Update of formalism for semileptonic hyperon decays

Varvara Batozskaya[1,2], Andrzej Kupść[2,3]

[1]Institute of High Energy Physics, Beijing, China
[2]National Centre for Nuclear Research, Warsaw, Poland
[3]Uppsala University, Uppsala, Sweden

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Semileptonic Λ decay

- Momenta and masses: $\Lambda(p_1, M_1) \to p(p_2, M_2) + e^-(p_e, m_e) + \bar{\nu}_e(p_{\bar{\nu}_e}, 0)$
- Standard set of invariant FF for weak current-induced baryonic $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions:

$$M_\mu = M_\mu^V + M_\mu^A = \langle p(p_2)|J_\mu^{V+A}|\Lambda(p_1)\rangle =$$

$$= \bar{u}(p_2)\left[\gamma_\mu\left(F_1^V(q^2) + F_1^A(q^2)\gamma_5\right) + \frac{i\sigma_{\mu\nu}q^\nu}{M_1}\left(F_2^V(q^2) + F_2^A(q^2)\gamma_5\right) + \frac{q_\mu}{M_1}\left(F_3^V(q^2) + F_3^A(q^2)\gamma_5\right)\right]u(p_1)$$

where $q_\mu = (p_1 - p_2)_\mu$

- For $\Lambda \to pe^-\bar{\nu}_e$ at $\mathcal{O}(\frac{m_e^2}{2q^2}) \sim 4 \cdot 10^{-6} \Rightarrow F_3^{V,A} \to 0$
- $\Lambda \to pe^-\bar{\nu}_e \Longrightarrow \Lambda \to pW^-(\to e^-\bar{\nu}_e)$
- Helicity amplitude is $H_{\lambda_2\lambda_W} = (H^V_{\lambda_2\lambda_W} + H^A_{\lambda_2\lambda_W})$ with $(\lambda_2 = \pm\frac{1}{2};\ \lambda_W = 0, \pm 1)$:

vector
$$H^V_{\frac{1}{2}1} = \sqrt{2Q_-}\left(-F_1^V - \frac{M_1 + M_2}{M_1}F_2^V\right),$$
$$H^V_{\frac{1}{2}0} = \frac{\sqrt{Q_-}}{\sqrt{q^2}}\left((M_1 + M_2)F_1^V + \frac{q^2}{M_1}F_2^V\right),$$

axial-vector
$$H^A_{\frac{1}{2}1} = \sqrt{2Q_+}\left(F_1^A - \frac{M_1 - M_2}{M_1}F_2^A\right),$$
$$H^A_{\frac{1}{2}0} = \frac{\sqrt{Q_+}}{\sqrt{q^2}}\left(-(M_1 - M_2)F_1^A + \frac{q^2}{M_1}F_2^A\right)$$

where $Q_\pm = (M_1 \pm M_2)^2 - q^2;\qquad H^{V,A}_{-\lambda_2,-\lambda_W} = \pm H^{V,A}_{\lambda_2\lambda_W}$

Form factors

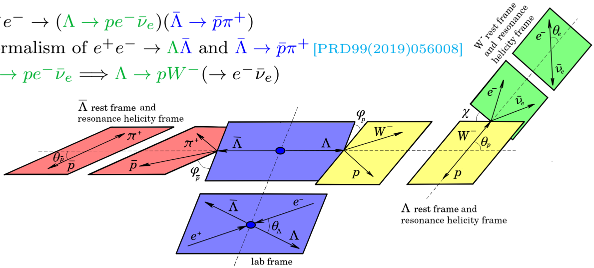
$$F_i^{V,A}(q^2) = F_i^{V,A}(0) \prod_{n=0}^{n_i} \frac{1}{1 - \frac{q^2}{m_{V,A}^2 + n\alpha'^{-1}}} \approx F_i^{V,A}(0)\left(1 + q^2 \sum_{n=0}^{n_i} \frac{1}{m_{V,A}^2 + n\alpha'^{-1}}\right)$$

	$F_i^{V,A}(0)(\Lambda \to p)$	$m_{V,A}$	α' [GeV^{-2}]	n_i
$F_1^V(q^2)$	$-\sqrt{\frac{3}{2}}$ [1]	$m_{K^*(892)^0}$ $(J^P = 1^-)$	0.9	$n_1 = 1$
$F_2^V(q^2)$	$\frac{M_\Lambda \mu_p}{2M_p} F_1^V(0)$ [2]			$n_2 = 2$
$F_3^V(q^2)$	0 [4]	-		$n_3 = 2$
$F_1^A(q^2)$	$0.719 F_1^V(0)$ [3]	$m_{K^*(1270)^0}$ $(J^P = 1^+)$		$n_1 = 1$
$F_2^A(q^2)$	0 [4]	–		$n_2 = 2$
$F_3^A(q^2)$	$\frac{M_\Lambda(M_\Lambda + M_p)}{(m_{K^-})^2} F_1^A(0)$ [4]	m_K $(J^P = 0^-)$		$n_3 = 2$

- [1] [PR135(1964)B1483], [PRL13(1964)264]
- [2] $\mu_p = 1.793$ [Lect.NotesPhys.222(1985)1], [Ann.Rev.Nucl.Part.Sci.53(2003)39], [JHEP0807(2008)132]
- [3] [PRD41(1990)780]
- [4] Vanish in the $SU(3)$ symmetry limit; Goldberger-Treiman relation [PR110(1958)1178], [PR111(1958)354]

Decay chain: $\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}$ (1)

- $e^+e^- \to (\Lambda \to pe^-\bar{\nu}_e)(\bar{\Lambda} \to \bar{p}\pi^+)$
- Formalism of $e^+e^- \to \Lambda\bar{\Lambda}$ and $\bar{\Lambda} \to \bar{p}\pi^+$ [PRD99(2019)056008]
- $\Lambda \to pe^-\bar{\nu}_e \Longrightarrow \Lambda \to pW^-(\to e^-\bar{\nu}_e)$



- Decay matrix or transition matrix $b_{\mu\nu}$ for $\{\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}\}$

$$\sigma_\mu \to \sum_{\nu=0}^{3} b_{\mu\nu}\sigma_\nu^{d-W} \Longrightarrow \quad b_{\mu\nu} = \frac{1}{4\pi}\sum_{\lambda_2=-1/2}^{1/2}\sum_{\lambda_W,\lambda'_W=-1}^{1} H_{\lambda_2\lambda_W}H^*_{\lambda_2\lambda'_W}\sum_{\kappa,\kappa'=-1/2}^{1/2}(\sigma_\mu)^{\kappa,\kappa'}(\sigma_\nu)^{\lambda_2-\lambda'_W,\lambda_2-\lambda_W}\mathcal{D}^{1/2*}_{\kappa,\lambda_2-\lambda_W}(\Omega)\mathcal{D}^{1/2}_{\kappa',\lambda_2-\lambda'_W}(\Omega)\times \sum_{\lambda_l,\lambda_\nu=-1/2}^{1/2}|h^l_{\lambda_l\lambda_\nu=\pm 1/2}|^2\mathcal{D}^{1*}_{\lambda_W,\lambda_l-\lambda_\nu}(\Omega')\mathcal{D}^{1}_{\lambda'_W,\lambda_l-\lambda_\nu}(\Omega'),$$

