Soft Gluon Resummation in Higgs plus jet Production

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MSU

Education

Sep.2000-Jun.2004 Dalian University of Technology

B.S

Applied physics

Sep.2005- Aug.2010 Graduate University of the Chinese Academy of Sciences

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Theoretical physics

Thesis Topic: Production and decay of heavy quarkonia

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Work Experience

Sep.2010-Aug.2012 Peking University in China
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The research of particle physics
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Post doctor
The research of particle physics
Supervisor: Feng Yuan

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Post doctor
The research of particle physics
Supervisor: C P Yuan

Outline

- QCD resummation
- soft gluon resummation in Higgs plus jet production
- Summary

A simple example

- The running of coupling constant

RGE:

$$\mu_R^2 \frac{d\alpha_s}{d\mu_R^2} = \beta(\alpha_s) = -(b_0\alpha_s^2 + b_1\alpha_s^3 + b_2\alpha_s^4 + \cdots)$$

By solving RGE

$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + \beta_0\, \alpha_s(\mu_0^2)\ln(\mu^2/\mu_0^2)} = \alpha_s(\mu_0^2)\sum_{n=0}^{\infty}\left(-\beta_0\, \alpha_s(\mu_0^2)\ln\frac{\mu^2}{\mu_0^2}\right)^n$$

QCD soft gluon resummation

- Consider the production process pp →H(Z)+X

$$\frac{d\sigma}{dQ_T^2} \sim \frac{1}{Q_T^2}\{ \alpha_S(L+1) \quad +\alpha_S^2(L^3+L^2) \quad +\alpha_S^3(L^5+L^4)+\alpha_S^4(L^7+L^6)+\dots$$
$$+\alpha_S^2(L+1) \quad +\alpha_S^3(L^3+L^2)+\alpha_S^4(L^5+L^4)+\dots$$
$$+\alpha_S^3(L+1)+\alpha_S^4(L^3+L^2)+\dots\}$$

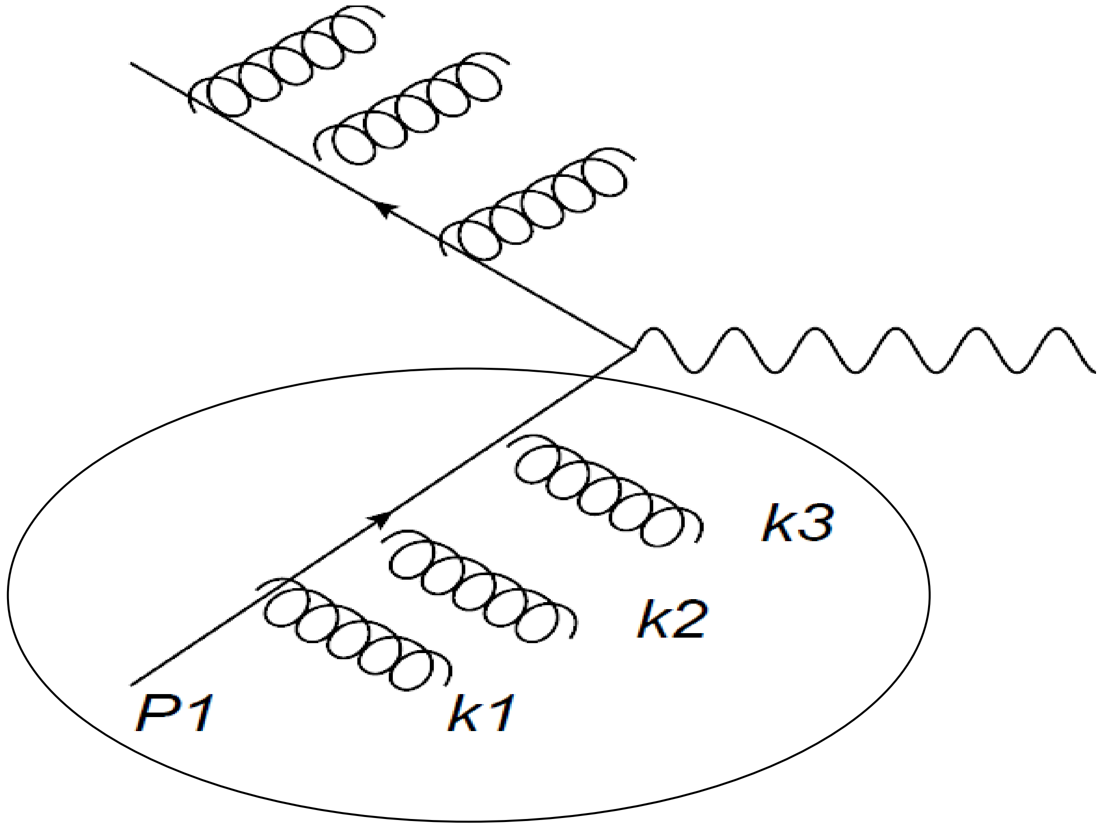
Where Q_T is the transverse momentum, and Q is the mass of H(Z), and L = Log[Q^2 / Q_T^2].

- We have to resum these large logs to make reliable predictions

$$W(Q,b) = e^{-\int_{1/b}^{Q}\frac{d\mu}{\mu}\left(\ln\frac{Q}{\mu}A+B\right)}\left[C\otimes f_1\right]\left[C\otimes f_2\right]$$

For g g → H + X $\quad$ $\mathbf{A^{(1)}=C_A\, a_s/\pi}$ $\quad$ $\mathbf{B^{(1)}=-2C_A\,\beta_0\, a_s/\pi}$

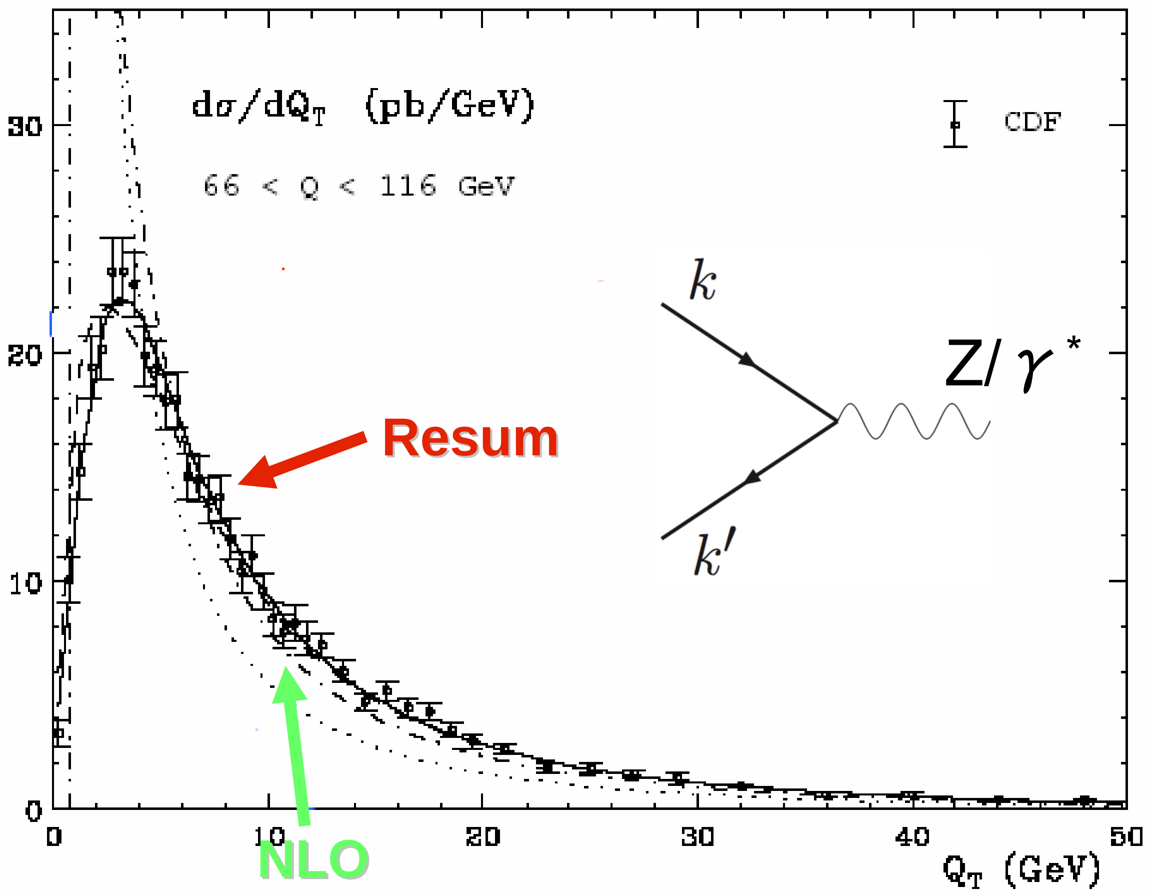
For q $\bar{q}$ → Z + X $\quad$ $\mathbf{A^{(1)}=C_F\, a_s/\pi}$ $\quad$ $\mathbf{B^{(1)}=-2C_F/3\, a_s/\pi}$



