Hadronic Inputs to the $(g\text{-}2)_\mu$ Puzzle

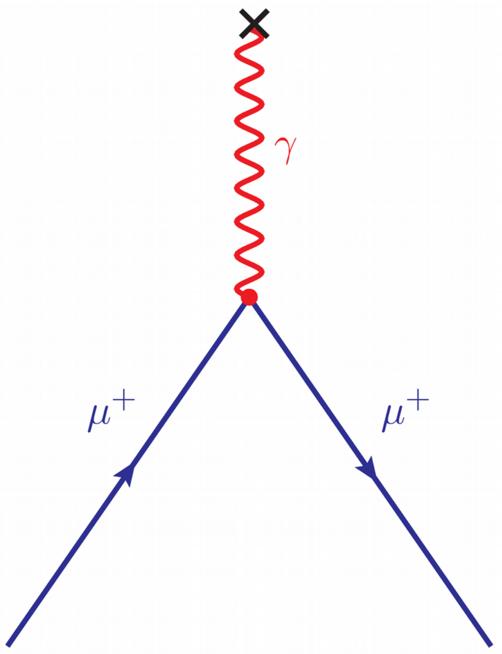
March 19, 2018 | Christoph Florian Redmer

2nd International Workshop on High Intensity Electron Positron Accelerator at China
Yanqihu Campus, UCAS

$(g\text{-}2)_\mu$

Magnetic moment of μ : $\vec{\mu}_\mu = g_\mu \frac{e}{2m_\mu}\vec{s}$

Dirac theory: $g_\mu = 2$



$(g\text{-}2)_\mu$

Magnetic moment of μ : $\vec{\mu}_\mu = g_\mu \frac{e}{2m_\mu}\vec{s}$

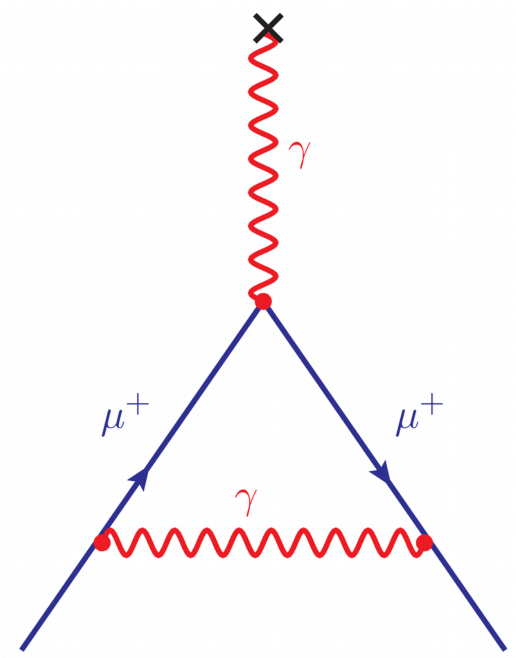
Dirac theory: $g_\mu = 2$

Quantum Field Theory: $g_\mu \neq 2$

Muon anomaly: $a_\mu = \frac{g_\mu - 2}{2}$

$$a_\mu^{theo} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadr}$$

Contribution	in units of 10^{-10}
Schwinger	11620000



J.S. Schwinger (1948):

$$a_\mu^{QED,LO} = \frac{\alpha}{2\pi}$$

$(g\text{-}2)_\mu$

Magnetic moment of μ : $\vec{\mu}_\mu = g_\mu \frac{e}{2m_\mu}\vec{s}$

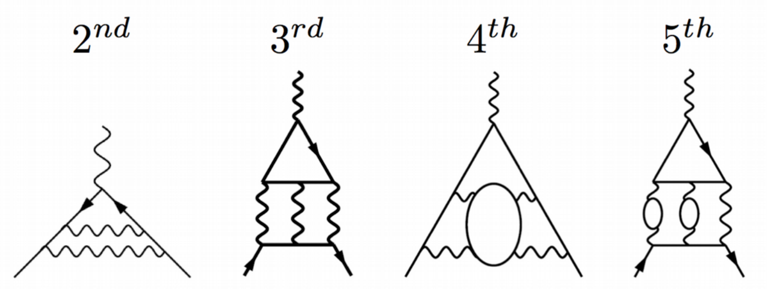
Dirac theory: $g_\mu = 2$

Quantum Field Theory: $g_\mu \neq 2$

Muon anomaly: $a_\mu = \frac{g_\mu - 2}{2}$

$$a_\mu^{theo} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadr}$$

Some examples for
higher order QED corrections:



Total number of diagrams:
7 72 891 12672

Contribution	in units of 10^{-10}
Schwinger	11620000
QED	11658471.895 ± 0.008

Kinoshita et al., PRL 109 (2012) 111808

$(g-2)_\mu$

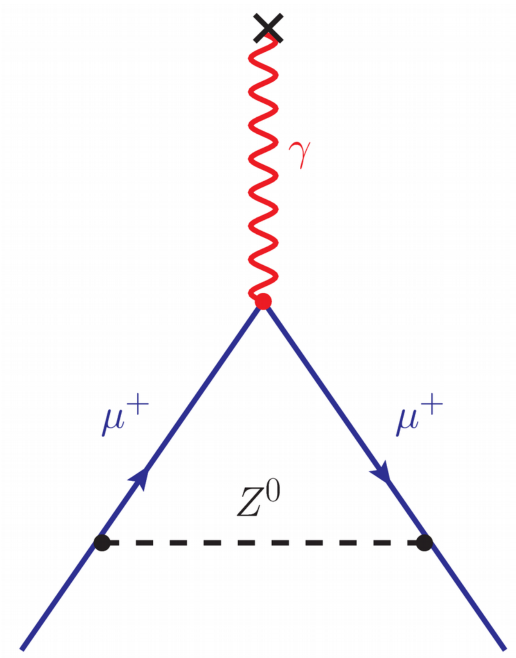
Magnetic moment of μ : $\vec{\mu}_\mu = g_\mu \frac{e}{2m_\mu}\vec{s}$