- Four helicity amplitudes: $H_{\frac{1}{2}0}, H_{-\frac{1}{2}0}, H_{\frac{1}{2}1}, H_{-\frac{1}{2}-1}$
- $\kappa, \kappa'; \lambda_2$ - index of mother hyperon (Λ) and daughter baryon (p)
- $\lambda_W, \lambda'_W; \lambda_l, \lambda_\nu$ - index of W^--boson, lepton and neutrino
- Kinematic variables: $\Omega = \{\phi_p, \theta_p, 0\}$, $\Omega' = \{\chi, \theta_l, 0\}$ and $q^2 \in (m_e^2, (M_\Lambda - M_p)^2)$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}$ (2)

- Relation between helicity amplitudes and decay parameters

$$\sigma_D^{sl} = \frac{1}{4}(1 \mp \cos\theta_l)^2 |H_{\frac{1}{2}1}|^2 + \frac{1}{4}(1 \pm \cos\theta_l)^2 |H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l (|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2),$$
$$\alpha_D^{sl} = \frac{1}{4}(1 \mp \cos\theta_l)^2 |H_{\frac{1}{2}1}|^2 - \frac{1}{4}(1 \pm \cos\theta_l)^2 |H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l (|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),$$
$$\beta_D^{sl} = \frac{1}{\sqrt{2}}\sin\theta_l ((1 \pm \cos\theta_l)\Im(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) + (1 \mp \cos\theta_l)\Im(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0})),$$
$$\gamma_D^{sl} = \frac{1}{\sqrt{2}}\sin\theta_l ((1 \pm \cos\theta_l)\Re(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) + (1 \mp \cos\theta_l)\Re(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0})).$$

- Non-zero elements of the decay matrix $b_{\mu\nu}$:

$$\begin{aligned}
b_{00} &= \sigma_D^{sl}, \\
b_{03} &= \alpha_D^{sl}, \\
b_{10} &= \alpha_D^{sl}\cos\phi_p \sin\theta_p, \\
b_{11} &= -(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\theta_p\cos\phi_p \\
&\quad +(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\sin\phi_p, \\
b_{12} &= \quad (\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\theta_p\cos\phi_p \\
&\quad +(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\sin\phi_p, \\
b_{13} &= \sigma_D^{sl}\sin\theta_p\cos\phi_p, \\
b_{20} &= \alpha_D^{sl}\sin\theta_p\sin\phi_p,
\end{aligned}$$

$$\begin{aligned}
b_{21} &= -(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\theta_p\sin\phi_p \\
&\quad -(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\phi_p, \\
b_{22} &= \quad (\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\cos\theta_p\sin\phi_p \\
&\quad -(\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\cos\phi_p, \\
b_{23} &= \sigma_D^{sl}\sin\theta_p\sin\phi_p, \\
b_{30} &= \alpha_D^{sl}\cos\theta_p, \\
b_{31} &= \quad (\gamma_D^{sl}\cos\chi + \beta_D^{sl}\sin\chi)\sin\theta_p, \\
b_{32} &= -(\gamma_D^{sl}\sin\chi - \beta_D^{sl}\cos\chi)\sin\theta_p, \\
b_{33} &= \sigma_D^{sl}\cos\theta_p.
\end{aligned}$$

- Main parameters:
  $\sigma_D^{sl} \equiv \sigma_D^{sl}(\theta_l, q^2)$, $\alpha_D^{sl} \equiv \alpha_D^{sl}(\theta_l, q^2)$, $\beta_D^{sl} \equiv \beta_D^{sl}(\theta_l, q^2)$, $\gamma_D^{sl} \equiv \gamma_D^{sl}(\theta_l, q^2)$
- Each element of $b_{\mu\nu}$ is multiplied by $p = \sqrt{Q_+ Q_-}/(2M_1)$

Intermediate step (1)

- σ_Λ^{sl}, α_Λ^{sl}, β_Λ^{sl} and $\gamma_\Lambda^{sl} \Rightarrow \{n, \alpha, \alpha', \alpha'', \beta_{1,2}, \gamma_{1,2}\}(q^2)$: $g_{av}^\Lambda(q^2)$, $g_w^\Lambda(q^2)$
- Introduce the intermediate parameters:

$$
\begin{aligned}
\text{normalization} \quad n &= |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 + 2(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2),\\
\alpha &= |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2 + 2(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),\\
\alpha' &= |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 - 2(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2),\\
\alpha'' &= |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2 - 2(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),\\
\beta_{1,2} &= 2(\Im(H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}-1}) \pm \Im(H^*_{\frac{1}{2}1}H_{\frac{1}{2}0})),\\
\gamma_{1,2} &= 2(\Re(H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}-1}) \pm \Re(H^*_{\frac{1}{2}1}H_{\frac{1}{2}0})),
\end{aligned}
$$

where $\beta_{1,2} = \frac{1}{2}\sqrt{n^2 - \alpha^2 - (\alpha')^2 + (\alpha'')^2}\sin\phi_{1,2}$ and $\gamma_{1,2} = \frac{1}{2}\sqrt{n^2 - \alpha^2 - (\alpha')^2 + (\alpha'')^2}\cos\phi_{1,2}$

- $\alpha^2 + (\alpha')^2 - (\alpha'')^2 + 2\sum_{i=1}^2(\gamma_i^2 + \beta_i^2) = n^2$
- Using the definition of helicity amplitudes the main parameters to describe semileptonic hyperon decays are:

- $F_1^V(0)$, $F_2^V(0)$, $F_1^A(0)$

or

- $g_{av}^D(0) = \dfrac{F_1^A(0)}{F_1^V(0)}$, $\quad g_w^D(0) = \dfrac{F_2^V(0)}{F_1^V(0)}$

Intermediate step (2)

- Relations between intermediate and decay parameters:

$$n = ((M_-M_+)^2 - q^4)(1 + (g^D_{av}(q^2))^2) + q^2\left(4M_1M_2((g^D_{av}(q^2))^2 - 1) + \frac{q^2}{M_1^2}Q_-((g^D_w(q^2))^2(M_+^2 + q^2) + 4g^D_w(q^2)M_+)\right)$$

