

Equivalent $SU(3)_f$ approaches for two-body Anti-triplet charmed baryon decays

萧佑国

Yu-Kuo Hsiao

Shanxi Normal university

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In collaboration with
Y.L. Wang and H.J. Zhao

Outline:

1. Introduction
2. Formalism
3. Results
4. Summary

Introduction (IRA and TDA)

- IRA: irreducible $SU(3)_f$ approach (amplitude)

$\mathbf{B}_c$, $\mathbf{B}$, M , and $\mathcal{H}_{eff}$ in the irreducible forms of $SU(3)_f$,
connected as the invariant $SU(3)_f$ amplitudes (a_i)

[Savage, Springer, PRD42, 1527 (1990)]

$$\mathcal{H}_{\text{IRA}} \sim c_- H(6) + c_+ H(\overline{15}), \quad c_- > c_+$$

a_1, a_2, a_3 , QCD-favored

a_4, a_5, a_6, a_7 , QCD-disfavored

C.D. Lu, W. Wang and F.S. Yu [PRD93, 056008 (2016)],

“Test flavor $SU(3)$ symmetry in exclusive Λ_c decays,”

Geng, Hsiao, Y.H. Lin and L.L. Liu [PLB776, 265 (2017)]

able to explain $\mathcal{B}(\mathbf{B}_c \rightarrow \mathbf{B}M)$

Introduction (IRA and TDA)

- One prefers topological diagrams for the decay: factorizable, non-factorizable, W -boson exchange
- BESIII [PLB783, 200 (2018)]

Feynman diagrams of $\Lambda_c^+ \rightarrow \Xi^{(*)0} K^+$

IRA: $\mathcal{M}(\Lambda_c^+ \rightarrow \Xi^0 K^+) = -2(a_2 - \frac{a_4 + a_7}{2})$

Pole model + current algebra:

J. Zou, F. Xu, G. Meng and H.Y. Cheng, PRD101, 014011 (2020),

“Two-body hadronic weak decays of antitriplet charmed baryons.”

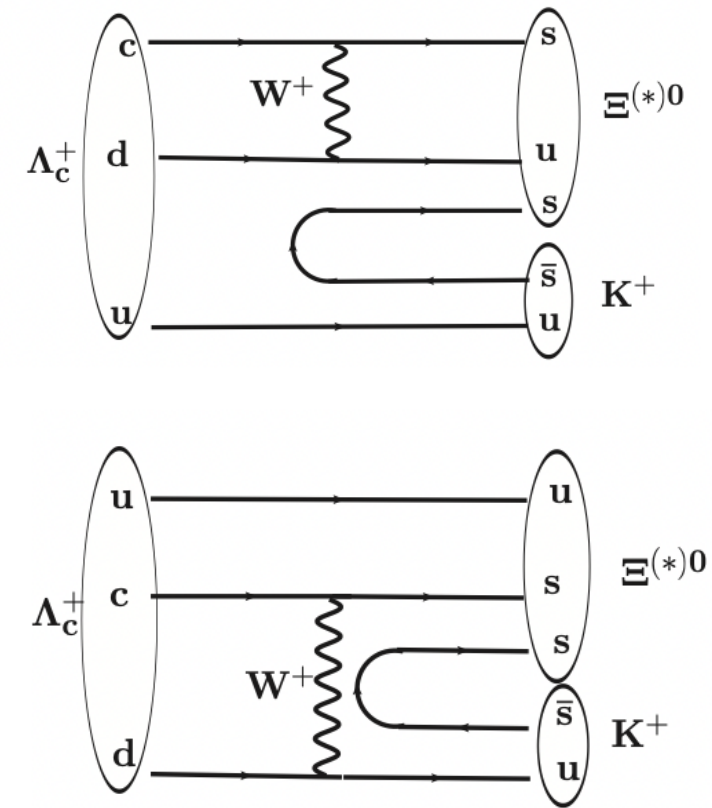
Long-distance triangle rescattering:

H.W. Ke and X.Q. Li [PRD102, 113013 (2020)]

“A natural interpretation on the data of $\Lambda_c \rightarrow \Sigma\pi$.”

