INDIANA UNIVERSITY
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Jefferson Lab

Rare Tau decays and CP violation searches with polarized beams

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Indiana University/Jefferson Laboratory

The 2nd international workshop on High Intensity Electron-Positron Accelerator (HIEPA) in China
Beijing, March 20, 2018

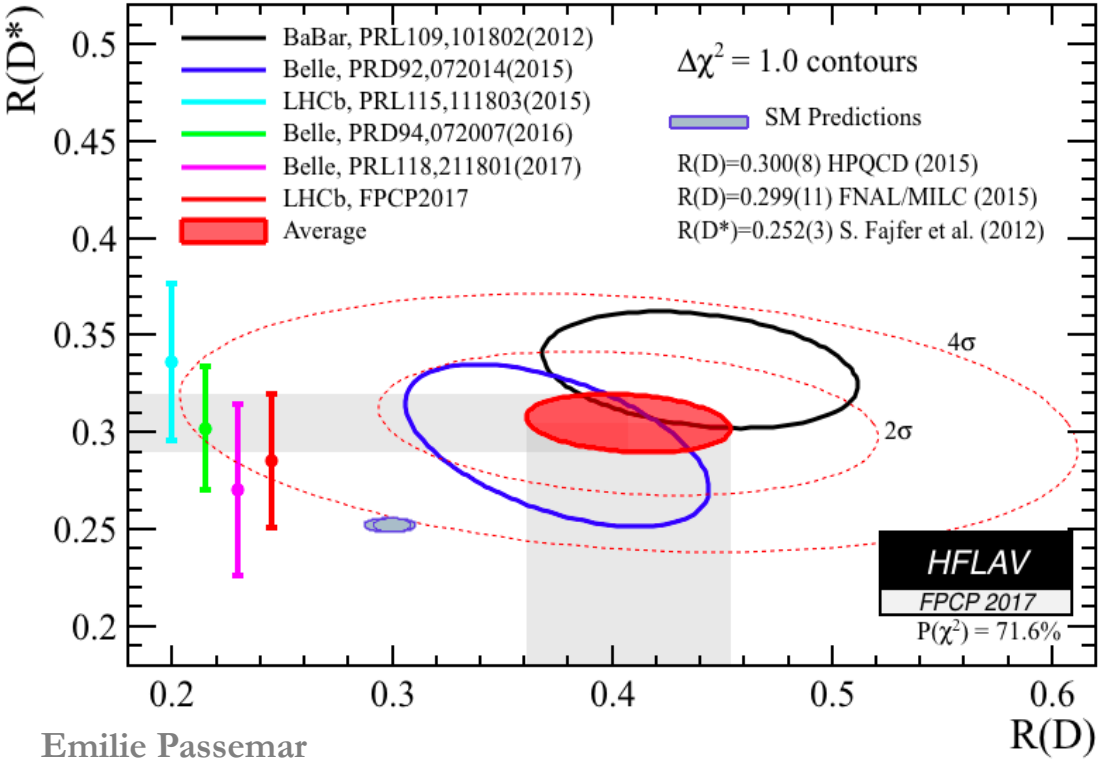
Outline :

1. Introduction and Motivation:
2. Lepton Flavour Violation
3. CP violation in tau decays
4. Conclusion and outlook

1.1 Quest for New Physics

- New era in particle physics :
  ➡ (unexpected) *success of the Standard Model*: a successful theory of microscopic phenomena with *no intrinsic energy limitation*

- *Where do we look?* Everywhere! ➡ search for New Physics with *broad search strategy* given lack of clear indications on the SM-EFT boundaries (*both in energies and effective couplings*)



- Hint from B physics anomalies?

b → c charged currents:
τ vs. light leptons (μ, e) [R(D), R(D*)]

$$R(X) = \frac{\Gamma(B \to X\tau\bar{\nu})}{\Gamma(B \to X\ell\bar{\nu})}$$

1.1 Quest for New Physics

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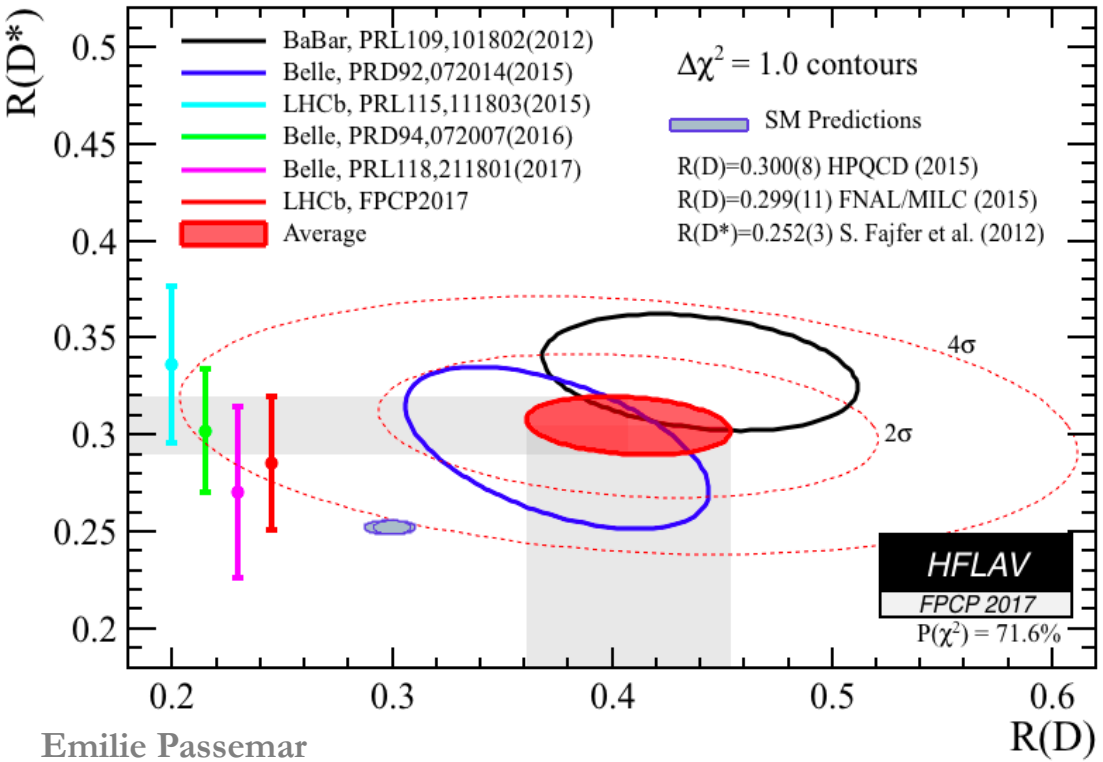

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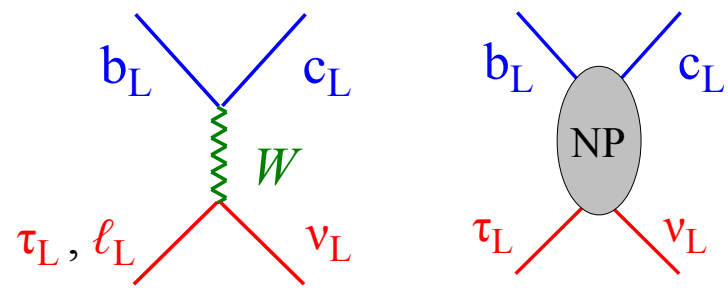
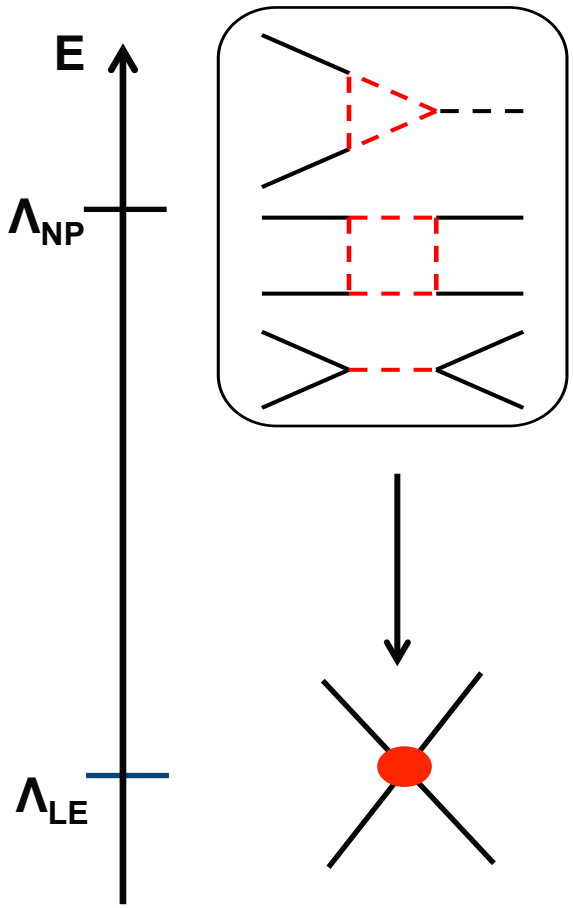


➡ Key unique role of *Tau physics*

1.2 τ lepton as a unique probe of new physics



- In the quest of New Physics, can be sensitive to very high scale:
  - Kaon physics: $[\varepsilon_K]$ $\quad \frac{s\bar{d}s\bar{d}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^5$ TeV
  - Tau Leptons: $[\tau \to \mu\gamma]$ $\quad \frac{\tau\bar{\mu} f\bar{f}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^2$ TeV
- At low energy: lots of experiments e.g., *BaBar*, *Belle*, *BESIII, LHCb* ⇨ important improvements on measurements and bounds obtained and more expected (*Belle II*, *HIEPA?*)
- In many cases no SM background: e.g., LFV, EDMs
- For some modes accurate calculations of hadronic uncertainties essential, e.g. CPV in hadronic Tau decays

⇨ Tau leptons very important to look for *New Physics*!

1.2 τ lepton as a unique probe of new physics

- A lot of progress in tau physics since its discovery on all the items described before ➡ important experimental efforts from *LEP*, *CLEO, B factories: Babar, Belle, BES, VEPP-2M, LHCb, neutrino experiments*,…

  ➡ More to come from *LHCb, BES, VEPP-2M, Belle II, CMS, ATLAS, HIEPA?*

Experiment	Number of τ pairs
LEP	~3×10^{5}
CLEO	~1×10^{7}
BaBar	~5×10^{8}
Belle	~9×10^{8}
Belle II	~10^{12}

- But τ physics has still potential "*unexplored frontiers*"

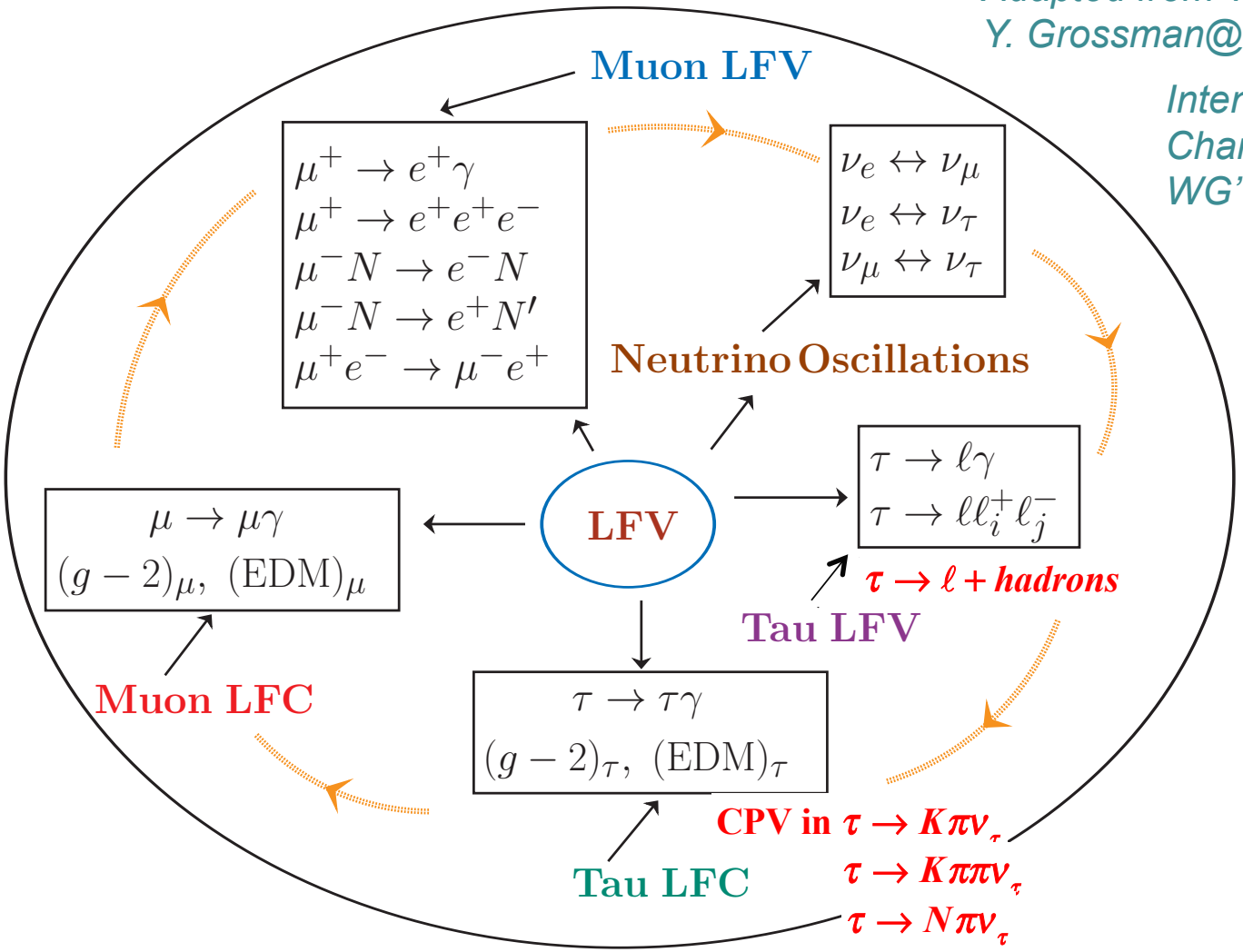
  ➡ deserve future exp. & th. efforts

- In the following, some selected examples

1.3 The Program

Adapted from Talk by Y. Grossman@CLFV2013

Intensity Frontier Charged Lepton WG'13

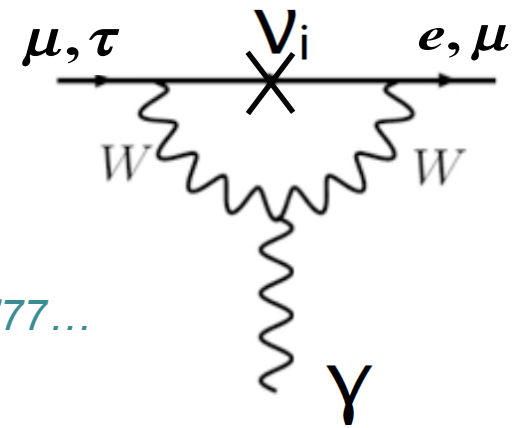


2. Charged Lepton-Flavour Violation

2.1 Introduction and Motivation

- Lepton Flavour Number is an « accidental » symmetry of the SM (m_ν=0)
- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ➡ *unobservably small rates!*

E.g.: $\boxed{\mu \to e\gamma}$



$$\boxed{Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W}\right|^2 < 10^{-54}}$$

Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br(\tau \to \mu\gamma) < 10^{-40}\right]$$

- Extremely *clean probe of beyond SM physics*
- In New Physics models: seazible effects
  Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

2.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

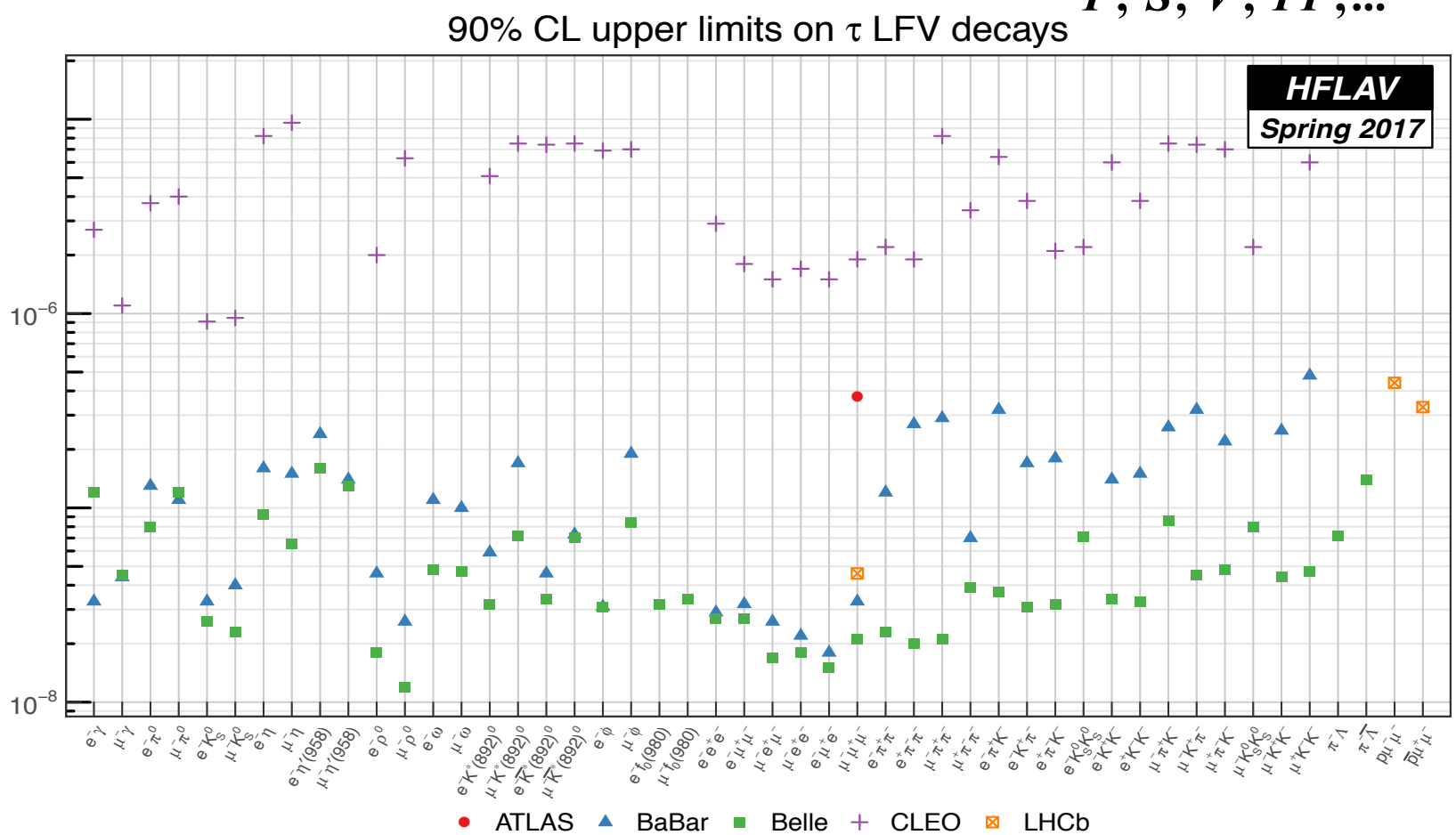
Talk by D. Hitlin @ CLFV2013

		$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

2.2 Tau LFV

- Several processes: $\tau \to \ell\gamma$, $\tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta$, $\tau \to \ell Y$ ← P, S, V, $P\bar{P}$,...