$$\frac{1}{(p1-k1)^2}\frac{1}{(p1-k1-k2)^2}\frac{1}{(p1-k1-k2-k3)^2}$$

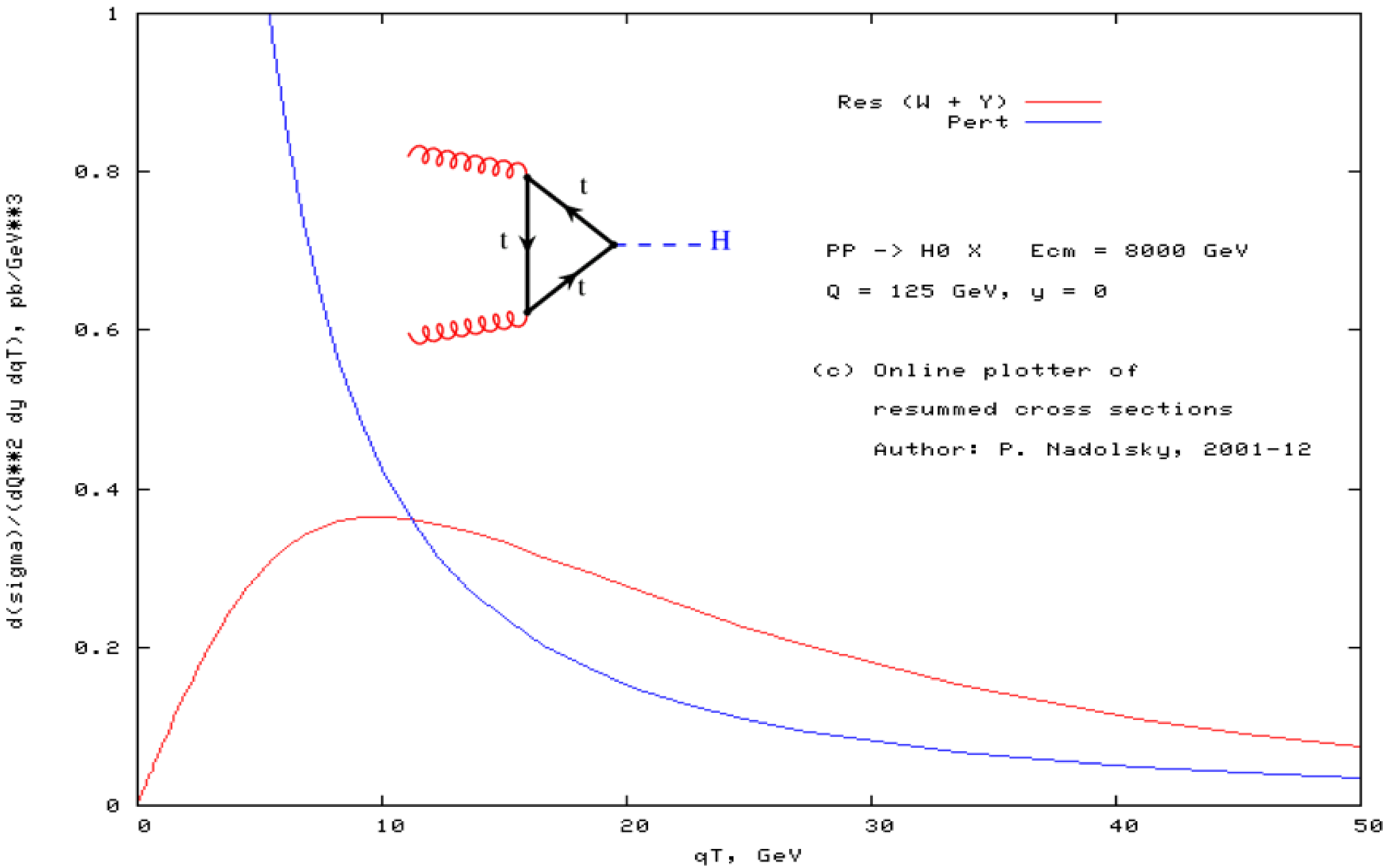
$$p1^2=0$$

How Large of the Resummation effects

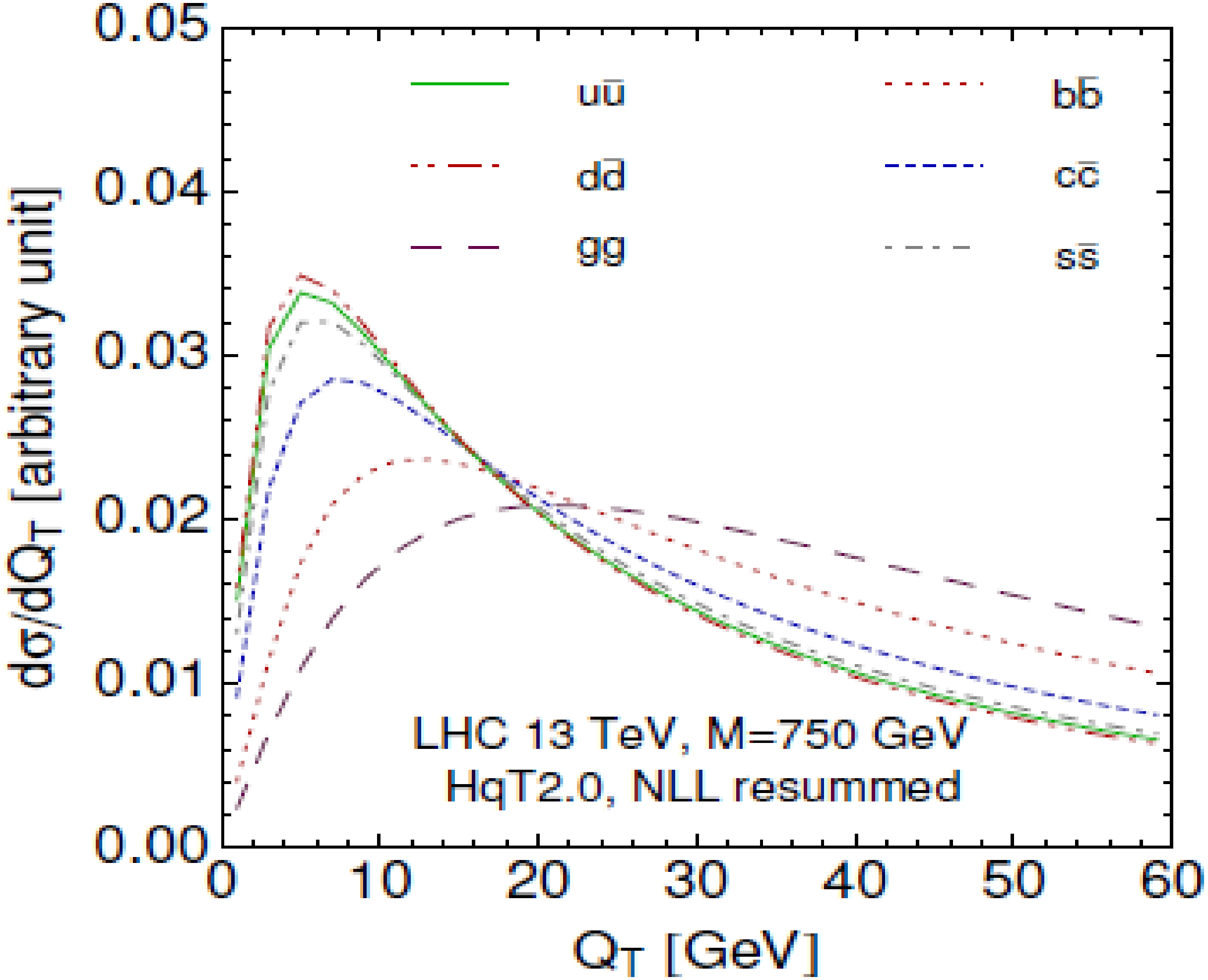


Kulesza, Sterman, Vogelsang, 02

Higgs production in pp collision



750 GeV pseudo scalar particle production at LHC

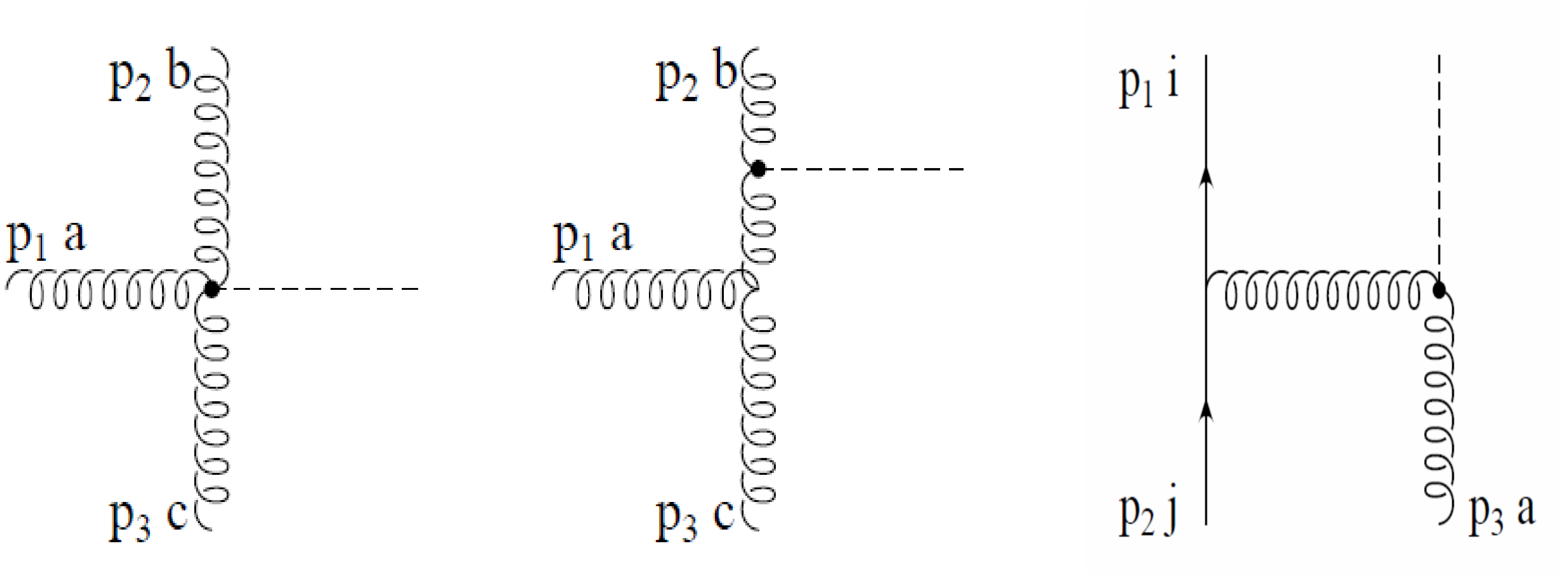


Jun Gao, Hao Zhang, Hua Xing Zhu **arXiv:1512.08478**

- The Sudakov factor knows about the color structure of the initial states

Higgs plus one jet productions in pp collision

- the leading order feynman diagrams



Sudakov factor in Higgs plus one jet process

$$S_{\text{Sud}}(Q^2, b_\perp) = \int_{b_0^2/b_\perp^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[\ln\left(\frac{Q^2}{\mu^2}\right) A + B + D \ln \frac{1}{R^2} \right]$$

- for g g → H g

$$A = C_A \frac{\alpha_s}{\pi}, \; B = -2C_A \beta_0 \frac{\alpha_s}{\pi}$$

- Additional term in Higgs plus jet:

$$D = C_A \frac{\alpha_s}{2\pi}$$

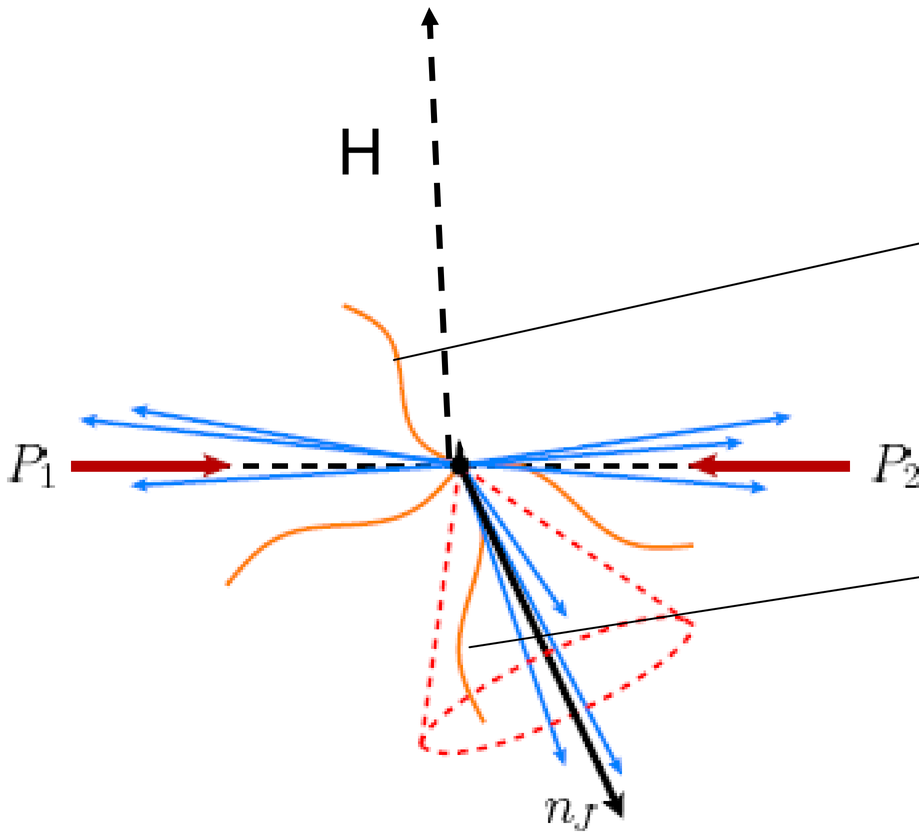
- for g q $\rightarrow$ H q

$$A = (C_F/2 + C_A/2)\frac{\alpha_s}{\pi}$$

$$B = (-C_A\beta_0 - 3/4C_F - (1/2)C_A \ln u/t + (1/2)C_F \ln u/t)\frac{\alpha_s}{\pi}$$

$$D = C_F\frac{\alpha_s}{2\pi}$$

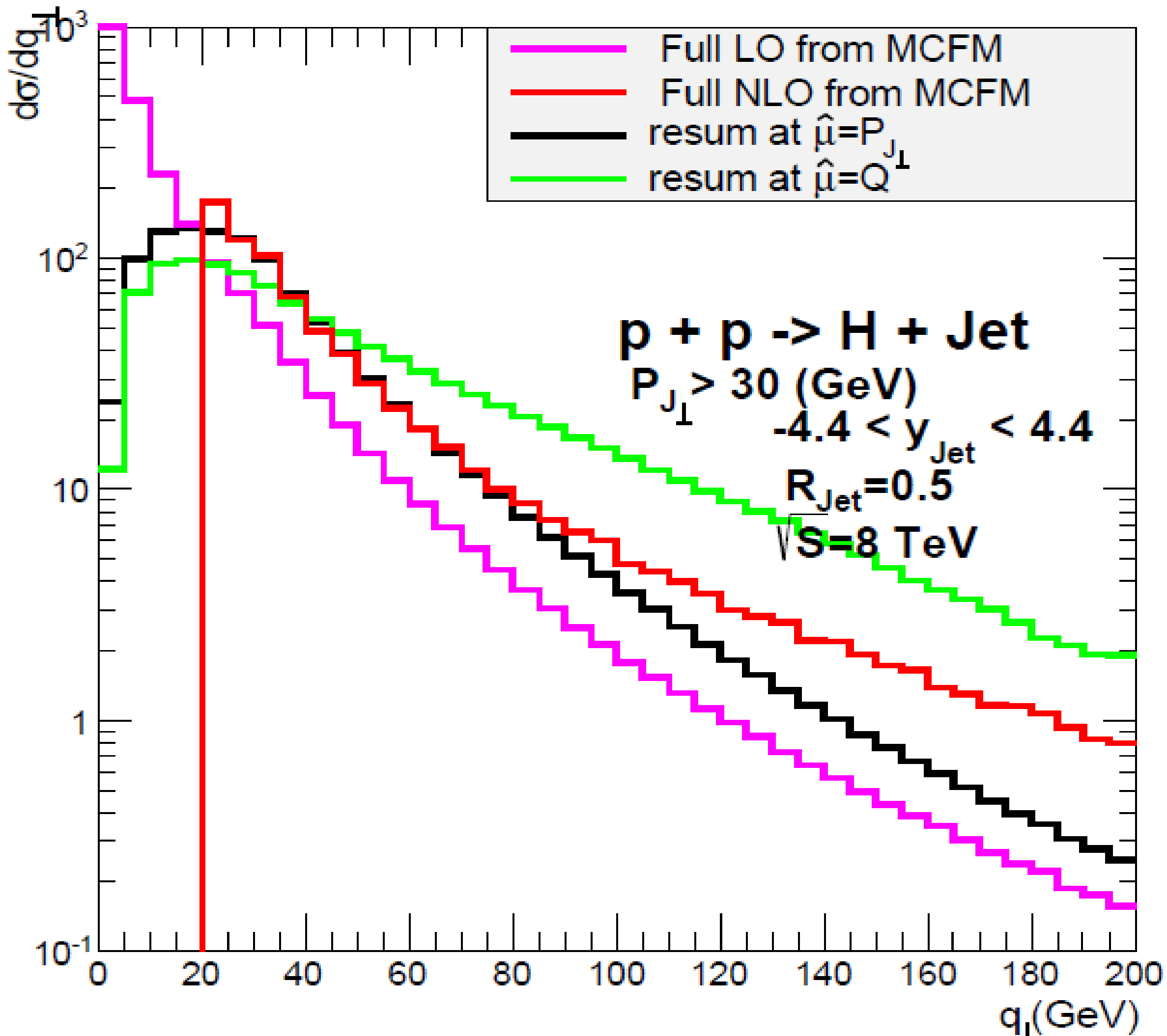
Sudakov knows about cone size



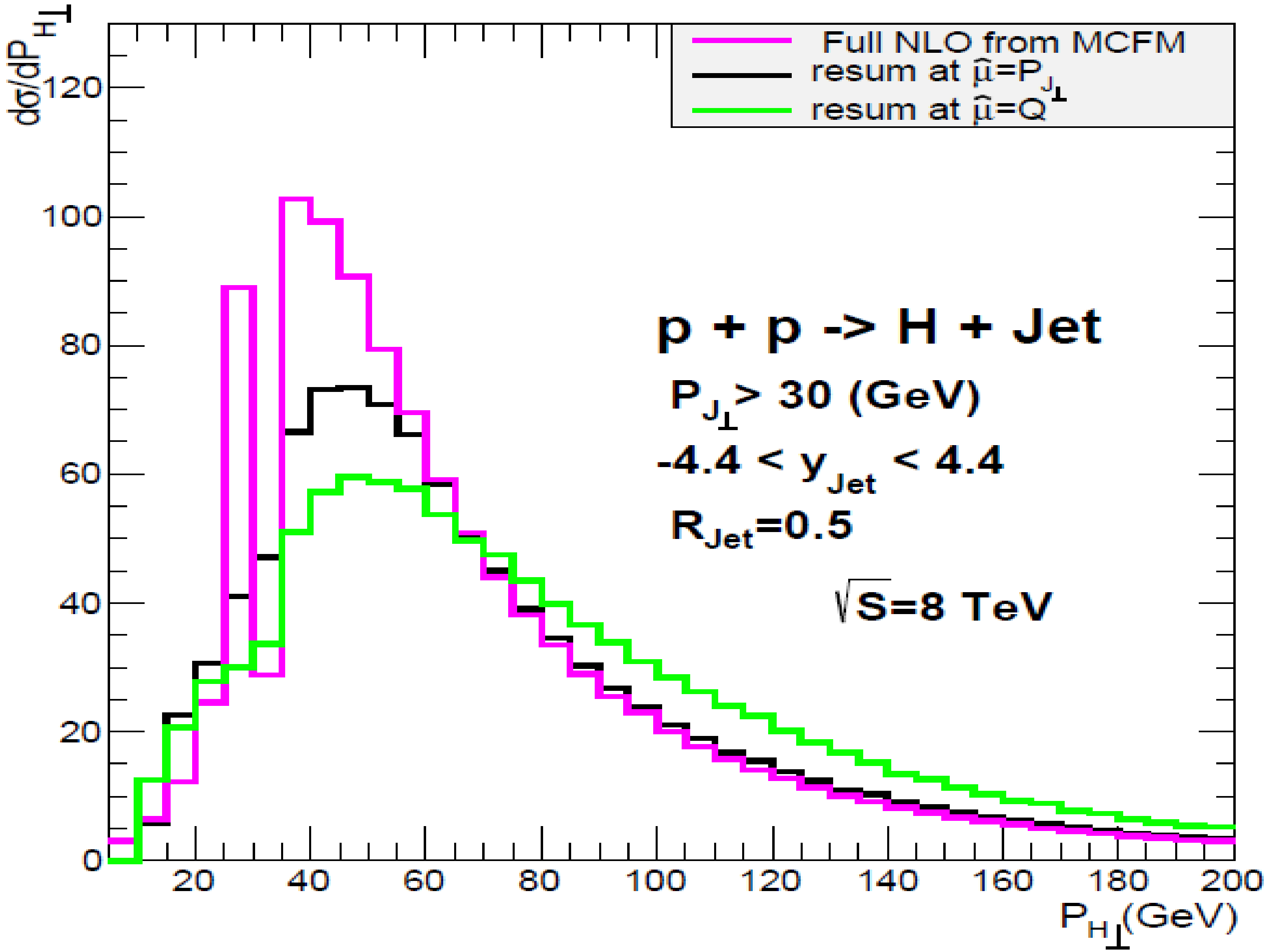
This will effect the system transverse momentum.

This will not

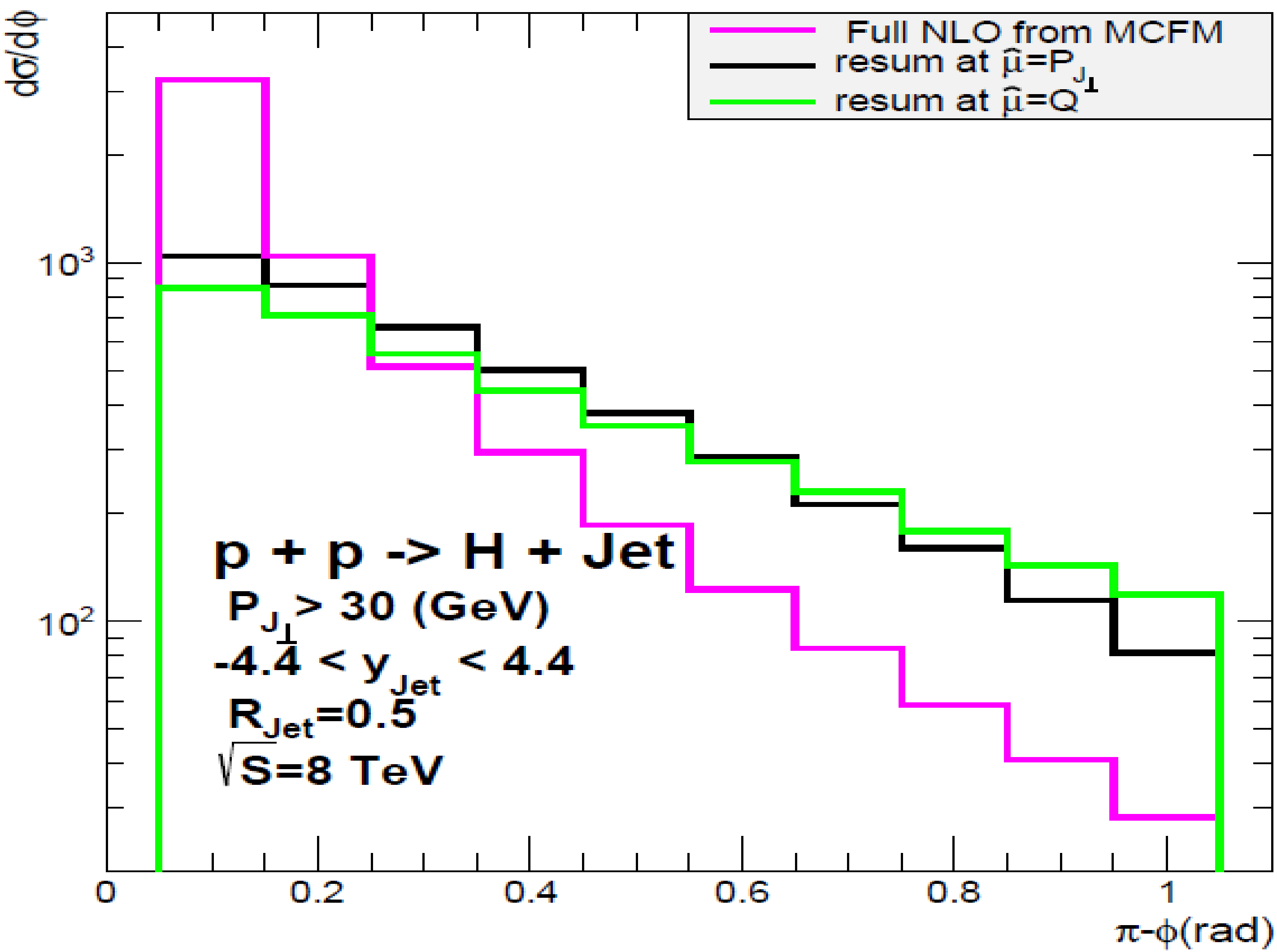
$q_\perp$ distribution of Higgs plus leading jet system



