北京航空航天大學
BEIHANG UNIVERSITY

Charmed Baryon Ξ_c Decays from Lattice QCD

Qi-An Zhang

Beihang University

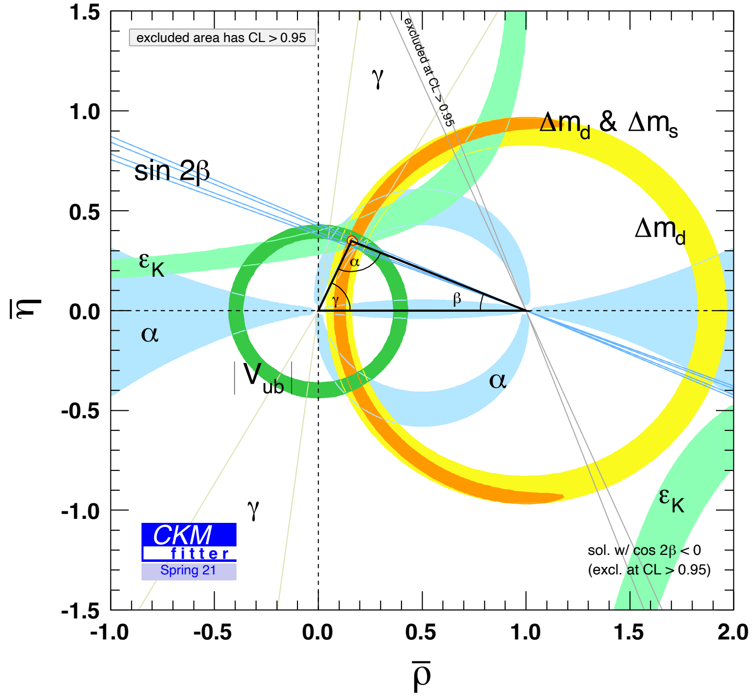
9 April 2023 @ BESIII粲强子物理研讨会

OUTLOOK

- Motivation
- Brief introduction of LQCD
- Semileptonic decays of Ξ_c
- $\Xi_c - \Xi_c{}'$ mixing
- Summary and outlook

Motivation: indirect test of SM

- **In the SM, The quark interaction strength is encoded in one unitary matrix (CKM):**



PDG2020

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$= \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

$\mathbf{0.9985 \pm 0.0005}$

$\Rightarrow$ $\mathbf{10^{-4}}$ **accuracy**

$\mathbf{1.025 \pm 0.022}$

$\Rightarrow$ $\mathbf{10^{-2}}$ **accuracy**

$\mathbf{0.9970 \pm 0.0018}$

$\Rightarrow$ $\mathbf{10^{-3}}$ **accuracy**

$\mathbf{1.026 \pm 0.022}$

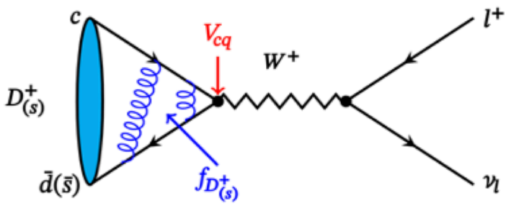
$\Rightarrow$ $\mathbf{10^{-2}}$ **accuracy**

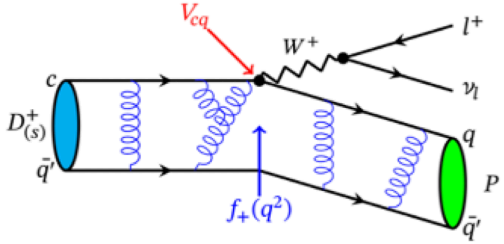
Motivation: indirect test of SM

Combing experiments + LQCD calculations of "Golden channels" provide the testing of CKM unitary.

Single hadron in initial state and at most one hadron in final state, both hadrons are stable in QCD

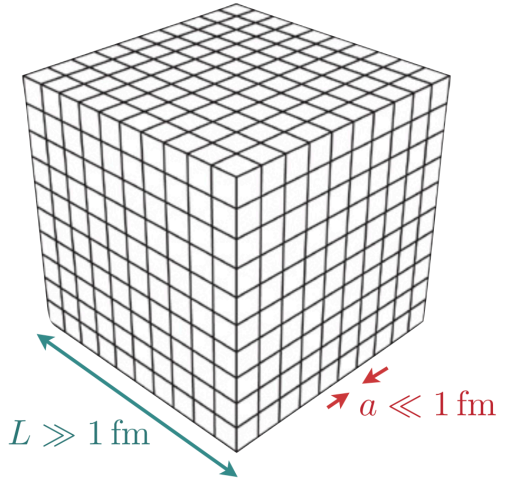
$$
V_{CKM} = \begin{pmatrix}
\begin{array}{c}\boldsymbol{V_{ud}} \\ \pi \to \ell\nu \end{array} & \begin{array}{c}\boldsymbol{V_{us}} \\ K \to \ell\nu, \pi\ell\nu \end{array} & \begin{array}{c}\boldsymbol{V_{ub}} \\ B \to \ell\nu, \pi\ell\nu \\ \Lambda_b \to \pi\ell\nu \end{array} \\
\begin{array}{c}\boldsymbol{V_{cd}} \\ D \to \ell\nu, \pi\ell\nu \end{array} & \begin{array}{c}\boldsymbol{V_{cs}} \\ D_s \to \ell\nu, D \to K\ell\nu \\ \Lambda_c \to \Lambda\ell\nu, \Xi_c \to \Xi\ell\nu \end{array} & \begin{array}{c}\boldsymbol{V_{cb}} \\ B \to D\ell\nu, D^*\ell\nu \\ \Lambda_b \to \Lambda_c\ell\nu \end{array} \\
\begin{array}{c}\boldsymbol{V_{td}} \\ \langle B_d|\bar{B}_d\rangle \\ B \to \pi\ell\ell \end{array} & \begin{array}{c}\boldsymbol{V_{ts}} \\ \langle B_s|\bar{B}_s\rangle \\ B \to K\ell\ell \end{array} & \boldsymbol{V_{tb}}
\end{pmatrix}
$$





Lattice QCD in a nutshell

Lattice v.s. Continuum	
We simulate:	**We want:**
😀 Euclidean space	🤔 Minkowski space
😀 Finite lattice spacing a	🤔 $a \to 0$
😀 Finite volume L^3	🤔 $L \to \infty$
😀 Lattice regularization	🤔 Some continuum scheme
😀 Some bare input quark masses: $am_l,\ am_s,\ am_c,\ am_b$ In general, $m_\pi^{\text{lat}} \neq m_\pi^{\text{phy}}$	🤔 $m_q^{\text{lat}} = m_q^{\text{phy}}$



➯ **Need to <u>control all limits</u>:** particularly simultaneously control FV and discretization

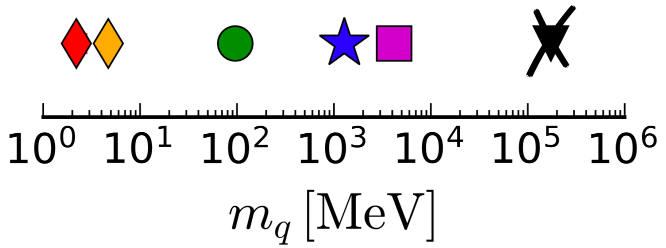
➯ **<u>Universality</u>:** different input parameters **must** give converge results.

