The measurement of R at BESIII

Weiping Wang

24-th, July. 2017

Outline

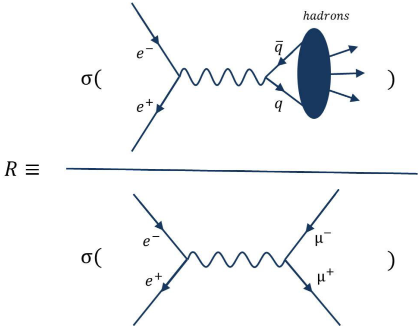
- The physics of R
- Introduction
- Data sets
- Event selection
- Background substraction
- Systematic uncertainty study
- Results
- Summary

The definition of R

R is defined as the ratio of the production rate of hadron and muon pairs:

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{1}$$

That is:



$$\equiv 3\sum_{flavor} Q_f^2 = \begin{cases} 2 & \text{for} \quad u,d,s \\ 10/3 & \text{for} \quad u,d,s,c \\ 11/3 & \text{for} \quad u,d,s,c,b \end{cases}$$

The definition of R

According to the QED theory, the cross section of $e^+e^- \to \mu^+\mu^-$ process can be precisely calculated. At the level of lowest oder,

$$\frac{d\sigma_{\mu\mu}}{d\Omega} = \frac{\alpha^2}{4s}\sqrt{1-\frac{4m_\mu^2}{s}}\left[\left(1+\frac{4m_\mu^2}{s}\right)+\left(1-\frac{4m_\mu^2}{s}\right)\cos^2\theta\right] \quad (2)$$

The total cross section is

$$\sigma_{total} = \frac{4\pi\alpha^2}{3s}\sqrt{1-\frac{4m_\mu^2}{s}}\left(1+\frac{2m_\mu^2}{s}\right) \quad (3)$$

In the high energy limit where $\sqrt{s} \gg 2m_\mu$, it turns out

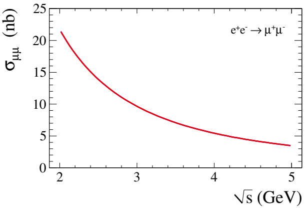
$$\frac{d\sigma_{\mu\mu}}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta) \quad \text{and} \quad \sigma_{total} = \frac{4\pi\alpha^2}{3s} \quad (4)$$

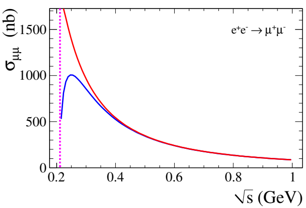
That is $\sigma(e^+e^- \to \mu^+\mu^-) = 4\pi\alpha^2/(3s) = 86.85\ \mathrm{nb}/s(\mathrm{GeV}^2)$

The definition of R

Regarding the denominator of the Eq. 1:

- The $\sigma(e^+e^- \to \mu^+\mu^-)$ is just a reference.
- It is OK only the lowest order diagram is included in the denominator.
- At $\sqrt{s} = 2 \sim 5$ GeV, the high energy limit can be achieved esaily.
- The calculation of QED provides analytical result.
- Not necessary to measure $\sigma(e^+e^- \to \mu^+\mu^-)$.

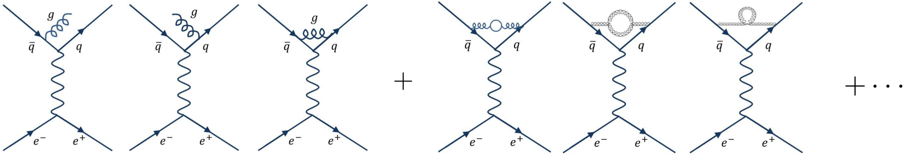




The definition of R

For the numerator of the Eq. 1:

- Higher order diagrams should be included.
- The QCD theory can realize third order (α_s^3) correction, with perturbation method.
- What we measured contains all the high order corrections.



