P_f(E)\, B\left(\omega \to \pi^0\gamma\right)$$

$$F_{\omega\pi\gamma}\left(E^2\right) = \frac{g_{\rho\omega\pi}}{f_\rho}\left(\frac{m_\rho^2}{D_\rho} + A_1 e^{i\varphi_1}\frac{m^2_{\rho(1450)}}{D_{\rho(1450)}} + A_2 e^{i\varphi_2}\frac{m^2_{\rho(1700)}}{D_{\rho(1700)}} + A_3 e^{i\varphi_3}\frac{m^2_{\rho(2100)}}{D_{\rho(2100)}}\right)$$

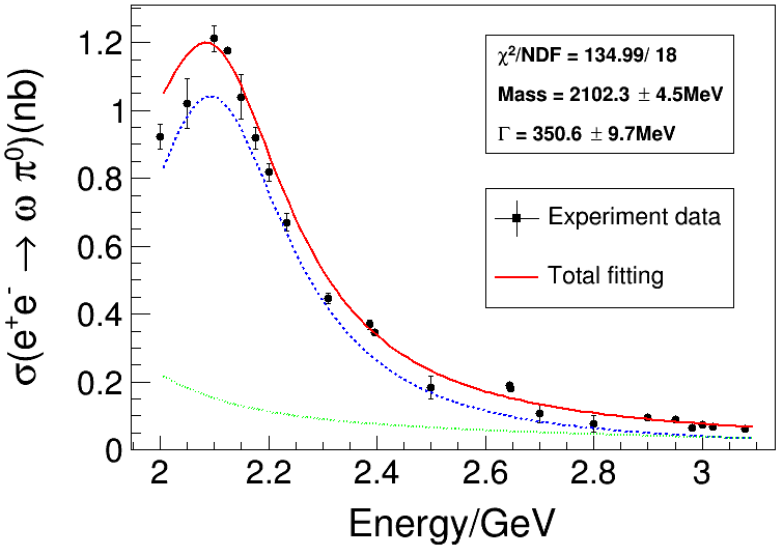
Other parameters are fixed using SND fitting result

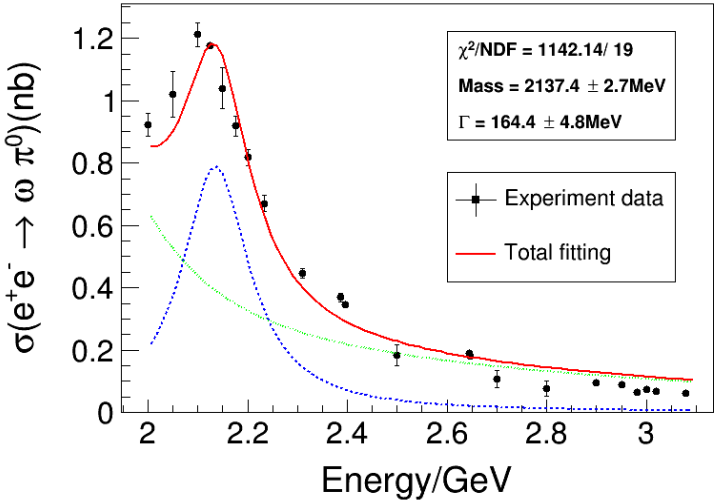
This item is to describe our "resonance".



Try other method (only statistics included)

Relative BW + Bkg shape(described with SND parameters)
And without interference



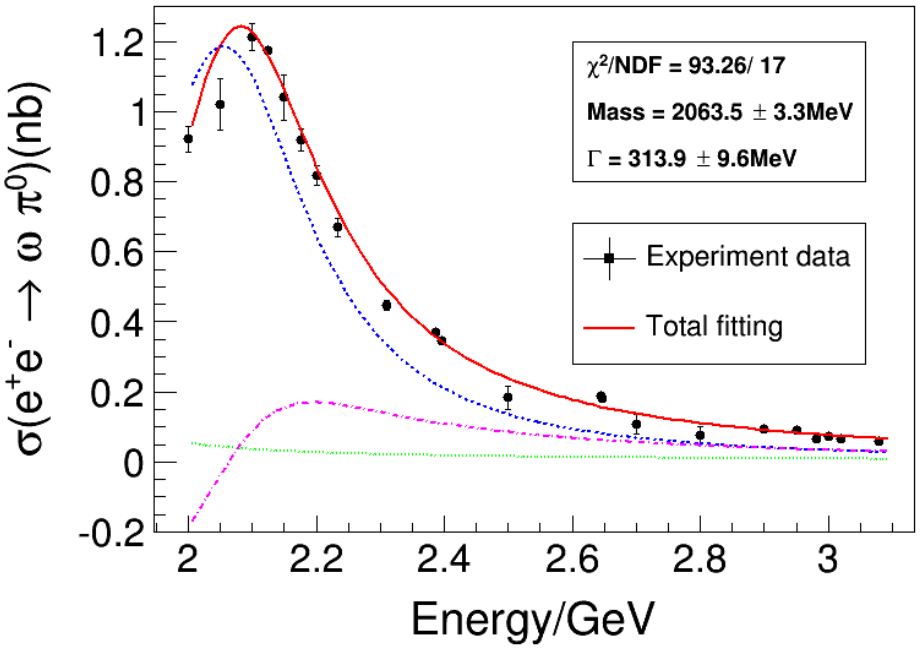


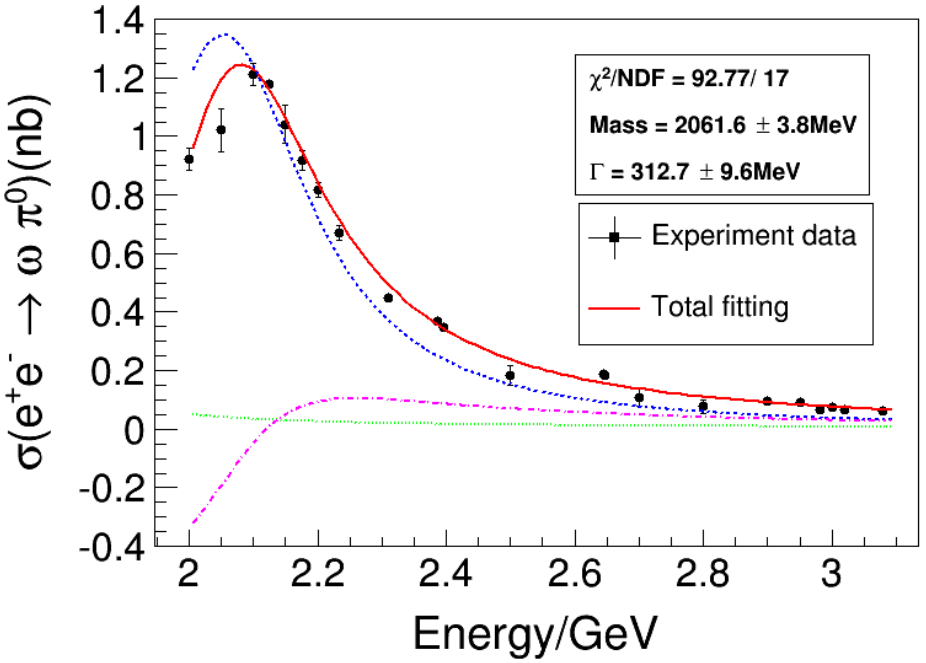
The contribution from Bkg is allowed to vary

The contribution from Bkg is fixed

Try other method (only statistics included)

Relative BW + Bkg shape(described with SND parameters)
And considering interference





Correlation

$$\chi^2 = \left(\vec{x} - \vec{\mu}\right)^T M^{-1} \left(\vec{x} - \vec{\mu}\right)$$

$$M = \begin{pmatrix} \sigma_1^2 & \sigma_1^{'}\sigma_2^{'} & \cdots & \sigma_1^{'}\sigma_{22}^{'} \\ \sigma_2^{'}\sigma_1^{'} & \sigma_2^2 & \cdots & \sigma_2^{'}\sigma_{22}^{'} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{22}^{'}\sigma_1^{'} & \sigma_{22}^{'}\sigma_2^{'} & \cdots & \sigma_{22}^2 \end{pmatrix}$$

Table 2: Summary of systematic uncertainties in cross section measurement of $e^+e^- \to \omega\pi^0$.

Source	Value
Tracking efficiency	2.0%
Particle identification	2.0%
Photon detection efficiency	4.0%
Branching fraction	0.7%
Luminosity	1.0%
$(1+\delta)^{ISR} \cdot (1+\delta)^{VP}$ correction	0.6%
4C kinematic fit	0.4%
π_1^0 mass window	0.2%
π_2^0 mass window	0.2%
$\cos\theta_{\pi_1^0} < 0.9$	0.1%
Signal shape	3.8%
Background shape	0.2%
Fitting range	0.3%
Total	6.4%