Challenges in heavy flavors from LQCD

🤔 **Problems of heavy quarks on discrete lattice:**

Care about both **IR (finite volume)** and **UV (discretization)** **regulators**:

$m_\pi L \gtrsim 4,$ and $a^{-1} \gg$ mass scale of interest

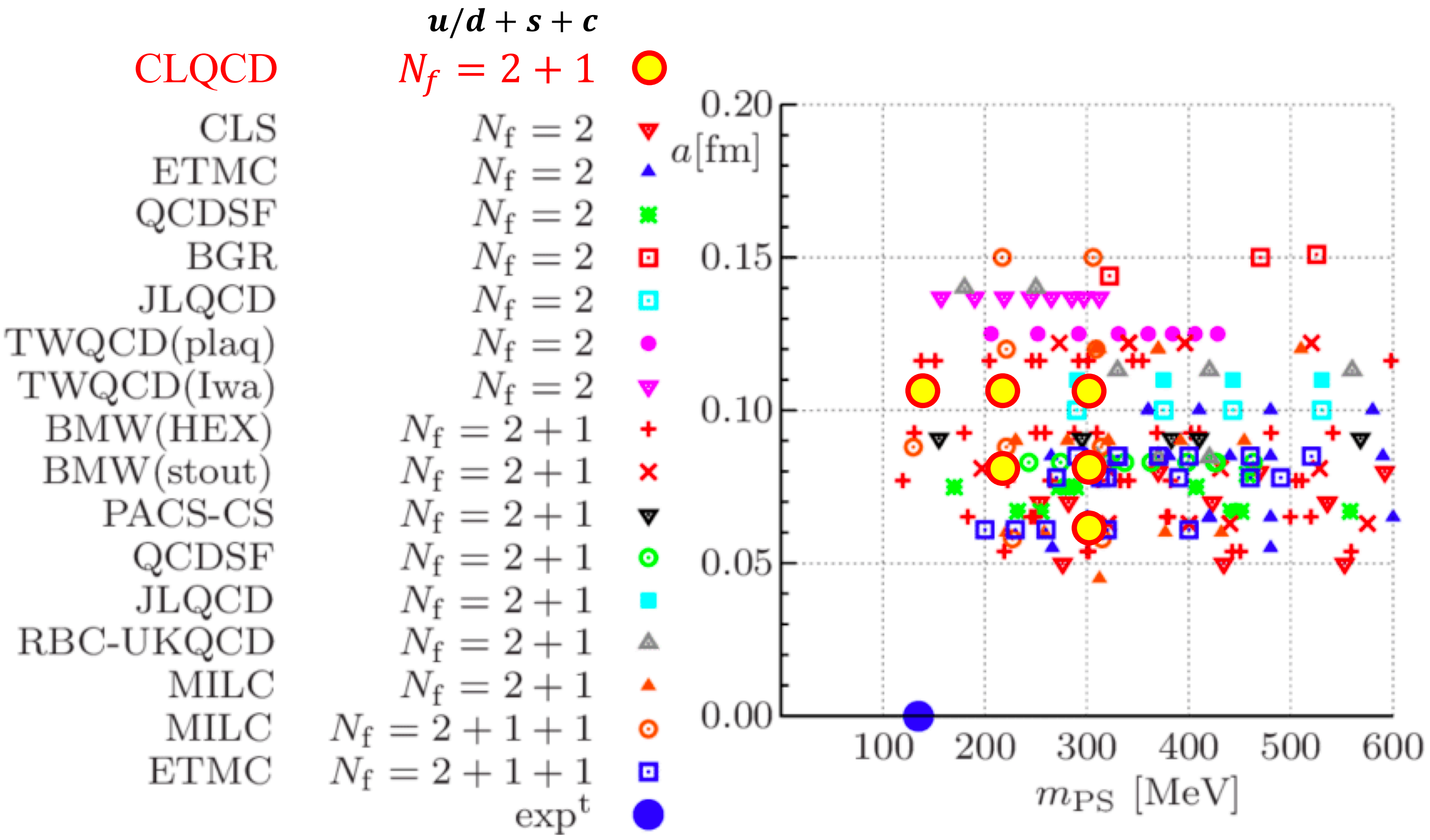


More flavors, need finer lattice

➾ For $m_\pi = m_\pi^{\rm phy} \sim 140\text{MeV}$, and $m_c \simeq 1.3\text{GeV}$, $m_b \simeq 4.2\text{GeV}$, that needs:

- $L \gtrsim 5.6\text{fm}$,
- $a^{-1} \gg 1.3\text{GeV} \simeq (0.15\text{fm})^{-1}$ **for charm** 🤔
- $a^{-1} \gg 4.2GeV \simeq (0.05fm)^{-1}$ **for bottom** 😰

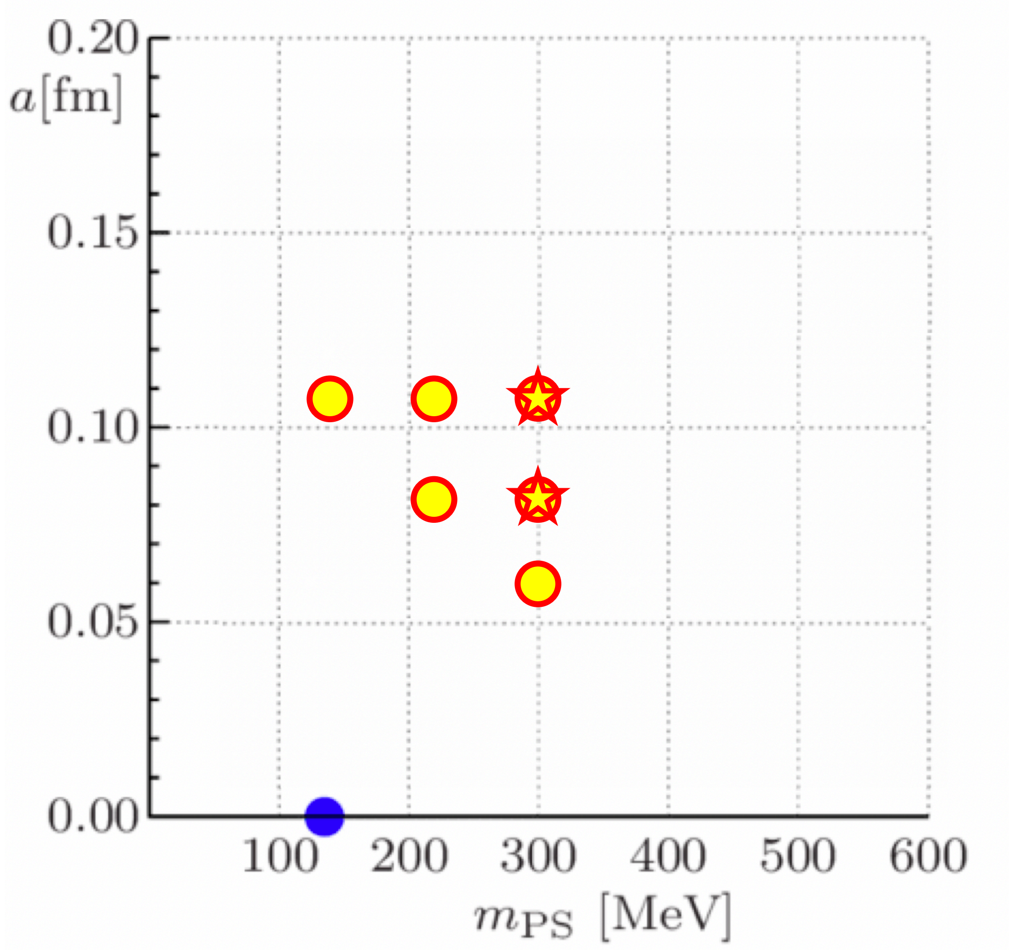
Recover to continuum physics



Source image from K. Jansen et al, added CLQCD (in preparation) in extras.

