

# Holographic vortex pair annihilation in superfluid turbulence

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Based mainly on **arXiv:1412.8417** with:  
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**HUST@Hefei,China**

## International School on NR and GWs

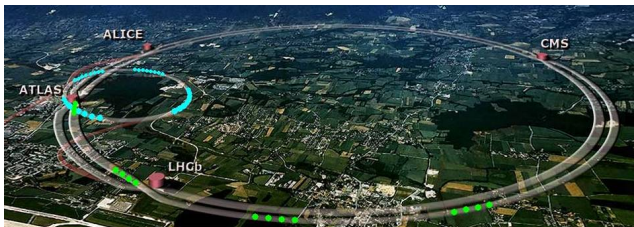
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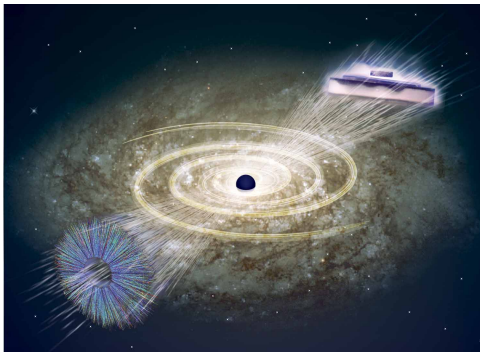
<https://www.apctp.org/plan.php/NRGW2015>



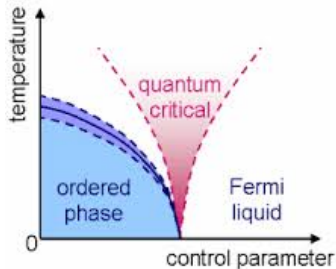
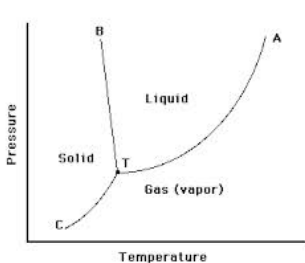
LIGO connected to LHC by holography!



Black hole can also answer condensed matter questions!



- The physical world is partially unified by remarkable **RG flow** in QFT
  - High Energy Physics: **IR**→**UV**(Reductionism)
  - Condensed Matter Physics: **UV**→**IR**(Emergence)
    - Thermal Phase Transition
    - Quantum Phase Transition



- Another seemingly distinct part is gravitation, which is understood as **geometry** by general relativity

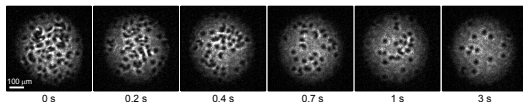
Remarkably, with AdS/CFT correspondence, general relativity can also geometrize renormalization flow in particular when the quantum field theory is strongly coupled, namely

$$GR = RG.$$

In this sense, the world is further unified by AdS/CFT duality. This talk will focus on its particular application to condensed matter physics by general relativity.

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[Shin *et.al.* arXiv:1403.4658]



Gross-Pitaevskii equation

$$(i - \eta)\hbar\partial_t\varphi = \left(-\frac{\nabla^2}{2m} + V(x, y, t) + g|\varphi|^2 - \mu\right)\varphi$$



## Here comes AdS/CFT I

It is a machine, mapping a hard quantum many-body problem to an easy classical few-body one.

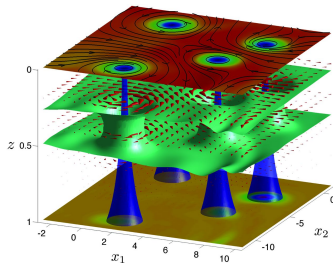


- Strongly coupled systems
- Non-equilibrium behaviors

## Here comes AdS/CFT I

[Adams, Chesler, Liu, arXiv:1212.0281]

- Kolmogorov scaling law:  $\epsilon_{kin}(k) \sim \varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$ ,
- A direct energy cascade from IR to UV.