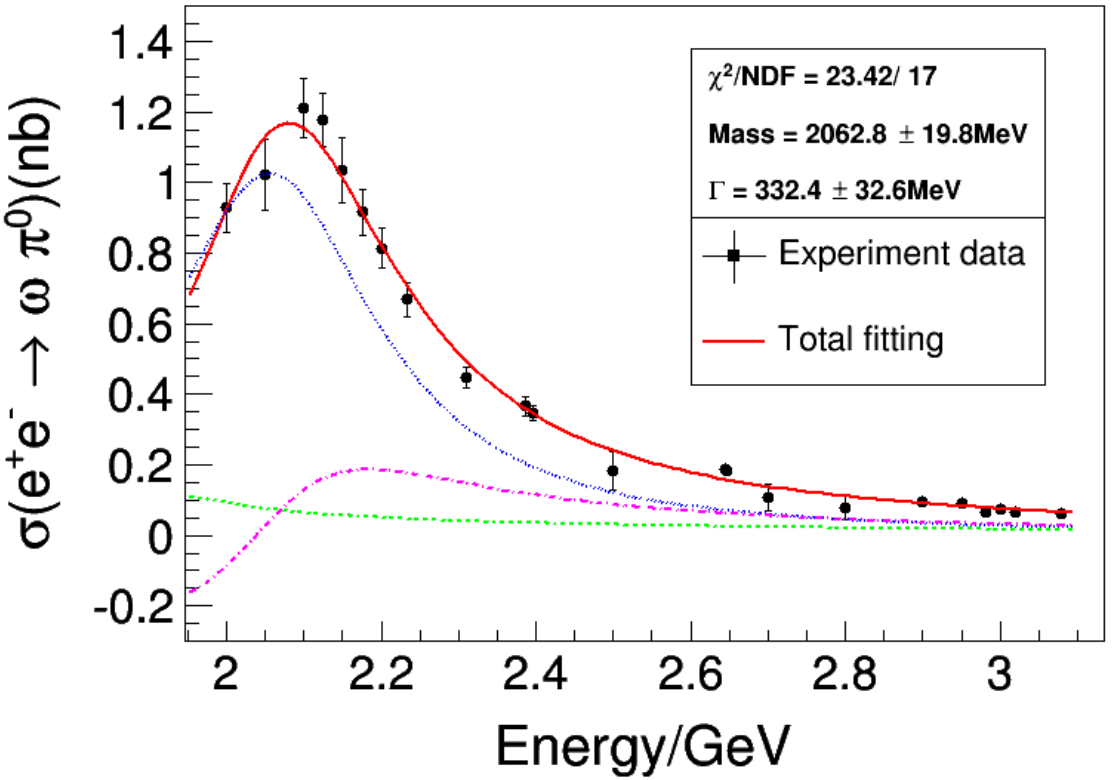
$\sigma_i^{'}\sigma_j^{'}$ are the crrelation between x_i and x_j

σ_i^2 are the total cross section uncertainty for x_i

Without correlation (including sys uncertainty)

$$\sigma(E)=\frac{4\pi\alpha^2}{3E^3}\left|F_{\omega\pi\gamma}\left(E^2\right)\right|^2 P_f(E)\,B\left(\omega\to\pi^0\gamma\right)$$

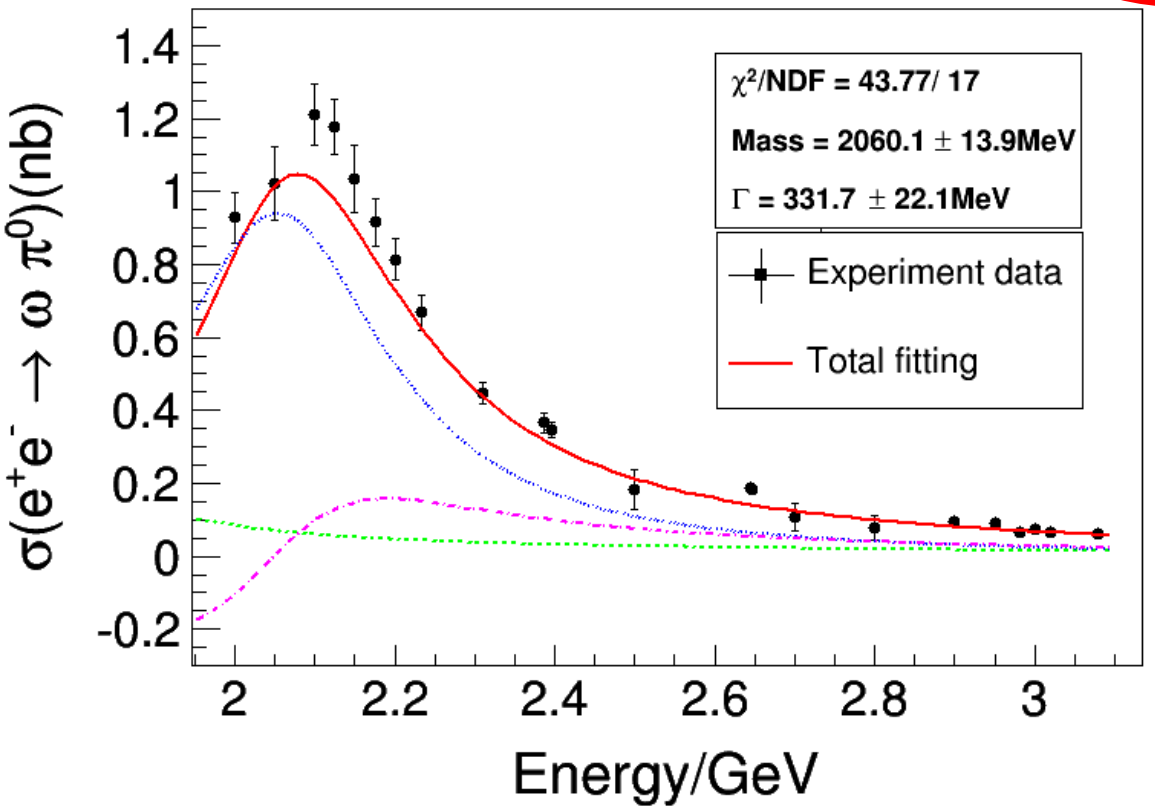
$$F_{\omega\pi\gamma}\left(E^2\right)=\frac{g_{\rho\omega\pi}}{f_\rho}\left(\frac{m_\rho^2}{D_\rho}+A_1e^{i\varphi_1}\frac{m_{\rho(1450)}^2}{D_{\rho(1450)}}+A_2e^{i\varphi_2}\frac{m_{\rho(1700)}^2}{D_{\rho(1700)}}+A_3e^{i\varphi_3}\frac{m_{\rho(2100)}^2}{D_{\rho(2100)}}\right)$$



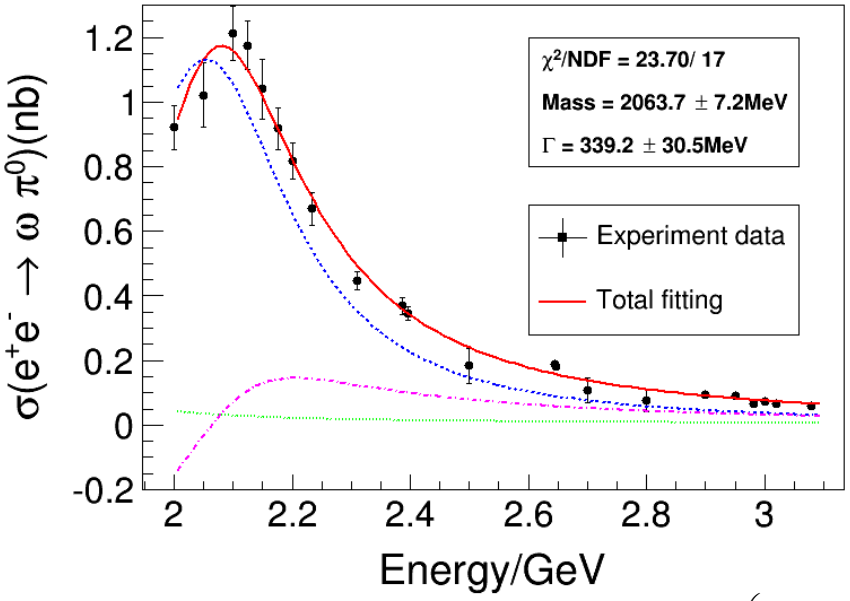
Considering correlation (including sys uncertainty)

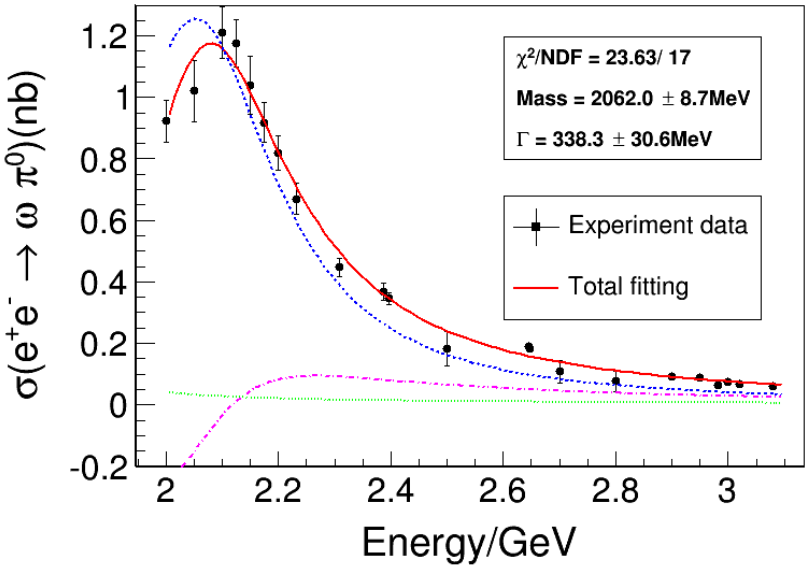
$$\sigma(E)=\frac{4\pi\alpha^2}{3E^3}\left|F_{\omega\pi\gamma}\left(E^2\right)\right|^2 P_f(E)\,B\left(\omega\to\pi^0\gamma\right)$$

$$F_{\omega\pi\gamma}\left(E^2\right)=\frac{g_{\rho\omega\pi}}{f_\rho}\left(\frac{m_\rho^2}{D_\rho}+A_1e^{i\varphi_1}\frac{m_{\rho(1450)}^2}{D_{\rho(1450)}}+A_2e^{i\varphi_2}\frac{m_{\rho(1700)}^2}{D_{\rho(1700)}}+A_3e^{i\varphi_3}\frac{m_{\rho(2100)}^2}{D_{\rho(2100)}}\right)$$



Without correlation (Including Sys uncertainty)