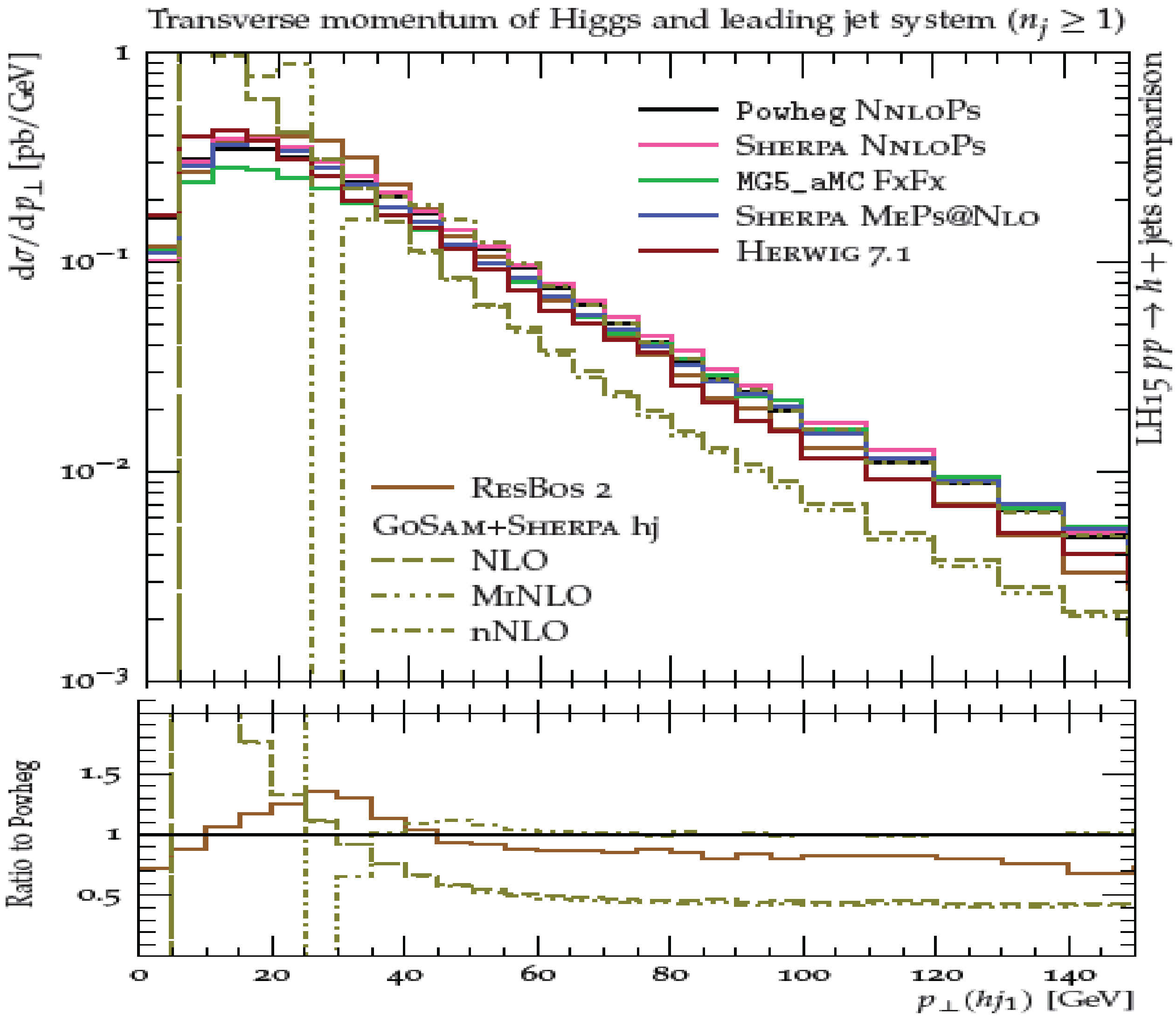
Higgs $P_\perp$ distribution



distribution of the azimuthal angle between Higgs and leading jet

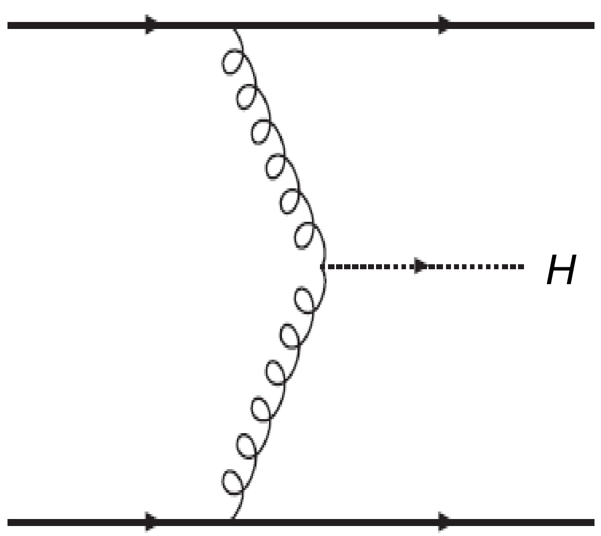


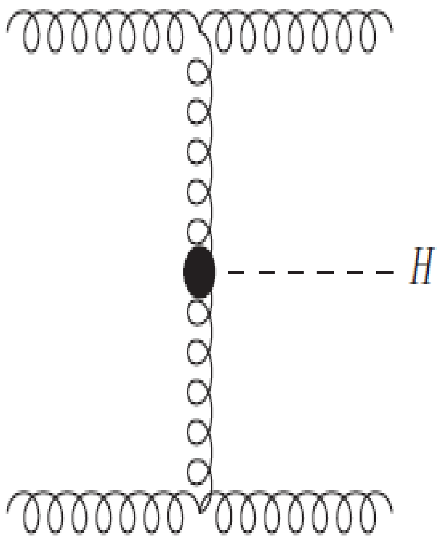
Comparison to MC generators and Fixed Order

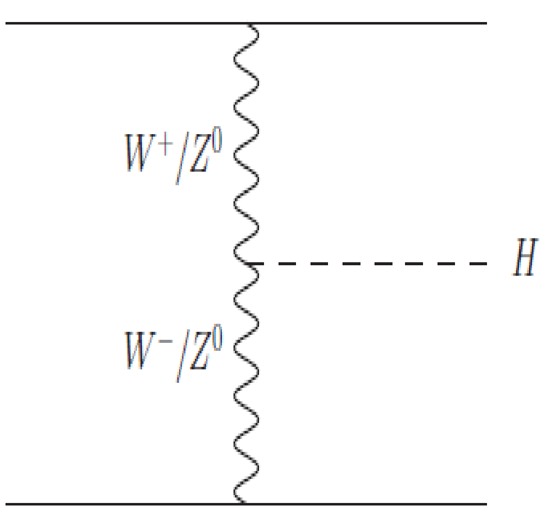


Higgs plus two jets production in pp collisions at large Δy_{jj} region

- The dominant contributions at tree level







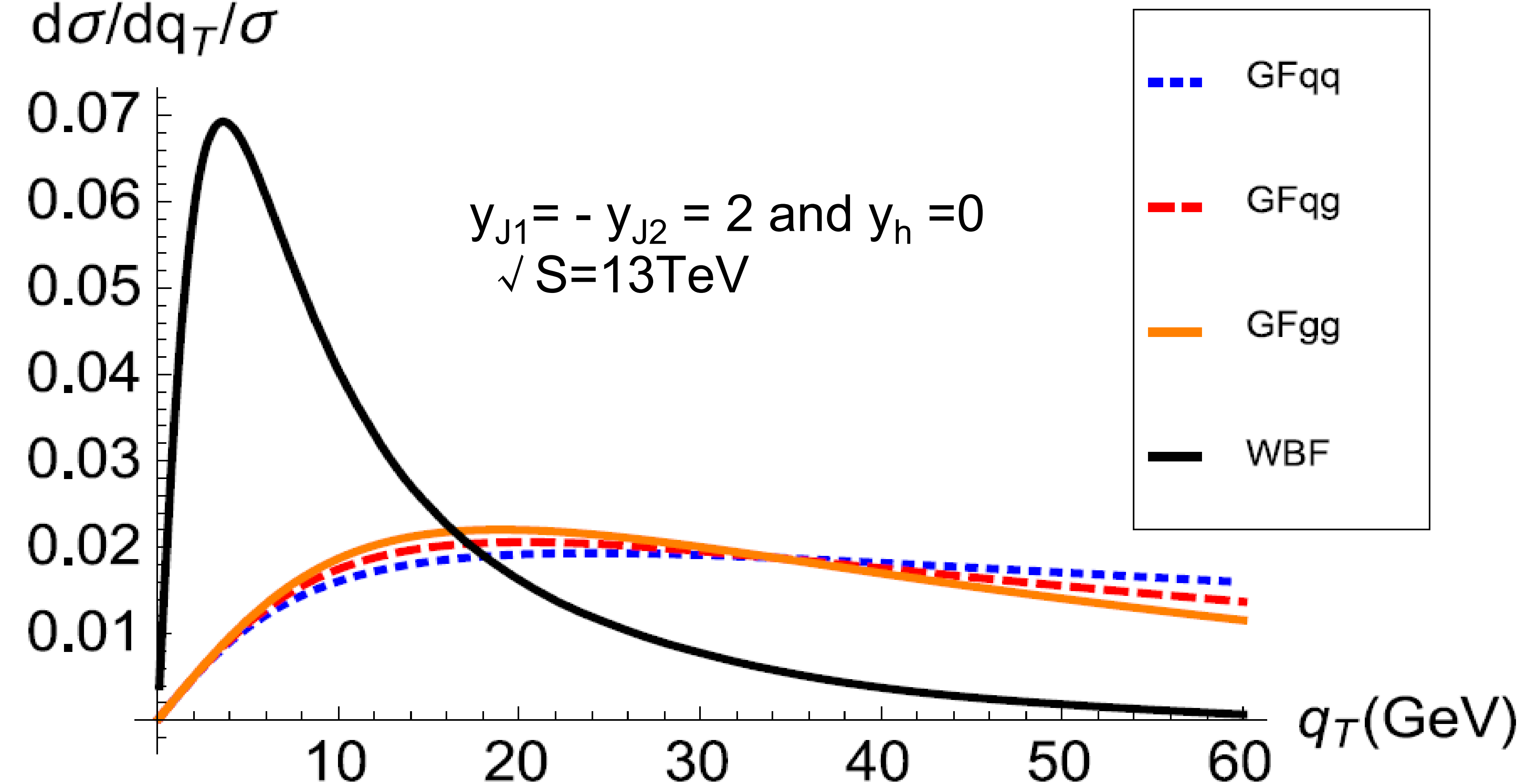
Sudakov factors

$$S_a(\hat{\mu}, b_\perp) = \int_{\mu_b^2}^{\hat{\mu}^2} \frac{d\mu^2}{\mu^2} \left[\ln\left(\frac{s}{\mu^2}\right) A_a + B_a + D_a \ln \frac{1}{R^2} + \gamma_a'^s \right]$$

- Where A and B coefficients are the same as Drell-Yan or Higgs plus 0 jet production.
- The coefficient D is decided by color structure of jet.

$$\gamma'^s_{qWBF} = -C_F \ln \frac{u_1}{t_1}, \quad \gamma'^s_{qGF} = (C_A - C_F) \ln \frac{u_1}{t_1}, \quad \gamma'^s_{gGF} = 0$$

- In the large Δy_{jj} region, $u_1 \gg t_1$



- q_T is the total transverse momentum of Higgs plus two leading jets

Summary

- The QCD resummation can give us a precise prediction of the SM
- The Sudakov factor knows about color structure and kinematics information of initial and final state particles.
- Such properties can help us to search for new physics signal

Thank you very much!

- Then, we consider the process:

$$H_1(P_1) + H_2(P_2) \to c(k_1) + \bar{c}(k_2) + X$$

The W term will became

$$W_{kl}(x_i, b) = x_1 f_l(x_1, b, \xi_1^2, \mu^2, \rho) x_2 f_k(x_2, b, \xi_2^2, \mu^2, \rho) \mathrm{Tr}\left[\mathbf{H}(Q^2, \mu^2, \rho) \mathbf{S}(b, \mu^2, \rho)\right]$$

The Hard and soft part have to be expanded by a group of color basis

For the channel $q(i) + \bar{q}(j) \to t(k) + \bar{t}(l)$, we adopt the bases

$$C_1(i,j,k,l) = \delta_{ij}\delta_{kl}, \qquad C_2(i,j,k,l) = T^d_{ij} T^d_{kl} .$$

While for $g(a) + g(b) \to t(k) + \bar{t}(l)$, we have the bases

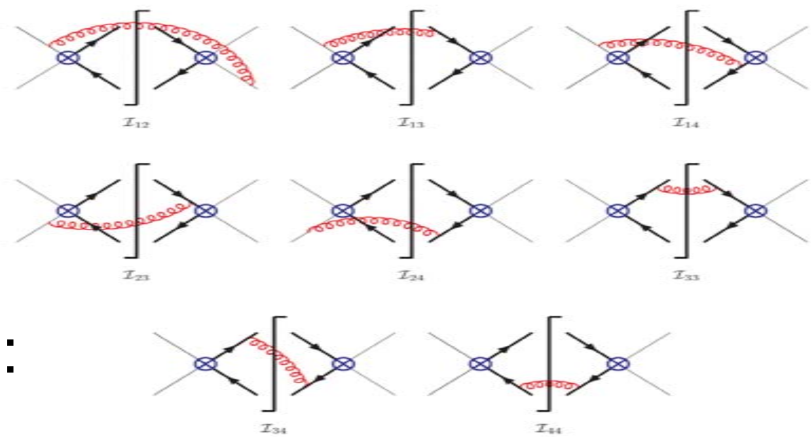
$$C_1(a,b,k,l) = \delta^{ab}\delta_{kl}, \qquad C_2(a,b,k,l) = i f^{abd} T^d_{kl}, \qquad C_3(a,b,k,l) = d^{abd} T^d_{kl}$$

The soft factor’s definition

We can definite the soft factor as:

$$S_{IJ} = \int_0^{\pi} \frac{(\sin\phi)^{-2\epsilon}}{\frac{\sqrt{\pi}\Gamma(\frac{1}{2}-\epsilon)}{\Gamma(1-\epsilon)}} d\phi \, C^{bb'}_{Iii'} C^{aa'}_{Jll'} \langle 0|\mathcal{L}^{\dagger}_{vcb'}\mathcal{L}_{vbc}\mathcal{L}^{\dagger}_{\bar{v}ca'}\mathcal{L}_{\bar{v}ac}\mathcal{L}^{\dagger}_{nji}\mathcal{L}_{ni'k}\mathcal{L}^{\dagger}_{nkl}\mathcal{L}_{nl'j}|0\rangle$$