But the temporal variation of vortex number density  $n(t)$  is more easily accessible by cold atom experiments. **So here we are!**

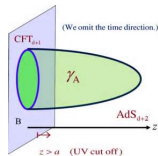
## What AdS/CFT is I: Dictionary



$$Z_{CFT}[J] = S_{AdS}[\phi](J = \phi)$$

## What AdS/CFT is II: Implications

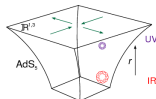
- Entanglement entropy for boundary QFT is equal to the extremal surface area in the bulk gravity



- Finite temperature field theory with finite chemical potential is dual to charged black hole

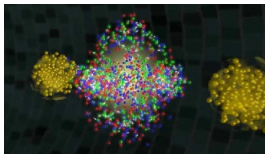


- AdS boundary corresponds to QFT at UV fixed point and the bulk horizon corresponds to IR fixed point



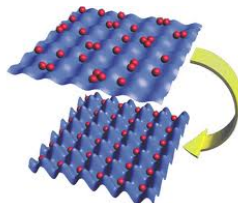
## Towards applied AdS/CFT

- AdS/QCD



- AdS/CMT

Non-Fermi liquids, superfluids and superconductors, charge density waves, thermalization and many-body localization...



- AdS/???

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- Action of model

[Hartnoll, Herzog, and Horowitz, arXiv:0803.3295, 0810.6513]

$$S = \frac{1}{16\pi G} \int_{\mathcal{M}} d^4x \sqrt{-g} \left[ R + \frac{6}{L^2} + \frac{1}{q^2} \left( -\frac{1}{4} F_{ab} F^{ab} - |D\Psi|^2 - m^2 |\Psi|^2 \right) \right]. \quad (1)$$

- Background metric

$$ds^2 = \frac{L^2}{z^2} [-f(z) dt^2 - 2dt dz + dx^2 + dy^2], \quad f(z) = 1 - \left( \frac{z}{z_h} \right)^3. \quad (2)$$

- Heat bath temperature

$$T = \frac{3}{4\pi z_h}. \quad (3)$$

- Equations of motion

$$D_a D_a \Psi - m^2 \Psi = 0, \quad \nabla_a F^{ab} = i(\bar{\Psi} D^b \Psi - \Psi \overline{D^b \Psi}). \quad (4)$$

- Asymptotical behavior at AdS boundary

$$A_\nu = a_\nu + b_\nu z + o(z), \quad (5)$$

$$\Psi = \frac{1}{L}[\phi z + z^2 \psi + o(z^2)]. \quad (6)$$

- AdS/CFT dictionary

$$\langle J^\nu \rangle = \frac{\delta S_{ren}}{\delta a_\nu} = \lim_{z \rightarrow 0} \frac{\sqrt{-g}}{q^2} F^{z\nu}, \quad (7)$$

$$\begin{aligned} \langle O \rangle &= \frac{\delta S_{ren}}{\delta \phi} = \lim_{z \rightarrow 0} \left[ \frac{z\sqrt{-g}}{Lq^2} D^z \bar{\Psi} - \frac{z\sqrt{-\gamma}}{L^2 q^2} \bar{\Psi} \right] \\ &= \frac{1}{q^2} (\bar{\psi} - \dot{\bar{\phi}} - i a_t \bar{\phi}), \end{aligned} \quad (8)$$

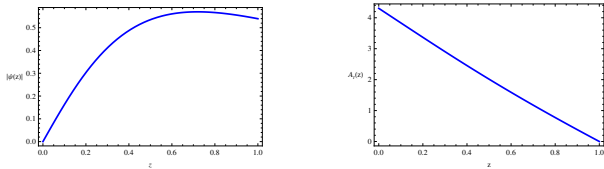
where

$$S_{ren} = S - \frac{1}{Lq^2} \int_{\mathcal{B}} \sqrt{-\gamma} |\Psi|^2 \quad (9)$$

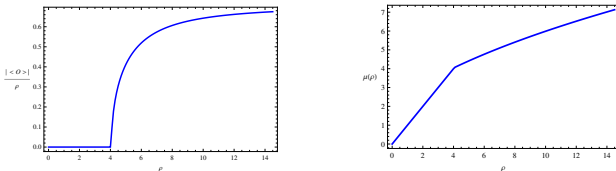
is the renormalized action by holography.



## Phase transition to a superfluid



**Figure:** The profile of amplitude of scalar field and electromagnetic potential for the superconducting phase at the charge density  $\rho = 4.7$ .



**Figure:** The condensate and chemical potential as a function of charge density with the critical charge density  $\rho_c = 4.06$  ( $\mu_c = 4.07$ ).