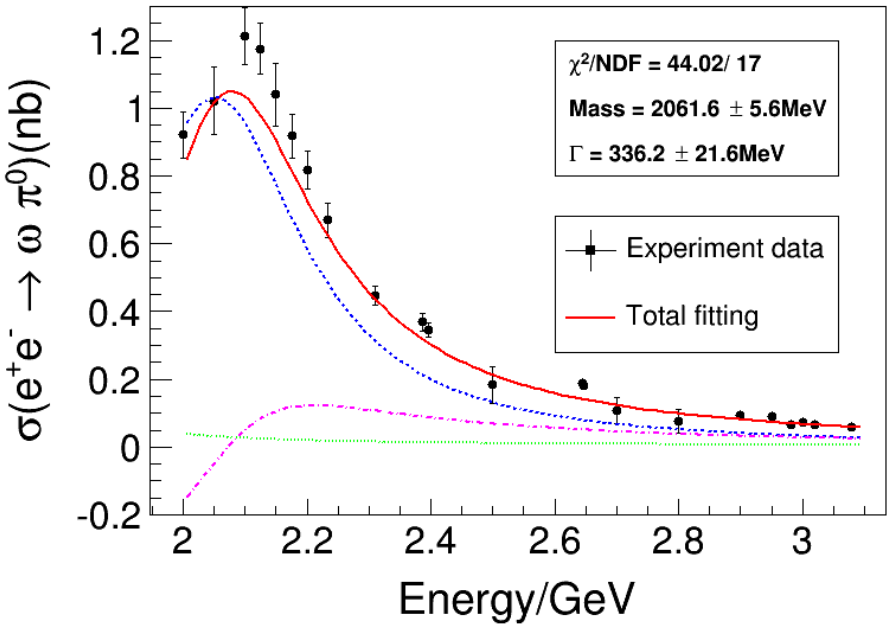


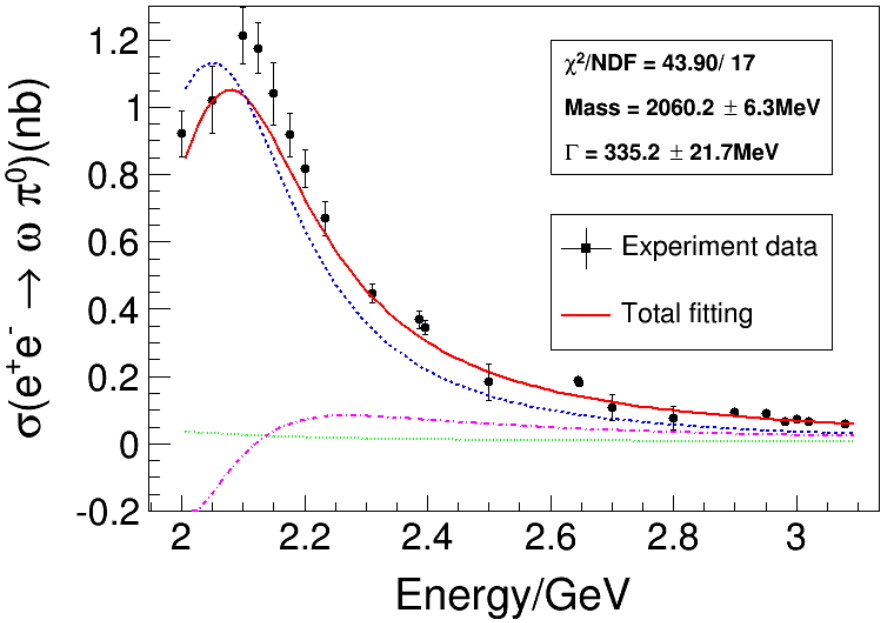
$$F_{\omega\pi\gamma}\left(E^2\right)=\frac{g_{\rho\omega\pi}}{f_\rho}\left(\frac{m_\rho^2}{D_\rho}+A_1e^{i\varphi_1}\frac{m_{\rho(1450)}^2}{D_{\rho(1450)}}+A_2e^{i\varphi_2}\frac{m_{\rho(1700)}^2}{D_{\rho(1700)}}\right)$$

$$poly=\frac{4\pi\alpha^2}{3E^3}\left|F_{\omega\pi\gamma}\left(E^2\right)\right|^2P_f\left(E\right)B\left(\omega\to\pi^0\gamma\right)$$

$$\sigma\left(E\right)=\left|\frac{m\Gamma\left(E\right)}{E-m+iE\Gamma\left(E\right)}+Aei^{\phi}\sqrt{poly}\right|^2$$

Considering correlation (Including Sys uncertainty)





$$F_{\omega\pi\gamma}\left(E^2\right)=\frac{g_{\rho\omega\pi}}{f_\rho}\left(\frac{m_\rho^2}{D_\rho}+A_1e^{i\varphi_1}\frac{m_{\rho(1450)}^2}{D_{\rho(1450)}}+A_2e^{i\varphi_2}\frac{m_{\rho(1700)}^2}{D_{\rho(1700)}}\right)$$

$$poly=\frac{4\pi\alpha^2}{3E^3}\left|F_{\omega\pi\gamma}\left(E^2\right)\right|^2P_f\left(E\right)B\left(\omega\to\pi^0\gamma\right)$$

$$\sigma\left(E\right)=\left|\frac{m\Gamma\left(E\right)}{E-m+iE\Gamma\left(E\right)}+Aei^{\phi}\sqrt{poly}\right|^2$$

Possible resonances from PDG

Table 14: possible ρ resonances

particle	Mass(MeV)	Width(MeV)
$\rho(770)$(PDG confirmed)	775.26±0.25	149.1±0.8
$\rho(1450)$(PDG confirmed)	1465±25	400±60
$\rho(1570)$	1570±72	144±86
$\rho(1700)$(PDG confirmed)	1720±20	250±100
$\rho(1900)$	1880±30	130±30
$\rho(1990)$	1982±14	188±24
$\rho(2000)$	2000±30	260±45
$\rho(2150)$	2155±21	320±70
$\rho(2250)$	2260±20	160±25
$\rho(2270)$	2265±40	325±80
$\rho(2350)$	2330±35	400±100
$\rho_2(1940)$	1940±40	155±40
$\rho_2(2225)$	2225±35	335^{+100}_{-50}
$\rho_3(1690)$(PDG confirmed)	1688.8±2.1	161±10
$\rho_4(2230)$	2230±25	210±30

Table 15: Other possible resonances

particle	Mass(MeV)	Width(MeV)
$b_1(1960)$	1960±35	230±50
$X(2100)$	2100±40	250±40
$X(2150)$	2150±10	260±10

Fit function and parameters in PDG

$$\sigma(\sqrt{s}) = \frac{4\pi\alpha^2}{3s^{3/2}} |F_{\omega\pi\gamma}(s)|^2 P_f(\sqrt{s}),$$

$$F_{\omega\pi\gamma}(s) = \frac{g_{\rho\omega\pi}}{f_\rho} \left(f_0 + A_1 e^{i\varphi_1} f_1 + A_2 e^{i\varphi_2} f_2 + A_3 e^{i\varphi_3} f_3 \right)$$

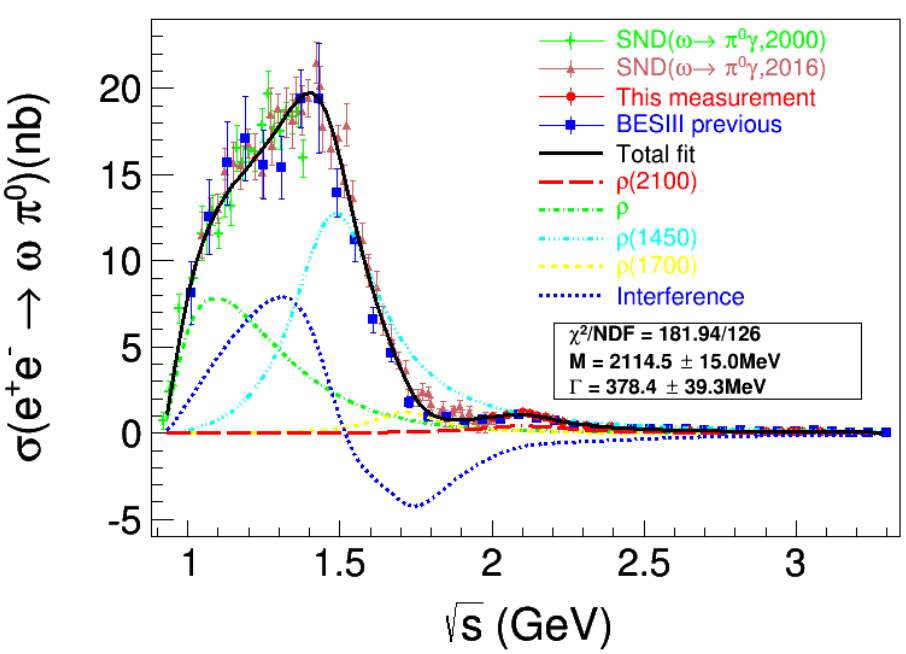
$$f_i(i = 0, 1, 2, 3) = \frac{m^2_{\rho_i}}{m^2_{\rho_i} - s - i\sqrt{s}\Gamma_{\rho_i}(\sqrt{s})}$$

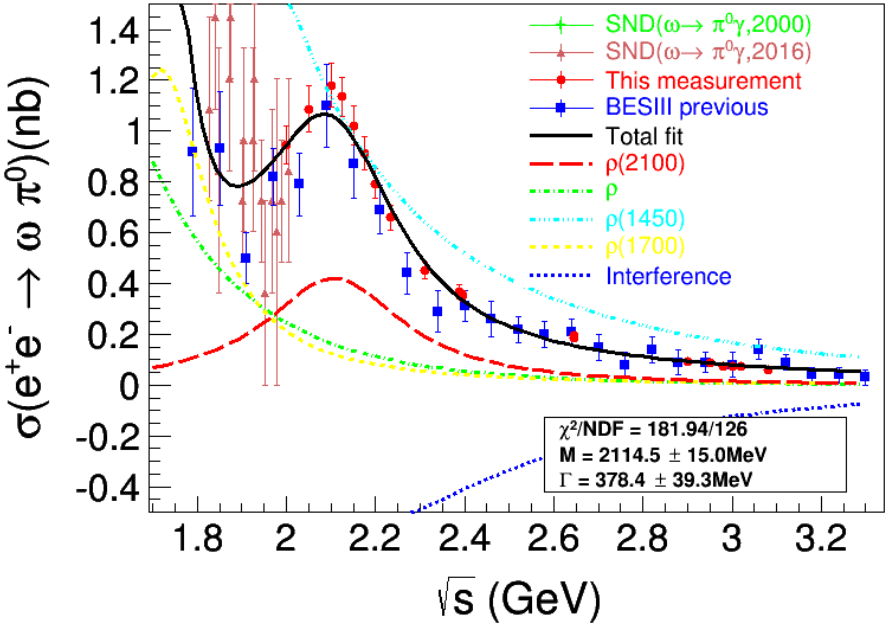
Table II. The parameters of ρ resonances in PDG.