Yu, Hsiao [PLB820, 136586 (2021)]

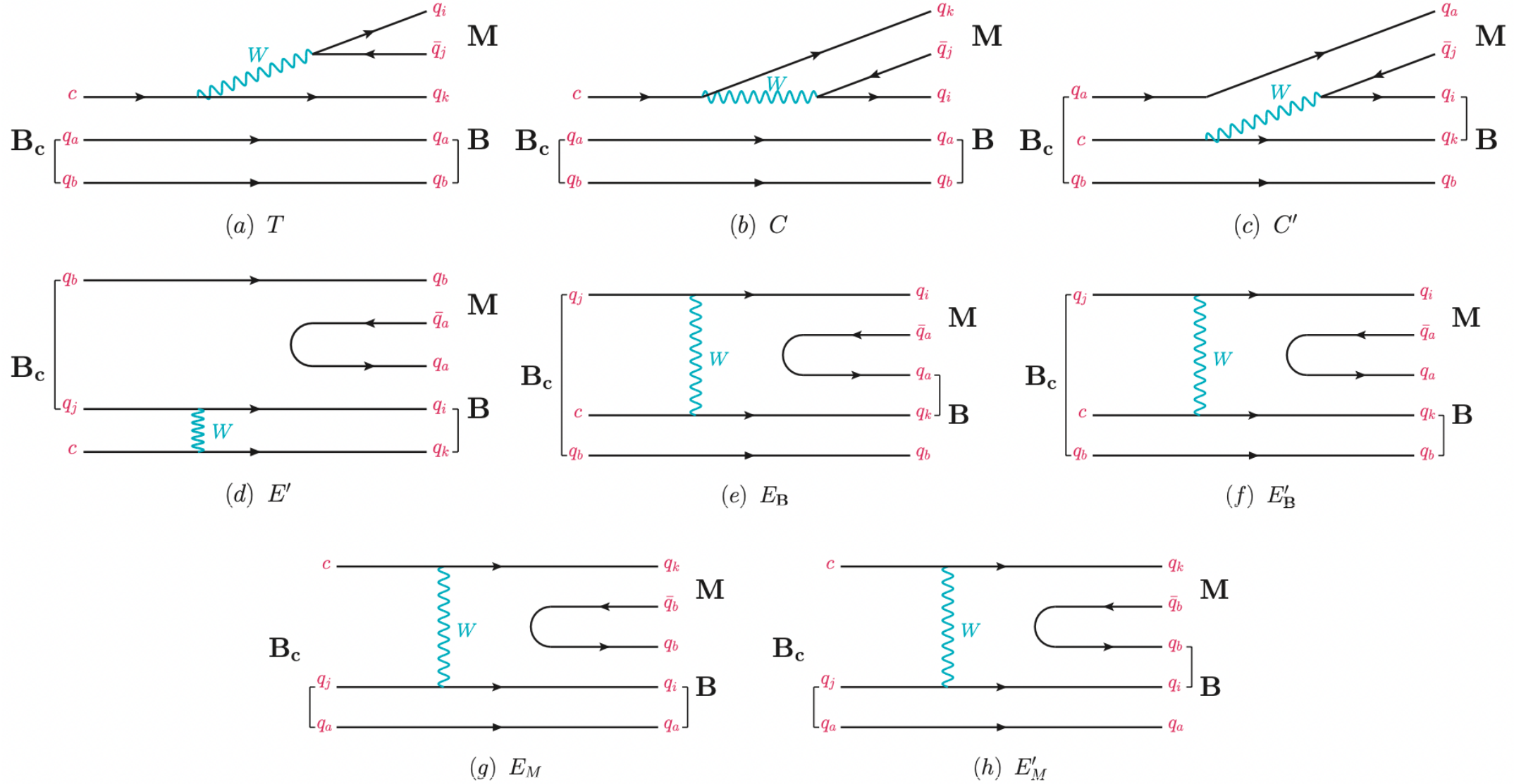
“Cabibbo-favored $\Lambda_c^+ \rightarrow \Lambda a_0(980)^+$ decay in the final state interaction.”