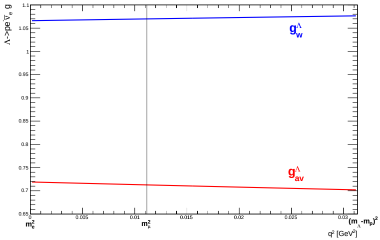
$$\alpha = 2\sqrt{Q_-Q_+}\left[g^D_{av}(q^2)(q^2 - M_-M_+) + 2g^D_{av}(q^2)g^D_w(q^2)q^2\frac{M_2}{M_1}\right]$$

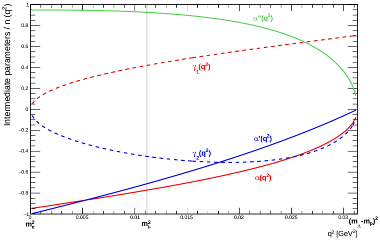
$$\alpha' = Q_-Q_+\left[-(1 + (g^D_{av}(q^2))^2) + (g^D_w(q^2))^2\frac{q^2}{M_1^2}\right]$$

$$\alpha'' = 2\sqrt{Q_-Q_+}\left[g^D_{av}(q^2)(q^2 + M_-M_+) + 2g^D_{av}(q^2)g^D_w(q^2)q^2\right]$$

where $M_- = M_1 - M_2$ and $M_+ = M_1 + M_2$ and $Q_\pm = M_\pm^2 - q^2$

- $\{\alpha, \alpha', \alpha'', \gamma_{1,2}\}(q^2)/n(q^2) \in [-1, +1]$





Intermediate step (3)

- Non-zero elements of the decay matrix $b_{\mu\nu}$:

$$
\begin{aligned}
&\boxed{b_{00} = 1,} && b_{21} = \mp A\cos\theta_p \sin\phi_p \mp B\cos\phi_p,\\
&b_{03} = a_D^{sl}, && b_{22} = \pm B\cos\theta_p \sin\phi_p \mp A\cos\phi_p,\\
&\boxed{b_{10} = a_D^{sl}\cos\phi_p \sin\theta_p,} && b_{23} = \sin\theta_p \sin\phi_p,\\
&b_{11} = \mp A\cos\theta_p \cos\phi_p \pm B\sin\phi_p, && \boxed{b_{30} = a_D^{sl}\cos\theta_p,}\\
&b_{12} = \pm B\cos\theta_p \cos\phi_p \pm A\sin\phi_p, && b_{31} = \pm A\sin\theta_p,\\
&b_{13} = \sin\theta_p \cos\phi_p, && b_{32} = \mp B\sin\theta_p,\\
&\boxed{b_{20} = a_D^{sl}\sin\theta_p \sin\phi_p,} && b_{33} = \cos\theta_p,
\end{aligned}
$$

$$
\text{where} \quad a_D^{sl} = \frac{\alpha_D^{sl}(\theta_l, q^2)}{\sigma_D^{sl}(\theta_l, q^2)} = \frac{\alpha + \alpha'' \cos^2\theta_l \mp (n + \alpha')\cos\theta_l}{n + \alpha' \cos^2\theta_l \mp (\alpha + \alpha'')\cos\theta_l},
$$

$$
A = \frac{1}{2\sqrt{2}} \frac{\sin\theta_l}{\sigma_D^{sl}(\theta_l, q^2)} [\cos\chi(\gamma_1 \pm \cos\theta_l \gamma_2) + \sin\chi(\beta_1 \pm \cos\theta_l \beta_2)],
$$

$$
B = \frac{1}{2\sqrt{2}} \frac{\sin\theta_l}{\sigma_D^{sl}(\theta_l, q^2)} [\sin\chi(\gamma_1 \pm \cos\theta_l \gamma_2) - \cos\chi(\beta_1 \pm \cos\theta_l \beta_2)].
$$

Sensitivity for extracted parameters (1)

- Study the importance of the individual parameters in joint angular distribution and its correlations using likelihood function [PRDD100(2019)11]

$$\mathcal{L}(\boldsymbol{\omega}) = \prod_{i=1}^{N} \mathcal{P}(\boldsymbol{\xi}_i, \boldsymbol{\omega}) \equiv \prod_{i=1}^{N} \frac{\mathcal{W}(\boldsymbol{\xi}_i, \boldsymbol{\omega})}{\int \mathcal{W}(\boldsymbol{\xi}, \boldsymbol{\omega}) d\boldsymbol{\xi}}$$

N is the number of events in the final selection
$\boldsymbol{\xi}_i$ is the full set of kinematic variables describing i-th event
$\boldsymbol{\omega}$ is the full set of individual parameters

- Reduced asymptotic expression of inverse covariant matrix element:

$$V_{kl}^{-1} = N \int \frac{1}{\mathcal{P}} \frac{\partial \mathcal{P}}{\partial \omega_k} \frac{\partial \mathcal{P}}{\partial \omega_l} d\boldsymbol{\xi}$$

- Sensitivity $= \sigma \times \sqrt{N}$
- To eliminate complicated calculation of derivatives the numerical differentiation is used [Numerical Analysis]:

$$\frac{\partial \mathcal{P}}{\partial \omega_k} = \sum_{\omega_1}^{\omega_n} \lim_{h \to 0} \frac{\mathcal{P}(\omega_k + h) - \mathcal{P}(\omega_k)}{h}$$

Definitions and input values

$$\boxed{A_D \equiv \frac{X_D + X_{\bar{D}}}{X_D - X_{\bar{D}}} \quad \text{and} \quad \langle X_D \rangle \equiv \frac{X_D - X_{\bar{D}}}{2}}$$

$$\text{where} \quad X_D = \alpha_\Lambda, g_{av}^\Lambda, g_w^\Lambda \quad \text{and} \quad X_{\bar{D}} = \alpha_{\bar{\Lambda}}, g_{av}^{\bar{\Lambda}}, g_w^{\bar{\Lambda}}$$

- Input values of decay parameters [NaturePhys.15(2019)631] and slide 3

Decay	α_ψ	$\Delta\Phi$	α_Λ	g_{av}	g_w
$J/\psi \to \Lambda\bar{\Lambda}$	$0.461 \pm 0.006 \pm 0.007$	$0.740 \pm 0.010 \pm 0.008$			
$\Lambda \to p\pi^-$			0.750 ± 0.010		
$\Lambda \to pe^-\bar{\nu}_e$				0.719	1.066

Sensitivity for extracted parameters ($\sigma \times \sqrt{N}$) (2)