Resonances	M(MeV/c^2)	Γ(MeV)
$\rho(770)$	775.26±0.25	149.1±0.8
$\rho(1450)$	1465±25	400±60
$\rho(1700)$	1720±20	250±100

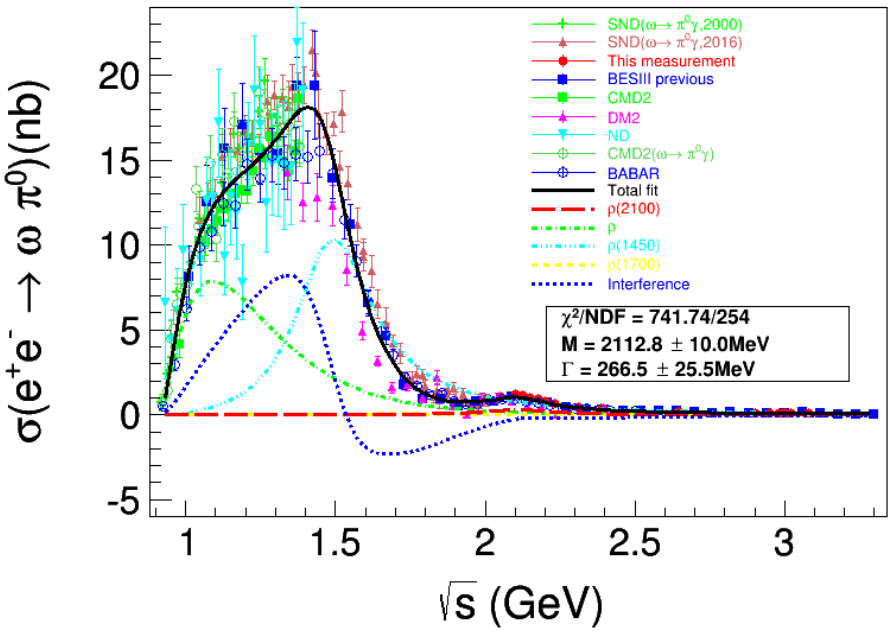
Model1: Parameters of rho(1700)'s mass and width fixed
Model2: Masses of rho(1700) and rho(1450) fixed
Model3: Both masses and widths of rho(1700) and rho(1450) fixed

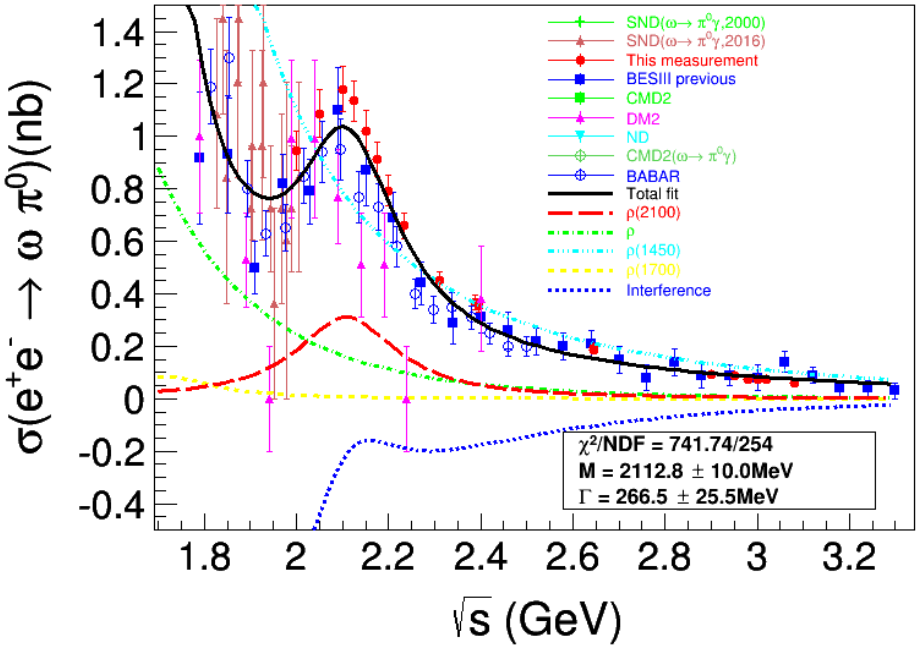
Model1: Parameters of rho(1700)'s mass and width fixed
Data: SND + BESIII



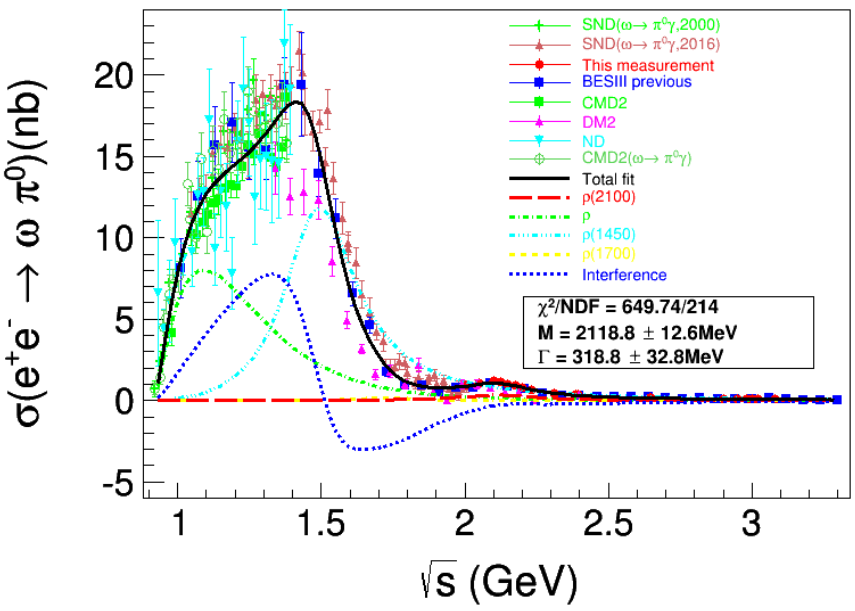


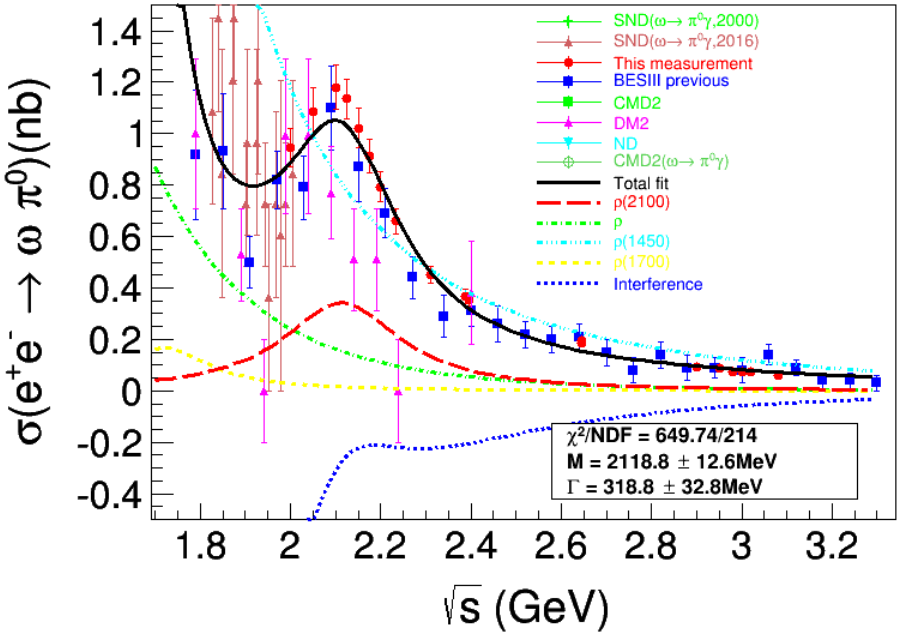
Model1: Parameters of rho(1700)'s mass and width fixed
Data: All the data





Model1: Parameters of rho(1700)'s mass and width fixed
Data: All the data except BABAR