- 48 LFV modes studied at Belle and BaBar

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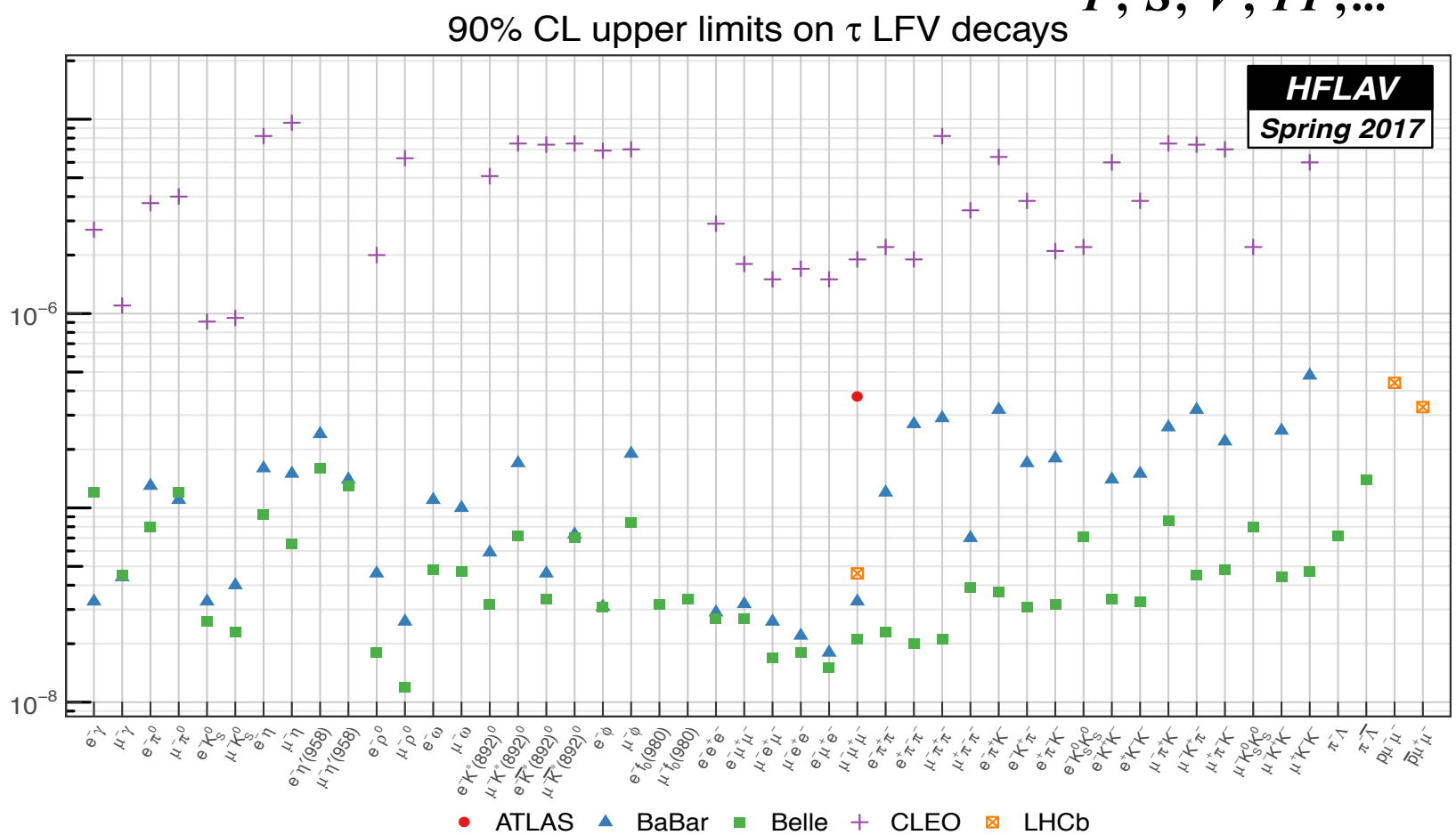
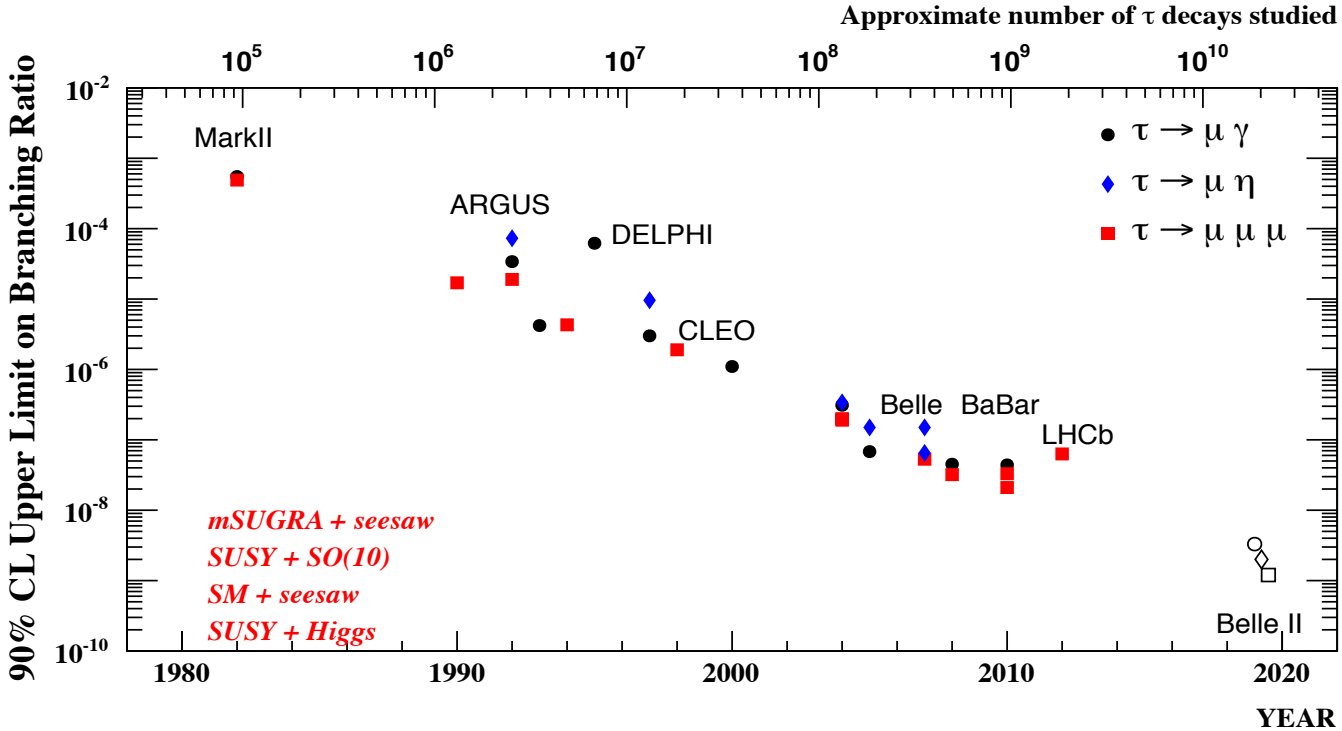
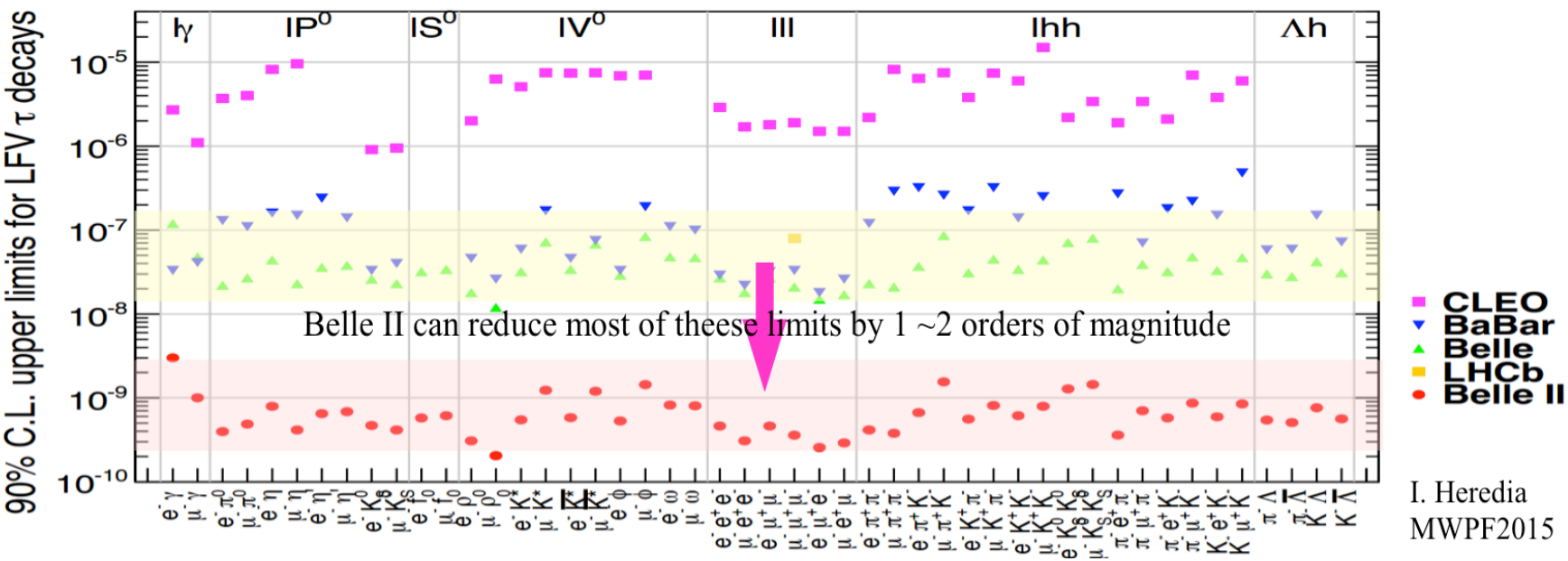


- Expected sensitivity 10^{-9} or better at *LHCb, Belle II?*



S. Banerjee'17



I. Heredia
MWPF2015

2.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

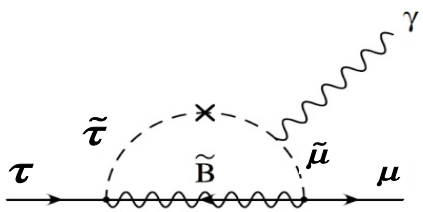
See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:

  - Dipole:

$$\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$$

e.g.



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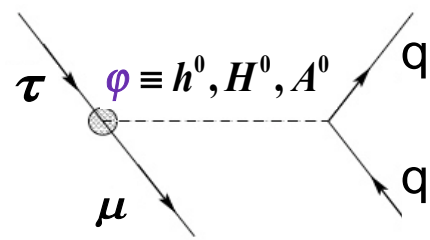
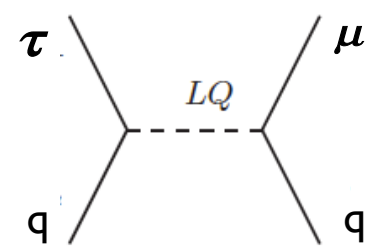
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\mathcal{L}_{eff}^{S,V} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{q} \Gamma q$$

e.g.



$\Gamma \equiv 1$



$\Gamma \equiv \gamma^\mu$

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  - Integrating out heavy quarks generates *gluonic operator*

$$\frac{1}{\Lambda^2} \bar{\mu} P_{L,R} \tau Q \bar{Q} \rightarrow \mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau\, G_{\mu\nu}^a G_a^{\mu\nu}$$

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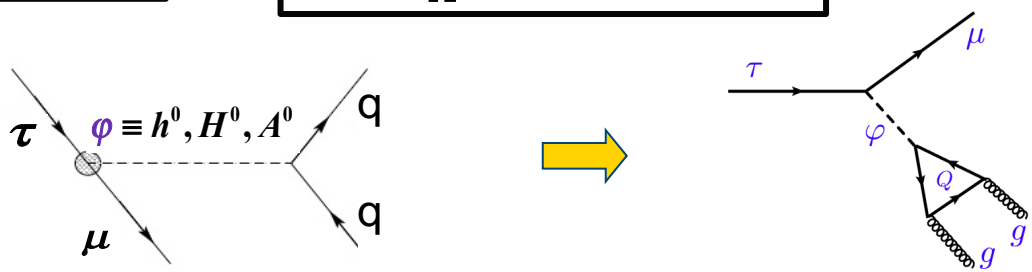

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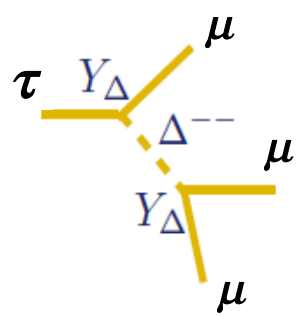
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{\mu}\, \Gamma P_{L,R} \mu$

$\Gamma \equiv 1, \gamma^\mu$

e.g.



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  - Lepton-gluon (Scalar, Pseudo-scalar): $\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau\, G_{\mu\nu}^a G_a^{\mu\nu}$
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- Each UV model generates a *specific pattern* of them

$\Gamma \equiv 1, \gamma^\mu$

2.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O^{4\ell}_{S,V}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O^q_V	–	–	✓ (I=1)	✓(I=0,1)	–	–
O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!

- But need reliable determinations of the hadronic part:
  form factors and *decay constants* (e.g. f_η, $f_{\eta'}$)

2.4 Model discriminating power of Tau processes

- Summary table: *Celis, Cirigliano, E.P.'14*

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O_D	✓	✓	✓	✓	–	–
O^q_V	–	–	✓ (I=1)	✓(I=0,1)	–	–
O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- Form factors for $\tau \to \mu(e)\pi\pi$ determined using *dispersive techniques*
- Hadronic part: *Donoghue, Gasser, Leutwyler'90*

$$H_\mu = \langle \pi\pi | \left(V_\mu - A_\mu\right) e^{iL_{QCD}} | 0 \rangle = \left(\textit{Lorentz struct.}\right)^i_\mu F_i(s)$$

with

$$s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

Moussallam'99
Daub et al'13
Celis, Cirigliano, E.P.'14

- 2-channel unitarity condition is solved with I=0 S-wave ππ and KK scattering data as input

$$\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)$$

$$n = \pi\pi, K\bar{K}$$

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O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*

2.5 Handles

- Two handles:
  - Branching ratios: $\boxed{R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}}$ with F_M dominant LFV mode for model M
  - Spectra for > 2 bodies in the final state: $\boxed{\frac{dBR(\tau \to \mu\mu\mu)}{d\sqrt{s}}}$

- Benchmarks:
  - Dipole model: $C_D \neq 0$, $C_{else} = 0$
  - Scalar model: $C_S \neq 0$, $C_{else} = 0$
  - Vector (gamma,Z) model: $C_V \neq 0$, $C_{else} = 0$
  - Gluonic model: $C_{GG} \neq 0$, $C_{else} = 0$

2.6 Model discriminating of BRs

Celis, Cirigliano, E.P.'14

- Two handles:
  - Branching ratios: $R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}$ with F_M dominant LFV mode for model M

		$\mu\pi^+\pi^-$	$\mu\rho$	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	0.26×10^{-2}	0.22×10^{-2}	0.13×10^{-3}	0.22×10^{-2}	1
	BR	$< 1.1 \times 10^{-10}$	$< 9.7 \times 10^{-11}$	$< 5.7 \times 10^{-12}$	$< 9.7 \times 10^{-11}$	$< 4.4 \times 10^{-8}$
S	$R_{F,S}$	1	0.28	0.7	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 5.9 \times 10^{-9}$	$< 1.47 \times 10^{-8}$	-	-
$V^{(\gamma)}$	$R_{F,V^{(\gamma)}}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
Z	$R_{F,Z}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
G	$R_{F,G}$	1	0.41	0.41	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 8.6 \times 10^{-9}$	$< 8.6 \times 10^{-9}$	-	-