QCD calculation of R

The calculation of the $R(s)$ is based on the perturbative QCD method and performed to α_s^3 order:

$$R(s) = 3 \sum_{flavor} Q_f^2 \left[1 + \frac{\alpha_s}{\pi} + r_1 \left(\frac{\alpha_s}{\pi} \right)^2 + r_2 \left(\frac{\alpha_s}{\pi} \right)^3 \right] + O(\alpha_s^4) \tag{5}$$

where α_s is the strong coupling constant and

$$r_1 = 1.9857 - 0.1153f \tag{6}$$

$$r_2 = -6.6368 - 1.2001f - 0.0052f^2 - 1.2395 \frac{(\sum Q_f)^2}{3 \sum Q_f^2} \tag{7}$$

where the f denotes the number of quarks can be produced at $\sqrt{s}$.

QCD calculation of R

- $\sqrt{s} < 3.7$ GeV, $f = 3$, $\sum Q_f = 0$, $\sum Q_f^2 = \frac{2}{3}$, $r_1 = 1.6398$, $r_2 = -10.284$
- $\sqrt{s} < 10$ GeV, $f = 4$, $\sum Q_f = \frac{2}{3}$, $\sum Q_f^2 = \frac{10}{9}$, $r_1 = 1.5245$, $r_2 = -11.686$
- $\sqrt{s} > 11$ GeV, $f = 5$, $\sum Q_f = \frac{1}{3}$, $\sum Q_f^2 = \frac{11}{9}$, $r_1 = 1.4092$, $r_2 = -12.805$

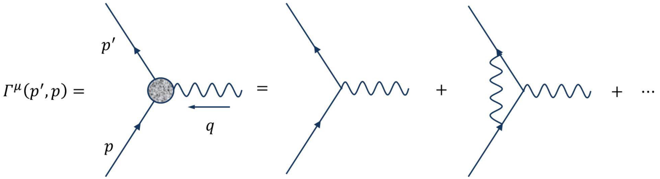
Therefore

$$R \equiv \begin{cases} 2\left[1 + \frac{\alpha_s}{\pi} + 1.64\left(\frac{\alpha_s}{\pi}\right)^2 - 10.3\left(\frac{\alpha_s}{\pi}\right)^3\right] + O(\alpha_s^4) & \text{for} \quad u,d,s \\ \frac{10}{3}\left[1 + \frac{\alpha_s}{\pi} + 1.52\left(\frac{\alpha_s}{\pi}\right)^2 - 11.7\left(\frac{\alpha_s}{\pi}\right)^3\right] + O(\alpha_s^4) & \text{for} \quad u,d,s,c \\ \frac{11}{3}\left[1 + \frac{\alpha_s}{\pi} + 1.41\left(\frac{\alpha_s}{\pi}\right)^2 - 12.8\left(\frac{\alpha_s}{\pi}\right)^3\right] + O(\alpha_s^4) & \text{for} \quad u,d,s,c,b \end{cases}$$

The measurement of R value can be used to determine the α_s at $\sqrt{s}$.

The corrected electron vertex function

The radiation correction to the electron vertex function:



After applying Lorentz invariance requirement and Ward identity, the corrected vertex function takes the form

$$\Gamma^\mu(p',p) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m} F_2(q^2) \tag{8}$$

- $F_1(q^2)$ and $F_2(q^2)$ are functions of q^2 called form factors. The loop diagrams will contribute to F_1 and F_2 at order α and higher.
- To lowest order, $F_1(q^2) \equiv 1$ and $F_2(q^2) \equiv 0$.
- $F_1(q^2)$ renormalizes the electric charge of the electron. While the $F_2(q^2)$ modifies the relative magnetic moment of the electron.

The anomalous magnetic moment

Since the actual magnetic moment is measured at non-relativistic energies with $q^2 \to 0$, the moment which can be compared to data is

$$g \equiv 2 + 2F_2(0) \tag{9}$$

Thus the anomalous magnetic moment of lepton is

$$a_\ell \equiv \frac{g-2}{2} = F_2(0) \tag{10}$$

In conclusion, the anomalous magnetic moment of lepton exactly comes from the high oder radiative corrections to the $\sigma^{\mu\nu}$ term of the lepton vertex function in Eq. 8.