Recover to continuum physics

This work based on the <u>first CLQCD ensembles</u>



Ensembles completed before 2022:

- ✓ **Semi-leptonic decays of Ξ_c**

  Q-A. Zhang *et al.*, CPC46, 011002 (2022)

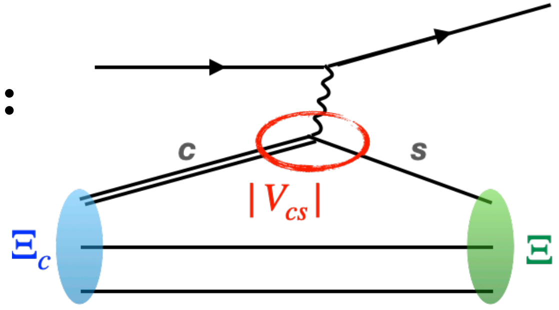
Ensembles completed after 2022:

- ✓ **$\Xi_c - \Xi_c{}'$ mixing**

  H. Liu *et al.*, arXiv: hep-lat/2303.17865

Semi-leptonic decays of Ξ_c

☞ **Determine the CKM matrix element $|V_{cs}|$;**



☞ **Ξ_c contains more versatile decay modes, will reveal more QCD dynamics:**

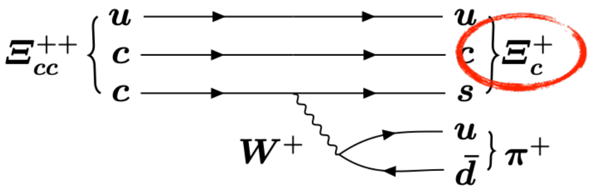
A different pattern between inclusive and exclusive decays of Λ_c and D:

$$\frac{\mathcal{B}(\Lambda_c^+ \to X e^+ \nu_e) = (3.95 \pm 0.34 \pm 0.09)\,\%}{\mathcal{B}(\Lambda_c^+ \to \Lambda e^+ \nu_e) = (3.63 \pm 0.38 \pm 0.20)\,\%} \quad \boxed{\sim 1} \qquad \frac{\mathcal{B}(D^0 \to X e^+ \nu_e) = (6.49 \pm 0.11)\,\%}{\mathcal{B}(D^0 \to K^- e^+ \nu_e) = (3.542 \pm 0.035)\,\%} \quad \boxed{\sim 2}$$

☞ **Experimental researches of heavy baryons:**

✓ Studies of doubly-charmed baryon Ξ_{cc}^{++} decay

R. Aaij *et al.* [LHCb], PRL121, 162002 (2018)

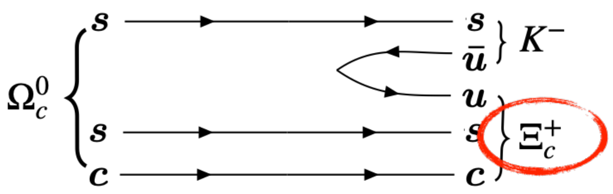


✓ Precision measurement of the lifetime of Ξ_b^0

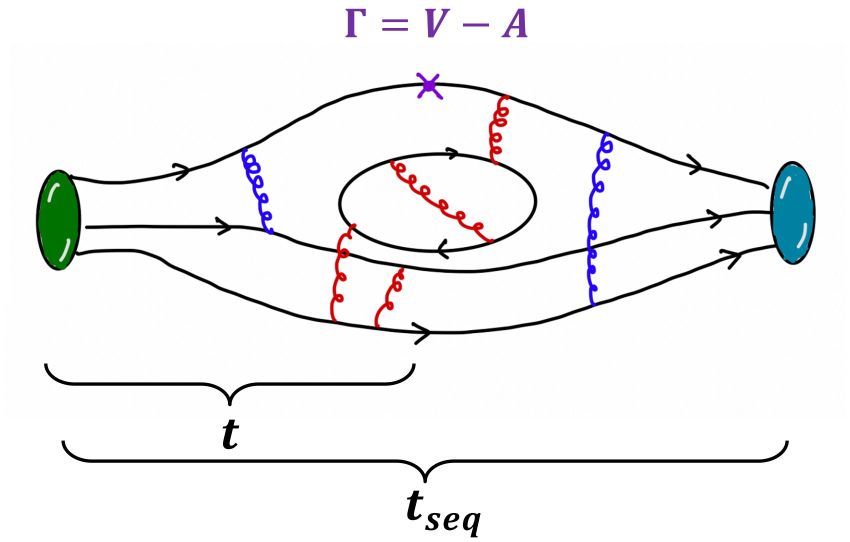
R. Aaij *et al.* [LHCb], PRL113, 032001 (2014)

✓ Discovery of new exotic hadron candidates Ω_c

R. Aaij et al. [LHCb], PRL118, 182001(2017)



✓ LQCD calculations of 2-point & 3-point function



- $t, t_{seq} \to \infty$ *to isolate the ground state;*
- $t_{seq} \gg t, (t_{seq} - t)$

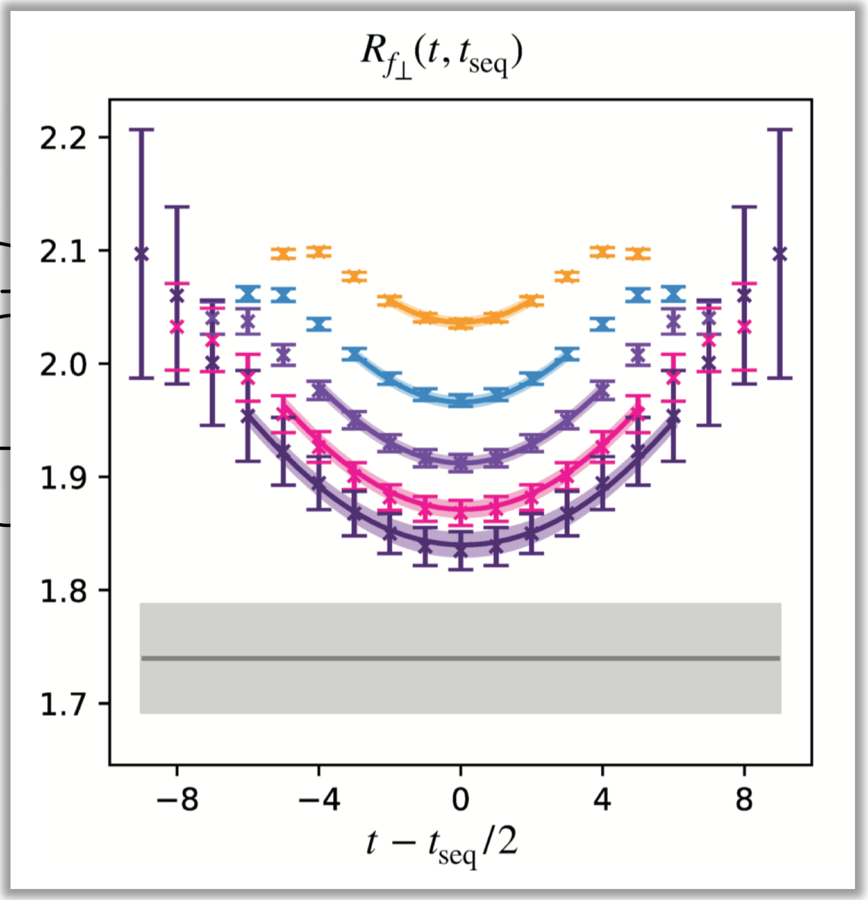
- Ratio between 3pt and 2pt related to the form factors:

