

Only SND data

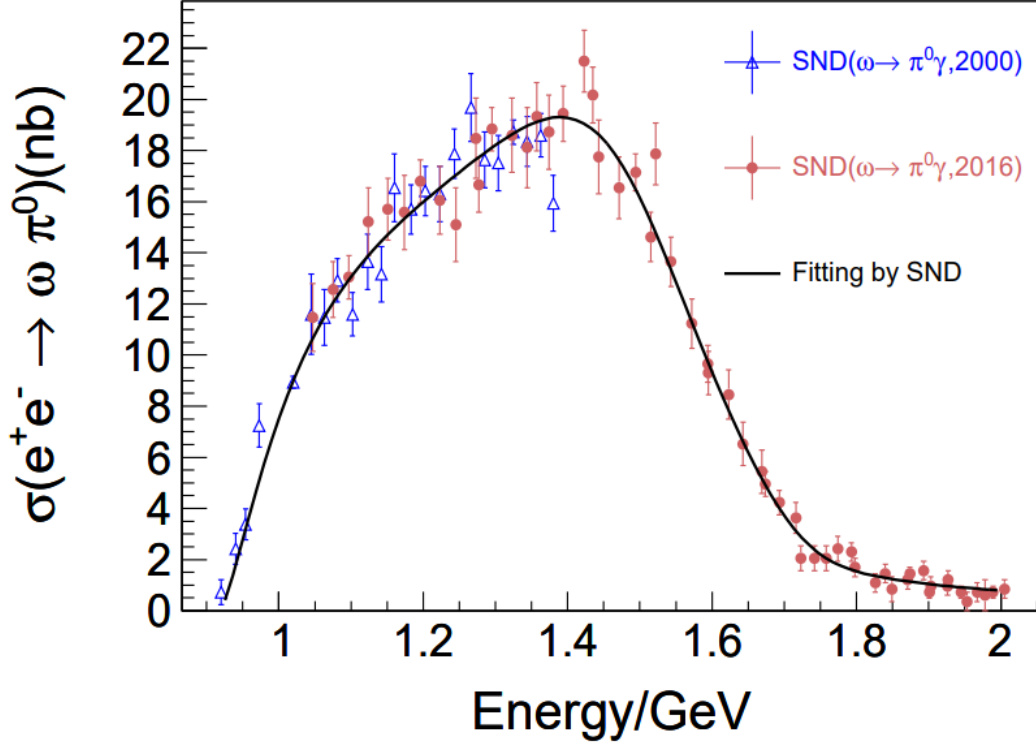


Table 6: Fitted parameters of $e^+e^- \rightarrow \omega\pi^0$ from SND.

Parameters	SND's values	PDG's values
$g_{\rho\omega\pi}, \text{GeV}^{-1}$	15.9 ± 0.4	-
A_1	0.175 ± 0.016	-
A_2	0.014 ± 0.004	-
$M_{\rho(1450)}, \text{MeV}/c^2$	1510 ± 7	1465 ± 25
$\Gamma_{\rho(1450)}, \text{MeV}$	440 ± 40	400 ± 60
$M_{\rho(1700)}, \text{MeV}/c^2$	$\equiv 1720$	1720 ± 20
$\Gamma_{\rho(1700)}, \text{MeV}$	$\equiv 250$	250 ± 100
$\varphi_1, \text{deg.}$	124 ± 17	-
$\varphi_2, \text{deg.}$	-63 ± 21	-
χ^2/ν	$71/73$	-

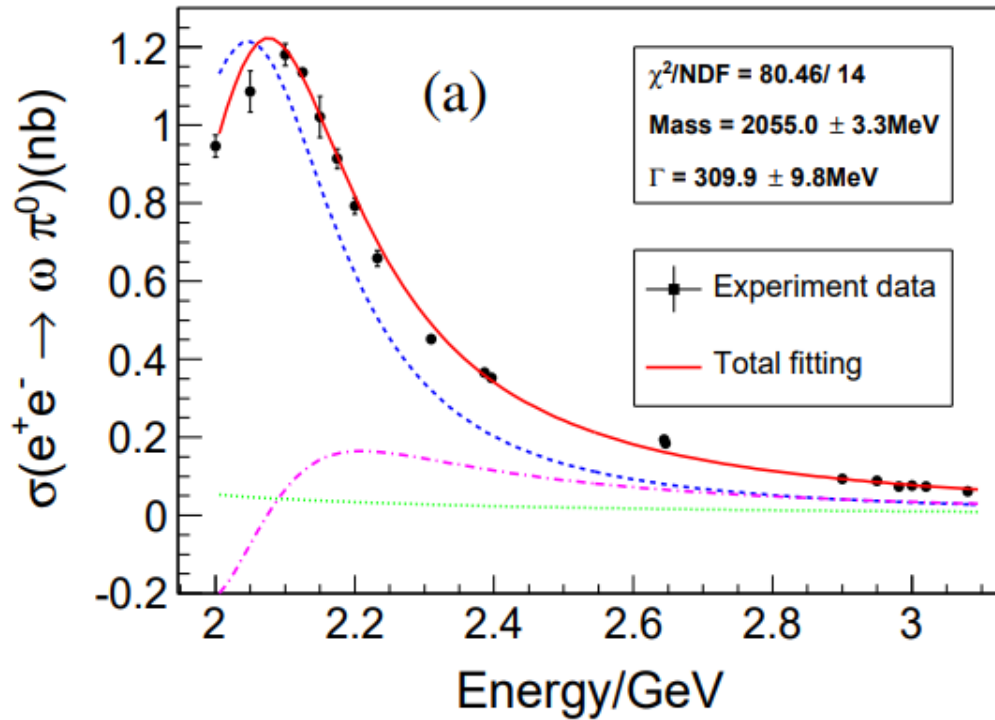
$$\sigma(\sqrt{s}) = \frac{4\pi\alpha^2}{3s^{\frac{3}{2}}} |F_{\omega\pi\gamma}(s)|^2 P_f(\sqrt{s})$$

$$F_{\omega\pi\gamma}(s) = \frac{g_{\rho\omega\pi}}{f_\rho} \left(\frac{m_\rho^2}{D_\rho} + A_1 e^{i\varphi_1} \frac{m_{\rho(1450)}^2}{D_{\rho(1450)}} + A_2 e^{i\varphi_2} \frac{m_{\rho(1700)}^2}{D_{\rho(1700)}} \right)$$

$$D_{\rho_i}(\sqrt{s}) = m_{\rho_i}^2 - s - i\sqrt{s}\Gamma_{\rho_i}(\sqrt{s})$$

Fit to $\omega\pi^0$ lineshape-I

(Only BESIII data)



