Update of formalism for semileptonic hyperon decays

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Semileptonic meeting
2 March 2021

Decay chain: $\frac{1}{2} \rightarrow \frac{1}{2} + \{0, \pm 1\}$ (1)

- Provide decay parameters to redefine long expressions:

$$\sigma_D^{sl} = \frac{1}{4}(1 \mp \cos\theta_l)^2 |H_{\frac{1}{2}1}|^2 + \frac{1}{4}(1 \pm \cos\theta_l)^2 |H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l (|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2),$$

$$\alpha_D^{sl} = \frac{1}{4}(1 \mp \cos\theta_l)^2 |H_{\frac{1}{2}1}|^2 - \frac{1}{4}(1 \pm \cos\theta_l)^2 |H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l (|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),$$

$$\beta_D^{sl} = \frac{1}{\sqrt{2}} \sin\theta_l ((1 \pm \cos\theta_l)\Im(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) + (1 \mp \cos\theta_l)\Im(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0})),$$

$$\gamma_D^{sl} = \frac{1}{\sqrt{2}} \sin\theta_l ((1 \pm \cos\theta_l)\Re(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) + (1 \mp \cos\theta_l)\Re(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0}))$$

- Relation between the helicity amplitudes and decay parameters doesn't perform $\rightarrow$ Rechecked using `Mathematica`

$$\cancel{\beta_D^{sl} = \sqrt{1 - (\alpha_D^{sl})^2} \sin\phi_D^{sl}} \text{ and } \cancel{\gamma_D^{sl} = \sqrt{1 - (\alpha_D^{sl})^2} \cos\phi_D^{sl}}$$

- Relation doesn't affect the definition of decay parameters
- Main difference: $b_{00} \neq 1 \longrightarrow b_{00} = \sigma_D^{sl}$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}$ (2)

- Non-zero elements of the decay matrix $b_{\mu\nu}$:

$$
\begin{aligned}
b_{00} &= \sigma_D^{sl}, \\
b_{03} &= \alpha_D^{sl}, \\
b_{10} &= \alpha_D^{sl} \cos\phi_p \sin\theta_p, \\
b_{11} &= -(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\theta_p\cos\phi_p \\
&\quad +(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\sin\phi_p, \\
b_{12} &= \quad (\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\theta_p\cos\phi_p \\
&\quad +(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\sin\phi_p, \\
b_{13} &= \sigma_D^{sl}\sin\theta_p\cos\phi_p, \\
b_{20} &= \alpha_D^{sl}\sin\theta_p\sin\phi_p,
\end{aligned}
$$

$$
\begin{aligned}
b_{21} &= -(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\theta_p\sin\phi_p \\
&\quad -(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\phi_p, \\
b_{22} &= \quad (\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\theta_p\sin\phi_p \\
&\quad -(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\phi_p, \\
b_{23} &= \sigma_D^{sl}\sin\theta_p\sin\phi_p, \\
b_{30} &= \alpha_D^{sl}\cos\theta_p, \\
b_{31} &= \quad (\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\sin\theta_p, \\
b_{32} &= -(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\sin\theta_p, \\
b_{33} &= \sigma_D^{sl}\cos\theta_p.
\end{aligned}
$$

- Main parameters: $\theta \equiv \theta_p$, $\phi \equiv \phi_p$,
$\sigma_D^{sl} \equiv \sigma_D^{sl}(\theta_l, q^2)$, $\alpha_D^{sl} \equiv \alpha_D^{sl}(\theta_l, q^2)$, $\beta_D^{sl} \equiv \beta_D^{sl}(\theta_l, q^2)$, $\gamma_D^{sl} \equiv \gamma_D^{sl}(\theta_l, q^2)$
- Each element of $b_{\mu\nu}$ is multiplied by $p = \sqrt{M_+(q^2)M_-(q^2)}/(2M_1)$

Intermediate step (1)

- σ_Λ^{sl}, α_Λ^{sl}, β_Λ^{sl} and γ_Λ^{sl} $\Rightarrow \{n, \alpha, \alpha', \alpha'', \beta_{1,2}, \gamma_{1,2}\}(q^2)$: $g_{av}^\Lambda(q^2)$, $g_w^\Lambda(q^2)$
- Introduce the intermediate parameters:

$$
\begin{aligned}
\text{normalization} \qquad n &= |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 + 2(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2),\\
\alpha &= |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2 + 2(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),\\
\alpha' &= |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 - 2(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2),\\
\alpha'' &= |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2 - 2(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),\\
\beta_{1,2} &= 2(\Im(H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}-1}) \pm \Im(H^*_{\frac{1}{2}1}H_{\frac{1}{2}0})),\\
\gamma_{1,2} &= 2(\Re(H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}-1}) \pm \Re(H^*_{\frac{1}{2}1}H_{\frac{1}{2}0})),
\end{aligned}
$$

where $\beta_{1,2} = \frac{1}{2}\sqrt{n^2 - \alpha^2 - (\alpha')^2 + (\alpha'')^2}\sin\phi_{1,2}$ and $\gamma_{1,2} = \frac{1}{2}\sqrt{n^2 - \alpha^2 - (\alpha')^2 + (\alpha'')^2}\cos\phi_{1,2}$

