$\tau^+\tau^-$ atom and τ mass

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Outline

1. Introduction
   - τ mass
   - $\tau^+\tau^-$ atom
2. The frame of Calculation
3. Numerical Result
4. Discussion

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τ

Need more precise measurements m_τ, Γ_τ, $(g-2)_\tau$ in PDG 2022

τ

$$J = \frac{1}{2}$$

Mass $m = 1776.86 \pm 0.12$ MeV

$(m_{\tau^+} - m_{\tau^-})/m_{\text{average}} < 2.8 \times 10^{-4}$, CL = 90%

Mean life $\tau = (290.3 \pm 0.5) \times 10^{-15}$ s

$c\tau = 87.03\ \mu$m

Magnetic moment anomaly > -0.052 and < 0.013, CL = 95%

$\text{Re}(d_\tau) = -0.220$ to 0.45×10^{-16} $e$cm, CL = 95%

$\text{Im}(d_\tau) = -0.250$ to 0.0080×10^{-16} $e$cm, CL = 95%

m_τ and lepton universality, 1405.1076

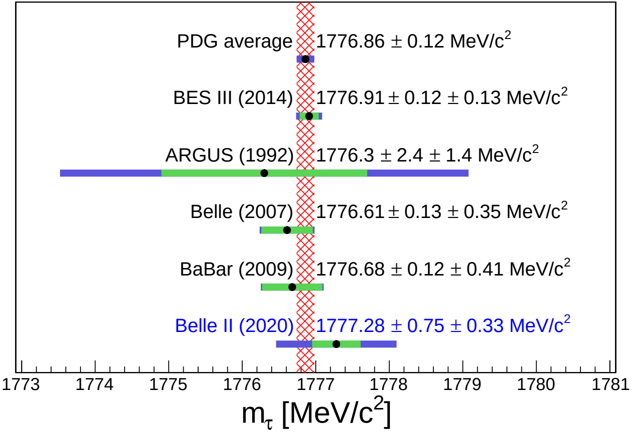
- Comparing the electronic branching fractions of τ and μ, lepton universality can be tested as

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\tau_\mu}{\tau_\tau}\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{B(\tau \to e\nu\bar{\nu})}{B(\mu \to e\nu\bar{\nu})}(1+F_W)(1+F_\gamma), \quad (1)$$

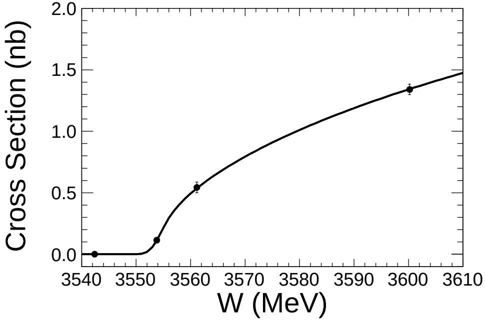
- BESIII measurement, 1405.1076

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = 1.0016 \pm 0.0042, \quad (2)$$

Measured m_τ, from Belle II 2008.04665



m_τ measurement at BESIII, 1405.1076



$$\sigma(E_{\rm CM}, m_\tau, \delta_w^{\rm BEMS}) = \frac{1}{\sqrt{2\pi}\delta_w^{\rm BEMS}} \int_{2m_\tau}^{\infty} dE'_{\rm CM} e^{\frac{-(E_{\rm CM}-E'_{\rm CM})^2}{2(\delta_w^{\rm BEMS})^2}} \int_0^{1-\frac{4m^2}{E'^2_{\rm CM}}} dx F(x, E'_{\rm CM}) \frac{\sigma_1(E'_{\rm CM}\sqrt{1-x}, m_\tau)}{|1-\prod(E_{\rm CM})|^2}$$