Benchmark

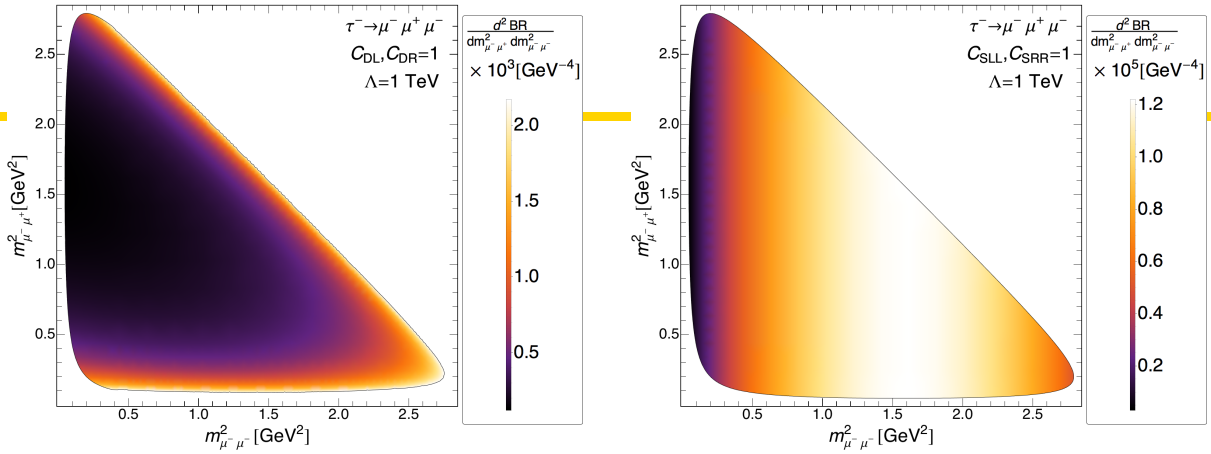
2.6 Model discriminating of BRs

- Studies in specific models

Buras et al.'10

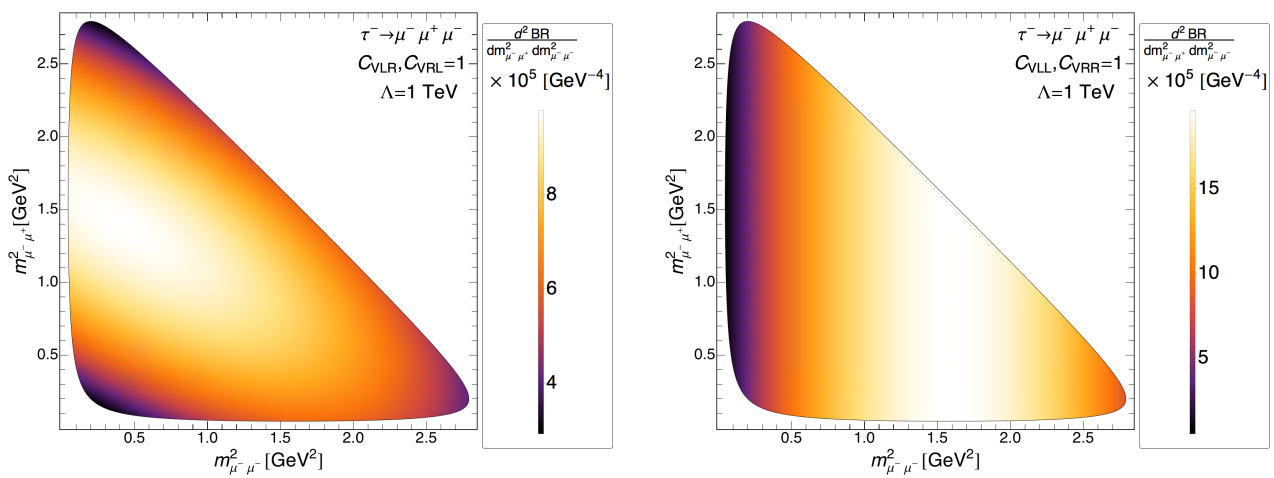
ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{\mathrm{Br}(\mu^- \to e^- e^+ e^-)}{\mathrm{Br}(\mu \to e\gamma)}$	$0.02 \ldots 1$	$\sim 6 \cdot 10^{-3}$	$\sim 6 \cdot 10^{-3}$	$0.06 \ldots 2.2$
$\frac{\mathrm{Br}(\tau^- \to e^- e^+ e^-)}{\mathrm{Br}(\tau \to e\gamma)}$	$0.04 \ldots 0.4$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	$0.07 \ldots 2.2$
$\frac{\mathrm{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\mathrm{Br}(\tau \to \mu\gamma)}$	$0.04 \ldots 0.4$	$\sim 2 \cdot 10^{-3}$	$0.06 \ldots 0.1$	$0.06 \ldots 2.2$
$\frac{\mathrm{Br}(\tau^- \to e^- \mu^+ \mu^-)}{\mathrm{Br}(\tau \to e\gamma)}$	$0.04 \ldots 0.3$	$\sim 2 \cdot 10^{-3}$	$0.02 \ldots 0.04$	$0.03 \ldots 1.3$
$\frac{\mathrm{Br}(\tau^- \to \mu^- e^+ e^-)}{\mathrm{Br}(\tau \to \mu\gamma)}$	$0.04 \ldots 0.3$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	$0.04 \ldots 1.4$
$\frac{\mathrm{Br}(\tau^- \to e^- e^+ e^-)}{\mathrm{Br}(\tau^- \to e^- \mu^+ \mu^-)}$	$0.8 \ldots 2$	~ 5	$0.3 \ldots 0.5$	$1.5 \ldots 2.3$
$\frac{\mathrm{Br}(\tau^- \to \mu^- \mu^+ \mu^-)}{\mathrm{Br}(\tau^- \to \mu^- e^+ e^-)}$	$0.7 \ldots 1.6$	~ 0.2	$5 \ldots 10$	$1.4 \ldots 1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti} \to e\mathrm{Ti})}{\mathrm{Br}(\mu \to e\gamma)}$	$10^{-3} \ldots 10^{2}$	$\sim 5 \cdot 10^{-3}$	$0.08 \ldots 0.15$	$10^{-12} \ldots 26$

⇒ Disentangle the *underlying dynamics* of NP



Dassinger, Feldman, Mannel, Turczyk' 07
Celis, Cirigliano, E.P.'14

Figure 3: *Dalitz plot for $\tau^- \to \mu^-\mu^+\mu^-$ decays when all operators are assumed to vanish with the exception of* $C_{DL,DR} = 1$ *(left) and* $C_{SLL,SRR} = 1$ *(right), taking $\Lambda = 1$ TeV in both cases. Colors denote the density for $d^2BR/(dm^2_{\mu^-\mu^+} dm^2_{\mu^-\mu^-})$, small values being represented by darker colors and large values in lighter ones. Here $m^2_{\mu^-\mu^+}$ represents m^2_{12} or m^2_{23}, defined in Sec. 3.1.*



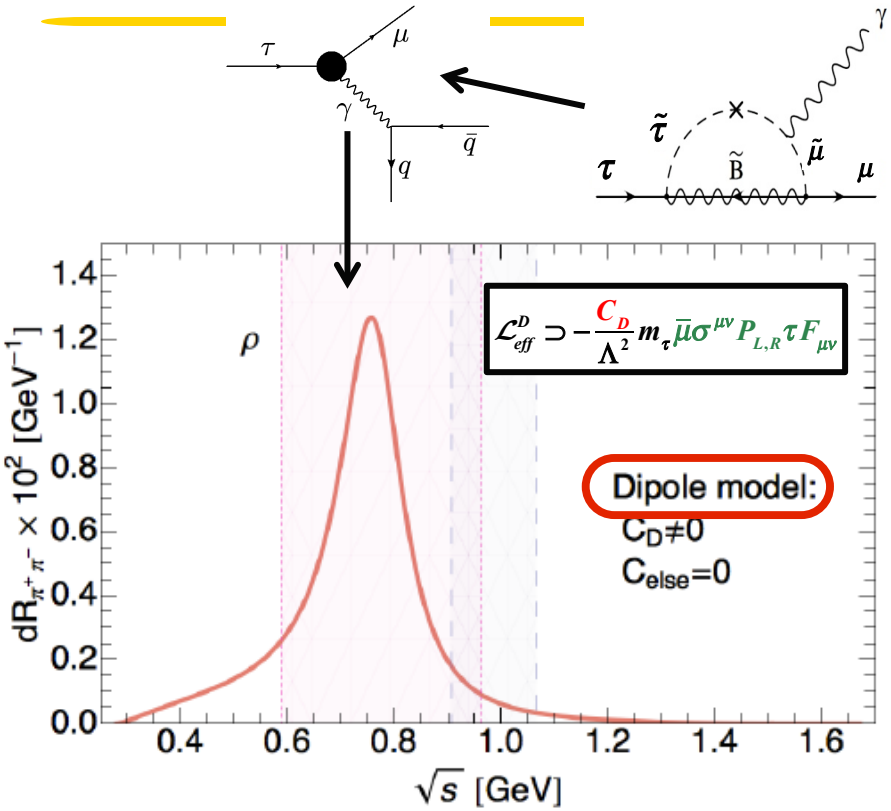
Angular analysis with polarized taus

Dassinger, Feldman, Mannel, Turczyk' 07

Figure 4: *Dalitz plot for $\tau^- \to \mu^-\mu^+\mu^-$ decays when all operators are assumed to vanish with the exception of* $C_{VRL,VLR} = 1$ *(left) and* $C_{VLL,VRR} = 1$ *(right), taking $\Lambda = 1$ TeV in both cases. Colors are defined as in Fig. 3.*

2.7 Discriminating power of $\tau \to \mu(e)\pi\pi$ decays

Celis, Cirigliano, E.P.'14



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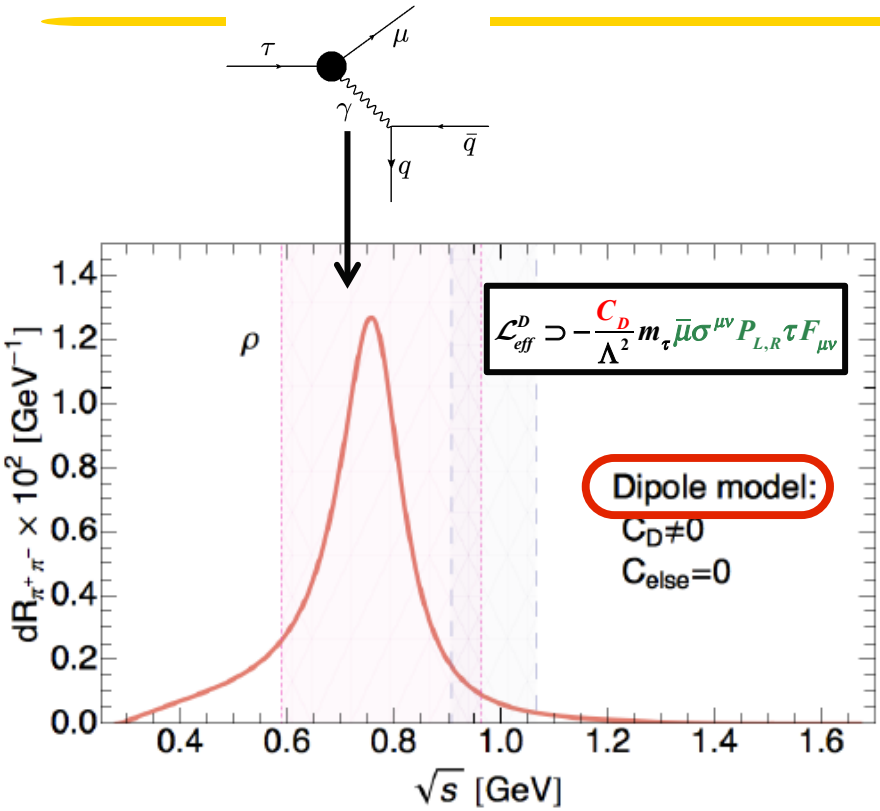
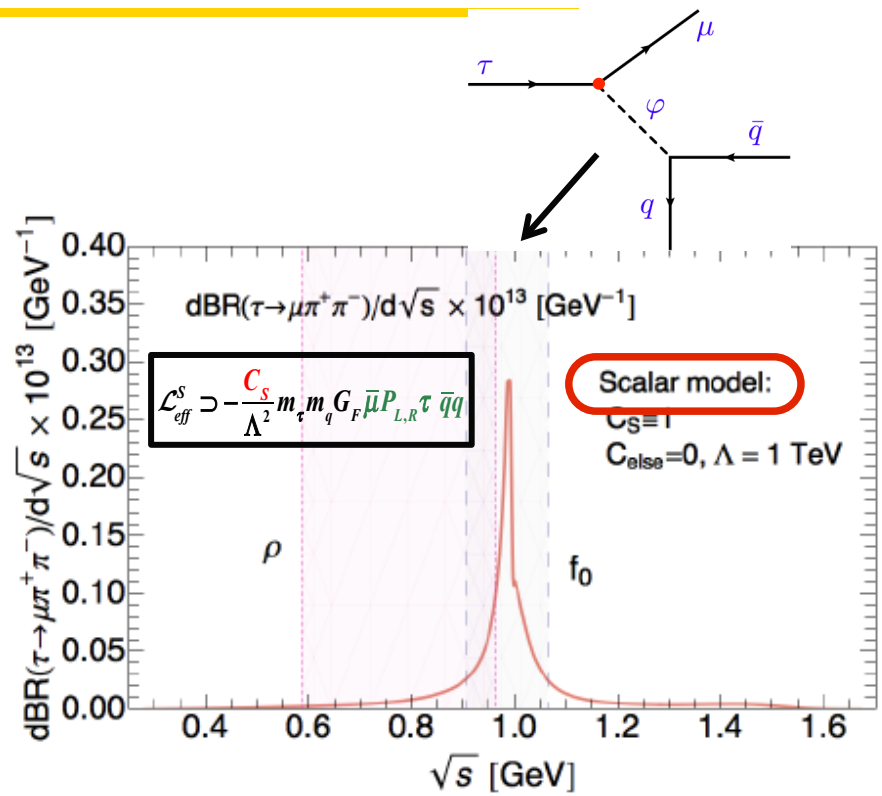
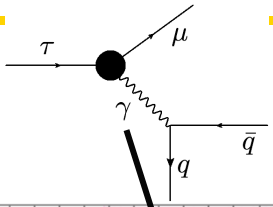
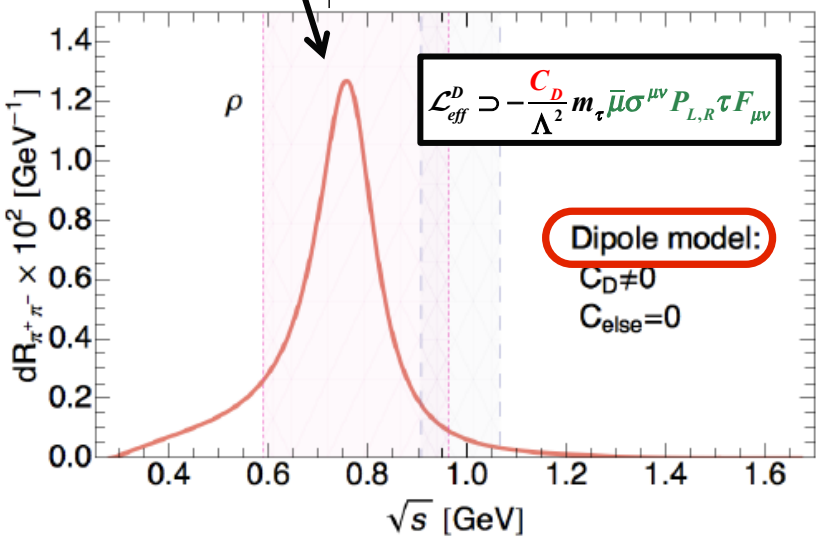
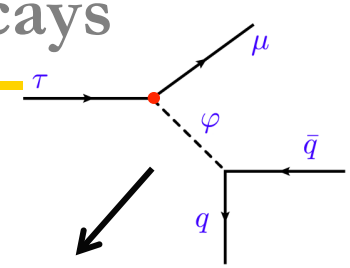


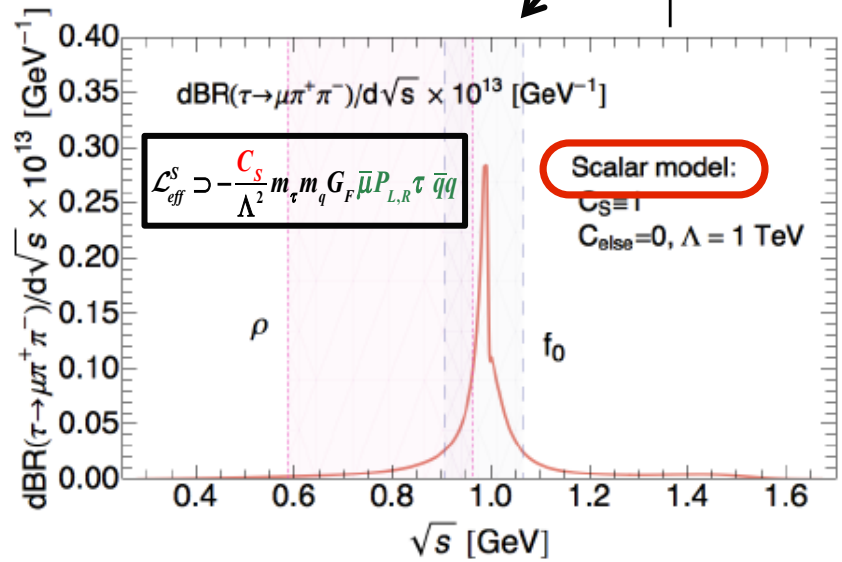


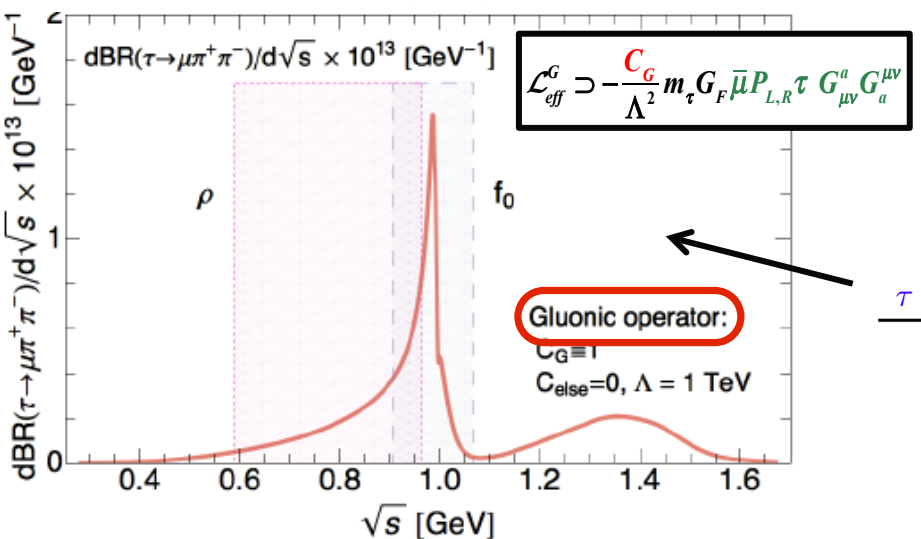
2.7 Discriminating power of $\tau \to \mu(e)\pi\pi$ decays



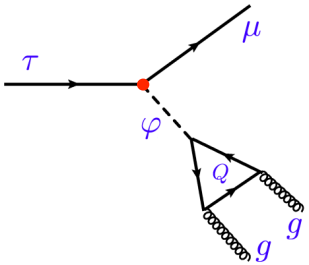








Different distributions according to the *operator!*



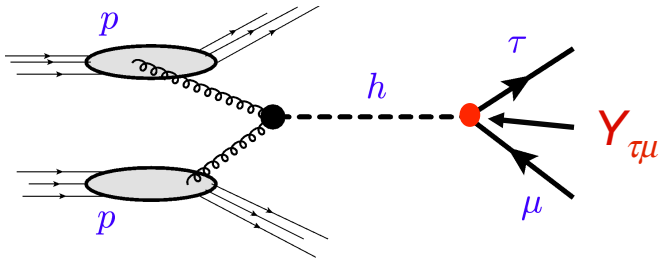
2.8 Non standard LFV Higgs coupling

- $$\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right) h$$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

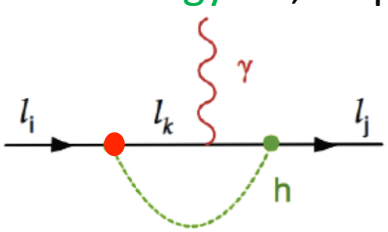
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

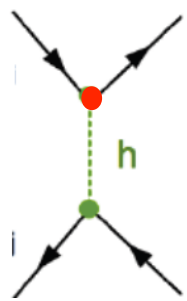
- High energy : LHC

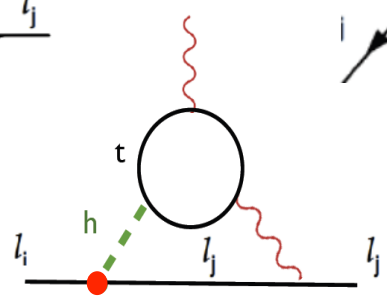


Hadronic part treated with perturbative QCD

- Low energy : D, S operators







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Goudelis, Lebedev, Park'11
Davidson, Grenier'10
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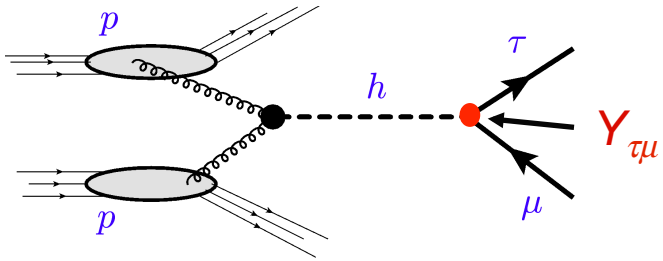
Arhrib, Cheng, Kong'12

