Mixing effects of $\Xi_c - \Xi_c'$ in decays

Resolving puzzle in $\Xi_c^0 \to \Xi^- e^+ \nu_e$ with $\Xi_c - \Xi_c'$ mixing within the quark model

Based on arXiv: 2210.07211 , 2211.12960 , 2212.02971

劉 佳 韋

UCAS

April 9, 2023

in decays

within the quark model

Research Motivation - Nonleptonic decays

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Asymmetries of anti-triplet charmed baryon decays

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Check for updates

Color symmetry + SU(3) flavor symmetry

PLB **794**, 19 (Feb 16, 2019)

Research Motivation - Nonleptonic decays

channel	data	$SU(3)_F$	Current Algebra
$10^4\mathcal{B}(\Lambda_c^+ \to \Sigma^0 K^+)^a$	$4.7 \pm 0.9 \pm 0.1 \pm 0.3$	5.4 ± 0.7	7.2
$10^4\mathcal{B}(\Lambda_c^+ \to \Sigma^+ K_S^0)^a$	$4.8 \pm 1.4 \pm 0.2 \pm 0.3$	5.4 ± 0.7	7.2
$10^4\mathcal{B}(\Lambda_c^+ \to n\pi^+)^b$	$6.6 \pm 1.2 \pm 0.4$	8.5 ± 2.0	2.7
$\alpha(\Lambda_c^+ \to pK_S^0)^c$	$0.18 \pm 0.43 \pm 0.14$	$-0.89^{+0.26}_{-0.11}$	-0.90
$\alpha(\Lambda_c^+ \to \Lambda K^+)^d$	$-0.585 \pm 0.049 \pm 0.018$	0.32 ± 0.32	-0.96
$\alpha(\Lambda_c^+ \to \Sigma^0 K^+)^d$	$-0.55 \pm 0.18 \pm 0.09,$	~ -1	-0.73
$100\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)^e$	1.43 ± 0.32	2.21 ± 0.11	6.47
$100\mathcal{R}(\Xi_c^0 \to \Xi^- K^+)$	$2.75 \pm 0.51 \pm 0.25$	4.4	6.0
$100\mathcal{R}(\Xi_c^0 \to \Sigma^0 K_s^0)^f$	$3.8 \pm 0.6 \pm 0.4$	2.3 ± 1.8	< 0.4
$10\mathcal{R}(\Xi_c^0 \to \Sigma^+ K^-)^f$	$1.23 \pm 0.07 \pm 0.10$	2.7 ± 0.5	0.71
$100\mathcal{B}(\Xi_c^+ \to \Xi^0 \pi^+)$	1.6 ± 0.8	0.38 ± 0.20	1.72
$\mathcal{R}(\Xi_c^+ \to \Xi^0 \pi^+)$	1.1 ± 0.6	0.17 ± 0.09	0.27

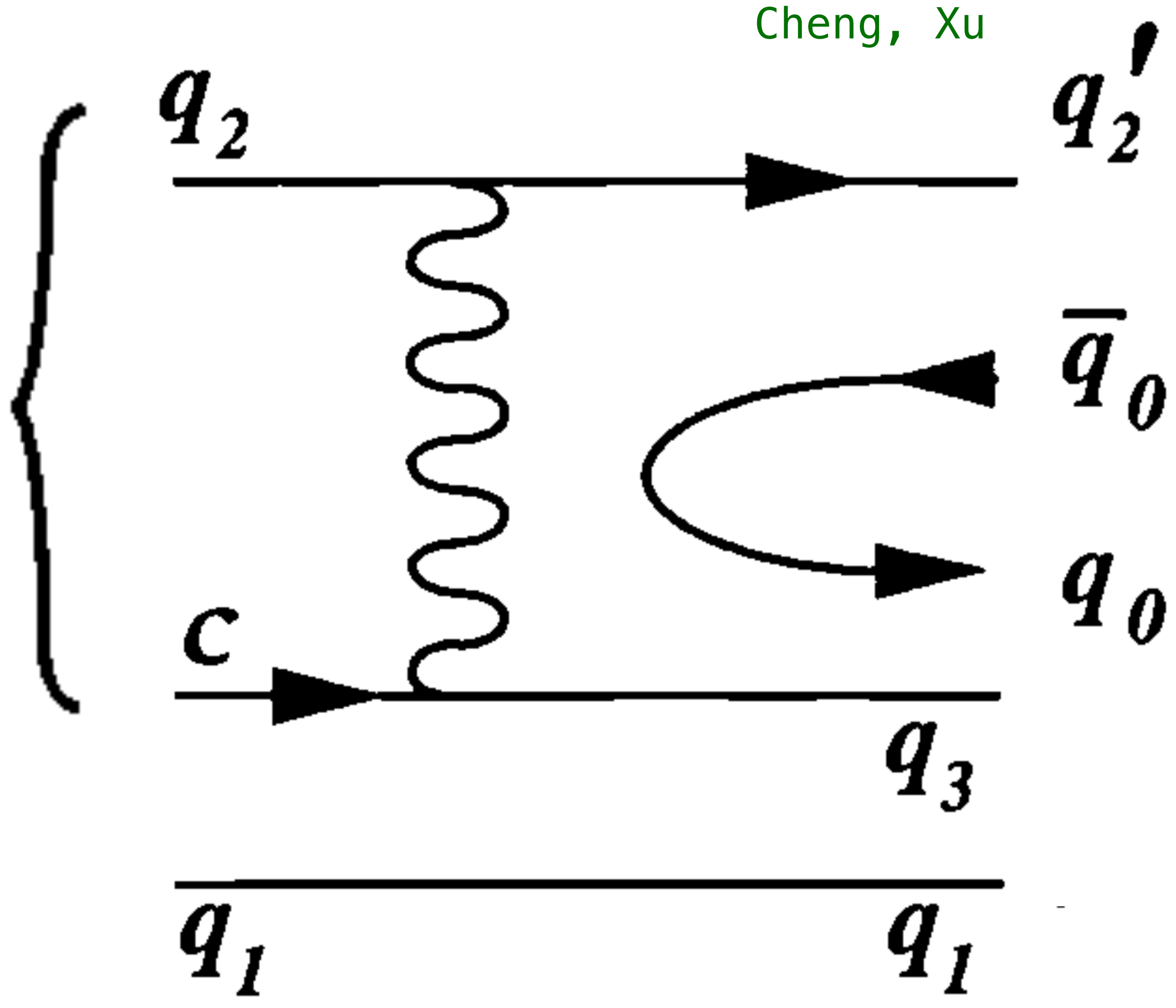
[a] Phys. Rev. D **106**, 052003 (2022).
[b] Phys. Rev. Lett. **128**, 142001 (2022).
[c] Phys. Rev. D **100**, 072004 (2019).
[d] arXiv:2208.08695 [hep-ex].
[e] Phys. Rev. Lett. **122**, 082001 (2019).
[f] Phys. Rev. D **105**, L011102 (2022).

$$\mathscr{R}(X) := \frac{\mathscr{B}(X)}{\mathscr{B}(\Xi_c^0 \to \Xi^- \pi^+)}$$

Input:
$$10^4 \mathscr{B}(\Lambda_c^+ \to \Sigma^0 K^+) = 5.2 \pm 0.8$$
$$100\mathscr{B}(\Xi_c^0 \to \Xi^- \pi^+) = 1.80 \pm 0.52$$

PRD **101** 014011 (2020); PRD **97** 074028 (2018)

Cheng, Xu



$= 0$

Chau, Cheng, Tseng

PRD **54**, 2132 (1996)

$$O_+ = \frac{1}{2}\sum_{q,q'=d,s} V_{uq}V_{cq'}^* \left((\bar{u}q)(\bar{q}'c)+(\bar{q}'q)(\bar{u}c)\right)$$

Körner-Pati-Woo theorem

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Research Motivation – Nonleptonic decays

See 胥英超's talk

channel	data	$SU(3)_F$	Current Algebra
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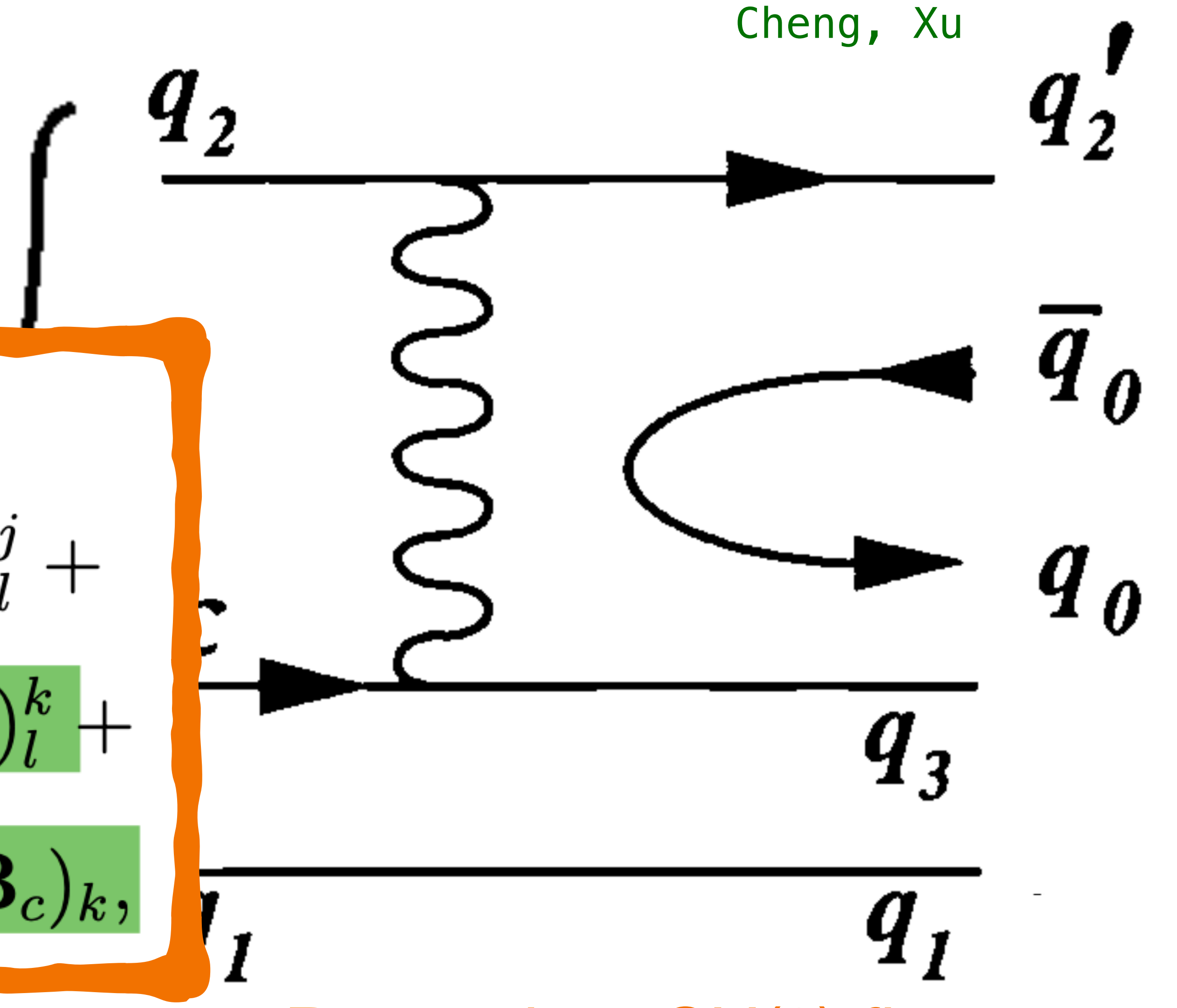
PRD **101** 014011 (2020); PRD **97** 074028 (2018)

Cheng, Xu

$$A_{(\mathbf{B}_c \to \mathbf{B}_n M)} =$$
$$a_0 H(6)_{ij}(\mathbf{B}_c')^{ik}(\mathbf{B}_n)^j_k (M)^l_l + a_1 H(6)_{ij}(\mathbf{B}_c')^{ik}(\mathbf{B}_n)^l_k (M)^j_l + a_2 H(6)_{ij}(\mathbf{B}_c')^{ik}(M)^l_k (\mathbf{B}_n)^j_l +$$
$$a_3 H(6)_{ij}(\mathbf{B}_n)^i_k (M)^j_l (\mathbf{B}_c')^{kl} + a_0'(\mathbf{B}_n)^i_j (M)^l_l H(\overline{15})^{jk}_i (\mathbf{B}_c)_k + a_4 H(\overline{15})^{li}_k (\mathbf{B}_c)_j (M)^j_i (\mathbf{B}_n)^k_l +$$
$$a_5(\mathbf{B}_n)^i_j (M)^l_i H(\overline{15})^{jk}_l (\mathbf{B}_c)_k + a_6 (\mathbf{B}_n)^j_i (M)^m_l H(\overline{15})^{li}_m (\mathbf{B}_c)_j + a_7 (\mathbf{B}_n)^l_i (M)^i_j H(\overline{15})^{jk}_l (\mathbf{B}_c)_k,$$



$= 0$

Chau, Cheng, Tseng

PRD 54, 2132 (1996)

Better than SU(3) flavor symmetry.

$$= \frac{1}{2}\sum_{q,q'=d,s} V_{uq}V^*_{cq'}\left((\bar{u}q)(\bar{q}'c) + (\bar{q}'q)(\bar{u}c)\right)$$

Körner-Pati-Woo theorem

- Test the KPW theorem directly
- Protected by the isospin symmetry
- The only two channels in $\mathscr{B}_c \to \mathscr{B}P$.

	$\mathcal{B}(\Lambda_c^+ \to \Sigma^0 K^+)$	$\mathcal{B}(\Lambda_c^+ \to \Sigma^+ K_S^0)$
QCD corrections [2]	2(8)	2(4)
MIT bag model [3]	7.2 ± 1.8	7.2 ± 1.8
Diagrammatic analysis [4]	5.5 ± 1.6	9.6 ± 2.4
$SU(3)_F$ flavor symmetry [5]	5.4 ± 0.7	5.4 ± 0.7
IRA method [6]	5.0 ± 0.6	6.2 ± 2.5
PDG 2020 [28]	5.2 ± 0.8	/

Research Motivation - Nonleptonic decays

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PRD **101** 014011 (2020); PRD **97** 074028 (2018)

Cheng, Xu

$\mathscr{B}(\Xi_c^0 \to \Sigma^0 K_s^0)$ is found to be 20 times smaller than $\mathscr{B}(\Xi_c^0 \to \Xi^- \pi^+)$

Cabibbo-favored

Cabibbo-suppressed, $V_{cd}^2 \approx 0.05$

Cabibbo-favored; large destructive interference

[a] Phys. Rev. D **106**, 052003 (2022).
[b] Phys. Rev. Lett. **128**, 142001 (2022).
[c] Phys. Rev. D **100**, 072004 (2019).
[d] arXiv:2208.08695 [hep-ex].
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$100\mathscr{B}(\Xi_c^0 \to \Xi^- \pi^+) = 1.80 \pm 0.52$

Inconsistencies are mainly found in the Ξ_c decays, whereas the Λ_c^+ ones fit well.