New data taking scenario at BESIII, from Zhang Jianyong TAU2018

Data comparison

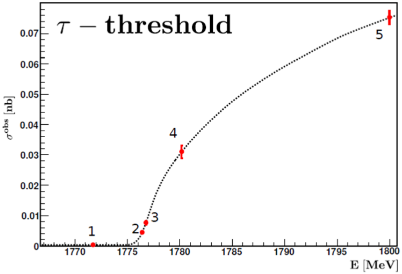
	J/ψ (pb^{-1})	ψ' (pb^{-1})	τ (pb^{-1})				
			3540 MeV	3553 MeV	3554 MeV	3560 MeV	3600 MeV
2011	1.5	7.5	4.3	0	5.6	3.9	9.6
2018	32.6	67.2	25.5	42.6	27.1	8.3	13.9

New data taking scenario at BESIII, from Zhang Jianyong TAU2018

Three energy regions:

- Low energy region
  Point 1, 14 pb^{-1}, to determine background
- Near threshold
  Point 2, 39 pb^{-1} and point 3, 26 pb^{-1}, to determine tau mass
- High energy region
  Point 4, 7 pb^{-1} for X^2 check
  Point 5, 14 pb^{-1} to determine detection efficiency

Total lum. ~100pb^{-1},
uncertainty: 0.1MeV



We obtain more than 130 pb^{-1} tau scan data!

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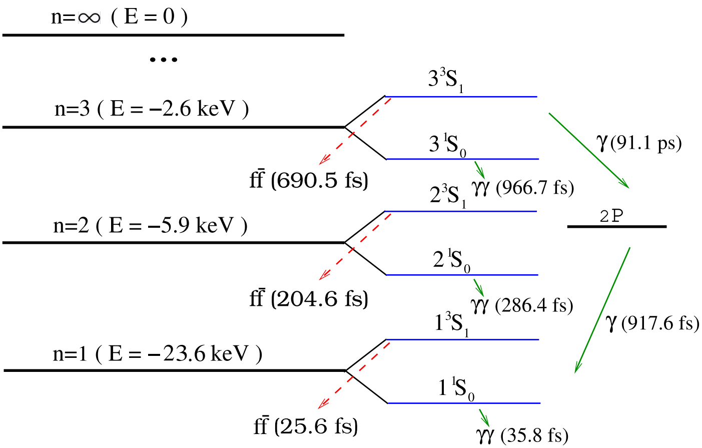
QED atom

1. QED atoms (e^+e^-, μ^+e^-, τ^+e^-, $\mu^+\mu^-$, $\tau^+\mu^-$, $\tau^+\tau^-$) are formed during QED interaction just as hydrogen.
2. The properties of QED atoms have been studied to test QED, fundamental symmetries, New Physics, gravity, and so on (hep-ex/0106103, 0912.0843, 1710.01833, 1802.01438, Phys.Rept. 975 (2022) 1-61).
3. Only positronium (e^+e^-) and muonium (μ^+e^-) had been discovered in 1951 and 1960 respectively.

$\tau^+\tau^-$ atom

1. $\tau^+\tau^-$ atom is the smallest QED atom for Bohr radius is 30.4 fm (Moffat:1975uw)
2. $\tau^+\tau^-$ atom is is named tauonium (Avilez:1977ai,Avilez:1978sa), ditauonium (2204.07269, 2209.11439), and true tauonium (2202.02316).
3. We named them following charmonium just as $J_\tau(nS)$ for $n^{2S+1}L_J = n^3S_1$ and $J^{PC} = 1^{--}$, $\chi_{\tau J}(nP)$ for $n^{2S+1}L_J = n+1^3P_J$ and $J^{PC} = J^{++}$.
4. The production of η_τ has been considered (2202.02316), and the production of J_τ in electron positron collisions has been estimated (0807.4114).
5. The spectroscopy of $\tau^+\tau^-$ atoms has been studies (2204.07269).