Signal function: relative Breit-Wigner

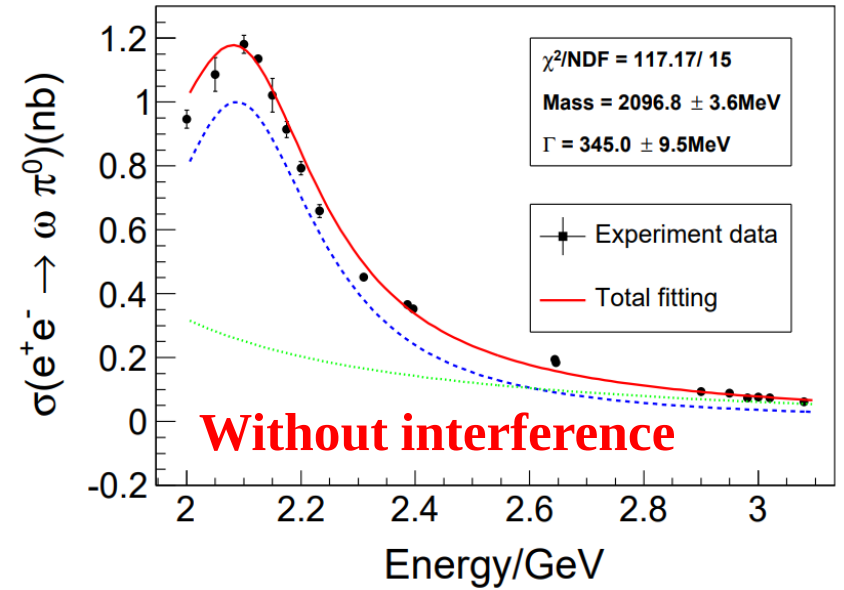
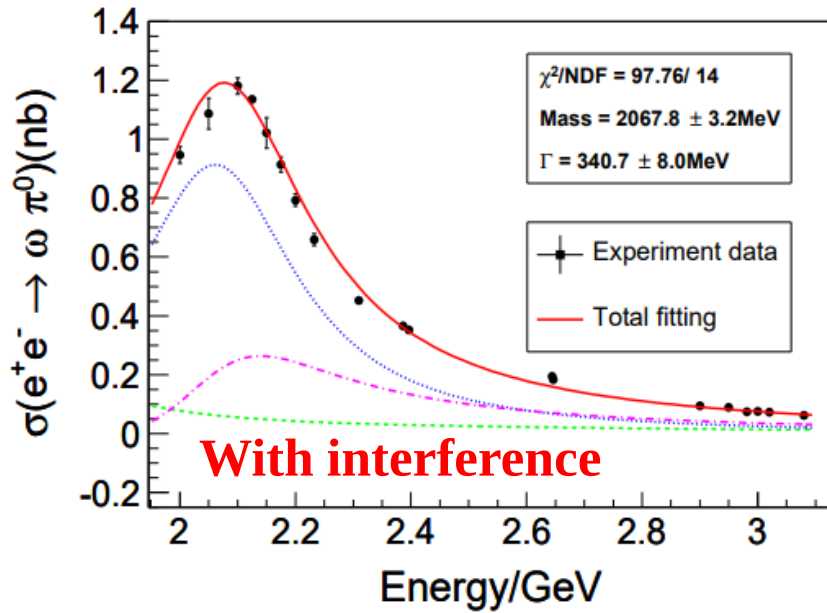
Background function: fixed shape from SND

$$BW = \frac{\sqrt{m\Gamma(m)}}{m^2 - m_0^2 + im\Gamma(m)}, \Gamma(m) = \Gamma_0 \left(\frac{m_0}{m}\right) \left(\frac{p}{p_0}\right)^{2l+1}$$

$$\sigma(e^+e^- \rightarrow \omega\pi^0) = \left| \sqrt{Poly} + Ae^{i\phi} \cdot BW \right|^2$$

Fit to $\omega\pi^0$ lineshape-II

(Only BESIII data)



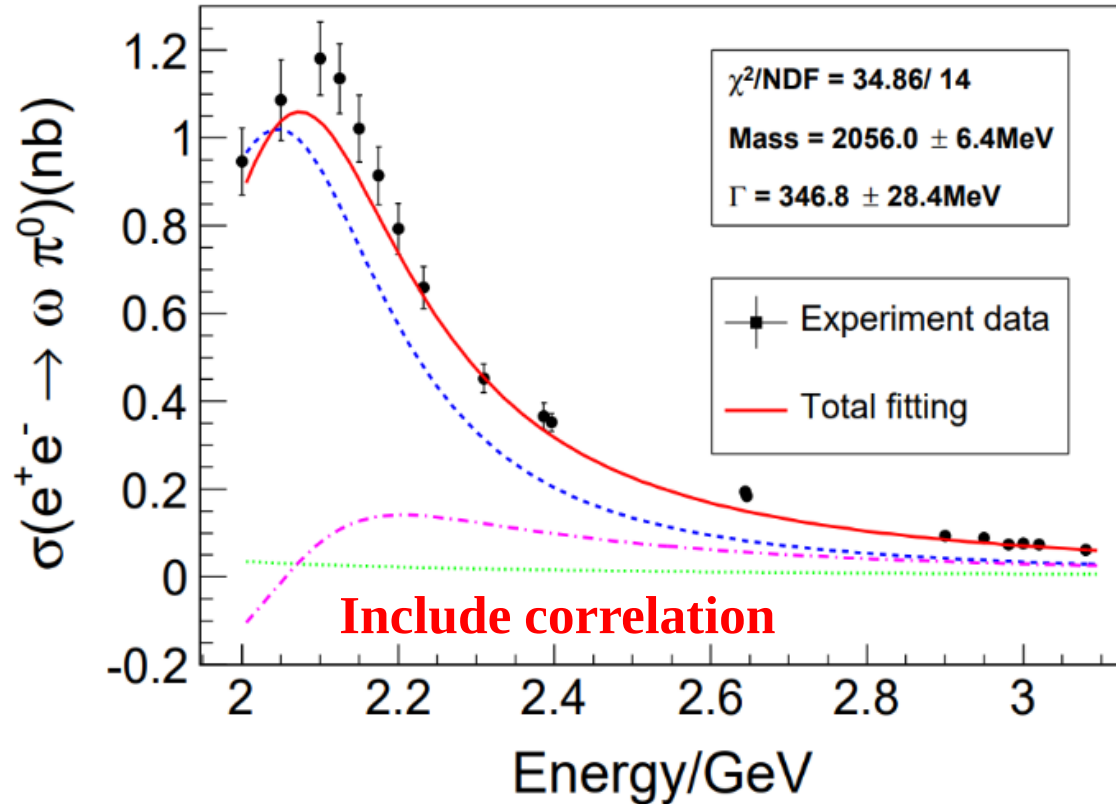
Background function: fixed shape from SND