- $\alpha^2 + (\alpha')^2 - (\alpha'')^2 + 2(\gamma_1^2 + \gamma_2^2 + \beta_1^2 + \beta_2^2) = n^2$
- Using the definition of helicity amplitudes the main parameters to describe semileptonic hyperon decays are:

- $F_1^V(0)$, $F_2^V(0)$, $F_1^A(0)$

or

- $g_{av}^D(0) = \dfrac{F_1^A(0)}{F_1^V(0)}$, $\quad g_w^D(0) = \dfrac{F_2^V(0)}{F_1^V(0)}$

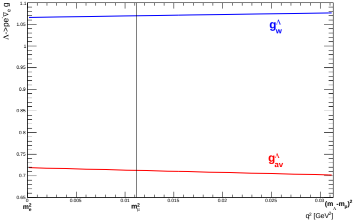
Intermediate step (2)

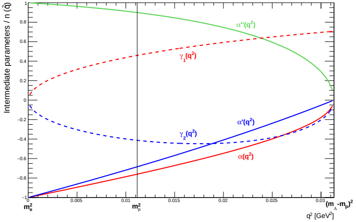
- Relations between intermediate and decay parameters:

$$
\begin{aligned}
n &= ((M_-M_+)^2 - q^4)(1 + (g_{av}^D(q^2))^2) \\
&\quad + q^2 \left(4M_1M_2((g_{av}^D(q^2))^2 - 1) + (g_w^D(q^2))^2 \frac{q^2}{M_1^2}(M_-^2 - q^2)(M_+^2 + q^2) + 4g_w^D(q^2)\frac{q^2}{M_1^2}(M_-^2 - q^2)M_+ \right) \\
\alpha &= 2\sqrt{(M_-M_+)^2 - 2q^2(M_1^2 + M_2^2) + q^4}\left[g_{av}^D(q^2)(q^2 - M_-M_+) + 2g_{av}^D(q^2)g_w^D(q^2)q^2\frac{M_2}{M_1} \right] \\
\alpha' &= (M_-^2 - q^2)(M_+^2 - q^2)\left[-(1 + (g_{av}^D(q^2))^2) + (g_w^D(q^2))^2\frac{q^2}{M_1^2} \right] \\
\alpha'' &= 2\sqrt{(M_-M_+)^2 - 2q^2(M_1^2 + M_2^2) + q^4}\left[g_{av}^D(q^2)(q^2 + M_-M_+) + 2g_{av}^D(q^2)g_w^D(q^2)q^2 \right]
\end{aligned}
$$

where $M_- = M_1 - M_2$ and $M_+ = M_1 + M_2$

- $\{\alpha, \alpha', \alpha'', \beta_{1,2}, \gamma_{1,2}\}(q^2)/n(q^2) \in [-1, +1]$





Intermediate step (3)

- Non-zero elements of the decay matrix $b_{\mu\nu}$:

$$
\begin{array}{ll}
\boxed{b_{00} = \sigma_D^{sl}(\theta_l, q^2),} & b_{21} = -A\cos\theta_p \sin\phi_p - B\cos\phi_p, \\
b_{03} = \alpha_D^{sl}(\theta_l, q^2), & b_{22} = \quad B\cos\theta_p \sin\phi_p - A\cos\phi_p, \\
\boxed{b_{10} = \alpha_D^{sl}(\theta_l, q^2)\cos\phi_p \sin\theta_p,} & b_{23} = \sigma_D^{sl}(\theta_l, q^2)\sin\theta_p \sin\phi_p, \\
b_{11} = -A\cos\theta_p \cos\phi_p + B\sin\phi_p, & \boxed{b_{30} = \alpha_D^{sl}(\theta_l, q^2)\cos\theta_p,} \\
b_{12} = \quad B\cos\theta_p \cos\phi_p + A\sin\phi_p, & b_{31} = \quad A\sin\theta_p, \\
b_{13} = \sigma_D^{sl}(\theta_l, q^2)\sin\theta_p \cos\phi_p, & b_{32} = -B\sin\theta_p, \\
\boxed{b_{20} = \alpha_D^{sl}(\theta_l, q^2)\sin\theta_p \sin\phi_p,} & b_{33} = \sigma_D^{sl}(\theta_l, q^2)\cos\theta_p,
\end{array}
$$

where $A = \frac{1}{2\sqrt{2}}\sin\theta_l[\cos\chi(\gamma_1 + \cos\theta_l\gamma_2) + \sin\chi(\beta_1 + \cos\theta_l\beta_2)]$

and $B = \frac{1}{2\sqrt{2}}\sin\theta_l[\sin\chi(\gamma_1 + \cos\theta_l\gamma_2) - \cos\chi(\beta_1 + \cos\theta_l\beta_2)]$