- TDA: topological $SU(3)_f$ approach (amplitude)

$SU(3)_f$ symmetry+topological diagrams

more approachable, more information



Ξ_c^0, Ξ_c^+ and Λ_c^+ : $(ds - sd)c$, $(su - us)c$ and $(ud - du)c$

$\mathbf{B} \sim (q_a - q_k)q_b [(q_i - q_a)q_b]$ for $E_{\mathbf{B}(M)}$, $\mathbf{B} \sim (q_k - q_b)q_a [(q_b - q_i)q_a]$ for $E'_{\mathbf{B}(M)}$

- Studies with TDA

Y. Kohara [PRD44, 2799 (1991)]

“Quark diagram analysis of charmed baryon decays,”

L.L. Chau, H.Y. Cheng and B. Tseng [PRD54, 2132 (1996)]

“Analysis of two-body decays of charmed baryons
using the quark diagram scheme”

H.J. Zhao, Y.L. Wang, Hsiao and Y. Yu [JHEP02, 165 (2020)]

“A diagrammatic analysis of two-body charmed baryon decays
with flavor symmetry.”

Hsiao, Q. Yi, S.T. Cai and H.J. Zhao [EPJC80, 1067 (2020)]

“Two-body charmed baryon decays involving decuplet baryon
in the quark-diagram scheme.”

Equivalent $SU(3)_f$ approaches for $B_c \rightarrow BM$

- Both based on $SU(3)_f$ symmetry,

TDA and IRA can be equivalent

X.G. He and W. Wang [CPC42, 103108 (2018)]

“Flavor $SU(3)$ Topological Diagram and

Irreducible Representation Amplitudes

for Heavy Meson Charmless Hadronic Decays: Mismatch and Equivalence,”

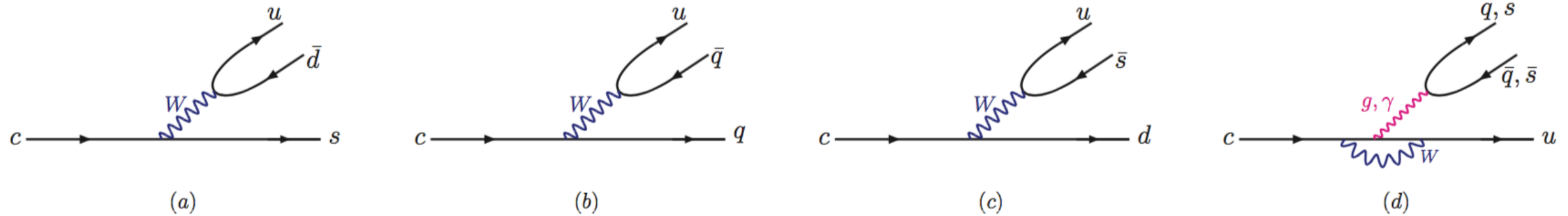
X.G. He, Y.J. Shi and W. Wang [EPJC80, 359 (2020)]

“Unification of Flavor $SU(3)$ Analyses of Heavy Hadron Weak Decays,”

Hsiao, Q. Yi, S.T. Cai and H.J. Zhao [EPJC80, 1067 (2020)]

“Two-body charmed baryon decays involving decuplet baryon
in the quark-diagram scheme.”

Formalism



• IRA

$$\mathcal{H}_{\text{IRA}} = c_- \frac{\epsilon^{ijl}}{2} H(6)_{lk} + c_+ H(\overline{15})_k^{ij}$$

$$\mathbf{B}_c(\mathbf{B}_{ci}) = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+)$$

$$\mathbf{B}_j^i : (n, p, \Sigma^{\pm,0}, \Xi^{-,0}, \Lambda)$$

$$M_j^i : (\pi^{\pm,0}, K^{\pm}, K^0, \bar{K}^0, \eta)$$

• TDA

$$\mathcal{H}_{\text{TDA}} = H_j^{ki} [H_2^{31} = 1]$$

$$\mathbf{B}_c(\mathbf{B}_c^{ij}) = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}$$