Dirac theory: $g_\mu = 2$

Quantum Field Theory: $g_\mu \neq 2$

Muon anomaly: $a_\mu = \frac{g_\mu - 2}{2}$

$$a_\mu^{theo} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadr}$$



Contribution	in units of 10^{-10}
QED	11658471.895 ± 0.008
Weak	15.4 ± 0.2

Kinoshita et al., PRL 109 (2012) 111808

Czarnecki et al., PRD 67 (2003) 073006 + Erratum

$(g\text{-}2)_\mu$

Magnetic moment of μ : $\vec{\mu}_\mu = g_\mu \frac{e}{2m_\mu}\vec{s}$

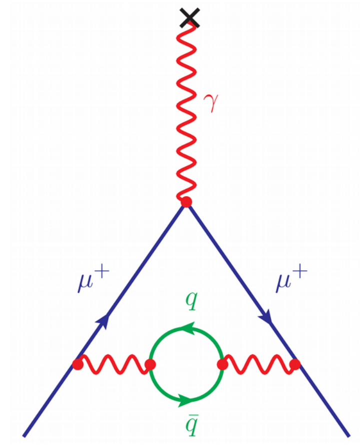
Dirac theory: $g_\mu = 2$

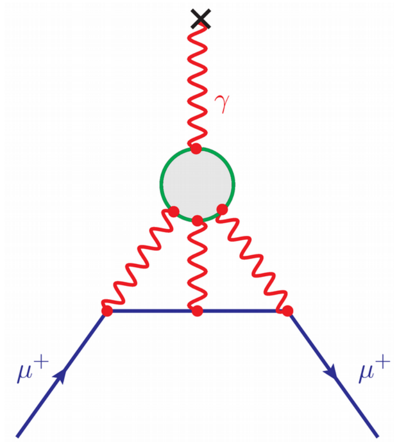
Quantum Field Theory: $g_\mu \neq 2$

Muon anomaly: $a_\mu = \frac{g_\mu - 2}{2}$

$$a_\mu^{theo} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadr}$$

Hadronic Vacuum Polarization

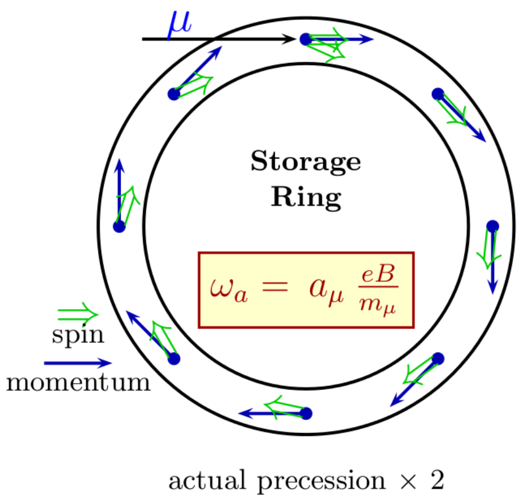




Hadronic Light-by-Light Scattering

Contribution	in units of 10^{-10}		
QED	11658471.895	± 0.008	Kinoshita et al., PRL 109 (2012) 111808
Weak	15.4	± 0.2	Czarnecki et al., PRD 67 (2003) 073006 + Erratum
HVP(leading order)	692.3	± 4.2	Davier et al., EPJC 17 (2011) 1515 + Erratum
HVP(higher order)	-9.79	± 0.07	Hagiwara et al., CPC 34 (2010) 728
HLBL	11.6	± 4.0	Jegerlehner, Nyffler, Phys.Rept. 477 (2009) 1
Total	11659181.4	± 5.8	

Direct Measurement

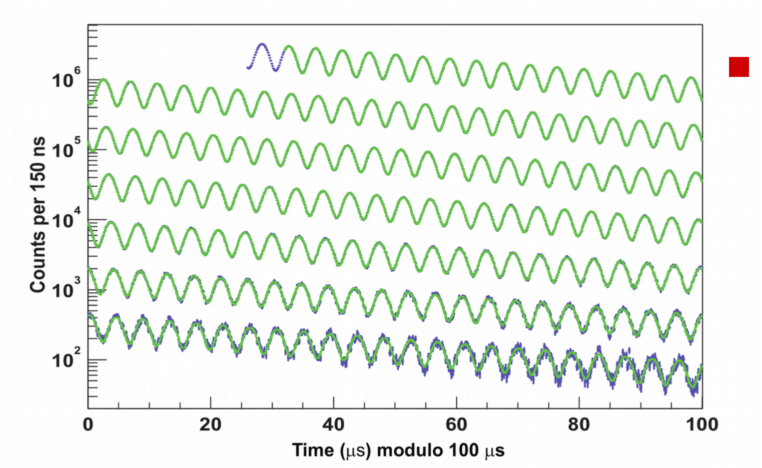


- $\pi^+ \to \mu^+ \nu_\mu$ (longitudinally polarized μ^+ due to P violation)
- Precession in magnetic field and focusing electric field

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c$$
$$= -\frac{e}{m_\mu}(a_\mu \vec{B} - [a_\mu - \frac{1}{\gamma^2 - 1}]\vec{v} \times \vec{E})$$

- Select " Magic γ " to be independent of $\vec{E}$

$$\gamma = \sqrt{1 + 1/a_\mu} = 29.3 \quad \Rightarrow \quad p_\mu = 3.094\mathrm{GeV/c}$$



Bennet et al., PRD 73 (2006) 072003

- Detect e^+ from $\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$
  - Direction of e^+ influenced by polarization of μ^+
  - Rate of measured e^+ modulated with $\vec{\omega}_a$

Direct Measurement

Long History of direct Measurements:

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL	1997	μ^+	11 659 251(150)	13
BNL	1998	μ^+	11 659 191(59)	5
BNL	1999	μ^+	11 659 202(15)	1.3
BNL	2000	μ^+	11 659 204(9)	0.73
BNL	2001	μ^-	11 659 214(9)	0.72
Average			11 659 208.0(6.3)	0.54

Latest High Precision Measurement of a_μ: BNL-E821

Bennet et al., PRD 73 (2006) 072003

$$a_\mu^{exp} = 11\,659\,208.9 \pm 6.3 \cdot 10^{-10}$$

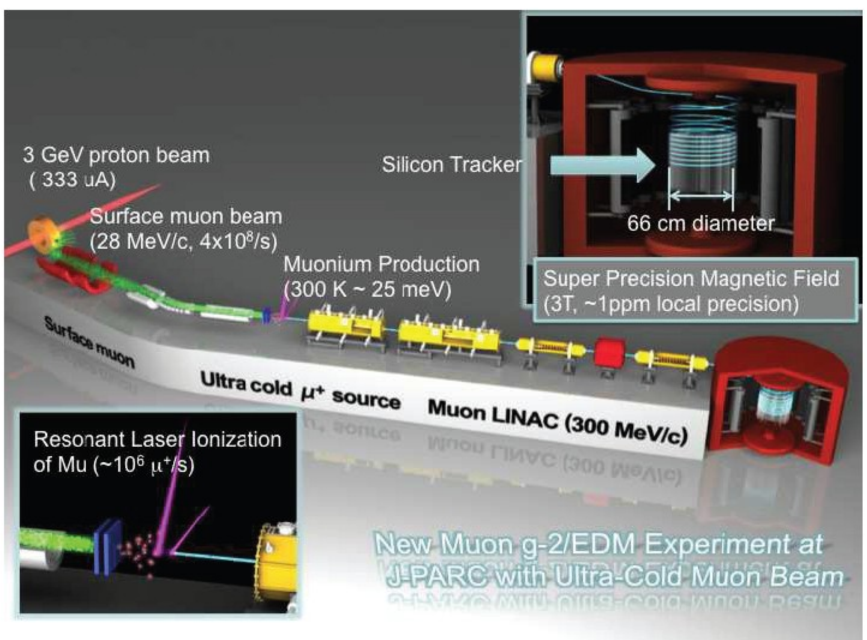
Discrepancy of 3 – 4σ compared to SM predictions!

Hint for New Physics?

New Measurements

Fermilab E989

- Reusing the BNL ring
- Higher statistics
- Improved systematics
- $\delta a_\mu \approx 1.6 \times 10^{-10}$



J-PARC

- Ultra cold muons
- No electric field
- $\delta a_\mu \sim 10 \times 10^{-11}$

Nucl.Phys.Proc.Suppl.218 (2011) 242

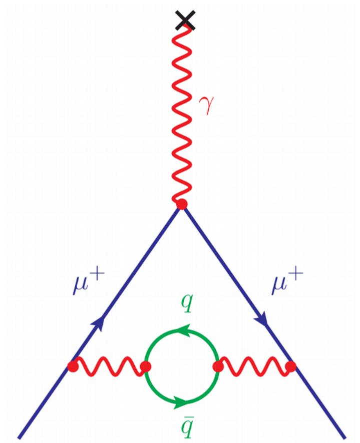
Experiment vs Theory

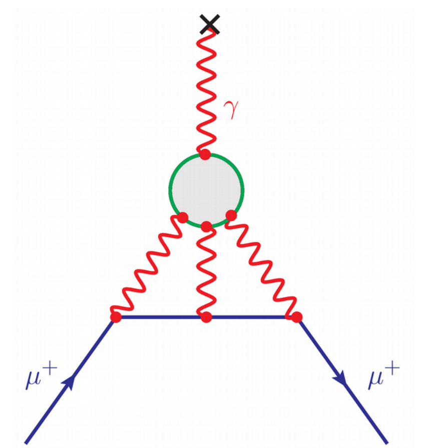
Improvement of δa_μ by a factor 4 by new experiments

→ Theory has to keep up with the precision!