The anomalous magnetic moment

There are several radiative corrections to the lepton vertex function:

$$a_\ell^{Exp.} = a_\ell^{SM} + a_\ell^{new} \tag{11}$$

where

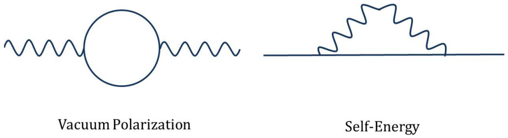
$$a_\ell^{SM} = a_\ell^{QED} + a_\ell^{Weak} + a_\ell^{Had.} \tag{12}$$

In principle, the a_ℓ^{QED} and a_ℓ^{Weak} can be precisely calculated to any order of α. While the $a_\ell^{Had.}$ should obtain from the measurement of the R value.

- $a_\ell^{new} = a_\ell^{Exp.} - a_\ell^{SM}$ reflects the contributions of physics beyond SM.
- a_ℓ^{QED} is the corrections from leptonic vacuum polarization, lepton self-energy and phton to the vertex function.
- a_ℓ^{Weak} is the corrections from $W^\pm$, Z^0 and H^0 interactions.
- $a_\ell^{Had.}$ is the contribution of hadronic vacuum polarization.

The anomalous magnetic moment: a_ℓ^{QED}

The main ingredients of QED radiative corrections are leptonic vacuum polarization and lepton self-energy:



Usually, the QED contributions to lepton anomalous magnetic moment can be expressed in the perturbative series

$$a_\ell^{QED} = \sum_{n=1}^{\infty} a_\ell^{(2n)} \left(\frac{\alpha}{\pi}\right)^n \tag{13}$$

- The nth term in Eq. 13 is called n-loop term or the $2n$th-order term, as it is $\mathcal{O}(\alpha^n) \sim \mathcal{O}(e^{2n})$
- The leading-order coefficient $a_\ell^{(2)} = 1/2 \equiv A_1^{(2)}$.

The anomalous magnetic moment: a_ℓ^{QED}

Since the dimensionful quantities of the radiative correction are expressed in unit of the mass of the external lepton, the coefficients of Eq. 13, *i.e.* $a_\ell^{(2n)}$, can be decomposed into four types according to the dependence on lepton mass ratios. We take electron as an example:

$$a_e^{(2n)} = A_1^{(2n)} + A_2^{(2n)}\left(\frac{m_e}{m_\mu}\right) + A_2^{(2n)}\left(\frac{m_e}{m_\tau}\right) + A_3^{(2n)}\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) \qquad (14)$$

where the subscript j attached to $A_j^{(2n)}$ denotes the number of leptons involved in its calculation.

- $A_1^{(2n)}$ is the contribution from electron only. It is mass-independent thus universally contributes to a_ℓ for all leptons.
- $A_2^{(2n)}\left(\frac{m_e}{m_\mu}\right)$ comes from at least one μ loop and without τ loop.
- $A_2^{(2n)}\left(\frac{m_e}{m_\tau}\right)$ comes from at least one τ loop and without μ loop.
- $A_3^{(2n)}\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\mu}\right)$ is the contribution of both one τ loop and one μ loop.

The anomalous magnetic moment: a_ℓ^{QED}

According to Eq. 14, we conclude that since $m_\mu \gg m_e$ and $m_\tau \gg m_e$, the $A_2^{(2n)}$ and $A_3^{(2n)}$ are significantly suppressed compared to $A_1^{(2n)}$.

Regarding the μ lepton:

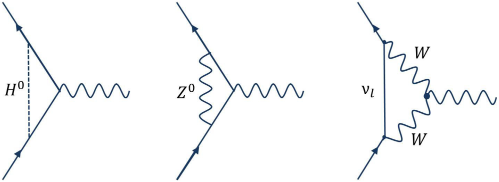
$$a_\mu^{(2n)} = A_1^{(2n)} + A_2^{(2n)}\left(\frac{m_\mu}{m_e}\right) + A_2^{(2n)}\left(\frac{m_\mu}{m_\tau}\right) + A_3^{(2n)}\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right) \tag{15}$$

This means that $A_2^{(2n)}\left(\frac{m_\mu}{m_e}\right)$ would be more contributive due to $m_\mu \gg m_e$.