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

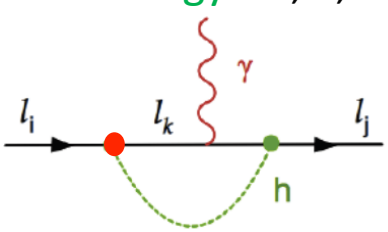
- High energy : LHC

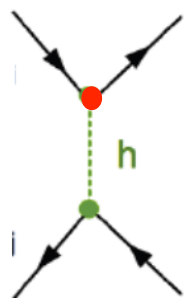


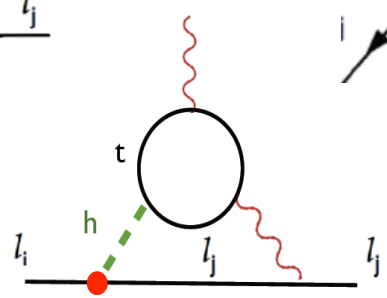
Hadronic part treated with perturbative QCD

Reverse the process

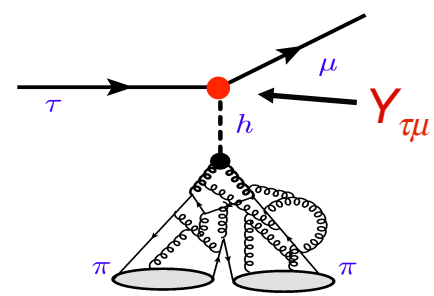
- Low energy : D, S, G operators







+



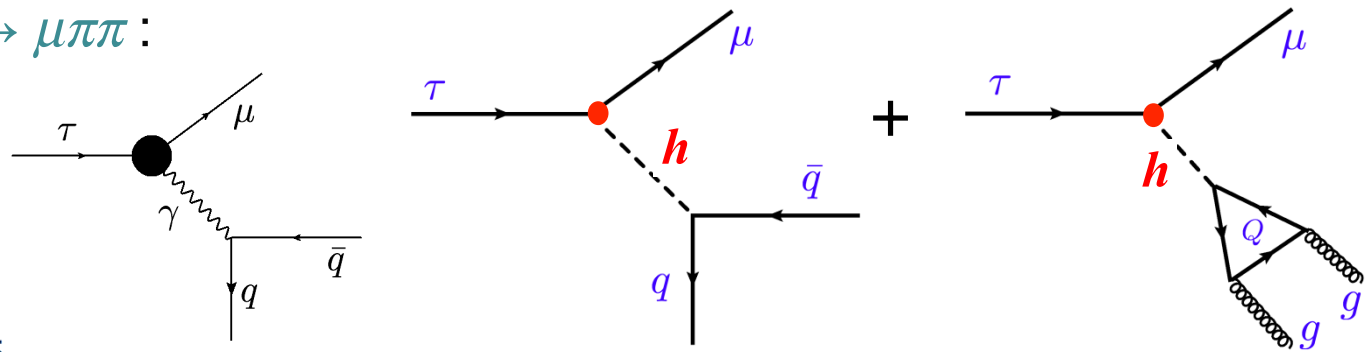
Hadronic part treated with non-perturbative QCD

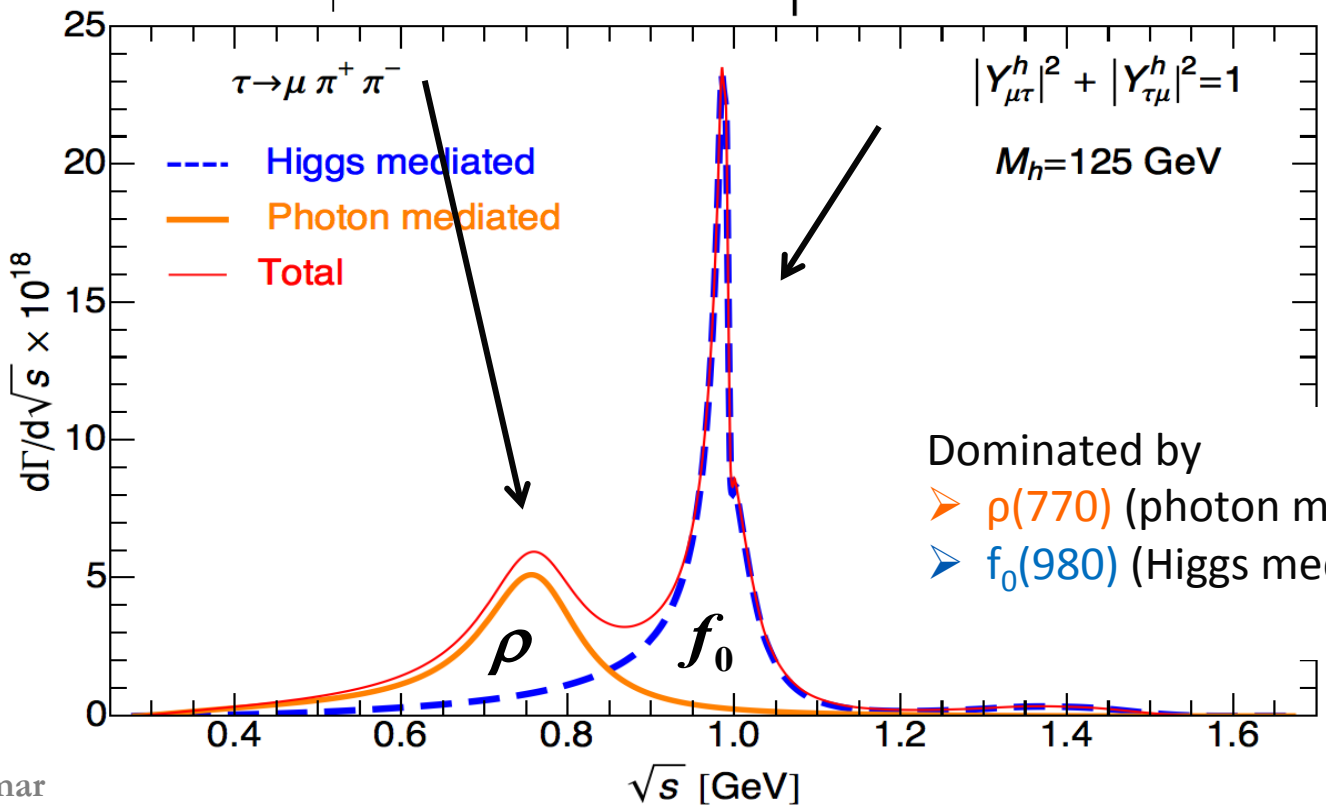
Constraints in the τμ sector

- At low energy

Cirigliano, Celis, E.P.'14

➢ $\tau \to \mu\pi\pi$:



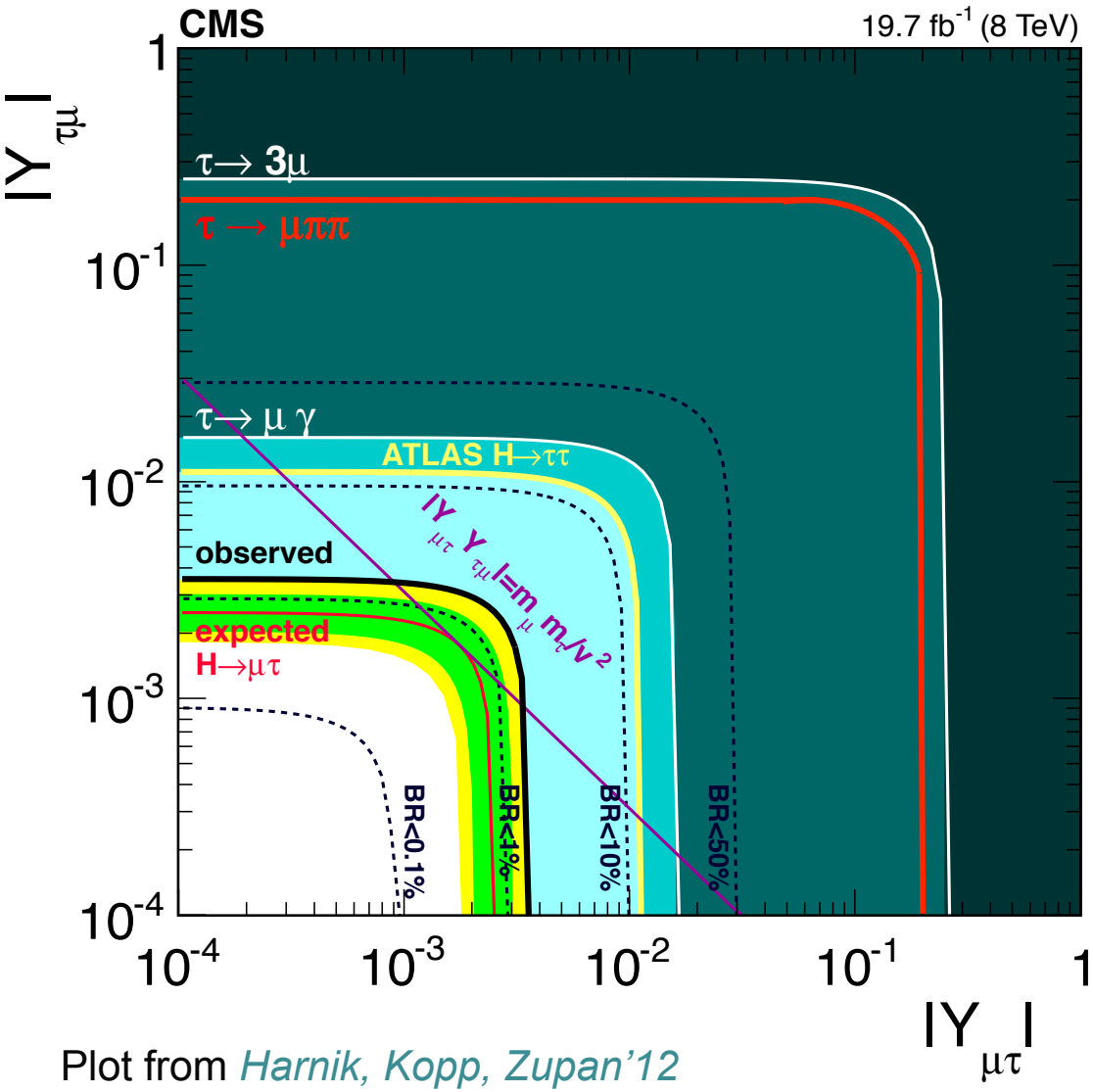


Dominated by

➢ ρ(770) (photon mediated)

➢ f_0(980) (Higgs mediated)

Constraints in the τμ sector



Plot from *Harnik, Kopp, Zupan'12* updated *by CMS'15*

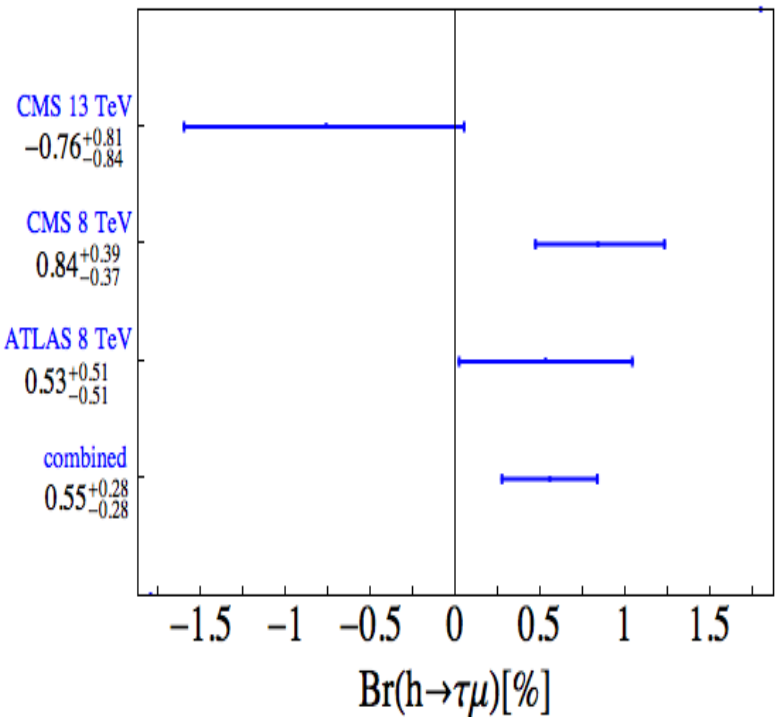
- Constraints from LE:
  - $\tau \to \mu\gamma$: best constraints but loop level
    ➡ sensitive to UV completion of the theory
  - $\tau \to \mu\pi\pi$: tree level diagrams
    ➡ robust handle on LFV
- Constraints from HE:
  LHC wins for τμ!
- Opposite situation for μe!
- For LFV Higgs and nothing else: LHC bound

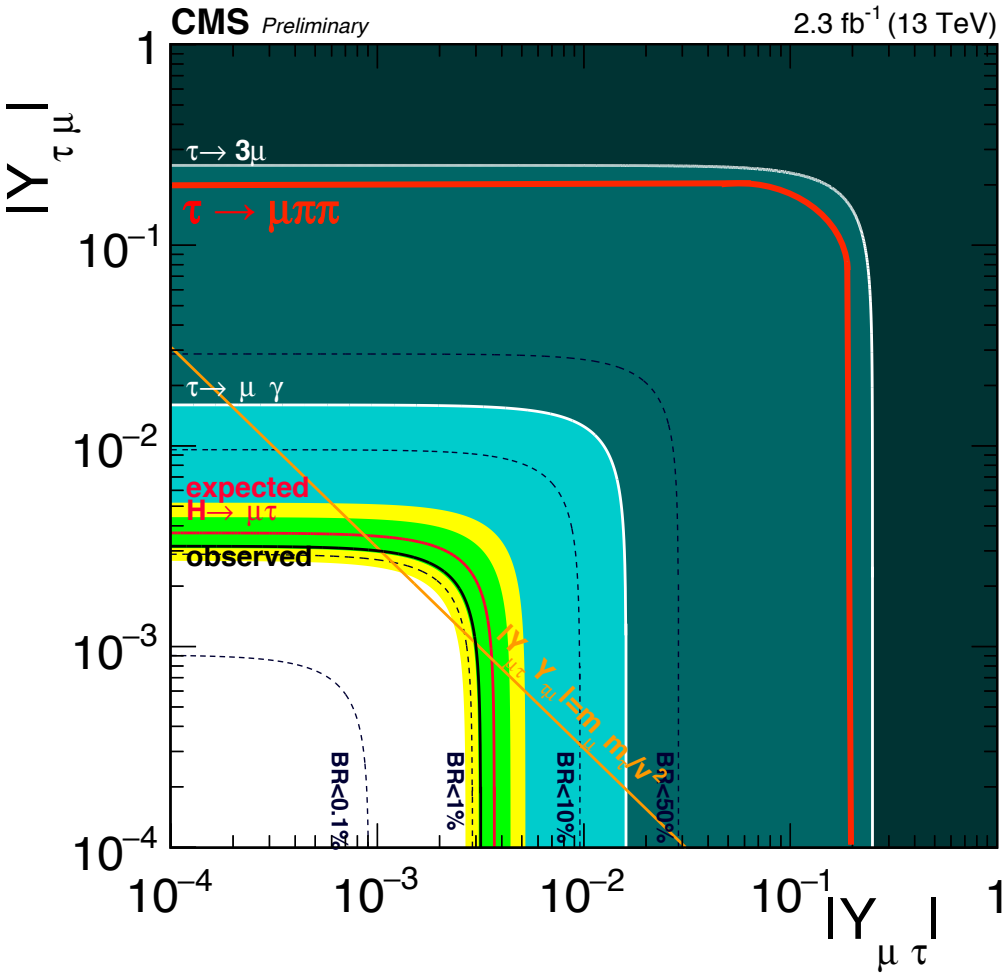
➡ $BR(\tau \to \mu\gamma) < 2.2 \times 10^{-9}$

$BR(\tau \to \mu\pi\pi) < 1.5 \times 10^{-11}$

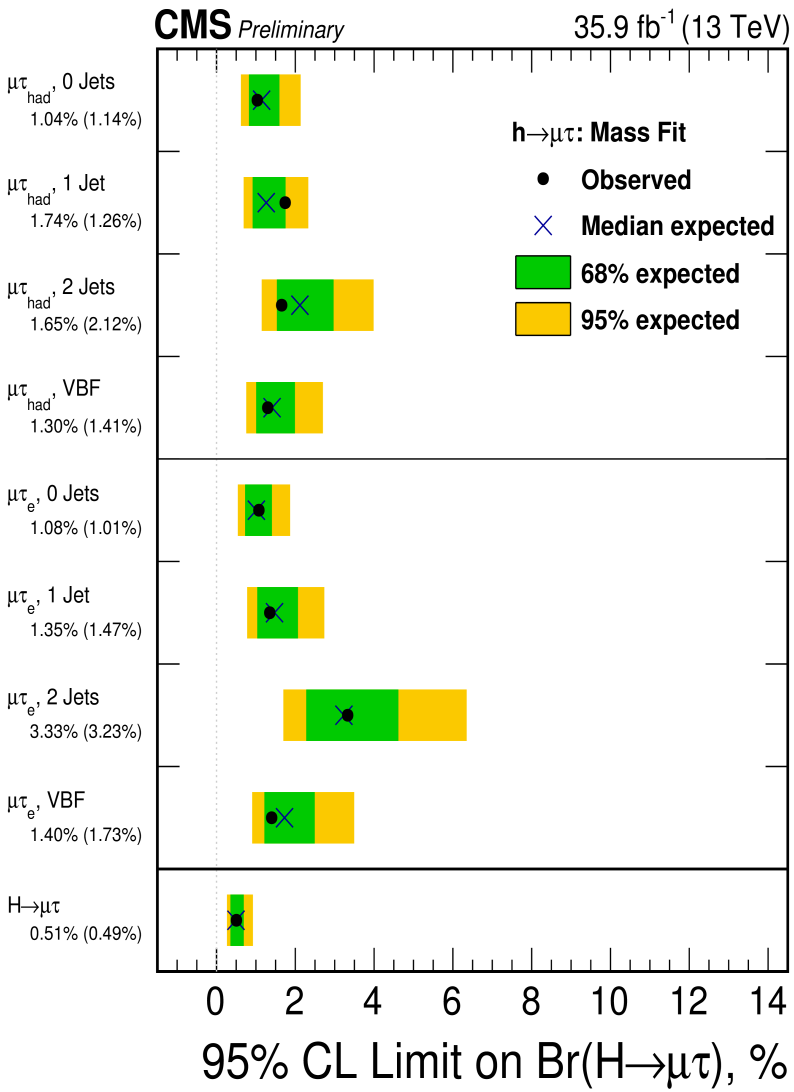
Hint of New Physics in h → τμ?