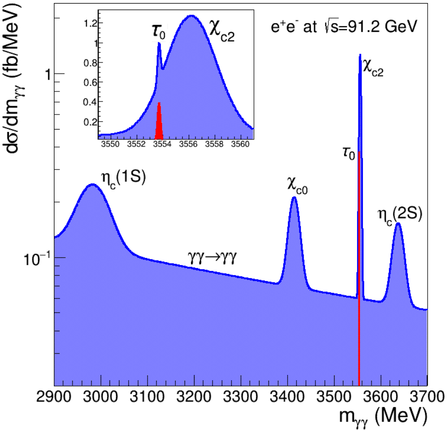
The spectroscopy of $\tau^+\tau^-$ atom, 2204.07269

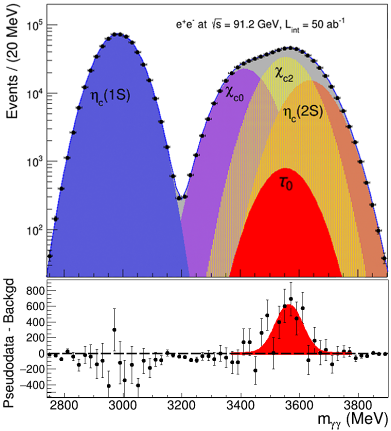


$\gamma\gamma \to \eta_\tau \to \gamma\gamma$, 2202.02316

Colliding system, c.m. energy, $\mathcal{L}_{\text{int}}$, exp.	$\sigma \times \mathcal{B}_{\gamma\gamma}$						$N \times \mathcal{B}_{\gamma\gamma}$	
	$\eta_c(1S)$	$\eta_c(2S)$	$\chi_{c,0}(1P)$	$\chi_{c,2}(1P)$	LbL	$\mathcal{T}_0$	$\mathcal{T}_0$	$\chi_{c,2}(1P)$
e^+e^- at 3.78 GeV, 20 fb^{-1}, BES III	120 fb	3.6 ab	15 ab	13 ab	30 ab	0.25 ab	–	–
e^+e^- at 10.6 GeV, 50 ab^{-1}, Belle II	1.7 fb	0.35 fb	0.52 fb	0.77 fb	1.7 fb	0.015 fb	750	38 500
e^+e^- at 91.2 GeV, 50 ab^{-1}, FCC-ee	11 fb	2.8 fb	3.9 fb	6.0 fb	12 fb	0.11 fb	5 600	$3 \cdot 10^5$
p-p at 14 TeV, 300 fb^{-1}, LHC	7.9 fb	2.0 fb	2.8 fb	4.3 fb	6.3 fb	0.08 fb	24	1290
p-Pb at 8.8 TeV, 0.6 pb^{-1}, LHC	25 pb	6.3 pb	8.7 pb	13 pb	21 pb	0.25 pb	0.15	8
Pb-Pb at 5.5 TeV, 2 nb^{-1}, LHC	61 nb	15 nb	21 nb	31 nb	62 nb	0.59 nb	1.2	62

$\gamma\gamma \to \eta_\tau \to \gamma\gamma$ at Z pole, 2202.02316





Recent progress: NNNLO

1. AMFlow: 2201.11669, 2201.11636, 2201.11637
2. $e^+e^- \to t\bar{t}$ at NNNLO in QCD, 2209.14259
3. $\Upsilon \to e^+e^-$, decay constant of B_c, 2207.14259, 2208.04302

$e^+e^- \to \tau^+\tau^- \to \nu X^- \bar{\nu} X^+$ around the $\tau^+\tau^-$ production threshold

1. Updated cross sections

$$\sigma_{ex}(W, m_\tau, \Gamma_\tau, \delta_w) = \int_{m(J_\tau)}^{\infty} dW' \frac{e^{-\frac{(W-W')^2}{2\delta_w^2}}}{\sqrt{2\pi}\delta_w} \int_0^{1-\frac{m(J_\tau)^2}{W'^2}} dx F(x, W') \frac{\bar{\sigma}(W'\sqrt{1-x}, m_\tau, \Gamma_\tau)}{|1-\Pi(W'\sqrt{1-x})|^2}.$$