For the τ lepton, a measurement of a_τ is very difficult due to the short τ-lifetime.

The anomalous magnetic moment: a_ℓ^{Weak}

The weak interaction contribution to a_ℓ is originated from the exchanges of gauge bosons $W^\pm$, Z^0 and the Higgs boson. To the leading order of weak interaction contributions, the vertex diagrams are

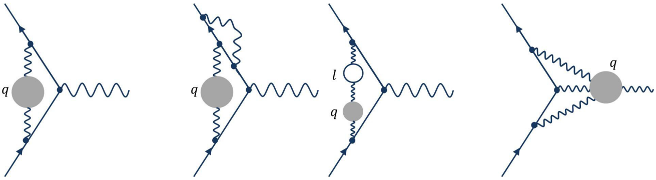


There is a factor of m_ℓ^2/m_W^2 in the weak interaction contributions to the a_ℓ. Therefore, the a_ℓ^{Weak} is very small comparing to the a_ℓ^{QED}. However, it cannot be ignored due to its comparable scale with respect to the $\delta a_\mu = a_\mu^{Exp.} - a_\mu^{SM}$.

Similarly, the size of a_ℓ^{Weak} is enhanced relative to a_ℓ^{QED} when the lepton mass becomes heavier.

The anomalous magnetic moment: $a_\ell^{Had.}$

The hadronic contribution to a_ℓ is originated from the hadronic vacuum polarization (VP). The lowest order of $a_\ell^{Had.}$ is of $\mathcal{O}(\alpha^2)$. Following diagrams show the $\mathcal{O}(\alpha^2)$- and $\mathcal{O}(\alpha^3)$-hadronic VP diagrams:



Above diagrams represents three types of hadronic contribution to $a_\ell^{Had.}$

$$a_\ell^{Had.} = a_\ell(\text{had. VP}) + a_\ell(\text{NLO had. VP}) + a_\ell(\text{had. l-l}) \tag{16}$$

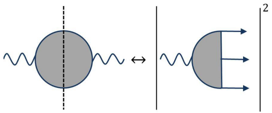
- a_ℓ(had. VP)$\sim\mathcal{O}(\alpha^2)$ and it also includes the $\mathcal{O}(\alpha^3)$-terms due to one virtual photon exchange between quarks in the grayed circle.
- a_ℓ(NLO had. VP) is obtained from the QED corrections to a photon line, lepton line or leptonic vertex of a_ℓ(had. VP).
- a_ℓ(had. l-l) is of $\mathcal{O}(\alpha^3)$ and is the contribution of light-by-light scattering.

The anomalous magnetic moment: $a_\ell^{Had.}$

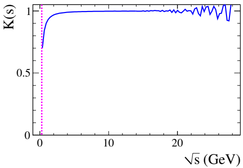
At the low energy region ($\sqrt{s} < 1.0$ GeV), the perturbation theory of QCD is not valid. Therefore the a_ℓ(had. VP) cannot be calculated analytically. Fortunately, this contribution may be calculated in terms of experimental total cross section. Take the μ lepton as an example:

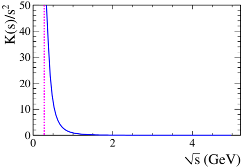
$$a_\mu(\text{had. VP}) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{R(s)K(s)}{s^2} \tag{17}$$

where the kernel $K(s)$ varying from 0.63 at threshold to 1.0 at $s = \infty$.



Optical Theorem



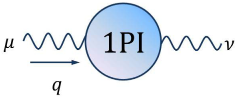


- The motivation of this integral is the optical theorem of the QFT.
- Only the R values below 1.0 GeV contribute the a_ℓ(had. VP) significantly.
- The kernel is numerically instable at large $\sqrt{s}$, as shown in right plot.
- After sufficient high energy, *e.g.* $\sqrt{s} = 40$ GeV, the a_ℓ(had. VP) is calculated using perturbative QCD thery.

