Electromagnetic form factor of baryons from Chiral Effective field theory

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with J.Haldenbauer and U.-G. Meissner Based on: JHEP1707 (2017) 078; PRD96 (2017) 116001; PRD98 (2018) 014005;

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Outlines



1. Introduction

- Nucleon anti-nucleon: one of the most typical strong interactions
- LEAR has done lots of measurements



Baryon anti-baryon spectrum

- Threshold enhancement of baryon anti-baryon is observed by lots of experiment: pp, $\Lambda\Lambda$, $\Sigma\Sigma$, $\Lambda_c\Lambda_c$, $\Xi\Xi(?)...$
- Their interactions are important to study the the structures in the baryon anti-baryon spectroscopy.
- BABAR, BESIII, BelleII...
- FAIR: PANDA (pp), PAX (polarized p)?

red: anti-baryon

Strategy

New insights in strong interactions?



2. NN scattering amplitudes

NN scattering amplitude, two parts

- elastic NN scattering:
 E.Epelbaum et.al., EPJA51 (2015), 53
 - pion(s) exchange: NN Chiral EFT+G-parity
 - LECs of contact term: to be fixed by data
- annihilation: unitarity, fit to the data



$$V^{NN} = V_{1\pi} + V_{2\pi} + V_{3\pi} + \dots + V_{cont}$$
$$V_{el}^{\bar{N}N} = -V_{1\pi} + V_{2\pi} - V_{3\pi} + \dots + V_{cont}$$
$$V_{ann}^{\bar{N}N} = \sum_{X} V^{\bar{N}N \to X}$$

J.Haidenbauer, talk at Bochum

ChEFT

- Up to N³LO, in time ordered ChEFT:
 - only irreducible diagrams contributes
 - Lippmann-Shwinger equation



ChEFT: potentials

pion(s) exchange potentials:

$$V_{1\pi}(q) = \left(\frac{g_A}{2F_{\pi}}\right)^2 \left(1 - \frac{p^2 + p'^2}{2m^2}\right) \tau_1 \cdot \tau_2 \frac{\sigma_1 \cdot \mathbf{q} \, \sigma_2 \cdot \mathbf{q}}{\mathbf{q}^2 + M_{\pi}^2}$$

 $V_{2\pi} = V_C + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_C + [V_S + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_S] \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + [V_T + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_T] \boldsymbol{\sigma}_1 \cdot \boldsymbol{q} \boldsymbol{\sigma}_2 \cdot \boldsymbol{q}$ $+ [V_{LS} + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_{LS}] i(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{q} \times \boldsymbol{k}),$

 Fourier transformation: change it into corordinat space to do regularization

$$\begin{split} V_{C}(q) &= 4\pi \int_{0}^{\infty} f(r) V_{C}(r) j_{0}(qr) r^{2} dr ,\\ V_{S}(q) &= 4\pi \int_{0}^{\infty} f(r) \left(V_{S}(r) j_{0}(qr) + \tilde{V}_{T}(r) j_{2}(qr) \right) r^{2} dr ,\\ V_{T}(q) &= -\frac{12\pi}{q^{2}} \int_{0}^{\infty} f(r) \tilde{V}_{T}(r) j_{2}(qr) r^{2} dr ,\\ V_{SL}(q) &= \frac{4\pi}{q} \int_{0}^{\infty} f(r) V_{LS}(r) j_{1}(qr) r^{3} dr .\\ \end{split}$$