$$
\begin{aligned}
R(q^2, t, t_{\text{seq}}; T, \Gamma) &= \sqrt{\frac{C_3(q^2, t, t_{\text{seq}}; T, \Gamma) C_3(q^2, t, t_{\text{seq}} - t; T, \Gamma)}{C_2^{\Xi_c}(t_{\text{seq}}; T) C_2^{\Xi}(t_{\text{seq}}; T)}} \\
&\approx F\left[1 + c_1\left(e^{-\Delta E t/2} + e^{-\Delta E\,(t_{\text{seq}} - t)/2}\right)\right]
\end{aligned}
$$

➢ F: ground state matrix element with specific Γ and projection T;

➢ Linear combinations of F give the form factors;

✓ **LQCD calculations of 2-point & 3-point function => Form factors**



- Ratio between 3pt and 2pt related to the form factors:

$$R(q^2, t, t_{\rm seq}; T, \Gamma) = \sqrt{\frac{C_3(q^2, t, t_{\rm seq}; T, \Gamma) C_3(q^2, t, t_{\rm seq} - t; T, \Gamma)}{C_2^{\Xi_c}(t_{\rm seq}; T) C_2^{\Xi}(t_{\rm seq}; T)}}$$

$$\approx F\left[1 + c_1\left(e^{-\Delta E t/2} + e^{-\Delta E (t_{\rm seq} - t)/2}\right)\right]$$

➢ F: ground state matrix element with specific Γ and projection T;

➢ Linear combinations of F give the form factors;

➢ "Just" do the fit.

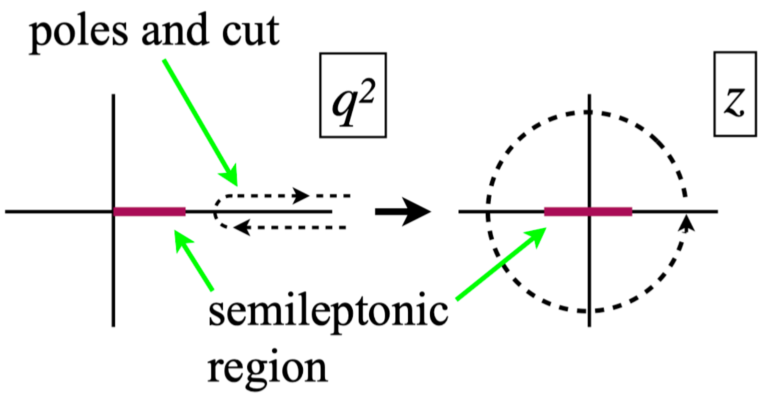
Parameterizing q^2 dependence: z-expansion

🤔 **Discrete datapoints → Continuous distribution**

Conformal mapping: z-expansion ⟶ wider kinematic range

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_0 = q^2_{\rm max} = (m_{\Xi_c} - m_{\Xi})^2, \quad t_+ = (m_D + m_K)^2$$

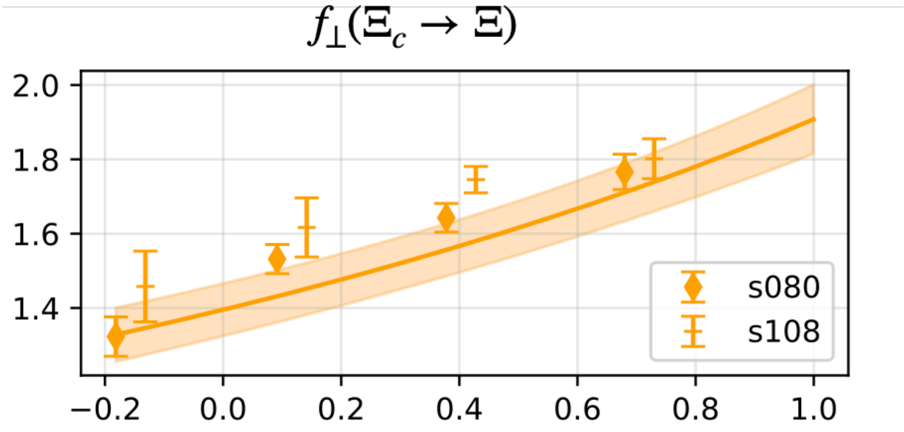


Parametrization of form factors: *PRD79,013008(2009)*

$$f(q^2) = \frac{1}{1 - q^2/(m^f_{\rm pole})^2} \sum_{n=0}^{n_{\rm max}} (c^f_n + d^f_n a^2)[z(q^2)]^n$$

➢ Use D_s pole mass in the form factors;

➢ Consider the discretization effects by estimating the d^f_n terms.



$\Xi_c \to \Xi$ form factors from LQCD

✓ **Branching fractions:**

$$\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = 2.38(0.30)(0.32)(0.07)\,\%$$

$$\mathscr{B}\left(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu\right) = 2.29(0.29)(0.30)(0.06)\,\%$$

$$\mathscr{B}\left(\Xi_c^+ \to \Xi^0 e^+ \nu_e\right) = 7.18(0.90)(0.96)(0.20)\,\%$$

$$\mathscr{B}\left(\Xi_c^+ \to \Xi^0 \mu^+ \nu_\mu\right) = 6.91(0.87)(0.91)(0.19)\,\%$$

PDG $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.8 \pm 1.2)\,\%$

Belle $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\,\%$

ALICE $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\,\%$

QCD SR $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.4 \pm 1.7)\,\%$

LF QM $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (3.49 \pm 0.95)\,\%$

LCSR $\mathscr{B}\left(\Xi_c^0 \to \Xi^- e^+ \nu_e\right) = (2.4^{+0.9}_{-1.0})\,\%$

✓ **Determination of $|V_{cs}|$:**

⇒ from ALICE results:

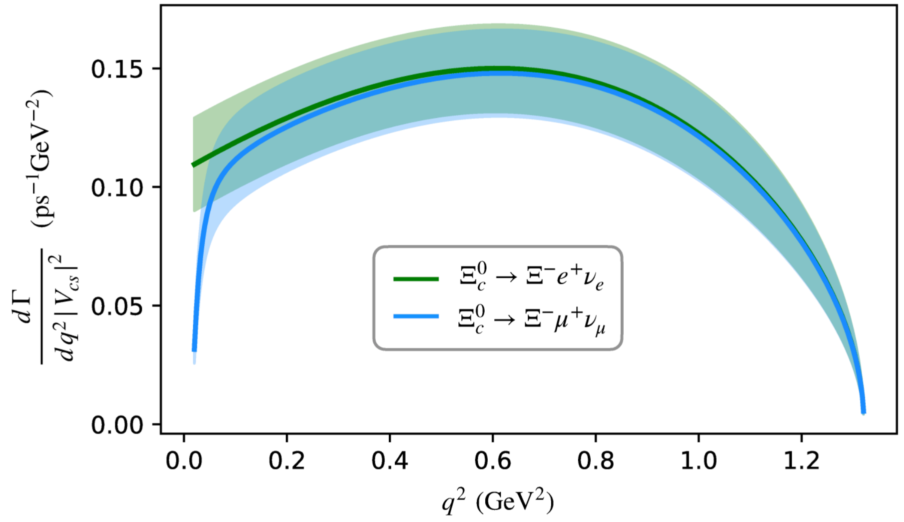
$$|V_{cs}| = 0.983(0.060)_{\text{stat.}}(0.065)_{\text{syst.}}(0.167)_{\text{exp.}}$$