$$\mathbf{B}_{ijk} = \epsilon_{ijl} \mathbf{B}_k^l$$

$$M_j^i : (\pi^{\pm,0}, K^{\pm}, K^0, \bar{K}^0, \eta)$$

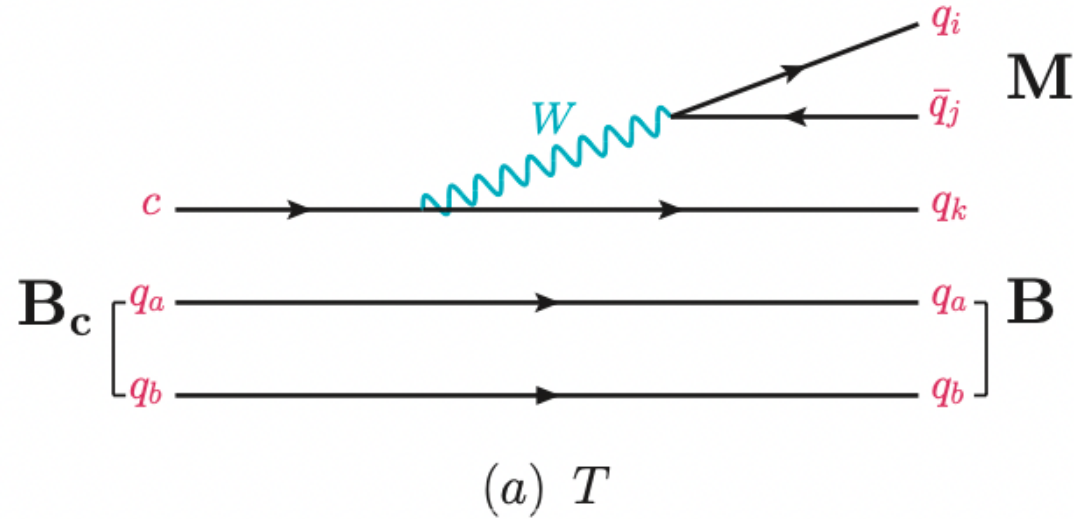
$$\mathcal{M}_{\text{IRA}} = \mathcal{M}_6 + \mathcal{M}_{\overline{15}},$$

$$\mathcal{M}_6 = a_1 H_{ij}(6) T^{ik} \mathbf{B}_k^l M_l^j + a_2 H_{ij}(6) T^{ik} M_k^l \mathbf{B}_l^j + a_3 H_{ij}(6) \mathbf{B}_k^i M_l^j T^{kl},$$

$$\begin{aligned} \mathcal{M}_{\overline{15}} = & a_4 H_k^{li}(\overline{15}) \mathbf{B}_{cj} M_i^j \mathbf{B}_k^l + a_5 \mathbf{B}_j^i M_i^l H(\overline{15})_l^{jk} \mathbf{B}_{ck} \\ & + a_6 \mathbf{B}_l^k M_j^i H(\overline{15})_i^{jl} \mathbf{B}_{ck} + a_7 \mathbf{B}_i^l M_j^i H(\overline{15})_l^{jk} \mathbf{B}_{ck}, \end{aligned}$$

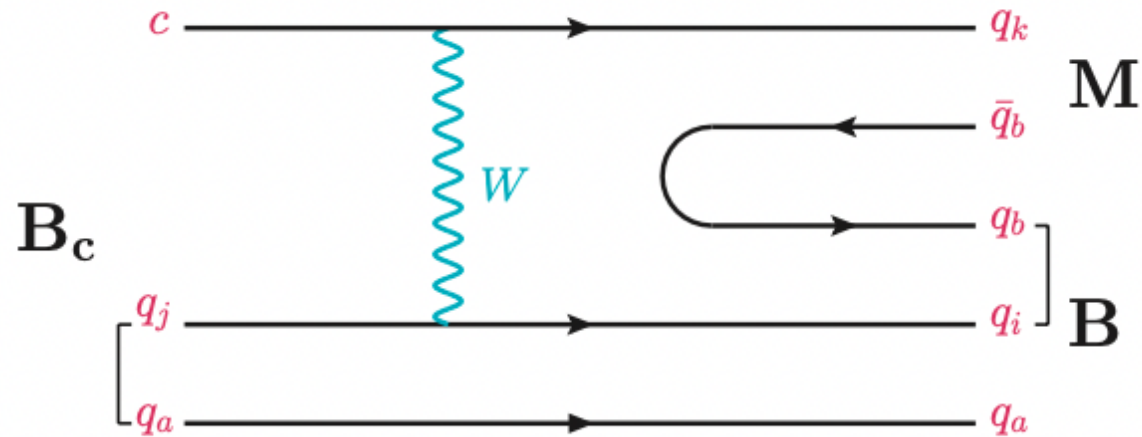
$$\begin{aligned} \mathcal{M}_{\text{TDA}} = & T \mathbf{B}_c^{ab} H_j^{ki} \mathbf{B}_{abk} M_j^i + C \mathbf{B}_c^{ab} H_j^{ki} \mathbf{B}_{abi} M_j^k + C' \mathbf{B}_c^{ab} H_j^{ki} \mathbf{B}_{ikb} M_j^a \\ & + E_{\mathbf{B}} \mathbf{B}_c^{jb} H_j^{ki} \mathbf{B}_{kab} M_a^i + E'_{\mathbf{B}} \mathbf{B}_c^{jb} H_j^{ki} \mathbf{B}_{kba} M_a^i \\ & + E_M \mathbf{B}_c^{jb} H_j^{ki} \mathbf{B}_{iba} M_a^k + E'_M \mathbf{B}_c^{jb} H_j^{ki} \mathbf{B}_{iab} M_a^k + E' \mathbf{B}_c^{jb} H_j^{ki} \mathbf{B}_{ika} M_a^b, \end{aligned}$$