$$\sigma(e^+e^- \rightarrow \omega\pi^0) = \frac{4\pi\alpha^2}{3s^{\frac{3}{2}}} \left| \frac{g_{\rho\omega\pi}}{f_\rho} \left(\frac{m_\rho^2}{D_\rho} + A_1 e^{i\varphi_1} \frac{m_{\rho(1450)}^2}{D_{\rho(1450)}} + A_2 e^{i\varphi_2} \frac{m_{\rho(1700)}^2}{D_{\rho(1700)}} + A_3 e^{i\varphi_3} \frac{m_{\rho(2100)}^2}{D_{\rho(2100)}} \right) \right|^2 P_f(\sqrt{s})$$

$$D_{\rho_i}(\sqrt{s}) = m_{\rho_i}^2 - s - i\sqrt{s}\Gamma_{\rho_i}(\sqrt{s})$$

Fit to $\omega\pi^0$ lineshape-III

(Only BESIII data)

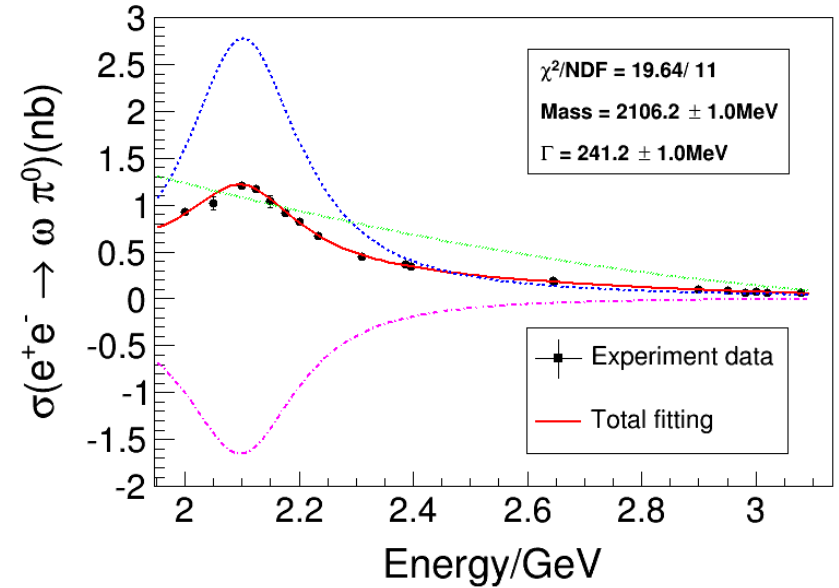
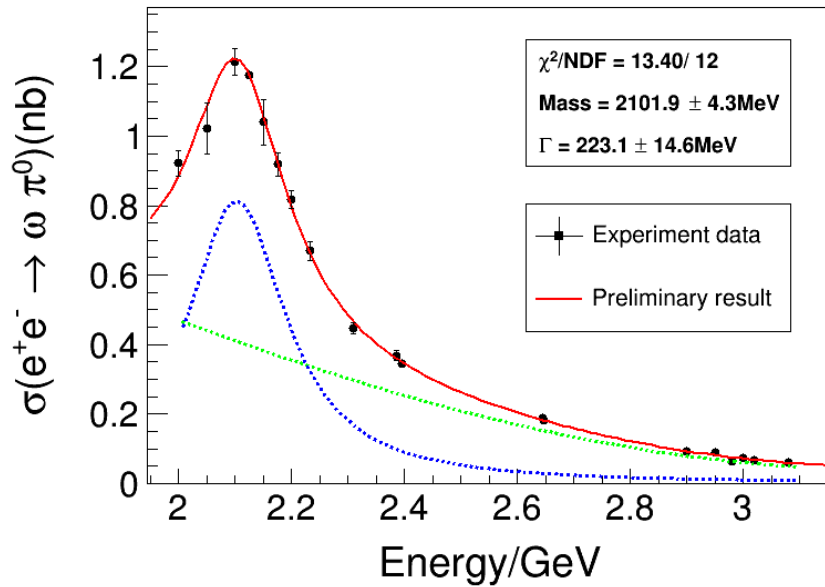


$$BW = \frac{\sqrt{m\Gamma(m)}}{m^2 - m_0^2 + im\Gamma(m)}, \Gamma(m) = \Gamma_0 \left(\frac{m_0}{m}\right) \left(\frac{p}{p_0}\right)^{2l+1}$$

$$\sigma(e^+e^- \rightarrow \omega\pi^0) = \left| \sqrt{Poly} + Ae^{i\phi} \cdot BW \right|^2$$

Fit to $\omega\pi^0$ lineshape-IV

(Only BESIII data)