⇒ from Belle results:

$$|V_{cs}| = 0.834(0.051)_{\text{stat.}}(0.056)_{\text{syst.}}(0.127)_{\text{exp.}}$$

⇒ PDG average:

$$|V_{cs}| = 0.97320 \pm 0.00011$$



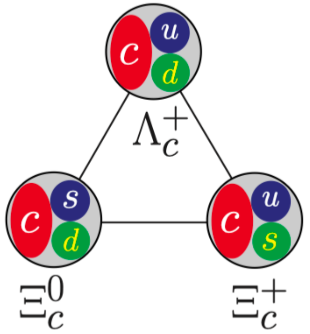
$\Xi_c - \Xi_c{}'$ mixing

Mixing between mass eigenstates and flavor eigenstates:

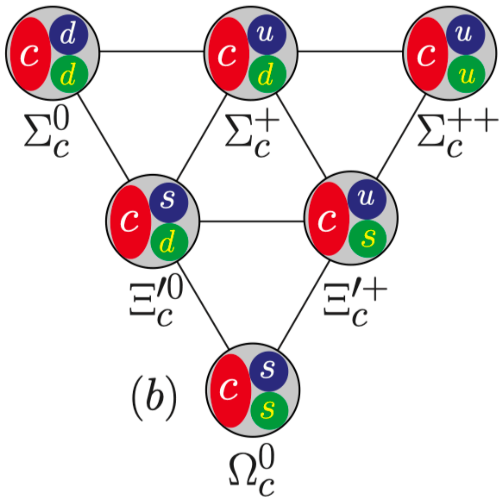
$$\begin{pmatrix} \Xi_c \\ \Xi_c' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \Xi_c^{\bar{\mathbf{3}}} \\ \Xi_c^{\mathbf{6}} \end{pmatrix}$$

PLB823,136765(2021)

channel	branching ratio(%)		
	experimental data	fit data(pole model)	fit data(constant).
$\Lambda_c^+ \to \Lambda^0 e^+ \nu_e$	3.6 ± 0.4	3.61 ± 0.32	3.62 ± 0.32
$\Lambda_c^+ \to \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5	3.48 ± 0.30	3.45 ± 0.30
$\Xi_c^+ \to \Xi^0 e^+ \nu_e$	2.3 ± 1.5	3.89 ± 0.73	3.92 ± 0.73
$\Xi_c^0 \to \Xi^- e^+ \nu_e$	1.54 ± 0.35	1.29 ± 0.24	1.31 ± 0.24
$\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44	1.24 ± 0.23	1.24 ± 0.23
fit parameter (pole model)	$f_1 = 1.01 \pm 0.87,\ \delta f_1 = -0.51 \pm 0.92$ $f_1' = 0.60 \pm 0.49,\ \delta f_1' = -0.23 \pm 0.41$		$\chi^2/d.o.f = 1.6$
fit parameter (constant)	$f_1 = 0.86 \pm 0.92,\ \delta f_1 = -0.25 \pm 0.88$ $f_1' = 0.85 \pm 0.36,\ \delta f_1' = -0.43 \pm 0.50$		$\chi^2/d.o.f = 1.9$



Triplet



Sextet

$$T_{c\bar{3}} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix} \quad T_{c6} = \begin{pmatrix} \Sigma_c^{++} & \frac{\Sigma_c^+}{\sqrt{2}} & \frac{\Xi_c^{+\prime}}{\sqrt{2}} \\ \frac{\Sigma_c^+}{\sqrt{2}} & \Sigma_c^0 & \frac{\Xi_c^{0\prime}}{\sqrt{2}} \\ \frac{\Xi_c^{+\prime}}{\sqrt{2}} & \frac{\Xi_c^{0\prime}}{\sqrt{2}} & \Omega_c^0 \end{pmatrix}$$

$\Xi_c - \Xi_c{}'$ mixing

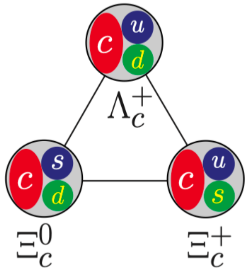
🤔 **Several works from various methods are controversial……**

Sum rule	$5.5^\circ \pm 1.8^\circ$	*Phys. Rev. D 83, 016008 (2011);*
HQET	$8.12^\circ \pm 0.80^\circ$	*Nucl. Phys. A 1008, 122139, (2021);*
Quark model	$16.27^\circ \pm 2.30^\circ$	*Phys. Rev. D 105, 096011 (2022);*
	$24.66^\circ \pm 0.90^\circ$	*Phys. Lett. B 838, 137736, (2023);* *Phys. Lett. B 839, 137831, (2023);*
Lattice QCD	**Negligibly small**	*Phys. Rev. D 90, 094507, (2014).*

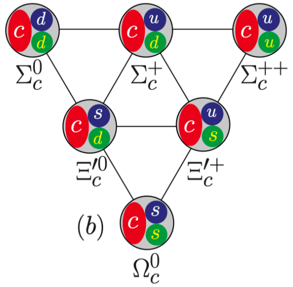
$\Xi_c - \Xi_c{}'$ mixing from LQCD

Baryonic currents of $SU(3)_F$ eigenstates: $\mathbf{3 \otimes 3 = \bar{3} \oplus 6}$

Phys. Lett. B, 285:254–262, 1992.



➢ Anti-symmetric triplet: $O^{\bar{3}} = \epsilon^{abc} \underbrace{\left(\ell^{Ta} C\gamma_5 s^b\right)}_{J=0} P_+ \tilde{c}^c,$



➢ Symmetric sextet: $O^6 = \epsilon^{abc} \underbrace{\left(\ell^{Ta} C\vec{\gamma} s^b\right)}_{J=1} \cdot \vec{\gamma}\gamma_5 P_+ \tilde{c}^c$

➾ Build the 2×2 correlation function matrix for lattice calculation:

$$\mathcal{C}(t, t_0) = \sum_{\vec{x}} \begin{pmatrix} \left\langle O_p^{\bar{3}}(\vec{x}, t)\bar{O}_w^{\bar{3}}(\vec{0}, t_0)\right\rangle & \left\langle O_p^{\bar{3}}(\vec{x}, t)\bar{O}_w^{6}(\vec{0}, t_0)\right\rangle \\ \left\langle O_p^{6}(\vec{x}, t)\bar{O}_w^{\bar{3}}(\vec{0}, t_0)\right\rangle & \left\langle O_p^{6}(\vec{x}, t)\bar{O}_w^{6}(\vec{0}, t_0)\right\rangle \end{pmatrix}$$