CMS'16

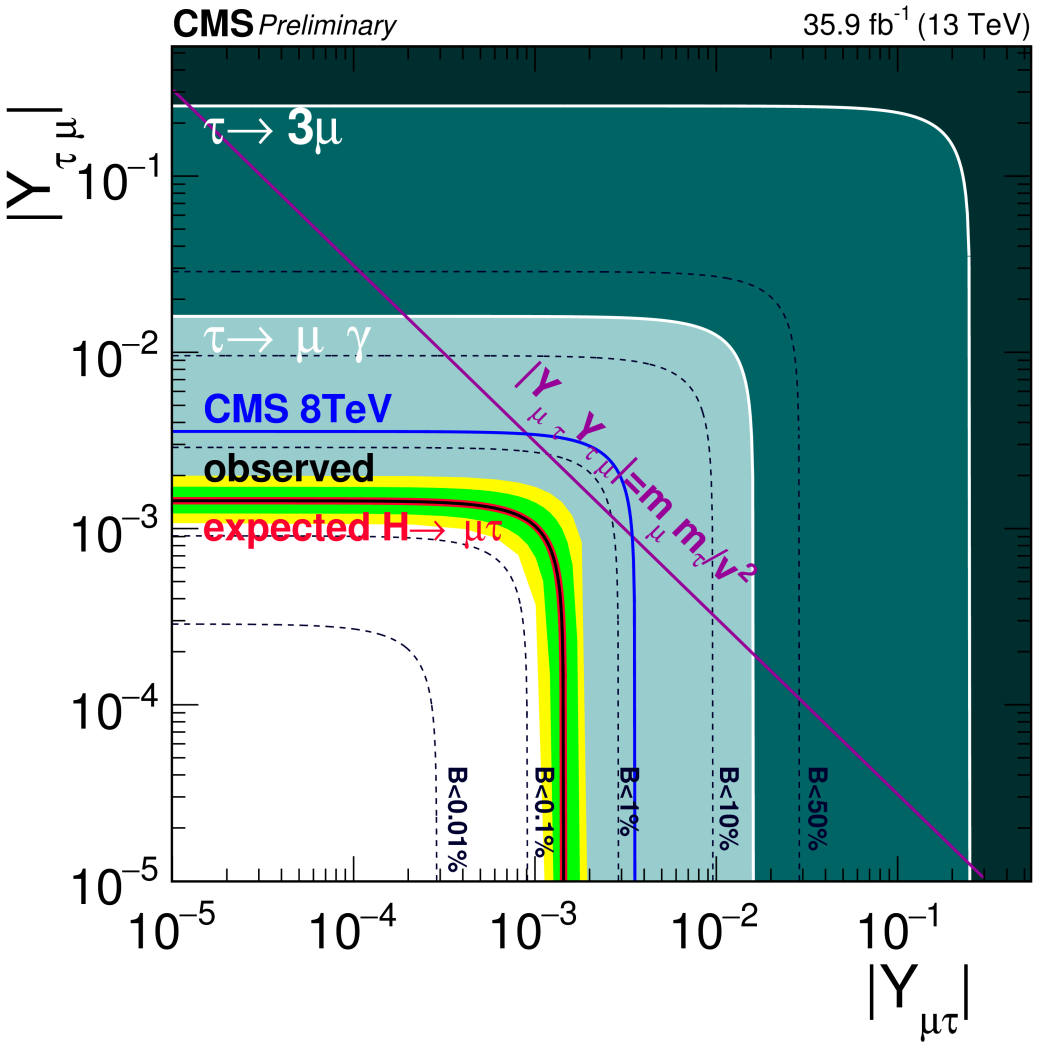




Hint of New Physics in h → τμ?



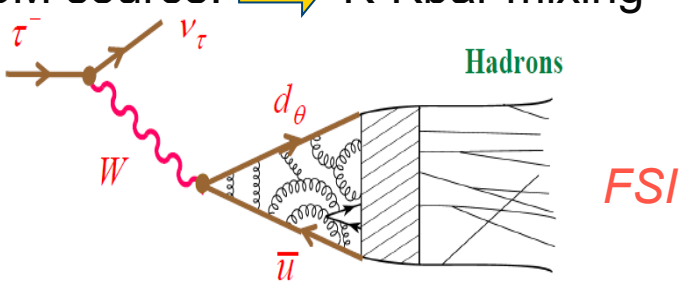
CMS'17



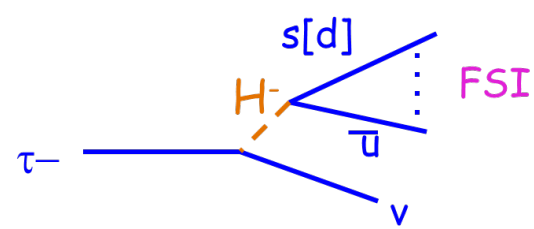
3. LFC processes: CPV in tau decays

3.1 Introduction

- CP violation measured in K and B decays ⇨ in agreement with SM
- Not enough CP violation to explain asymmetry matter/anti-matter
- Look elsewhere:
  - Neutrinos
  - Charged leptons
  - Electric dipole moments
- Aim: pin down new sources of CPV in the lepton sector and discriminating between NP scenarios
- Study of tau decays:
  - CPV in tau pair production ($e^+e^- \rightarrow \tau^+\tau^-$) ⇨ EDM
  - CPV in hadronic tau decays
    - SM source: ⇨ K-Kbar mixing



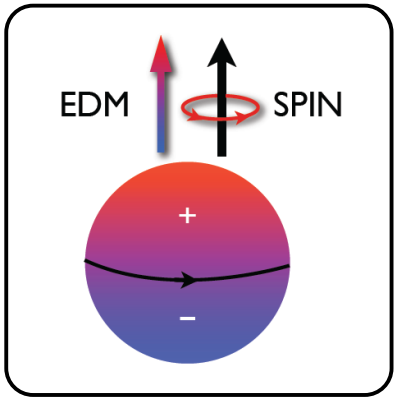
NP source: In the vertex



3.2 EDM of the Tau

- Probe to test new sources of CP violation

No SM background



P and T violation:

$$\vec{d} \propto \vec{J}$$

$$\mathcal{H} \sim d\,\vec{J}\cdot\vec{E}$$

EDMs in $e \cdot cm$ *From V. Cirigliano*

System	current	projected	SM (CKM)
e	$\sim 10^{-28}$	10^{-29}	$\sim 10^{-38}$
μ	$\sim 10^{-19}$		$\sim 10^{-35}$
τ	$\sim 10^{-16}$		$\sim 10^{-34}$
n	$\sim 10^{-26}$	10^{-28}	$\sim 10^{-31}$
p	$\sim 10^{-23}$	10^{-29} **	$\sim 10^{-31}$
^{199}Hg	$\sim 10^{-29}$	10^{-30}	$\sim 10^{-33}$
^{129}Xe	$\sim 10^{-27}$	10^{-29}	$\sim 10^{-33}$
^{225}Ra	$\sim 10^{-23}$	10^{-26}	$\sim 10^{-33}$
...	...		...

- CPV in tau pair production ($e^+e^- \rightarrow \tau^+\tau^-$) ⇒ EDM
- Very challenging measurement for τ
- Measured using spin correlations of decay product the taus
- Help of polarized beams?

3.2 EDM of the Tau

- The squared spin density matrix for $e^+(\mathbf{p})\, e^-(-\mathbf{p}) \to \gamma^* \to \tau^+(\mathbf{k},\mathbf{S}_+)\, \tau^-(-\mathbf{k},\mathbf{S}_-)$

$$\mathcal{M}^2 = \mathcal{M}^2_{SM} + \mathrm{Re}(\mathrm{d}_\tau)\mathcal{M}^2_{\mathrm{Re}} + \mathrm{Im}(\mathrm{d}_\tau)\mathcal{M}^2_{\mathrm{Im}} + |\mathrm{d}_\tau|^2\mathcal{M}^2_{\mathrm{d}^2}$$

- Study of spin momentum correlations:

$$\boxed{\begin{aligned} \mathcal{M}^2_{\mathrm{Re}} &\propto (S_+ \times S_-)\cdot\hat{k}, \quad (S_+ \times S_-)\cdot\hat{p}, \\ \mathcal{M}^2_{\mathrm{Im}} &\propto (S_+ - S_-)\cdot\hat{k}, \quad (S_+ - S_-)\cdot\hat{p}, \end{aligned}}$$

τ spin vector

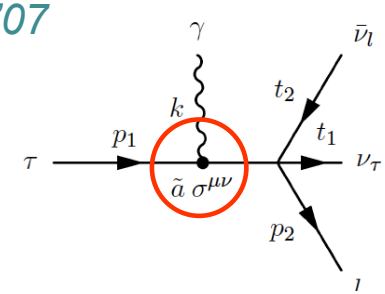
unit vectors of momenta of τ and e^- in CMS

Belle'02

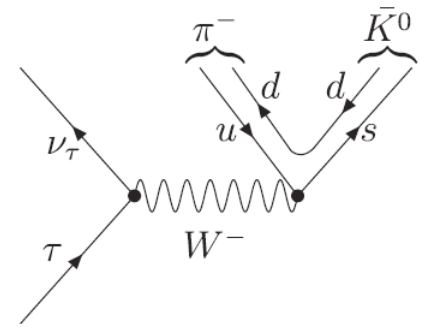
$$\boxed{-0.22 < \mathrm{Re}\,(d_\tau) < 0.45 \quad (10^{-16} e\cdot\mathrm{cm})} \quad \text{and} \quad \boxed{-0.25 < \mathrm{Im}\,(d_\tau) < 0.08 \quad (10^{-16} e\cdot\mathrm{cm})}$$

- Polarized beams will help since the decay products of only one tau could be studied *Bernabeu, Gonzalez-Sprinberg, Vidal'04,'07*

- Radiative decay possibility
  Eidelmana, Epifanov, Fael, Mercolli, Passera'16



3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry



- $$A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}$$

$$= |p|^2 - |q|^2 \approx (0.36 \pm 0.01)\%$$ in the SM

Bigi & Sanda'05
Grossman & Nir'11

$$\left|K_S^0\right\rangle = p\left|K^0\right\rangle + q\left|\overline{K}^0\right\rangle$$
$$\left|K_L^0\right\rangle = p\left|K^0\right\rangle - q\left|\overline{K}^0\right\rangle$$

$$\left\langle K_L \middle| K_S \right\rangle = |p|^2 - |q|^2 \simeq 2\,\mathrm{Re}\left(\varepsilon_K\right)$$

- Experimental measurement : *BaBar'11*

$$A_{Q\,\mathrm{exp}} = \left(-0.36 \pm 0.23_{\mathrm{stat}} \pm 0.11_{\mathrm{syst}}\right)\%$$ ⇨ 2.8σ from the SM!

- CP violation in the tau decays should be of opposite sign compared to the one in D decays in the SM

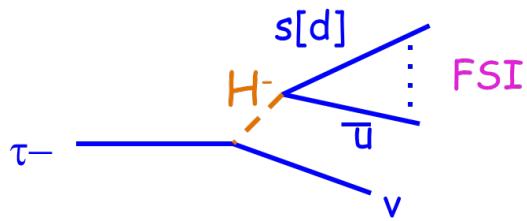
Grossman & Nir'11

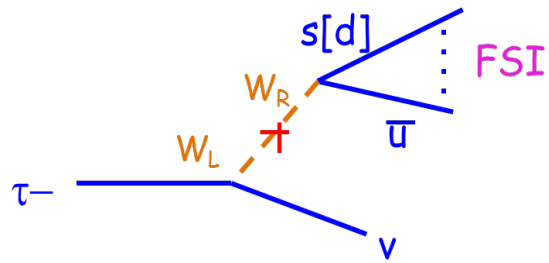
$$A_D = \frac{\Gamma\left(D^+ \to \pi^+ K_S^0\right) - \Gamma\left(D^- \to \pi^- K_S^0\right)}{\Gamma\left(D^+ \to \pi^+ K_S^0\right) + \Gamma\left(D^- \to \pi^- K_S^0\right)} = (-0.54 \pm 0.14)\%$$

Belle, Babar, CLEO, FOCUS

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- New physics? Charged Higgs, W_L-W_R mixings, leptoquarks, tensor interactions (*Devi, Dhargyal, Sinha'14, Cirigliano, Crivellin, Hoferichter'17*)?





Bigi'Tau12

Very difficult to explain!

- Need to investigate how large can be the prediction in realistic new physics models: it looks like *a tensor interaction* can explain the effect but in conflict with bounds from neutron EDM and $D\bar{D}$ mixing

Cirigliano, Crivellin, Hoferichter'17

➡ light BSM physics?

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- In this measurement, need to know hadronic part ⇨ form factors

$$\langle K\pi | \bar{s}\gamma_\mu u | 0 \rangle = \left[(p_K - p_\pi)_\mu + \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu \right] f_+(s) - \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu f_0(s)$$

vector scalar

with $s = Q^2 = (p_K + p_\pi)^2$

$$\Delta_{K\pi} = \left(M_K^2 - M_\pi^2\right)$$

3.3 $\tau \to K\pi\nu_\tau$ angular CP violating asymmetry

- Measurement of the angular CP asymmetry from Belle:

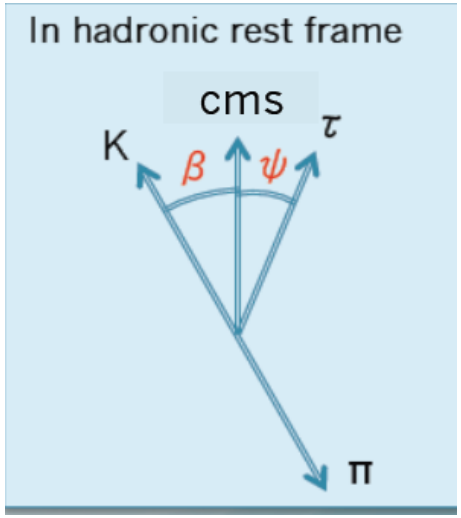
$$\frac{d\Gamma\left(\tau^- \to K\pi^-\nu_\tau\right)}{d\sqrt{Q^2}\,d\cos\theta\;d\cos\beta} = \left[A(Q^2) - B(Q^2)\left(3\cos^2\psi - 1\right)\left(3\cos^2\beta - 1\right)\right]\left|f_+(s)\right|^2 + m_\tau^2\left|\tilde{f}_0(s)\right|^2 - C(Q^2)\cos\psi\cos\beta\,\mathrm{Re}\left(f_+(s)\tilde{f}_0^*(s)\right)$$

CP violating term
S-P interference

– $A(Q^2)$, $B(Q^2)$, $C(Q^2)$: kinematic factors

– Angles:
in $K\pi$ rest frame
  - β: angle between kaon and e^+e^- CMS frame
  - Ψ: angle between τ and CMS frame

in τ rest frame
  - θ: angle between τ direction in CMS and direction of $K\pi$ system (dependence with Ψ)



3.3 $\tau \to K\pi\nu_\tau$ angular CP violating asymmetry

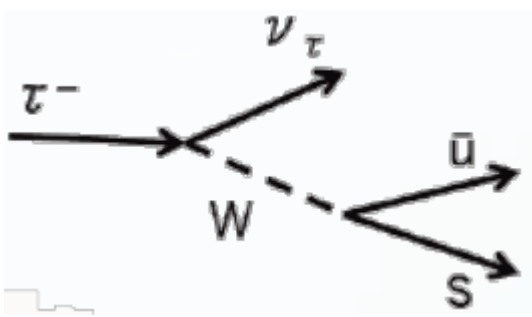
- Measurement of the angular CP asymmetry from Belle:

$$\frac{d\Gamma\left(\tau^- \to K\pi^-\nu_\tau\right)}{d\sqrt{Q^2}\, d\cos\theta\; d\cos\beta} = \left[A(Q^2) - B(Q^2)\left(3\cos^2\psi - 1\right)\left(3\cos^2\beta - 1\right)\right]\left|f_+(s)\right|^2 + m_\tau^2\left|\tilde{f}_0(s)\right|^2 - C(Q^2)\cos\psi\cos\beta\,\mathrm{Re}\left(f_+(s)\tilde{f}_0^*(s)\right)$$

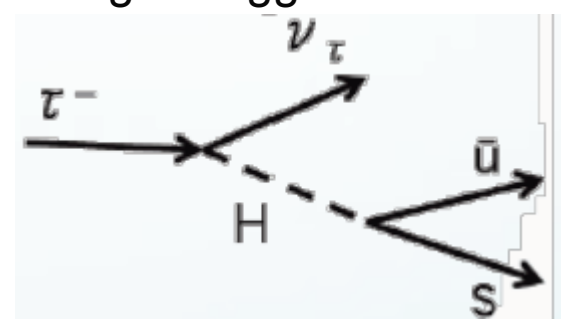
CP violating term
S-P interference

– $A(Q^2)$, $B(Q^2)$, $C(Q^2)$: kinematic factors

Charged Higgs contribution



+



Khün & Mirkes'05

$$\tilde{f}_0(s) = f_0(s) + \frac{\eta^2}{m_\tau^2} f_H(s)$$

with $$f_H(s) = \frac{s}{m_u - m_s} f_0(s)$$

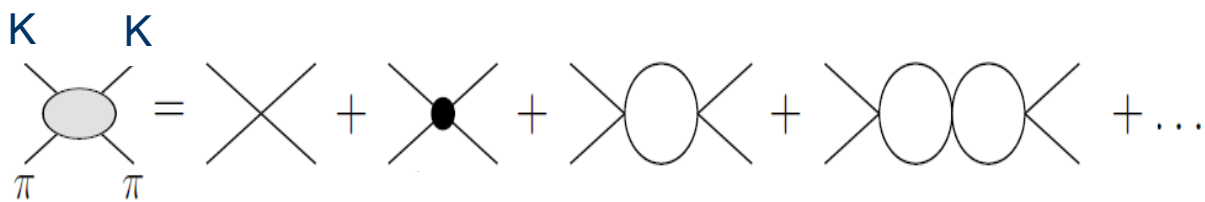
3.3 $\tau \to K\pi\nu_\tau$ angular CP violating asymmetry

- Belle uses sum of BWs to fit the invariant mass distribution *Belle'08*

$$F_V = \frac{1}{1+\beta+\chi}\left[BW_{K^*(892)}(s) + \beta BW_{K^*(1410)}(s) + \chi BW_{K^*(1680)}(s)\right]$$

$$F_S = \varkappa \frac{s}{M^2_{K_0^*(800)}} BW_{K_0^*(800)}(s) + \gamma \frac{s}{M^2_{K_0^*(1430)}} BW_{K_0^*(1430)}(s)$$

- Can be justified for the vector but not for the scalar!
  ➡ Use a parametrization relying on dispersion relations instead:
  - Resum all final state $K\pi$ rescattering