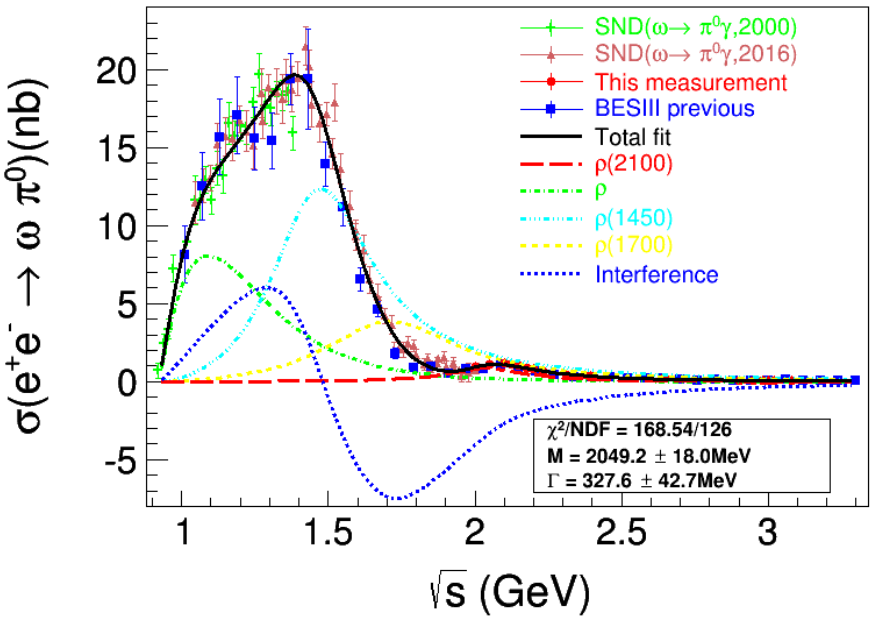


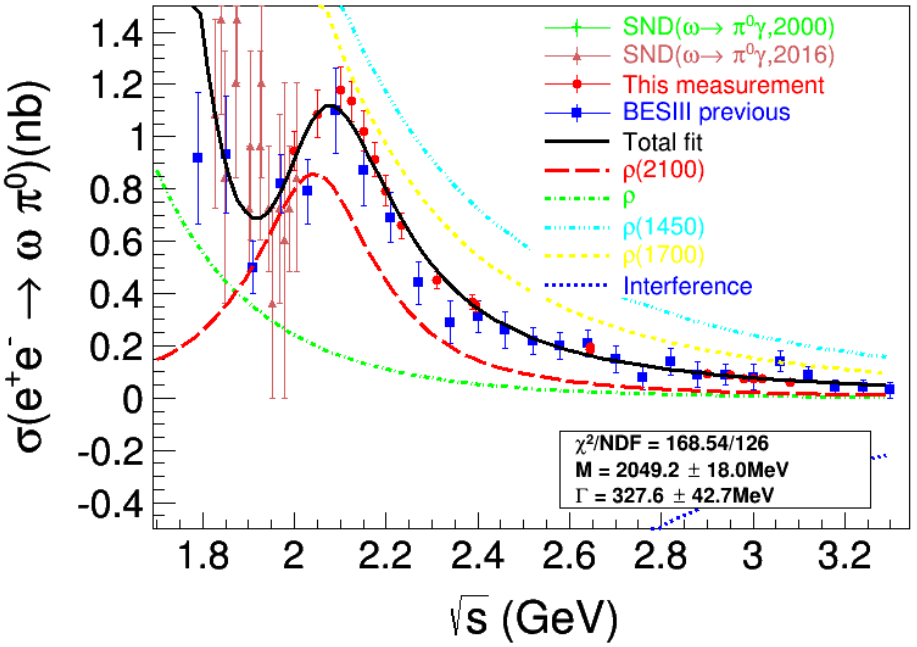


Model1: Parameters of rho(1700)'s mass and width fixed

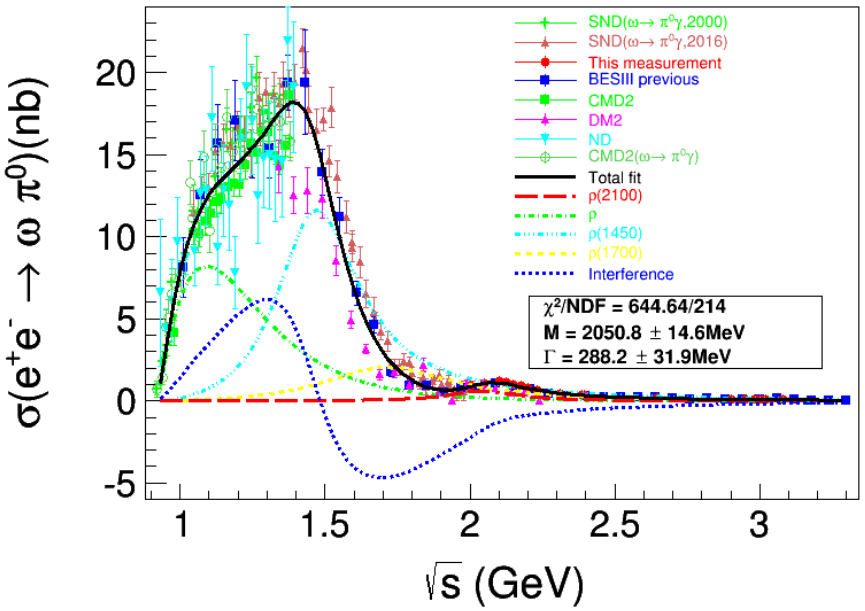
Parameters	Model1 $M_{\rho(1700)}$ & $\Gamma_{\rho(1700)}$ fixed	
	SND+BESIII	All data - BABAR
$g_{\rho\omega\pi}$ (GeV^{-1})	16.9±0.3	17.2±0.2
A_1	0.13±0.01	0.10±0.01
A_2	0.021±0.004	0.008±0.003
A_3	0.013±0.002	0.010±0.001
ϕ_1 (deg.)	116±11	123±8
ϕ_2 (deg.)	-95±19	-105±22
ϕ_3 (deg.)	-141±21	-150±21
$M_{\rho(770)}$ (MeV)	≡775.26±0.25	≡775.26±0.25
$\Gamma_{\rho(770)}$ (MeV)	≡149.1±0.8	≡149.1±0.8
$M_{\rho(1450)}$ (MeV)	1479.8±5.8	1490.2±4.3
$\Gamma_{\rho(1450)}$ (MeV)	353±22	308±15
$M_{\rho(1700)}$ (MeV)	≡1720	≡1720
$\Gamma_{\rho(1700)}$ (MeV)	≡250	≡250
$X_{\rho(2100)}$ (MeV)	2115±15	2119±13
$\Gamma_{\rho(1450)}$ (MeV)	378±38	319±33
$(\chi^2/ndf)_{Tot}$	182/126=1.44	650/214=3.04
$(\chi^2/ndf)_{This}$	24/18=1.33	35/18=1.94

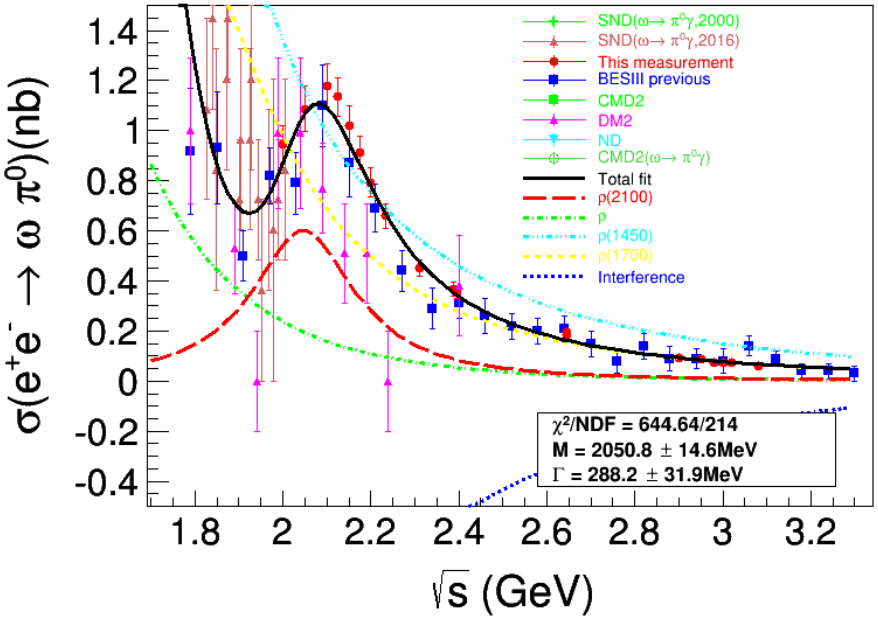
Model2: Masses of rho(1700) and rho(1450) fixed
Data: SND + BESIII