Mixing angle from correlated joint fit

- **Parametrization form of correlation function matrix elements:**

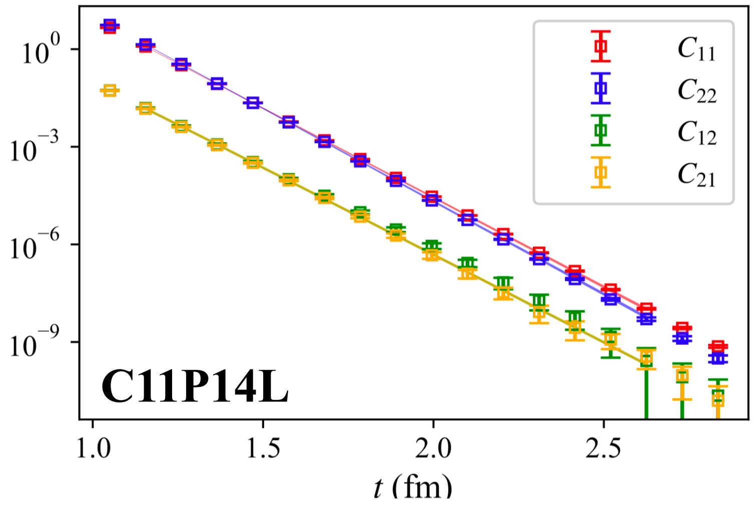
$$C_{11}(t,t_0) = A_p A_w^\dagger \left[\frac{\cos^2\theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} + \frac{\sin^2\theta}{2m_{\Xi_c'}} e^{-m_{\Xi_c'}(t-t_0)} \right],$$

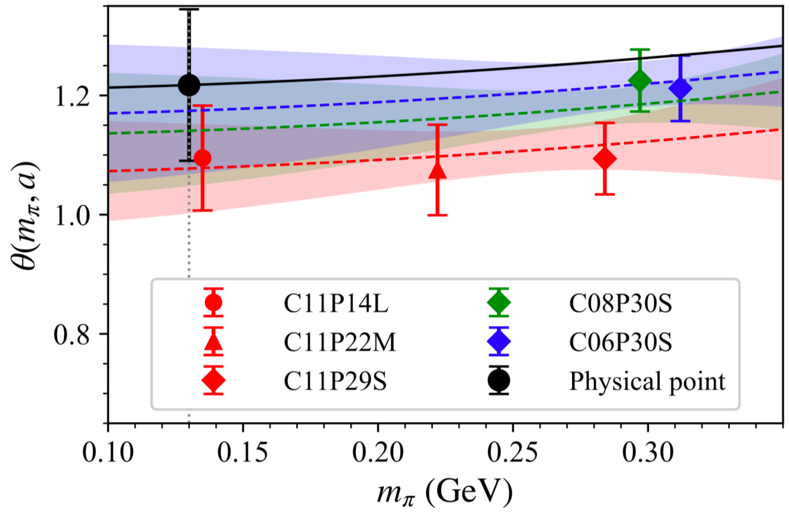
$$C_{12}(t,t_0) = A_p B_w^\dagger \left[\frac{\cos\theta\sin\theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} - \frac{\cos\theta\sin\theta}{2m_{\Xi_c'}} e^{-m_{\Xi_c'}(t-t_0)} \right],$$

$$C_{21}(t,t_0) = B_p A_w^\dagger \left[\frac{\cos\theta\sin\theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} - \frac{\cos\theta\sin\theta}{2m_{\Xi_c'}} e^{-m_{\Xi_c'}(t-t_0)} \right],$$

$$C_{22}(t,t_0) = B_p B_w^\dagger \left[\frac{\sin^2\theta}{2m_{\Xi_c}} e^{-m_{\Xi_c}(t-t_0)} + \frac{\cos^2\theta}{2m_{\Xi_c'}} e^{-m_{\Xi_c'}(t-t_0)} \right],$$

➢ Parameters: $A_w, A_p, B_w, B_p, m_{\Xi_c}, m_{\Xi_c'}, \theta$





Mixing angle from generalized eigenvalue problem (GEVP)

Solving the generalized eigenvalue problem:

$$\mathcal{C}(t)v_n(t) = \lambda_n(t)\mathcal{C}(t_r)v_n(t),$$

Reference time

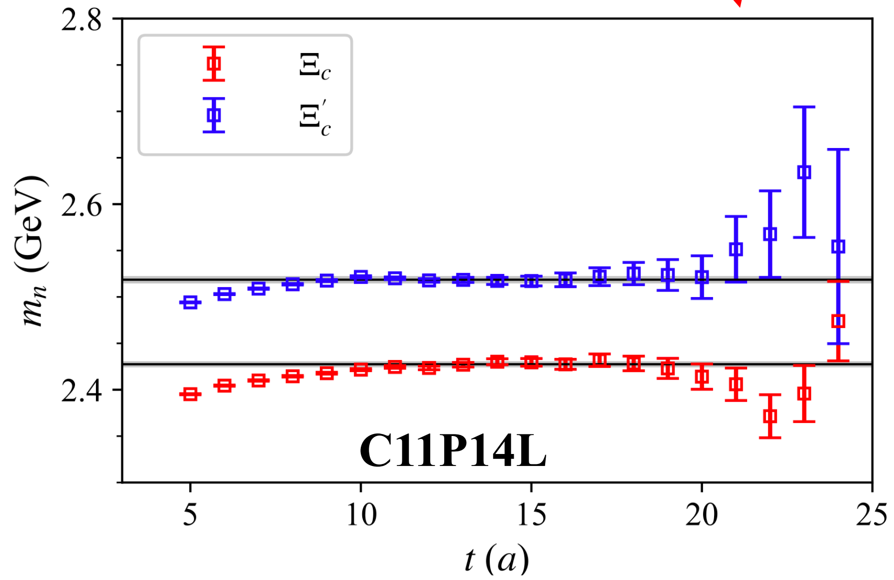
Mass from eigenvalues

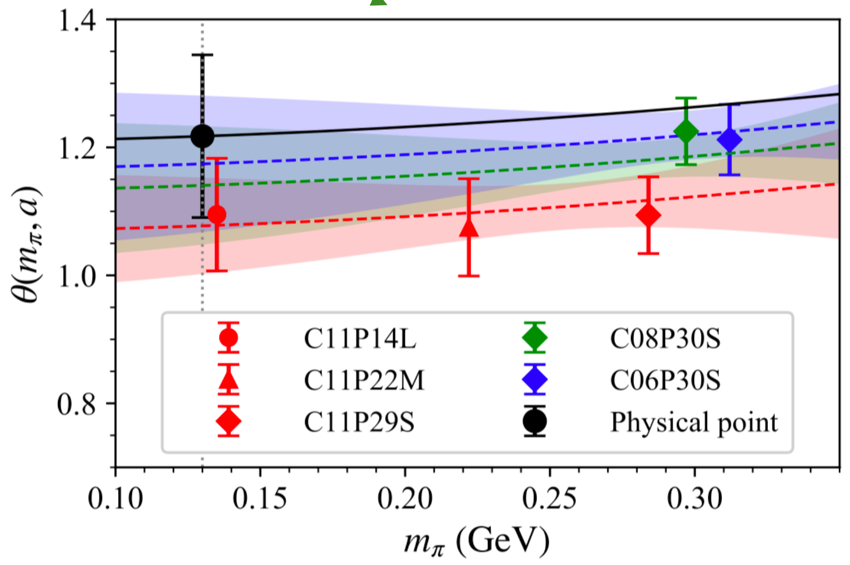
$$\lambda_n(t) = c_0 e^{-m_n(t-t_r)}\left(1 + c_1 e^{-\Delta E(t-t_r)}\right)$$

$$v_1 = \sqrt{1 + \frac{A_p^2 \cot^2\theta}{B_p^2}}\begin{pmatrix}\frac{A_p}{B_p}\cot\theta \\ 1\end{pmatrix},$$