Contribution	in units of 10^{-10}
QED	11658471.895 ± 0.008
Weak	15.4 ± 0.2
HVP(leading order)	692.3 ± 4.2
HVP(higher order)	-9.79 ± 0.07
HLBL	11.6 ± 4.0
Total	11659181.4 ± 5.8

Hadronic Vacuum Polarization





Hadronic Light-by-Light Scattering

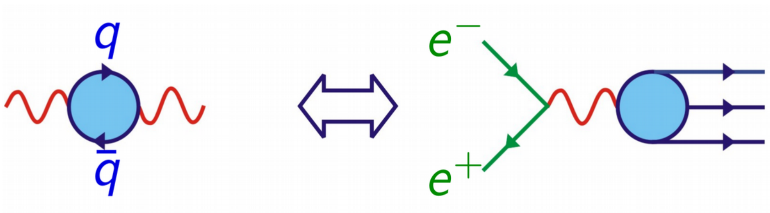
Hadronic contributions completely dominate the uncertainty of the Standard Model prediction!

Challenge: Perturbative methods cannot be applied in the relevant energy regime

Experimental Input needed!

Hadronic Vacuum Polarization

related to hadronic cross sections by optical theorem



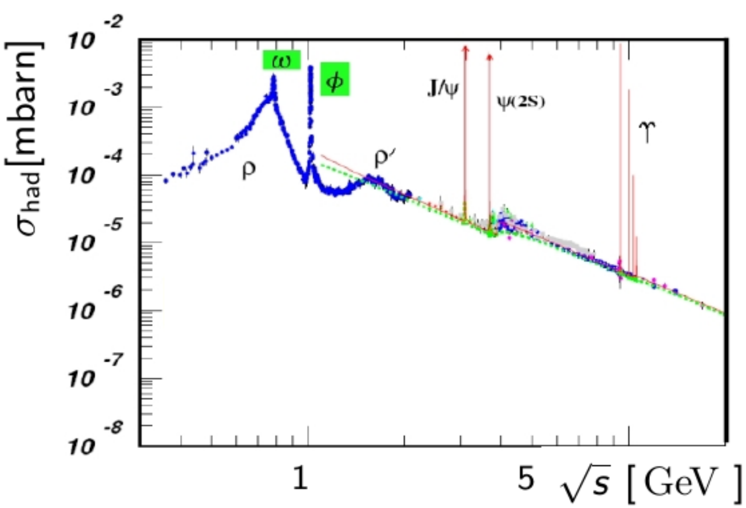
Dispersion Integral :

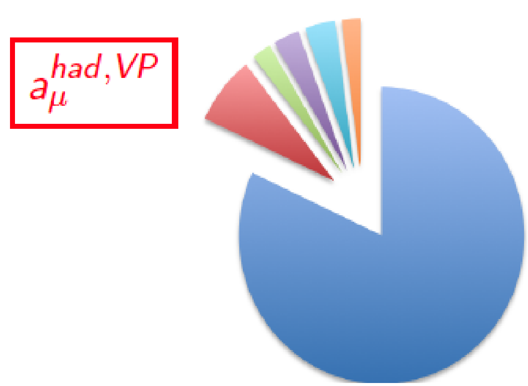
$$a_\mu^{\text{hVP,LO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} K(s)\sigma(e^+e^- \to \text{hadr})ds$$

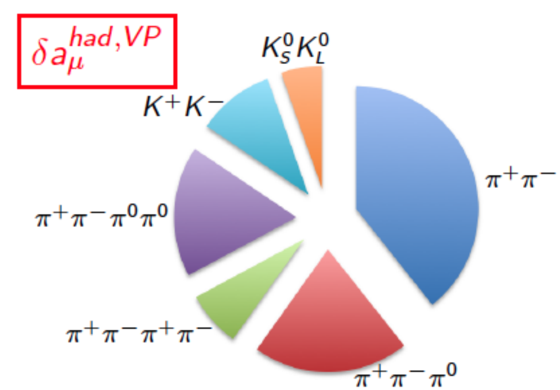
$$K(s) \sim \frac{1}{s}$$

$$\sigma(e^+e^- \to \text{hadr}) \sim \frac{1}{s}$$

} Low energy contributions dominate !

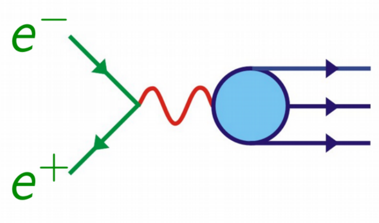


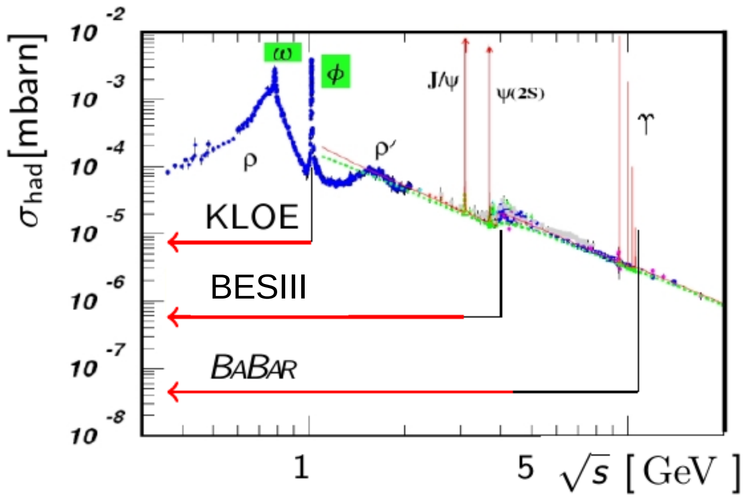


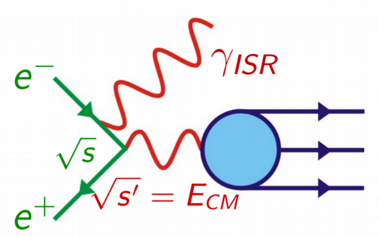


Hadronic Vacuum Polarization

Energy Scan Measurements:
CMD, SND (Novosibirsk)
BESIII (Beijing)







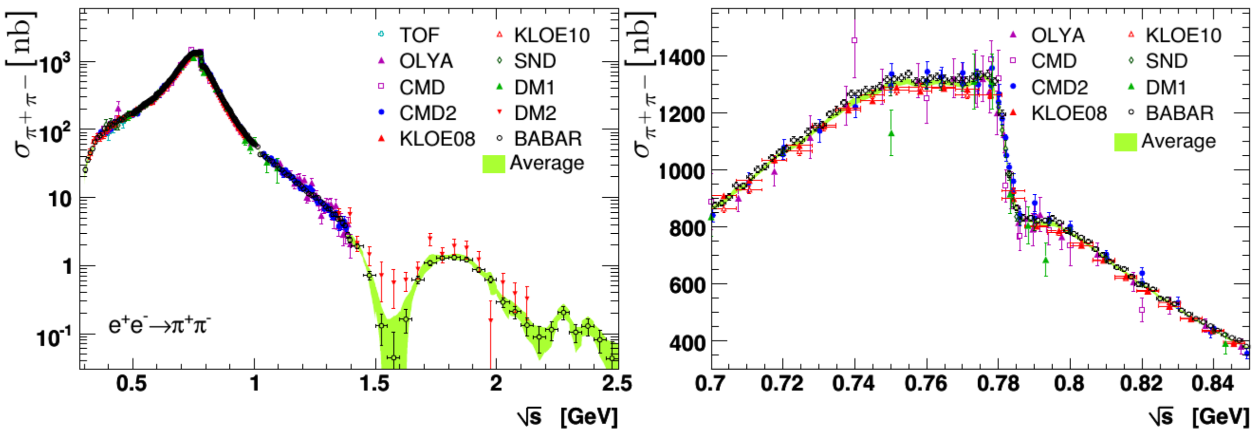
Initial State Radiation Measurements:

- Photon emitted in initial state
- Measurement a different energy possible

KLOE (Frascati)
BaBar (Stanford)
BESIII (Beijing)
Belle2 (Tsukuba)

Hadronic Vacuum Polarization

$e^+e^- \to \pi^+\pi^-$ accounts for 75% of a_μ^{hVP} ⟶ Good knowledge important !