$$T^{ij} \equiv \mathbf{B}_{ck} \epsilon^{ijk},$$



Decay mode	M_{TDA}	M_{IRA}
$\Lambda_c^+ \rightarrow \Lambda^0 \pi^+$	$-\frac{1}{\sqrt{6}}(4T + C' - E_{\mathbf{B}} - E')$	$-\frac{\sqrt{6}}{3}(a_1 + a_2 + a_3 - \frac{a_5 - 2a_6 + a_7}{2})$
$\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$	$\frac{1}{\sqrt{2}}(C' + E_{\mathbf{B}} - E')$	$-\sqrt{2}(a_1 - a_2 - a_3 - \frac{a_5 - a_7}{2})$
$\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$	$-\frac{1}{\sqrt{2}}(C' + E_{\mathbf{B}} - E')$	$\sqrt{2}(a_1 - a_2 - a_3 - \frac{a_5 - a_7}{2})$
$\Lambda_c^+ \rightarrow \Xi^0 K^+$	$E'^{(s)}$	$-2(a_2 - \frac{a_4 + a_7}{2})$
$\Lambda_c^+ \rightarrow p \bar{K}^0$	$2C - E'_M$	$-2(a_1 - \frac{a_5 + a_6}{2})$
$\Lambda_c^+ \rightarrow \Sigma^+ \eta$	$[\frac{1}{\sqrt{2}}(C' - E_{\mathbf{B}} + E')c\phi - E'_M{}^{(s)}s\phi]$	$\sqrt{2}c\phi(-a_1 - a_2 + a_3 + \frac{a_5 + a_7}{2}) - s\phi a_4$
$\Xi_c^+ \rightarrow \Sigma^+ \bar{K}^0$	$-2C + C'$	$2(a_3 - \frac{a_4 + a_6}{2})$
$\Xi_c^+ \rightarrow \Xi^0 \pi^+$	$-2T - C'$	$2(a_3 + \frac{a_4 + a_6}{2})$
$\Xi_c^0 \rightarrow \Lambda^0 \bar{K}^0$	$\frac{1}{\sqrt{6}}(2C + C' - E_M - 2E'_M - E')$	$-\frac{\sqrt{6}}{3}(2a_1 - a_2 - a_3 + \frac{2a_5 - a_6 - a_7}{2})$
$\Xi_c^0 \rightarrow \Sigma^0 \bar{K}^0$	$\frac{1}{\sqrt{2}}(2C - C' + E_M + E')$	$-\sqrt{2}(a_2 + a_3 - \frac{a_6 - a_7}{2})$
$\Xi_c^0 \rightarrow \Sigma^+ K^-$	$-E_M - E'$	$2(a_2 + \frac{a_4 + a_7}{2})$
$\Xi_c^0 \rightarrow \Xi^0 \pi^0$	$\frac{1}{\sqrt{2}}(E_{\mathbf{B}} + C')$	$-\sqrt{2}(a_1 - a_3 - \frac{a_4 - a_5}{2})$
$\Xi_c^0 \rightarrow \Xi^- \pi^+$	$2T - E_{\mathbf{B}}$	$2(a_1 + \frac{a_5 + a_6}{2})$
$\Xi_c^0 \rightarrow \Xi^0 \eta$	$[\frac{1}{\sqrt{2}}(E_{\mathbf{B}} - C')c\phi + (E_M^{(s)} + E'_M{}^{(s)} + E'^{(s)})s\phi]$	$\sqrt{2}c\phi(a_1 - a_3 + \frac{a_4 + a_5}{2}) - 2s\phi(a_2 + \frac{a_7}{2})$