- $\sigma_D^{sl}(\theta_l, q^2) = n + \alpha'\cos^2\theta_l - (\alpha + \alpha'')\cos\theta_l$
- $\alpha_D^{sl}(\theta_l, q^2) = \alpha + \alpha''\cos^2\theta_l - (n + \alpha')\cos\theta_l$

- To stabilize behavior of decay matrix elements $\Longrightarrow$ normalize all b_{ij} as b_{ij}/b_{00} $\Longrightarrow b_{00} = 1$

Joint angular distribution

- Process $e^+e^- \to (\Lambda \to pe^-\bar{\nu}_e)(\bar{\Lambda} \to \bar{p}\pi^+)$

$$\mathrm{Tr}\rho_{pW\bar{p}} \propto \mathcal{W}(\boldsymbol{\xi};\boldsymbol{\omega}) = \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}} b^{\Lambda}_{\mu 0} a^{\bar{\Lambda}}_{\bar{\nu}0}$$

  - $C_{\mu\bar{\nu}}(\theta_\Lambda;\alpha_\psi,\Delta\Phi)$
  - $b_{\mu 0}$ matrices for $1/2 \to 1/2 + \{0,\pm 1\}$ decays $\Leftrightarrow$ $b^{\Lambda}_{\mu 0} \equiv b_{\mu 0}(\theta_p,\varphi_p,\theta_e,\chi,q^2;\boxed{g^{\Lambda}_{av}, g^{\Lambda}_{w}})$
  - $a_{\bar{\nu}0}$ matrices for $1/2 \to 1/2 + 0$ decays $\Leftrightarrow$ $a^{\bar{\Lambda}}_{\bar{\nu}0} \equiv a_{\bar{\nu}0}(\theta_{\bar{p}},\varphi_{\bar{p}};\boxed{\alpha_{\bar{\Lambda}}})$
    - $\boldsymbol{\xi} \equiv (\theta_\Lambda,\theta_p,\varphi_p,\theta_e,\chi,q^2,\theta_{\bar{p}},\varphi_{\bar{p}})$
    - $\boldsymbol{\omega} \equiv (\alpha_\psi,\Delta\Phi,\boxed{g^{\Lambda}_{av}, g^{\Lambda}_{w}},\boxed{\alpha_{\bar{\Lambda}}})$

- Range of $q^2 \in (m_l^2, (M_1 - M_2)^2)$ is specific for each decay

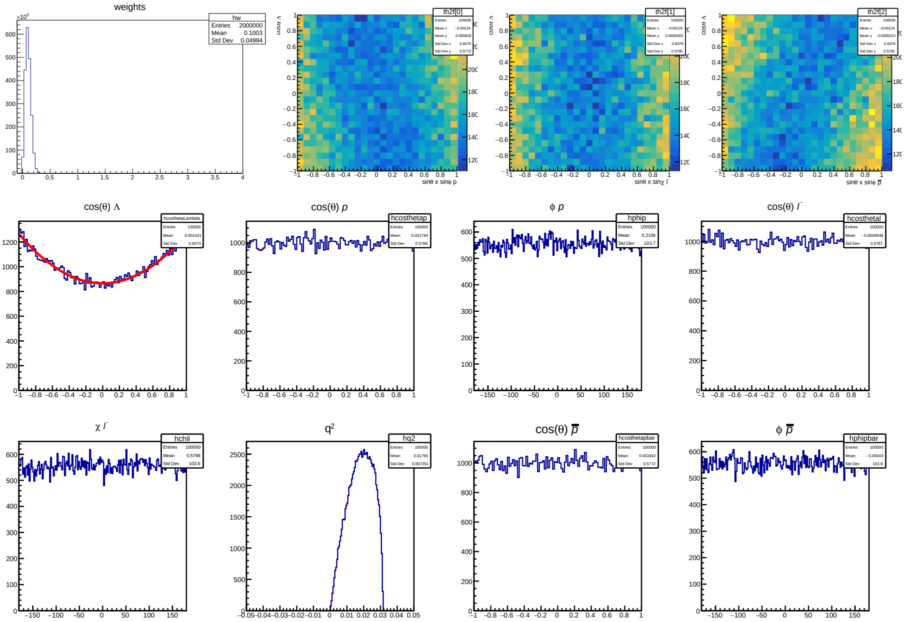
MC generator (step 1)