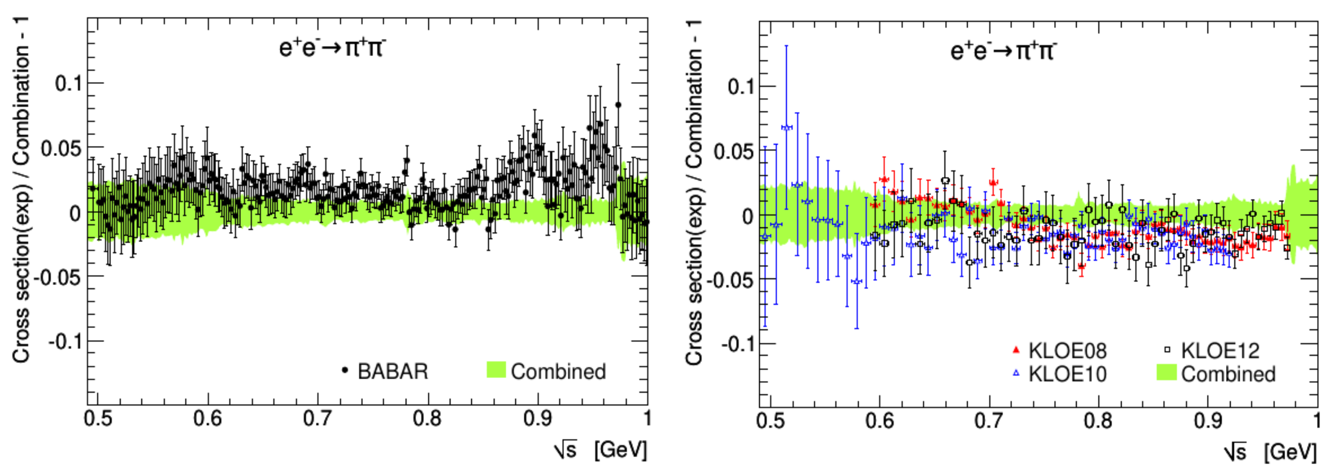


Systematic uncertainties:

0.5% BaBar
0.8% KLOE
0.8% CMD
1.5% SND

CMD, SND: } Limited by statistics

KLOE and BaBar measurements dominate world average



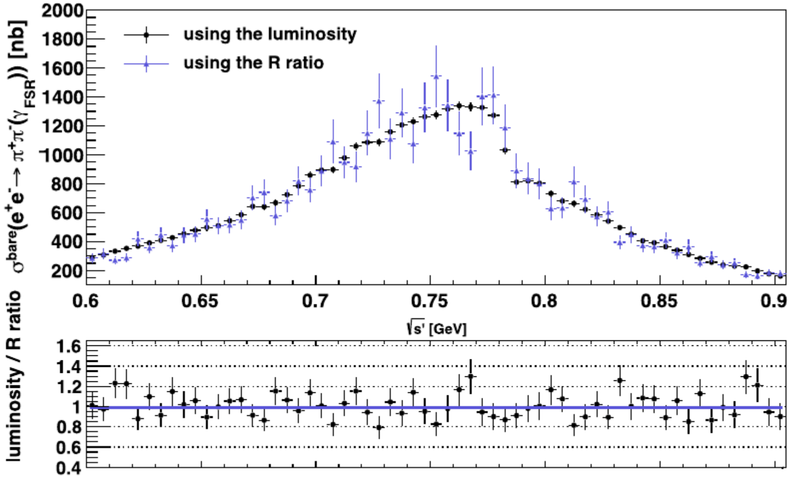
systematic differences ⟶ large uncertainty for a_μ^{hVP}

Hadronic Vacuum Polarization

$e^+e^- \to \pi^+\pi^-$ measurement at BESIII

Phys.Lett.B753 (2016) 629

- 2.9 fb^{-1} on ψ(3770) peak
- Tagged ISR technique
- $\mu - \pi$ separation with ANN
- Careful evaluation of systematics
  - Total uncertainty of 0.9% achieved
  - Dominated by:
    - Luminosity measurement (0.5%)
    - Uncertainty of radiator function (0.5%)



Normalization to $\mu^+\mu^-$ limited by statistics

- Systematic uncertainties cancel
- 20 fb^{-1} needed
- Approx. 5 years data taking at BESIII

- Evaluation for $0.6 \le m_{\pi\pi} \le 0.9$
  - 70% of total 2π contribution
  - 50% of a_μ^{hVP} contribution

Hadronic Vacuum Polarization

$e^+e^- \to \pi^+\pi^-$ measurement at BESIII

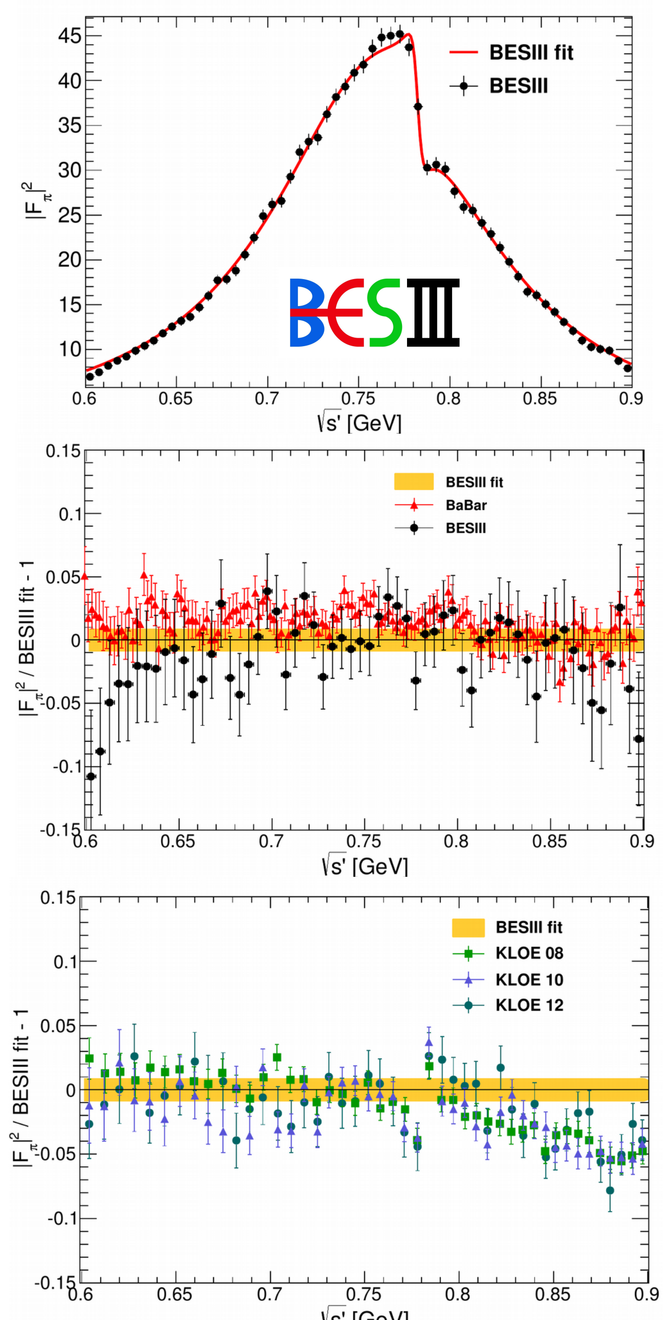
Phys.Lett.B753 (2016) 629

- 2.9 fb^{-1} on ψ(3770) peak
- Tagged ISR technique
- $\mu - \pi$ separation with ANN
- Careful evaluation of systematics
  - Total uncertainty of 0.9% achieved
  - Dominated by:
    - Luminosity measurement (0.5%)
    - Uncertainty of radiator function (0.5%)