- Sensitivity for $\Lambda\bar{\Lambda} \to (p\pi^-)(\bar{p}\pi^+)$ is in an agreement with [PRD100(2019)114005]

Decay	α_ψ	$\Delta\Phi$	α_Λ	g_{av}^Λ	g_w^Λ	$\langle\alpha_\Lambda\rangle$	A_Λ	$\langle g_{av}^\Lambda\rangle$	A_{av}^Λ	$\langle g_w^\Lambda\rangle$	A_w^Λ
$\Lambda\bar{\Lambda} \to (p\pi^-)(\bar{p}\pi^+)$	3.43	7.47	6.83			1.76	8.81				
$\Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}\pi^+)$ $(q^2=0, g_w=0)$	3.19	7.13	6.97	21.0							
$\Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}\pi^+)(g_w=0)$	3.46	7.57	3.46	10.6							
$\Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}\pi^+)$	3.50	7.74	3.73	25.9	120						
$\Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}e^+\nu_e)$ $(q^2=0, g_w=0)$	2.79	6.61		21.1				2.98	28.9		
$\Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}e^+\nu_e)(g_w=0)$	3.42	7.42		10.6				7.33	10.6		
$\Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}e^+\nu_e)$	3.43	7.44		24.2	111			15.6	22.9	83.5	69.5

$(q^2=0, g_w=0)$

	g_{av}^Λ	$g_{av}^{\bar{\Lambda}}$	$\alpha_{\bar{\Lambda}}$	α_ψ	$\Delta\Phi$
g_{av}^Λ	1	$\boxed{0.96}$		-0.02	0.02
$g_{av}^{\bar{\Lambda}}$		1		0.05	0.03
$\alpha_{\bar{\Lambda}}$	$\boxed{0.93}$		1		
α_ψ	-0.06		0.01	1	0.02
$\Delta\Phi$	0.04		0.10	0.18	1

$(g_w=0)$

	g_{av}^Λ	$g_{av}^{\bar{\Lambda}}$	$\alpha_{\bar{\Lambda}}$	α_ψ	$\Delta\Phi$
g_{av}^Λ	1	0.04		0.04	0.05
$g_{av}^{\bar{\Lambda}}$		1		-0.04	-0.05
$\alpha_{\bar{\Lambda}}$	-0.18		1		
α_ψ	0.01		0.19	1	0.24
$\Delta\Phi$	0.01		0.25	0.27	1

	g_{av}^Λ	$g_{av}^{\bar{\Lambda}}$	g_w^Λ	$g_w^{\bar{\Lambda}}$	$\alpha_{\bar{\Lambda}}$	α_ψ	$\Delta\Phi$
g_{av}^Λ	1	0.05	$\boxed{-0.86}$	0.08		-0.04	-0.04
$g_{av}^{\bar{\Lambda}}$		1	-0.08	0.91		0.04	0.05
g_w^Λ	$\boxed{-0.87}$		1	-0.12		0.06	0.07
$g_w^{\bar{\Lambda}}$				1		0.06	0.08
$\alpha_{\bar{\Lambda}}$	0.21		-0.31		1		
α_ψ	-0.01		0.01		0.17	1	0.25
$\Delta\Phi$	-0.00		0.00		0.24	0.28	1

* $\Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}e^+\nu_e)$ (above the diagonal) and $\Lambda\bar{\Lambda} \to (pe^-\bar{\nu}_e)(\bar{p}\pi^+)$ (below the diagonal)

Helicity amplitudes of the lepton pair $h^l_{\lambda_l \lambda_\nu}$

- Lepton and anti-neutrino spinors

$$\bar{u}_{l^-}(\mp\frac{1}{2}, p_{l^-}) = \sqrt{E_l + m_l}\left(\chi^\dagger_\mp, \frac{\pm|\vec{p}_{l^-}|}{E_l + m_l}\chi^\dagger_\mp\right),$$

$$v_{\bar{\nu}}(\frac{1}{2}) = \sqrt{E_\nu}\begin{pmatrix}\chi_+ \\ -\chi_+\end{pmatrix},$$

where $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$ and $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$ are Pauli two-spinors

- SM form of the lepton current ($\lambda_W = \lambda_{l^-} - \lambda_{\bar{\nu}}$)

$$h^l_{\lambda_{l^-}=\mp 1/2, \lambda_{\bar{\nu}}=1/2} = \bar{u}_{l^-}(\mp\frac{1}{2})\gamma^\mu(1+\gamma_5)v_{\bar{\nu}}(\frac{1}{2})\begin{Bmatrix}\epsilon_\mu(-1) \\ \epsilon_\mu(t), \epsilon_\mu(0)\end{Bmatrix}$$

where $\epsilon^\mu(t) = (1;0,0,0), \epsilon^\mu(0) = (0;0,0,1)$ and $\epsilon^\mu(\mp 1) = (0;\mp 1, -i, 0)/\sqrt{2}$

- Moduli squared of the helicity amplitudes [EPJ C59 (2009) 27]

$$\text{nonflip}(\lambda_W = \mp 1) : |h^l_{\lambda_l=\mp\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8(q^2 - m_l^2),$$

$$\text{flip}(\lambda_W = 0, t) : |h^l_{\lambda_l=\pm\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8\frac{m_l^2}{2q^2}(q^2 - m_l^2)$$

- Upper and lower signs refer to the configurations $(l^-, \bar{\nu}_l)$ ($\lambda_\nu = 1/2$) and (l^+, ν_l) ($\lambda_\nu = -1/2$), respectively
- In case of the e-mode only nonflip transition remains under assumption $\frac{m_e^2}{2q^2} \to 0$

Semileptonic Λ decay[flip]

- For $\Lambda \to p e^- \bar{\nu}_e$ at $\mathcal{O}(\frac{m_e^2}{2q^2}) \sim 4 \cdot 10^{-6} \Rightarrow F_3^{V,A} \to 0$
- For $\Lambda \to p \mu^- \bar{\nu}_\mu$ at $\mathcal{O}(\frac{m_\mu^2}{2q^2}) \sim 0.18 \Rightarrow F_3^{V,A} \neq 0$