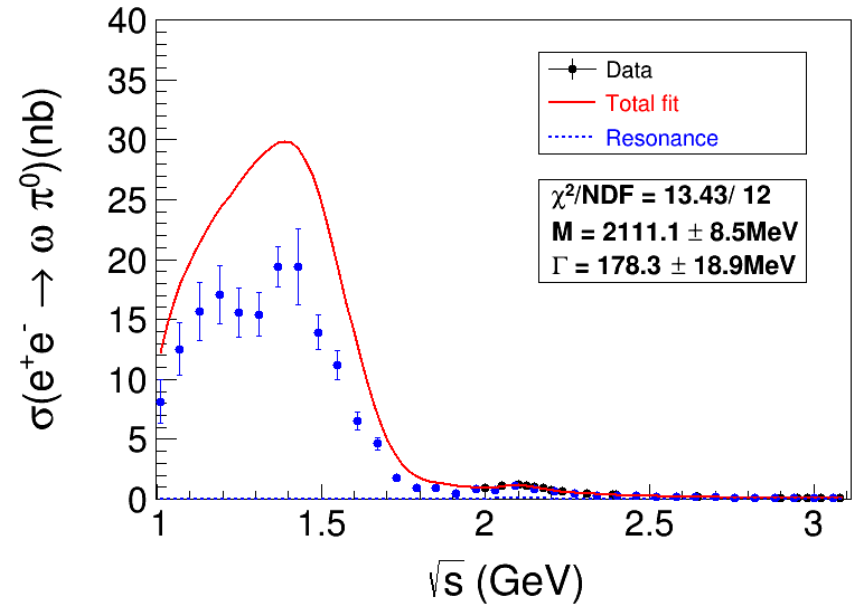
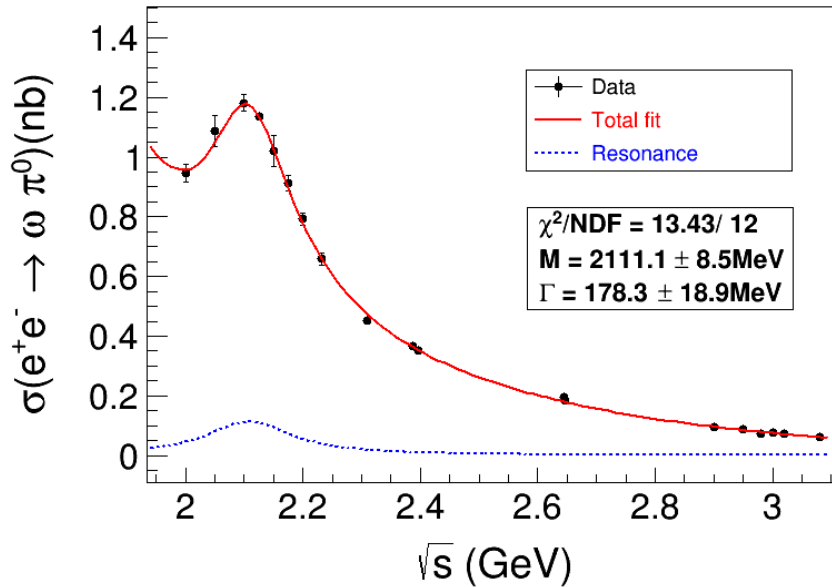
$$BW = \frac{\sqrt{m\Gamma(m)}}{m^2 - m_0^2 + im\Gamma(m)}, \Gamma(m) = \Gamma_0 \left(\frac{m_0}{m}\right) \left(\frac{p}{p_0}\right)^{2l+1}$$

Background function: 3rd Chebychev Polynomial

Fit to $\omega\pi^0$ lineshape-V (1)

(Only BESIII data)

Fixed parameters + $s^{-1/2}$ + BW (Include interference)



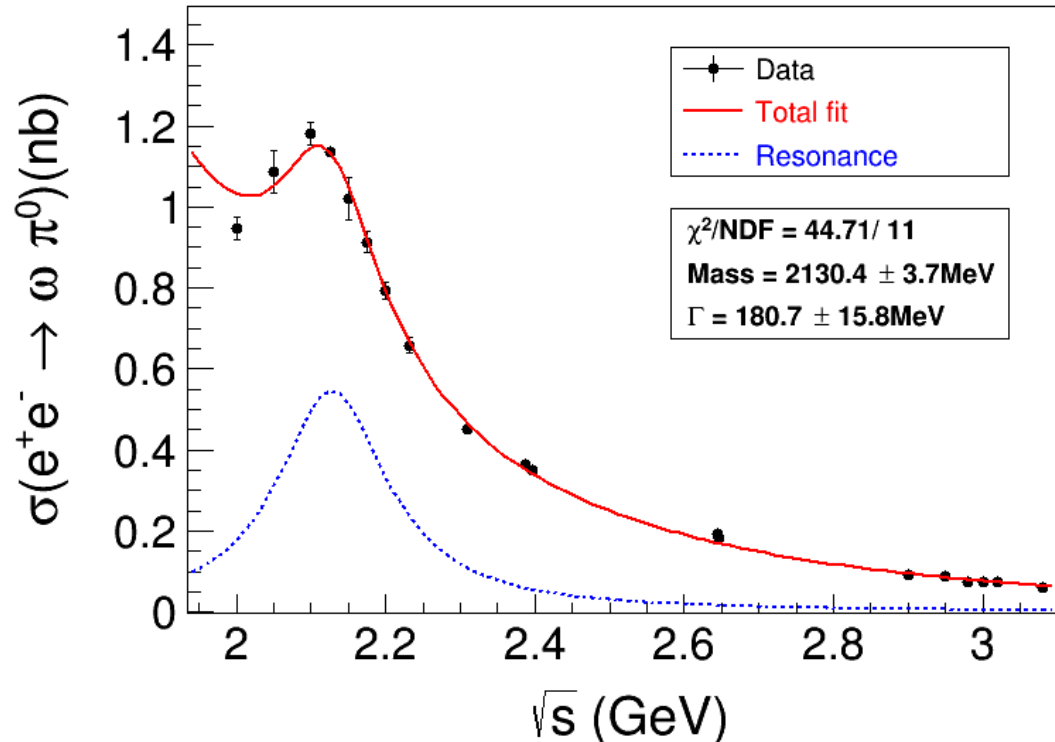
$$Poly = \frac{4\pi\alpha^2}{3s^{\frac{3}{2}}} \left| \frac{g_{\rho\omega\pi}}{f_\rho} \left(\frac{m_\rho^2}{D_\rho} + A_1 e^{i\varphi_1} \frac{m_{\rho(1450)}^2}{D_{\rho(1450)}} + A_2 e^{i\varphi_2} \frac{m_{\rho(1700)}^2}{D_{\rho(1700)}} \right) \right|^2 P_f(\sqrt{s})$$

$$BW = \frac{\sqrt{m\Gamma(m)}}{m^2 - m_0^2 + im\Gamma(m)}, \Gamma(m) = \Gamma_0 \left(\frac{m_0}{m} \right) \left(\frac{p}{p_0} \right)^{2l+1}$$

$$\sigma(e^+e^- \rightarrow \omega\pi^0) = \left| BW + A_1 e^{i\phi_1} \cdot \sqrt{Poly} + A_2 e^{i\phi_2} \cdot \sqrt{1/s^\alpha} \right|^2$$