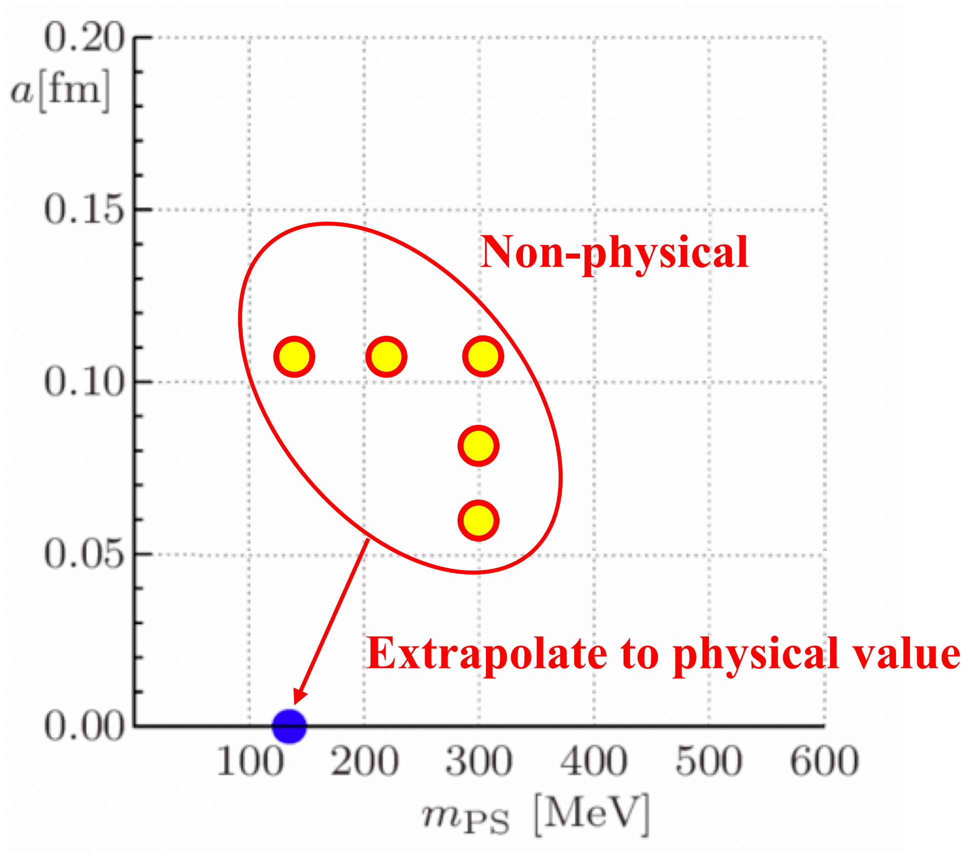
$$v_2 = \sqrt{1 + \frac{A_p^2 \tan^2\theta}{B_p^2}}\begin{pmatrix}-\frac{A_p}{B_p}\tan\theta \\ 1\end{pmatrix},$$

Mixing angle from eigenvectors

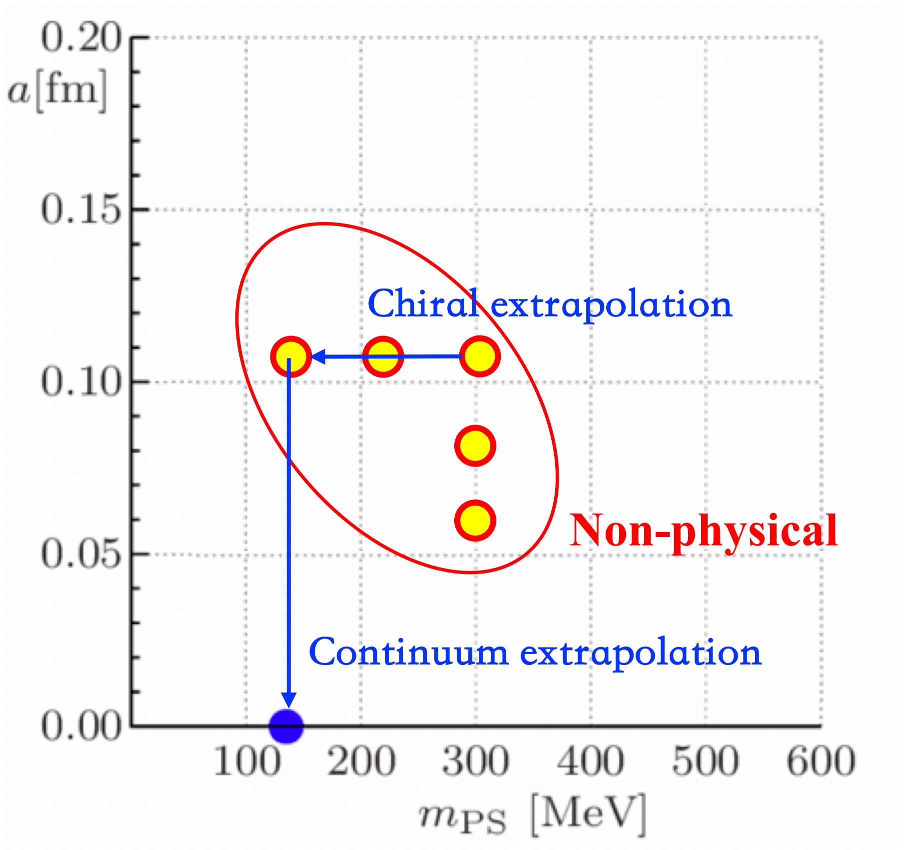




Chiral and continuum extrapolation



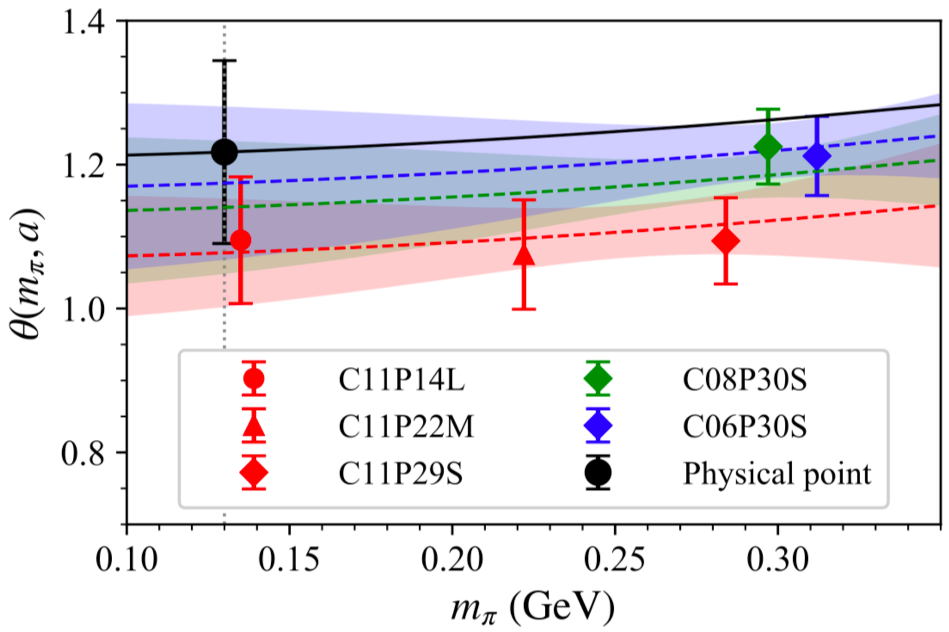
Chiral and continuum extrapolation



Extrapolation formula:

$$\theta(m_\pi, a) = \theta_{\rm phy} + c_1\left(m_\pi^2 - m_{\pi,\rm phy}^2\right) + c_2 a^2,$$
$$m_n(m_\pi, a) = m_{n,\rm phy} + c_1\left(m_\pi^2 - m_{\pi,\rm phy}^2\right) + c_2 a^2.$$