Research Motivation - Semileptonic decays

$\Xi_c^0 \to \Xi^- e^+ \nu_e$

$\mathscr{B}_{\text{Belle}} = (1.31 \pm 0.04 \pm 0.07 \pm 0.38)\,\%$

PRL **127** 121803 (2021)

$\mathscr{B}_{\text{ALICE}} = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\,\%$

PRL **127** 272001 (2021)

$\mathscr{B}_{\text{LQCD}} = (2.38 \pm 0.30 \pm 0.32 \pm 0.07)\,\%$

CPC **46** 011002 (2022)

$\mathscr{B}_{SU(3)} = (4.05 \pm 0.15)\,\%$

From $\mathscr{B}(\Lambda_c^+ \to \Lambda e^+ \nu_e) = (3.56 \pm 0.11 \pm 0.07)\,\%$

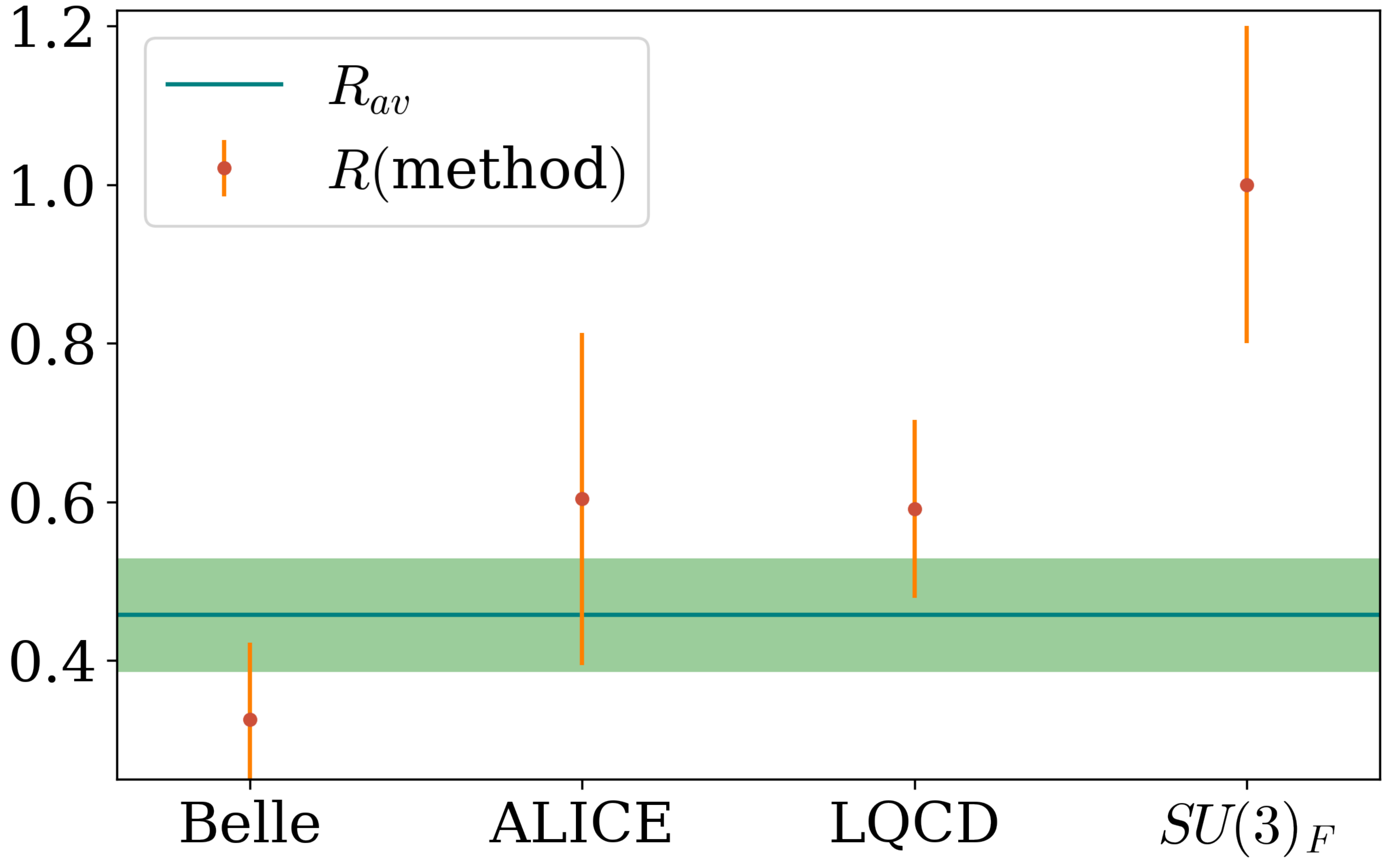
PRL **129**, 231803 (2022)

$$\frac{\Gamma(D_s^+ \to \phi e^+ \nu_e)}{\Gamma(D^+ \to \overline{K}^{*0} e^+ \nu_e)} = 0.91 \pm 0.06\,, \quad \frac{1}{2}\frac{\Gamma(D_s^+ \to K^0 e^+ \nu_e)}{\Gamma(D^+ \to \pi^0 e^+ \nu_e)} = 0.94 \pm 0.10\,,$$

X. G. He, Fei Huang, W. W., Zhi-Peng Xing

PLB **823**, 136765 (2021)

$$R(\text{method}) = \frac{2\tau_{\Lambda_c^+}}{3\tau_{\Xi_c^0}} \frac{\mathscr{B}_{\text{method}}(\Xi_c^0 \to \Xi^- e^+ \nu_e)}{\mathscr{B}(\Lambda_c \to \Lambda e^+ \nu_e)}$$



Research Motivation - S

$\underline{\Xi_c^0 \to \Xi^- e^+ \nu_e}$

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PLB **823**, 136765 (2021)

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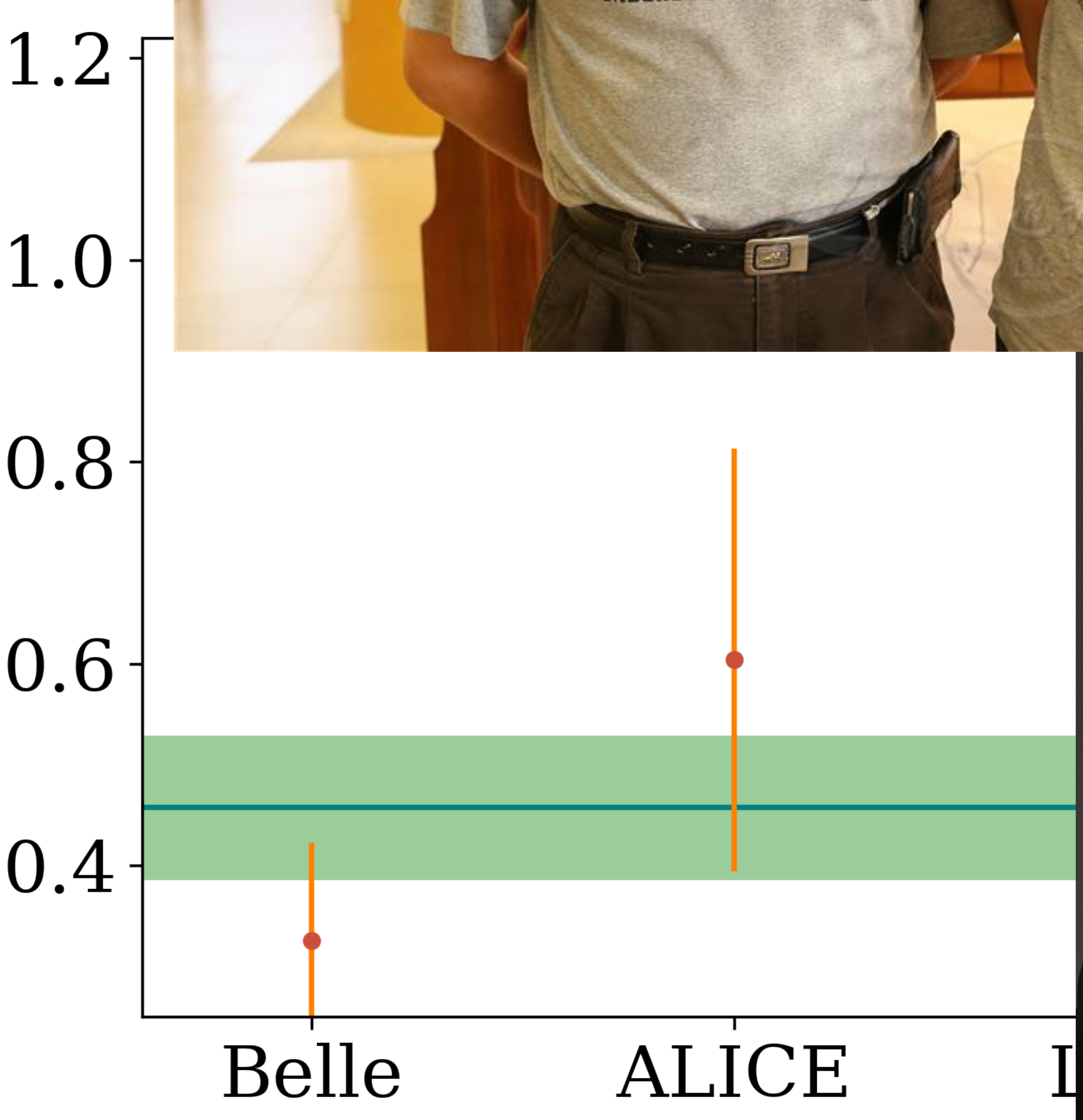
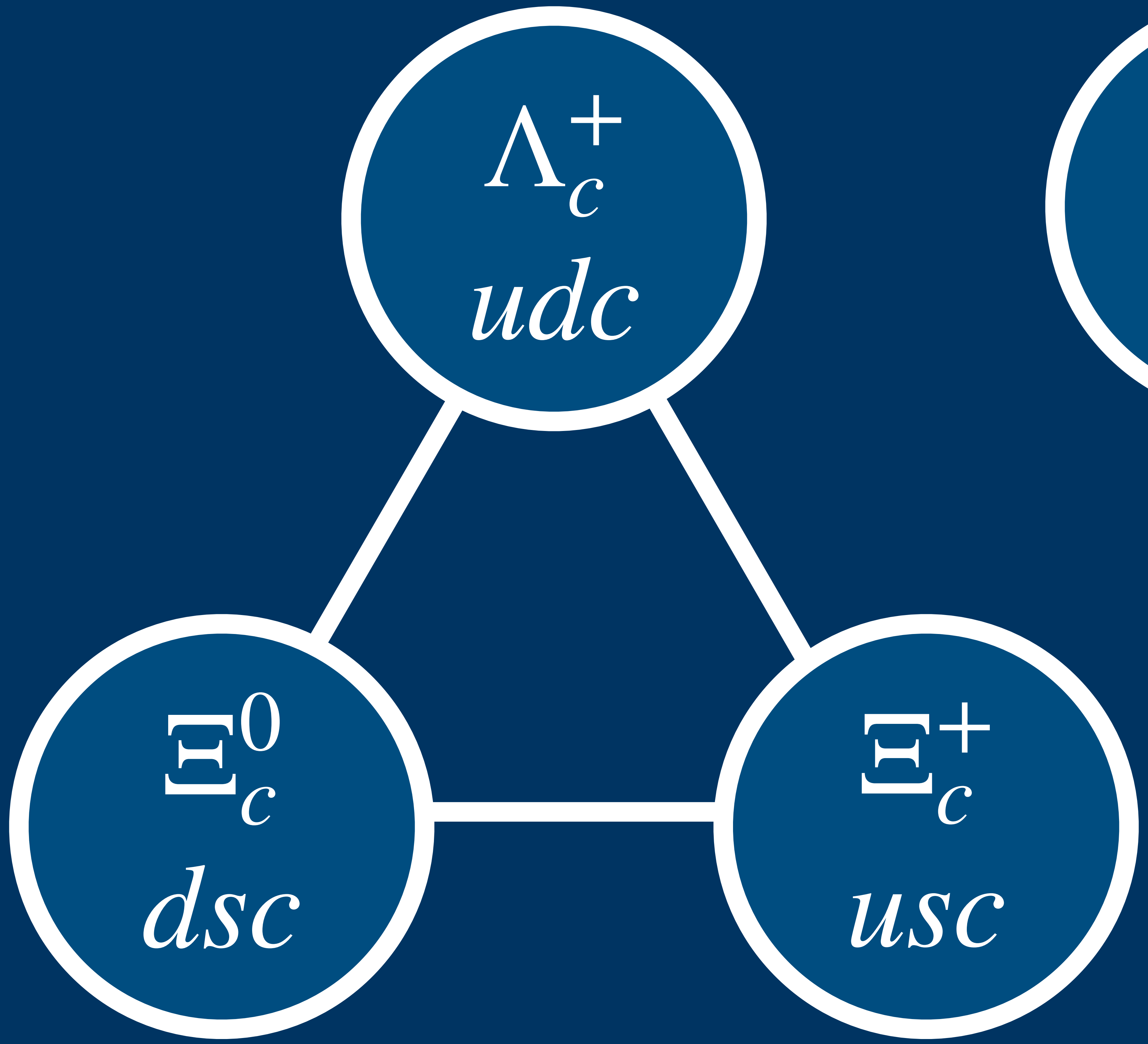
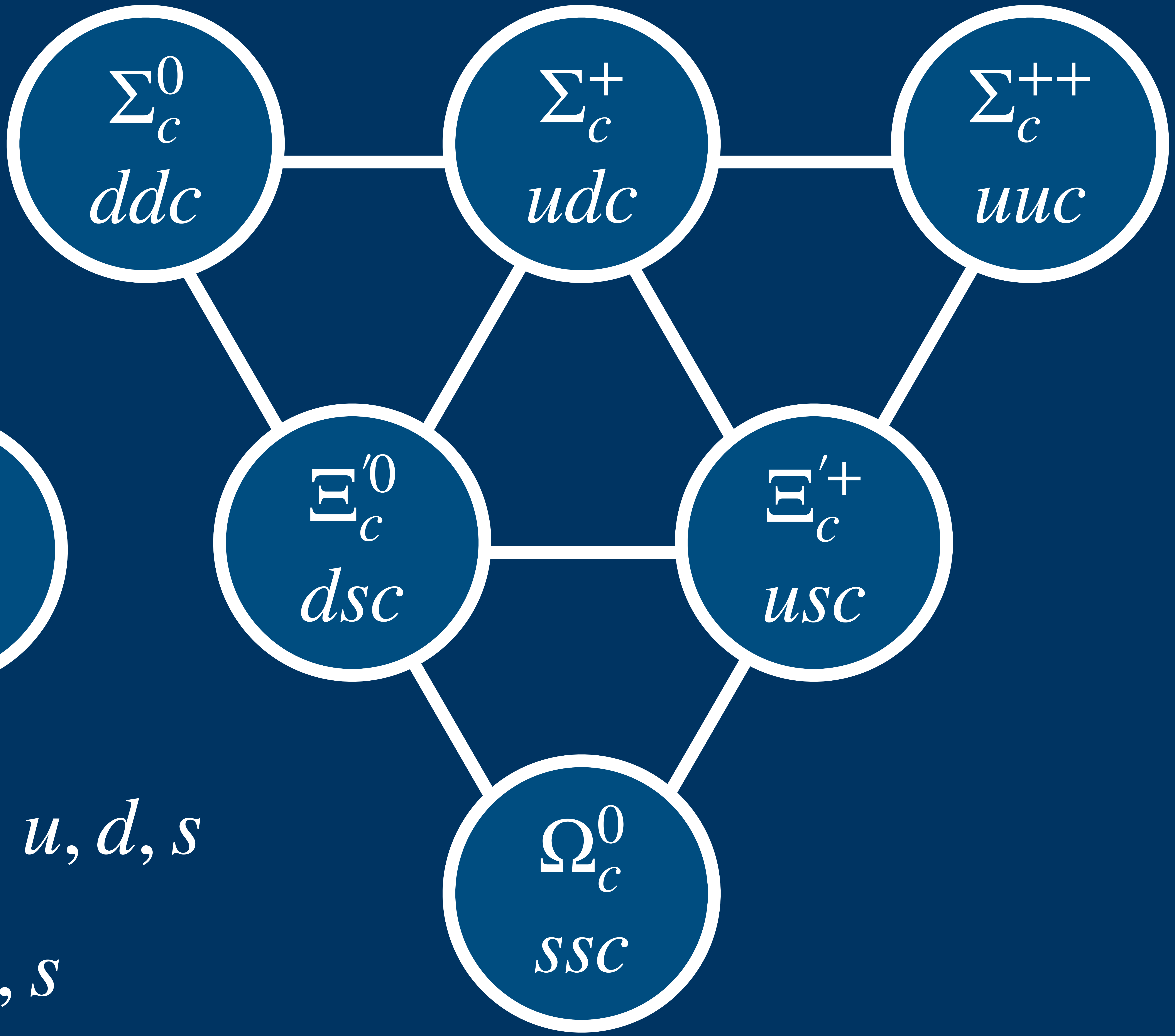