ChEFT: potentials

• Contact terms: short distance $V({}^{1}S_{0}) = \tilde{C}_{1S_{0}} + C_{1S_{0}}(p^{2} + p'^{2}) + D^{1}{}_{1S_{0}}p^{2}p'^{2} + D^{2}{}_{1S_{0}}(p^{4} + p'^{4}),$ $V({}^{3}S_{1}) = \tilde{C}_{3S_{1}} + C_{3S_{1}}(p^{2} + p'^{2}) + D^{1}{}_{3S_{1}}p^{2}p'^{2} + D^{2}{}_{3S_{1}}(p^{4} + p'^{4}),$ $V({}^{1}P_{1}) = C_{1P_{1}}pp' + D_{1P_{1}}pp'(p^{2} + p'^{2}),$ $V({}^{3}P_{1}) = C_{3P_{0}}pp' + D_{3P_{1}}pp'(p^{2} + p'^{2}),$ $V({}^{3}P_{0}) = C_{3P_{0}}pp' + D_{3P_{0}}pp'(p^{2} + p'^{2}),$ $V({}^{3}P_{2}) = C_{3P_{2}}pp' + D_{3P_{2}}pp'(p^{2} + p'^{2}),$ $V({}^{3}D_{1} - {}^{3}S_{1}) = C_{\epsilon_{1}}p'^{2} + D^{1}{}_{\epsilon_{1}}p^{2}p'^{2} + D^{2}{}_{\epsilon_{1}}p'^{4},$ $V({}^{3}S_{1} - {}^{3}D_{1}) = C_{\epsilon_{1}}p'^{2} + D^{1}{}_{\epsilon_{1}}p^{2}p'^{2} + D^{2}{}_{\epsilon_{2}}p^{4},$

Non-local regularization

$$f(p',p) = \exp\left(-rac{p'^m + p^m}{\Lambda^m}
ight)$$

 Annihilation terms: short distance physics, around 1 fm or less
 the same form as that of contact terms

$$V_{\rm ann} = V_{\bar{N}N \to X} G_X V_{X \to \bar{N}N}$$

Ignore the transition between annihilation channels

ChEFT: potentials

- Imaginary term: -i, required by unitarity
- Real term: be absorbed into terms of momentum

$$\begin{split} V_{ann}^{L=0} &= -i(\tilde{C}_{1_{S_0}}^a + C_{1_{S_0}}^a p^2 + D_{1_{S_0}}^a p^4)(\tilde{C}_{1_{S_0}}^a + C_{1_{S_0}}^a p'^2 + D_{1_{S_0}}^a p'^4), \\ V_{ann}^{L=1} &= -i(C_{\alpha}^a p + D_{\alpha}^a p^3)(C_{\alpha}^a p' + D_{\alpha}^a p'^3), \\ V_{ann}^{L=2} &= -i(D_{\beta}^a)^2 p^2 p'^2, \\ V_{ann}^{L=3} &= -i(D_{\gamma}^a)^2 p^3 p'^3, \\ V_{ann}^{S\to S} &= -i(\tilde{C}_{3_{S_1}}^a + C_{3_{S_1}}^a p^2 + D_{3_{S_1}}^a p^4)(\tilde{C}_{3_{S_1}}^a + C_{3_{S_1}}^a p'^2 + D_{3_{S_1}}^a p'^4), \\ V_{ann}^{S\to D} &= -i(\tilde{C}_{3_{S_1}}^a + C_{3_{S_1}}^a p^2 D_{3_{S_1}}^a p^4)C_{\epsilon_1}^a p'^2, \\ V_{ann}^{D\to S} &= -iC_{\epsilon_1}^a p^2(\tilde{C}_{3_{S_1}}^a + C_{3_{S_1}}^a p'^2 + D_{3_{S_1}}^a p'^4), \\ V_{ann}^{D\to D} &= -i[(C_{\epsilon_1}^a)^2 + (C_{3_{D_1}}^a)^2]p^2 p'^2. \end{split}$$

Brs.

Higer power appears due to unitarity

Fits to Nijmegen's phase shifts

PW projection, Lippmann-Shwinger equation

 $T_{L''L'}(p'',p';E_k) = V_{L''L'}(p'',p') + \sum_{L} \int_0^\infty \frac{dpp^2}{(2\pi)^3} V_{L''L}(p'',p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p,p';E_k)$