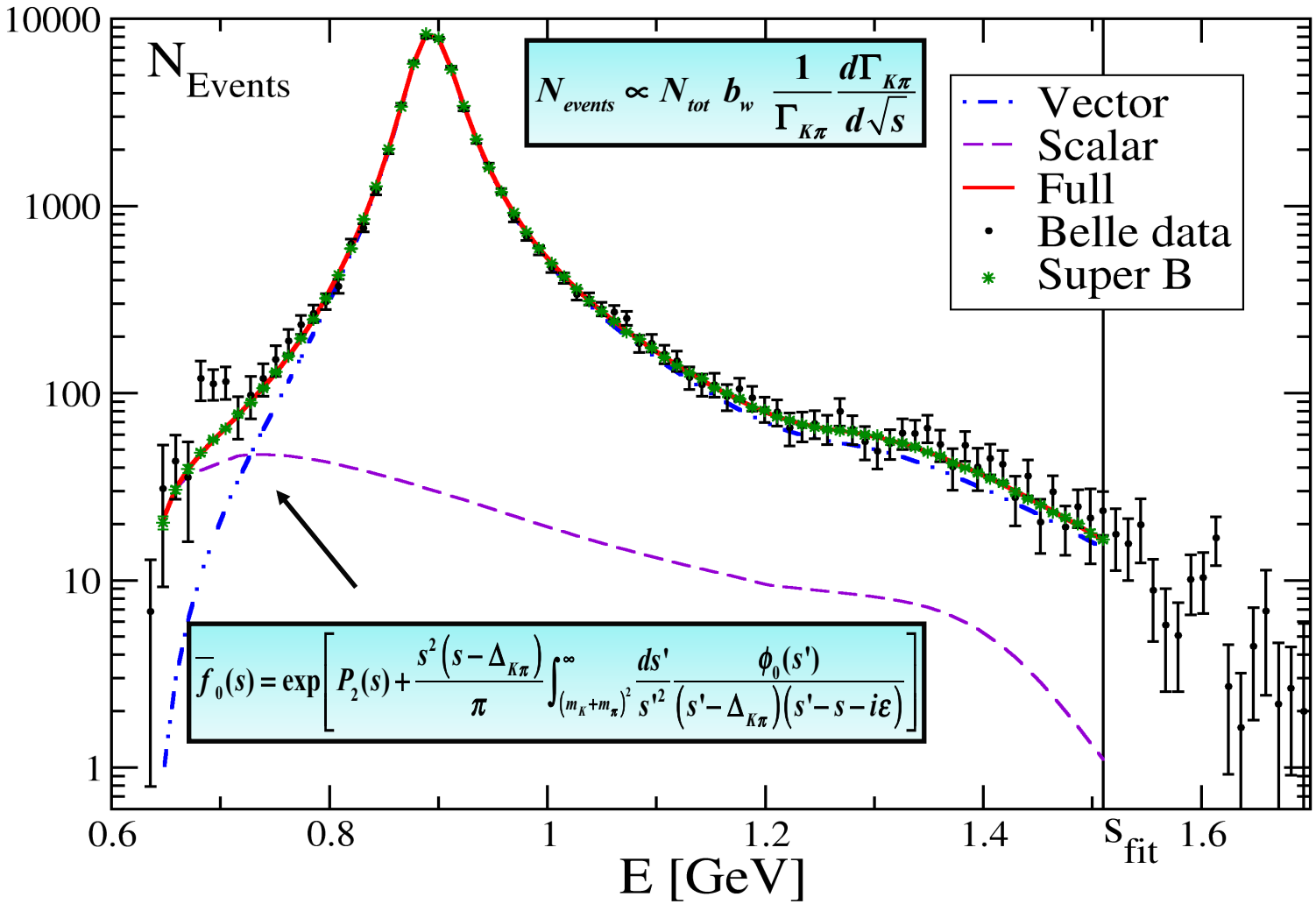
  - Allow to combine with $K \to \pi l \nu_l$ precise measurements

- Several theoretical parametrizations proposed: All rely on analyticity and unitarity and crossing symmetry *Jamin, Pich, Portolés'06,'08, Moussallam'08, Boito, Escribano, Jamin'09,'10, Bernard, Boito, E.P.'11, Bernard'14*

3.3 $\tau \to K\pi\nu_\tau$ angular CP violating asymmetry

Bernard, Boito, E.P.'11

Antonelli, Cirigliano, Lusiani, E.P.'13



3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- In this measurement, need to know hadronic part ⇨ form factors

$$\langle K\pi | \bar{s}\gamma_\mu u | 0 \rangle = \left[(p_K - p_\pi)_\mu + \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu \right] f_+(s) - \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu f_0(s)$$

with $s = Q^2 = (p_K + p_\pi)^2$

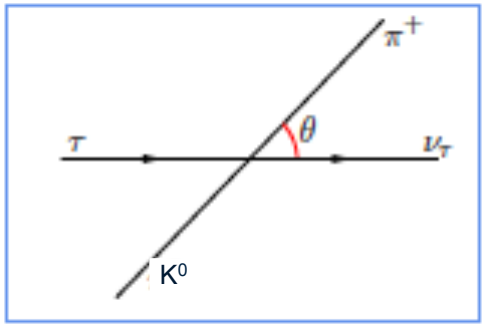
vector

scalar

- Up to now know from decay spectrum but difficult to disentangle scalar and vector form factor ⇨ consider the FB asymmetry instead

$$A_{FB} = \frac{d\Gamma(\cos\theta) - d\Gamma(-\cos\theta)}{d\Gamma(\cos\theta) + d\Gamma(-\cos\theta)}$$

Beldjoudi & Truong'94
Moussallam, B2TIP



- Formula: can disentangle scalar and vector FF easily

$$A_{FB}(s) = \frac{3\Delta_{\pi^+K^0}\sqrt{\lambda_{\pi^+K^0}(s)}|f_V^{K\pi}(s)||f_0^{K\pi}(s)|\cos\left(\delta_1^{1/2} - \delta_0^{1/2}\right)}{\underbrace{|f_V^{K\pi}(s)|^2\lambda_{\pi^+K^0}(s)}_{\text{vanishes at threshold}}(1 + 2s/m_\tau^2) + 3|f_0^{K\pi}(s)|^2\Delta^2_{\pi^+K^0}}.$$

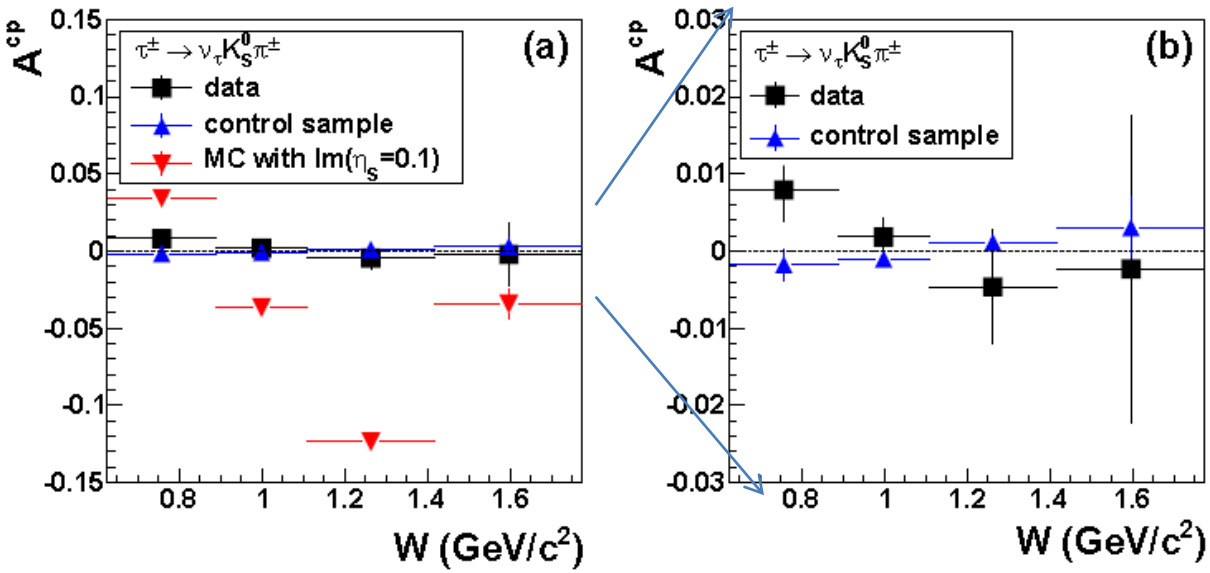
⇨ Never done before: Feasible at HIEPA?

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- Measurement of the direct contribution of NP in the angular CP violating asymmetry done by *CLEO* and *Belle*

  ➡ Belle does not see any asymmetry at the *0.2 - 0.3% level*

Belle'11



- Problem with this measurement? ➡ It would be great to have other experimental measurements from *Belle II or HIEPA*

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- The angular CP asymmetry from Belle:

$$\frac{d\Gamma\left(\tau^- \to K\pi^-\nu_\tau\right)}{d\sqrt{Q^2}\, d\cos\theta\; d\cos\beta} = \left[A(Q^2) - B(Q^2)\left(3\cos^2\psi - 1\right)\left(3\cos^2\beta - 1\right)\right]\left|f_+(s)\right|^2 + m_\tau^2\left|\tilde{f}_0(s)\right|^2 - C(Q^2)\cos\psi\cos\beta\,\mathrm{Re}\left(f_+(s)\tilde{f}_0^*(s)\right)$$

CP violating term
S-P interference

- When integrating on the angle the interference term between scalar and vector vanishes

$$\frac{d\Gamma}{d\sqrt{Q^2}} = \frac{G_F^2 \sin^2\theta_c m_\tau^3}{3\times 2^5 \times \pi^3 Q^2}\left(1 - \frac{Q^2}{m_\tau^2}\right)^2\left(1 + \frac{2Q^2}{m_\tau^2}\right) \times q_1(Q^2)\left\{q_1(Q^2)^2\,|F_V|^2 + \frac{3}{4}\frac{Q^2}{\left(1 + 2Q^2/m_\tau^2\right)}|F_S|^2\right\}$$

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

Devi, Dhargyal, Sinha'14
Cirigliano, Crivellin, Hoferichter'17

- We need a tensor interaction to get some interference:

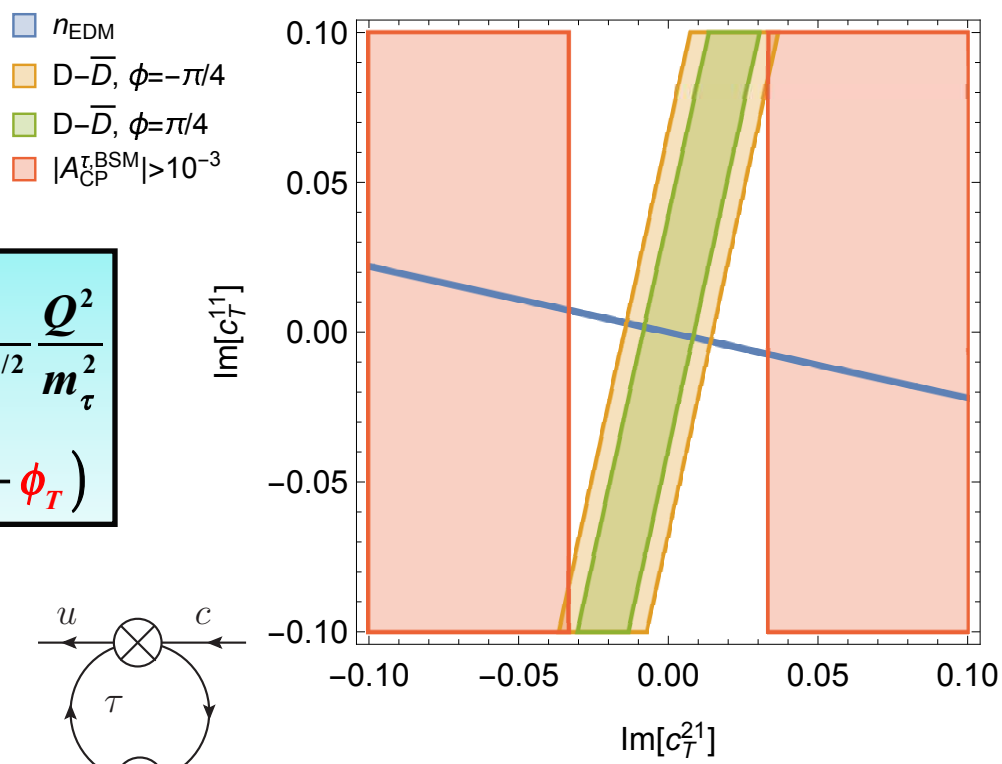
$$\mathcal{H}_T^{eff} \equiv G'(\bar{s}\sigma_{\mu\nu}u)(\bar{\nu}_\tau(1+\gamma_5)\sigma^{\mu\nu}\tau)$$

with $G' = \frac{G_F}{\sqrt{2}} C_T, \quad C_T = |C_T| e^{i\phi_T}$

- When integrating the interference term between vector and tenson does not vanish:

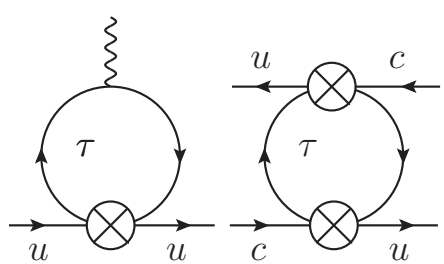
$$\frac{d\Gamma}{dQ^2} = \frac{d\Gamma_{SM}}{dQ^2} + \frac{d\Gamma_T}{dQ^2} + \frac{d\Gamma_{V-T}}{dQ^2}$$

$$\frac{d\Gamma_{V-T}}{dQ^2} = G_F^2 \sin^2\theta_C \frac{m_\tau^3}{32\pi^3}\left(\frac{m_\tau^2 - Q^2}{m_\tau^2}\right)^2 \frac{q_1^3}{(Q^2)^{3/2}} \frac{Q^2}{m_\tau^2} \times |C_T||F_V(s)||F_T(s)|\cos(\delta_T(s) - \delta_V(s) + \phi_T)$$



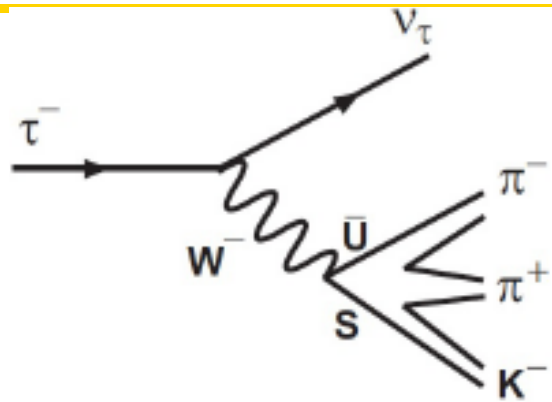
In conflict with bounds from neutron EDM and DD mixing

Cirigliano, Crivellin, Hoferichter'17



3.4 Three body CP asymmetries

- Ex: $\tau \to K\pi\pi\nu_\tau$



e.g., Choi, Hagiwara and Tanabashi'98
Kiers, Little, Datta, London et al.,'08
Mileo, Kiers and, Szynkman'14

- A variety of CPV observables can be studied : $\tau \to K\pi\pi\nu_\tau$, $\tau \to \pi\pi\pi\nu_\tau$ rate, angular asymmetries, triple products,….

  Same principle as in charm, *see Bevan'15*

  Difficulty : Treatement of the hadronic part
  Hadronic final state interactions have to be taken into account!
  ➡ Disentangle weak and strong phases

- More form factors, more asymmetries to build but same principles as for 2 bodies

4. Conclusion and outlook

Conclusion and outlook

- Direct searches for new physics at the TeV-scale at LHC by ATLAS and CMS ⇨ energy frontier

- Probing new physics orders of magnitude beyond that scale and helping to decipher possible TeV-scale new physics requires to work hard on the *intensity* and *precision frontiers*

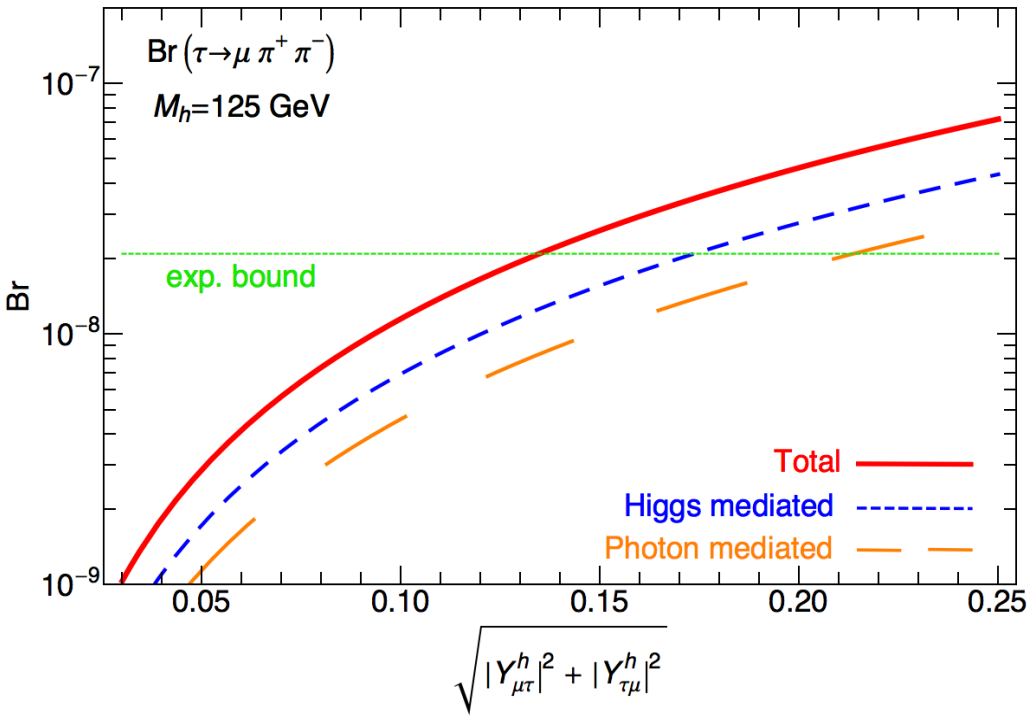
- Charged leptons and in particular *tau physics* offer an important spectrum of possibilities:
  - LFV measurement has SM-free signal
  - Current experiments and mature proposals promise orders of magnitude sensitivity improvements (*Belle II*, *Tau-Charm factory*, etc.)
  - There is a hint of new dynamics in CPV asymmetries in the tau sector
  - Progress towards a better knowledge of hadronic uncertainties
  - New physics models usually strongly correlate the flavours sectors

Conclusion and outlook

- We show how CLFV decays offer excellent model discriminating tools giving indications on
  - the *mediator* (operator structure)
  - the *source of flavour breaking* (comparison $\tau\mu$ vs. τe vs. μe)
- Interplay low energy and collider physics: LFV of the Higgs boson
- We discussed the possibilities to look for CP violation in the tau sector: BaBar result does not agree with SM expectation but needs to be confirmed ⇨ A lot of new measurements possible (A_{CP}, A_{FB}, etc.) to shed light on CP violation in the tau sector: combine strong and weak phase determination
- EDM of the tau also very interesting to study but difficult
- Several topics extremely interesting to study at the tau-charm factories that I did not address:
  - α_S, $|V_{us}|$ and m_s from hadronic tau decays
  - Lepton universality tests, Michel parameters…
- A lot of very interesting physics remains to be done in the tau sector!