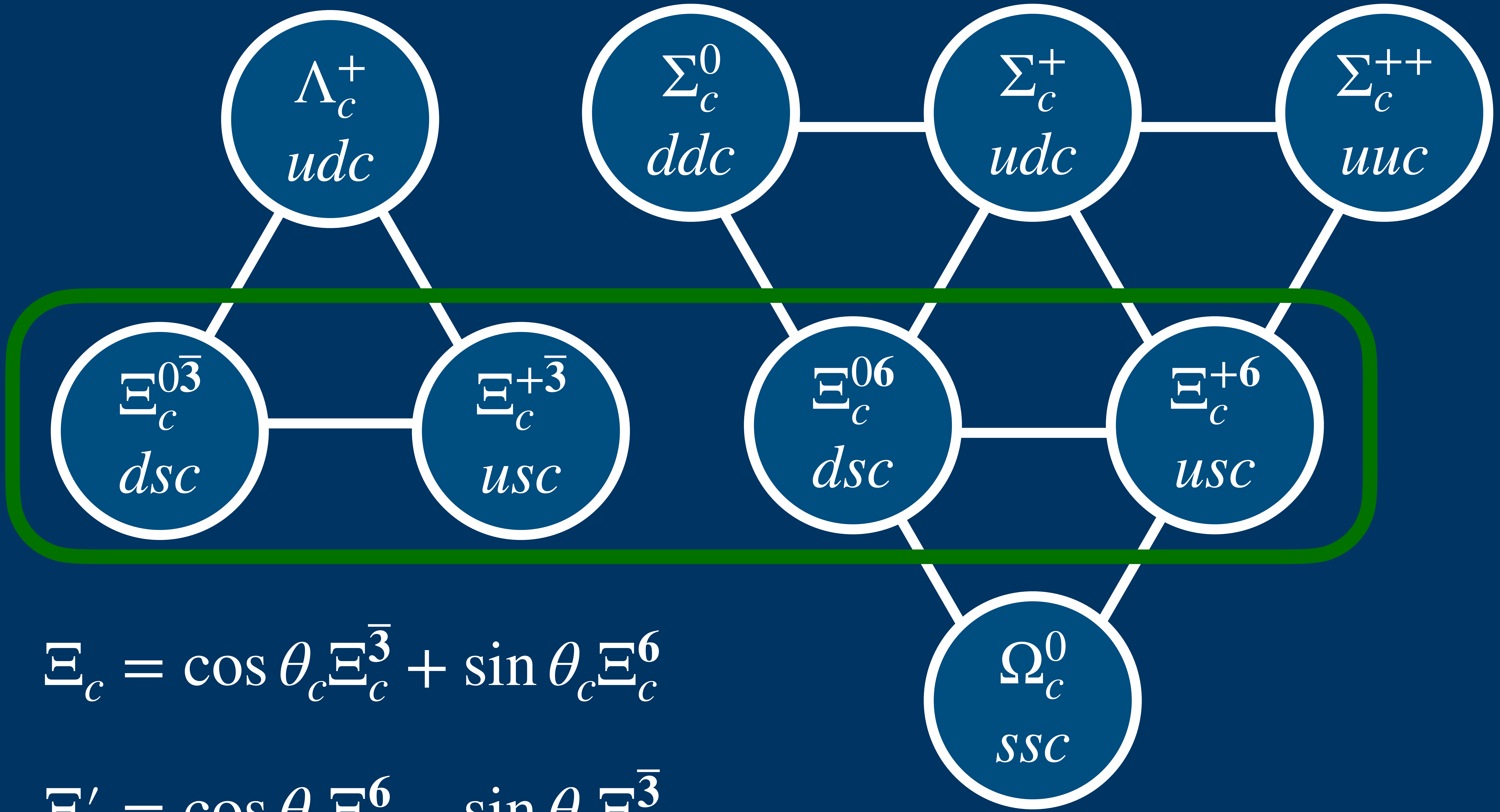




$\overline{\mathbf{3}}$: antisymmetric in u, d, s

$\mathbf{6}$: symmetric in u, d, s

Categorized according to the $SU(3)_F$ group



$$\Xi_c = \cos\theta_c \Xi_c^{\bar{\mathbf{3}}} + \sin\theta_c \Xi_c^{\mathbf{6}}$$

$$\Xi_c' = \cos\theta_c \Xi_c^{\mathbf{6}} - \sin\theta_c \Xi_c^{\bar{\mathbf{3}}} .$$

Mass relations

Model-Independent Bottom Baryon Mass Predictions in the $1/N_c$ Expansion

Elizabeth E. Jenkins

Phys. Rev. D **54**, 4515 (1996), Phys. Rev. D **55**, 10 (1997), Phys. Rev. D **77**, 034012 (2008)

Breaking operators

Orders of errors

$(SU(3), J_Q)$	Mass Combination	Theory	$Q=c$	$Q=b$
$(1,0)$	$\frac{1}{3}\left(\Lambda_Q + 2\Xi_Q^{\bar{\mathbf{3}}}\right)$	$m_Q + N_c\Lambda$	2380*	5687
$(1,0)$	$-\frac{1}{3}\left(\Lambda_Q + 2\Xi_Q^{\bar{\mathbf{3}}}\right) + \frac{1}{18}\left[3\left(\Sigma_Q + 2\Sigma_Q^*\right) + 2\left(\Xi_Q^{\mathbf{6}} + 2\Xi_Q^*\right) + \left(\Omega_Q + 2\Omega_Q^*\right)\right]$	$2\frac{1}{N_c}\Lambda$	207	207
$(1,1)$	$\frac{1}{6}\left[3\left(\Sigma_Q^* - \Sigma_Q\right) + 2\left(\Xi_Q^* - \Xi_Q^{\mathbf{6}}\right) + \left(\Omega_Q^* - \Omega_Q\right)\right]$	$3\frac{1}{N_c}\frac{\Lambda^2}{m_Q}$	66*	20
$(8,0)$	$\left(\Lambda_Q - \Xi_Q^{\bar{\mathbf{3}}}\right)$	$\frac{3}{2\sqrt{3}}(\epsilon\Lambda_\chi)$	−195*	−195
$(8,0)$	$-\frac{5}{8}\left(\Lambda_Q - \Xi_Q^{\bar{\mathbf{3}}}\right) + \frac{1}{24}\left[3\left(\Sigma_Q + 2\Sigma_Q^*\right) - \left(\Xi_Q^{\mathbf{6}} + 2\Xi_Q^*\right) - 2\left(\Omega_Q + 2\Omega_Q^*\right)\right]$	$\frac{1}{2\sqrt{3}}\frac{15}{8}\frac{1}{N_c}(\epsilon\Lambda_\chi)$	40.6	40.6
$(8,1)$	$\Xi_Q^{\overline{\mathbf{36}}}$	$\frac{3}{4}\frac{1}{N_c}\frac{\Lambda}{m_Q}(\epsilon\Lambda_\chi)$	11.8	3.5
$(8,1)$	$\frac{1}{6}\left[3\left(\Sigma_Q^* - \Sigma_Q\right) - \left(\Xi_Q^* - \Xi_Q^{\mathbf{6}}\right) - 2\left(\Omega_Q^* - \Omega_Q\right)\right] - \frac{5}{2\sqrt{3}}\Xi_Q^{\overline{\mathbf{36}}}$	$\frac{1}{2\sqrt{3}}\frac{15}{2}\frac{1}{N_c^2}\frac{\Lambda}{m_Q}(\epsilon\Lambda_\chi)$	11.3	3.4
$(27,0)$	$\frac{1}{6}\left[\left(\Sigma_Q + 2\Sigma_Q^*\right) - 2\left(\Xi_Q^{\mathbf{6}} + 2\Xi_Q^*\right) + \left(\Omega_Q + 2\Omega_Q^*\right)\right]$ Independent of m_Q	$\frac{3}{2}\frac{1}{N_c}(\epsilon^2\Lambda_\chi)$	−4.4*	−4.4
$(27,1)$	$\frac{1}{4}\left[\left(\Sigma_Q^* - \Sigma_Q\right) - 2\left(\Xi_Q^* - \Xi_Q^{\mathbf{6}}\right) + \left(\Omega_Q^* - \Omega_Q\right)\right]$	$\frac{9}{8}\frac{1}{N_c^2}\frac{\Lambda}{m_Q}(\epsilon^2\Lambda_\chi)$	0.23	0.07

$(\Lambda, \epsilon\Lambda_\chi) \approx (300,\ 225)$ MeV

$$\epsilon = \frac{1}{4} \sim \frac{m_s}{\Lambda_\chi}$$

$\Xi_c^{\mathbf{6}} = 2.5600(11)$ GeV

$\Xi_c' = 2.5784(4)$ GeV

$\Xi_b^{\mathbf{6}} = 5.9315(18)$ GeV

$\Xi_b' = 5.93502(5)$ GeV

$\Xi_Q^{\mathbf{6}} = \Xi_Q'$ if $\theta_Q = 0$

Improved equal spacing rule

$$\Xi_c^{\mathbf{6}} = \Xi_c^* - \frac{1}{2}\left(\Sigma_c^* - \Sigma_c + \Omega_c^* - \Omega_c\right)$$

$$\Xi_b^{\mathbf{6}} = \frac{1}{2}\left(\Sigma_b + \Omega_b\right) - \frac{1}{2}\left(\Sigma_c^* - 2\Xi_c^* + \Omega_c^*\right)$$

Mass relations

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$(1,1)$	$\frac{1}{6}\left[3\left(\Sigma_Q^*-\Sigma_Q\right)+2\left(\Xi_Q^*-\Xi_Q^{\mathbf{6}}\right)+\left(\Omega_Q^*-\Omega_Q\right)\right]$	$3\frac{1}{N_c}\frac{\Lambda^2}{m_Q}$	66*	20
$(8,0)$	$\left(\Lambda_Q-\Xi_Q^{\bar{\mathbf{3}}}\right)$	$\frac{3}{2\sqrt{3}}(\epsilon\Lambda_\chi)$	-195*	-195
$(8,0)$	$-\frac{5}{8}\left(\Lambda_Q-\Xi_Q^{\bar{\mathbf{3}}}\right)+\frac{1}{24}\left[3\left(\Sigma_Q+2\Sigma_Q^*\right)-\left(\Xi_Q^{\mathbf{6}}+2\Xi_Q^*\right)-2\left(\Omega_Q+2\Omega_Q^*\right)\right]$	$\frac{1}{2\sqrt{3}}\frac{15}{8}\frac{1}{N_c}(\epsilon\Lambda_\chi)$	40.6	40.6
$(8,1)$	$\Xi_Q^{\overline{\mathbf{36}}}$	$\frac{3}{4}\frac{1}{N_c}\frac{\Lambda}{m_Q}(\epsilon\Lambda_\chi)$	11.8	3.5
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$(\Lambda, \epsilon\Lambda_\chi) \approx (300,\ 225)$ MeV

$$\epsilon = \frac{1}{4} \sim \frac{m_s}{\Lambda_\chi}$$

$\Xi_c^{\mathbf{6}} = 2.5600(11)$ GeV

$|\theta_c| = 0.137(5)\pi$

$\Xi_b^{\mathbf{6}} = 5.9315(18)$ GeV

$|\theta_b| = 0.049(13)\pi$

$\theta_b/\theta_c = m_c/m_b \approx 0.3$

$\Xi_c^{\overline{\mathbf{36}}} \approx 40$ MeV

Improved equal spacing rule

$$\Xi_c^{\mathbf{6}} = \Xi_c^* - \frac{1}{2}\left(\Sigma_c^* - \Sigma_c + \Omega_c^* - \Omega_c\right)$$

$$\Xi_b^{\mathbf{6}} = \frac{1}{2}\left(\Sigma_b + \Omega_b\right) - \frac{1}{2}\left(\Sigma_c^* - 2\Xi_c^* + \Omega_c^*\right)$$

Mass relations

obtained from mass relations : $|\theta_c| \approx 25^\circ$

Mixing Angle of $\Xi_Q - \Xi'_Q$
in Heavy Quark Effective Therory

Yoshimitsu Matsui *
Faculty of Law, Aichi University,
Nagoya, Aichi 453-8777, Japan

$$
\begin{aligned}
M(\Xi_Q) &= m_Q + \cos^2\theta_Q\,\bar{\Lambda}'_{0+} + \sin^2\theta_Q\,\bar{\Lambda}'_{1+} \\
&\quad - \cos^2\theta_Q \frac{\lambda_1^{0^+}}{2m_Q} - \sin^2\theta_Q\left(\frac{\lambda_1^{1^+}}{2m_Q} \overset{+}{-} 4\frac{\lambda_2^{1^+}}{2m_Q}\right) \\
M(\Xi'_Q) &= m_Q + \sin^2\theta_Q\,\bar{\Lambda}'_{0+} + \cos^2\theta_Q\,\bar{\Lambda}'_{1+} \\
&\quad - \sin^2\theta_Q \frac{\lambda_1^{0^+}}{2m_Q} - \cos^2\theta_Q\left(\frac{\lambda_1^{1^+}}{2m_Q} \overset{+}{-} 4\frac{\lambda_2^{1^+}}{2m_Q}\right) \\
M(\Xi^*_Q) &= m_Q + \bar{\Lambda}'_{1+} - \frac{\lambda_1^{1^+}}{2m_Q} + 2\frac{\lambda_2^{1^+}}{2m_Q}.
\end{aligned}
$$

Assume $\lambda'_2 = \lambda_2$, where $\lambda_2^{(\prime)}$ describes the chromo-interaction among the heavy and light quarks.