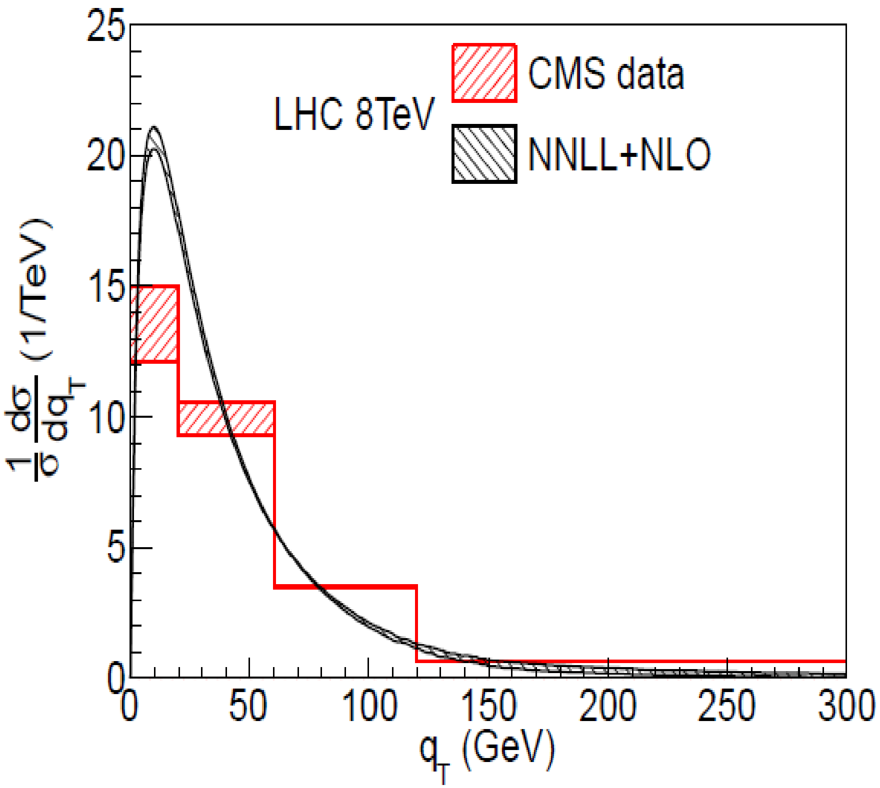
the evolution equation:

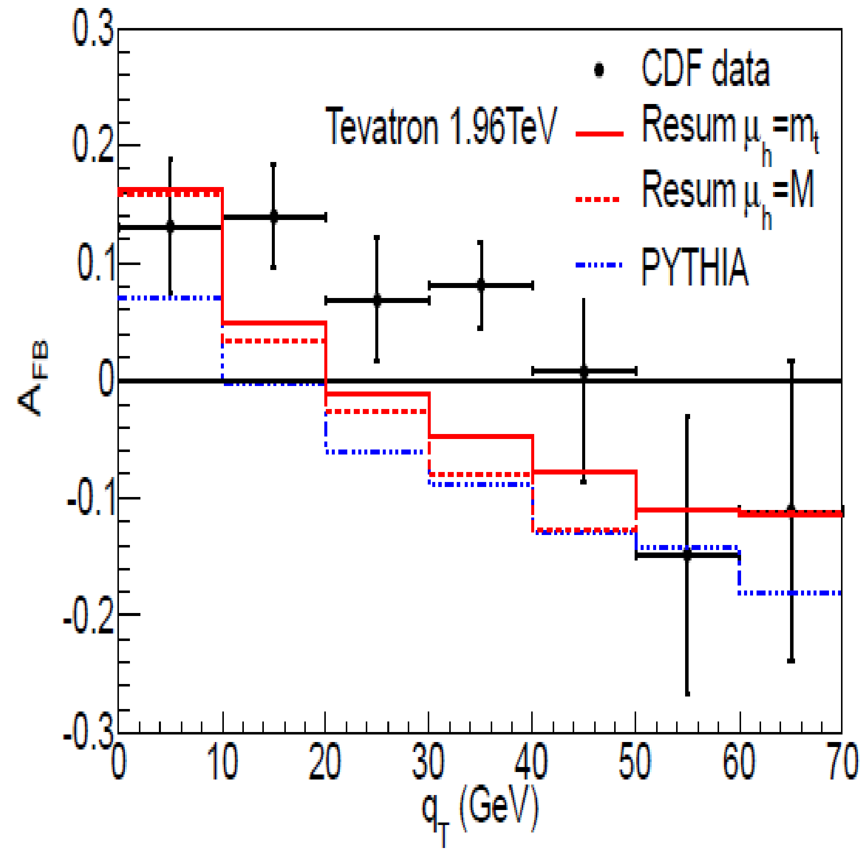


$$\frac{d}{d\ln\mu} S_{i\bar{i}}(\mu) = -\gamma^{s\dagger}_{i\bar{i}} S_{i\bar{i}}(\mu) - S_{i\bar{i}}(\mu)\gamma^{s}_{i\bar{i}}$$

Then you can get the W funtion:

$$W_{kl}\left(x_i, b_\perp, \frac{C_1^2}{C_2^2 b_\perp^2}\right) = f_k(x_A, C_1^2/(C_2^2/b_\perp^2)) f_l(x_B, C_1^2/(C_2^2/b_\perp^2))$$
$$\times Tr\left[\mathbf{H}(M^2_{c\bar{c}}, M^2_{c\bar{c}})\mathrm{EXP}\{-\int_{C_1^2/b_\perp^2}^{M^2_{c\bar{c}}} \frac{d\mu}{\mu}\gamma^{s\dagger}_{i\bar{i}}\}\mathbf{S}(b, \frac{C_1^2}{C_2^2 b_\perp^2})\mathrm{EXP}\{-\int_{C_1^2/b_\perp^2}^{M^2_{c\bar{c}}} \frac{d\mu}{\mu}\gamma^{s}_{i\bar{i}}\}\right]$$





These two pictures come from the paper:
C. S. Li et al Phys.Rev. D88 (2013) 074004
Where they used the SCET.
Our analytic result is consistent with theirs.

$$A_{FB}(q_T) = \frac{\sigma_F(q_T) - \sigma_B(q_T)}{\sigma_F(q_T) + \sigma_B(q_T)}$$

$$\sigma_F(q_T) = \int_0^1 d\cos\theta \, \frac{d^2\sigma}{d\cos\theta dq_T}$$

$$\sigma_B(q_T) = \int_{-1}^0 d\cos\theta \, \frac{d^2\sigma}{d\cos\theta dq_T}$$

Soft Gluon Resummations in Dijet Azimuthal Angular Correlations at the Collider

Peng Sun,[1] C.-P. Yuan,[2] and Feng Yuan[1]

Abstract

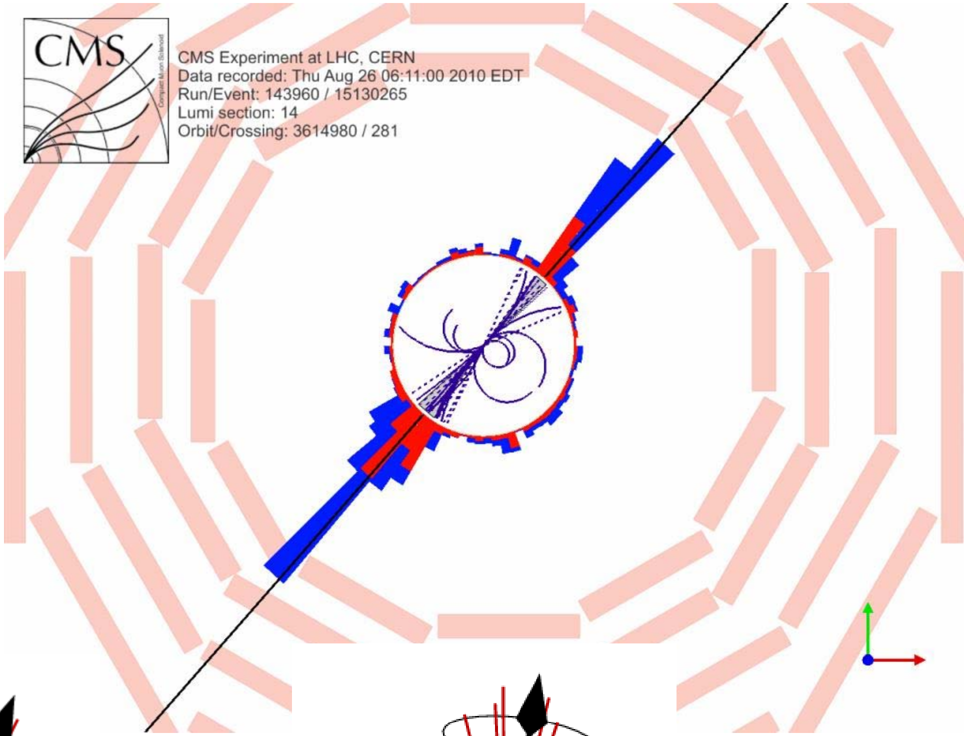
We derive the all order soft gluon resummation in dijet azimuthal angular correlation in pp collisions at the next-to-leading logarithmic level. The relevant coefficients for the resummation Sudakov factor, and the soft and hard factors are calculated. The theory predictions agree well with the experimental data from D0 collaboration at the Tevatron.

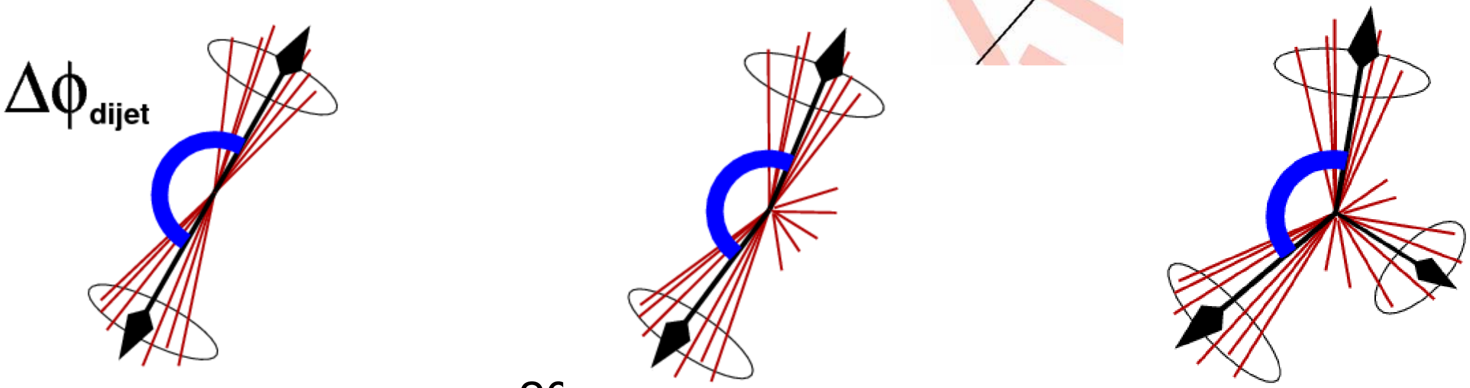
Motivations:

reummation of large logs in dijet production
factorization breaking effects
Collins-Qiu, 2007; Vogelsang-Yuan, 2007; Rogers-Mulders 2010

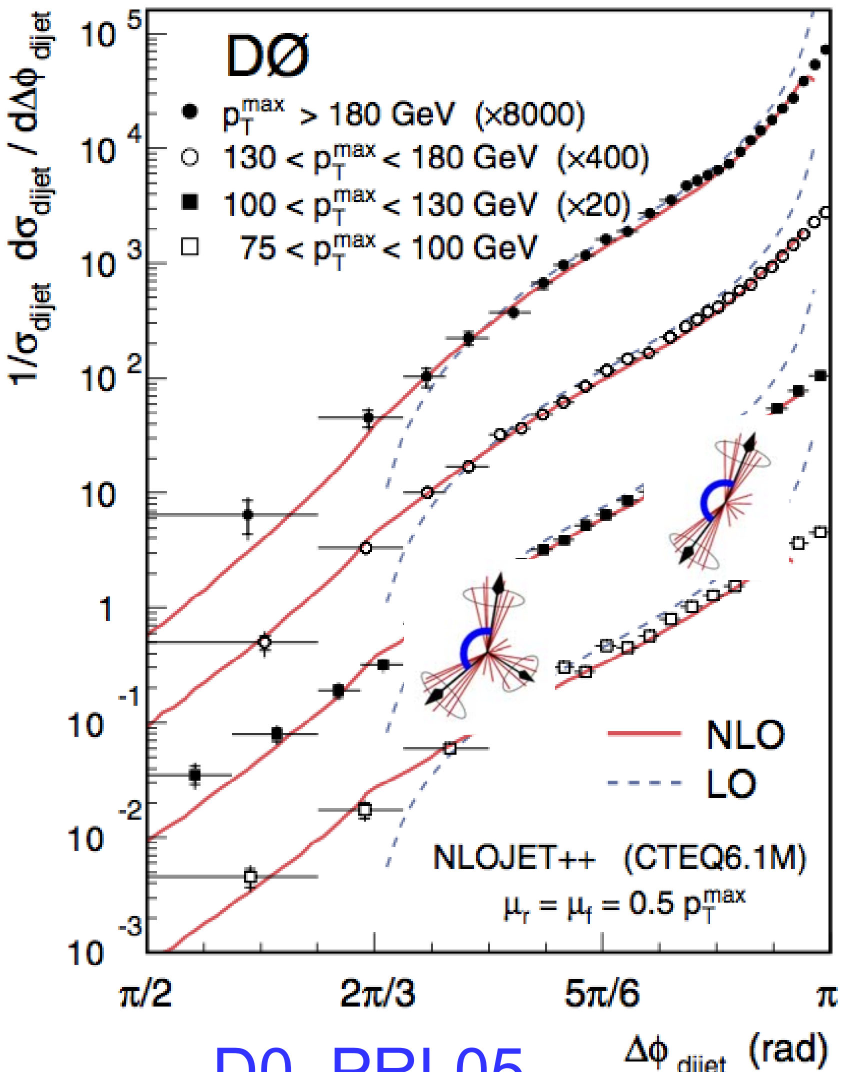
Dijet production at the hadron colliders

- Most abundant events
- Almost back-to-back
- De-correlation comes
    - Hard gluon jet
    - Soft gluon radiation



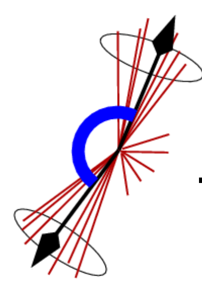


QCD calculations



D0, PRL05

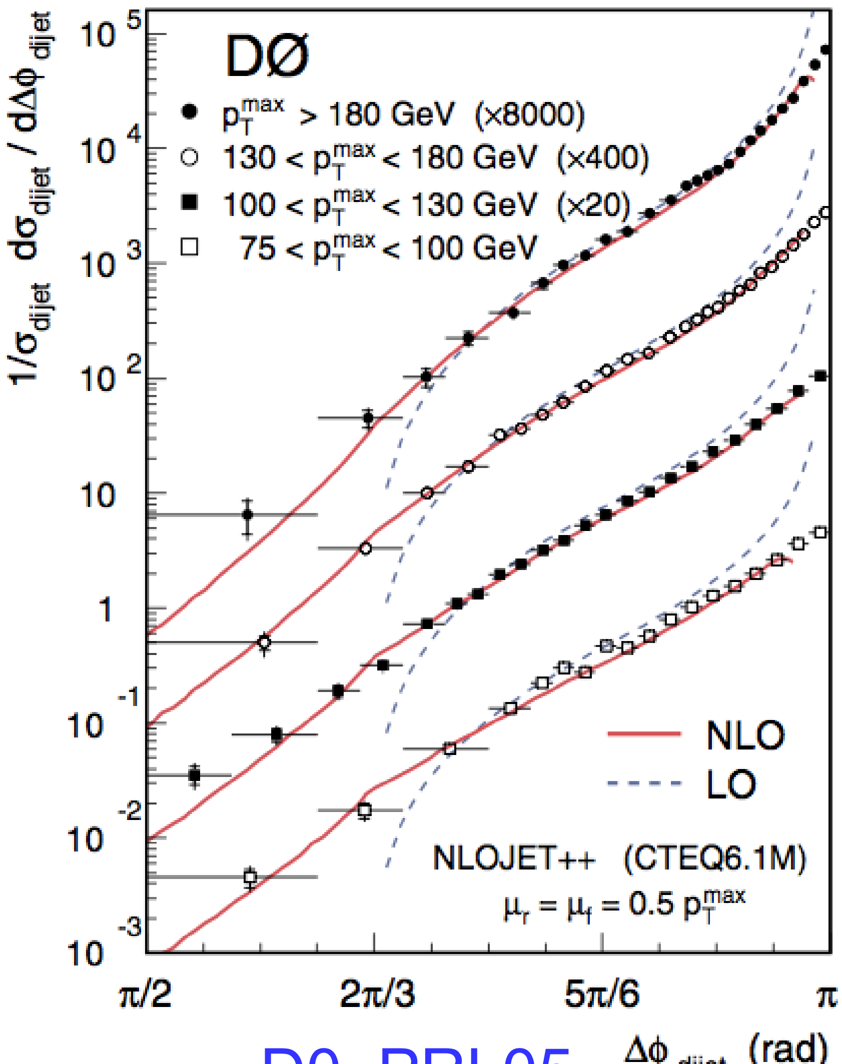
- Fixed order calculations divergent around π , where soft gluon radiation dominate
- All order resummation is needed to understand the physics around here
  - Two separate scales $P_T >> q_T$