• The total χ^2 for different cutoffs

R=0.7 fm	$R=0.8\mathrm{fm}$	$R=0.9\mathrm{fm}$	$R=1.0\mathrm{fm}$	R=1.1 fm	R=1.2 fm
0.002	0.003	0.004	0.004	0.019	0.036
0.032	0.023	0.025	0.036	0.090	0.176
0.143	0.106	0.115	0.177	0.312	0.626
2.855	2.012	2.171	3.383	5.531	9.479
	R=0.7 fm 0.002 0.032 0.143 2.855	R=0.7 fm R=0.8 fm 0.002 0.003 0.032 0.023 0.143 0.106 2.855 2.012	R=0.7 fm R=0.8 fm R=0.9 fm 0.002 0.003 0.004 0.032 0.023 0.025 0.143 0.106 0.115 2.855 2.012 2.171	R=0.7 fm R=0.8 fm R=0.9 fm R=1.0 fm 0.002 0.003 0.004 0.004 0.032 0.023 0.025 0.036 0.143 0.106 0.115 0.177 2.855 2.012 2.171 3.383	R=0.7 fm R=0.8 fm R=0.9 fm R=1.0 fm R=1.1 fm 0.002 0.003 0.004 0.004 0.019 0.032 0.023 0.025 0.036 0.090 0.143 0.106 0.115 0.177 0.312 2.855 2.012 2.171 3.383 5.531

Cutoffs: 0.9, 1.0 fm,
 ChEFT does not work
 above k=500 MeV



Phase shifts of different cutoff

Lowest partial waves are perfect up to 300 MeV



Phase shifts

Higher partial waves may be worse above 200 MeV



Error bands of different order

- Convergent when increasing orders, even in resonance region
- LO provides very limited description, only S-wave
- N³LO, up to 300 MeV for S- and P- waves



Error bands of different order

 Higher partial waves are well described at least up to 200 or 250 MeV



Observables Cross sections 500 500 Nakamura 1984 Bruckner 1987 $\sigma_{_{ann}}$ $\sigma_{_{tot}}$ Hamilton 1980 Spencer 1970 400 400-Brando 1991 Clough 1984 Bugg 1987 PWA 2012 300 300-(qm) PWA 2012 200-200b 100 100-0-0 30 150 Alston 1975 Chaloupka 1976 $\boldsymbol{\sigma}_{_{el}}$ $\boldsymbol{\sigma}_{_{cex}}$ Sakamoto 1982 Hamilton 1980 Nakamura 1984 Coupland 1977 20-100-PWA 2012 PWA 2012 a (mb) 10-50-0+ 0 0+ 200 400 600 200 400 800 600 800 0 p_{lab} (MeV/c) p_{lab} (MeV/c)

Angular distribution

 Differential cross section, analyzing power and correlation parameters are helpful for PWA



pp annihilation cross section

- Check in low energy region
- The 1/β² (not 1/β) anomalous threshold behaviour is caused by attractive Coulumn interaction



np annihilation cross section

Purely isospin one components, helpful for PWA



Energy shifts

- Hadronic shifts and broadenings in hyperfine states of pH
- Energy shifts and widths: Dese-Trueman formula

		NLO	N ² LO	N ³ LO	$N^{2}LO$ [42]	Experiment
$\Delta E_S + i \frac{\Gamma_S}{2} = -\frac{4}{M r^3} a_S^{sc} \left(1 - \frac{a_S^{sc}}{r_s} \beta \right)$	$E_{1_{S_0}}$ (eV)	-448	-446	-443	-436	-440(75) [98]
	/					-740(150) [97]
$\Delta E_P + i \frac{\Gamma_P}{2} = -\frac{3}{8M_p r_p^5} a_P^{sc}$	$\Gamma_{^{1}S_{0}}$ (eV)	1155	1183	1171	1174	1200(250) [98]
P B	ő.	3			S	1600(400) [97]
	$E_{{}^{3}S_{1}}$ (eV)	-742	-766	-770	-756	-785(35) [98]
\mathbf{I}						-850(42) [99]
T.L. Trueman, NP26 (1961) 57	$\Gamma_{{}^{3}S_{1}}$ (eV)	1106	1136	1161	1120	940(80) [98]
J. Carbonell <i>et.al.</i> , ZPA 343						770(150) [99]
(1992) 325	$E_{^{3}P_{0}} (\mathrm{meV})$	17	12	8	16	139(28) [100]
	$\Gamma_{{}^{\mathfrak{I}}P_0}$ (meV)	194	195	188	169	120(25) [100]
	E_{1S} (eV)	-670	-688	-690	-676	-721(14) [98]
X.W. Kang et.al. JHEP1402	Γ_{1S} (eV)	1118	1148	1164	1134	1097(42) [98]
(2014) 113	E_{2P} (meV)	1.3	2.8	4.7	2.3	15(20) [100]
	Γ_{2P} (meV)	36.2	37.4	37.9	27	38.0(2.8) [100]

NN FSI: proton FF

- Proton form factor: probe of the inner structure of proton
- $ee \rightarrow pp$, helpful for determine the space-like region FF



new measurement

CMD-3 has excellent measurement in low energy regionBESIII's in the region near threshold?