- Using **`YYbar_example`** package by Patrik, the presented process can be generated
  - MC samples: $N_{\mathrm{evt}}^{\mathrm{sig}}$=$10^5$ and $N_{\mathrm{evt}}^{\mathrm{phsp}}$=$10^6$
  - Generate $q^2 \in (m_e^2, (M_\Lambda - M_p)^2)$
  - Set of input values:

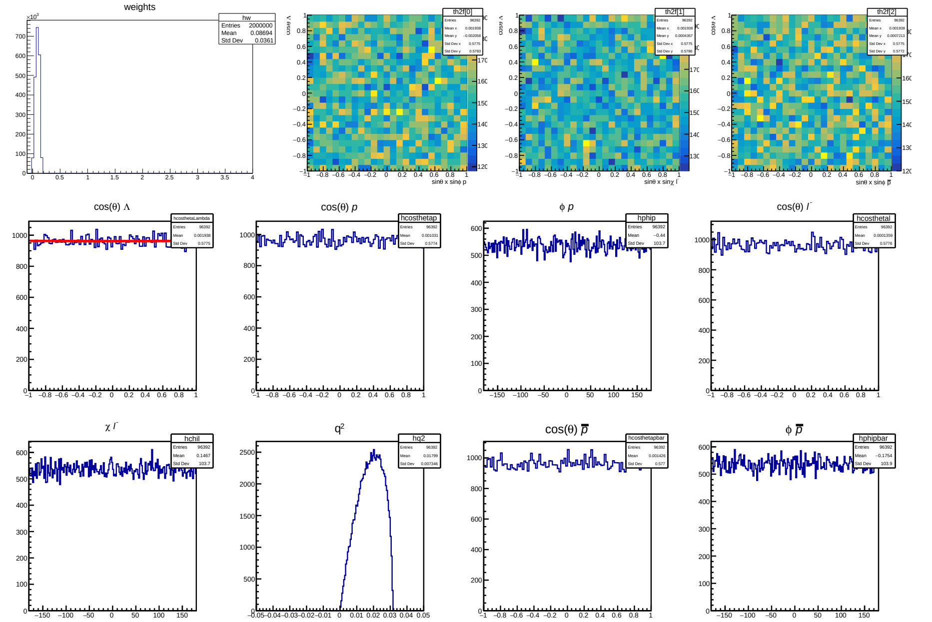
```
pp[0] =  0.461;      // alpha_J/psi (arxiv:1808.08917)
pp[1] =  0.74;       // Delta_Phi (arxiv:1808.08917, equal 42.4deg)
pp[2] =  0.719;       // gav Lambda->p e- nu_ebar
pp[3] =  1.066;       // gw Lambda->p e- nu_ebar
pp[4] = -0.758;       // alpha_D LambdaBar->pbar pi+ (arxiv:1808.08917)
```

- No negative weights are observed
- Maximal weight ∼0.36

Random samples ($N_{sig} = 10^5$) (step 1)



Random samples ($N_{phsp} = 10^6$) (step 1)



Run through fit method (step 2)

- Set input values in the fit

```
void mainMLL(){

    // instantiating the values to be measured
    Double_t pp[4];
    for( int i = 0; i < 4; i++ ) pp[i]=0;

    // starting values for fit
    Double_t alpha_jpsi      = 0.461;    // alpha_J/Psi
    Double_t dphi_jpsi       = 0.74;      // relative phase, Dphi_J/Psi
    Double_t gav_lam_plnu    = 0.719;     // gav (Lam->p l nubar_l)
    Double_t gw_lam_plnu     = 1.066;     // gw (Lam->p l nubar_l)
    Double_t alpha_lam_pbarpip  = -0.758;   // alpha (Lambar->pbar pi+)

    alpha_jpsi        = gRandom->Rndm();
    dphi_jpsi         = gRandom->Rndm();
    gav_lam_plnu    = 0.719;
    gw_lam_plnu    = 1.066;
    alpha_lam_pbarpip = -0.758;

    ReadData();
    ReadMC();
```

- Output value of fit method

Parameter	$N_{sig}^{MC} = 10^5, N_{phsp}^{MC} = 10^6$
α_Ψ	0.4570±0.0110
$\Delta\Phi$	0.7927±0.0252
g_{av}	0.6601±0.0506
g_w	1.1251±0.2727
$\alpha_{\bar{\Lambda}}$	-0.7527±0.0121

	α_Ψ	$\Delta\Phi$	g_{av}	g_w	$\alpha_{\bar{\Lambda}}$
α_Ψ	1	0.281	-0.026	0.032	0.151
$\Delta\Phi$		1	0.016	-0.013	0.269
g_{av}			1	-0.766	0.323
g_w				1	-0.411
$\alpha_{\bar{\Lambda}}$					1