- Helicity amplitude is $H_{\lambda_2 \lambda_W} = (H^V_{\lambda_2 \lambda_W} + H^A_{\lambda_2 \lambda_W})$ with $(\lambda_2 = \pm\frac{1}{2};\ \lambda_W = t, 0, \pm 1)$:

$$\text{vector}\left|\begin{aligned} H^V_{\frac{1}{2}1} &= \sqrt{2Q_-}\left(-F_1^V - \frac{M_1+M_2}{M_1}F_2^V\right), \\ H^V_{\frac{1}{2}0} &= \frac{\sqrt{Q_-}}{\sqrt{q^2}}\left((M_1+M_2)F_1^V + \frac{q^2}{M_1}F_2^V\right), \\ H^V_{\frac{1}{2}t} &= \frac{\sqrt{Q_+}}{\sqrt{q^2}}\left((M_1-M_2)F_1^V + \frac{q^2}{M_1}F_3^V\right), \end{aligned}\right. \qquad \text{axial-vector}\left|\begin{aligned} H^A_{\frac{1}{2}1} &= \sqrt{2Q_+}\left(F_1^A - \frac{M_1-M_2}{M_1}F_2^A\right), \\ H^A_{\frac{1}{2}0} &= \frac{\sqrt{Q_+}}{\sqrt{q^2}}\left(-(M_1-M_2)F_1^A + \frac{q^2}{M_1}F_2^A\right), \\ H^A_{\frac{1}{2}t} &= \frac{\sqrt{Q_-}}{\sqrt{q^2}}\left(-(M_1+M_2)F_1^A + \frac{q^2}{M_1}F_3^A\right), \end{aligned}\right.$$

where $Q_\pm = (M_1 \pm M_2)^2 - q^2$; $\qquad H^{V,A}_{-\lambda_2, -\lambda_W} = \pm H^{V,A}_{\lambda_2 \lambda_W}$

- SU(3) symmetry limit $\Longrightarrow F_3^V = 0$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + \{t, 0, \pm 1\}$ [flip]

- Six helicity amplitudes: $H_{\frac{1}{2}0}, H_{-\frac{1}{2}0}, H_{\frac{1}{2}1}, H_{-\frac{1}{2}-1}, H_{\frac{1}{2}t}, H_{-\frac{1}{2}t}$
- Relation between helicity amplitudes and decay parameters

$$\begin{aligned}
\sigma_D'^{sl} &= \frac{1}{2}\sin^2\theta_l(|H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2) + \cos^2\theta_l(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2) + |H_{-\frac{1}{2}t}|^2 + |H_{\frac{1}{2}t}|^2 - 2\cos\theta_l\Re(H_{-\frac{1}{2}t}H^*_{-\frac{1}{2}0} + H_{\frac{1}{2}t}H^*_{\frac{1}{2}0}),\\
\alpha_D'^{sl} &= \frac{1}{2}\sin^2\theta_l(|H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2) + \cos^2\theta_l(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2) + |H_{-\frac{1}{2}t}|^2 - |H_{\frac{1}{2}t}|^2 - 2\cos\theta_l\Re(H_{-\frac{1}{2}t}H^*_{-\frac{1}{2}0} - H_{\frac{1}{2}t}H^*_{\frac{1}{2}0}),\\
\beta_{3,4} &= 2\Im(H^*_{-\frac{1}{2}t}H_{-\frac{1}{2}-1} \pm H^*_{\frac{1}{2}1}H_{\frac{1}{2}t}),\\
\gamma_{3,4} &= 2\Re(H^*_{-\frac{1}{2}t}H_{-\frac{1}{2}-1} \pm H^*_{\frac{1}{2}1}H_{\frac{1}{2}t}).
\end{aligned}$$

- Non-zero elements of the decay matrix $b'_{\mu\nu}$:

$$\begin{aligned}
b'_{00} &= 1, & b'_{21} &= B'\cos\phi_p + A'\sin\phi_p\cos\theta_p,\\
b'_{03} &= a_D'^{sl}, & b'_{22} &= A'\cos\phi_p - B'\sin\phi_p\cos\theta_p,\\
b'_{10} &= a_D'^{sl}\cos\phi_p\sin\theta_p, & b'_{23} &= \sin\theta_p\sin\phi_p,\\
b'_{11} &= A'\cos\theta_p\cos\phi_p - B'\sin\phi_p, & b'_{30} &= a_D'^{sl}\cos\theta_p,\\
b'_{12} &= -B'\cos\theta_p\cos\phi_p - A'\sin\phi_p, & b'_{31} &= -A'\sin\theta_p,\\
b'_{13} &= \sin\theta_p\cos\phi_p, & b'_{32} &= B'\sin\theta_p,\\
b'_{20} &= a_D'^{sl}\sin\theta_p\sin\phi_p, & b'_{33} &= \cos\theta_p,
\end{aligned}$$

$$\text{where } a_D'^{sl} = \frac{\alpha_D'^{sl}(\theta_l, q^2)}{\sigma_D'^{sl}(\theta_l, q^2)}, \quad A' = \frac{1}{\sqrt{2}}\frac{\sin\theta_l}{\sigma_D'^{sl}(\theta_l, q^2)}[\cos\chi(\cos\theta_l\gamma_2 - \gamma_4) + \sin\chi(\cos\theta_l\beta_2 - \beta_4)],$$
$$B' = \frac{1}{\sqrt{2}}\frac{\sin\theta_l}{\sigma_D'^{sl}(\theta_l, q^2)}[\sin\chi(\cos\theta_l\gamma_2 - \gamma_4) - \cos\chi(\cos\theta_l\beta_2 - \beta_4)].$$

- Each element of $b'_{\mu\nu}$ is multiplied by $p = \sqrt{Q_+Q_-}/(2M_1)$ and $\frac{m_l^2}{2q^2}$

Intermediate step[flip] (1)

- σ'^{sl}_{Λ} and $\alpha'^{sl}_{\Lambda} \Rightarrow \{n, \alpha, \alpha', \alpha'', \beta_{1,2}, \gamma_{1,2}\}(q^2) + \{\epsilon, \epsilon', \beta_{3,4}, \gamma_{3,4}\}(q^2)$: $g^{\Lambda}_{av}(q^2)$, $g^{\Lambda}_{av3}(q^2)$
- Additional intermediate parameters:

$$\epsilon = 4(|H_{-\frac{1}{2}t}|^2 + |H_{\frac{1}{2}t}|^2),$$
$$\epsilon' = 4(|H_{-\frac{1}{2}t}|^2 - |H_{\frac{1}{2}t}|^2).$$

- $(n+\alpha')(n-\alpha'+\epsilon) - (\alpha+\alpha'')(\alpha-\alpha''+\epsilon') - 2\sum_{i=1}^{4}(\gamma_i^2+\beta_i^2) = 0$
  If no flip transition $\Longrightarrow$ relation on slide 6
- Introduce additional main parameter of semileptonic hyperon decay:

or

- $F_3^A(0)$
- $g^D_{av3}(0) = \dfrac{F_3^A(0)}{F_1^V(0)}$

Intermediate step$^{[\text{nonflip+flip}]}$ (2)