$E'_M{}^s$ for $g \rightarrow s\bar{s}$



(h) E'_M

Using $\mathcal{M}_{\text{IRA}} = \mathcal{M}_{\text{TDA}}$

$$(T, C, C') = (a_1 + \frac{a_4 + a_6}{2}, -a_1 + \frac{a_4 + a_6}{2}, -2a_1 + 2a_3)$$

$$(E_{\text{B}}, E_M, E') = (a_4, -2a_4 - 2a_7, -2a_2 + a_4 + a_7)$$

$$E'_M = E_{\text{B}}$$

$$(a_1, a_2, a_3) = (\frac{T-C}{2}, -\frac{E_M + 2E'}{4}, \frac{T-C+C'}{2})$$

$$(a_4, a_5, a_6, a_7) = (E_{\text{B}}, 0, T + C - E_{\text{B}}, -E_{\text{B}} - \frac{E_M}{2})$$

such that we “topologize” the $SU(3)_f$ invariant amplitudes.

Explain the data

- $\Xi_c^0 \rightarrow \Xi^- K^+$ and $\Xi_c^0 \rightarrow \Xi^- \pi^+$

data: $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- K^+)/\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+) = 0.02$

theory: $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- K^+)/\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+) \simeq (-\sin \theta_c)^2 \simeq 0.05$

$\Rightarrow SU(3)_f$ symmetry breaking

IRA: at least 3 new parameters for breaking.

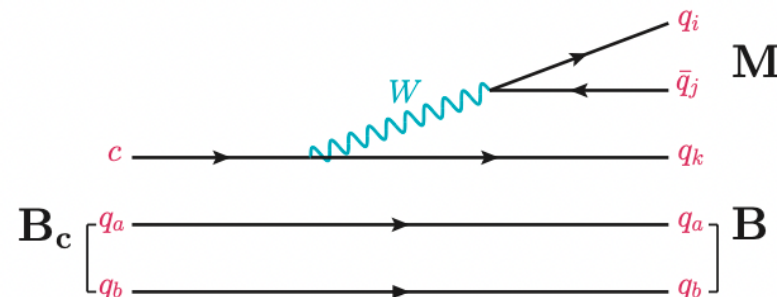
Savage [PLB257, 414 (1991)]

“SU(3) violations in the nonleptonic decay of charmed hadrons,”

Geng, Hsiao, C.W. Liu and T.H. Tsai [EPJC78, 593 (2018)]

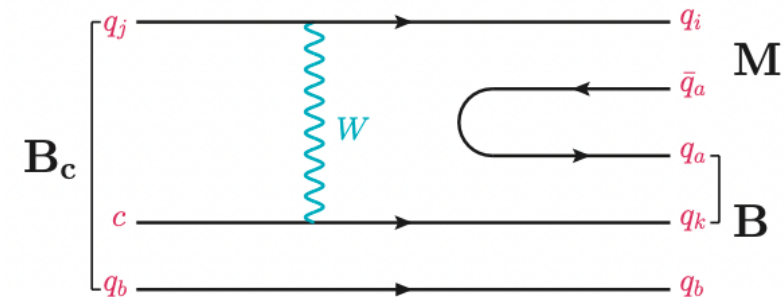
“SU(3) symmetry breaking in charmed baryon decays.”

$$\text{TDA: } \mathcal{R}(\Xi_c^0) \equiv \frac{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- K^+)}{\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)} = s_c^2 \frac{(2T - E_{\mathbf{B}}^{(s)})^2}{(2T - E_{\mathbf{B}})^2}$$