HQET : $\left|\theta_c\right| = 8.12^\circ \pm 0.80^\circ$

$$
\begin{aligned}
M(\Lambda_Q) &= m_Q + \bar{\Lambda}_0 - \frac{\lambda_1}{2m_Q} \\
M(\Sigma_Q) &= m_Q + \bar{\Lambda}_1 - \frac{\lambda_1}{2m_Q} - 4\frac{\lambda_2}{2m_Q} \\
M(\Sigma^*_Q) &= m_Q + \bar{\Lambda}_1 - \frac{\lambda_1}{2m_Q} + 2\frac{\lambda_2}{2m_Q}. \\
M(\Omega_Q) &= m_Q + \bar{\Lambda}''_1 - \frac{\lambda_1}{2m_Q} - 4\frac{\lambda'_2}{2m_Q} \\
M(\Omega^*_Q) &= m_Q + \bar{\Lambda}''_1 - \frac{\lambda_1}{2m_Q} + 2\frac{\lambda'_2}{2m_Q} \\
M(\Xi_Q) &= m_Q + \cos^2\theta_Q\bar{\Lambda}'_0 + \sin^2\theta_Q\bar{\Lambda}'_1 \\
&\quad - \cos^2\theta_Q\frac{\lambda_1}{2m_Q} - \sin^2\theta_Q\left(\frac{\lambda_1}{2m_Q} + \frac{\lambda'_2+\lambda_2}{m_Q}\right) \boxed{- \sin(2\theta_Q)\sqrt{3}\frac{(\lambda'_2-\lambda_2)}{4m_Q}} \\
M(\Xi'_Q) &= m_Q + \sin^2\theta_Q\bar{\Lambda}'_0 + \cos^2\theta_Q\bar{\Lambda}'_1 \\
&\quad - \sin^2\theta_Q\frac{\lambda_1}{2m_Q} - \cos^2\theta_Q\left(\frac{\lambda_1}{2m_Q} + \frac{\lambda'_2+\lambda_2}{m_Q}\right) \boxed{+ \sin(2\theta_Q)\sqrt{3}\frac{(\lambda'_2-\lambda_2)}{4m_Q}} \\
M(\Xi^*_Q) &= m_Q + \bar{\Lambda}'_1 - \frac{\lambda_1}{2m_Q} + \frac{\lambda'_2+\lambda_2}{2m_Q}
\end{aligned}
$$

$\theta_Q \propto \dfrac{m_s}{m_Q}$

Missing in the paper $O\left(\dfrac{m_s}{m_Q}\right)$

HQET after revised* : $\theta_c \approx -24^\circ$

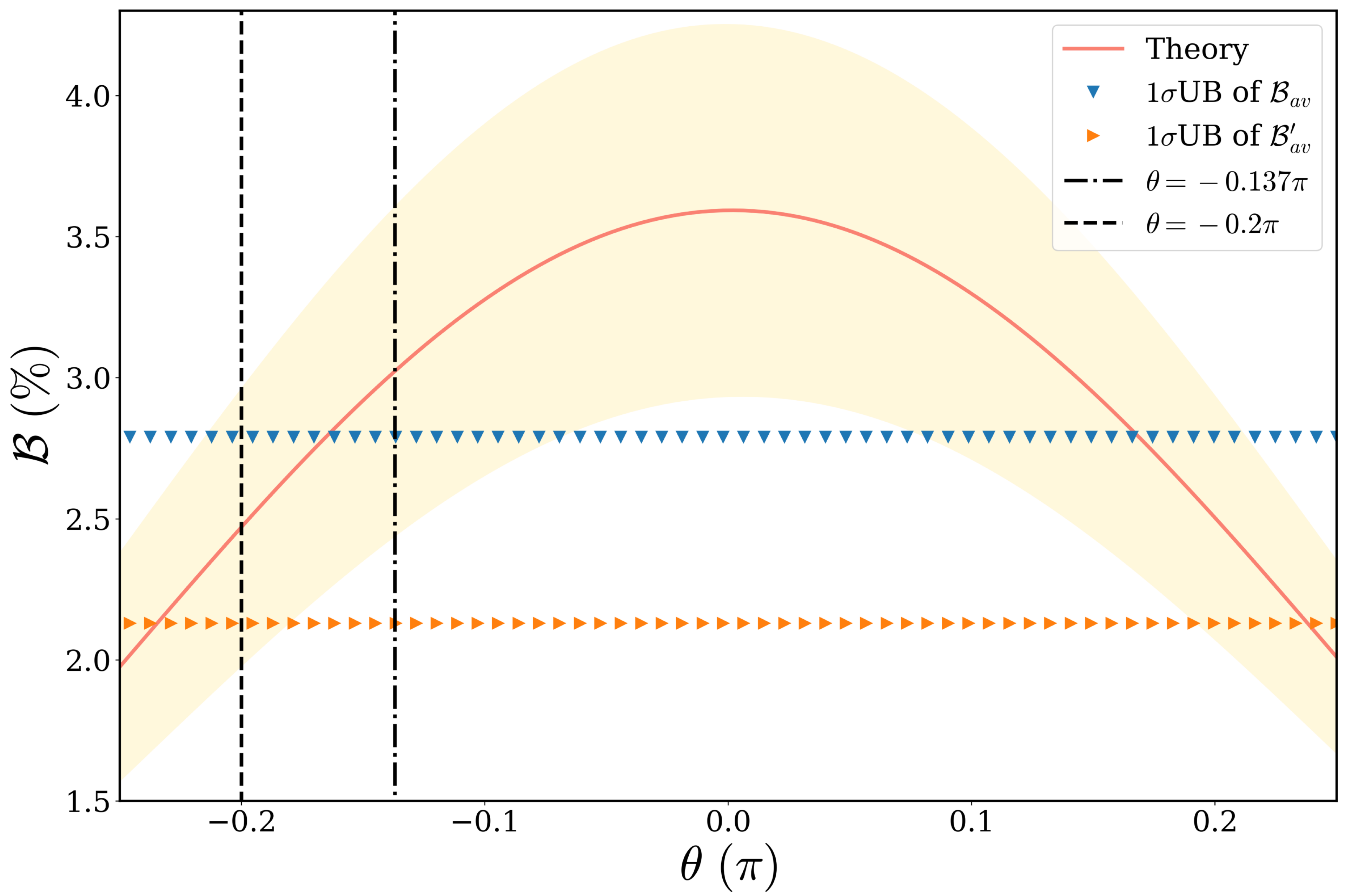
*The sign depends on the convention

The mixing effects in Ξ_c semileptonic decays

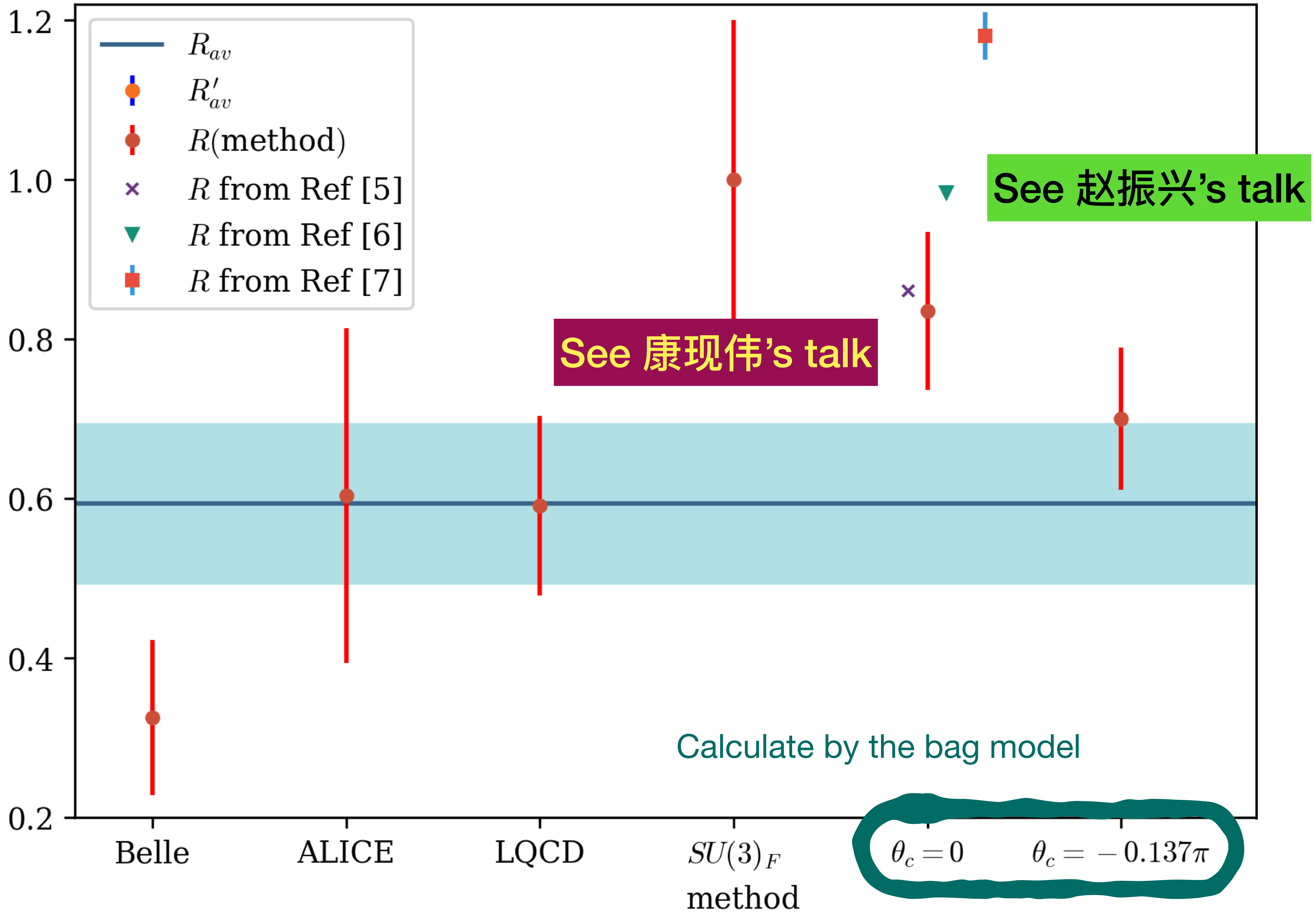
$$\mathscr{A}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = \cos\theta_c \mathscr{A}(\Xi_c^{0\bar{3}} \to \Xi^- e^+ \nu_e)$$
$$+\sin\theta_c \mathscr{A}(\Xi_c^{06} \to \Xi^- e^+ \nu_e)$$

$$\mathscr{A}(\Xi_c^0 \to \Xi^- e^+ \nu_e) \approx \cos\theta_c \mathscr{A}(\Xi_c^{0\bar{3}} \to \Xi^- e^+ \nu_e)$$

$$\Gamma \to \cos^2\theta_c \Gamma \approx 0.8\Gamma$$



See 邢志鹏's talk



Relativistic quark model Ref. [5] Particles **3**, 208 (2020)

Light-front quark model Ref. [6] CPC **42**, 093101 (2018)

Ref. [7] PRD **103**, 054018 (2021)