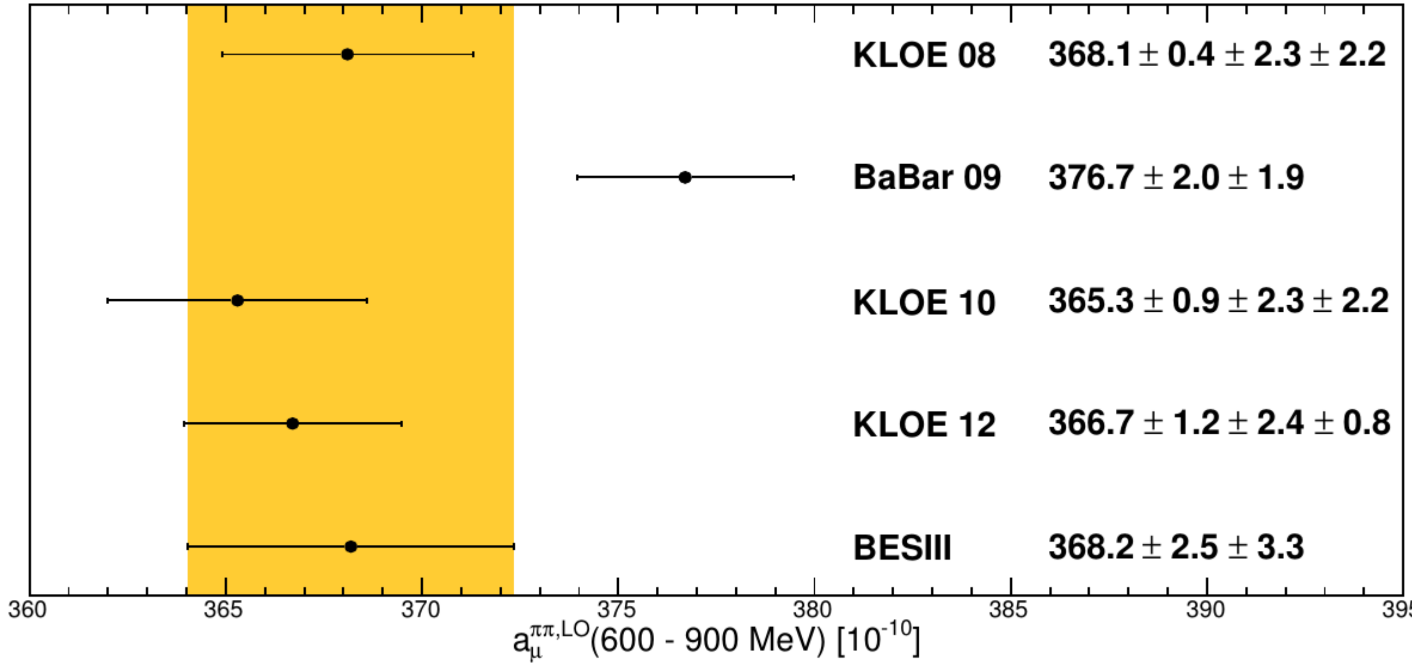
- Evaluation for $0.6 \leq m_{\pi\pi} \leq 0.9$
  - 70% of total 2π contribution
  - 50% of a_μ^{hVP} contribution

Comparison to previous measurements:

- Systematic shift in pion form factor
  - below ρ/ω interference w.r.t. BaBar
  - above ρ/ω interference w.r.t. KLOE



Hadronic Vacuum Polarization



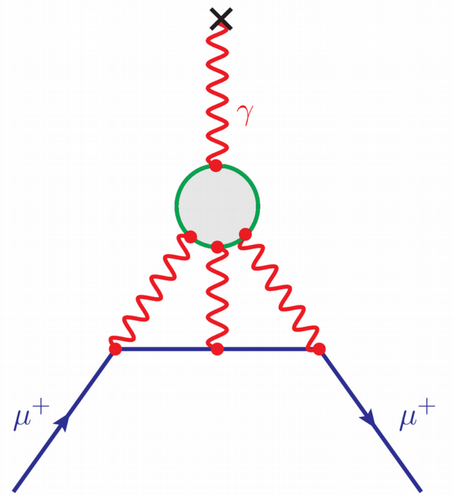
Ablikim et al., Phys.Lett.B753 (2016) 629

- Precision competitive to measurements by BaBar and KLOE
- Good agreement with all KLOE results
- BESIII result confirms $a_\mu^{\text{theo,SM}} - a_\mu^{\text{exp}} > 3\sigma$
- New evaluations of a_μ^{hVP} including BESIII result available

Davier et al. EPJ C77 (2018) 822
Teubner et al. arXiv:1802.02995

Hadronic Light-by-Light

a_μ^{hLBL} not directly related to measurable quantities



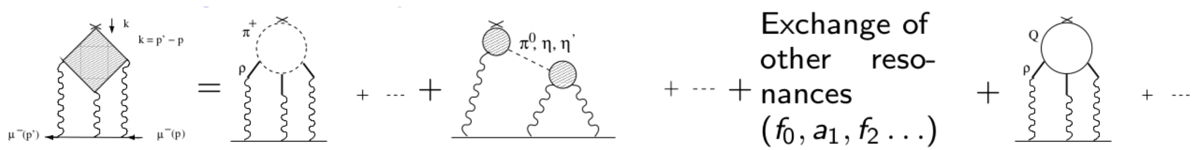
- Interaction of virtual mesons with real/virtual photons
  - ChPT at lowest energies
  - pQCD at high energies
  - Intermediate region ?

- "classic" approach: Hadronic models
  - "Glasgow Consensus" arXiv:0901.0306
  - Jegerlehner,Nyffeler Phys.Rep.477 (2009) 1

- Models can be validated with experimental data
- Error estimates for a_μ^{hLBL} are model dependent

Relevant Processes and Energies

Counting scheme for contributions to a_μ^{hLBL} (de Rafael, Phys.Lett. B322 (1994) 239)



Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C

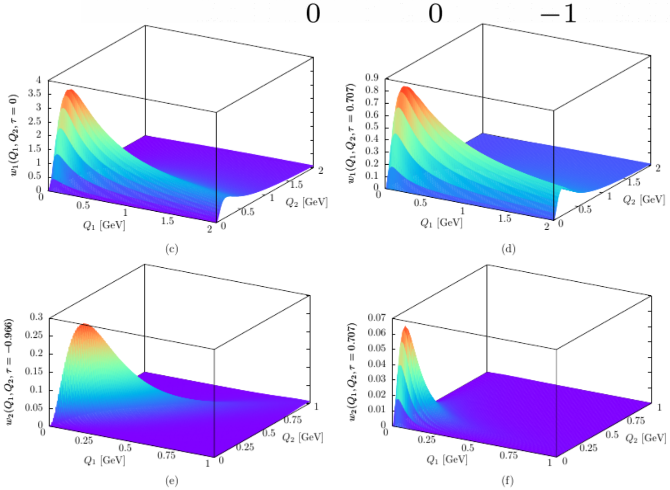
Dominating contributions:
- Pion loop
- PS meson exchange