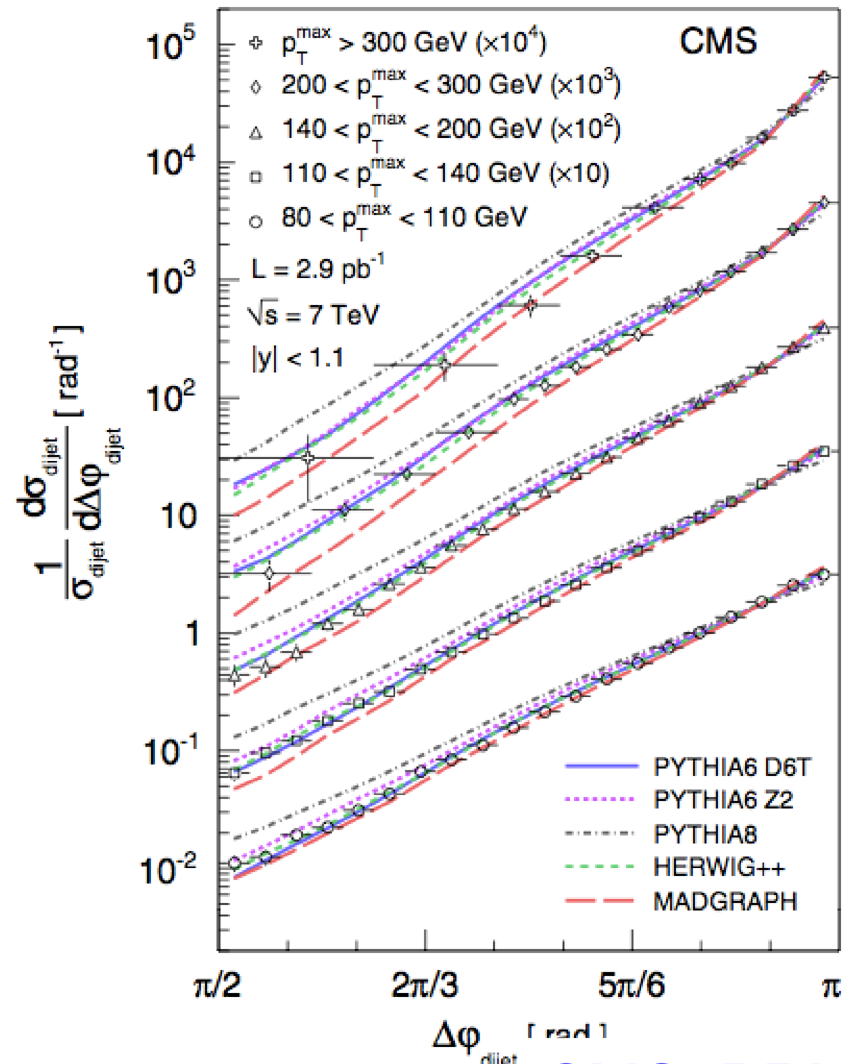
Leading P_T

Total $q_T \approx P_T \mathrm{Sin}(\pi - \Delta\phi)$

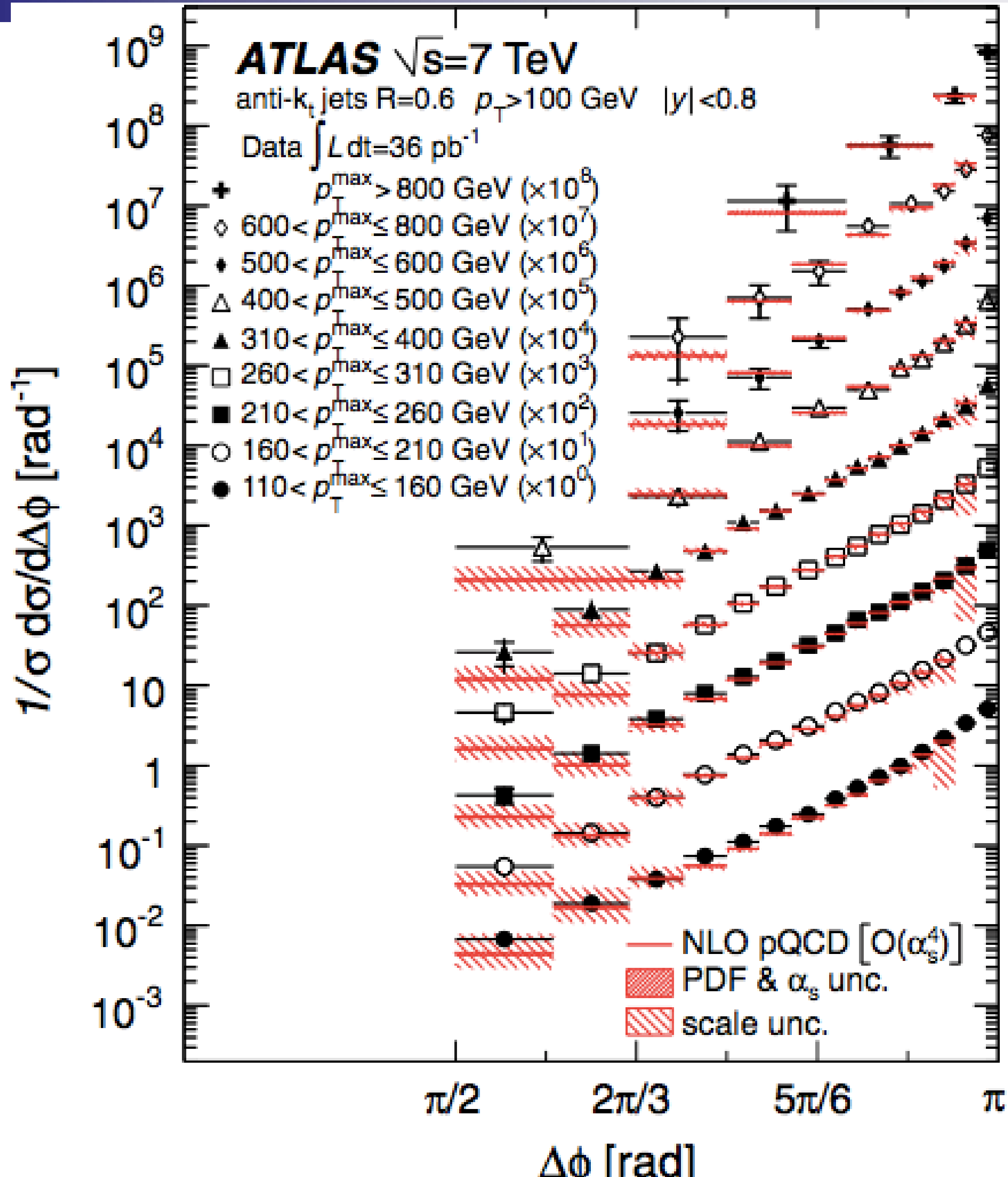
Beautiful data from Tevatron/LHC



D0, PRL05



CMS, PRL11



ATLAS $\sqrt{s}$=7 TeV
anti-k_t jets R=0.6 p_T>100 GeV |y|<0.8
Data $\int L\,dt$=36 pb^{-1}
p_T^{max}> 800 GeV (×10^8)
600< p_T^{max}≤ 800 GeV (×10^7)
500< p_T^{max}≤ 600 GeV (×10^6)
400< p_T^{max}≤ 500 GeV (×10^5)
310< p_T^{max}≤ 400 GeV (×10^4)
260< p_T^{max}≤ 310 GeV (×10^3)
210< p_T^{max}≤ 260 GeV (×10^2)
160< p_T^{max}≤ 210 GeV (×10^1)
110< p_T^{max}≤ 160 GeV (×10^0)
NLO pQCD [O(α_s^4)]
PDF & α_s unc.
scale unc.
1/σ dσ/dΔφ [rad^{-1}]
Δφ [rad]
π/2
2π/3
5π/6
π

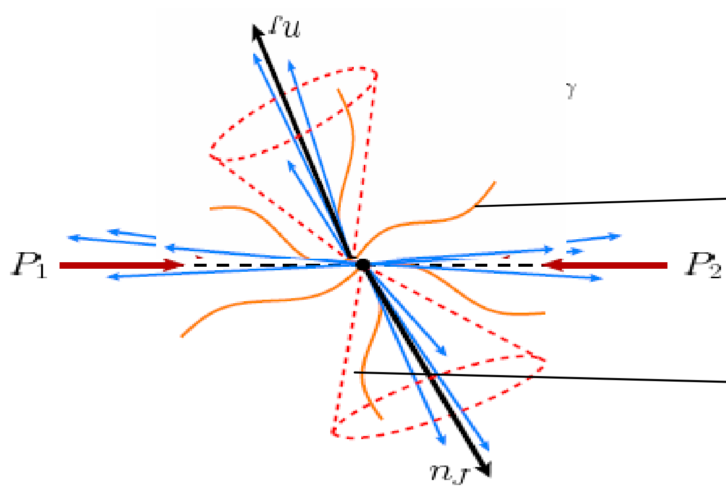
ATLAS, PRL11

There three kinds of large logarithms in the processes:
$(\mathrm{Log}(q_\perp/P_J))^2$, $\mathrm{Log}(q_\perp/P_J)$ and $\mathrm{Log}(R)\mathrm{Log}(q_\perp/P_J)$

$$\frac{d^4\sigma}{dy_1 dy_2 dP_J^2 d^2q_\perp} = \sum_{ab} \sigma_0 \left[\int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} W_{ab\to cd}(x_1, x_2, b_\perp) + Y_{ab\to cd} \right]$$

where

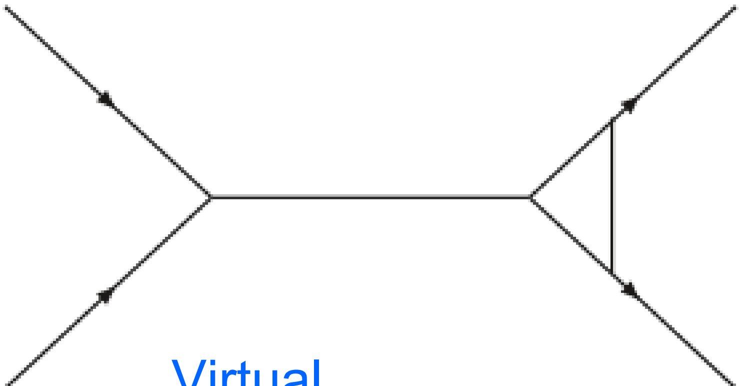
$$W_{ab\to cd}(x_i, b) = x_1 f_a(x_1, b, \zeta^2, \mu^2, \rho) x_2 f_b(x_2, b, \bar{\zeta}^2, \mu^2, \rho) \mathrm{Tr}\left[\mathbf{H}_{ab\to cd}(Q^2, \mu^2, \rho)\mathbf{S}_{ab\to cd}(b, \mu^2, \rho)\right]$$



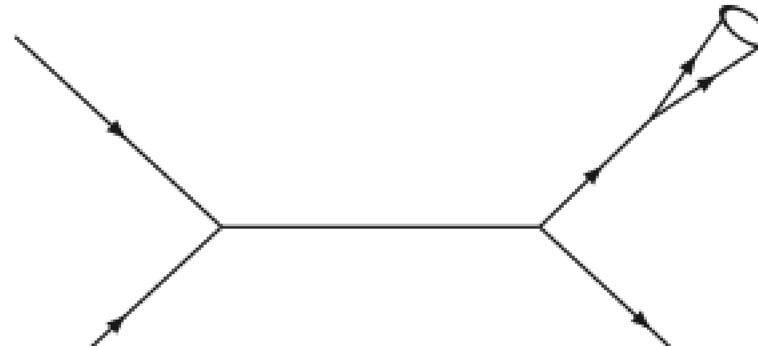
It will contribute to $\Delta\ \phi$ distribution.

It will not

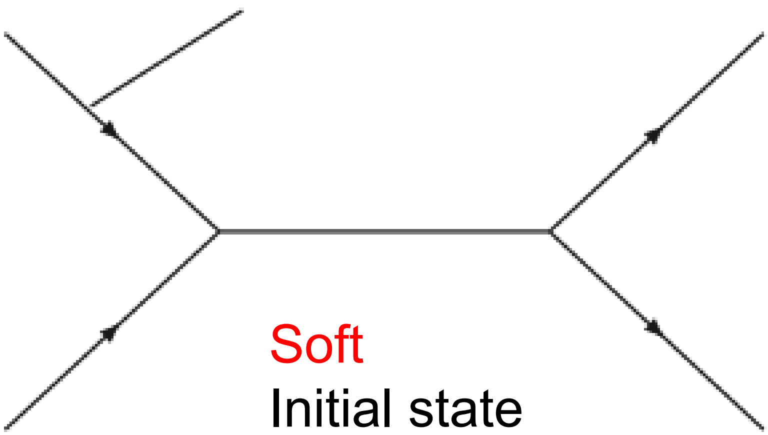
Soft and collinear gluon at one-loop



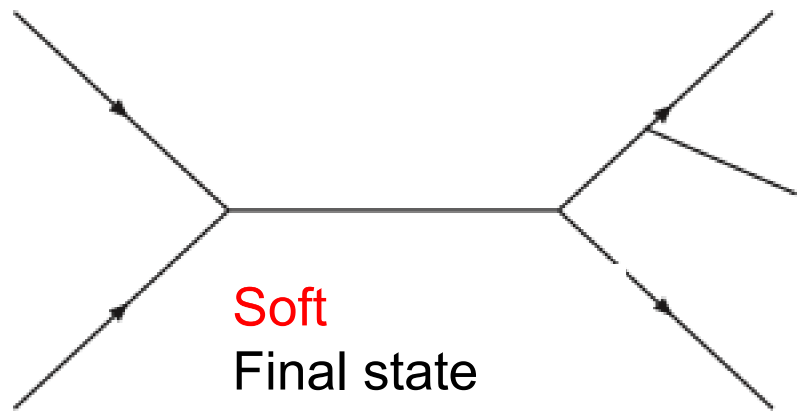
Virtual
Ellis-Sexton 86



Jet (Narrow Jet Approx.)
Jager-Stratmann-Vogelsang
2004



Soft
Initial state



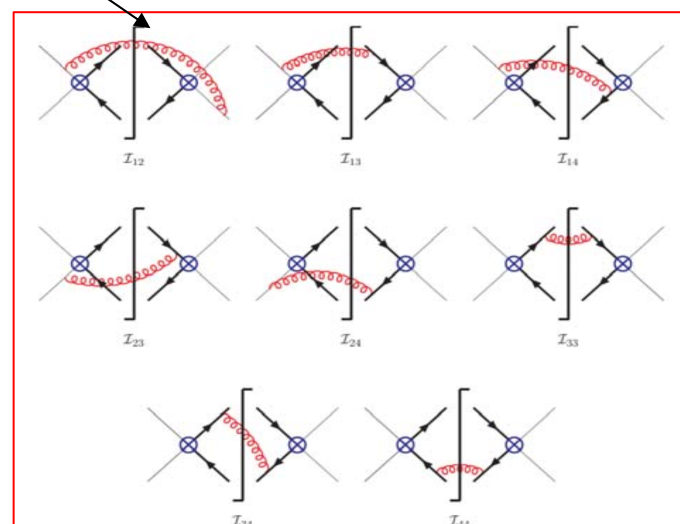
Soft
Final state
(out of jet cone)

$$S_{IJ} = \int_0^\pi \frac{d\phi_0}{\pi} C^{bb'}_{Iii'} \boxed{C^{aa'}_{Jll'}} \langle 0|\mathcal{L}^\dagger_{vcb'}(b_\perp)\mathcal{L}_{vbc}(b_\perp)\mathcal{L}^\dagger_{\bar{v}ca'}(0)\mathcal{L}_{\bar{v}ac}(0)\mathcal{L}^\dagger_{nji}(b_\perp)\mathcal{L}_{\bar{n}i'k}(b_\perp)\mathcal{L}^\dagger_{\bar{n}kl}(0)\mathcal{L}_{nl'j}(0)|0\rangle$$

The soft factor satisfies

$$\frac{d}{d\ln\mu} S_{IJ}(\mu) = -\boxed{\Gamma^{s\dagger}_{IJ'}} S_{J'J}(\mu) - S_{IJ'}(\mu)\Gamma^{s}_{J'J}$$

$$c_1 = f^{a_1a_2c_1} f_{a_3a_4c_1}\,, \quad c_2 = f^{a_1a_3c_1} f_{a_2a_4c_1} + f^{a_1a_4c_1} f_{a_2a_3c_1}\,, \quad c_3 = d^{a_1a_2c_1} f_{a_3a_4c_1}\,,$$
$$c_4 = f^{a_1a_2c_1} d_{a_3a_4c_1}\,, \quad c_5 = d^{a_1a_4c_1} f_{a_2a_3c_1}\,, \quad c_6 = \delta^{a_1a_2}\delta^{a_3a_4}\,, \quad c_7 = \delta^{a_1a_3}\delta^{a_2a_4}\,, \quad c_8 = \delta^{a_1a_4}\delta^{a_2a_3}$$



$$c_1 = \delta^{a_1a_2}\delta_{a_3a_4}\,, \qquad c_2 = i f^{a_1a_2c}\, t^c_{a_3a_4}\,, \qquad c_3 = d^{a_1a_2c}\, t^c_{a_3a_4}$$