The mixing effects in Ξ_c semileptonic decays

$$\mathscr{A}(\Xi_c \to \Xi' e^+ \nu_e) = \cos\theta_c \mathscr{A}(\Xi_c^{\bar{\mathbf{3}}} \to \Xi' e^+ \nu_e) + \sin\theta_c \mathscr{A}(\Xi_c^{\mathbf{6}} \to \Xi' e^+ \nu_e)$$

$$= \sin\theta_c \mathscr{A}(\Xi_c^{\mathbf{6}} \to \Xi' e^+ \nu_e)$$

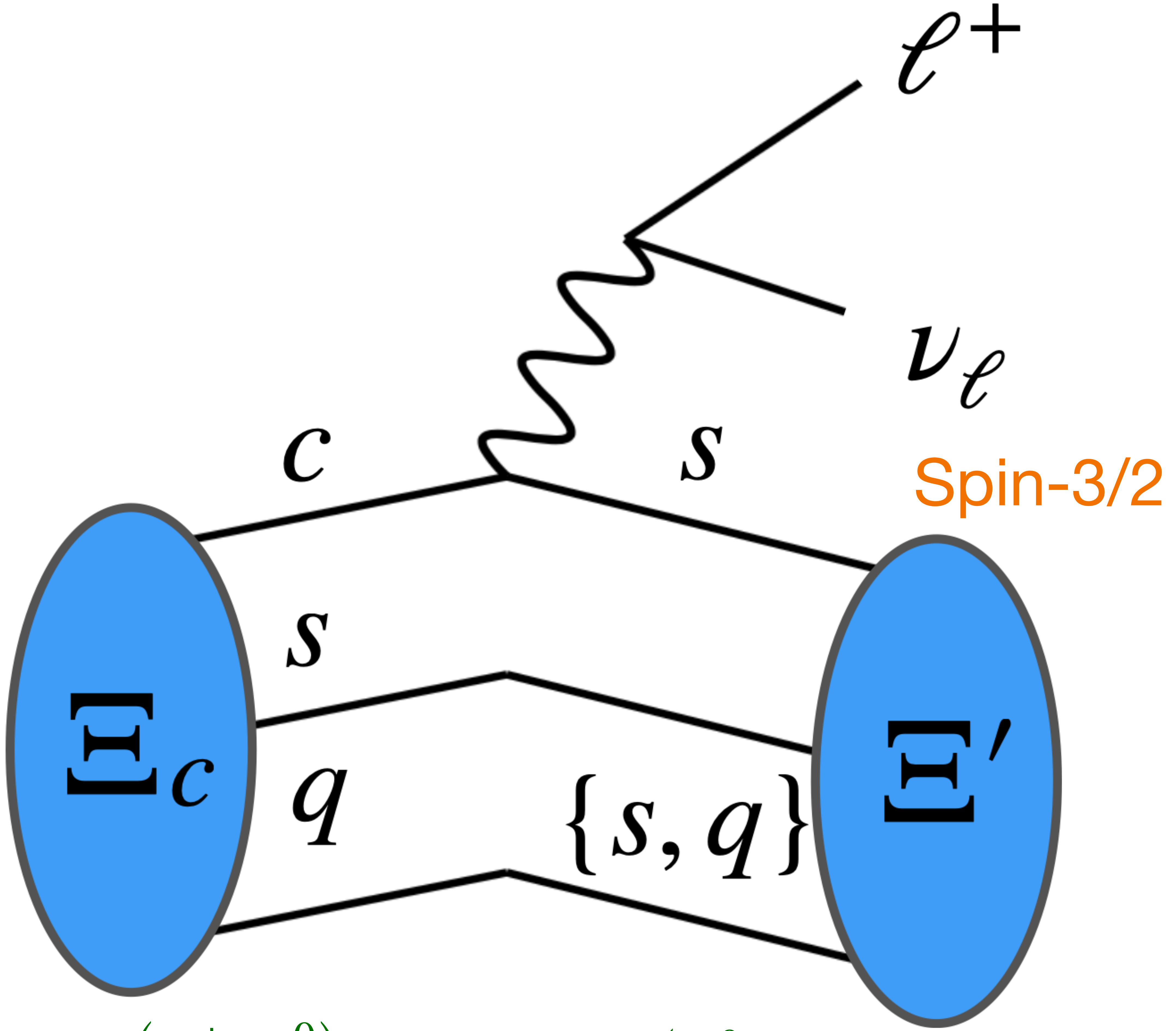
$$\Gamma(\Xi_c \to \Xi e^+ \nu_e) = \sin^2\theta_c \Gamma(\Xi_c^{\mathbf{6}} \to \Xi' e^+ \nu_e)$$

Forbidden for $\theta_c = 0$

Prediction

$\mathscr{B}(\Xi_c^0 \to \Xi'^- e^+ \nu_e) = (4.4 \pm 0.5) \times 10^{-3}$

$\mathscr{B}(\Xi_c^+ \to \Xi'^0 e^+ \nu_e) = (1.3 \pm 0.2)\,\%$



$\Xi_c = \left(\Xi_c^+, \Xi_c^0\right)$ $\Xi' = \left(\Xi^0(1530), \Xi^-(1530)\right)$

Can be recommended to lattice QCD.

The mixing effects in Ξ_c nonleptonic decays

See 徐繁荣's slide $(6.88^{+16.62}_{-16.04})^\circ$

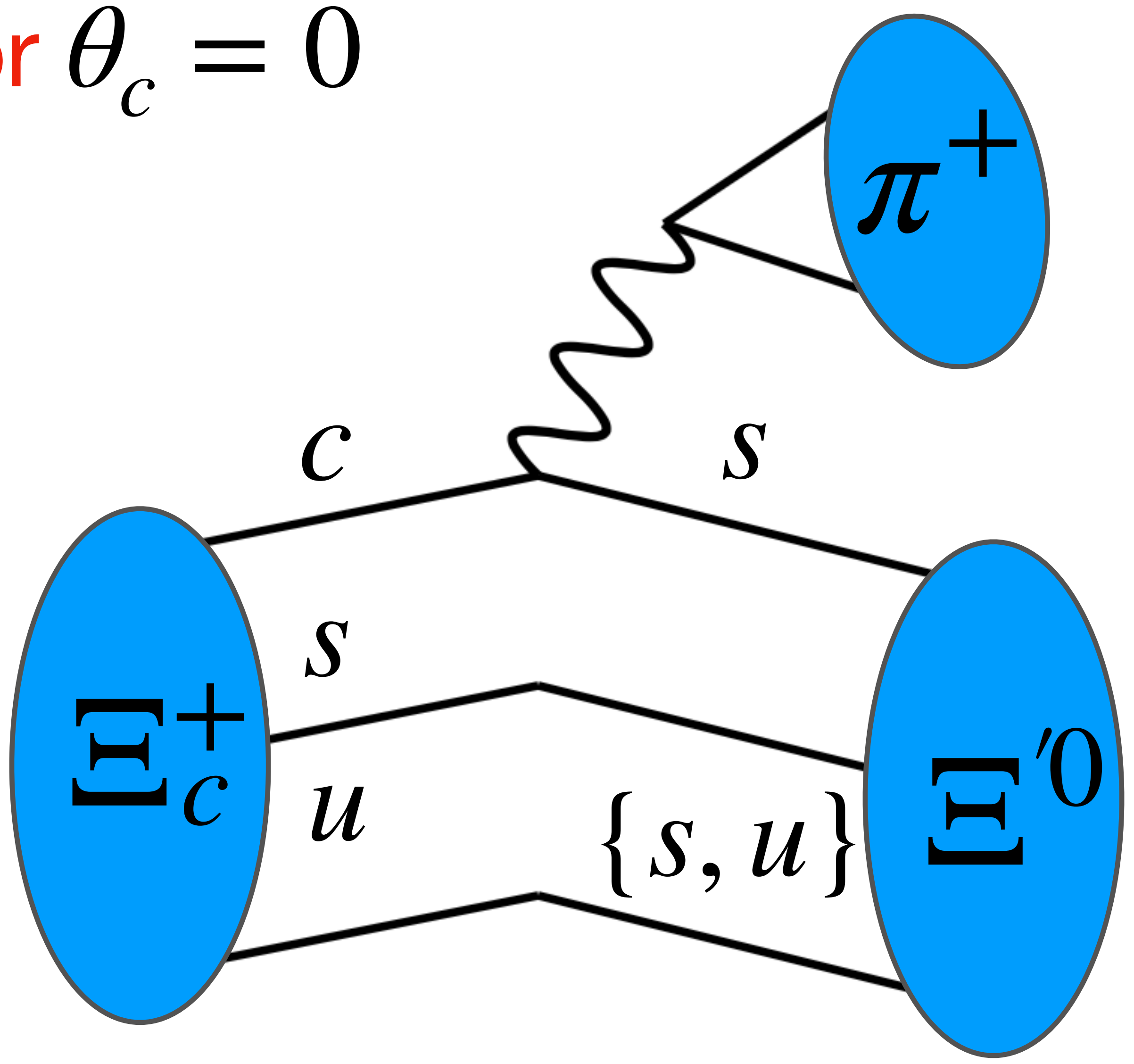
Might solve the discrepancy

$100\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)^e$	1.43 ± 0.32	2.21 ± 0.11	6.47
$100\mathcal{R}(\Xi_c^0 \to \Xi^- K^+)$	$2.75 \pm 0.51 \pm 0.25$	4.4	6.0
$100\mathcal{R}(\Xi_c^0 \to \Sigma^0 K_s^0)^f$	$3.8 \pm 0.6 \pm 0.4$	2.3 ± 1.8	< 0.4

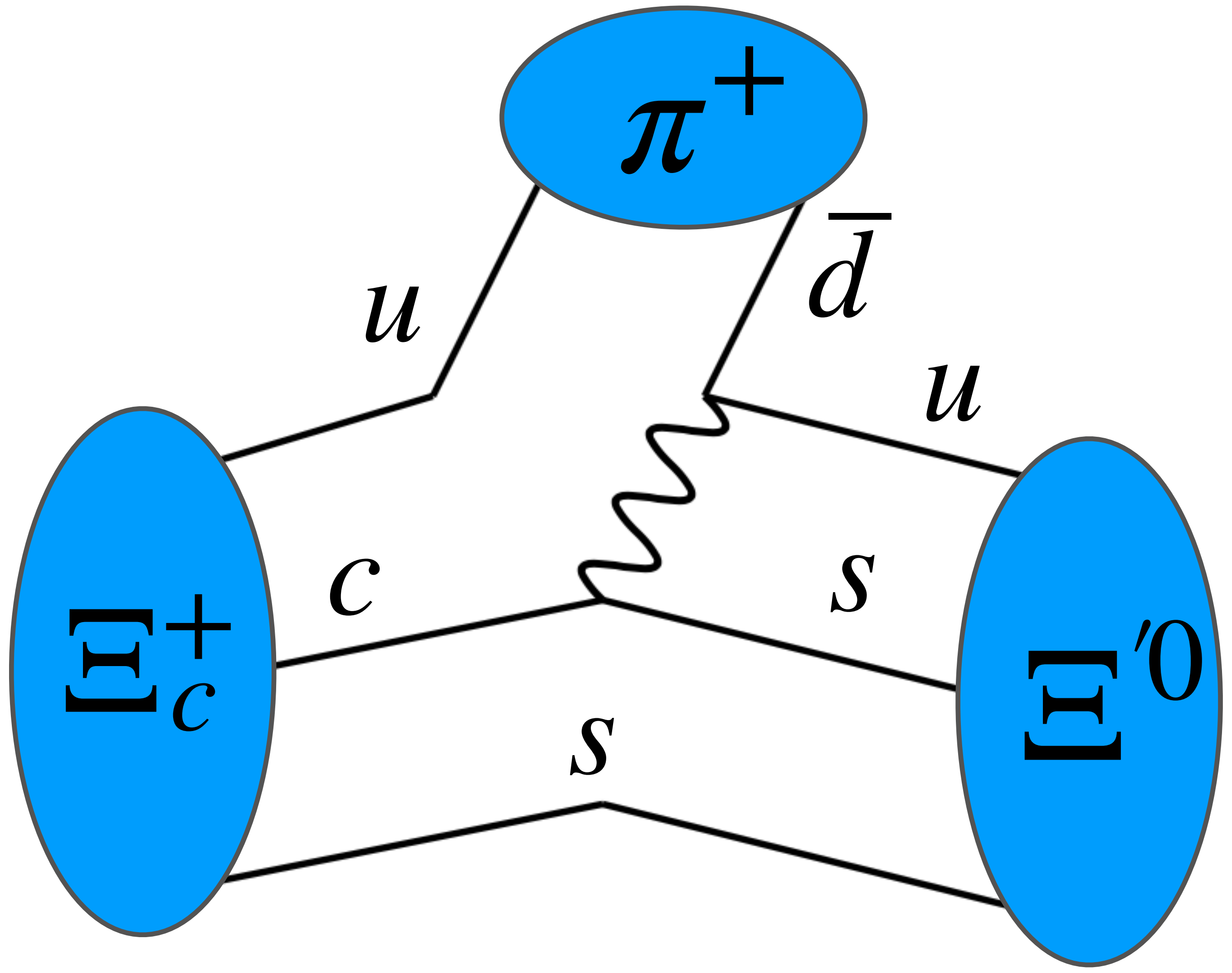
$$V_{cs}\mathscr{A}(\Xi_c^0 \to \Xi^-\pi^+) \neq V_{cd}\mathscr{A}(\Xi_c^0 \to \Xi^- K^+)$$

$$\mathscr{B}\left(\Xi_c^+ \to \Xi'^0\pi^+\right) = \sin^2\theta_c \mathscr{B}\left(\Xi_c^{6+} \to \Xi'^0\pi^+\right) = (3.8 \pm 0.5) \times 10^{-3}$$

Forbidden for $\theta_c = 0$



Forbidden by Körner-Pati-Woo theorem



Phys. Rev. D **46**, 3836 (1992); Z. Phys. C **55**, 659 (1992); Phys. Rev. D **55**, 7067 (1997)

The mixing ef

Either there is mixing or the KPW theorem is invalid

Observation of $\Xi(1620)^0$ and Evidence for $\Xi(1690)^0$ in $\Xi_c^+ \to \Xi^-\pi^+\pi^+$ Decays