3D integral representation for pion-pole contribution (Nyffeler, Phys.Rev. D94 (2016) 093006)

$$a_\mu^{\mathrm{HLbL};\pi^0(1)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau\, w_1(Q_1,Q_2,\tau)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-(Q_1+Q_2)^2)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_2^2,0)$$

$$a_\mu^{\mathrm{HLbL};\pi^0(2)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau\, w_2(Q_1,Q_2,\tau)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-Q_2^2)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-(Q_1+Q_2)^2,0)$$

$$Q_i^2 = -q_i^2$$



$w_i(Q_1, Q_2, \tau)$: Universal weighting functions

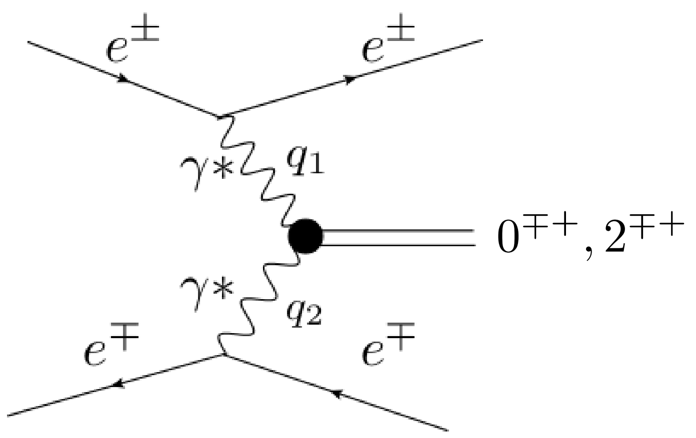
$\mathcal{F}_{\pi^0\gamma*\gamma*}$: Transition form factor

relevant momentum region to measure Transition Form Factor
0.25 – 1.25 GeV

space-like Transition Form Factors

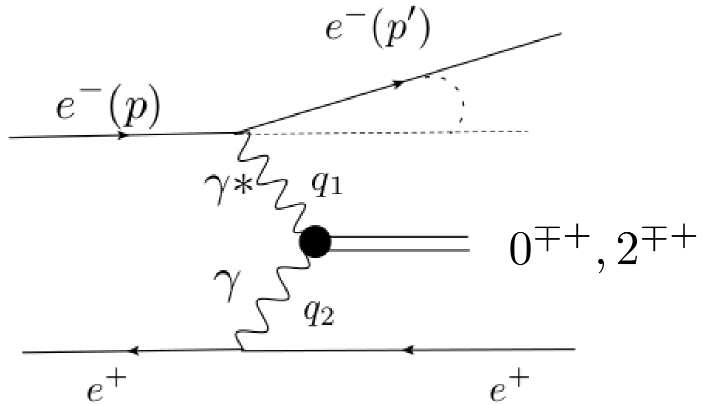
Can be investigated at e^+e^- colliders:

- Exchange of two photons in e^+e^- collisions
- Pseudoscalar, axial, and tensor states accessible
- $\sigma \propto \alpha^2 \ln^2 E$
- $\sigma \propto F^2(Q_1^2, Q_2^2)$, with $Q_i^2 = -q_i^2$



- Forward peaked kinematic
  - Experimentally challenging

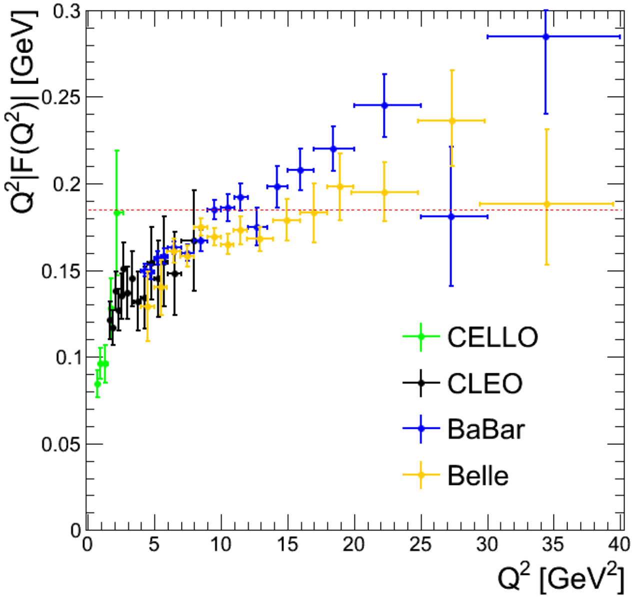
- Single-tag to study momentum dependence
  - Detect only one scattered lepton
  - Require small virtuality for second photon
    - $F^2(Q_1^2, Q_2^2) \Rightarrow F^2(Q_1^2, 0) \Rightarrow F^2(Q^2)$



- TFF should factorize at lowest energies

$$F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2) \sim F_{\pi\gamma^*\gamma}(Q_1^2, 0) \times F_{\pi\gamma^*\gamma}(0, Q_2^2)$$

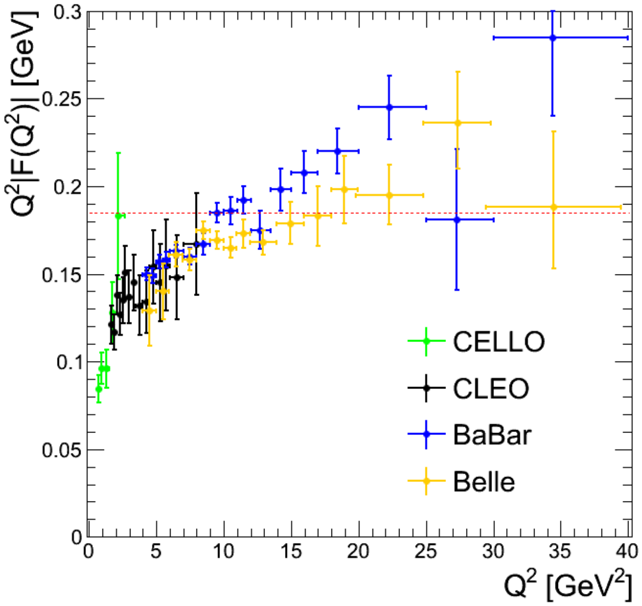
π^0 Transition Form Factor



- Results from B-factories cover large Q^2 ($5 < Q^2$ [GeV^2] < 40)
  - Discrepancy for π^0 between BaBar and Belle
- Data scarce at lowest Q^2
  - Region of relevance for $(g-2)\mu$

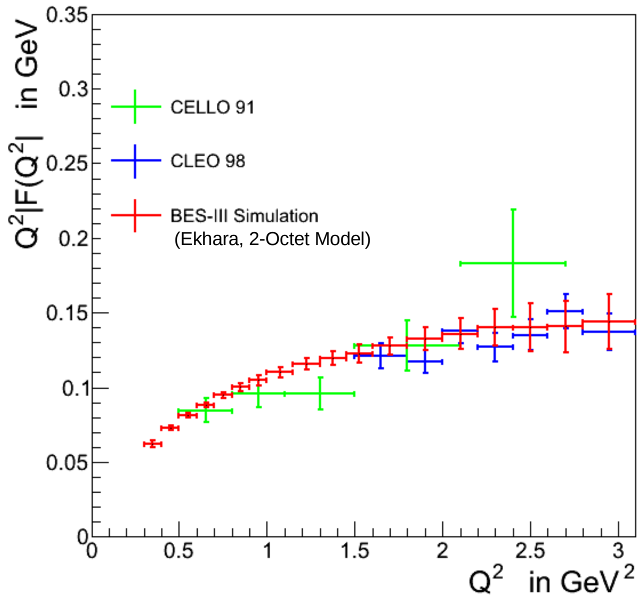
CELLO: Z.Phys.C49 (1991) 401
CLEO: Phys.Rev.D57 (1998) 33
BaBar: Phys.Rev.D80 (2009) 052002
Belle: Phys.Rev.D86 (2012) 092007

π^0 Transition Form Factor



- Results from B-factories cover large Q^2 ($5 < Q^2$ [GeV^2] < 40)
  - Discrepancy for π^0 between BaBar and Belle
- Data scarce at lowest Q^2
  - Region of relevance for (g-2)µ

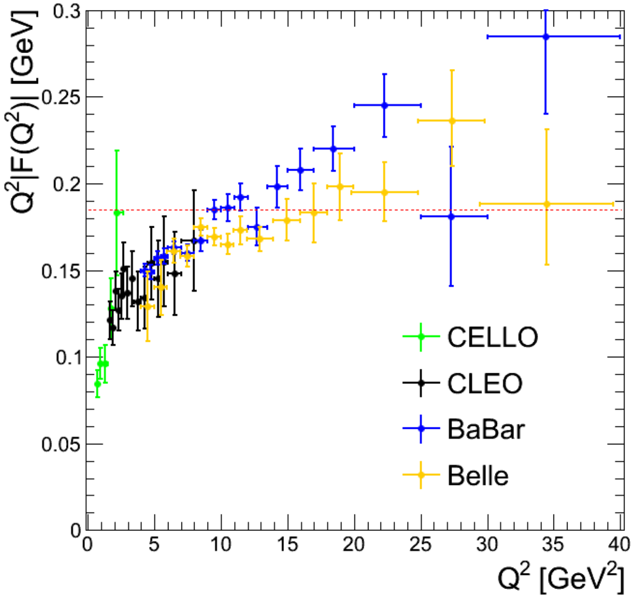
CELLO: Z.Phys.C49 (1991) 401
CLEO: Phys.Rev.D57 (1998) 33
BaBar: Phys.Rev.D80 (2009) 052002
Belle: Phys.Rev.D86 (2012) 092007