Fit to $\omega\pi^0$ lineshape-V (1)

(Only BESIII data)

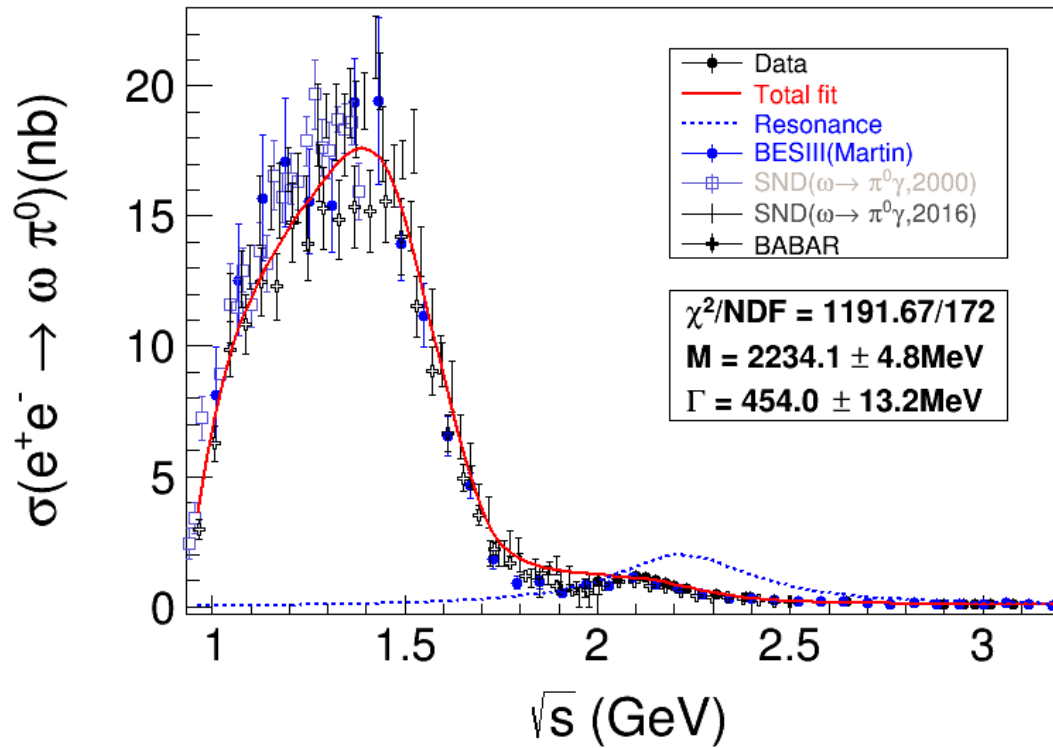


$$Poly = \frac{4\pi\alpha^2}{3s^{\frac{3}{2}}} \left| \frac{g_{\rho\omega\pi}}{f_\rho} \left(\frac{m_\rho^2}{D_\rho} + A_1 e^{i\varphi_1} \frac{m_{\rho(1450)}^2}{D_{\rho(1450)}} + A_2 e^{i\varphi_2} \frac{m_{\rho(1700)}^2}{D_{\rho(1700)}} \right) \right|^2 P_f(\sqrt{s})$$

$$BW = \frac{\sqrt{m\Gamma(m)}}{m^2 - m_0^2 + im\Gamma(m)}, \Gamma(m) = \Gamma_0 \left(\frac{m_0}{m} \right) \left(\frac{p}{p_0} \right)^{2l+1}$$

$$\sigma(e^+e^- \rightarrow \omega\pi^0) = |BW|^2 + A_1 \left| \sqrt{Poly} \right|^2 + A_2 \left| \sqrt{1/s^\alpha} \right|^2$$

Combined data

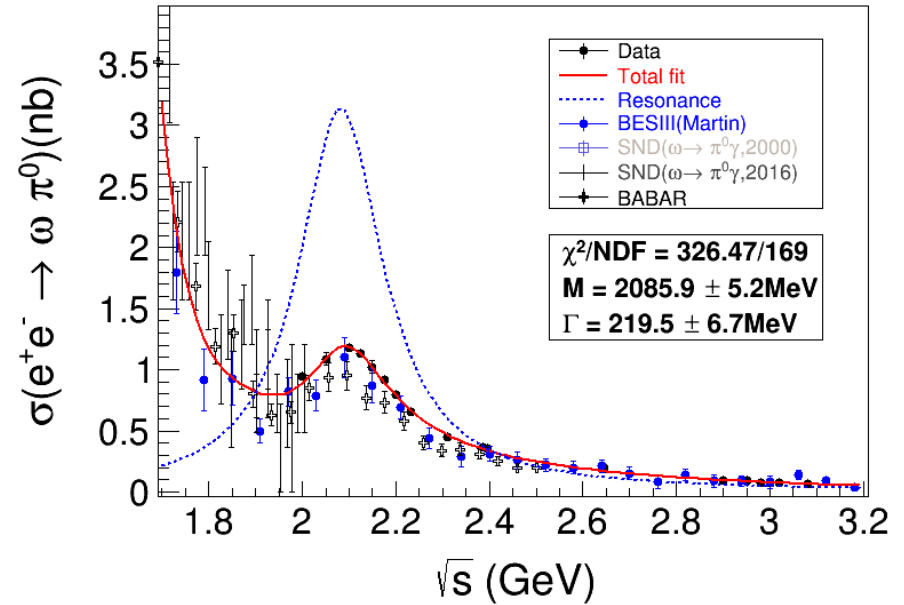
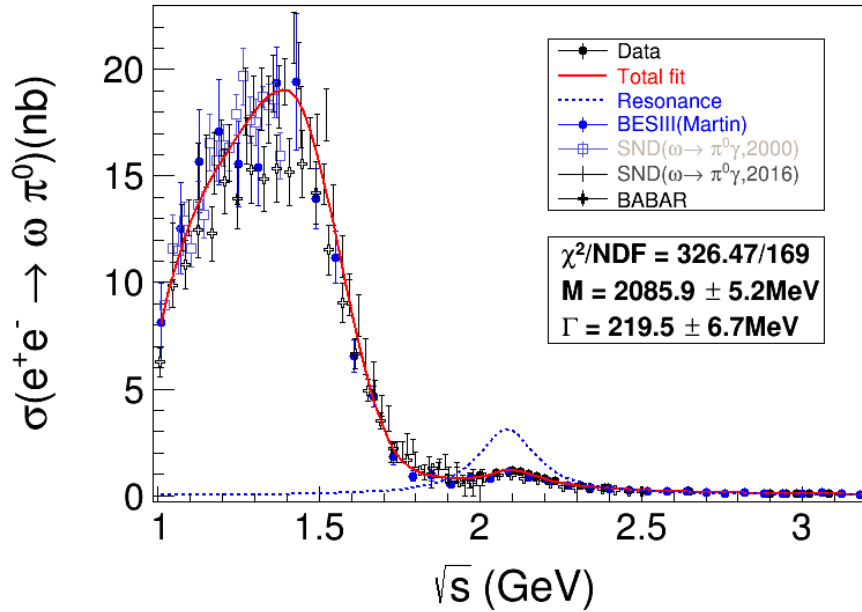