Cross checks

- Divergences cancelled out between virtual, jet, sot contributions (dimension regulation applied)
- Final results :double logs, single logs, ..

$$W^{(1)}(b_\perp)|_{logs.} = \frac{\alpha_s}{2\pi}\left\{h^{(0)}_{q_iq_j\to q_iq_j}\left[-\ln\left(\frac{\mu^2 b_\perp^2}{b_0^2}\right)(\mathcal{P}_{qq}(\xi)\delta(1-\xi')+\mathcal{P}_{qq}(\xi')\delta(1-\xi))-\delta(1-\xi)\right.\right.$$

$$\left.\times\delta(1-\xi')\left(\underline{C_F\ln^2\left(\frac{Q^2b_\perp^2}{b_0^2}\right)}+\underline{\ln\left(\frac{Q^2b_\perp^2}{b_0^2}\right)\Big(-3C_F}+C_F\ln\frac{1}{R_1^2}+C_F\ln\frac{1}{R_2^2}\Big)\right)\right]$$

$$\left.-\delta(1-\xi)\delta(1-\xi')\underline{\ln\left(\frac{Q^2b_\perp^2}{b_0^2}\right)\Gamma^{(qq')}_{sn}}\right\}, \tag{71}$$

Quark channel: $q_iq_j \to q_iq_j$

After solving the evolution equations

$$W_{ab\to cd}(x_1,x_2,b) = x_1 f_a(x_1,\mu=c_0/b_\perp)x_2 f_b(x_2,\mu=c_0/b_\perp)e^{-S_{Sud}(Q^2,b_\perp)} \times \mathrm{Tr}\left[\mathbf{H}_{ab\to cd}\exp[-\int_{c_0/b_\perp}^{Q}\frac{d\mu}{\mu}\gamma^{s\dagger}]\mathbf{S}_{ab\to cd}\exp[-\int_{c_0/b_\perp}^{Q}\frac{d\mu}{\mu}\gamma^{s}]\right]$$

where

$$S_{Sud}(Q^2,b_\perp,C_1,C_2) = \int_{C_1^2/b_\perp^2}^{C_2^2Q^2}\frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{Q^2}{\mu^2}\right)A + B + D_1\ln\boxed{\frac{2}{R_1}} + D_2\ln\frac{2}{R_2}\right]$$

For g g → jj $A_{gg}=C_A\, a_s/\pi$ $B_{gg}=-2C_A\,\beta_0\, a_s/\pi$

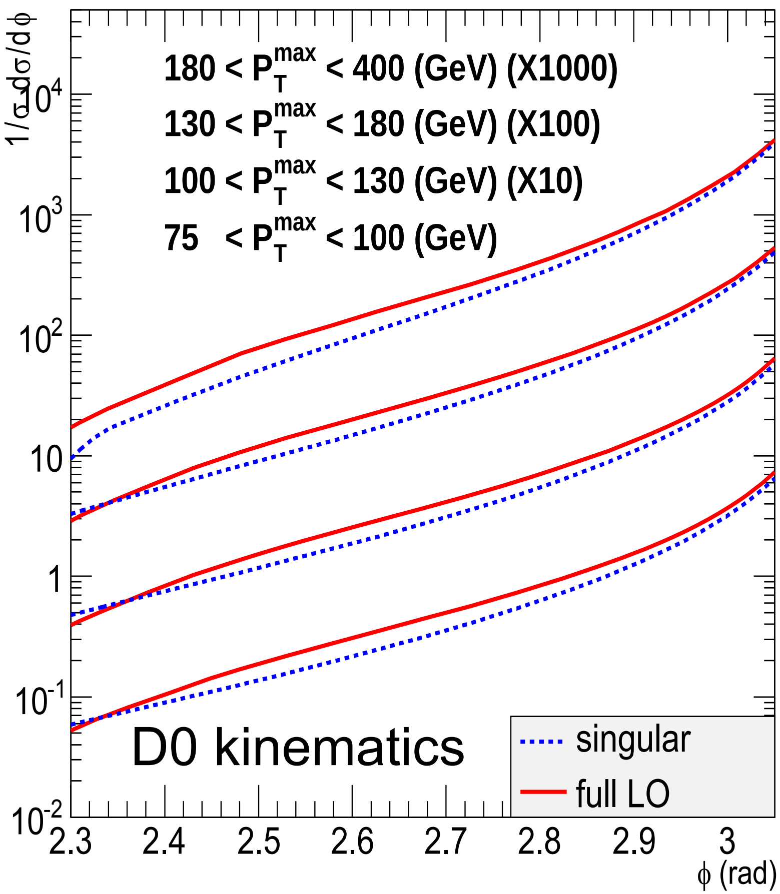
For q q → jj $A_{qq}=C_F\, a_s/\pi$ $B_{qq}=-2C_F/3\; a_s/\pi$

For q g → jj $A_{qg}=(A_{gg}+A_{qq})/2$ $B_{qg}= =(B_{gg}+B_{qq})/2$

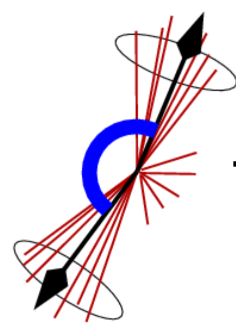
for quark jet $D_i=C_F\, a_s/\pi$

for gluon jet $D_i=C_A\, a_s/\pi$

Compared to full calculations



$$
\frac{\alpha_s}{2\pi^2}\frac{1}{q_\perp^2}\sum_{ab,a'b'}\sigma_0\int\frac{dx_1'}{x_1'}\frac{dx_2'}{x_2'}x_1' f_a(x_1',\mu)x_2' f_b(x_2',\mu)
$$
$$
\times\Big\{h^{(0)}_{a'b'\to cd}\big[\xi_1\mathcal{P}_{a'/a}(\xi_1)\delta(1-\xi_2)+\xi_2\mathcal{P}_{b'/b}(\xi_2)\delta(1-\xi_1)
$$
$$
+\delta(1-\xi_1)\delta(1-\xi_2)\delta_{aa'}\delta_{bb'}\left((C_a+C_b)\ln\frac{Q^2}{q_\perp^2}+C_c\ln\frac{1}{R_1^2}+C_d\ln\frac{1}{R_2^2}\right)\Big]
$$
$$
+\delta(1-\xi_1)\delta(1-\xi_2)\delta_{aa'}\delta_{bb'}\Gamma^{ab\to cd}_{sn}\Big\}\ , \qquad (10
$$

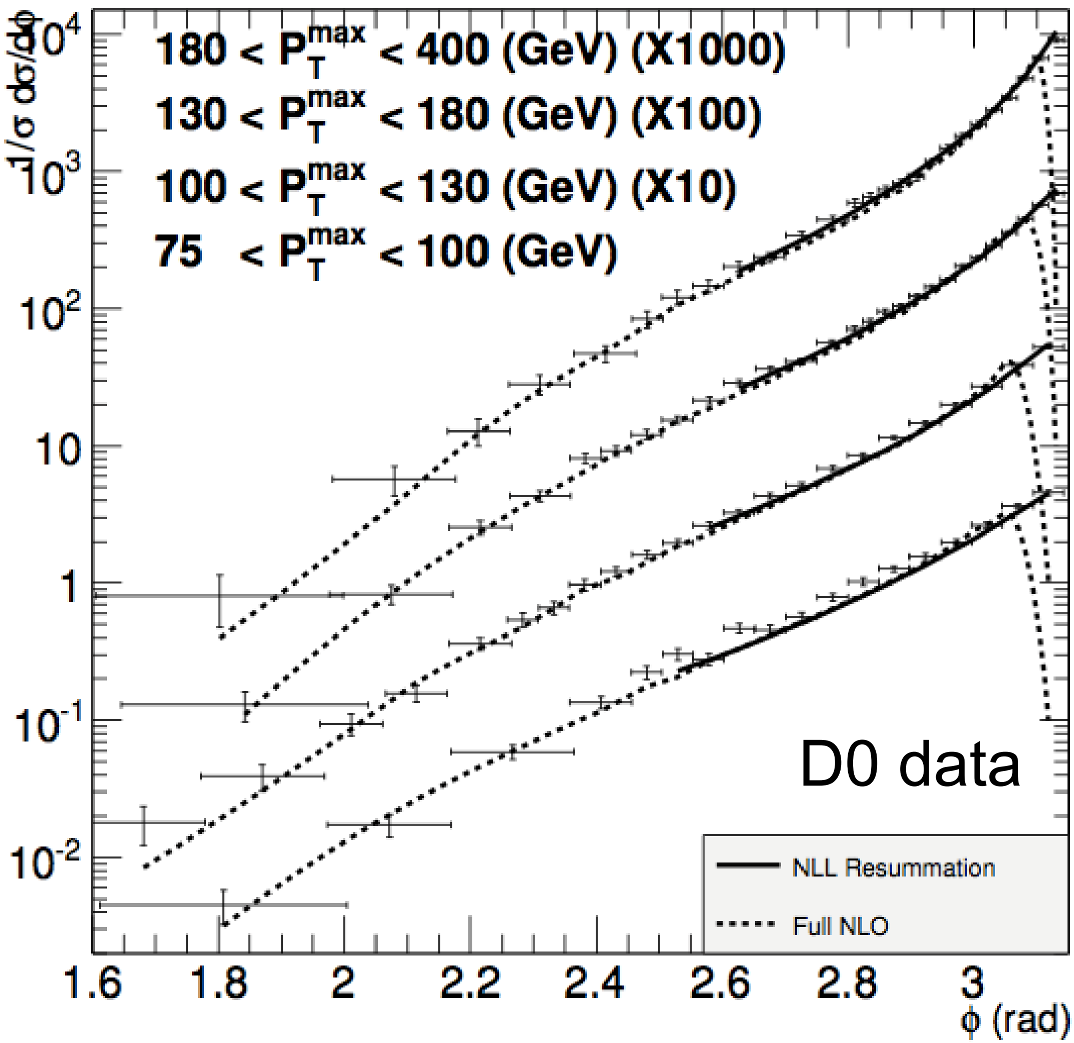


Leading q_T

Total $q_T \approx P_T \mathrm{Sin}(\pi - \phi)$

full LO: Nagy 2002, NLOJET++

Compared to the data



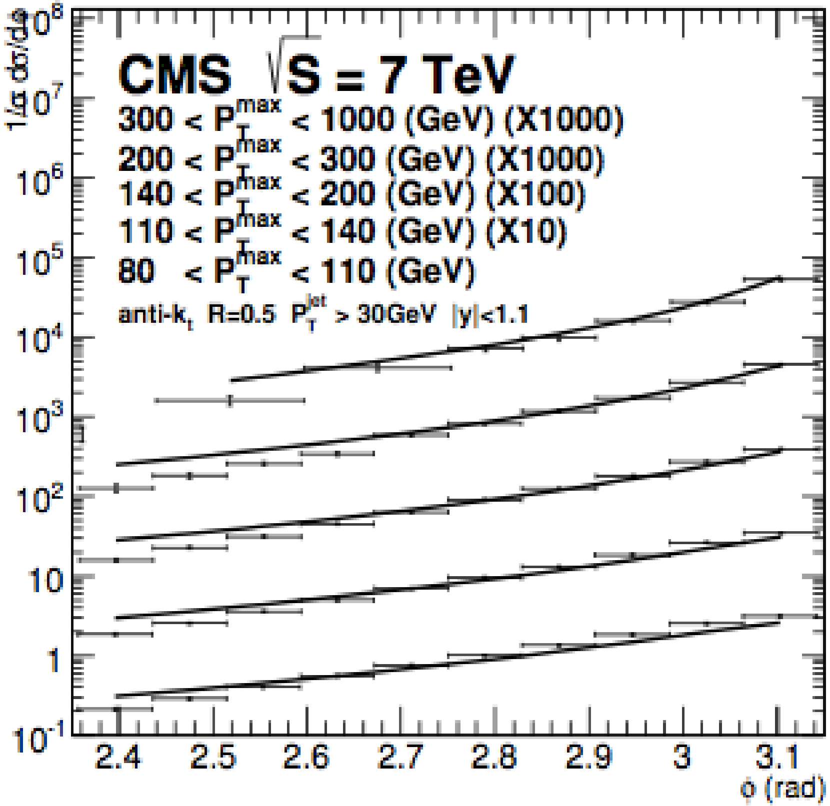
NLL Resummation:
Sun,C.P.Yuan, F.Yuan, PRL2014

$$x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\rm Sud}(Q^2, b_\perp)}$$

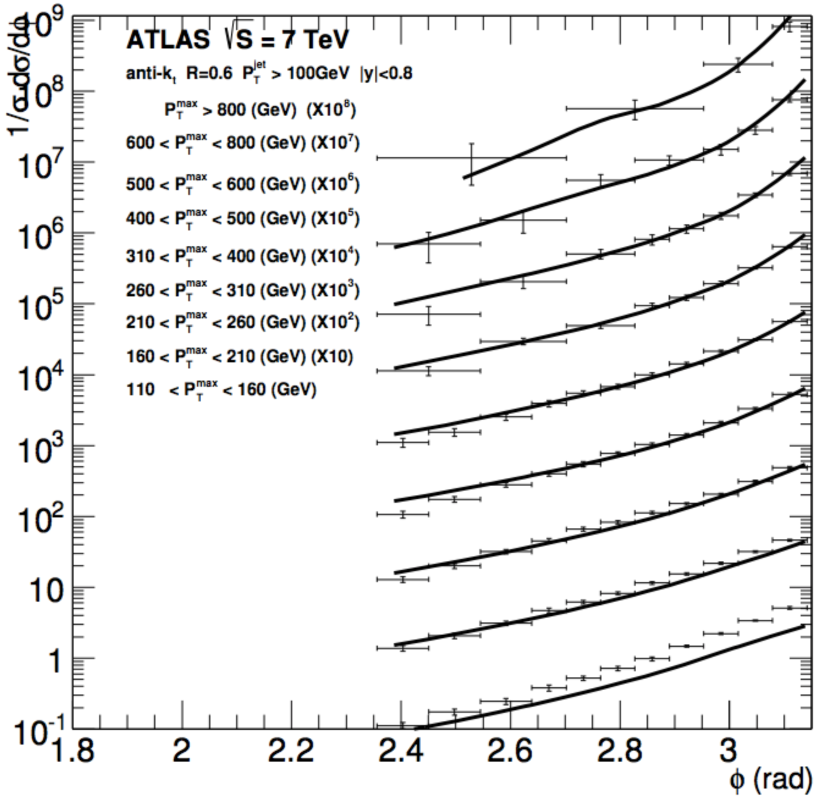
$$\mathrm{Tr}\left[\mathbf{H}_{ab\to cd} \exp[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s\dagger}] \mathbf{S}_{ab\to cd} \exp[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s}]\right]$$