- Relations between intermediate and decay parameters:

$$n = ((M_-M_+)^2 - q^4)(1 + (g^D_{av}(q^2))^2) + q^2\left(4M_1M_2((g^D_{av}(q^2))^2 - 1) + \frac{q^2}{M_1^2}Q_-((g^D_w(q^2))^2(M_+^2 + q^2) + 4g^D_w(q^2)M_+)\right)$$

$$\alpha = 2\sqrt{Q_-Q_+}\left[g^D_{av}(q^2)(q^2 - M_-M_+) + 2g^D_{av}(q^2)g^D_w(q^2)q^2\frac{M_2}{M_1}\right]$$

$$\alpha' = Q_-Q_+\left[-(1 + (g^D_{av}(q^2))^2) + (g^D_w(q^2))^2\frac{q^2}{M_1^2}\right]$$

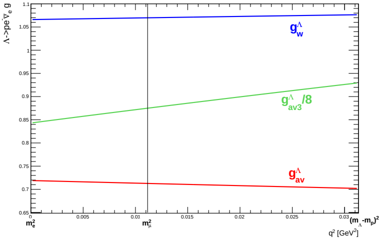
$$\alpha'' = 2\sqrt{Q_-Q_+}\left[g^D_{av}(q^2)(q^2 + M_-M_+) + 2g^D_{av}(q^2)g^D_w(q^2)q^2\right]$$

$$\epsilon = (M_-M_+)^2(1 + (g^D_{av}(q^2))^2) - q^2\left(M_-^2 + M_+^2(g^D_{av}(q^2))^2 + \frac{Q_-}{M_1}(2g^D_{av}(q^2)g^D_{av3}(q^2)M_+ - (g^D_{av3}(q^2))^2q^2)\right)$$

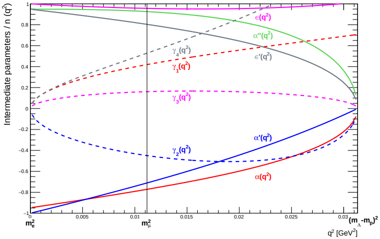
$$\epsilon' = 2M_-\sqrt{Q_-Q_+}\left(g^D_{av}(q^2)M_+ - \frac{q^2}{M_1}g^D_{av3}(q^2)\right)$$

where $M_- = M_1 - M_2$ and $M_+ = M_1 + M_2$ and $Q_\pm = M_\pm^2 - q^2$

- $\{\alpha, \alpha', \alpha'', \gamma_{1,2}\}(q^2) + \{\epsilon, \epsilon', \gamma_{3,4}\}(q^2)/n(q^2) \in [-1, +1]$



Preliminary



Joint angular distribution

- Full decay matrix of semileptonic hyperon decay:

$$b^f_{\mu\nu} = \sum_{\mu,\nu=0}^{3}\left(b_{\mu\nu} + \frac{m_l^2}{2q^2}b'_{\mu\nu}\right)$$

where $b_{\mu\nu}$ and $b'_{\mu\nu}$ are nonflip (slide 8) and flip (slide 14) transitions

- Process $e^+e^- \to (\Lambda \to pe^-\bar{\nu}_e)(\bar{\Lambda} \to \bar{p}\pi^+)$

$$\mathrm{Tr}\rho_{pW\bar{p}} \propto \mathcal{W}(\boldsymbol{\xi};\boldsymbol{\omega}) = \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}} b^{\Lambda}_{\mu 0} a^{\bar{\Lambda}}_{\bar{\nu}0}$$

  - $C_{\mu\bar{\nu}}(\theta_\Lambda;\alpha_\psi,\Delta\Phi)$
  - $b^f_{\mu 0}$ matrices for $1/2 \to 1/2 + \{t,0,\pm1\}$ decays $\Leftrightarrow$ $b^{\Lambda}_{\mu 0} \equiv b^f_{\mu 0}(\theta_p,\varphi_p,\theta_e,\chi,q^2;\boxed{g^\Lambda_{av},g^\Lambda_w,g^\Lambda_{av3}})$
  - $a_{\bar{\nu}0}$ matrices for $1/2 \to 1/2 + 0$ decays $\Leftrightarrow$ $a^{\bar{\Lambda}}_{\bar{\nu}0} \equiv a_{\bar{\nu}0}(\theta_{\bar{p}},\varphi_{\bar{p}};\boxed{\alpha_{\bar{\Lambda}}})$
    - $\boldsymbol{\xi} \equiv (\theta_\Lambda,\theta_p,\varphi_p,\theta_e,\chi,q^2,\theta_{\bar{p}},\varphi_{\bar{p}})$
    - $\boldsymbol{\omega} \equiv (\alpha_\psi,\Delta\Phi,\boxed{g^\Lambda_{av},g^\Lambda_w,g^\Lambda_{av3}},\boxed{\alpha_{\bar{\Lambda}}})$

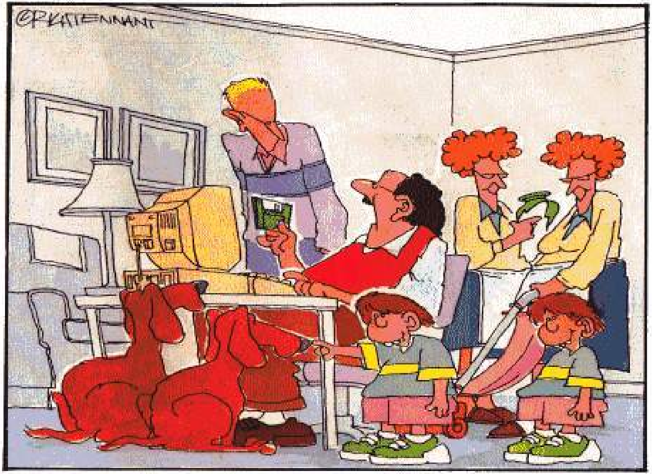
- If flip transition is taking into account, $g^D_{av3} \neq 0$
- Range of $q^2 \in (m_l^2,(M_1-M_2)^2)$ is specific for each decay

ToDo list and next steps

- Test formalism using
  1. Production of `mDIY` and `MC PhSp` for $e^+e^- \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$
     ∘ true and reco `mDIY` and `MC PhSp`
     ∘ allow to extract true and reco values of α_Λ and $\alpha_{\bar{\Lambda}}$ decay parameters
     ♠ $\alpha_\Lambda^{\rm true}$ and $\alpha_{\bar{\Lambda}}^{\rm true}$ are extracted and verified
  2. Modification of `mDIY` to include the semileptonic decay formalism
     ♠ Done
  3. Production of true and reco `mDIY` and `MC PhSp` for $\Lambda \to pe^-\bar{\nu}_e$
     ∘ extraction of the g_{av}^Λ and g_w^Λ decay parameters
     ♣ In progress
  4. If all steps work, consider more difficult scenario, mixed `MC` samples
  5. If previous step works, move to the real data