$$BW = \frac{\sqrt{m\Gamma(m)}}{m^2 - m_0^2 + im\Gamma(m)}, \Gamma(m) = \Gamma_0 \left(\frac{m_0}{m}\right) \left(\frac{p}{p_0}\right)^{2l+1}$$

$$\sigma(e^+e^- \rightarrow \omega \pi^0) = \left| \boxed{\sqrt{Poly}} + Ae^{i\phi} \cdot BW \right|^2$$

Parameters fixed

Combined data



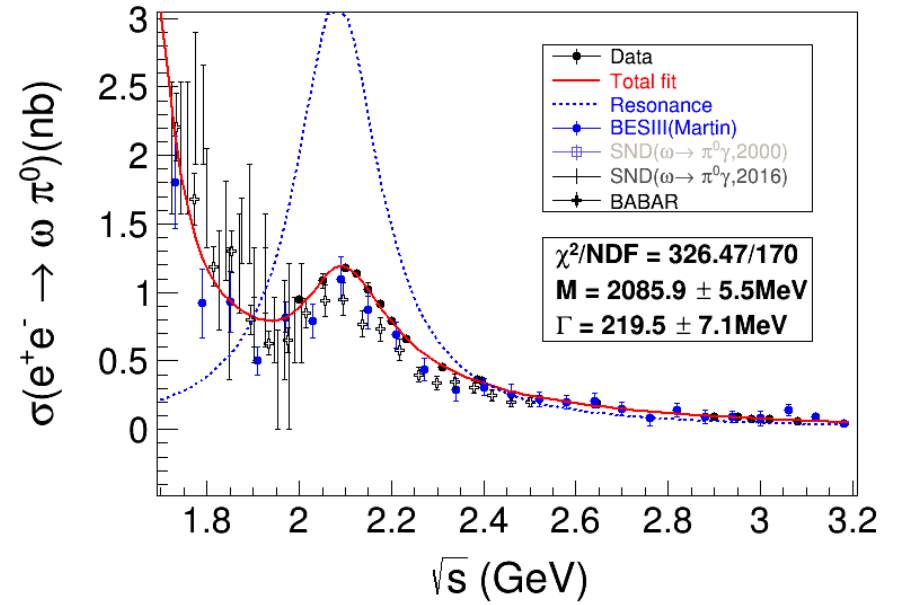
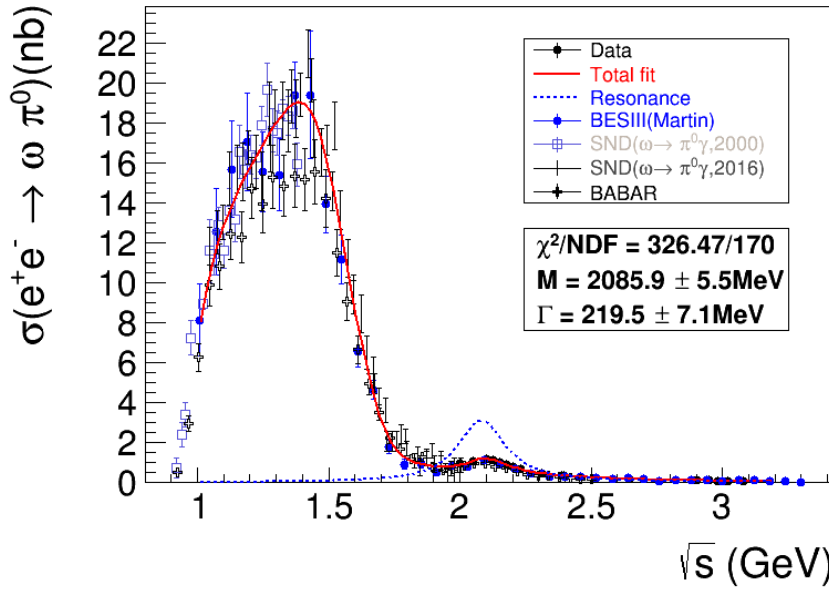
$$Poly = \frac{4\pi\alpha^2}{3s^{\frac{3}{2}}} \left| \frac{g_{\rho\omega\pi}}{f_\rho} \left(\frac{m_\rho^2}{D_\rho} + A_1 e^{i\varphi_1} \frac{m_{\rho(1450)}^2}{D_{\rho(1450)}} + A_2 e^{i\varphi_2} \frac{m_{\rho(1700)}^2}{D_{\rho(1700)}} \right) \right|^2 P_f(\sqrt{s})$$

$$BW = \frac{\sqrt{m\Gamma(m)}}{m^2 - m_0^2 + im\Gamma(m)}, \Gamma(m) = \Gamma_0 \left(\frac{m_0}{m} \right) \left(\frac{p}{p_0} \right)^{2l+1}$$

$$\sigma(e^+e^- \rightarrow \omega \pi^0) = \left| BW + A_1 e^{i\phi_1} \cdot \boxed{\sqrt{Poly}} + A_2 e^{i\phi_2} \cdot \sqrt{1/s^\alpha} \right|^2$$