(a) T

$E_{\mathbf{B}}^s$ for $g \rightarrow s\bar{s}$



(e) $E_{\mathbf{B}}$

	TDA (S1)	TDA (S2)
χ^2	4.5	5.7
$n.d.f$	5	4
$ T $	0.23 ± 0.02	0.24 ± 0.02
$ C $	0.26 ± 0.01	0.23 ± 0.02
$ C' $	0.34 ± 0.02	0.32 ± 0.03
$ E_{\mathbf{B}} $	0.22 ± 0.03	0.22 ± 0.05
$ E_{\mathbf{B}}^s $		0.37 ± 0.06
$ E_M $	0.40 ± 0.03	0.38 ± 0.03
$ E' $	0.24 ± 0.02	0.23 ± 0.02
δ_C	$(183.2 \pm 9.6)^\circ$	$(179.5 \pm 12.9)^\circ$
$\delta_{C'}$	$(163.7 \pm 5.0)^\circ$	$(149.7 \pm 6.7)^\circ$
$\delta_{E_{\mathbf{B}}}$	$(-100.3 \pm 7.1)^\circ$	$(-93.6 \pm 8.2)^\circ$
$\delta_{E_{\mathbf{B}}^s}$		$(43.3 \pm 8.0)^\circ$
δ_{E_M}	$(100.3 \pm 8.0)^\circ$	$(113.2 \pm 10.7)^\circ$
$\delta_{E'}$	$(-71.1 \pm 6.7)^\circ$	$(-50.1 \pm 12.3)^\circ$

$$\mathcal{B}_{S1}(\Xi_c^0 \rightarrow \Xi^- K^+) = (11.8 \pm 1.8) \times 10^{-4}$$

$$\mathcal{B}_{S2}(\Xi_c^0 \rightarrow \Xi^- K^+) = (4.1 \pm 2.8) \times 10^{-4}$$

$$\mathcal{B}_{ex}(\Xi_c^0 \rightarrow \Xi^- K^+) = (3.9 \pm 1.2) \times 10^{-4}$$

$$E_{\mathbf{B}}^s = |n_q| e^{i\delta_{n_q}} E_{\mathbf{B}}$$

$$|n_q| = 1.2 \pm 0.4, \delta_{n_q} = (50.3 \pm 11.5)^\circ$$

Explain the data

- CA $\Xi_c^0 \rightarrow \mathbf{B}M$

$$\mathcal{B}_{ex}(\Xi_c^0 \rightarrow \Lambda^0 \bar{K}^0, \Sigma^0 \bar{K}^0, \Sigma^+ K^-)$$

$$= (8.24 \pm 2.44, 1.38 \pm 0.48, 2.21 \pm 0.68) \times 10^{-3}$$

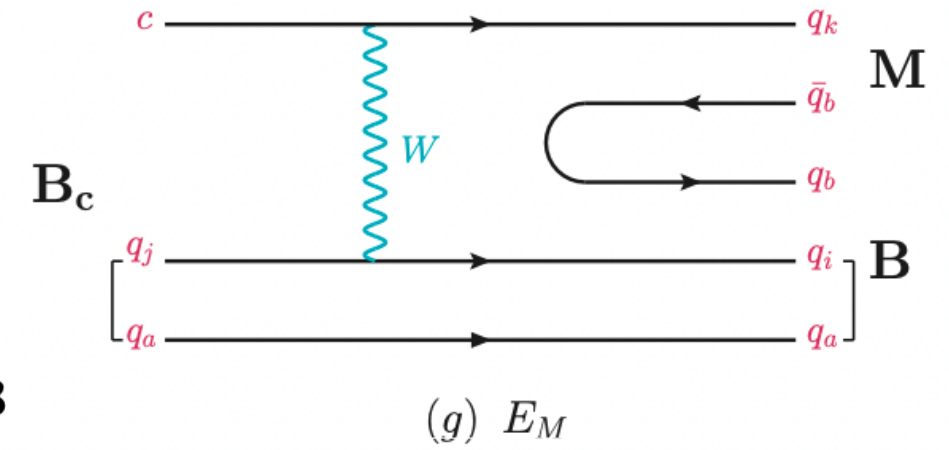
Belle [PRD105, L011102 (2022)]

$$\text{TDA: } \mathcal{M}(\Xi_c^0 \rightarrow \Lambda^0 \bar{K}^0, \Sigma^0 \bar{K}^0, \Sigma^+ K^-) = \frac{1}{\sqrt{6}}(2C + C' - E_M - 2E'_M - E'),$$

$$\frac{1}{\sqrt{2}}(2C - C' + E_M + E'), -E_M - E'$$

E_M only appearing in $\Xi_c^0 \rightarrow \mathbf{B}M$, instead of $\Xi_c^+, \Lambda_c^+ \rightarrow \mathbf{B}M$,

cannot be neglected ($E_M = -2a_4 - 2a_7$).