FSI

- To analyze $ee \rightarrow pp$, we need to consider FSI
- Distorted-wave Born approximation (DWBA):

$$T_{\bar{N}N\to\bar{N}N} = V_{\bar{N}N\to\bar{N}N} + V_{\bar{N}N\to\bar{N}N}G_0T_{\bar{N}N\to\bar{N}N},$$

$$T_{e^+e^-\to\bar{N}N} = V_{e^+e^-\to\bar{N}N} + V_{e^+e^-\to\bar{N}N}G_0T_{\bar{N}N\to\bar{N}N},$$



Vector meson dominance: ³S₁-³D₁

J.Haidenbauer, X.-W. Kang, U.-G. Meißner, NPA 929 (2014), PRD91 (2015) 074003.

N²LO

N²LO result. N³LO's is coming

$$\sigma_{e^+e^- \to \overline{p}p} = \frac{4\pi\alpha^2\beta}{3s} C_p(s) \left[|G_M(s)|^2 + \frac{2M_p^2}{s} |G_E(s)|^2 \right]$$
$$|G_{\text{eff}}(s)| = \sqrt{\frac{\sigma_{e^+e^- \to \overline{p}p}(s)}{\frac{4\pi\alpha^2\beta}{3s} C_p(s) \left[1 + \frac{2M_p^2}{s}\right]}} J.\text{Hardenbauer, X.-W. Kang, U.-G.}$$

Meißner, NPA 929 (2014)



X1835?

- The amplitude gives information of structures, vice versa
- There is a structure around pp threshold in many channels



DWBA

• The intermaediate states NN rescattering is included $J/\psi \rightarrow \gamma \bar{N}N \rightarrow \gamma \eta' \pi^+ \pi^-$

- We need distorted-wave Born approximation (DWBA) $T_{\bar{N}N\to\bar{N}N} = V_{\bar{N}N\to\bar{N}N} + V_{\bar{N}N\to\bar{N}N}G_0T_{\bar{N}N\to\bar{N}N}, BR(\bar{p}p\to\eta'\pi^+\pi^-) = 0.626\%$ $T_{\bar{N}N\to\eta'\pi\pi} = V_{\bar{N}N\to\eta'\pi\pi} + T_{\bar{N}N\to\bar{N}N}G_0V_{\bar{N}N\to\eta'\pi\pi}, A_{J/\psi\to\gamma\bar{N}N} = A_{J/\psi\to\gamma\bar{N}N}^0 + A_{J/\psi\to\gamma\bar{N}N}^0 G_0T_{\bar{N}N\to\bar{N}N}, A_{J/\psi\to\gamma\eta'\pi\pi} = A_{J/\psi\to\gamma\eta'\pi\pi}^0 + A_{J/\psi\to\gamma\bar{N}N}^0 G_0T_{\bar{N}N\to\eta'\pi\pi} + A_{J/\psi\to\gamma\bar{N}N}^0 G_0T_{\bar{N}N\to\eta'\pi\pi} + A_{J/\psi\to\gamma\bar{N}N}^0 G_0T_{\bar{N}N\to\eta'\pi\pi}, X.W.Kang et.al., PRD91 (2015) 074003.$
- $J/\psi \rightarrow \omega pp$, ϕpp channels: isospin 0 component does not contribute to the enhancement near the threshold

 $T_{\bar{p}p} = (T^{I=0} + T^{I=1})/2 \longrightarrow T = (0.4 T^0 + 0.6 T^1)$

Modified isospin 1 NN amplitude

- Isospin 1 amplitude is modified slightly to fit to the J/ $\psi \rightarrow \gamma p p$ data
- Low energy part is the same as previous analysis
- After modification, we find a isospin 1 resonance: binding energy (-50.8-i 40.9) MeV, too wide to be seen