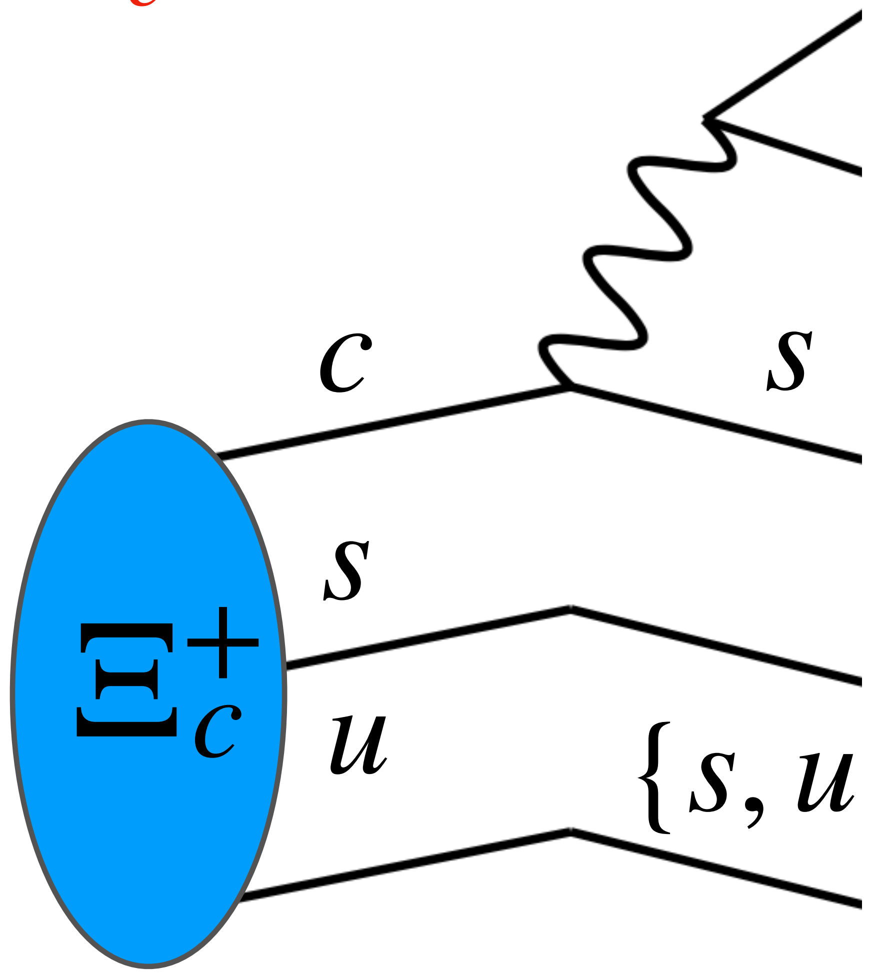
See 徐繁荣's slide

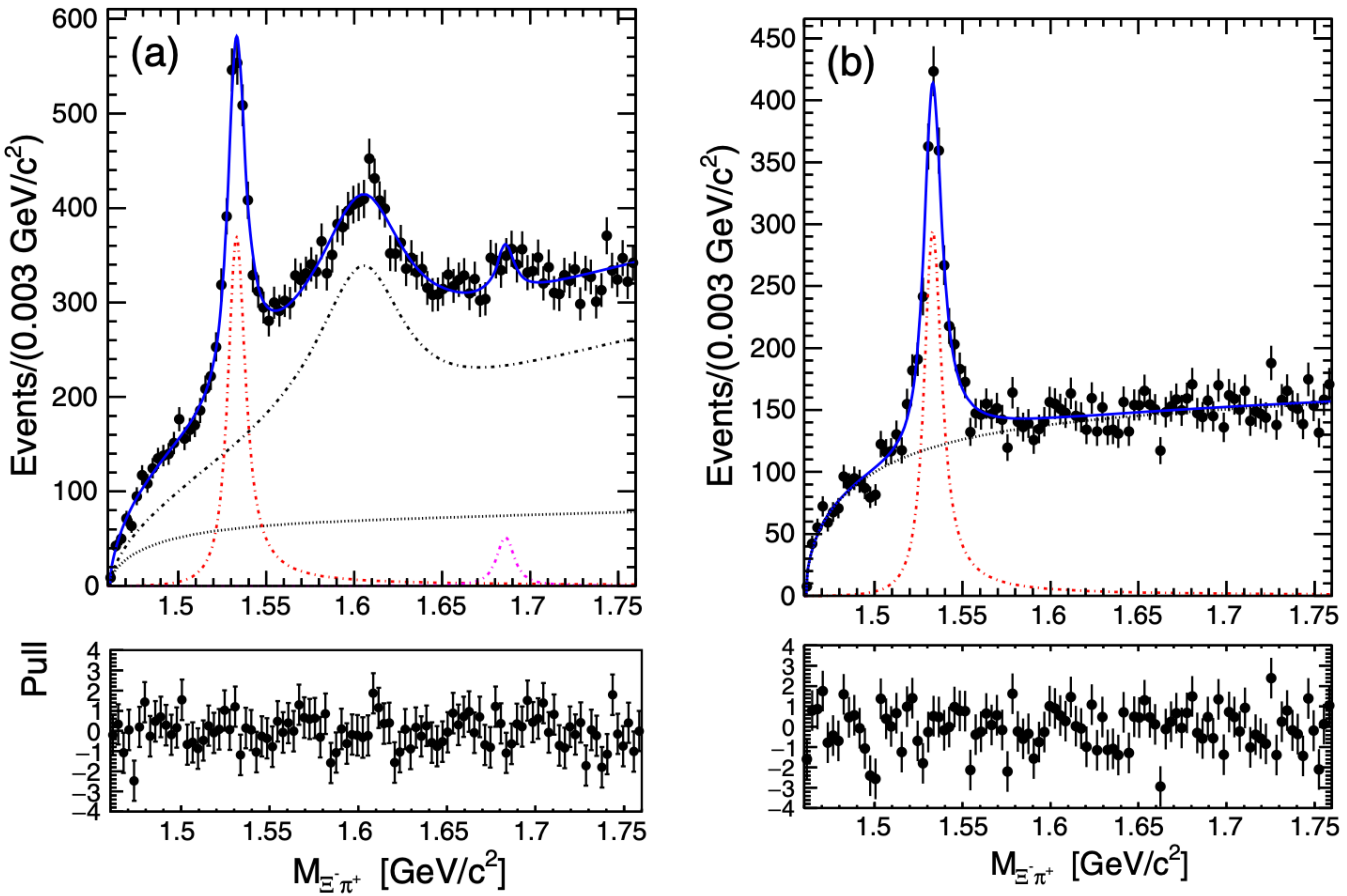
$(6.88^{+1}_{-1}$

$V_{cs}\mathscr{A}(\Xi_c^0 \to \Xi^-\pi^+) \neq$

$\mathscr{B}\left(\Xi_c^+ \to \Xi'^0\pi^+\right)$

Forbidden for $\theta_c = 0$





The mixing effects in Ξ_{cc} semileptonic decays

$$\mathscr{R}(\theta_c) = \frac{\Gamma'}{\Gamma} = \frac{\Gamma(\Xi_{cc} \to \Xi_c' e^+ \nu_e)}{\Gamma(\Xi_{cc} \to \Xi_c e^+ \nu_e)}$$

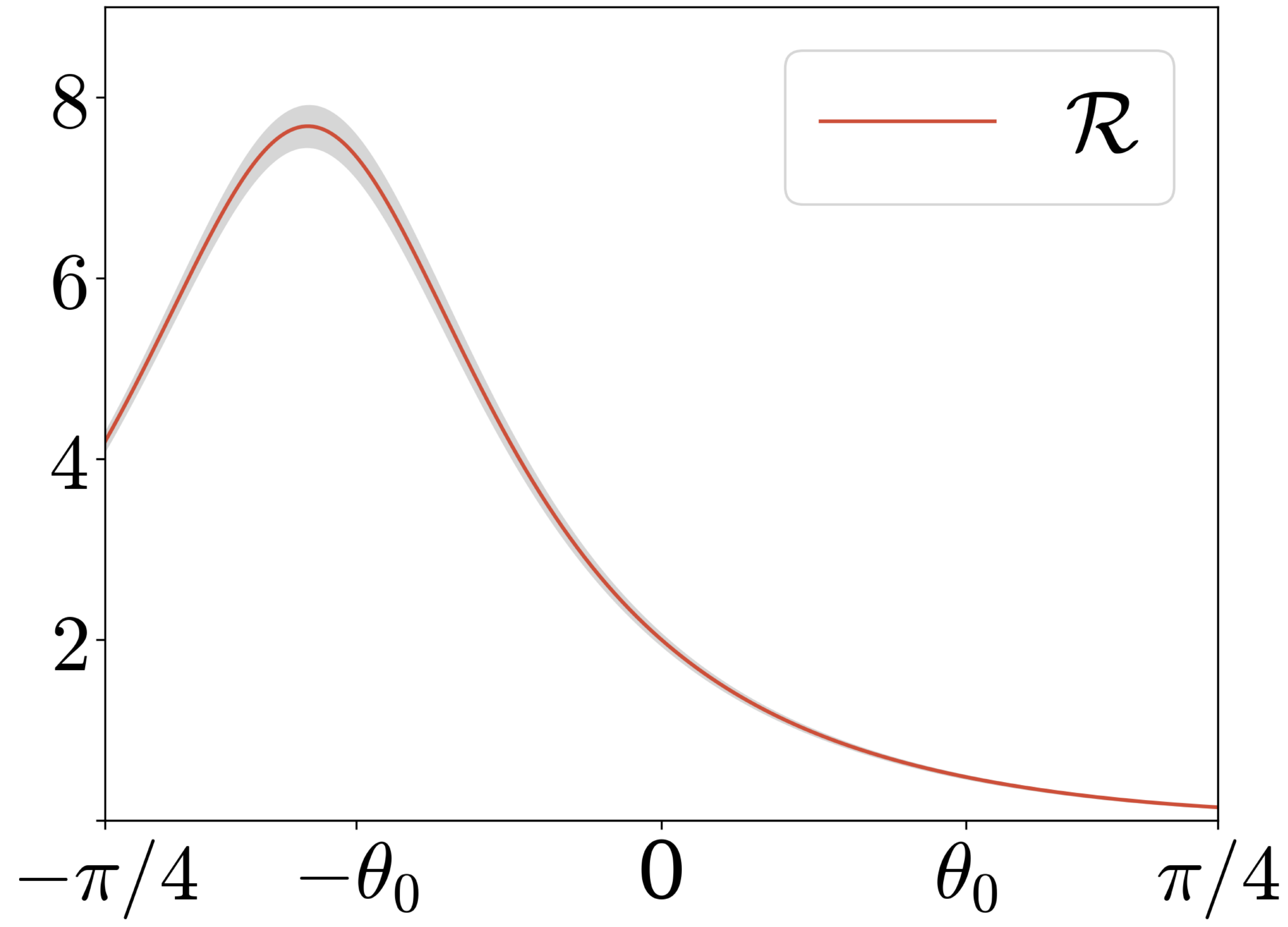
$$\alpha^{(\prime)} = \frac{\Gamma^{(\prime)}(\lambda = 1/2) - \Gamma^{(\prime)}(\lambda = -1/2)}{\Gamma^{(\prime)}(\lambda = 1/2) + \Gamma^{(\prime)}(\lambda = -1/2)}$$

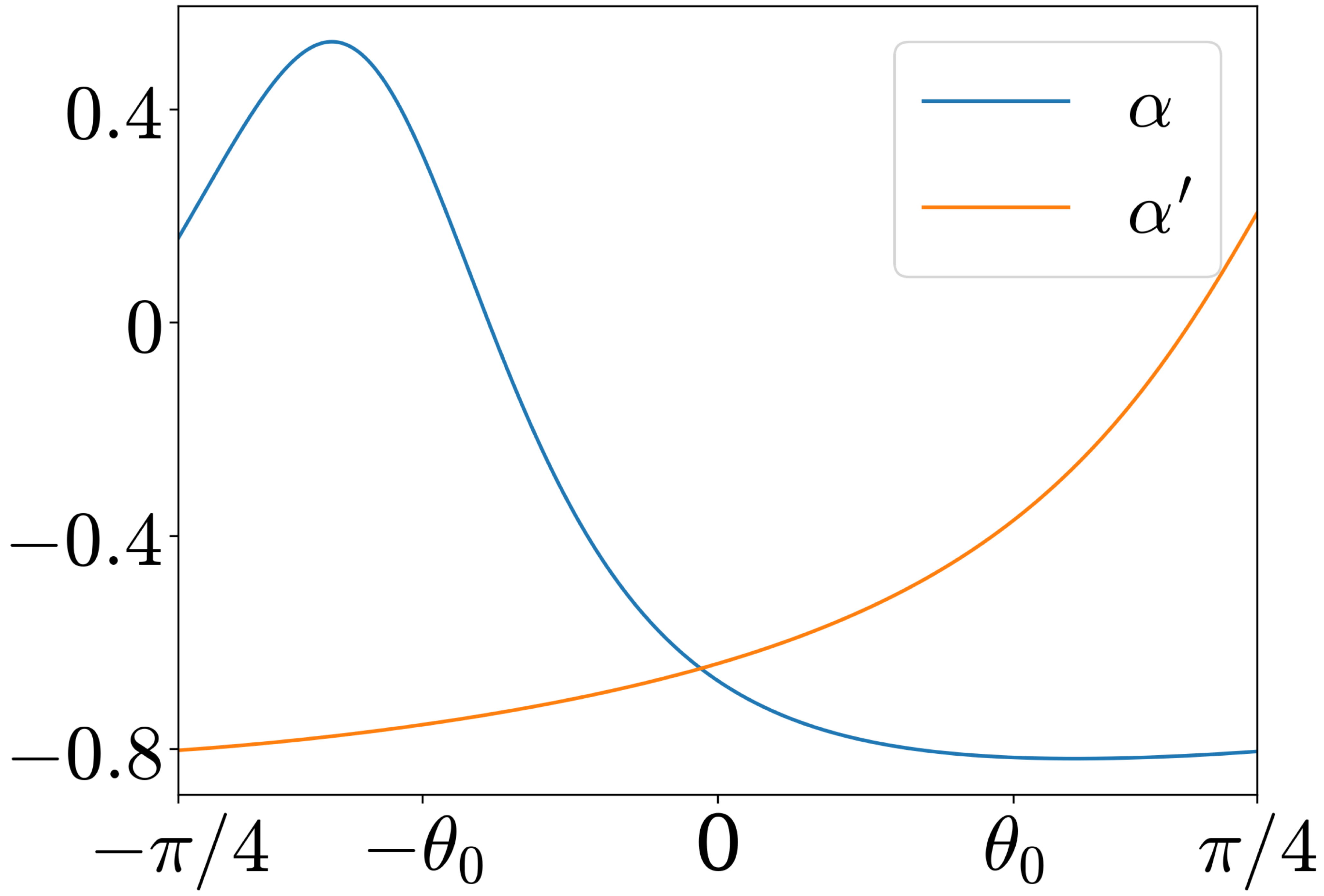
For $\theta_c = -0.137\pi$

$\mathscr{R}, \alpha, \alpha' = 7.33 \pm 0.23,\ 0.32,\ -0.76$

For $\theta_c = 0.137\pi$

$\mathscr{R}, \alpha, \alpha' = 0.46 \pm 0.01,\ -0.82,\ -0.38$





The mixing effects in Ξ_{cc} nonleptonic decays

Current algebra (S wave) + Pole model (P wave)

Table from PRD **101** 014011 (2020)