Sensitivity for extracted parameters (1)

- Study the importance of the individual parameters in joint angular distribution and its correlations using likelihood function [PRDD100(2019)11]

$$\mathcal{L}(\boldsymbol{\omega}) = \prod_{i=1}^{N} \mathcal{P}(\boldsymbol{\xi}_i, \boldsymbol{\omega}) \equiv \prod_{i=1}^{N} \frac{\mathcal{W}(\boldsymbol{\xi}_i, \boldsymbol{\omega})}{\int \mathcal{W}(\boldsymbol{\xi}, \boldsymbol{\omega}) d\boldsymbol{\xi}}$$

N is the number of events in the final selection
$\boldsymbol{\xi}_i$ is the full set of kinematic variables describing i-th event
$\boldsymbol{\omega}$ is the full set of individual parameters

- Reduced asymptotic expression of inverse covariant matrix element:

$$V_{kl}^{-1} = N \int \frac{1}{\mathcal{P}} \frac{\partial \mathcal{P}}{\partial \omega_k} \frac{\partial \mathcal{P}}{\partial \omega_l} d\boldsymbol{\xi}$$

- Sensitivity $= \sigma \times \sqrt{N}$
- To eliminate complicated calculation of derivatives the numerical differentiation is used [Numerical Analysis]:

$$\frac{\partial \mathcal{P}}{\partial \omega_k} = \sum_{\omega_1}^{\omega_n} \lim_{h \to 0} \frac{\mathcal{P}(\omega_k + h) - \mathcal{P}(\omega_k)}{h}$$

Sensitivity for extracted parameters (2)

- To simplify estimations, a case where g_w and q^2 equal 0 is considered
- CP asymmetry A_D and A_{av}^D:

$$A_D \equiv \frac{\alpha_D + \alpha_{\bar{D}}}{\alpha_D - \alpha_{\bar{D}}} \quad \text{and} \quad A_{av}^D \equiv \frac{g_{av}^D + g_{av}^{\bar{D}}}{g_{av}^D - g_{av}^{\bar{D}}}$$

- Input values of decay parameters [NaturePhys.15(2019)631]

Decay	α_ψ	$\Delta\Phi$	α_Λ	g_{av}^Λ	g_w^Λ
$J/\psi \to \Lambda\bar{\Lambda}$	0.461 ± 0.006	0.740 ± 0.010			
$\Lambda \to p\pi^-$			0.750 ± 0.010		
$\Lambda \to pe^-\bar{\nu}_e$				0.719	0

- Sensitivities ($\sigma \times \sqrt{N}$) for extracted parameters (preliminary)

Decay	α_ψ	$\Delta\Phi$	α_Λ	g_{av}^Λ	$\langle\alpha_\Lambda\rangle$	A_Λ	$\langle g_{av}^\Lambda\rangle$	A_{av}^Λ
$\Lambda\bar{\Lambda} \to (p\pi^-)(\bar{p}\pi^+)$	3.43	7.47	6.83		1.76	8.81		
$\Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}\pi^+)$ $(q^2 = 0, g_w = 0)$	3.19	7.13	6.97	21.0				
$\Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}e^+\nu_e)$ $(q^2 = 0, g_w = 0)$	2.79	6.61		21.1			2.98	28.9

$J/\psi \to \Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}e^+\nu_e)$

	g_{av}^Λ	$g_{av}^{\bar{\Lambda}}$	$\alpha_{\bar{\Lambda}}$	α_ψ	$\Delta\Phi$
g_{av}^Λ	1	0.96		-0.02	0.02
$g_{av}^{\bar{\Lambda}}$		1		0.05	0.03
$\alpha_{\bar{\Lambda}}$	0.93		1		
α_ψ	-0.06		0.01	1	0.02
$\Delta\Phi$	0.04		0.10	0.18	1

$J/\psi \to \Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}\pi^+)$

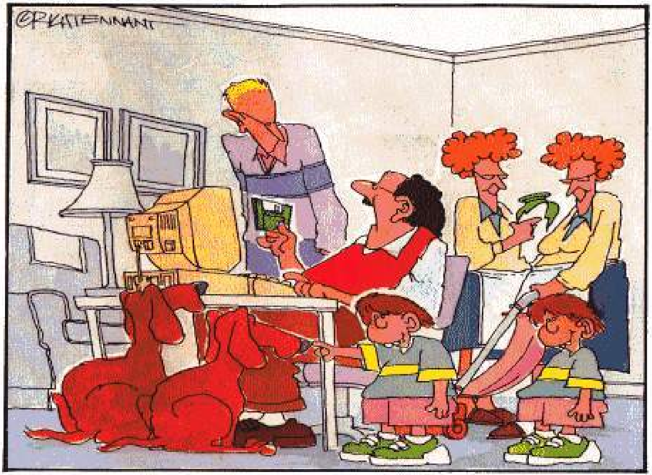
* large correlation between g_{av}^Λ, $g_{av}^{\bar{\Lambda}}$ and $\alpha_{\bar{\Lambda}}$