2. Cross sections in BESIII, 1405.1076

$$\sigma(E_{\rm CM}, m_\tau, \delta_w^{\rm BEMS}) = \frac{1}{\sqrt{2\pi}\delta_w^{\rm BEMS}} \int_{2m_\tau}^{\infty} dE'_{\rm CM} e^{\frac{-(E_{\rm CM}-E'_{\rm CM})^2}{2(\delta_w^{\rm BEMS})^2}} \int_0^{1-\frac{4m^2}{E'^2_{\rm CM}}} dx F(x, E'_{\rm CM}) \frac{\sigma_1(E'_{\rm CM}\sqrt{1-x}, m_\tau)}{|1-\prod(E_{\rm CM})|^2}$$

3. Difference: shift $2m_\tau$ to $m(J_\tau)$ in the range of integration and add Γ_τ as a variable of the cross sections after including $J_\tau(nS)$ atom.

$\bar{\sigma}(W, m_\tau, \Gamma_\tau)$, 1312.4791

1. $\bar{\sigma}(W, m_\tau, \Gamma_\tau)$

$$\bar{\sigma}(W, m_\tau, \Gamma_\tau) = \frac{4\pi\alpha^2}{3W^2}\frac{24\pi}{W^2}\mathrm{Im}\left[G_{\bar{\nu}X^+\nu X^-}(0,0,W-2m_\tau)\right], \quad (3)$$

2. $G_{\bar{\nu}X^+\nu X^-}(\vec{r}, \vec{r}\,', E)$ represents a Green function of $\tau^+\tau^-$ currents in the non-relativistic effective theory, where $\tau^+\tau^-$ decay to $\bar{\nu}X^+\nu X^-$

$$G_{\bar{\nu}X^+\nu X^-}(\vec{r}, \vec{r}\,', E) = \sum_n \frac{\psi_n(\vec{r})\psi_n^*(\vec{r}\,')}{E_n - E - i\epsilon} Br[n \to \bar{\nu}X^+\nu X^-] + \int \frac{d^3\vec{k}}{2\pi^3}\frac{\psi_{\vec{k}}(\vec{r})\psi_{\vec{k}}^*(\vec{r}\,')}{E_{\vec{k}} - E - i\epsilon}, \quad (4)$$

3. Then

$$\bar{\sigma}(W) = \bar{\sigma}^{J_\tau}(W) + \bar{\sigma}(W)_{continue} \quad (5)$$

Breit-Wigner formula

1. Green function approach to bound states is consistent with Breit-Wigner formula for a narrow bound states

$$\bar{\sigma}^{J_\tau}(W) = \sum_n \frac{6\pi^2}{W^2}\delta(W - m(J_\tau(nS)))\Gamma(J_\tau(nS) \to e^+e^-)Br(J_\tau(nS) \to \bar{\nu}X^+\nu X^-) \quad (6)$$

2. Ignore the binding Energy of $J_\tau(nS)$ for it much less than δ_w

$$\bar{\sigma}^{J_\tau}(W) = \frac{6\pi^2}{W^2}\delta(W - 2m_\tau)\sum_n \Gamma(J_\tau(nS) \to e^+e^-)Br(J_\tau(nS) \to \bar{\nu}X^+\nu X^-) \quad (7)$$

Decay mode of $J_\tau(nS)$

$$\begin{aligned} \Gamma_{total}(J_\tau(nS)) &= \Gamma_{Ani}(J_\tau(nS)) + \Gamma_{Weak}(J_\tau(nS)) + \Gamma_{E1}(J_\tau(nS)) \\ \Gamma_{Ani}(J_\tau(nS)) &= 4.2\Gamma(J_\tau(nS) \to e^+e^-) \\ \Gamma_{Weak}(J_\tau(nS)) &= 2\Gamma(\tau \to \nu X^-) \end{aligned} \tag{8}$$