Threshold effects

- The inteference between background and cusp constructs the structre around pp threshold
- Bound state description is not necessary



Prediction

- We suggest BESIII to study the process of $J/\psi \rightarrow \gamma \omega \rho^0$
- More narrow bound state has more pronouned structure



4. $\Lambda_c \Lambda_c$ FSI: Λ_c FF

• We need to clarify the resonances appear in $\Lambda_c \Lambda_c$ scattering amplitude



Hadronic scattering amplitude

- Isospin 0, no OPE contribution
- VMD, ³S₁-³D₁

$$\begin{split} V(^3S_1)(p',p) &= \tilde{C}_{^3S_1} + C_{^3S_1}(p'^2 + p^2) - \mathrm{i}\,(\tilde{C}^a_{^3S_1} + C^a_{^3S_1}p'^2)\,(\tilde{C}^a_{^3S_1} + C^a_{^3S_1}p^2),\\ V(^3D_1 - {}^3S_1)(p',p) &= C_{\epsilon_1}\,p'^2 - \mathrm{i}\,C^a_{\epsilon_1}\,p'^2\,(\tilde{C}^a_{^3S_1} + C^a_{^3S_1}p^2),\\ V(^3S_1 - {}^3D_1)(p',p) &= C_{\epsilon_1}\,p^2 - \mathrm{i}\,(\tilde{C}^a_{^3S_1} + C^a_{^3S_1}p'^2)\,C^a_{\epsilon_1}\,p^2,\\ V(^3D_1)(p',p) &= 0\,, \end{split}$$

One resonance exchange in ChEFT

$$\mathcal{L} = g_{V} \overline{\Psi} \gamma^{\mu} \Psi \phi_{\mu} + \frac{f_{V}}{4 M_{\Lambda_{c}}} \overline{\Psi} \sigma^{\mu\nu} \Psi \left(\partial_{\mu} \phi_{V} - \partial_{V} \phi_{\mu} \right) + \text{H.c.}$$

$$V_{3S_{1}}(p', p; E) = \frac{4}{9m_{V}(E - m_{V})} \left[g_{V} \left(1 + \frac{M_{\Lambda_{c}}}{2E_{p'}} \right) + f_{V} \left(\frac{E}{4M_{\Lambda_{c}}} + \frac{E}{2E_{p'}} \right) \right] \left[g_{V} \left(1 + \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{4M_{\Lambda_{c}}} + \frac{E}{2E_{p'}} \right) \right] \left[g_{V} \left(1 + \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p'}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p'}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 + \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p'}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 + \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{4M_{\Lambda_{c}}} + \frac{E}{2E_{p'}} \right) \right] \left[g_{V} \left(1 + \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{4M_{\Lambda_{c}}} + \frac{E}{2E_{p'}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{2E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{E_{p}} \right) + f_{V} \left(\frac{E}{2E_{p}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \left[g_{V} \left$$

Born term and DWBA

Born amplitude, including pole term and nonpole term

 $F_{3S_{1}}^{0}(p',p;E) = -\frac{4\alpha}{9} \left\{ G_{ee} \left(1 + \frac{M_{\Lambda_{c}}}{2E_{p'}} \right) + \frac{g_{ee}}{m_{V}(E-m_{V})} \left[g_{V} \left(1 + \frac{M_{\Lambda_{c}}}{2E_{p'}} \right) + f_{V} \left(\frac{E}{4M_{\Lambda_{c}}} + \frac{E}{2E_{p'}} \right) \right] \right\} \left(1 + \frac{m_{e}}{2E_{p}} \right),$ $F_{3D_{1}}^{0}(p',p;E) = -\frac{2\alpha}{9} \left\{ G_{ee} \left(1 - \frac{M_{\Lambda_{c}}}{E_{p'}} \right) + \frac{g_{ee}}{m_{V}(E-m_{V})} \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{E_{p'}} \right) + f_{V} \left(\frac{E}{2E_{p'}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \right\} \left(1 - \frac{m_{e}}{E_{p}} \right),$ $F_{3D_{1}-3S_{1}}^{0}(p',p;E) = -\frac{2\sqrt{2}\alpha}{9} \left\{ G_{ee} \left(1 - \frac{M_{\Lambda_{c}}}{E_{p'}} \right) + \frac{g_{ee}}{m_{V}(E-m_{V})} \left[g_{V} \left(1 - \frac{M_{\Lambda_{c}}}{E_{p'}} \right) + f_{V} \left(\frac{E}{2E_{p'}} - \frac{E}{2M_{\Lambda_{c}}} \right) \right] \right\} \left(1 + \frac{m_{e}}{2E_{p}} \right),$ $F_{3S_{1}-3D_{1}}^{0}(p',p;E) = -\frac{2\sqrt{2}\alpha}{9} \left\{ G_{ee} \left(1 + \frac{M_{\Lambda_{c}}}{2E_{p'}} \right) + \frac{g_{ee}}{m_{V}(E-m_{V})} \left[g_{V} \left(1 + \frac{M_{\Lambda_{c}}}{2E_{p'}} \right) + f_{V} \left(\frac{E}{4M_{\Lambda_{c}}} + \frac{E}{2E_{p'}} \right) \right] \right\} \left(1 - \frac{m_{e}}{E_{p}} \right).$ = LS and DWBA