ToDo list and next steps

- Test formalism using
  1. Production of `mDIY` and `MC PhSp` for $e^+e^- \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$
     - ◦ true and reco `mDIY` and `MC PhSp`
     - ◦ allow to extract true and reco values of α_Λ and $\alpha_{\bar{\Lambda}}$ decay parameters
     - ♠ $\alpha_\Lambda^{\text{true}}$ and $\alpha_{\bar{\Lambda}}^{\text{true}}$ are extracted and verified
  2. Modification of `mDIY` to include the semileptonic decay formalism
     - ♣ Need to rewrite
  3. Production of true and reco `mDIY` and `MC PhSp` for $\Lambda \to pe^-\bar{\nu}_e$
     - ◦ extraction of the g_{av}^Λ and g_w^Λ decay parameters
     - ♣ Need to reprocess
  4. If all steps work, consider more difficult scenario, mixed `MC` samples
  5. If previous step works, move to the real data

- Additional steps:
  - Estimate sensitivity to g_{av}^Λ and g_w^Λ decay parameters
    - ♠ Preliminary result for g_{av}^Λ when $g_w^\Lambda = q^2 = 0$
    - ◦ Include dependence on q^2
    - ◦ Include g_w^Λ

Backups



"I ALWAYS BACK UP EVERYTHING."

CP violation in non-leptonic decays

[Andrzej's slides]

$$A_\Lambda = \frac{\alpha_\Lambda + \alpha_{\bar{\Lambda}}}{\alpha_\Lambda - \alpha_{\bar{\Lambda}}}$$

- BESIII result: $A_\Lambda = -0.006 \pm 0.012 \pm 0.007$ [NaturePhys.15(2019)631]
- CKM: $-3 \cdot 10^{-5} \leq A_\Lambda \leq 4 \cdot 10^{-5}$ [PRD67(2003)056001]
  Extensions of SM: $A_{\Lambda \to p\pi^-} \sim (0.05 - 1.2) \cdot 10^{-4}$ [Chin.Phys.C42(2018)013101]

Experiment	N_{evt}	$\sigma(A_\Lambda)$	
BESIII (2018)	$4.2 \cdot 10^5$	$1.2 \cdot 10^{-2}$	$N_{J/\psi} = 1.31 \cdot 10^9$
Some estimations			
BESIII	$3 \cdot 10^6$	$5 \cdot 10^{-3}$	$N_{J/\psi} = 10^{10}$ $\mathcal{L} = 0.47 \cdot 10^{33}/\mathrm{cm}^2/\mathrm{s}$, ΔE=0.9 MeV
SuperTauCharm	$6 \cdot 10^8$	$3 \cdot 10^{-4}$	$N_{J/\psi} = 2 \cdot 10^{12}$ $\mathcal{L} = 10^{35}/\mathrm{cm}^2/\mathrm{s}$, ΔE=0.9 MeV
SuperTauCharm + reduced ΔE	$3 \cdot 10^9$	$1.4 \cdot 10^{-4}$	$N_{J/\psi} = 10^{13}$ $\mathcal{L} = 10^{35}/\mathrm{cm}^2/\mathrm{s}$, ΔE<0.9 MeV (?)

CP symmetry test A_{Λ}^{av}

$$A_{\Lambda}^{av} = \frac{g_{av}^{\Lambda}(0) + g_{av}^{\bar{\Lambda}}(0)}{g_{av}^{\Lambda}(0) - g_{av}^{\bar{\Lambda}}(0)}$$

- $g_{av}(0) = \frac{F_1^A(0)}{F_1^V(0)}$
  - Determination of $F_1^V(0)$ value by CVC hypothesis: $F_1^V(0) < 0$
  - No determination of $F_1^A(0)$ value by any general theoretical arguments
- Some assumptions:
  - $F_1^A(0) \lesseqgtr 0$ then $g_{av}(0) \gtreqless 0$
  - ? Naïve assumption $g_{av}^{\Lambda} = -g_{av}^{\bar{\Lambda}}$

    $\Longrightarrow$ Need to be care with sign of g_{av}

Experiment	N_{evt}	Result	Reference
E555 (Fermilab)	37286	$g_{av}(0) = 0.719 \pm 0.016 \pm 0.012$ constrain on $g_w(0) = 0.97$	[PRD41(1990)780]
SPS (CERN)	7111	$g_{av}(0) = 0.70 \pm 0.03$ measured $F_2^V(0) = 1.32 \pm 0.81$	[ZPC21(1983)1]
AGS (BNL)	10^4	$\lvert g_{av}(0)\rvert = 0.734 \pm 0.031$ used $g_w(0) = 0.97$	[PLB98(1981)123]