➩ Extrapolated results:



m_c dependence

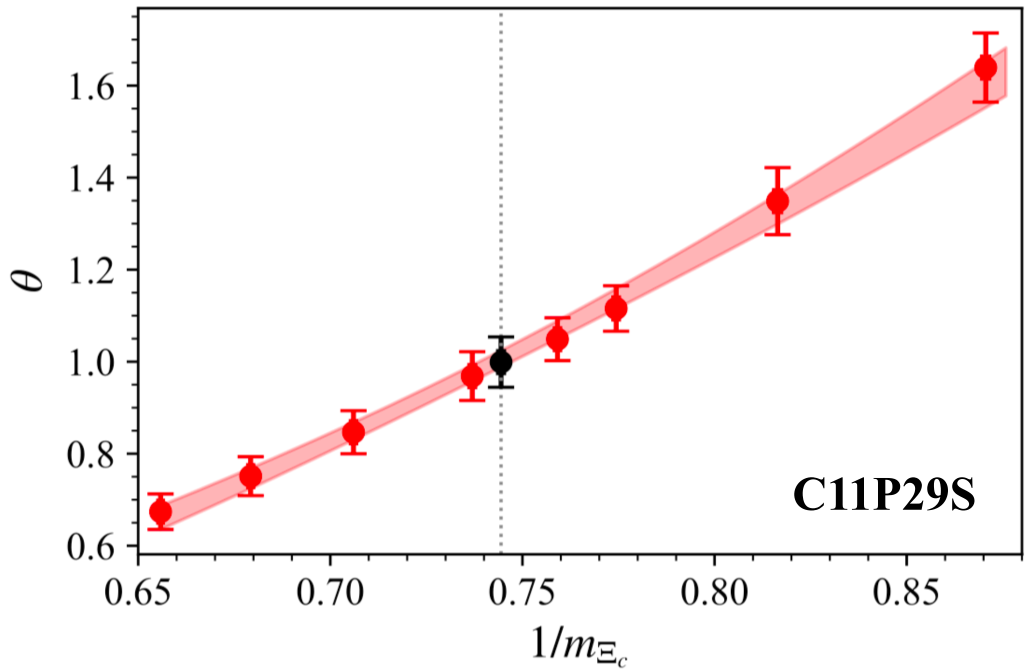
In HQET, the mixing would vanish in the heavy-quark limit.

🤔 Valence quark mass is tunable in lattice QCD⋯

➯ Solve the baryon masses and mixing angle from different charm quark masses;

➯ Then fit the m_c dependence of θ:

$$\theta = \frac{B_1}{m_{\Xi_c}} + \frac{B_2}{m_{\Xi_c}^2}.$$



$B_1 = -2.78(52)\mathrm{GeV}$ and $B_2 = 12.9(1.3)\mathrm{GeV}^2$ with $\chi^2/\mathrm{d.o.f} = 0.11.$

Summary and outlook

- Lattice QCD calculations of Ξ_c semi-leptonic decays and $\Xi_c - \Xi_c'$ mixing;
- The first practice of CLQCD ensembles;
- Charmed Hadrons: testing SM, probing NP, understanding QCD……

Summary and outlook

- ➢ Lattice QCD calculations of Ξ_c semi-leptonic decays and $\Xi_c - \Xi_c{}'$ mixing;
- ➢ The first practice of CLQCD ensembles;
- ➢ Charmed Hadrons: testing SM, probing NP, understanding QCD……

🤔 **More challenges:**

- More precise results of Λ_c and Ξ_c decays;
- Ω_c decay and form factors;
- Other matrix elements: lifetime (4-quark current), inclusive decays (4pt) ……

Summary and outlook

- Lattice QCD calculations of Ξ_c semi-leptonic decays and $\Xi_c - \Xi_c'$ mixing;
- The first practice of CLQCD ensembles;
- Charmed Hadrons: testing SM, probing NP, understanding QCD……

🤔 **More challenges:**

- More precise results of Λ_c and Ξ_c decays;
- Ω_c decay and form factors;
- Other matrix elements: lifetime (4-quark current), inclusive decays (4pt) ……

Lattice calculations have achieved significant progress in the past years.

Extensive analyses already in light meson sector.

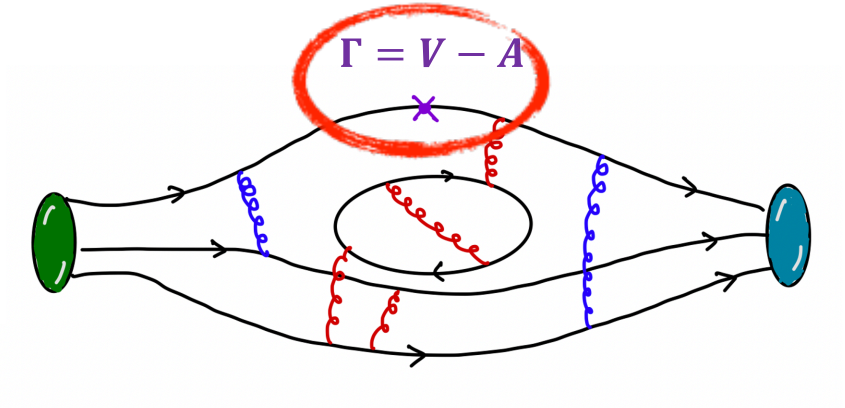
While heavy flavor physics on lattice is underway......

Thank you for your attention!

Backup slides

Non-perturbative renormalization

Vector and axial-vector $c \to s$ current



- Z_V from conserved and local vector currents;
- Z_A/Z_V from off-shell quark matrix elements.

☞ **Like Z_V:**

$$R_V^{q_1\to q_2}(t) = \frac{\left\langle M_1\left(\frac{T}{2}\right)\sum_{\vec{x}} V_c^{q_1\to q_2}(\vec{x},t) M_2(0)\right\rangle}{\left\langle M_1\left(\frac{T}{2}\right)\sum_{\vec{x}} V^{q_1\to q_2}(\vec{x},t) M_2(0)\right\rangle} = Z_V^{q_1\to q_2} + \mathcal{O}\left(e^{-\frac{T}{4\Delta E}}\right)$$

$$\text{with} \quad (M_1, M_2) = (\eta_s, D_s) \quad v.s. \quad \sqrt{(\eta_s,\eta_s) * (\eta_c,\eta_c)}$$