Full NLO: Nagy 2002, NLOJET++

At the LHC

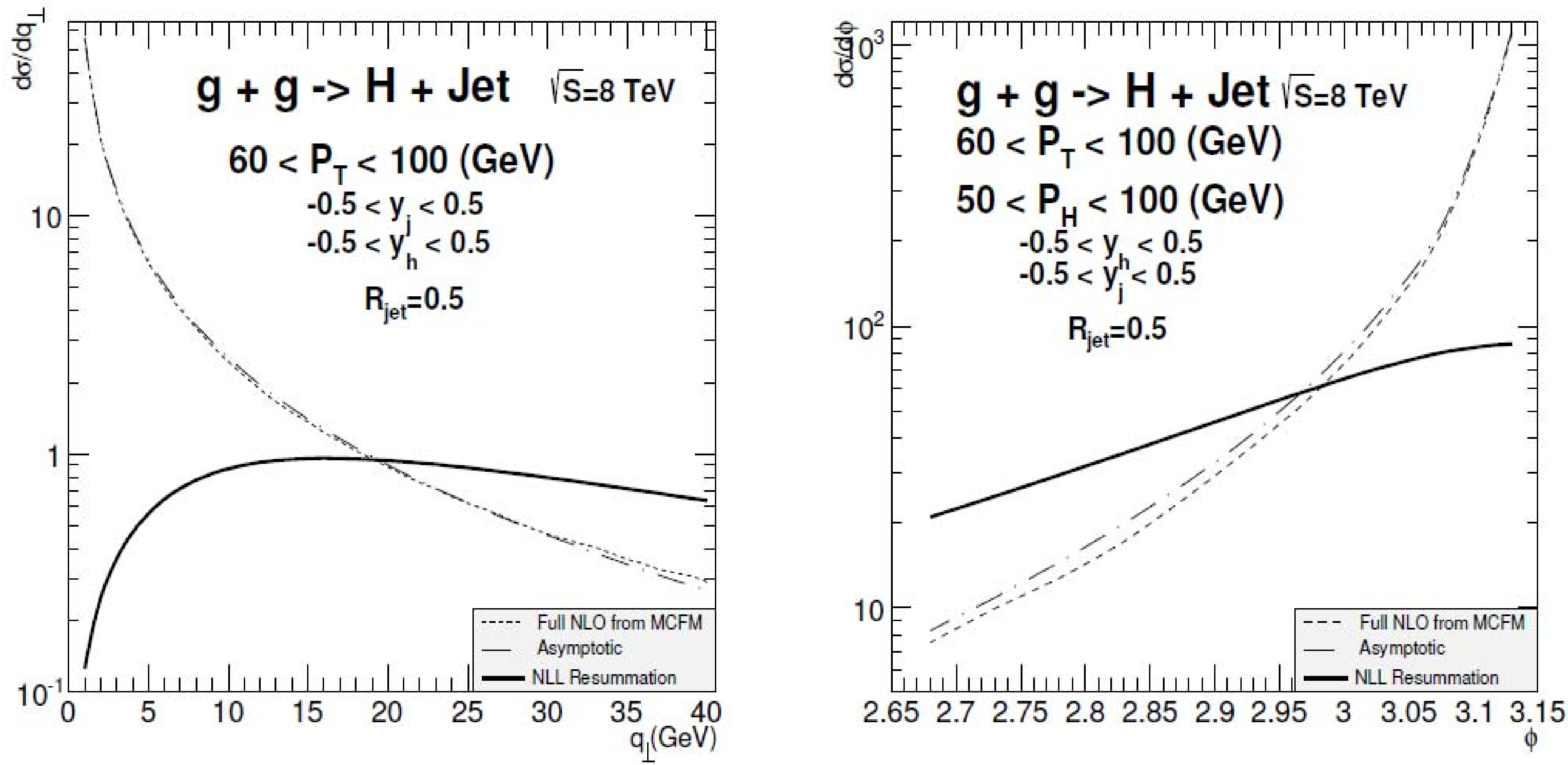


CMS √S = 7 TeV
300 < P_T^max < 1000 (GeV) (X1000)
200 < P_T^max < 300 (GeV) (X1000)
140 < P_T^max < 200 (GeV) (X100)
110 < P_T^max < 140 (GeV) (X10)
80 < P_T^max < 110 (GeV)
anti-k_t R=0.5 P_T^jet > 30GeV |y|<1.1
1/σ dσ/dφ
φ (rad)



ATLAS √S = 7 TeV
anti-k_t R=0.6 P_T^jet > 100GeV |y|<0.8
P_T^max > 800 (GeV) (X10^8)
600 < P_T^max < 800 (GeV) (X10^7)
500 < P_T^max < 600 (GeV) (X10^6)
400 < P_T^max < 500 (GeV) (X10^5)
310 < P_T^max < 400 (GeV) (X10^4)
260 < P_T^max < 310 (GeV) (X10^3)
210 < P_T^max < 260 (GeV) (X10^2)
160 < P_T^max < 210 (GeV) (X10)
110 < P_T^max < 160 (GeV)
1/σ dσ/dφ
φ (rad)

Higgs + jet production in pp collision

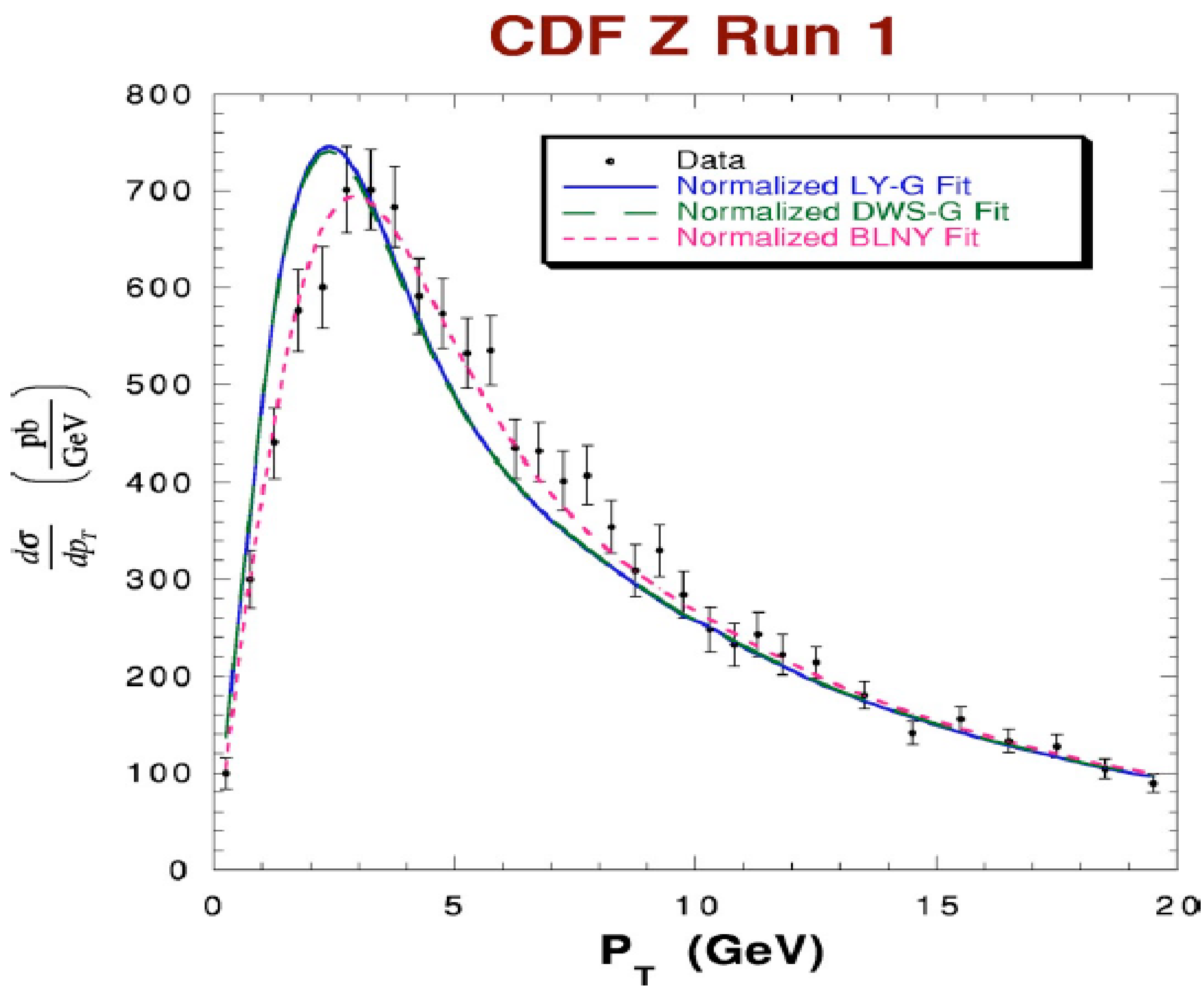


- Higgs+Jet, Sun, C.-P. Yuan, F. Yuan, Phys.Rev.Lett. 114 (2015) 202001

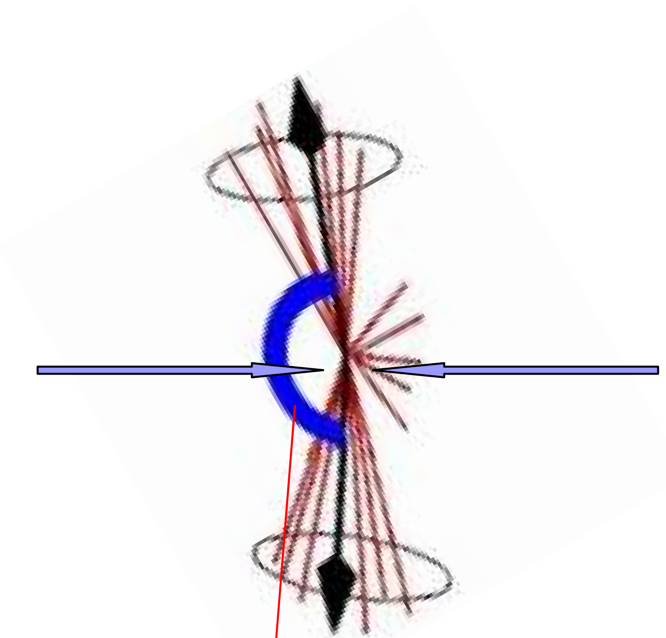
Summary

- Soft gluon resummation for dijet correlation at the next-to-leading logarithmic order agrees well with the experimental data.
- Extending to EW boson plus jet production will be interesting to follow

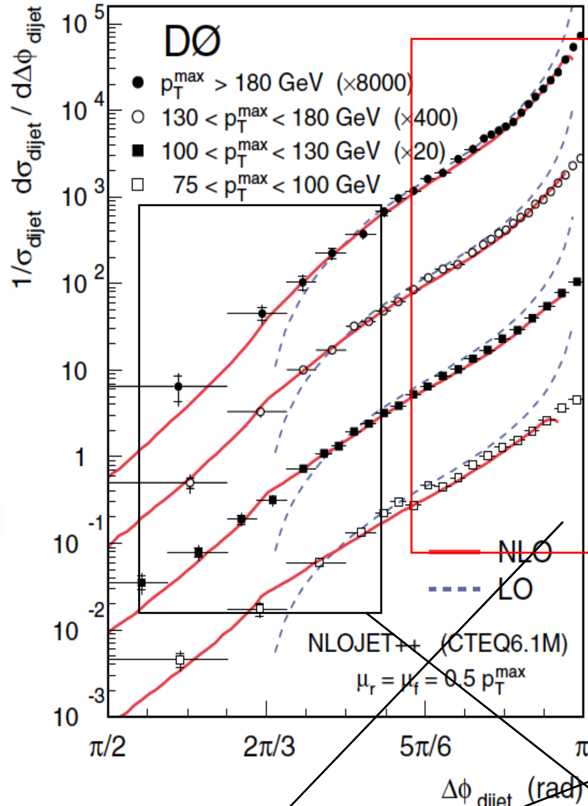
Thank you very much!

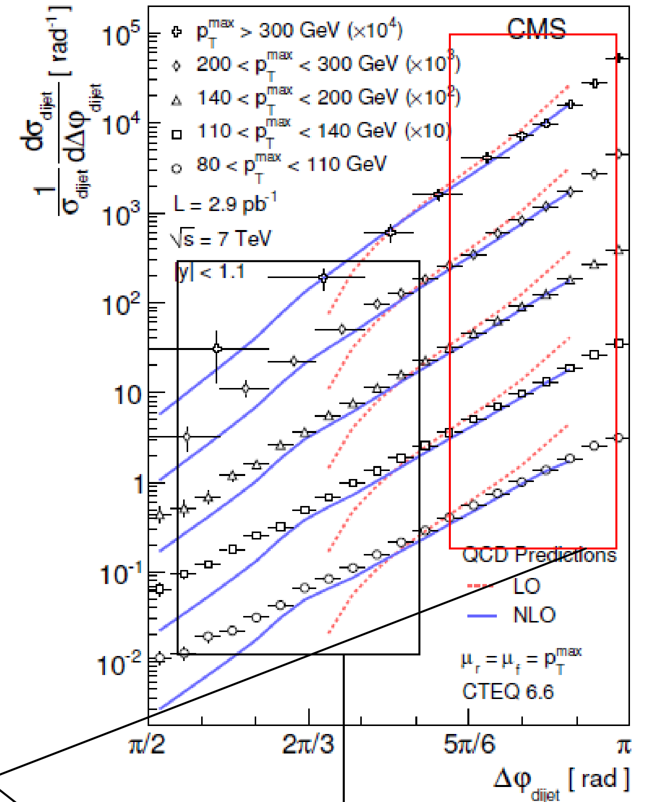


C. P. Yuan et al PRD 67, 073016

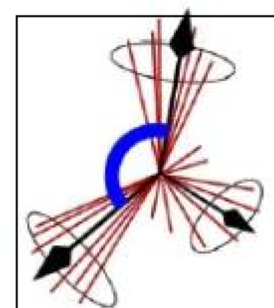


$\Delta\ \phi$

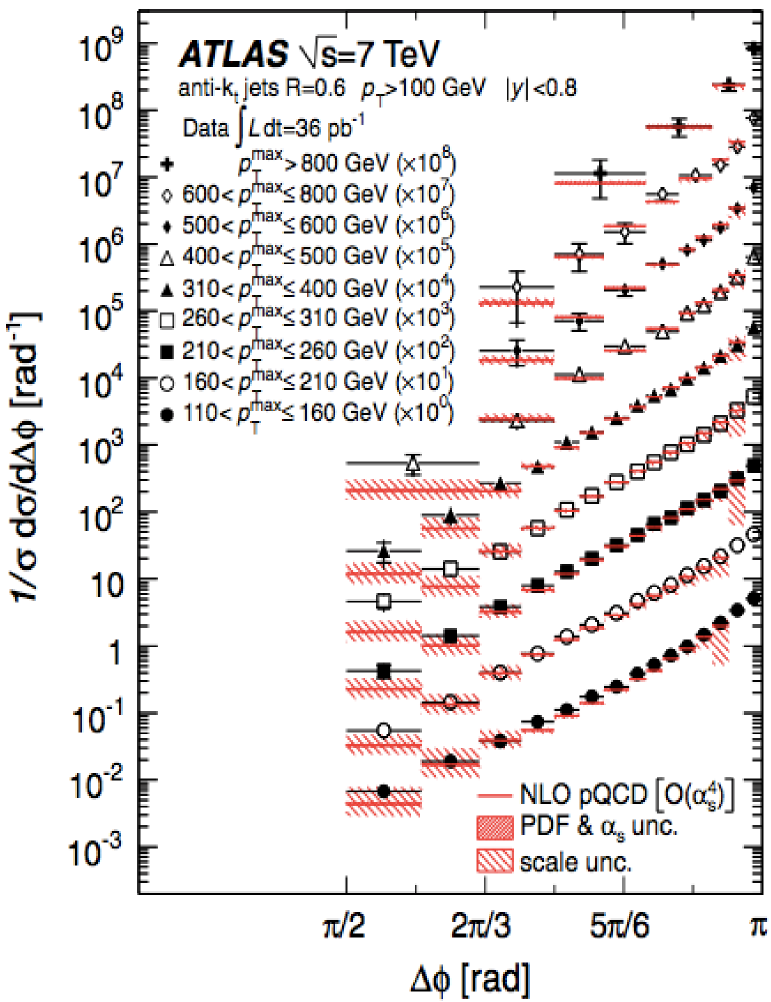


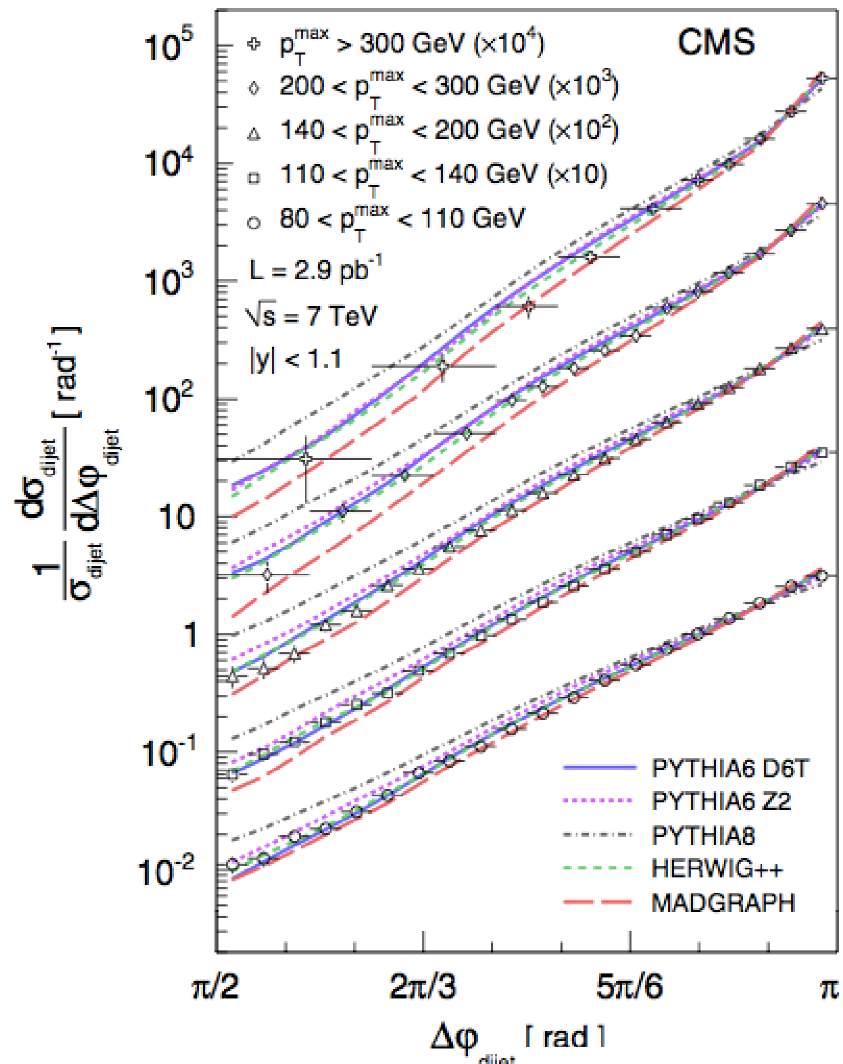


Here cross section is dominated by processes with only two jets.
The azimuthal angular distribution comes form soft gluon radiation.
QCD resummation is needed.