Prospects for BESIII

- 2.9 fb^{-1} analyzed at ψ(3770) peak
- Covering $0.3 < Q^2$ [GeV^2] < 3.1
- Unprecedented statistical accuracy expected for $Q^2 < 1.5$ GeV^2
- Limited by statistics above 3 GeV^2

π^0 Transition Form Factor

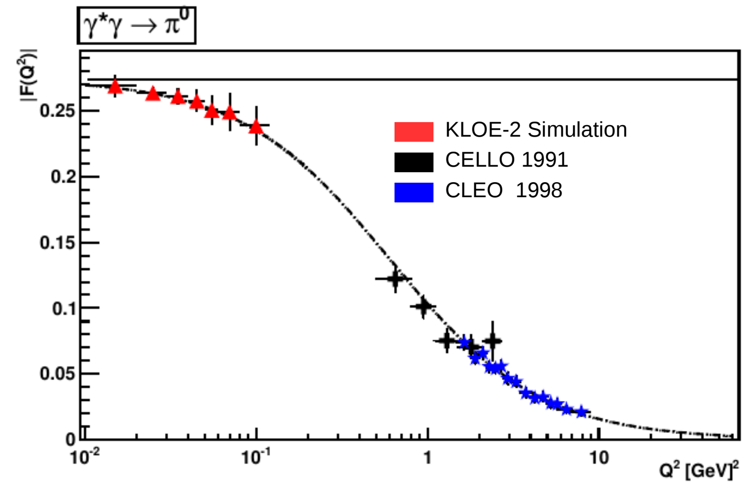


- Results from B-factories cover large Q^2 ($5 < Q^2$ [GeV^2] < 40)
  - Discrepancy for π^0 between BaBar and Belle
- Data scarce at lowest Q^2
  - Region of relevance for (g-2)µ

CELLO: Z.Phys.C49 (1991) 401
CLEO: Phys.Rev.D57 (1998) 33
BaBar: Phys.Rev.D80 (2009) 052002
Belle: Phys.Rev.D86 (2012) 092007

Prospects for KLOE-2

- Special tagging detectors installed
- Covering $0.01 < Q^2$ [GeV^2] < 0.1
- 6% statistical accuracy expected from 5 fb^{-1} at ϕ peak



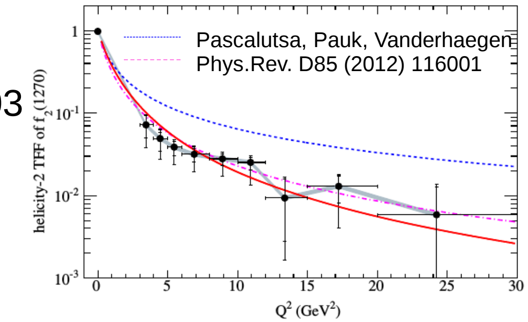
Babusci et al., EPJC 72 (2012) 1917

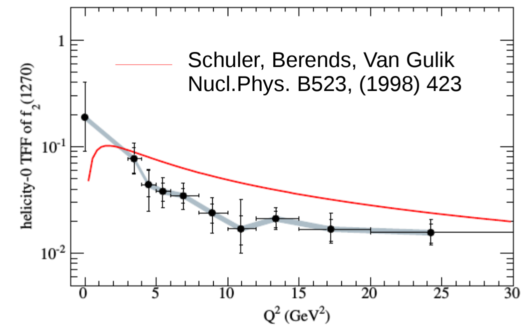
$\gamma^*\gamma \rightarrow \pi\pi$

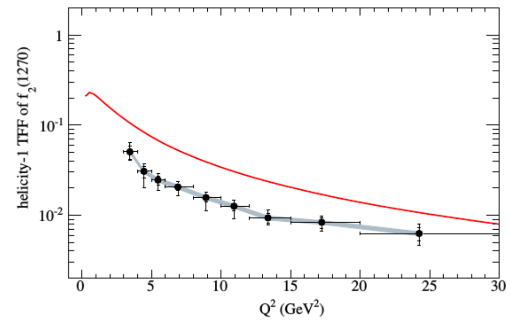
First single-tag measurement by Belle

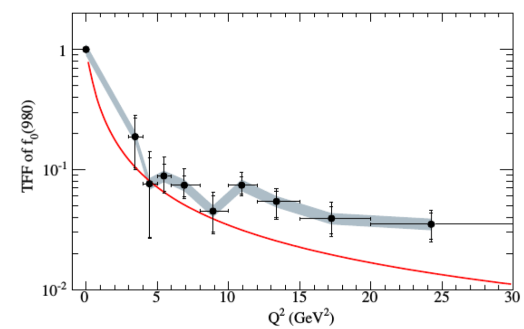
Phys.Rev.D93 (2016) 032003

- $\gamma^*\gamma \rightarrow \pi^0\pi^0$
- 759 fb^{-1}
- $3 < Q^2$ [GeV²] < 30
- $0.5 < W$ [GeV/c²] < 2.1
- $|\cos\theta^*| < 1.0$
- Determination of partial-wave amplitudes
- Measurement of TFF for $f_2(1270)$ and $f_0(980)$







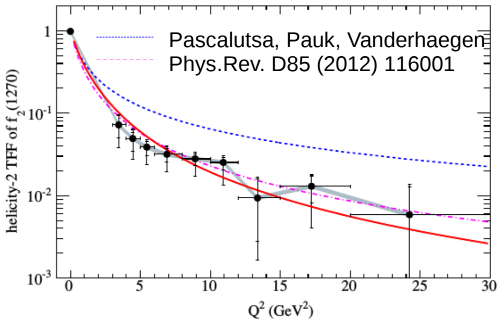


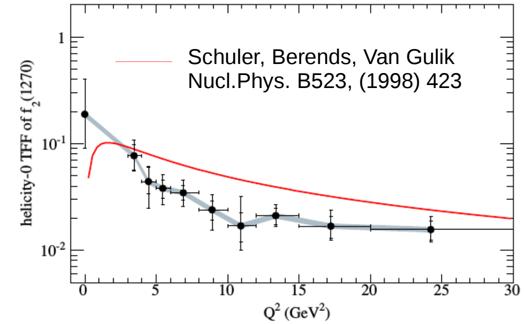
$\gamma^*\gamma \to \pi\pi$

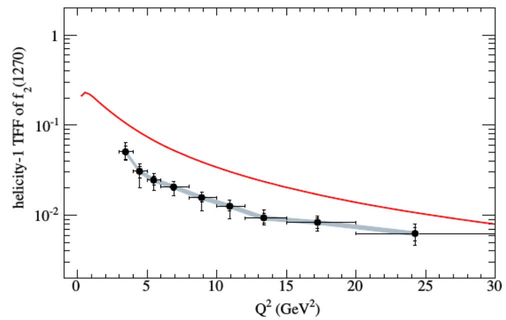
First single-tag measurement by Belle

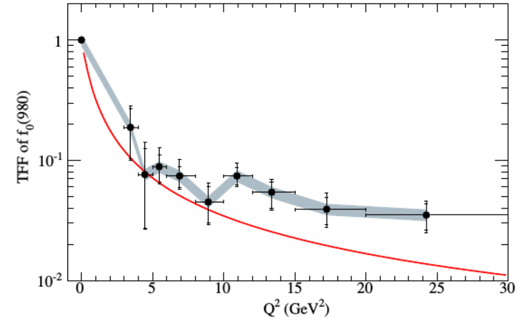
Phys.Rev.D93 (2016) 032003

- $\gamma^*\gamma \to \pi^0\pi^0$
- 759 fb^{-1}
- $3 < Q^2$ [GeV²] < 30
- $0.5 < W$ [GeV/c²] < 2.1
- $|\cos\theta^*| < 1.0$
- Determination of partial-wave amplitudes
- Measurement of TFF for $f_2(1270)$ and $f_0(980)$