5. Back-up

3.5 Results



Bound:

$$\sqrt{\left|Y^h_{\mu\tau}\right|^2 + \left|Y^h_{\tau\mu}\right|^2} \leq 0.13$$

Process	(BR $\times 10^8$) 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2 + \lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau \to \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \to \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\pi^0\pi^0$	$< 1.4 \times 10^3$ [87]	< 6.3	Scalar, Gluon

Less stringent but more robust handle on LFV Higgs couplings

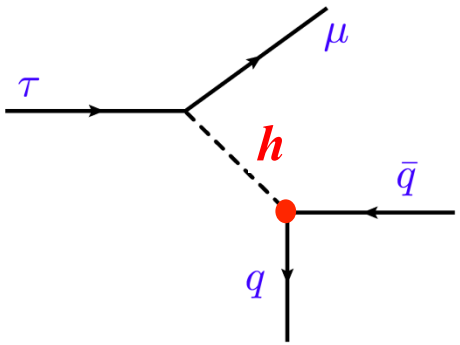
?

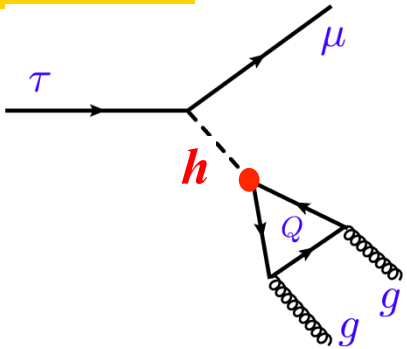
Belle'08'11'12 except last from *CLEO'97*

3.5 What if $\tau \to \mu(e)\pi\pi$ observed?

Reinterpreting *Celis, Cirigliano, E.P'14*

Talk by J. Zupan @ KEK-FF2014FALL





- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!
- $Y_{u,d,s}$ poorly bounded
- For $Y_{u,d,s}$ at their SM values :

$$Br(\tau \to \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, Br(\tau \to \mu\pi^0\pi^0) < 4.6 \times 10^{-12}$$
$$Br(\tau \to e\pi^+\pi^-) < 2.3 \times 10^{-10}, Br(\tau \to e\pi^0\pi^0) < 6.9 \times 10^{-11}$$

- But for $Y_{u,d,s}$ at their upper bound:

$$\boxed{\begin{array}{l} Br(\tau \to \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, Br(\tau \to \mu\pi^0\pi^0) < 1.5 \times 10^{-8} \\ Br(\tau \to e\pi^+\pi^-) < 4.3 \times 10^{-7}, Br(\tau \to e\pi^0\pi^0) < 2.1 \times 10^{-7} \end{array}}$$

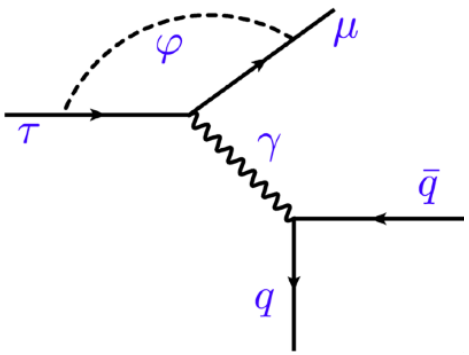
below present experimental limits!

- If discovered ⇒ among other things *upper limit* on $Y_{u,d,s}$!
  ⇒ Interplay between high-energy and low-energy constraints!

3.1 Constraints from $\tau \to \mu\pi\pi$

- Photon mediated contribution requires the pion vector form factor:

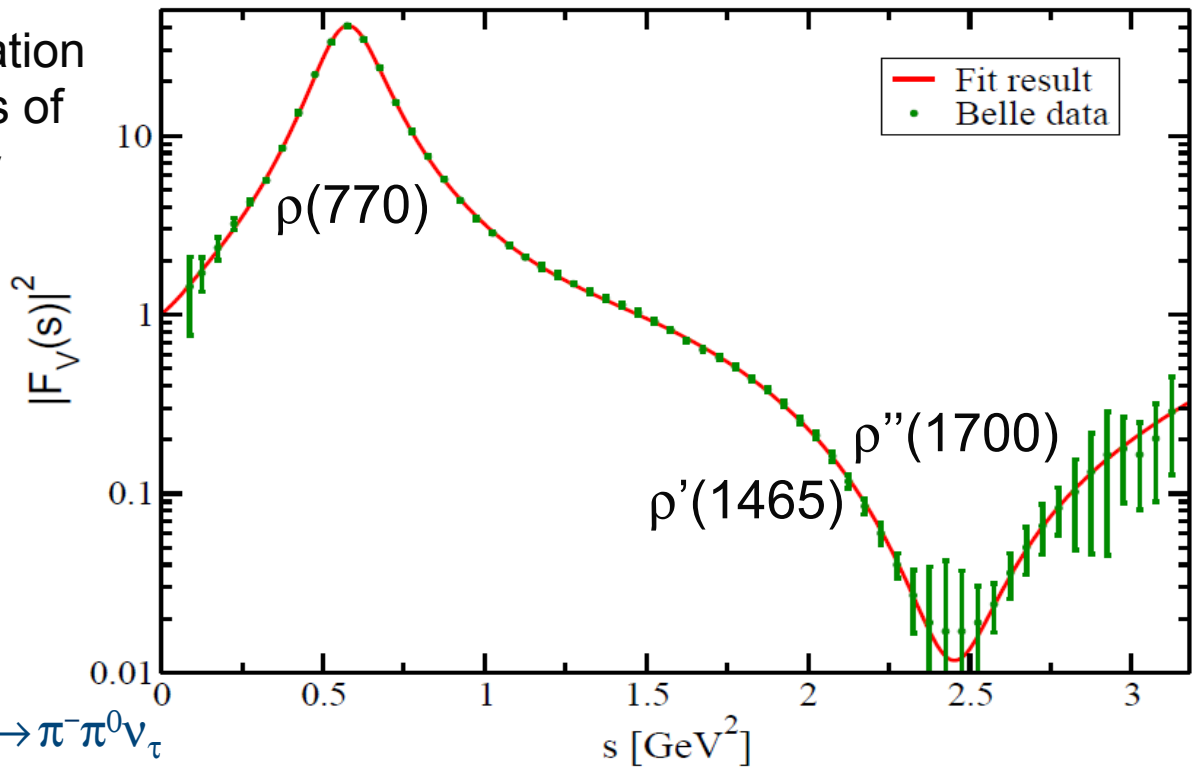
$$\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-})|\tfrac{1}{2}(\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d)|0\rangle \equiv F_V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$



- Dispersive parametrization following the properties of analyticity and unitarity of the Form Factor

Gasser, Meißner´91
Guerrero, Pich'97
Oller, Oset, Palomar´01
Pich, Portolés ´08
Gómez Dumm&Roig'13
...

- Determined from a fit to the Belle data on $\tau^- \to \pi^-\pi^0\nu_\tau$



Celis, Cirigliano, E.P.'14

Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements
    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)
  - Theoretically: Dispersive parametrization for $F_V(s)$

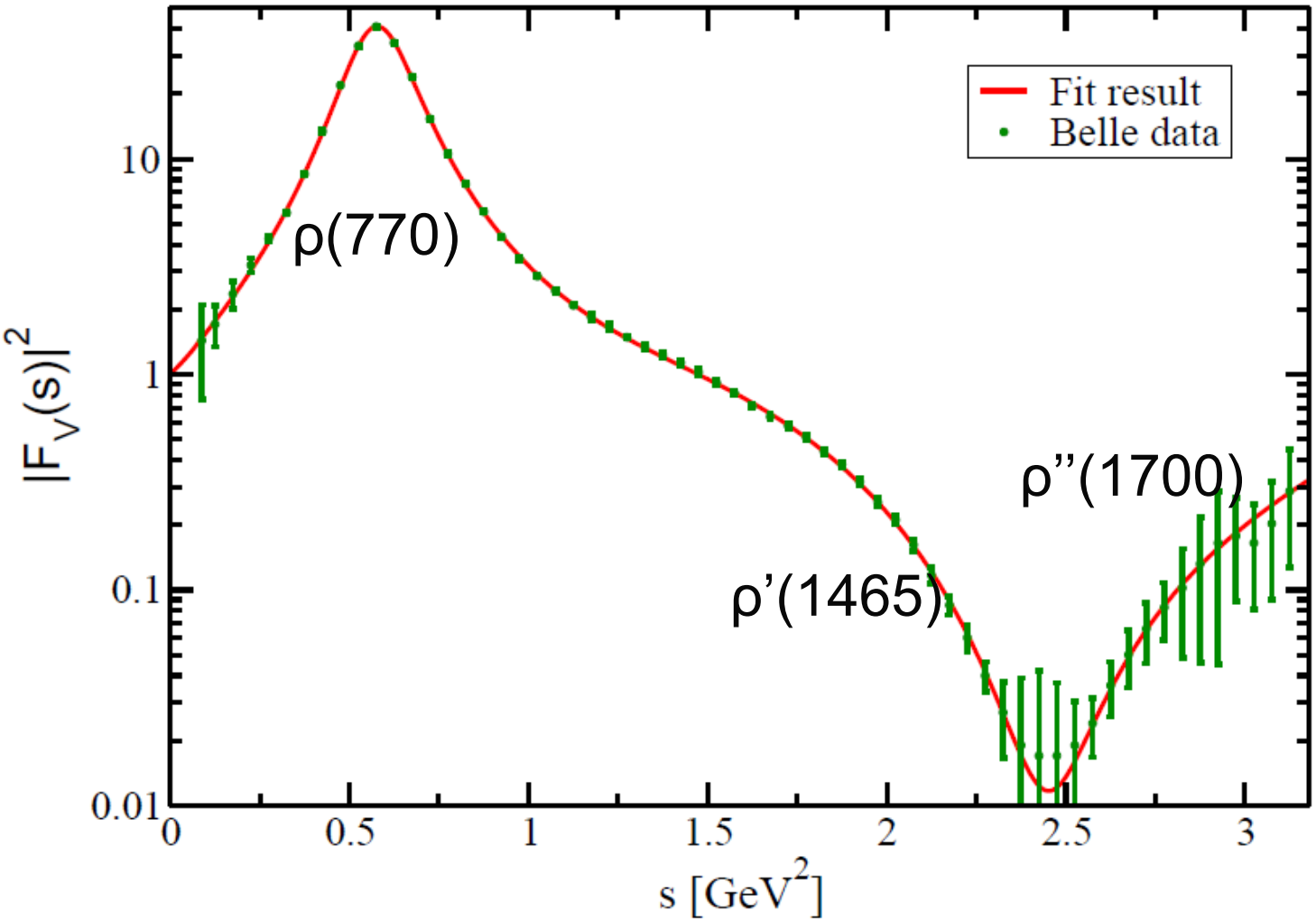
Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$F_V(s) = \exp\left[\lambda_V' \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_V'' - \lambda_V'^2\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{(s'-s-i\varepsilon)}\right]$$

Extracted from a model including 3 resonances ρ(770), ρ'(1465) and ρ''(1700) fitted to the data

  - Subtraction polynomial + phase determined from a *fit* to the *Belle data* $\tau^- \to \pi^0\pi^-\nu_\tau$

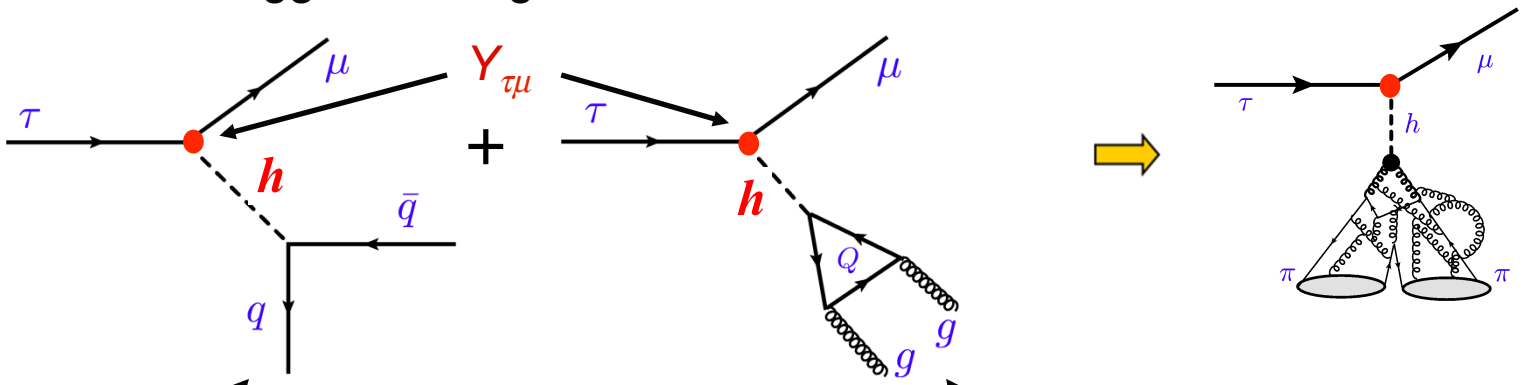
Determination of $F_V(s)$



Determination of $F_V(s)$ thanks to precise measurements from Belle!

3.1 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange



$$s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

$$\langle \pi^+\pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s) \qquad \langle \pi^+\pi^- | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$$

Voloshin'85

$$\langle \pi^+\pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$$

$$\theta^\mu_\mu = -9\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s} m_q \bar{q}q$$

$$\frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}} = \frac{(m_\tau^2 - s)^2 \sqrt{s - 4m_\pi^2}}{256\pi^3 m_\tau^3} \frac{\left(|Y^h_{\tau\mu}|^2 + |Y^h_{\mu\tau}|^2\right)}{M_h^4 v^2} \left|\mathcal{K}_\Delta \Delta_\pi(s) + \mathcal{K}_\Gamma \Gamma_\pi(s) + \mathcal{K}_\theta \theta_\pi(s)\right|^2$$

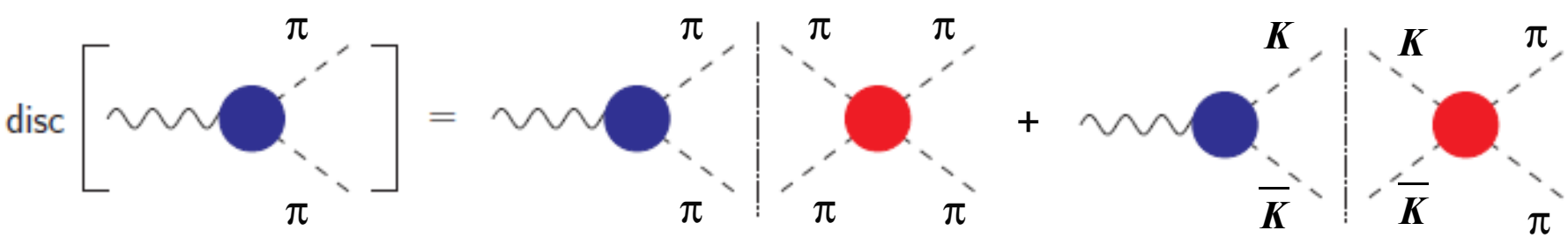
$$f\left(y^h_q\right)$$

Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- No experimental data for the other FFs ⇒ *Coupled channel analysis*
  up to $\sqrt{s}$~1.4 GeV
  Inputs: I=0, S-wave $\pi\pi$ and KK data

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13

- Unitarity:



⇒

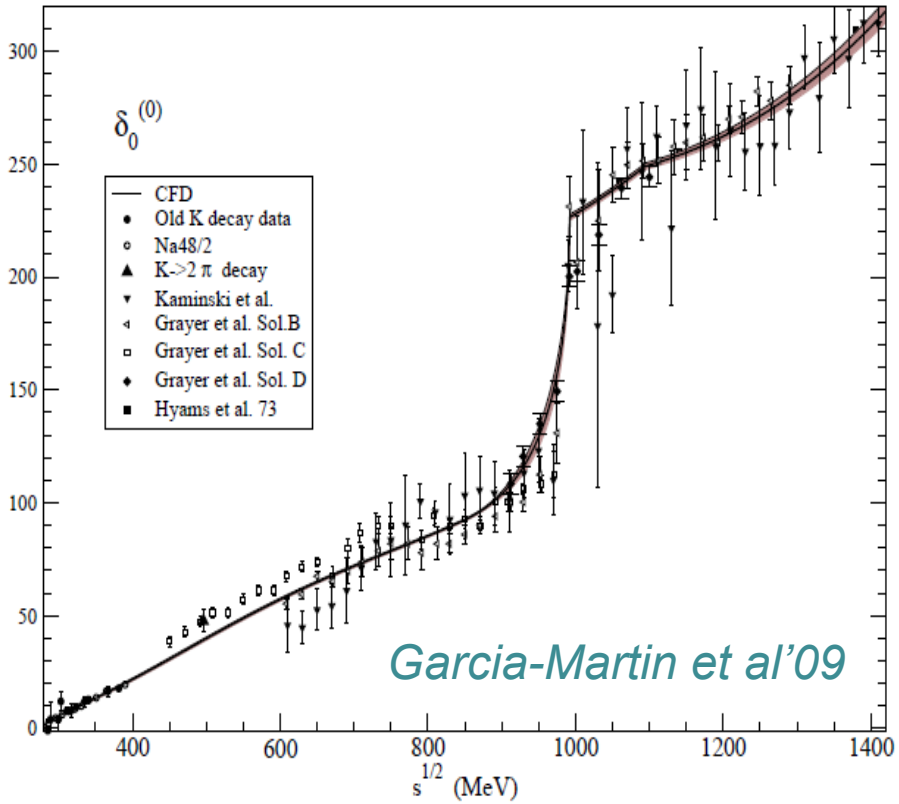
$$\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)$$