Beautiful data from LHC

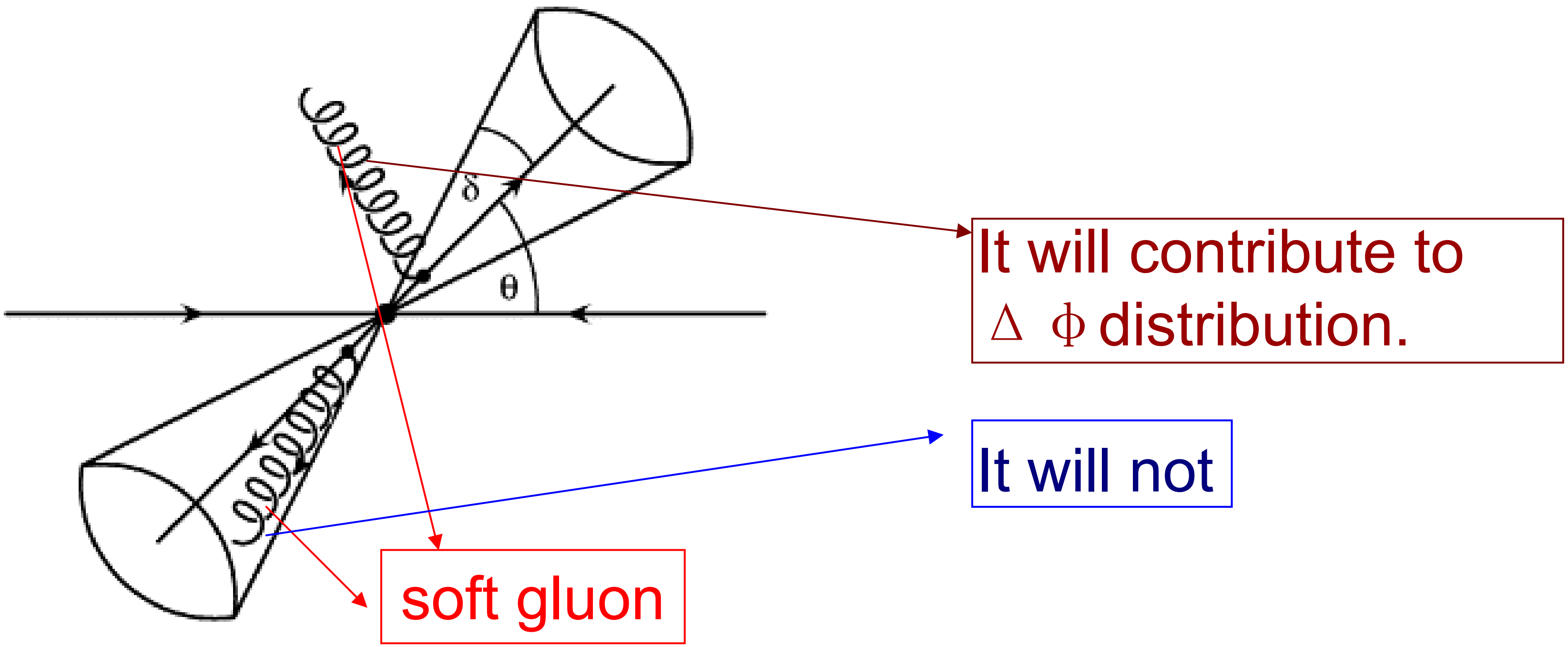




D0, PRL05

2016/6/9

CMS, PRL11



δ = R sin(θ), the R is the jet cone size, and its definition is:

$$R = (\Delta\phi^2 + \Delta y^2)^{1/2}$$

We take the small cone assumption. So R and δ are small value.

So, you have to cut off the contribution form soft gluon in the jet cone. There are two ways to do this.

A

$$n_j = (1, 0\ , \ 0\ \ , 1\ \ ) \rightarrow n_j^2\ \ = 0$$

$$n_g = (\ 1, 0\ , \sin(x)\ , \cos(x)\ \ )\ \text{ with }\ x > \delta$$

$$n_j\ .\ n_g\ \ =\ 1 - \cos(x) > \ \delta^{\,2}/2$$

B

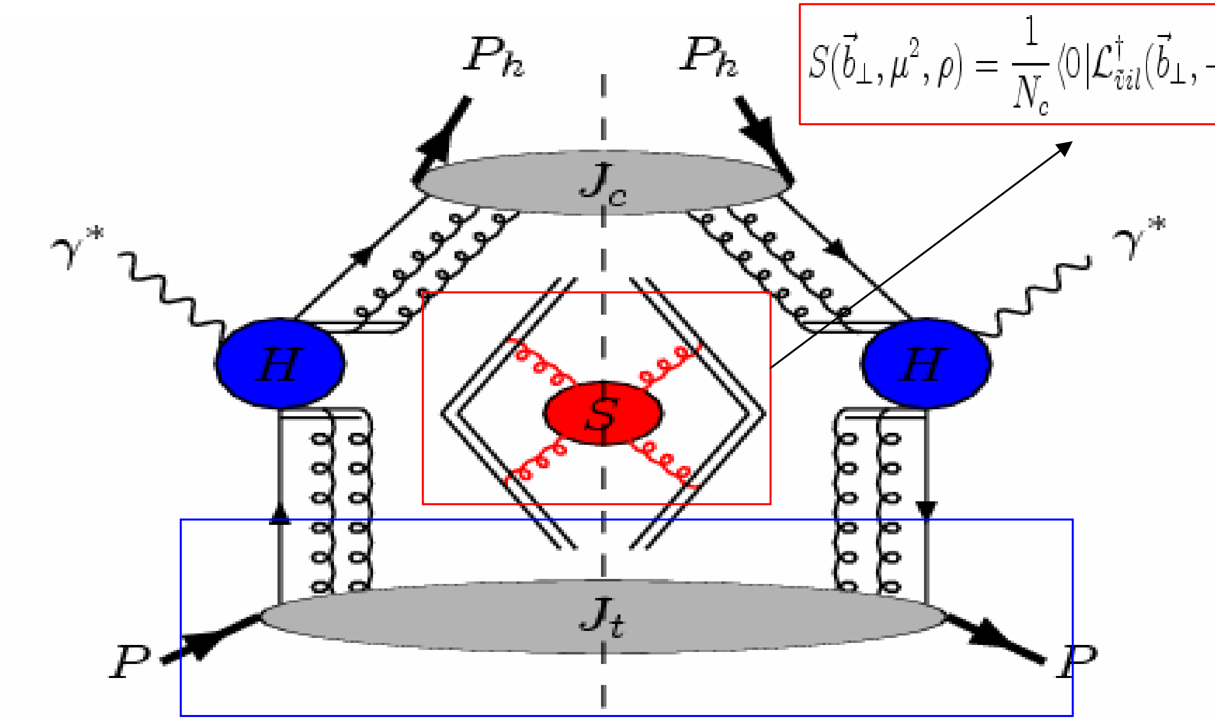
$$n_j\ = (1^+, 0_\perp,\ \ \delta^{\,2}/2\ ) \ \rightarrow n_j^2 = \ \delta^{\,2}$$

Then for any n_g ($n_g^2 = 0$), we have :

$$n_j\ .\ n_g\ \ >\ \delta^{\,2}/2$$

The difference between these two ways is proportional to δ .

TMD Factorization



$$S(\vec{b}_\perp,\mu^2,\rho) = \frac{1}{N_c}\langle 0|\mathcal{L}^\dagger_{\tilde{v}il}(\vec{b}_\perp,-\infty)\mathcal{L}^\dagger_{vlj}(\infty;\vec{b}_\perp)\mathcal{L}_{vjk}(\infty;0)\mathcal{L}_{\tilde{v}ki}(0;-\infty)|0\rangle$$

- Leading order in kt/Q
- Additional soft factor

Collins-Soper, 81
Ji-Ma-Yuan, 04
Collins-Metz 04
Cherednikov-Stefanis, 07

$$\mathcal{Q}(x,k_\perp,\mu,x\zeta) = \frac{1}{2}\int\frac{d\xi^-}{2\pi}e^{-ix\xi^-P^+}\int\frac{d^2\vec{b}_\perp}{(2\pi)^2}e^{i\vec{b}_\perp\cdot\vec{k}_\perp} \times \left\langle P\left|\overline{\psi}_q(\xi^-,0,\vec{b}_\perp)\mathcal{L}_v^\dagger(\infty;\xi^-,0,\vec{b}_\perp)\gamma^+\mathcal{L}_v(\infty;0)\psi_q(0)\right|P\right\rangle$$

$\sigma = H(Q,\mu)\, f(x,k_T,Q,\mu)\, d(z,k_T,Q,\mu)\, S(k_T,\mu)$

TMD factorization

At the $\boxed{\text{small transverse}}$ momentum limitation

$$W_{(\text{Drell-Yan})} = H(M, \mu)\, f_1(x_1\boxed{,b,}M, \mu)\, f_2(x_2, b, M, \mu)\, S(b, \mu)$$

Fourier transformation $\Updownarrow$

$\boxed{Q_t}$

W satisfies CSS evolution equation

$$\frac{\partial W(x_i, b, M^2)}{\partial \ln M^2} = (K + G')W(x_i, b, M^2)$$

At one-loop order for Drell-Yan process

$$K(b,\mu) = -\frac{\alpha_s C_F}{\pi} \ln \frac{b^2\mu^2}{c_0^2} \qquad G(Q,\mu) = -\frac{\alpha_s C_F}{\pi}\left(\ln \frac{Q^2}{\mu^2} - \frac{3}{2}\right)$$

TMD factorization for heavy quark pair production

- For the process:

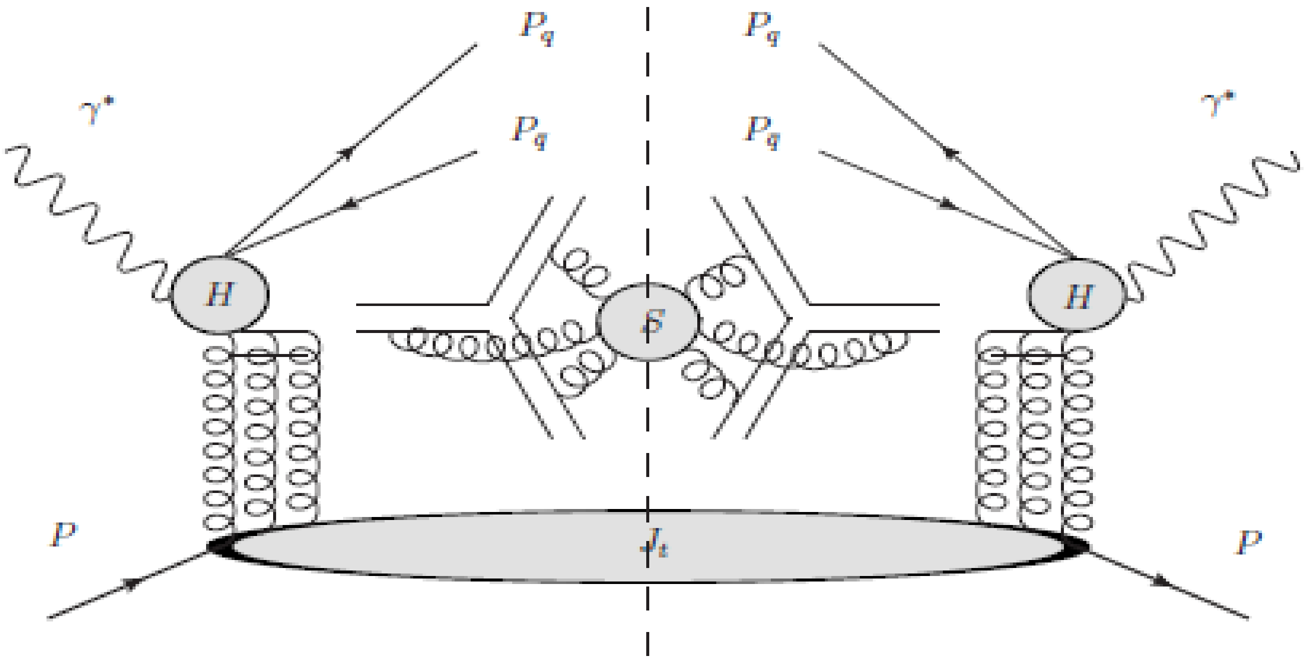
$$\gamma^* + p \to c\bar{c}[M_{c\bar{c}}, p_\perp] + X$$

There large logarithm $\log(M_{cc}/p_\perp)$ at the small $p_\perp$ region.

- For this process, you can not apply the TMD factorization formulism of SIDIS directly.

- It is because the finial state is color-octet

R. Zhu, P. Sun and F. Yuan **Phys.Lett. B727 (2013)**

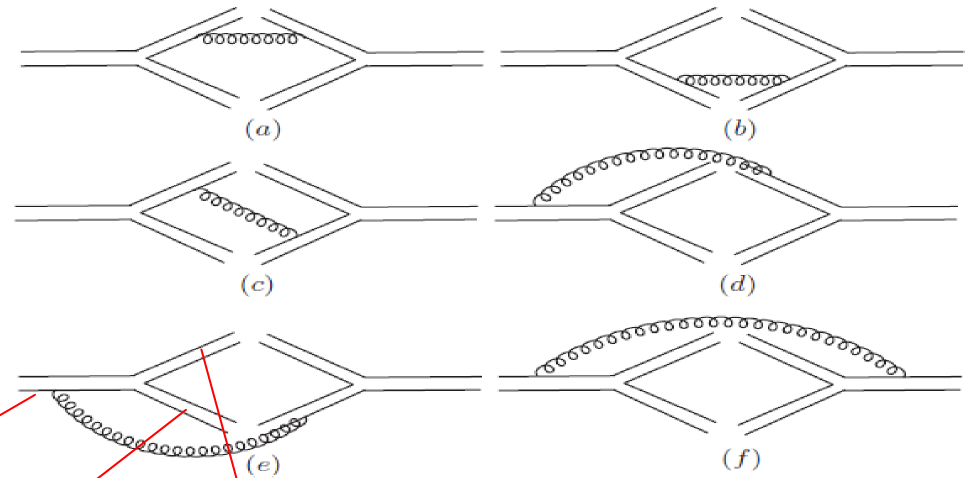


$$W_{c\bar{c}}(x,b_\perp)=H(\widetilde{Q},\mu)xg(x,b_\perp,\widetilde{Q},\mu)\overline{S}(b_\perp,\mu)$$

The TMD PDF still has the universality, but we should make a new definition for soft factor

There is no light cone singularity in heavy quark jet.

The soft factor



The definition of soft factor:

$$\overline{S}(b_\perp,\mu,\rho) = \frac{\int_0^\pi \frac{(\sin\phi)^{-2\epsilon}}{a_1} d\phi \, \langle 0|\mathcal{L}^\dagger_{\bar{v}ca'}(b_\perp)\,\mathrm{Tr}\left[\mathcal{L}^\dagger_{n_c}(b_\perp)\mathrm{T}^{a'}\mathcal{L}^\dagger_{n_{\bar{c}}}(b_\perp)\mathcal{L}_{n_{\bar{c}}}(0)\mathrm{T}^{a}\mathcal{L}_{n_c}(0)\right]\mathcal{L}_{\bar{v}ac}(0)|0\rangle}{\mathrm{Tr}[\mathrm{T}^d\mathrm{T}^d]}$$

At the one loop order

$$\overline{S}^{(1)}_{\mathrm{JMY}}(b_\perp,\mu,\rho) = \frac{\alpha_s}{2\pi}\{C_A \ln\frac{c_0^2}{b_\perp^2\mu^2}\,(B_{final} + \ln\rho^2 + \ln\frac{\tilde{Q}^2}{\zeta^2} - 1) + C_{final}\}$$

Sudakov form factor:

$$\gamma_K(\mu) = \frac{2\alpha_s(\mu)C_A}{\pi},$$

$$S_{sud} = -\int_{\tilde{Q}_0}^{\tilde{Q}} \frac{d\mu}{\mu}\left(\ln\frac{\tilde{Q}}{\mu}\gamma_K(\mu) - \gamma_S(\mu,1) + \frac{\alpha_s C_A}{\pi}(1 - 2\beta_0 - \ln\frac{\tilde{Q}_0^2 b_\perp^2}{c_0^2})\right)$$

$$\gamma_S(\mu,\rho) = -\frac{\alpha_s(\mu)C_A}{\pi}(B_{final} + \ln\rho - 1)$$