Decay mode of $J_\tau(nS)$

TABLE I: $\Gamma(J_\tau(nS))$ (meV)

n	$\Gamma(e^+e^-)$	Γ_{Weak}	Γ_{E1}	Γ_{total}	$\Gamma(e^+e^-)Br(\bar{\nu}X^+\nu X^-)$
1	6.1362	4.5346	0.00000	30.3066	0.9181
2	0.7671	4.5346	0.00000	7.7561	0.4484
3	0.2273	4.5346	0.00724	5.4964	0.1874
4	0.0959	4.5346	0.00506	4.9424	0.0880
5	0.0491	4.5346	0.00325	4.7440	0.0449
6	0.0284	4.5346	0.00214	4.6561	0.0277
7	0.0179	4.5346	0.00146	4.6112	0.0176
8	0.0120	4.5346	0.00104	4.5849	0.0119
9	0.0084	4.5346	0.00076	4.5700	0.0084

Cross sections from $J_\tau(nS)$

1. Then we get the $J_\tau(nS)$ contribution the cross section in Eq.11

$$\bar{\sigma}^{J_\tau}(W) = 3.26\,\delta(W - 2m_\tau)\ \mathrm{pb\ MeV} \tag{9}$$

2. Updated $\bar{\sigma}(W, m_\tau, \Gamma_\tau)$

$$\bar{\sigma}(W) = 3.26\delta(W - 2m_\tau)\ \mathrm{pb\ MeV} + \theta(W - 2m_\tau)\bar{\sigma}_{Continue} \tag{10}$$

Nomber of enents of $J_\tau \to \nu X^- \bar{\nu} X^+$ at BESIII

BESIII collect 42.6 pb^{-1} data at 3553 MeV and 27.1 pb^{-1} data at 3554 MeV in 2018. And $\delta_W = 1.2$ MeV. Then the nomber of enents of $J_\tau \to \nu X^- \bar{\nu} X^+$ at BESIII is

$$N_{J_\tau} \sim 50 \times Exp[-(W - 2m_\tau)^2/(2.88\ \mathrm{MeV}^2)] \tag{11}$$

We can discovery J_τ at BESIII.

m_τ Estimate

1. The data include contribution from $J_\tau(nS)$, so the continue contribution will be suppress.
2. m_τ will move from 1776.91 MeV to about 1777.77 MeV during BESIII data in 2011.
3. Γ_τ and $(g-2)_\tau$ will be measured at STCF.
4. Updated BESIII measurement, 1405.1076

$$\left(\frac{g_\tau}{g_\mu}\right)^2_{Updated} = \frac{1776.91^5}{1777.77^5} \times 1.0016 \pm 0.0042 = 0.9992 \pm 0.0042, \qquad (12)$$

Discussion

1. J_τ may be discovered at BESIII.
2. m_τ will be enlarged.
3. Γ_τ and $(g-2)_\tau$ will be measured at STCF.

Thanks

Process, c.m. energy, $\mathcal{L}_{\text{int}}$, exp.	$\sigma[\mu^+\mu^- + LH]$			$N[\mu^+\mu^- + LH]$			Significance
	$\psi(2S)$	Continue	J_τ	$\psi(2S)$	Continue	J_τ	
$\mu^+\mu^-$ at 3.554 GeV, 100 fb^{-1}, BES III	-	6.9 nb	0.94 pb	-	0.69E9	94E3	3.6 σ
$\mu^+\mu^-$ or LH at 3.554 GeV, 100 fb^{-1}, BES III	-	21 nb	3.0 pb	-	2.2E9	300E3	6.4σ
$\mu^+\mu^-$ via ISR at 10.6 GeV, 50 ab^{-1}, Belle II	108 fb	8.7 pb	7.7 ab	5.4E6	440E6	380	-
$\mu^+\mu^-$ via ISR at 91.2 GeV, 50 ab^{-1}, FCC-ee	1.6 fb	12.7 fb	0.12 ab	80E3	640E3	6	-