Channel	$A^{\rm fac}$	$A^{\rm com}$	$A^{\rm tot}$	$B^{\rm fac}$	$B^{\rm ca}$	$B^{\rm tot}$	$\mathcal{B}_{\rm theo}$	$\mathcal{B}_{\rm exp}$ [7]	$\alpha_{\rm theo}$	$\alpha_{\rm exp}$
$\Lambda_c^+ \to p\overline{K}^0$	3.45	4.48	7.93	-6.98	-2.06	-9.04	2.11×10^{-2}	$(3.18\pm0.16)10^{-2}$	-0.75	0.18 ± 0.45
$\Lambda_c^+ \to \Lambda\pi^+$	5.34	0	5.34	-14.11	3.60	-10.51	1.30×10^{-2}	$(1.30\pm0.07)10^{-2}$	-0.93	-0.84 ± 0.09
$\Lambda_c^+ \to \Sigma^0\pi^+$	0	7.68	7.68	0	-11.38	-11.38	2.24×10^{-2}	$(1.29\pm0.07)10^{-2}$	-0.76	-0.73 ± 0.18
$\Lambda_c^+ \to \Sigma^+\pi^0$	0	-7.68	-7.68	0	11.34	11.34	2.24×10^{-2}	$(1.25\pm0.10)10^{-2}$	-0.76	-0.55 ± 0.11
$\Lambda_c^+ \to \Xi^0K^+$	0	-4.48	-4.48	0	-12.10	-12.10	0.73×10^{-2}	$(0.55\pm0.07)10^{-2}$	0.90	
$\Lambda_c^+ \to \Sigma^+\eta$	0	3.10	3.10	0	-15.54	-15.54	0.74×10^{-2}	$(0.53\pm0.15)10^{-2}$	-0.95	
$\Lambda_c^+ \to p\pi^0$	0.41	-0.81	-0.40	-0.87	2.07	1.21	1.26×10^{-4}	$<2.7\times10^{-4}$	-0.97	
$\Lambda_c^+ \to p\eta$	-0.96	-1.11	-2.08	1.93	-0.34	1.59	1.28×10^{-3}	$(1.24\pm0.29)10^{-3}$	-0.55	
$\Lambda_c^+ \to n\pi^+$	1.64	-1.15	0.50	-3.45	2.93	-0.52		–		
$\Lambda_c^+ \to \Lambda K^+$	1.66	-0.08	1.58	-4.43	0.55	-3.70	1.07×10^{-3}	$(6.1\pm1.2)10^{-4}$	-0.96	
$\Lambda_c^+ \to \Sigma^0K^+$	0	1.49	1.49	0	-2.29	-2.29	7.23×10^{-4}	$(5.2\pm0.8)10^{-4}$	-0.73	
$\Lambda_c^+ \to \Sigma^+K^0$	0	2.10	2.10	0	-3.24	-3.24	1.44×10^{-3}	–	-0.73	

$$\mathscr{R}(\Xi_{cc}\to\Xi_c\pi) \equiv \frac{\mathscr{B}(\Xi_{cc}\to\Xi_c'\pi)}{\mathscr{B}(\Xi_{cc}\to\Xi_c\pi)}$$

$$\mathscr{R}(\Xi_{cc}^{++}\to\Xi_c^+\pi^+) = 1.41\pm0.17\pm0.10$$

LHCb, JHEP **46** 011002 (2022)

$$\mathscr{R}(\Xi_{cc}^{++}\to\Xi_c^+\pi^+) = 6.74$$

Current algebra, PRD **01** 034034 (2022)

The mixing effects in Ξ_{cc} nonleptonic decays

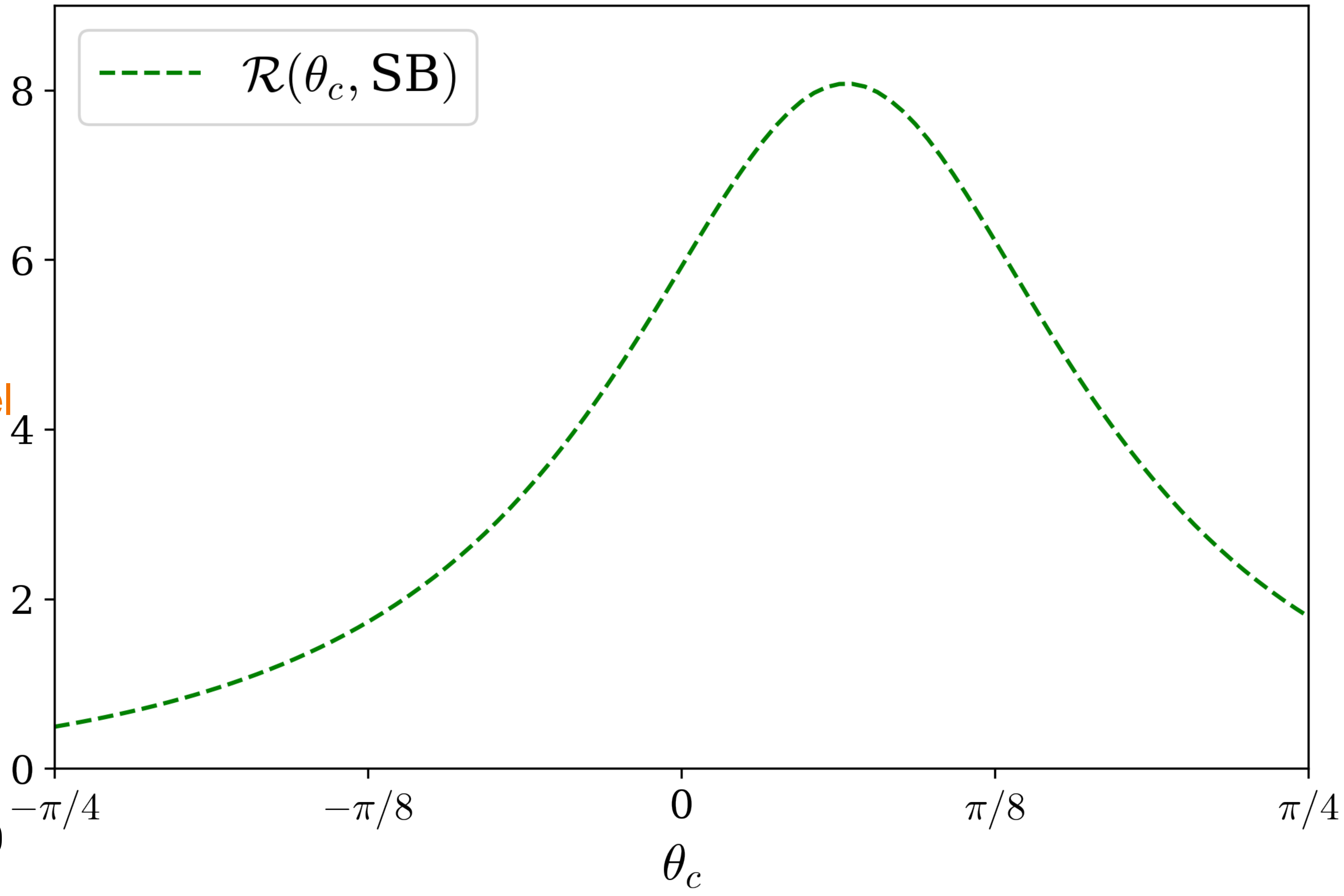
$\theta_c = -0.137\pi$

	SB $\theta_c = -\theta_0$			Cheng *et al.* [12]			Gutsche *et al.* [10]		
	$\mathcal{B}$	α	$\mathcal{R}$	$\mathcal{B}$	α	$\mathcal{R}$	$\mathcal{B}$	α	$\mathcal{R}$
$\Xi_{cc}^{++} \to \Xi_c^{+}\pi^+$	2.24	-93	1.45	0.69	-41	6.74	0.71	-57	4.77
$\Xi_{cc}^{++} \to \Xi_c^{\prime +}\pi^+$	3.25	-63		4.65	-84		3.39	-93	
$\Xi_{cc}^{+} \to \Xi_c^{0}\pi^+$	2.26	31	1.17	3.84	-31	0.40			
$\Xi_{cc}^{+} \to \Xi_c^{\prime 0}\pi+$	2.64	-99		1.55	-73				
$\Xi_{cc}^{+} \to \Xi_c^{+}\pi^0$	2.01	-5	0.25	2.38	-25	0.07			
$\Xi_{cc}^{+} \to \Xi_c^{\prime +}\pi^0$	0.51	-65		0.17	-3				

Covariant quark model to three-loop order

$\theta_c = 0$

For comparison with the others and references, please consult arXiv: 2211.12960



$$\theta_c = -0.090(12)\pi, \quad -0.48(1)\pi$$

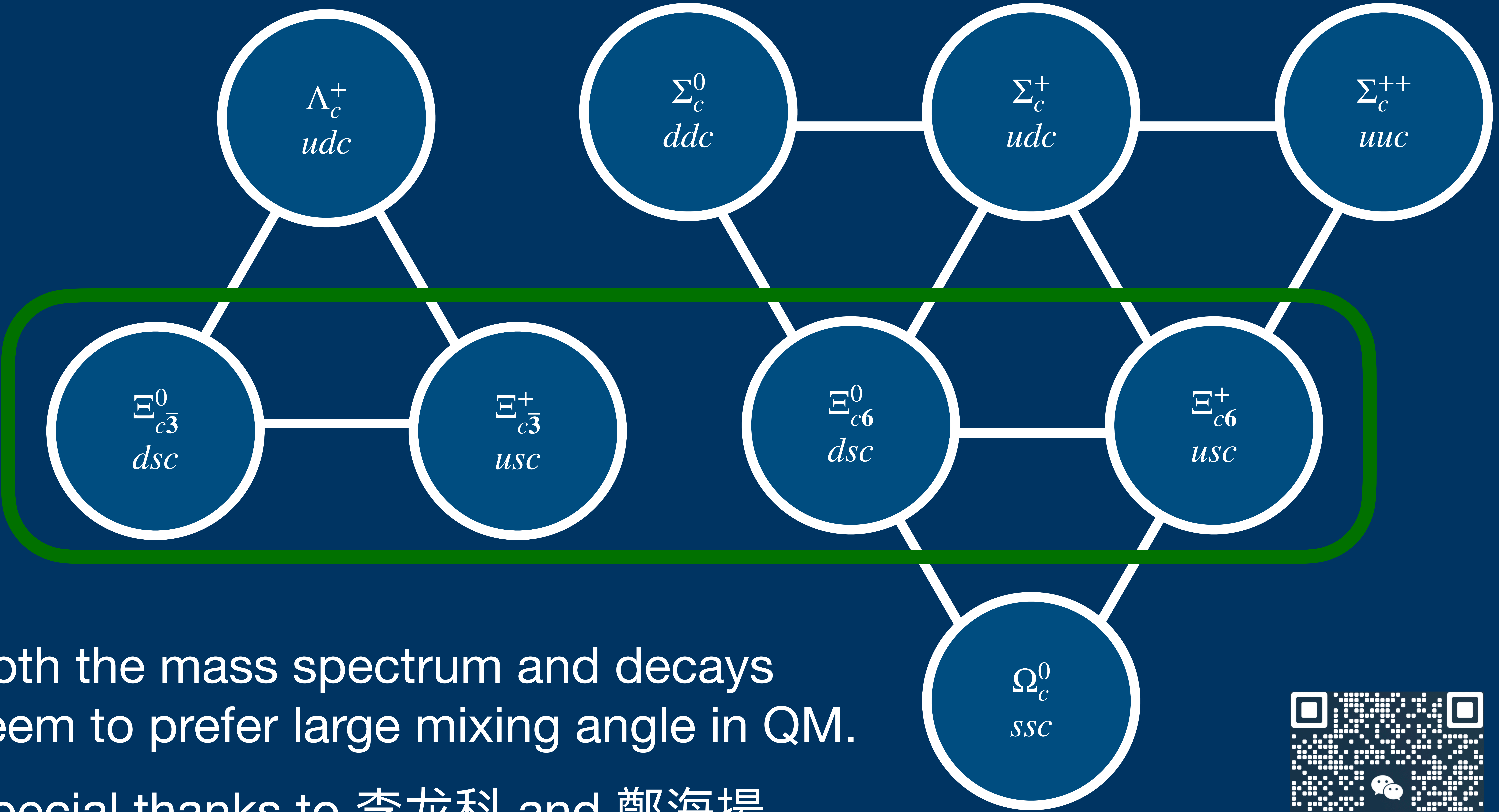
Light-front quark model, PRD **01** 034034 (2022)

$$\mathcal{R}(\Xi_{cc}^{++} \to \Xi_c^{+}\pi^{+}) = 1.41 \pm 0.17 \pm 0.10$$

LHCb, JHEP **46** 011002 (2022)

$$\mathcal{R}(\Xi_{cc}^{++} \to \Xi_c^{+}\pi^{+}) = 6.74$$

Current algebra, PRD **01** 034034 (2022)



- Both the mass spectrum and decays seem to prefer large mixing angle in QM.
- Special thanks to 李龙科 and 鄭海揚

Back-up slide: Validity of Körner-Pati-Woo theorem

Eur. Phys. J. C (2022) 82:297
https://doi.org/10.1140/epjc/s10052-022-10224-0

THE EUROPEAN PHYSICAL JOURNAL C

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Review

Topological tensor invariants and the current algebra approach: analysis of 196 nonleptonic two-body decays of single and double charm baryons – a review

Stefan Groote[1,a], **Jürgen G. Körner**[2]

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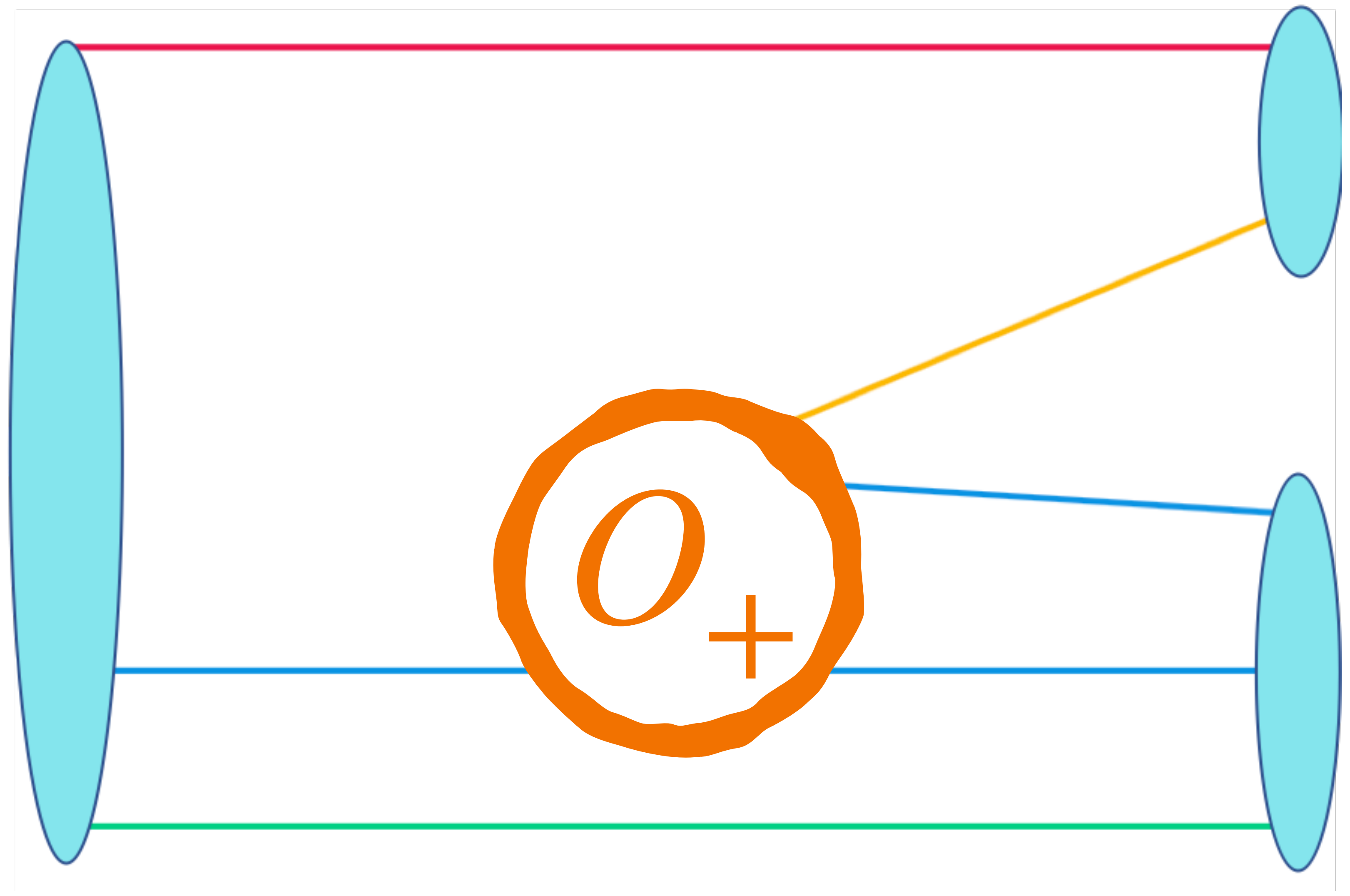
[2] PRISMA Cluster of Excellence, Institut für Physik, Johannes-Gutenberg-Universität, Staudinger Weg 7, 55099 Mainz, Germany