Model2: Masses of rho(1700) and rho(1450) fixed
Data: All the data except BABAR

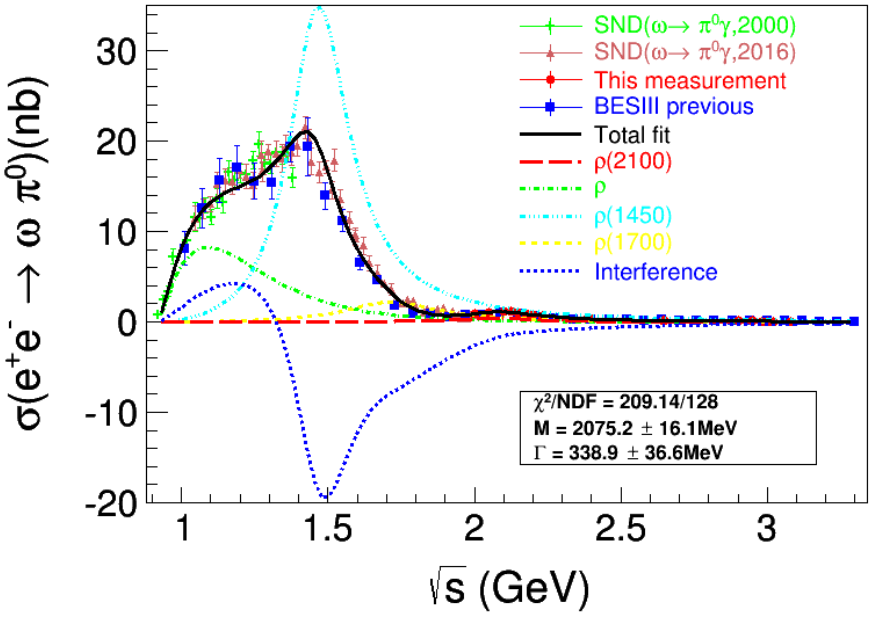


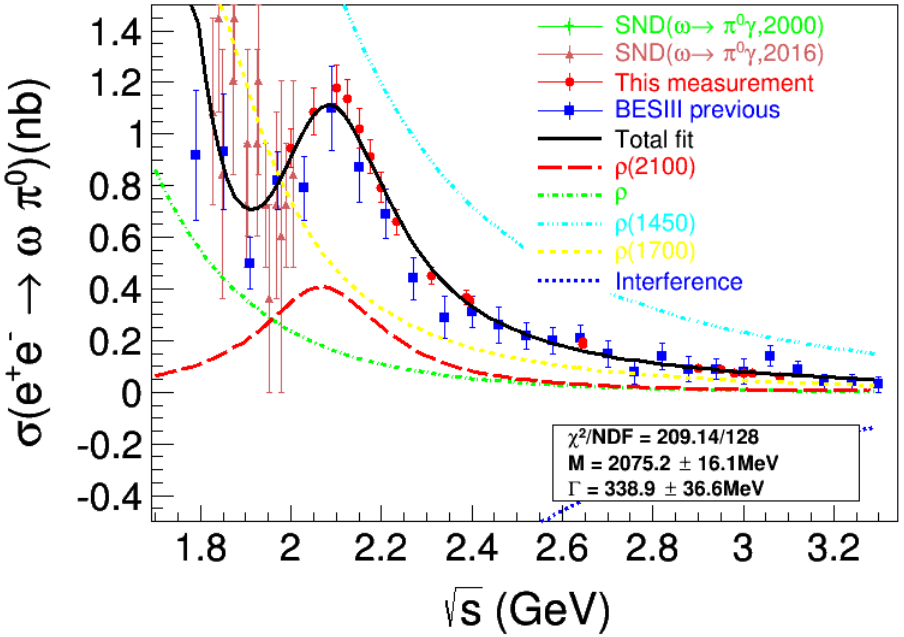


Model2: Masses of rho(1700) and rho(1450) fixed
Data: SND + BESIII

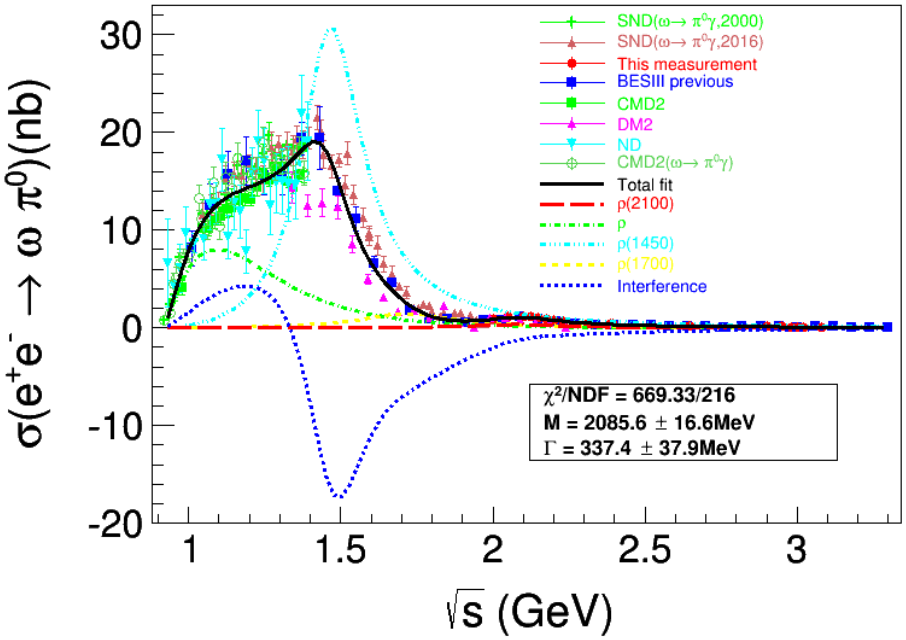
Parameters	Model1 $M_{\rho(1700)}$ & $\Gamma_{\rho(1700)}$ fixed		Model2 $M_{\rho(1700)}$ & $M_{\rho(1450)}$ fixed	
	SND+BESIII	All data - BABAR	SND+BESIII	All data - BABAR
$g_{\rho\omega\pi}$ (GeV^{-1})	16.9±0.3	17.2±0.2	17.2±0.4	17.4±0.2
A_1	0.13±0.01	0.10±0.01	0.16±0.02	0.12±0.01
A_2	0.021±0.004	0.008±0.003	0.081±0.021	0.057±0.009
A_3	0.013±0.002	0.010±0.001	0.017±0.003	0.012±0.002
ϕ_1 (deg.)	116±11	123±8	84±10	101±6
ϕ_2 (deg.)	-95±19	-105±22	-116±9	-108±9
ϕ_3 (deg.)	-141±21	-150±21	145±24	171±8
$M_{\rho(770)}$ (MeV)	≡775.26±0.25	≡775.26±0.25	≡775.26±0.25	≡775.26±0.25
$\Gamma_{\rho(770)}$ (MeV)	≡149.1±0.8	≡149.1±0.8	≡149.1±0.8	≡149.1±0.8
$M_{\rho(1450)}$ (MeV)	1479.8±5.8	1490.2±4.3	=1465	≡1465
$\Gamma_{\rho(1450)}$ (MeV)	353±22	308±15	434±36	351±18
$M_{\rho(1700)}$ (MeV)	≡1720	≡1720	≡1720	≡1720
$\Gamma_{\rho(1700)}$ (MeV)	≡250	≡250	562±88	550±31
$X_{\rho(2100)}$ (MeV)	2115±15	2119±13	2049±18	2051±15
$\Gamma_{\rho(1450)}$ (MeV)	378±38	319±33	328±43	288±32
$(\chi^2/ndf)_{Tot}$	182/126=1.44	650/214=3.04	169/126=1.34	644/214=3.01
$(\chi^2/ndf)_{This}$	24/18=1.33	35/18=1.94	17/18=0.94	18/18=1.00

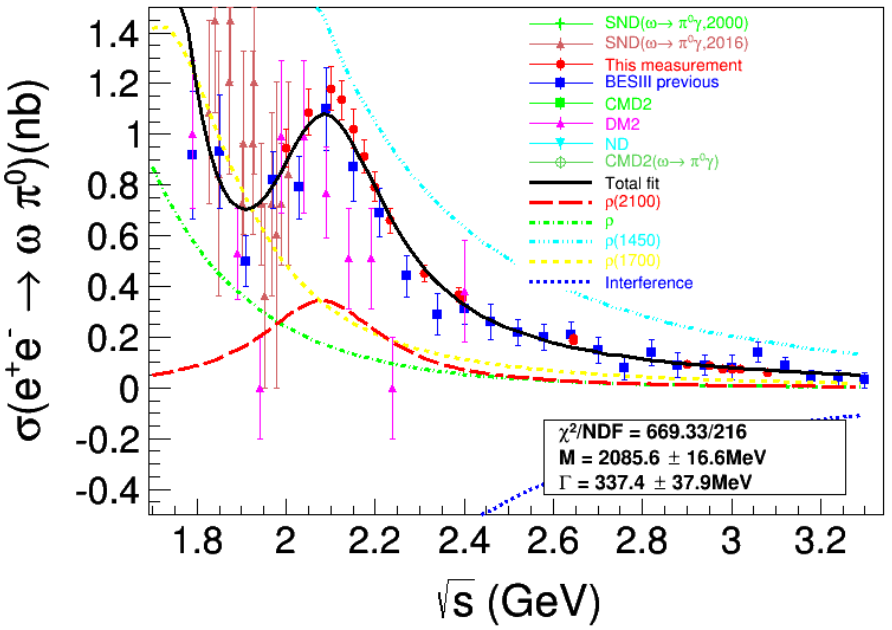
Model3: Both mass and width of rho(1700) and rho(1450) fixed
Data: SND + BESIII





Model3: Both mass and width of rho(1700) and rho(1450) fixed
Data: All data except BABAR





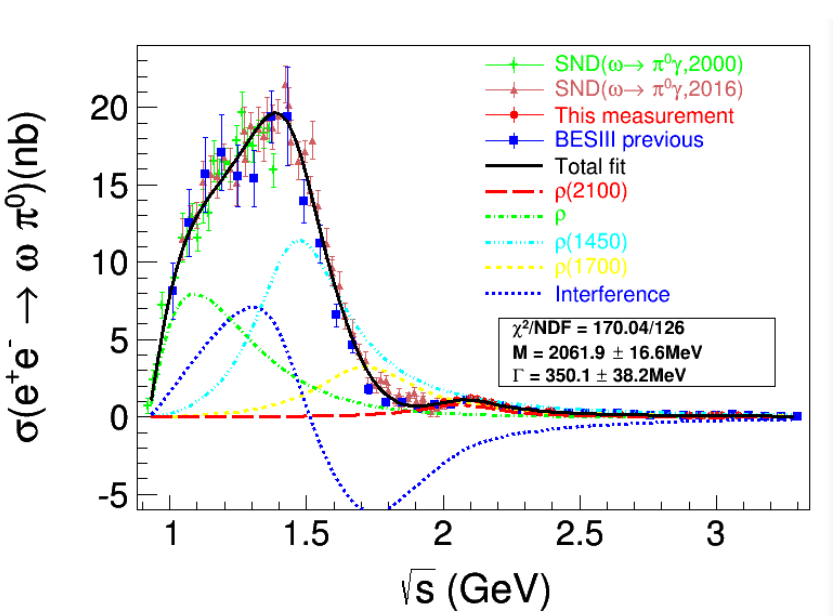
Model3: Both mass and width of rho(1700) and rho(1450) fixed
Data: SND + BESIII