$$\mathcal{B}_{S1}(\Xi_c^0 \rightarrow \Lambda^0 \bar{K}^0, \Sigma^0 \bar{K}^0, \Sigma^+ K^-)$$

$$= (9.85 \pm 2.26, 1.48_{-0.92}^{+1.27}, 2.21_{-0.16}^{+0.37}) \times 10^{-3}$$

$$\mathcal{B}_{S2}(\Xi_c^0 \rightarrow \Lambda^0 \bar{K}^0, \Sigma^0 \bar{K}^0, \Sigma^+ K^-)$$

$$= (10.0 \pm 2.9, 1.46_{-1.09}^{+1.57}, 2.25_{-1.1}^{+1.8}) \times 10^{-3}$$

Explain the data

- $\Lambda_c^+ \rightarrow p\pi^0$ and $\Lambda_c^+ \rightarrow n\pi^+$

$$\mathcal{B}_{ex}(\Lambda_c^+ \rightarrow p\pi^0) > 3 \times 10^{-4}, \text{ BESIII [PRD95, 111102 (2017)]}$$

$$\mathcal{B}_{ex}(\Lambda_c^+ \rightarrow p\pi^0) > 0.8 \times 10^{-4}, \text{ Belle [PRD103, 072004 (2021)]}$$

$$\mathcal{B}_{ex}(\Lambda_c^+ \rightarrow n\pi^+) = (6.6 \pm 1.3) \times 10^{-4}, \text{ BESIII [PRL128, 142001 (2022)]}$$

- $\mathcal{B}_{ex}(\Lambda_c^+ \rightarrow n\pi^+) \gg \mathcal{B}_{ex}(\Lambda_c^+ \rightarrow p\pi^0)$

$$\text{IRA: } \mathcal{M}(\Lambda_c^+ \rightarrow n\pi^+, p\pi^0) \sim A + B, (A - B)/\sqrt{2}$$

$$(A, B) = (a_2 + a_3 - a_7/2, a_6/2)$$

Causing the constructive and destructive interferences.

- Theoretical results

$$\mathcal{B}_{S1,S2}(\Lambda_c^+ \rightarrow p\pi^0) = (0.3_{-0.3}^{+1.0}, 0.4_{-0.4}^{+1.7}) \times 10^{-4}$$

$$\mathcal{B}_{S1,S2}(\Lambda_c^+ \rightarrow n\pi^+) = (7.6 \pm 1.7, 8.3 \pm 2.6) \times 10^{-4}$$

Consistent with the analyses with IRA

Partly restoring the QCD-disfavored parameters

Geng, C.W. Liu and T.H. Tsai [PLB794, 19 (2019)]

With all parameters

C.P. Jia, D. Wang and F.S. Yu [NPB956, 115048 (2020)]

F. Huang, Z.P. Xing and X.G. He [JHEP **03**, 143 (2022)]

With all parameters+ $SU(3)_f$ breaking

H. Zhong, F. Xu, Q. Wen and Y. Gu [JHEP02, 235 (2023)]

Summary

- We demonstrated that IRA and TDA for $\mathbf{B}_c \rightarrow \mathbf{B}M$ Equivalent $SU(3)_f$ approaches.
- We explained $\mathcal{B}(\Lambda_c^+ \rightarrow \Xi^0 K^+, \Xi^0 \pi^+)$ with $SU(3)_f$ symmetry breaking.
- $\mathcal{B}(\Lambda_c^+ \rightarrow p\pi^0)$ and $\mathcal{B}(\Lambda_c^+ \rightarrow n\pi^+)$ receive the contributions from the destructive and constructive interfering effects, respectively.
- The exchange topology E_M plays a key role in the $\Xi_c^0 \rightarrow \Lambda^0 \bar{K}^0, \Sigma^0 \bar{K}^0, \Sigma^+ K^-$ decays.

Thank You