Parameters fixed

Combined data



$$Poly = \frac{4\pi\alpha^2}{3s^{\frac{3}{2}}} \left| \frac{g_{\rho\omega\pi}}{f_\rho} \left(\frac{m_\rho^2}{D_\rho} + A_1 e^{i\varphi_1} \frac{m_{\rho(1450)}^2}{D_{\rho(1450)}} + A_2 e^{i\varphi_2} \frac{m_{\rho(1700)}^2}{D_{\rho(1700)}} \right) \right|^2 P_f(\sqrt{s})$$

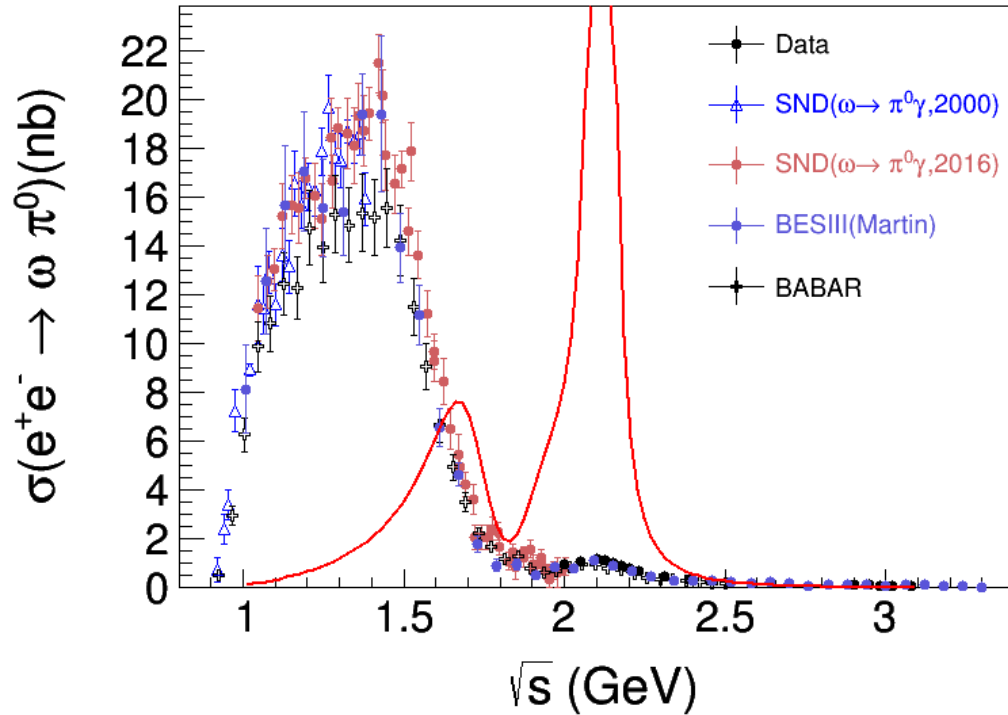
$$BW = \frac{\sqrt{m\Gamma(m)}}{m^2 - m_0^2 + im\Gamma(m)}, \Gamma(m) = \Gamma_0 \left(\frac{m_0}{m} \right) \left(\frac{p}{p_0} \right)^{2l+1}$$

$$\sigma(e^+e^- \rightarrow \omega \pi^0) = \left| BW + A_1 e^{i\phi_1} \cdot \sqrt{Poly} + A_2 e^{i\phi_2} \cdot \sqrt{1/s} \right|^2$$

Parameters fixed

Backup

Combined data



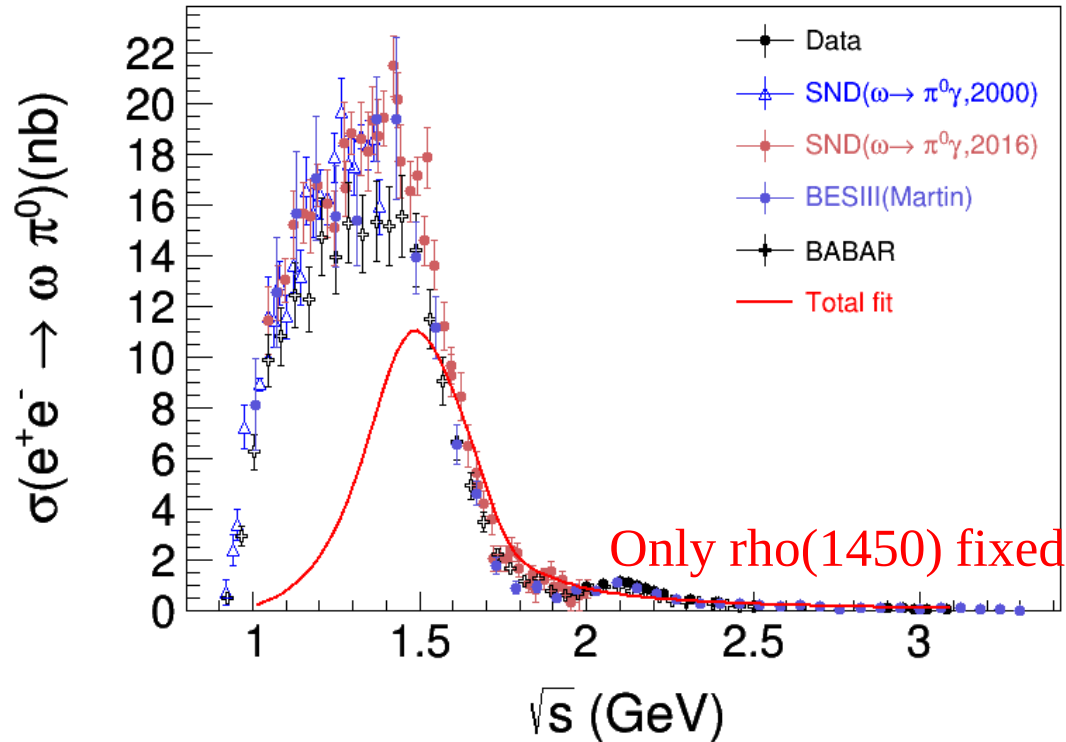
$$BW = \frac{\sqrt{m\Gamma(m)}}{m^2 - m_0^2 + im\Gamma(m)}, \Gamma(m) = \Gamma_0 \left(\frac{m_0}{m}\right) \left(\frac{p}{p_0}\right)^{2l+1}$$

$$Poly = \frac{4\pi\alpha^2}{3s^{\frac{3}{2}}} \left| \frac{g_{\rho\omega\pi}}{f_\rho} \left(\frac{m_\rho^2}{D_\rho} + A_1 e^{i\varphi_1} \frac{m_{\rho(1450)}^2}{D_{\rho(1450)}} + A_2 e^{i\varphi_2} \frac{m_{\rho(1700)}^2}{D_{\rho(1700)}} \right) \right|^2 P_f(\sqrt{s})$$

$$\sigma(e^+e^- \rightarrow \omega \pi^0) = \left| BW + A_1 e^{i\phi_1} \cdot \sqrt{Poly} + A_2 e^{i\phi_2} \cdot \sqrt{1/s^\alpha} \right|^2$$