$$T_{L''L'}(p'',p';E) = V_{L''L'}(p'',p';E) + \sum_{L} \int_{0}^{\infty} \frac{dpp^2}{(2\pi)^3} V_{L''L}(p'',p;E) \frac{1}{E - 2E_p + i0^+} T_{LL'}(p,p';E)$$

 $F_{L''L'}^{\Lambda_e^+\bar{\Lambda}_e^-,e^+e^-}(k,k_e;E) = F_{L''L'}^0(k,k_e;E) + \sum_L \int_0^\infty \frac{dpp^2}{(2\pi)^3} T_{L''L}(k,p;E) \frac{1}{E - 2E_p + i0^+} F_{LL'}^0(p,k_e;E)$

ee→<mark>∧</mark>_c∧_c fit

- LO $\Lambda_c \Lambda_c$ elastic scattering potential, without annihilation
- Varying the cutoffs



ee→<mark>∧</mark>_c∧_c fit

- NLO $\Lambda_c \Lambda_c$ elastic scattering potential, with annihilation
- varying the number of LECs



poles and scattering lengths

 Poles and scattering lengths are stable with nonpole term

Λ (GeV)	0.45	0.50	0.55	0.65	0.75	0.85
with pole term, see	e Eq. 🖪					
\tilde{C}_{3S_1}	191.8	110.1	61.27	7.853	-19.17	-34. <mark>4</mark> 8
g_V	-8.734	-8.123	-7.625	-6.837	-6.218	-5.706
m_V (GeV)	4.6344	4.6364	4.6383	4.6419	4.6448	4.6472
$g_{ee}(\times 10^{-3} \text{GeV}^2)$	1.052	1.067	1.081	1.102	1.116	1.126
χ^2	0.1	0.3	0.4	0.6	0.7	0.8
pole (GeV)	4.6550	4.6534	4.6514	4.6482	4.6462	4.6451
	-i0.0264	-i0.0311	-i0.0343	-i0.0376	-i0.0389	-i0.0394
<i>a</i> (fm)	-0.269	-0.485	-0.634	-0.818	-0.927	-1.002
with pole and non-	pole contribu	tion, see Eq. (4)				••••
C3 51	191.9	111.9	65.65	-0.0100	-11.76	-26.96
g_V	-8.808	-7.964	-7.356	-6.490	-5.899	-5.415
m_V (GeV)	4.6328	4.6398	4.6443	4.6473	4.6542	4.6572
$g_{ee}(\times 10^{-3} \text{GeV}^2)$	1.055	1.052	1.042	1.045	1.004	0.987
$G_{ee}(\times 10^{-3})$	0.272	-0.578(658)	-1.035	-1.100	-1.672	-1.787
χ^2	0.1	0.1	0.2	0.2	0.2	0.2
pole (GeV)	4.6543	4.6552	4.6554	4.6532	4.6550	4.6549
	-i0.0276	-i0.0284	-i 0.0295	-i0.0304	-i0.0314	-i0.0319
a (fm)	-0.325	-0.360	-0.403	-0.641	-0.538	-0.581