- Additional steps:
  - Sensitivity for g_i^Λ and A_i^Λ $(i = av, w)$:
    ♠ Preliminary result for three cases:
    ∘ $g_w^\Lambda = q^2 = 0$
    ∘ $g_w^\Lambda = 0$ and full q^2 range
    ∘ $g_w^\Lambda \neq 0$
  - Formalism with flip transition $(m_l^2/(2q^2) \neq 0)$:
    ♣ In progress

Backups



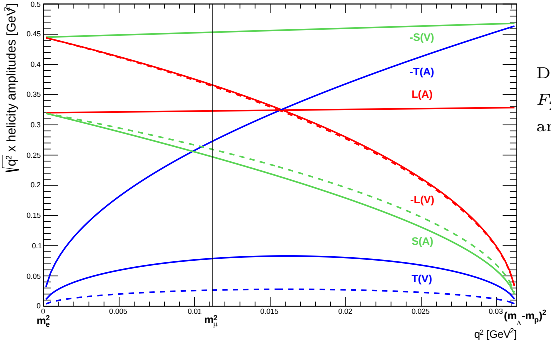
" I ALWAYS BACK UP EVERYTHING."

Size estimations of helicity amplitudes

$\mathrm{T(V,A)} := \sqrt{q^2} H^{V,A}_{\frac{1}{2}1}$

$\mathrm{L(V,A)} := \sqrt{q^2} H^{V,A}_{\frac{1}{2}0}$

$\mathrm{S(V,A)} := \sqrt{q^2} H^{V,A}_{\frac{1}{2}t}$



Dashed lines: $F_2^V(q^2)$ and $F_3^A(q^2)$ are switched off

- If $q^2 = m_e^2 \Rightarrow H^V_{\frac{1}{2}0} \sim H^V_{\frac{1}{2}t}$ and $H^A_{\frac{1}{2}0} \sim H^A_{\frac{1}{2}t}$ are dominated
- If $q^2 = (M_\Lambda - M_p)^2 \Rightarrow H^V_{\frac{1}{2}t}$ and $H^A_{\frac{1}{2}1} = -\sqrt{2} H^A_{\frac{1}{2}0}$ are dominated
- Using data of the E-555 experiment (Fermilab) [PRD41 (1990) 780]
  $\Rightarrow \frac{F_1^A(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.731 \pm 0.016$ and $\frac{F_2^V(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.15 \pm 0.30$
  $\Rightarrow \frac{F_1^A(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.719 \pm 0.016$ with constraint $\frac{F_2^V(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) \to 0.97$ (CVC)

CP violation in non-leptonic decays

[Andrzej's slides]

$$A_\Lambda = \frac{\alpha_\Lambda + \alpha_{\bar{\Lambda}}}{\alpha_\Lambda - \alpha_{\bar{\Lambda}}}$$

- BESIII result: $A_\Lambda = -0.006 \pm 0.012 \pm 0.007$ [NaturePhys.15(2019)631]
- CKM: $-3 \cdot 10^{-5} \le A_\Lambda \le 4 \cdot 10^{-5}$ [PRD67(2003)056001]
  Extensions of SM: $A_{\Lambda \to p\pi^-} \sim (0.05 - 1.2) \cdot 10^{-4}$ [Chin.Phys.C42(2018)013101]

Experiment	N_{evt}	$\sigma(A_\Lambda)$	
BESIII (2018)	$4.2 \cdot 10^5$	$1.2 \cdot 10^{-2}$	$N_{J/\psi} = 1.31 \cdot 10^9$
Some estimations			
BESIII	$3 \cdot 10^6$	$5 \cdot 10^{-3}$	$N_{J/\psi} = 10^{10}$ $\mathcal{L} = 0.47 \cdot 10^{33}/\text{cm}^2/\text{s}$, ΔE=0.9 MeV
SuperTauCharm	$6 \cdot 10^8$	$3 \cdot 10^{-4}$	$N_{J/\psi} = 2 \cdot 10^{12}$ $\mathcal{L} = 10^{35}/\text{cm}^2/\text{s}$, ΔE=0.9 MeV
SuperTauCharm + reduced ΔE	$3 \cdot 10^9$	$1.4 \cdot 10^{-4}$	$N_{J/\psi} = 10^{13}$ $\mathcal{L} = 10^{35}/\text{cm}^2/\text{s}$, ΔE<0.9 MeV (?)

CP symmetry test A_Λ^{av}

$$A_\Lambda^{av} = \frac{g_{av}^{\Lambda}(0) + g_{av}^{\bar{\Lambda}}(0)}{g_{av}^{\Lambda}(0) - g_{av}^{\bar{\Lambda}}(0)}$$

- $g_{av}(0) = \frac{F_1^A(0)}{F_1^V(0)}$
  - Determination of $F_1^V(0)$ value by CVC hypothesis: $F_1^V(0) < 0$
  - No determination of $F_1^A(0)$ value by any general theoretical arguments
- Some assumptions:
  - $F_1^A(0) \lessgtr 0$ then $g_{av}(0) \gtrless 0$
  - ? Naïve assumption $g_{av}^{\Lambda} = -g_{av}^{\bar{\Lambda}}$
    $\Longrightarrow$ Need to be care with sign of g_{av}

Experiment	N_{evt}	Result	Reference
E555 (Fermilab)	37286	$g_{av}(0) = 0.719 \pm 0.016 \pm 0.012$ constrain on $g_w(0) = 0.97$	[PRD41(1990)780]
SPS (CERN)	7111	$g_{av}(0) = 0.70 \pm 0.03$ measured $F_2^V(0) = 1.32 \pm 0.81$	[ZPC21(1983)1]
AGS (BNL)	10^4	$\lvert g_{av}(0)\rvert = 0.734 \pm 0.031$ used $g_w(0) = 0.97$	[PLB98(1981)123]