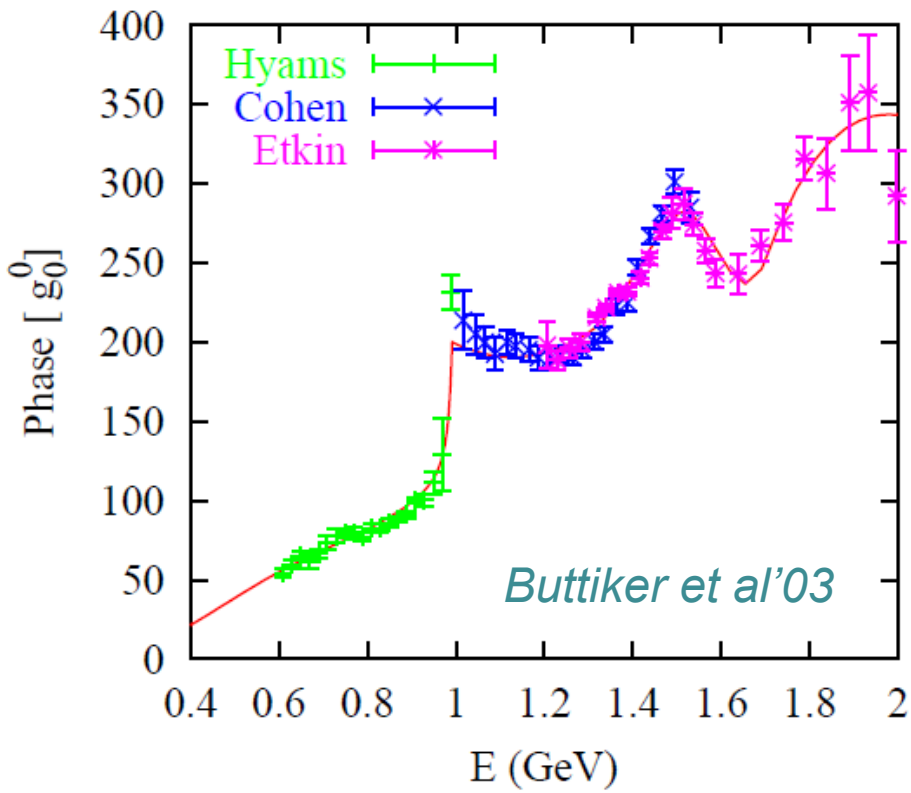
$$n = \pi\pi, K\bar{K}$$

Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

Celis, Cirigliano, E.P.'14

- Inputs : $\pi\pi \to \pi\pi$, KK





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buttiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree
- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- General solution:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$\boldsymbol{X(s) = C(s), D(s)}$$

$$\mathrm{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} \mathrm{Re}\left\{ T^*_{nm} \sigma_m(s) X_m^{(N)} \right\} \longrightarrow \mathrm{Re} X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \mathrm{Im} X_n^{(N+1)}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- Fix the polynomial with requiring $F_P(s) \to 1/s$ (*Brodsky & Lepage*) + ChPT:

Feynman-Hellmann theorem: ⇨

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_u + m_d)\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_u + m_s)\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_d + m_s)\, B_0 + O(m^2)$$

⇨

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- At LO in ChPT:

$$M^2_{\pi^+} = (m_u + m_d)\, B_0 + O(m^2)$$
$$M^2_{K^+} = (m_u + m_s)\, B_0 + O(m^2)$$
$$M^2_{K^0} = (m_d + m_s)\, B_0 + O(m^2)$$

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M^2_\pi + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M^2_\pi + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M^2_K - \frac{1}{2} M^2_\pi \right) + \cdots \end{aligned}$$

- Problem: large corrections in the case of the kaons!
  ⇒ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\; M^2_\pi$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05} \left(M^2_K - 1/2 M^2_\pi \right)$$

Dreiner, Hanart, Kubis, Meissner'13
Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

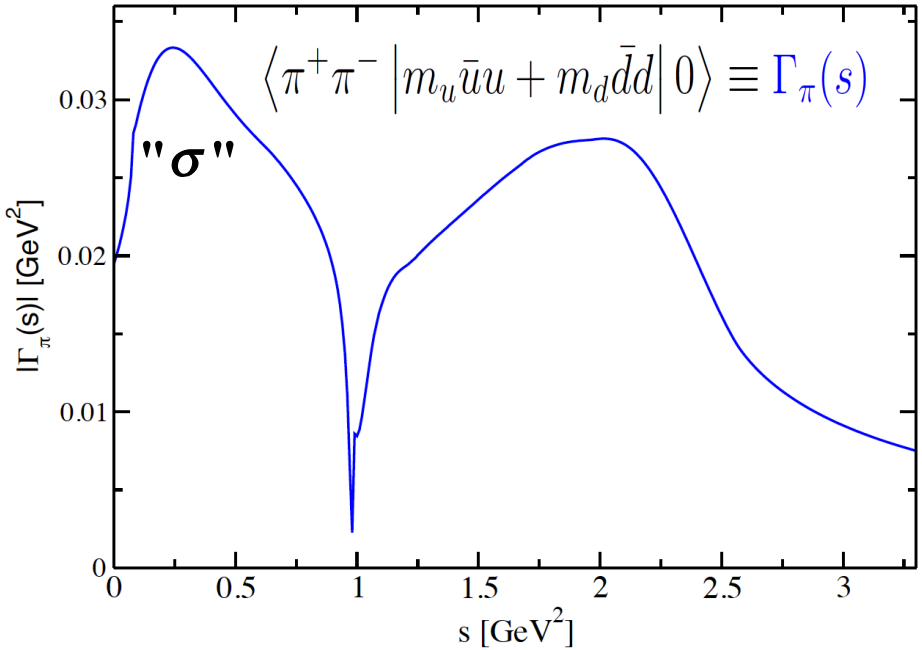
- General solution

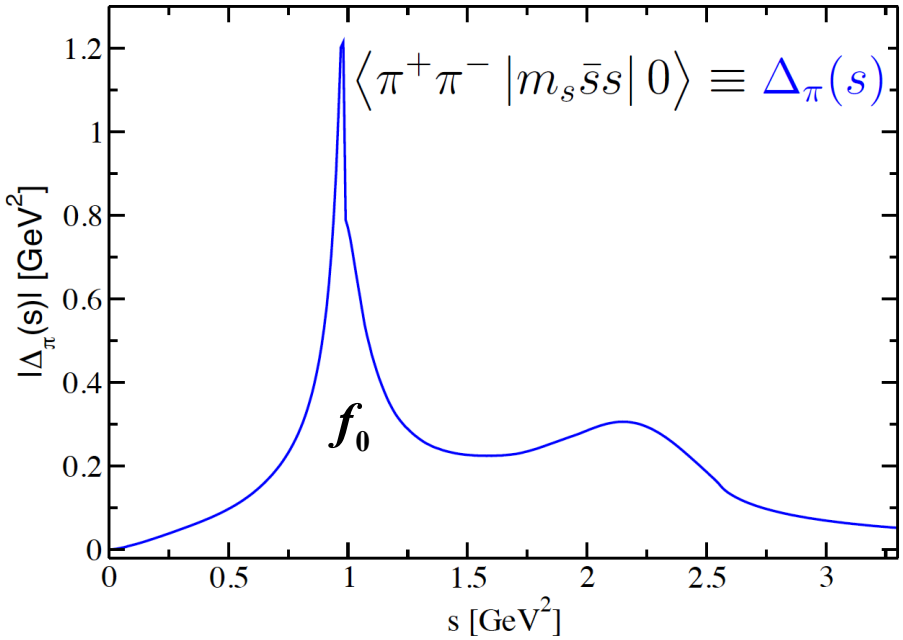
$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

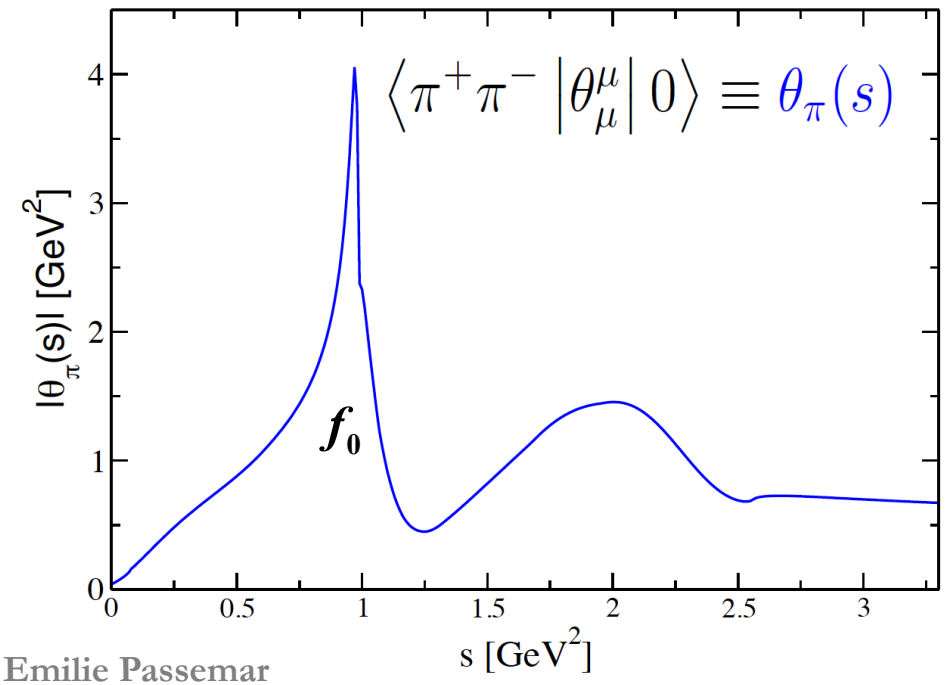
- For θ_P enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

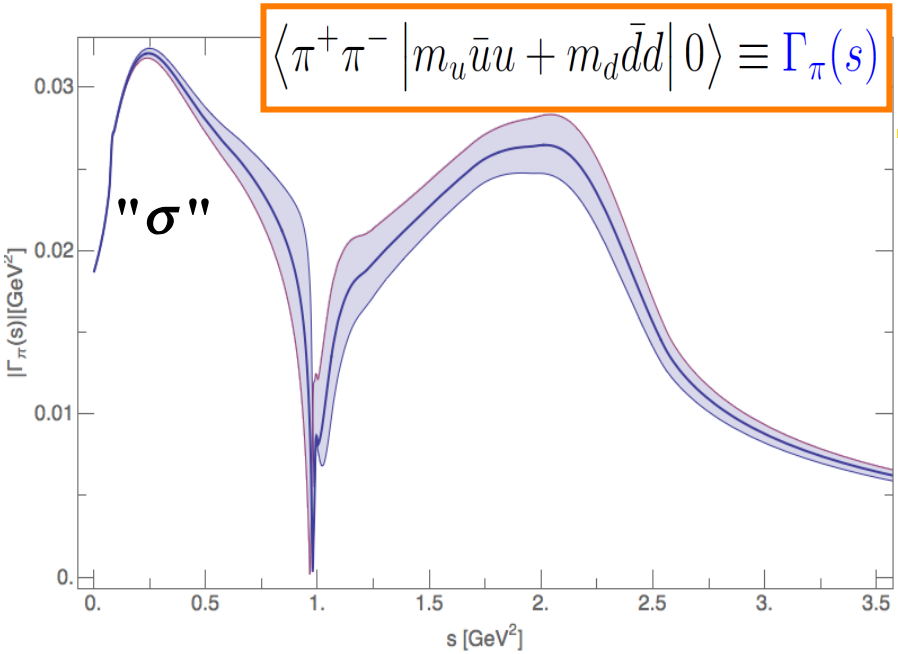
  ➡ Relax the constraints and match to ChPT

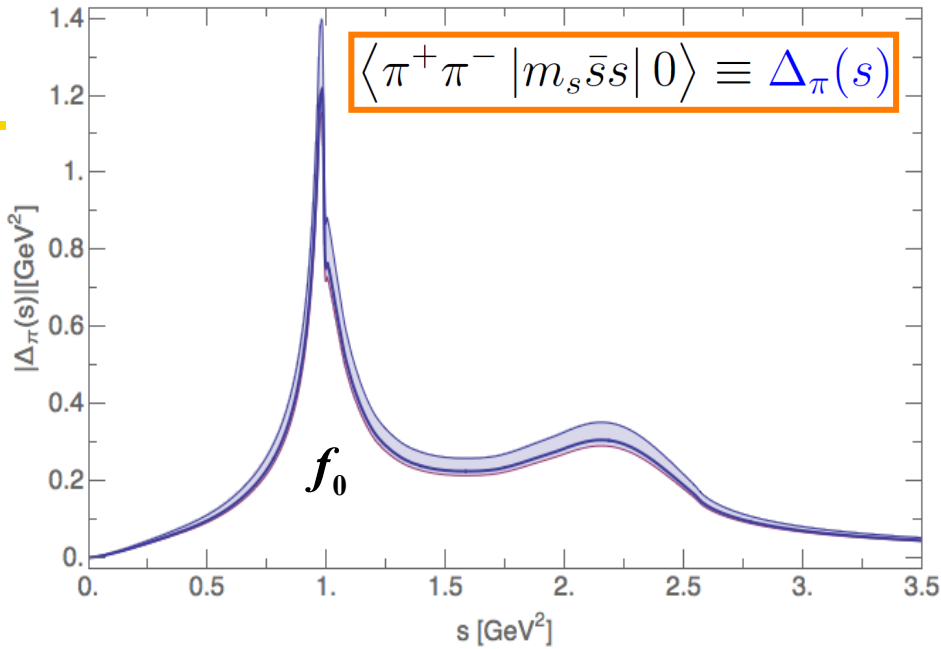
$$\begin{aligned} P_\theta(s) &= 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1\right) s \\ Q_\theta(s) &= \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2\right) s \end{aligned}$$

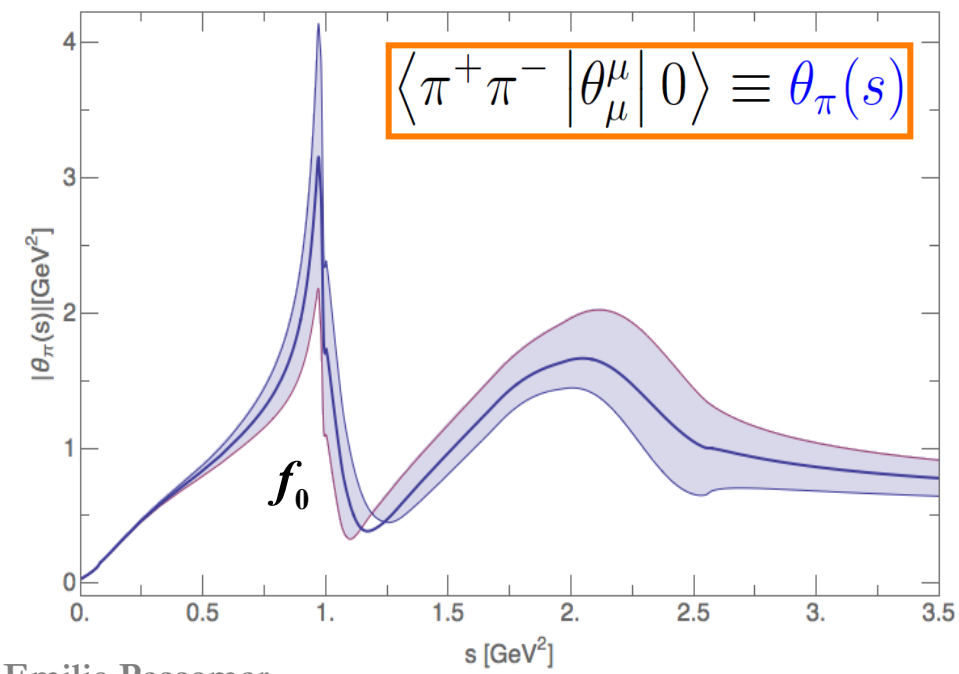

$\langle \pi^+\pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s)$
"σ"
$|\Gamma_\pi(s)|$ [GeV^2]
s [GeV^2]


$\langle \pi^+\pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$
f_0
$|\Delta_\pi(s)|$ [GeV^2]
s [GeV^2]

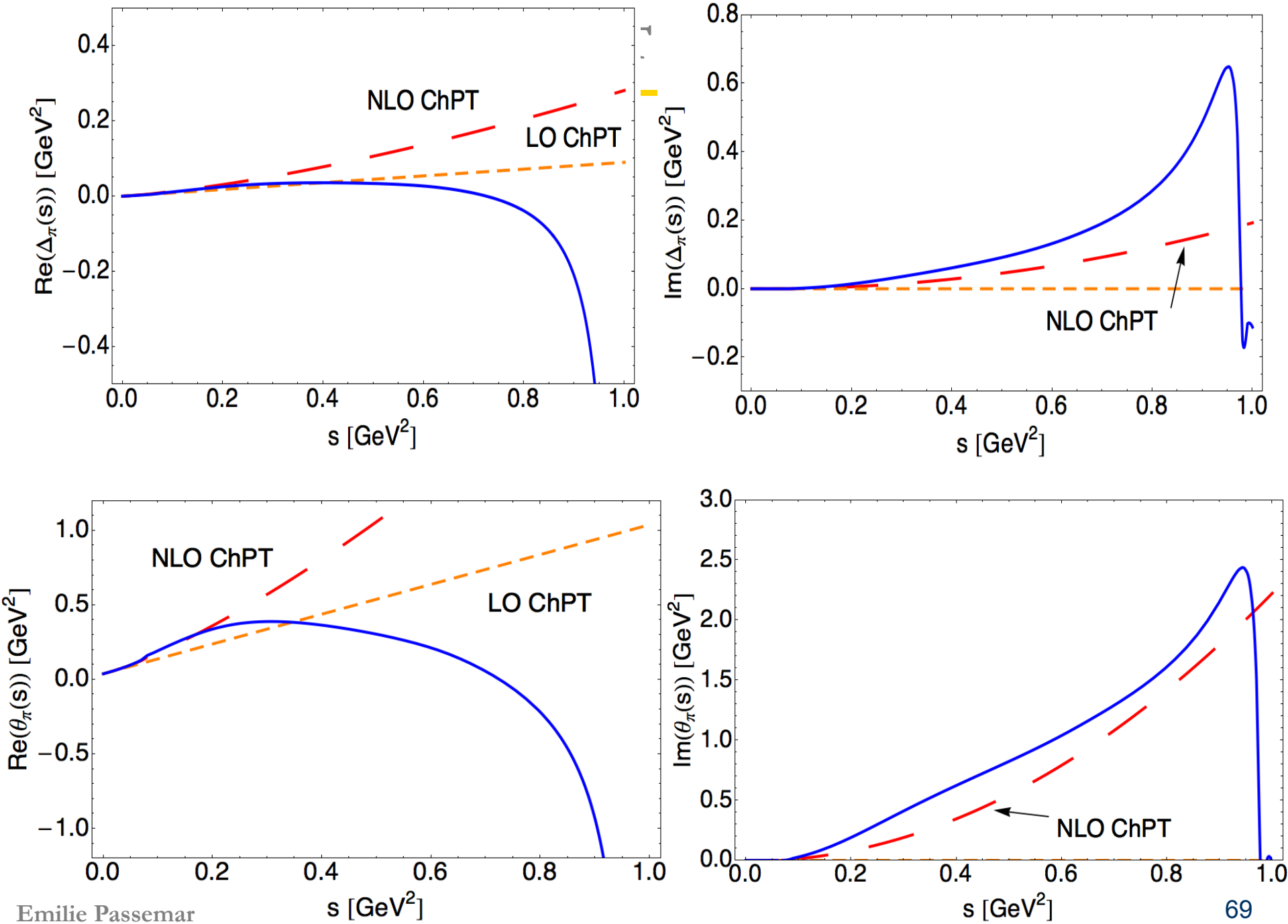

$\langle \pi^+\pi^- | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$
f_0
$|\theta_\pi(s)|$ [GeV^2]
s [GeV^2]







- Uncertainties:
  - Varying s_{cut} (1.4 GeV^2 - 1.8 GeV^2)
  - Varying the matching conditions
  - T matrix inputs


NLO ChPT
LO ChPT
Re(Δ_π(s)) [GeV^2]
s [GeV^2]
Im(Δ_π(s)) [GeV^2]
NLO ChPT
s [GeV^2]
NLO ChPT
LO ChPT
Re(θ_π(s)) [GeV^2]
s [GeV^2]
Im(θ_π(s)) [GeV^2]
NLO ChPT
s [GeV^2]