poles and scattering lengths

Poles are not sensitive to the strength of annihilation

	1 LEC	2 LECs	3 LECs	4 LECs
with pole term.				
$\tilde{C}_{3_{S_1}}$ (GeV ⁻²)	-19.17	-19.23	-0.1001	-49.78
C_{3S_1} (GeV ⁻⁴)	-	-	-191.3	-146.4
$\tilde{C}^{a}_{3S_{1}}$ (GeV ⁻¹)	0.42	0.1661	-0.5353	-1159
$C^{a}_{3S_{1}}$ (GeV ⁻³)	(-)	-	-	4567
g_V	-6.218	-6.218	-5.071	-4.705
m_V (GeV)	4.6448	4.6448	4.6386	4.6362
$g_{ee}(\times 10^{-3} {\rm GeV^2})$	1.116	1.116	1.079	1.171
χ^2	0.7	0.7	0.3	0.1
pole (GeV)	$4.6462 - {\rm i} 0.0389$	4.6455 - i0.0390	4.6501 - i0.0396	4.6506 - i0.0397
$a \ (fm)$	-0.927	-0.928	-0.726	$-0.916 - {\rm i} 0.844$
with pole and nor	n-pole contribution	n.		
	1 LEC	2 LECs	3 LECs	4 LECs
\tilde{C}_{3S_1} (GeV ⁻²)	-11.76	-11.74	-0.0135	-60.76
C_{3S_1} (GeV ⁻⁴)	1.000	(m)	-187.9	-74.23
$\tilde{C}^{a}_{3S_{1}}$ (GeV ⁻¹)		0.6595	0.0503	-1185
$C^{a}_{3S_{1}}$ (GeV ⁻³)		12.0	<u>27</u>	5455
g_V	-5.899	-5.897	-5.012	-4.858
m_V (GeV)	4.6542	4.6542	4.6414	4.6342
$g_{ee}(\times 10^{-3} \text{GeV}^2)$	1.004	1.003	1.063	1.200
$G_{ee}(\times 10^{-3})$	-1.672	-1.679	-0.455	0.512
χ^2	0.2	0.2	0.2	0.1
pole (GeV)	4.6550 - i0.0314	$4.6546 - {\rm i} 0.0312$	$4.6520 - {\rm i} 0.0285$	$4.6482 - {\rm i} 0.0341$
$a \ (fm)$	-0.000	-0.557	-0.032	-0.981 - 10.714

	present analysis	Belle [8]	Belle [7]	BABAR [6]
reaction	$e^+e^- \rightarrow \Lambda_c^+ \overline{\Lambda}_c^-$	$e^+e^- \rightarrow \Lambda_c^+ \overline{\Lambda}_c^-$	$e^+e^- \to \pi^+\pi^-\psi(2S)$	$e^+e^- \to \pi^+\pi^-\psi(2S)$
mass <mark>M(</mark> MeV)	4652.5 ± 3.4 ± 1.1	4634+8+5	4652 ± 10 ± 8	4669 ± 21 ± 3
width Γ (MeV)	62.6 ± 5.6 ± 4.3	92 ⁺⁴⁰⁺¹⁰ -24-21	$68 \pm 11 \pm 1$	$104\pm48\pm10$

Electromagnetic formfactor

- Effective form factors for LO, NLO
- BESIII's has a different trend comparing to Belle's
- The future measurement of BESIII up to 4.65 GeV is important



6. Summary

NN Amplitude

Up to N³LO, we fit ChEFT amplitude to PWA's. It works well at PLab<300 MeV for S- and P-waves, and a bit lower for higher partial waves. Its low energy part is also consistent with the antiprotonic hydrogen phenomenology.

We are studying the proton FF from $ee \rightarrow pp$ up to N3LO, with LS equation and DWBA method. The structure around pp threshold exhibits a cusp-like behavior.

 $\Lambda_{c}\Lambda_{c}$

NN

We study $e+e- \rightarrow \Lambda_c \Lambda_c$ process close to the theshold. the pole location of X(4630/4660) is obtained. The Λ_c FF is predicted.

Prospects

Amplitude analysis, analytical continuation to obtain the FF in unphysical region?



Thank You For your patience!