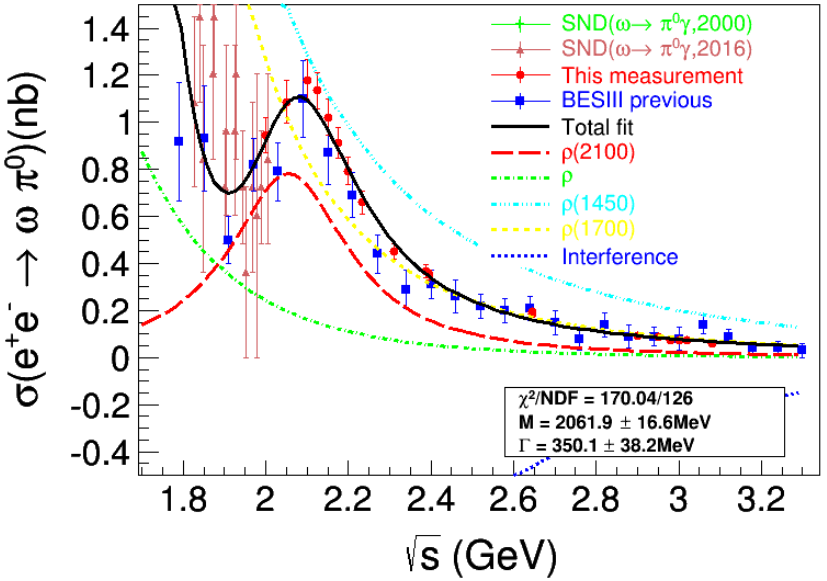
Table III. The fitted parameters to the $e^+e^- \to \omega\pi^0$ Born cross section.

Parameters	Model1 $M_{\rho(1700)}$ & $\Gamma_{\rho(1700)}$ fixed		Model2 $M_{\rho(1700)}$ & $M_{\rho(1450)}$ fixed		Model3 $M_{\rho(1700),\rho(1450)}$ & $\Gamma_{\rho(1700),\rho(1450)}$ fixed	
	SND+BESIII	All data - BABAR	SND+BESIII	All data - BABAR	SND+BESIII	All data - BABAR
$g_{\rho\omega\pi}$ (GeV^{-1})	16.9±0.3	17.2±0.2	17.2±0.4	17.4±0.2	17.4±0.2	17.2±0.2
A_1	0.13±0.01	0.10±0.01	0.16±0.02	0.12±0.01	0.15±0.00	0.14±0.00
A_2	0.021±0.004	0.008±0.003	0.081±0.021	0.057±0.009	0.043±0.003	0.035±0.003
A_3	0.013±0.002	0.010±0.001	0.017±0.003	0.012±0.002	0.011±0.002	0.011±0.001
ϕ_1 (deg.)	116±11	123±8	84±10	101±6	166±4	168±4
ϕ_2 (deg.)	-95±19	-105±22	-116±9	-108±9	1±9	5±11
ϕ_3 (deg.)	-141±21	-150±21	145±24	171±8	260±9	264±10
$M_{\rho(770)}$ (MeV)	≡775.26±0.25	≡775.26±0.25	≡775.26±0.25	≡775.26±0.25	≡775.26±0.25	≡775.26±0.25
$\Gamma_{\rho(770)}$ (MeV)	≡149.1±0.8	≡149.1±0.8	≡149.1±0.8	≡149.1±0.8	≡149.1±0.8	≡149.1±0.8
$M_{\rho(1450)}$ (MeV)	1479.8±5.8	1490.2±4.3	≡1465	≡1465	≡1465	≡1465
$\Gamma_{\rho(1450)}$ (MeV)	353±22	308±15	434±36	351±18	≡250±0	≡250±0
$M_{\rho(1700)}$ (MeV)	≡1720	≡1720	≡1720	≡1720	≡1720	≡1720
$\Gamma_{\rho(1700)}$ (MeV)	≡250	≡250	562±88	550±31	≡400±0	≡400±0
$X_{\rho(2100)}$ (MeV)	2115±15	2119±13	2049±18	2051±15	2075±16	2086±17
$\Gamma_{\rho(1450)}$ (MeV)	378±38	319±33	328±43	288±32	339±37	337±38
$(\chi^2/ndf)_{Tot}$	182/126=1.44	650/214=3.04	169/126=1.34	644/214=3.01	209/128=1.63	669/216=3.10
$(\chi^2/ndf)_{This}$	24/18=1.33	35/18=1.94	17/18=0.94	18/18=1.00	20/18=1.11	24/18=1.33

Backup

Model2: Masses of rho(1700) and rho(1450) fixed
Data: SND + BESIII

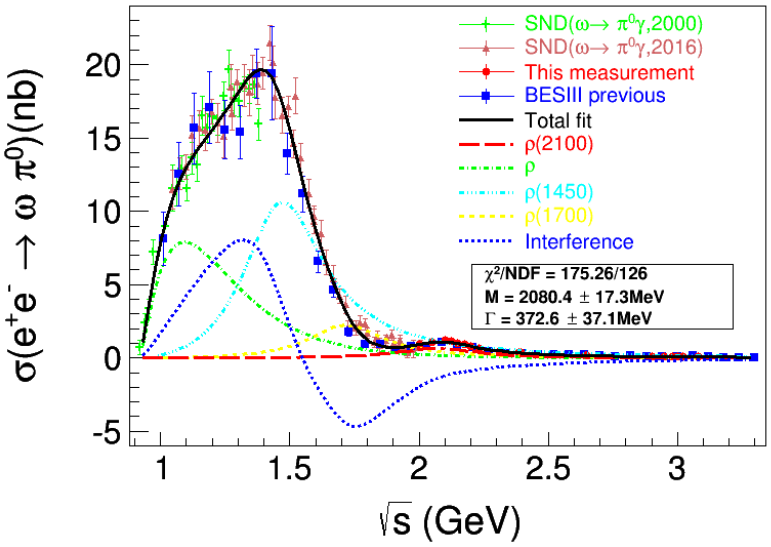


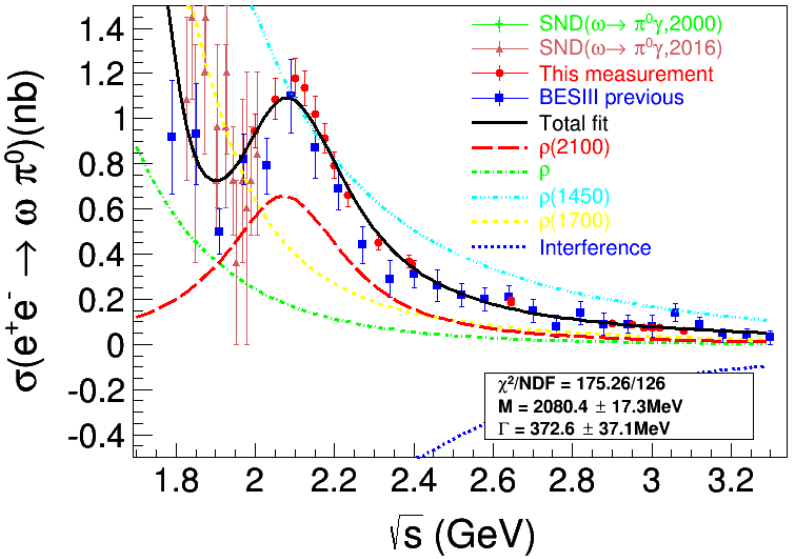


170/126 = 1.35

2Sigma variation allowed for widths of rho(1450) and rho(1700)

Model2: Masses of rho(1700) and rho(1450) fixed
Data: SND + BESIII

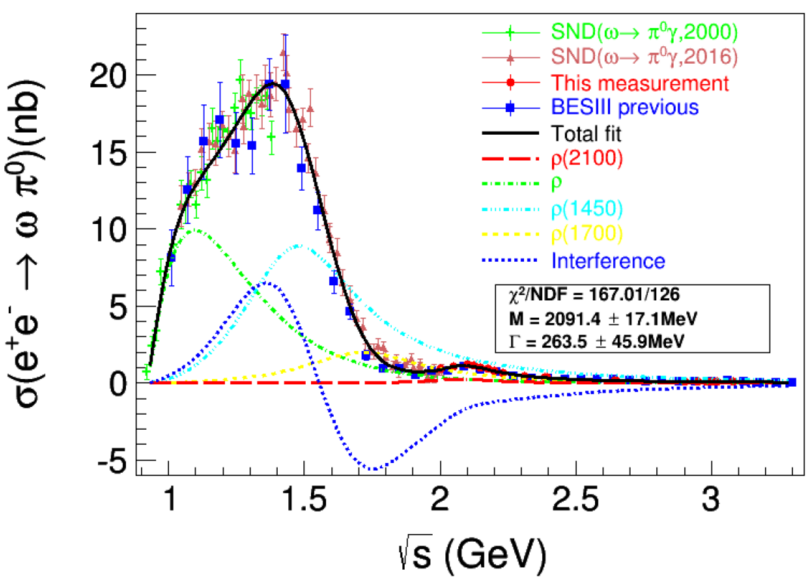


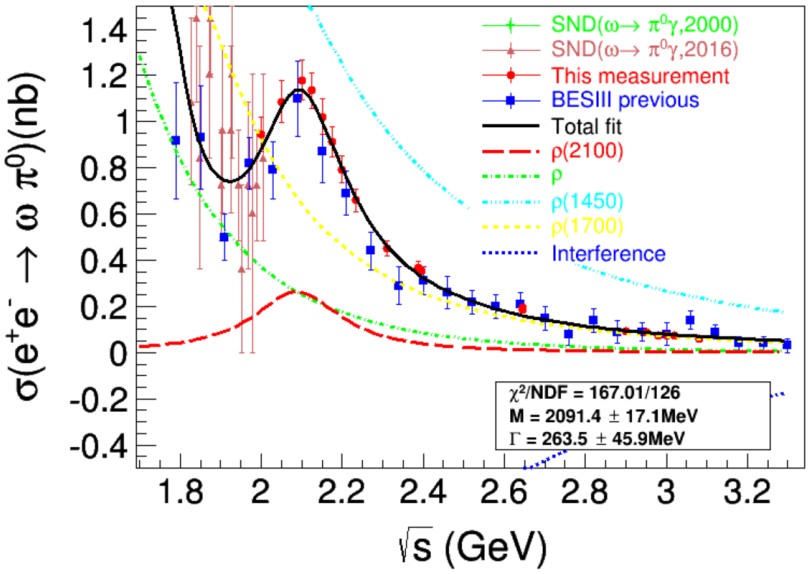


175/126 = 1.39

1Sigma variation allowed for widths of rho(1450) and rho(1700)