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## Quantized vortex in superfluids

With the superfluid velocity defined as

$$\mathbf{u} = \frac{\mathbf{j}}{|\psi|^2}, \mathbf{j} = \frac{i}{2}(\bar{\psi}\partial\psi - \psi\partial\bar{\psi}), \quad (10)$$

the winding number  $w$  of a vortex is determined by

$$w = \frac{1}{2\pi} \oint_{\gamma} d\mathbf{x} \cdot \mathbf{u}, \quad (11)$$

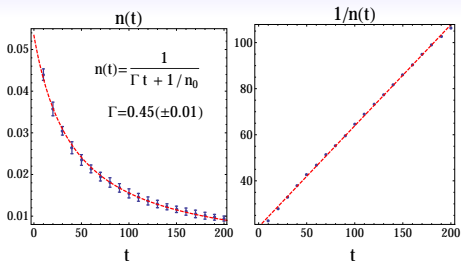
In particular, close to the core of a single vortex with winding number  $w$ , the condensate

$$\bar{\psi} \propto (\mathbf{z} - \mathbf{z}_0)^w, w > 0 \quad (12)$$

$$\psi \propto (\mathbf{z} - \mathbf{z}_0)^{-w}, w < 0 \quad (13)$$

with  $\mathbf{z}$  the complex coordinate and  $\mathbf{z}_0$  the location of the core.

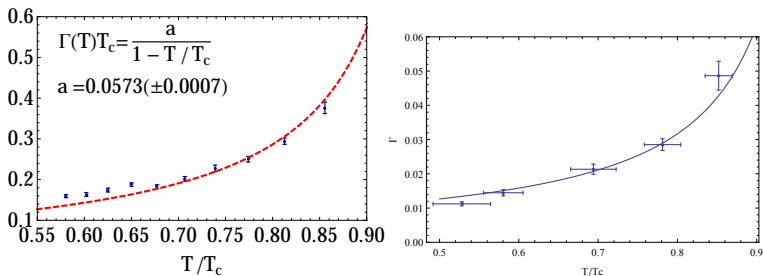
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**Figure:** The temporal evolution of averaged vortex number density in the turbulent superfluid over 12 groups of data with randomly prepared initial conditions at the chemical potential  $\mu = 6.25$

$$\frac{dn(t)}{dt} = -\Gamma n(t)^2, \quad (14)$$

where  $\Gamma = \frac{vd}{2}$  with  $v$  the velocity of vortices and  $d$  cross section if the vortices can be regarded as a gas of particles.



**Figure:** The variation of decay rate with respect to the temperature. The data near the critical point is fit by the effective field formula  $\Gamma \propto |O|^{-2}$  [Chesler, Lucas, arXiv:1411.2610].

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## Conclusion

- The decrease of vortex number can be well described by **two-body decay** due to vortex pair annihilation from a very early time on.
- The decay rate increases with the temperature.
- The decay rate near the critical temperature is **in good agreement** with the effective field theory calculation and the preliminary experimental data.
- Holography offers a **first principles** method for one to understand vortex dynamics by its gravity dual and may have an important impact on the upcoming experiments.



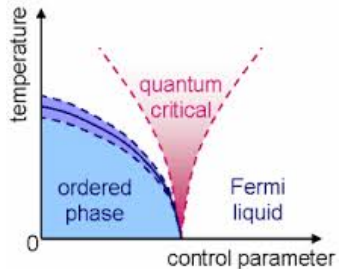
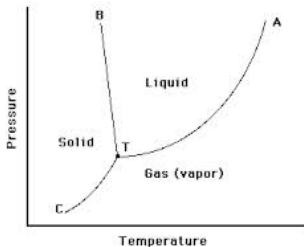
## Outlook

- Low temperature behavior, where  $T^2$  behavior can be reproduced?
- Other phenomena related to vortex dynamics such as snake instability, where the challenge arises mainly in the non-trivial boundary conditions.
- Back reaction effect, where one is require to go to fully numerical relativity regime.

Thanks for your attention!

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## Why AdS/CFT I: String theory

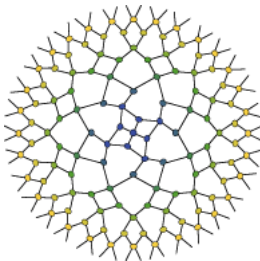
- Maldacena duality
- ABJM duality
- High spin gravity

## Why AdS/CFT II: General relativity

- Bousso's covariant entropy bound
- Bekenstein-Hawking's black hole thermodynamics
- Brown-York's surface tensor formulation of quasilocal energy and conserved charges
- Brown-Henneaux's asymptotic symmetry analysis for three dimensional gravity
- Minwalla's Gravity/Fluid correspondence

## Why AdS/CFT III: Quantum field theory

- MERA



- EHM