Back-up slide: Validity of Körner-Pati-Woo theorem

channel	data	$SU(3)_F$	Current Algebra
$10^4\mathcal{B}(\Lambda_c^+ \to \Sigma^0 K^+)^a$	$4.7 \pm 0.9 \pm 0.1 \pm 0.3$	5.4 ± 0.7	7.2
$10^4\mathcal{B}(\Lambda_c^+ \to \Sigma^+ K_S^0)^a$	$4.8 \pm 1.4 \pm 0.2 \pm 0.3$	5.4 ± 0.7	7.2
$10^4\mathcal{B}(\Lambda_c^+ \to n\pi^+)^b$	$6.6 \pm 1.2 \pm 0.4$	8.5 ± 2.0	2.7
$\alpha(\Lambda_c^+ \to pK_S^0)^c$	$0.18 \pm 0.43 \pm 0.14$	$-0.89^{+0.26}_{-0.11}$	-0.90
$\alpha(\Lambda_c^+ \to \Lambda K^+)^d$	$-0.585 \pm 0.049 \pm 0.018$	0.32 ± 0.32	-0.96
$\alpha(\Lambda_c^+ \to \Sigma^0 K^+)^d$	$-0.55 \pm 0.18 \pm 0.09,$	~ -1	-0.73
$100\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)^e$	1.43 ± 0.32	2.21 ± 0.11	6.47
$100\mathcal{R}(\Xi_c^0 \to \Xi^- K^+)$	$2.75 \pm 0.51 \pm 0.25$	4.4	6.0
$100\mathcal{R}(\Xi_c^0 \to \Sigma^0 K_s^0)^f$	$3.8 \pm 0.6 \pm 0.4$	2.3 ± 1.8	< 0.4
$10\mathcal{R}(\Xi_c^0 \to \Sigma^+ K^-)^f$	$1.23 \pm 0.07 \pm 0.10$	2.7 ± 0.5	0.71
$100\mathcal{B}(\Xi_c^+ \to \Xi^0\pi^+)$	1.6 ± 0.8	0.38 ± 0.20	1.72
$\mathcal{R}(\Xi_c^+ \to \Xi^0\pi^+)$	1.1 ± 0.6	0.17 ± 0.09	0.27

[a] Phys. Rev. D **106**, 052003 (2022).
[b] Phys. Rev. Lett. **128**, 142001 (2022).
[c] Phys. Rev. D **100**, 072004 (2019).
[d] arXiv:2208.08695 [hep-ex].
[e] Phys. Rev. Lett. **122**, 082001 (2019).
[f] Phys. Rev. D **105**, L011102 (2022).

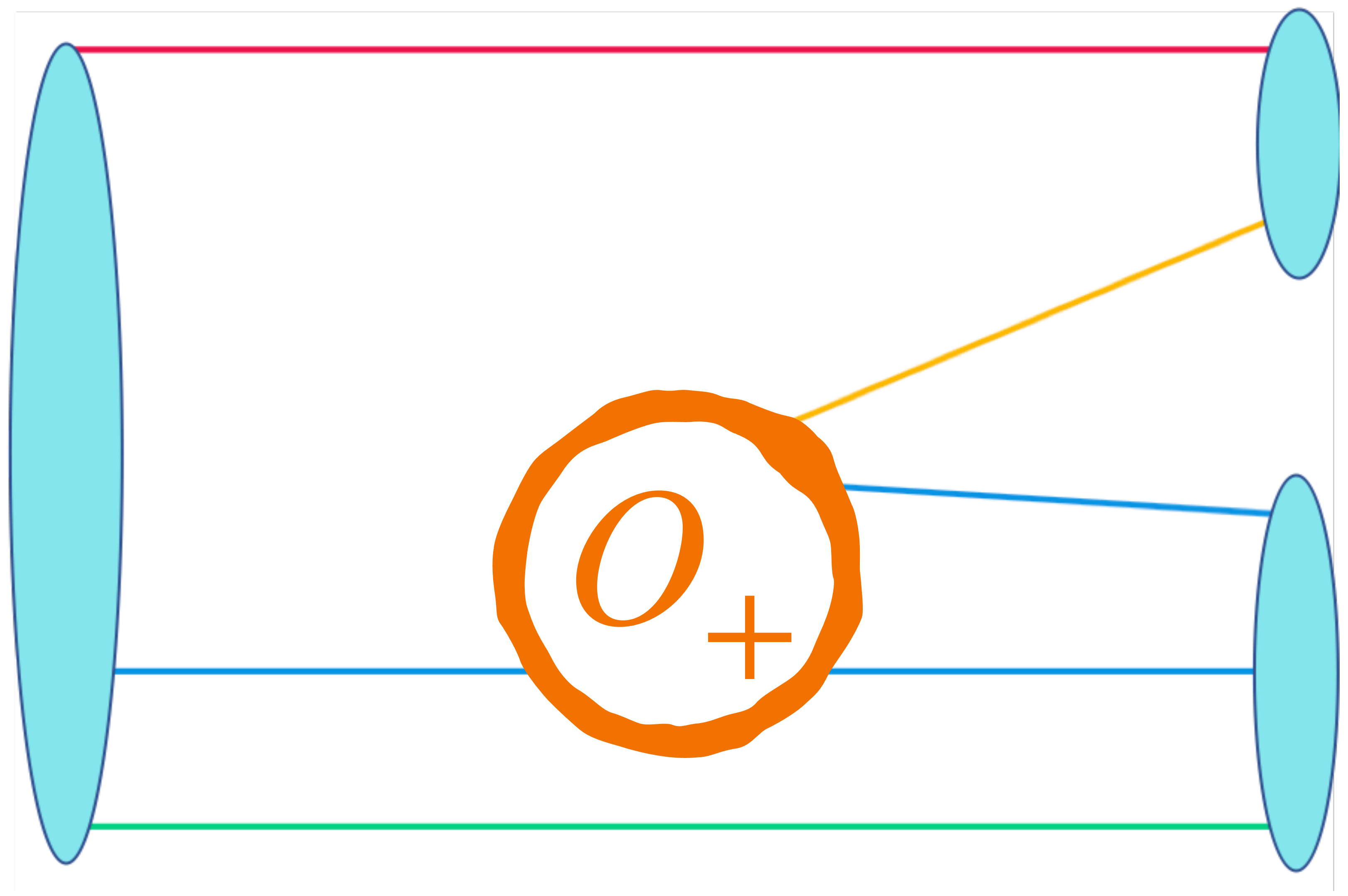
$$\mathscr{R}(X) := \frac{\mathscr{B}(X)}{\mathscr{B}(\Xi_c^0 \to \Xi^-\pi^+)}$$

Input:

$10^4\mathscr{B}(\Lambda_c^+ \to \Sigma^0 K^+) = 5.2 \pm 0.8$

$100\mathscr{B}(\Xi_c^0 \to \Xi^-\pi^+) = 1.80 \pm 0.52$

PRD **101** 014011 (2020); PRD **97** 074028 (2018)



$= 0$

$$O_+ = \frac{1}{2}\sum_{q,q'=d,s} V_{uq}V^*_{cq'}\left((\bar{u}q)(\bar{q}'c)+(\bar{q}'q)(\bar{u}c)\right)$$

Körner-Pati-Woo theorem

	$\mathcal{B}(\Lambda_c^+ \to \Sigma^0 K^+)$	$\mathcal{B}(\Lambda_c^+ \to \Sigma^+ K_S^0)$
QCD corrections [2]	2(8)	2(4)
MIT bag model [3]	7.2 ± 1.8	7.2 ± 1.8
Diagrammatic analysis [4]	5.5 ± 1.6	9.6 ± 2.4
$SU(3)_F$ flavor symmetry [5]	5.4 ± 0.7	5.4 ± 0.7
IRA method [6]	5.0 ± 0.6	1.0 ± 0.4
PDG 2020 [28]	5.2 ± 0.8	/

Mass relations

Model-Independent Bottom Baryon Mass Predictions in the $1/N_c$ Expansion

Elizabeth E. Jenkins

Phys. Rev. D **54**, 4515 (1996), Phys. Rev. D **55**, 10 (1997),
Phys. Rev. D **77**, 034012 (2008)

Breaking operators

Orders of errors

$(SU(3), J_Q)$	Mass Combination	Theory	$Q=c$	$Q=b$
$(1,0)$	$\frac{1}{3}\left(\Lambda_Q + 2\Xi_Q^{\bar{\mathbf{3}}}\right)$	$m_Q + N_c\Lambda$	2380^*	5687
$(1,0)$	$-\frac{1}{3}\left(\Lambda_Q + 2\Xi_Q^{\bar{\mathbf{3}}}\right) + \frac{1}{18}\left[3\left(\Sigma_Q + 2\Sigma_Q^*\right) + 2\left(\Xi_Q^{\mathbf{6}} + 2\Xi_Q^*\right) + \left(\Omega_Q + 2\Omega_Q^*\right)\right]$	$2\frac{1}{N_c}\Lambda$	207	207
$(1,1)$	$\frac{1}{6}\left[3\left(\Sigma_Q^* - \Sigma_Q\right) + 2\left(\Xi_Q^* - \Xi_Q^{\mathbf{6}}\right) + \left(\Omega_Q^* - \Omega_Q\right)\right]$	$3\frac{1}{N_c}\frac{\Lambda^2}{m_Q}$	66^*	20
$(8,0)$	$\left(\Lambda_Q - \Xi_Q^{\bar{\mathbf{3}}}\right)$	$\frac{3}{2\sqrt{3}}(\epsilon\Lambda_\chi)$	-195^*	-195
$(8,0)$	$-\frac{5}{8}\left(\Lambda_Q - \Xi_Q^{\bar{\mathbf{3}}}\right) + \frac{1}{24}\left[3\left(\Sigma_Q + 2\Sigma_Q^*\right) - \left(\Xi_Q^{\mathbf{6}} + 2\Xi_Q^*\right) - 2\left(\Omega_Q + 2\Omega_Q^*\right)\right]$	$\frac{1}{2\sqrt{3}}\frac{15}{8}\frac{1}{N_c}(\epsilon\Lambda_\chi)$	40.6	40.6
$(8,1)$	$\Xi_Q^{\mathbf{\bar{3}6}}$	$\frac{3}{4}\frac{1}{N_c}\frac{\Lambda}{m_Q}(\epsilon\Lambda_\chi)$	11.8	3.5
$(8,1)$	$\frac{1}{6}\left[3\left(\Sigma_Q^* - \Sigma_Q\right) - \left(\Xi_Q^* - \Xi_Q^{\mathbf{6}}\right) - 2\left(\Omega_Q^* - \Omega_Q\right)\right] - \frac{5}{2\sqrt{3}}\Xi_Q^{\mathbf{\bar{3}6}}$	$\frac{1}{2\sqrt{3}}\frac{15}{2}\frac{1}{N_c^2}\frac{\Lambda}{m_Q}(\epsilon\Lambda_\chi)$	11.3	3.4
$(27,0)$	$\frac{1}{6}\left[\left(\Sigma_Q + 2\Sigma_Q^*\right) - 2\left(\Xi_Q^{\mathbf{6}} + 2\Xi_Q^*\right) + \left(\Omega_Q + 2\Omega_Q^*\right)\right]$	$\frac{3}{2}\frac{1}{N_c}(\epsilon^2\Lambda_\chi)$	-4.4^*	-4.4
$(27,1)$	$\frac{1}{4}\left[\left(\Sigma_Q^* - \Sigma_Q\right) - 2\left(\Xi_Q^* - \Xi_Q^{\mathbf{6}}\right) + \left(\Omega_Q^* - \Omega_Q\right)\right]$	$\frac{9}{8}\frac{1}{N_c^2}\frac{\Lambda}{m_Q}(\epsilon^2\Lambda_\chi)$	0.23	0.07

Independent of m_Q

$(\Lambda, \epsilon\Lambda_\chi) \approx (300,\ 225)$ MeV

$$\epsilon = \frac{1}{4} \sim \frac{m_s}{\Lambda_\chi}$$

$$\Xi_c^{\mathbf{6}} = \Xi_c^* - \frac{1}{2}\left(\Sigma_c^* - \Sigma_c + \Omega_c^* - \Omega_c\right)$$

$$\left(\Sigma_b + 2\Sigma_b^*\right) - 2\left(\Xi_b' + 2\Xi_b^*\right) + \left(\Omega_b + 2\Omega_b^*\right)$$
$$= \left(\Sigma_c + 2\Sigma_c^*\right) - 2\left(\Xi_c' + 2\Xi_c^*\right) + \left(\Omega_c + 2\Omega_c^*\right) \pm 0.3\text{MeV}$$

Improved equal spacing rule

$$M_Q = \begin{pmatrix} \Xi_Q^{\bar{\mathbf{3}}} & \Xi_Q^{\mathbf{\bar{3}6}} \\ \Xi_Q^{\mathbf{\bar{3}6}} & \Xi_Q^{\mathbf{6}} \end{pmatrix} = \begin{pmatrix} -\cos 2\theta_Q & -\sin 2\theta_Q \\ -\sin 2\theta_Q & \cos 2\theta_Q \end{pmatrix} M_Q^{\Delta} + M_Q^0$$

$$\begin{pmatrix} \Xi_Q \\ \Xi_Q' \end{pmatrix} = \begin{pmatrix} \cos\theta_Q & \sin\theta_Q \\ -\sin\theta_Q & \cos\theta_Q \end{pmatrix} \begin{pmatrix} \Xi_Q^{\bar{\mathbf{3}}} \\ \Xi_Q^{\mathbf{6}} \end{pmatrix}$$

$$M_Q^{\Delta} = \frac{1}{2}(\Xi_Q' - \Xi_Q)$$

$$M_Q^0 = \frac{1}{2}(\Xi_Q + \Xi_Q')$$