Prospects for BESIII: $\gamma^*\gamma \to \pi^+\pi^-/\pi^0\pi^0$

- $0.2 < Q^2$ [GeV²] < 2.0
- $m_{\pi+\pi-} < M$ [GeV] < 2.0
- $|\cos\theta^*| < 1.0$

Data Driven Approaches

- Padé – Approximants
  - Parametrize TFF by series of rational approximants
  - Fit free parameters to experimental data
  - Estimate for systematic uncertainty provided
  - Space-like and time-like data can be used

Escribano, Masjuan, et al.
PRD 86 (2012) 094021
EPJC 75 (2015) 414

- Dispersive approaches to a_μ^{hLBL}
  - Describe dominating contributions with dispersion relations
  - Relation to measurable quantities
  - Reduce model dependency
  - Give more reliable error estimates
    - Goal 10 – 20 %

Bern (Colangelo, Hoferichter, et al.)

JHEP 1409 (2014) 091
PLB 738 (2014) 6
EPJ C74 (2014) 3180
JHEP 1509 (2015) 074

Mainz (Pauk, Vanderhaegen, et al.)

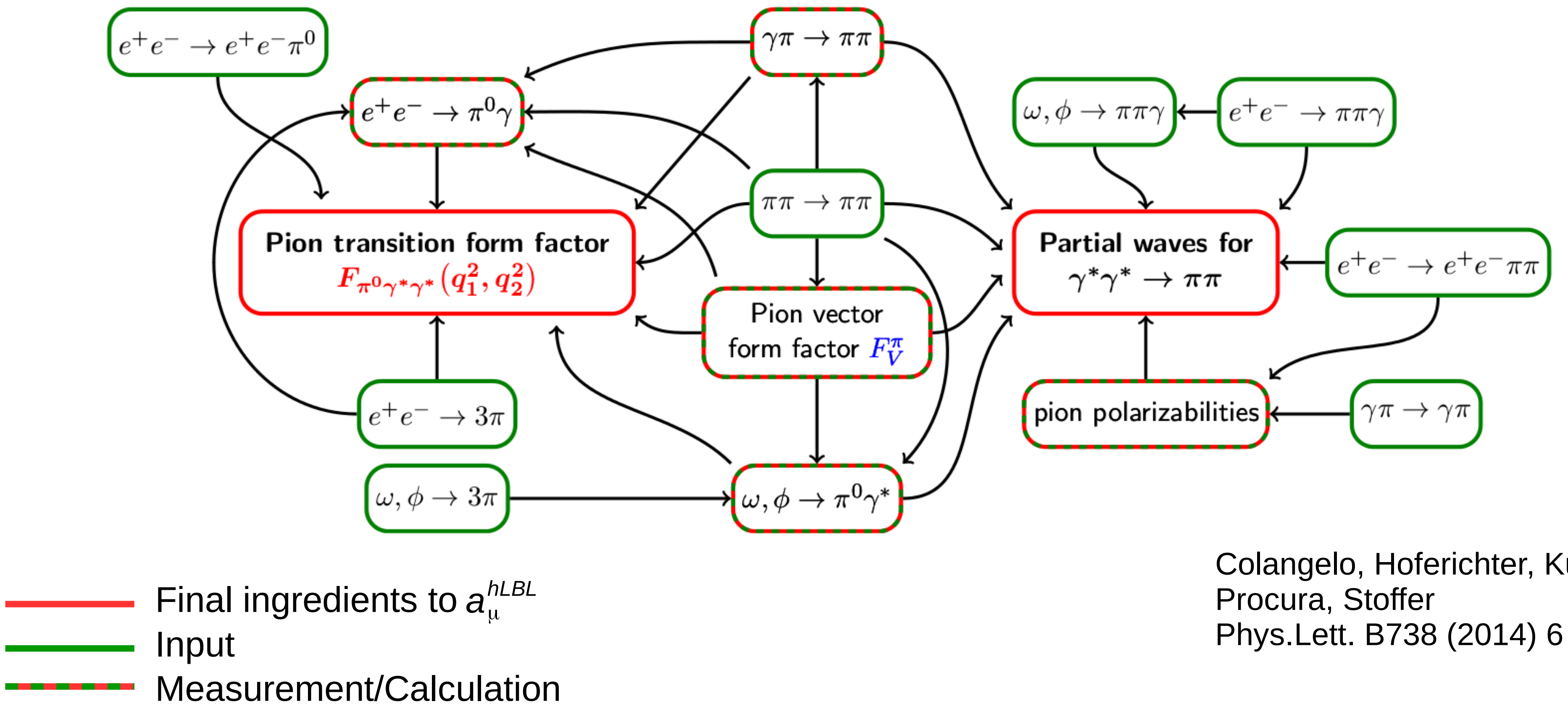
PRD 90 (2014) 113012
hep-ph:1403.7503

Dispersive Approach

Measurable quantities needed:

- TFF $F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ for arbitrary virtualities
- Partial waves for $\gamma^*\gamma^* \to \pi\pi$

Both can be constructed from other input:

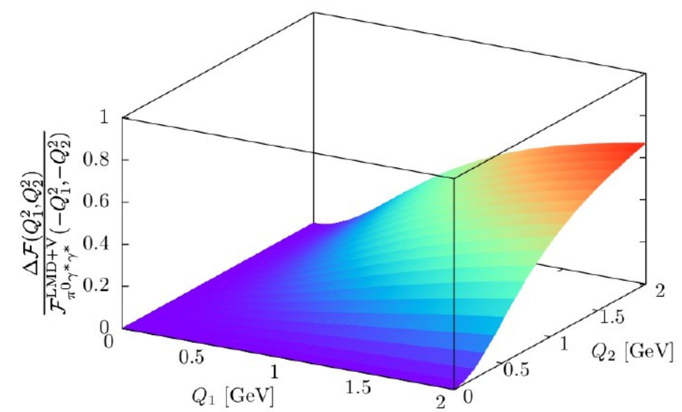


Colangelo, Hoferichter, Kubis, Procura, Stoffer
Phys.Lett. B738 (2014) 6

Double-tagged Measurements

Measurement of $F_{\pi^0\gamma\gamma}(Q_1^2, Q_2^2)$

- Full information
- Model independent information



Calculations: A. Nyffeler
Phys.Rev. D94, 2016, 053006

First test at BESIII

- ~ 10 fb^{-1} between 3.773 and 4.6 GeV
- Only $\mathcal{O}(10^2)$ event candidates expected

Clearly a case for a Super Tau Charm Factory!

Summary

SM prediction of a_μ limited by hadronic contributions

Experimental input needed to solve the puzzle

- hadronic Vacuum Polarization a_μ^{hVP}
  - Direct relation to hadronic cross sections $\sigma(\mathrm{e}^+\mathrm{e}^- \to \mathrm{hadrons})$
  - High precision data needed ⟶ HIEPA
- hadronic Light-by-Light scattering a_μ^{hLbL}
  - Realistic error estimates from data-driven approaches
    - Transition form factor $F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ and partial waves $\gamma^*\gamma^* \to \pi\pi$
      - High intensity machine with tagging detectors needed ⟶ HIEPA
  - In view of anticipated experimental accuracy $\delta a_\mu^{\mathrm{exp}} \sim 1.6 \times 10^{-10}$ contributions of η and η' become relevant!

$a_\mu^{hLbL,\eta} \sim 1.5 \times 10^{-10}$ Knecht,Nyffeler Phys.Rev.D65 (2002) 073034

$a_\mu^{hLbL,\eta'} \sim 1.5 \times 10^{-10}$

Great prospects and opportunities for a Super Tau Charm Factory!