Parameters free

Combined data



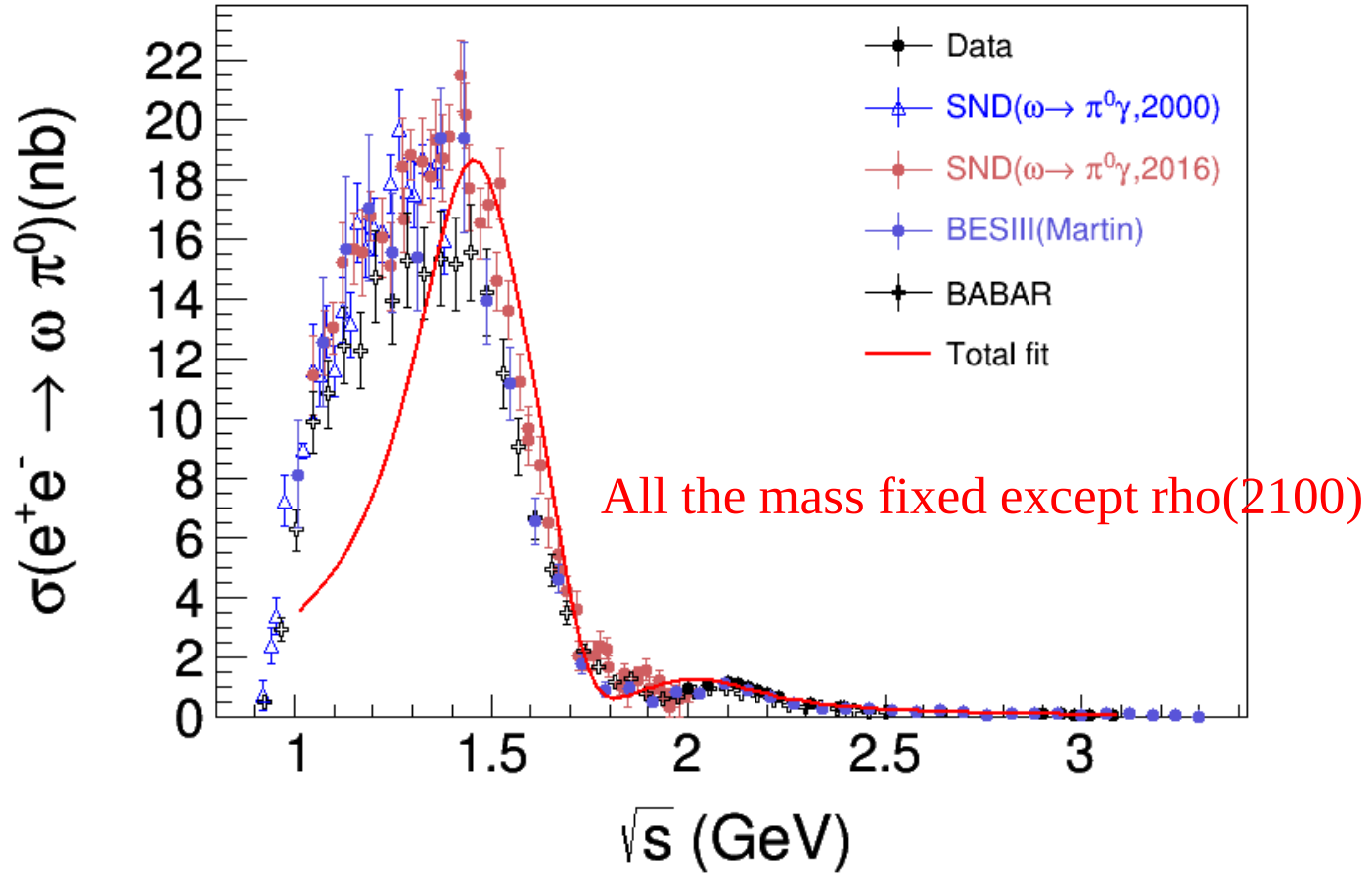
$$Poly = \frac{4\pi\alpha^2}{3s^{\frac{3}{2}}} \left| \frac{g_{\rho\omega\pi}}{f_\rho} \left(\frac{m_\rho^2}{D_\rho} + A_1 e^{i\varphi_1} \frac{m_{\rho(1450)}^2}{D_{\rho(1450)}} + A_2 e^{i\varphi_2} \frac{m_{\rho(1700)}^2}{D_{\rho(1700)}} \right) \right|^2 P_f(\sqrt{s})$$

$$BW = \frac{\sqrt{m\Gamma(m)}}{m^2 - m_0^2 + im\Gamma(m)}, \Gamma(m) = \Gamma_0 \left(\frac{m_0}{m} \right) \left(\frac{p}{p_0} \right)^{2l+1}$$

$$\sigma(e^+e^- \rightarrow \omega \pi^0) = \left| BW + A_1 e^{i\phi_1} \cdot \boxed{\sqrt{Poly}} + A_2 e^{i\phi_2} \cdot \sqrt{1/s^\alpha} \right|^2$$

Parameters free

Combined data



$$\sigma(e^+e^- \rightarrow \omega \pi^0) = \frac{4\pi\alpha^2}{3s^{\frac{3}{2}}} \left| \frac{g_{\rho\omega\pi}}{f_\rho} \left(\frac{m_\rho^2}{D_\rho} + A_1 e^{i\varphi_1} \frac{m_{\rho(1450)}^2}{D_{\rho(1450)}} + A_2 e^{i\varphi_2} \frac{m_{\rho(1700)}^2}{D_{\rho(1700)}} \right) + A_3 e^{i\varphi_3} \frac{m_{\rho(2100)}^2}{D_{\rho(2100)}} \right|^2 P_f(\sqrt{s})$$

Mass fixed