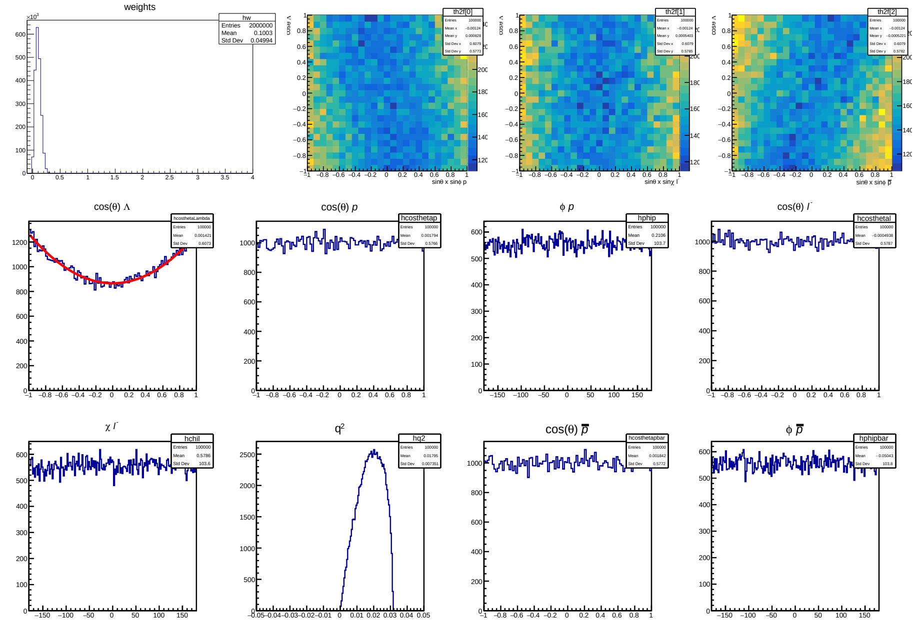
MC generator (step 1)

- Using **YYbar_example** package by Patrik, the presented process can be generated
  - MC samples: $N_{\rm evt}^{\rm sig}$=10^5 and $N_{\rm evt}^{\rm phsp}$=10^6
  - Generate $q^2 \in (m_e^2, (M_\Lambda - M_p)^2)$
  - Set of input values:

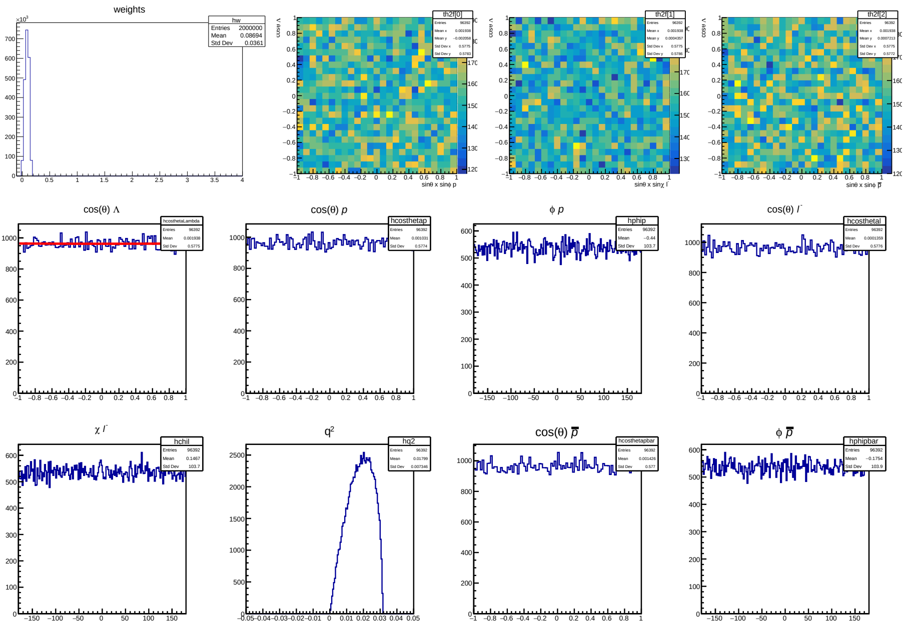
```
pp[0] =  0.461;      // alpha_J/psi (arxiv:1808.08917)
pp[1] =  0.74;       // Delta_Phi (arxiv:1808.08917, equal 42.4deg)
pp[2] =  0.719;       // gav Lambda->p e- nu_ebar
pp[3] =  1.066;       // gw Lambda->p e- nu_ebar
pp[4] = -0.758;       // alpha_D LambdaBar->pbar pi+ (arxiv:1808.08917)
```

- No negative weights are observed
- Maximal weight $\sim$0.36

Random samples ($N_{sig} = 10^5$) (step 1)



Random samples ($N_{phsp} = 10^6$) (step 1)



Run through fit method (step 2)

- Set input values in the fit

```
void mainMLL(){

      // instantiating the values to be measured
      Double_t pp[4];
      for( int i = 0; i < 4; i++ ) pp[i]=0;

      // starting values for fit
      Double_t alpha_jpsi     = 0.461;     // alpha_J/Psi
      Double_t dphi_jpsi      = 0.74;       // relative phase, Dphi_J/Psi
      Double_t gav_lam_plnu   = 0.719;      // gav (Lam->p l nubar_l)
      Double_t gw_lam_plnu    = 1.066;      // gw (Lam->p l nubar_l)
      Double_t alpha_lam_pbarpip  = -0.758;   // alpha (Lambar->pbar pi+)

      alpha_jpsi        = gRandom->Rndm();
      dphi_jpsi         = gRandom->Rndm();
      gav_lam_plnu    = 0.719;
      gw_lam_plnu   = 1.066;
      alpha_lam_pbarpip = -0.758;

      ReadData();
      ReadMC();
```

- Output value of fit method

Parameter	$N_{sig}^{MC} = 10^5, N_{phsp}^{MC} = 10^6$
α_Ψ	0.4570±0.0110
$\Delta\Phi$	0.7927±0.0252
g_{av}	0.6601±0.0506
g_w	1.1251±0.2727
$\alpha_{\bar{\Lambda}}$	-0.7527±0.0121

	α_Ψ	$\Delta\Phi$	g_{av}	g_w	$\alpha_{\bar{\Lambda}}$
α_Ψ	1	0.281	-0.026	0.032	0.151
$\Delta\Phi$		1	0.016	-0.013	0.269
g_{av}			1	-0.766	0.323
g_w				1	-0.411
$\alpha_{\bar{\Lambda}}$					1

Boundary case: $q^2_{\min}$

- $q^2_{\min} = m_e^2 \longrightarrow 0$:

$$
\begin{aligned}
b_{00} &= (1 + (g_{av}^D(0))^2)\sin^2\theta_l, \\
b_{03} &= -2g_{av}^D(0)\sin^2\theta_l, \\
b_{10} &= b_{03}\sin\theta_p\cos\phi_p, \\
b_{13} &= b_{00}\sin\theta_p\cos\phi_p, \\
b_{20} &= b_{03}\sin\theta_p\sin\phi_p, \\
b_{23} &= b_{00}\sin\theta_p\sin\phi_p, \\
b_{30} &= b_{03}\cos\theta_p, \\
b_{33} &= b_{00}\cos\theta_p
\end{aligned}
$$