Semileptonic Λ decay

- Momenta and masses: $\Lambda(p_1, M_1) \to p(p_2, M_2) + e^-(p_e, m_e) + \bar{\nu}_e(p_{\bar{\nu}_e}, 0)$
- Standard set of invariant FF for weak current-induced baryonic $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions:

$$M_\mu = M_\mu^V + M_\mu^A = \langle p(p_2)|J_\mu^{V+A}|\Lambda(p_1)\rangle =$$

$$= \bar{u}(p_2)\left[\gamma_\mu\left(F_1^V(q^2) + F_1^A(q^2)\gamma_5\right) + \frac{i\sigma_{\mu\nu}q^\nu}{M_1}\left(F_2^V(q^2) + F_2^A(q^2)\gamma_5\right) + \frac{q_\mu}{M_1}\left(F_3^V(q^2) + F_3^A(q^2)\gamma_5\right)\right]u(p_1)$$

where $q_\mu = (p_1 - p_2)_\mu$

- For $\Lambda \to pe^-\bar{\nu}_e$ at $\mathcal{O}(\frac{m_e^2}{2q^2}) \to 0 \Rightarrow F_3^{V,A} \to 0$

- Helicity amplitude is $H_{\lambda_2\lambda_W} = (H^V_{\lambda_2\lambda_W} + H^A_{\lambda_2\lambda_W})$ with $(\lambda_2 = \pm\frac{1}{2};\ \lambda_W = 0, \pm 1)$:

vector

$$H^V_{\frac{1}{2}1} = \sqrt{2M_-}\left(-F_1^V - \frac{M_1 + M_2}{M_1}F_2^V\right),$$

$$H^V_{\frac{1}{2}0} = \frac{\sqrt{M_-}}{\sqrt{q^2}}\left((M_1 + M_2)F_1^V + \frac{q^2}{M_1}F_2^V\right),$$

axial-vector

$$H^A_{\frac{1}{2}1} = \sqrt{2M_+}\left(F_1^A - \frac{M_1 - M_2}{M_1}F_2^A\right),$$

$$H^A_{\frac{1}{2}0} = \frac{\sqrt{M_+}}{\sqrt{q^2}}\left(-(M_1 - M_2)F_1^A + \frac{q^2}{M_1}F_2^A\right).$$

where $M_\pm(q^2) = (M_1 \pm M_2)^2 - q^2$; $H^{V,A}_{-\lambda_2,-\lambda_W} = \pm H^{V,A}_{\lambda_2\lambda_W}$

Form factors

$$F_i^{V,A}(q^2) = F_i^{V,A}(0)\prod_{n=0}^{n_i}\frac{1}{1-\frac{q^2}{m^2_{V,A}+n\alpha'^{-1}}} \approx F_i^{V,A}(0)\left(1+q^2\sum_{n=0}^{n_i}\frac{1}{m^2_{V,A}+n\alpha'^{-1}}\right) \equiv F_i^{V,A}(0)c_i^{V,A}(q^2)$$

	$F_i^{V,A}(0)(\Lambda \to p)$	$m_{V,A}$	α' [GeV^{-2}]	n_i
$F_1^V(q^2)$	$-\sqrt{\frac{3}{2}}$[1]	$m_{K^*(892)^0}$ ($J^P = 1^-$)	0.9	$n_1 = 1$
$F_2^V(q^2)$	$\frac{M_\Lambda \mu_p}{2M_p}F_1^V(0)$[2]			$n_2 = 2$
$F_3^V(q^2)$	0[4]	-		$n_3 = 2$
$F_1^A(q^2)$	$0.719F_1^V(0)$[3]	$m_{K^*(1270)^0}$ ($J^P = 1^+$)		$n_1 = 1$
$F_2^A(q^2)$	0[4]	–		$n_2 = 2$
$F_3^A(q^2)$	$\frac{M_\Lambda(M_\Lambda+M_p)}{(m_{K^-})^2}F_1^A(0)$[4]	m_K ($J^P = 0^-$)		$n_3 = 2$

- [1] [PR135(1964)B1483], [PRL13(1964)264]
- [2] $\mu_p = 1.793$ [Lect.NotesPhys.222(1985)1], [Ann.Rev.Nucl.Part.Sci.53(2003)39], [JHEP0807(2008)132]
- [3] [PRD41(1990)780]
- [4] Vanish in the $SU(3)$ symmetry limit; Goldberger-Treiman relation [PR110(1958)1178], [PR111(1958)354]