The effective coupling constant $\alpha(s)$

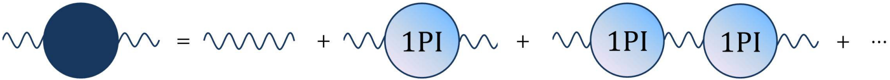
The vacuum polarization diagram (also known as the photon self-energy) can be reviewed as a modification to the photon structure by a virtual lepton pair. This diagram will alter the effective field $A^{\mu}(x)$ seen by the scattered electron. It can potentially shift the over all strength of this field, and can also change its dependence on space-time.

The vacuum polarization diagram is consistent of inserting all one-particle-irreducible (1PI) diagrams into photon propagator:

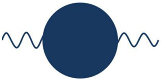


$$\equiv i\Pi^{\mu\nu}(q) \equiv i(q^2 g^{\mu\nu} - q^{\mu}q^{\nu})\Pi(q^2)$$

Therefore, the exact photon propagator function is:



After substituting the express of 1PI diagram and applying the Ward identity, the exact photon propagator takes the form:



$$\equiv \frac{-ig^{\mu\nu}}{q^2(1-\Pi(q^2))}$$

The effective coupling constant $\alpha(s)$

Above exact photon propagator function creates two effects:

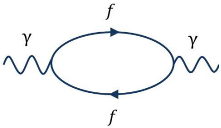
- Renormalize the electric charge of the theory: bare charge to physical charge, *i.e.* $e_0 \mapsto e \equiv e_0 \sqrt{\frac{1}{1-\Pi(0)}}$. This is a constant shift in the strength of the electric charge.
- The q^2-dependent effective coupling 'constant': $\alpha_0 \equiv \frac{e_0^2}{4\pi} \mapsto \alpha_{\text{eff.}}(q^2) \equiv \frac{e_0^2}{4\pi}\frac{1}{1-\Pi(q^2)}$, where the α_0 is the fine structure constant.

In practice, the effective fine structure constant at scale $\sqrt{s}$ is given by:

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)} \tag{18}$$

where $\Delta\alpha(s) = \Pi(s)$, and is the contribution from all one-particle-irreducible diagrams to the photon vacuum polarization. The $\Delta\alpha(s)$ is large at $s = m_Z^2$ due to the large change of the scale.

In the perturbative theory, the leading order corrections to $\Delta\alpha(s)$ is:

$\Delta\alpha(s) = \Sigma_f$ 

The effective coupling constant $\alpha(s)$

The contributions to $\Delta\alpha(s)$ can be distinguished to the contributions from leptons, the five active quarks and the top quark:

$$\Delta\alpha = \Delta\alpha_\ell + \Delta\alpha^{(5)}_{\mathrm{Had.}} + \Delta\alpha_{\mathrm{top}} \tag{19}$$

- Since the top quark is heavy, the $\Delta\alpha_{\mathrm{top}}$ is small comparing with other light fermions.
- $\Delta\alpha_\ell$ can be calculated analytically using the perturbative theory.
- The $\Delta\alpha^{(5)}_{\mathrm{Had.}}$ cannot be reliably calculated by using perturbative QCD at low energy scale.

Fortunately, one can evaluate this hadronic term $\Delta\alpha^{(5)}_{\mathrm{Had.}}$ from hadronic e^+e^- annihilation data by using dispersion relation together with the optical theorem which results in the integral:

$$\Delta\alpha^{(5)}_{\mathrm{Had.}}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\,\mathrm{Re}\int_{4m_\pi^2}^{\infty} \mathrm{d}s\,\frac{R(s)}{s(s-M_Z^2-i\varepsilon)} \tag{20}$$

- The integral is similar to the one in a_ℓ (Eq. 17) but with different kernel function.
- $\Delta\alpha^{(5)}_{\mathrm{Had.}}(M_Z^2)$ gets significant contributions from a broad energy range up to $\sqrt{s} \simeq M_Z/2$.