Model2: Masses of rho(1700) and rho(1450) fixed
Data: SND + BESIII



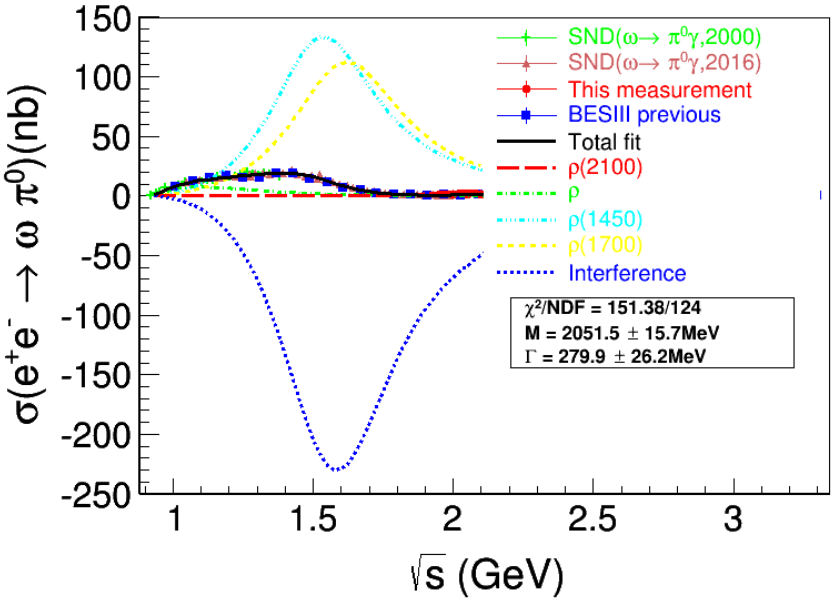


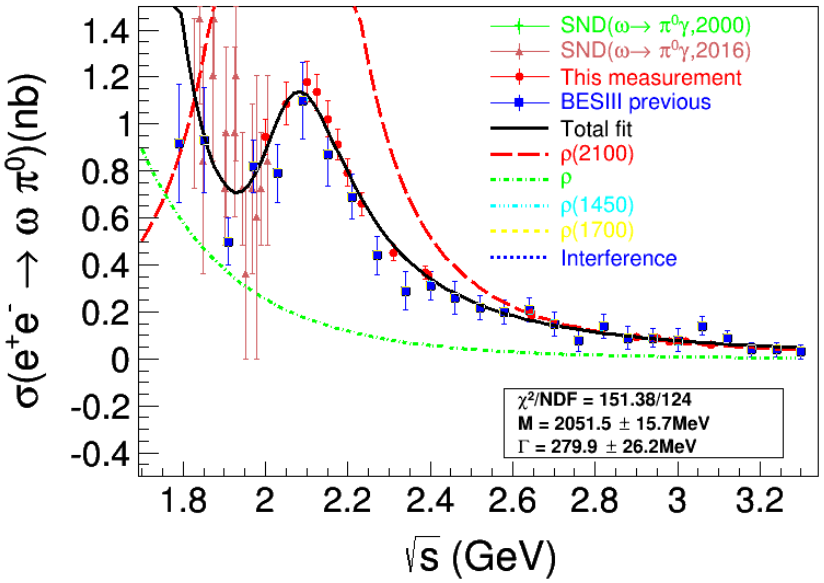
167/126 = 1.33

Have the total scale factor

g factor fixed from SND's result

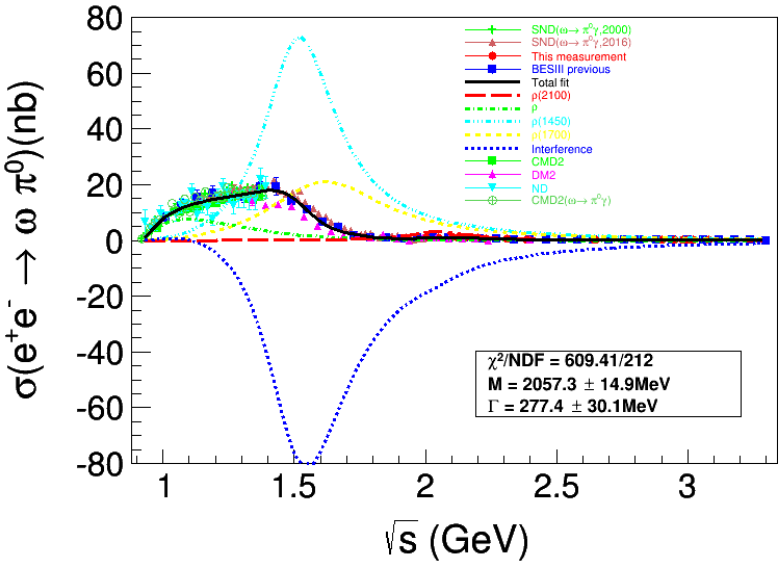
All the parameters free
Two collaborations
call limit

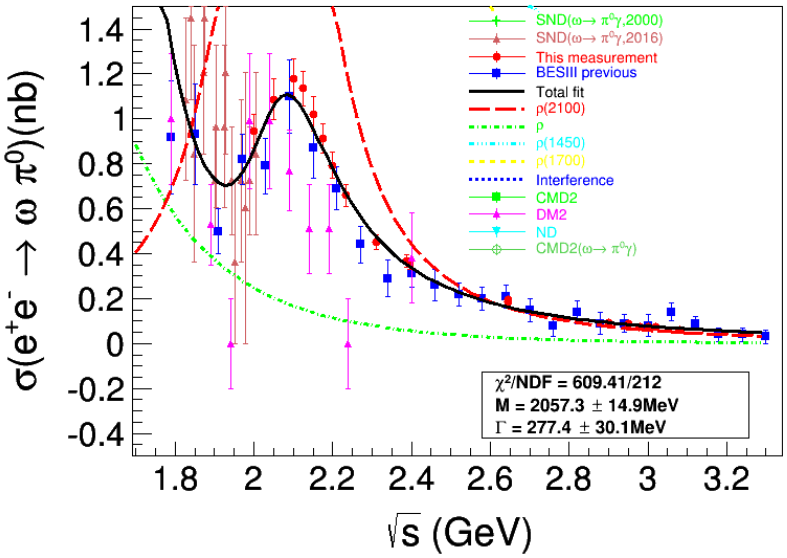





```
Total_Chisq: 15.3017
```

All the parameters free
All data except BABR
Call limit






```
Total Chisq: 18.1984
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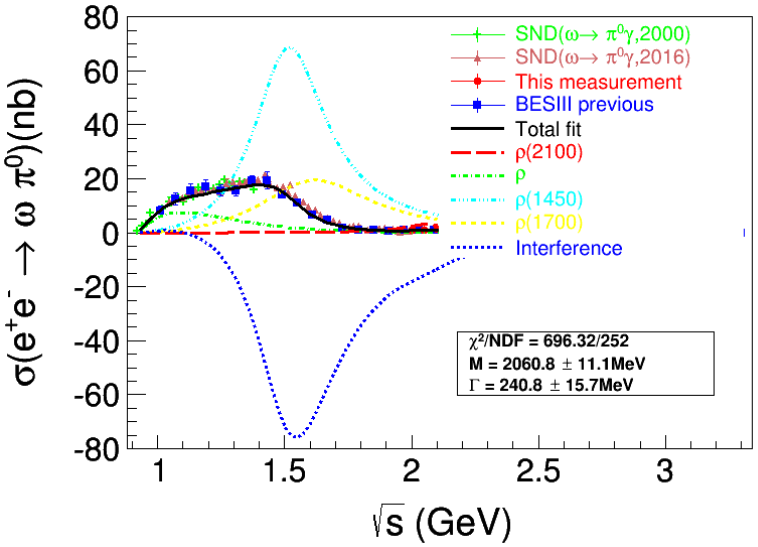
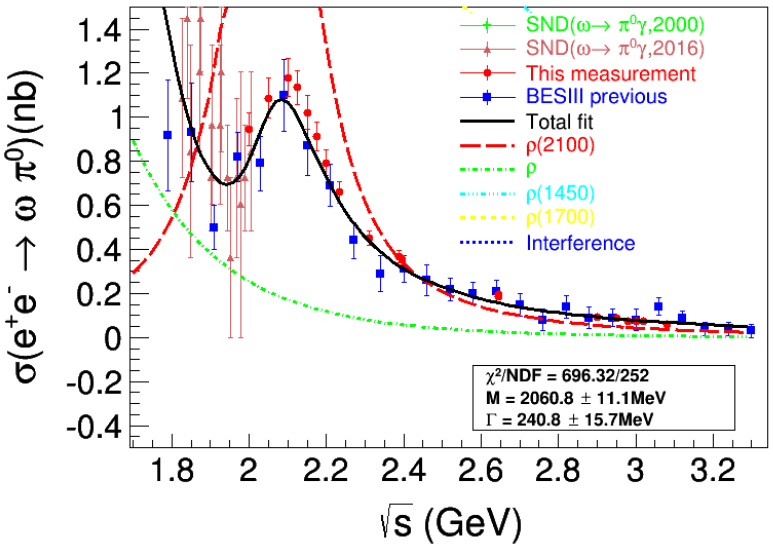
All the parameters free
All data
Call limit






```
Total_Chisq: 37.1266
```