Helicity amplitudes of the lepton pair $h^l_{\lambda_l \lambda_\nu}$

- Lepton and antineutrino spinors

$$\bar{u}_{l^-}(\mp\frac{1}{2}, p_{l^-}) = \sqrt{E_l + m_l}\left(\chi^\dagger_\mp, \frac{\pm|\vec{p}_{l^-}|}{E_l + m_l}\chi^\dagger_\mp\right),$$

$$v_{\bar{\nu}}(\frac{1}{2}) = \sqrt{E_\nu}\begin{pmatrix}\chi_+ \\ -\chi_+\end{pmatrix},$$

where $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$ and $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$ are Pauli two-spinors

- SM form of the lepton current ($\lambda_W = \lambda_{l^-} - \lambda_{\bar{\nu}}$)

$$h^l_{\lambda_{l^-}=\mp 1/2, \lambda_{\bar{\nu}}=1/2} = \bar{u}_{l^-}(\mp\frac{1}{2})\gamma^\mu(1+\gamma_5)v_{\bar{\nu}}(\frac{1}{2})\begin{Bmatrix}\epsilon_\mu(-1)\\ \epsilon_\mu(0)\end{Bmatrix}$$

where $\epsilon^\mu(0) = (0;0,0,1)$ and $\epsilon^\mu(\mp 1) = (0;\mp 1, -i, 0)/\sqrt{2}$

- Moduli squared of the helicity amplitudes [EPJ C59 (2009) 27]

$$\text{nonflip}(\lambda_W = \mp 1) : |h^l_{\lambda_l=\mp\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8(q^2 - m_l^2),$$

$$\text{flip}(\lambda_W = 0) : |h^l_{\lambda_l=\pm\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8\frac{m_l^2}{2q^2}(q^2 - m_l^2)$$

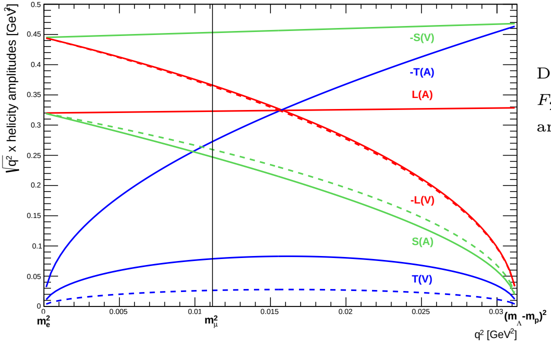
- Upper and lower signs refer to the configurations $(l^-, \bar{\nu}_l)$ $(\lambda_\nu = 1/2)$ and (l^+, ν_l) $(\lambda_\nu = -1/2)$, respectively
- In case of the e-mode only nonflip transition remains under assumption $\frac{m_e^2}{2q^2} \to 0$

Size estimations of helicity amplitudes

$\mathrm{T(V,A)} := \sqrt{q^2}\, H^{V,A}_{\frac{1}{2}1}$

$\mathrm{L(V,A)} := \sqrt{q^2}\, H^{V,A}_{\frac{1}{2}0}$

$\mathrm{S(V,A)} := \sqrt{q^2}\, H^{V,A}_{\frac{1}{2}t}$



Dashed lines: $F_2^V(q^2)$ and $F_3^A(q^2)$ are switched off

- If $q^2 = m_e^2 \Rightarrow H^V_{\frac{1}{2}0}$ and $H^A_{\frac{1}{2}0}$ are dominated
- If $q^2 = (M_\Lambda - M_p)^2 \Rightarrow H^A_{\frac{1}{2}1} = -\sqrt{2} H^A_{\frac{1}{2}0}$ are dominated
- Using data of the E-555 experiment (Fermilab) [PRD41 (1990) 780]

  $\Rightarrow \frac{F_1^A(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.731 \pm 0.016$ and $\frac{F_2^V(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.15 \pm 0.30$

  $\Rightarrow \frac{F_1^A(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.719 \pm 0.016$ with constraint $\frac{F_2^V(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) \to 0.97$ (CVC)

Boundary case: $q^2_{\min}$

- $q^2_{\min} = m_e^2 \longrightarrow 0$:

$$\begin{aligned}
b_{00} &= (1 + (g_{av}^D(0))^2)\sin^2\theta_l, \\
b_{03} &= -2g_{av}^D(0)\sin^2\theta_l, \\
b_{10} &= b_{03}\sin\theta_p\cos\phi_p, \\
b_{13} &= b_{00}\sin\theta_p\cos\phi_p, \\
b_{20} &= b_{03}\sin\theta_p\sin\phi_p, \\
b_{23} &= b_{00}\sin\theta_p\sin\phi_p, \\
b_{30} &= b_{03}\cos\theta_p, \\
b_{33} &= b_{00}\cos\theta_p
\end{aligned}$$