Partial Wave Analysis Amplitude Check

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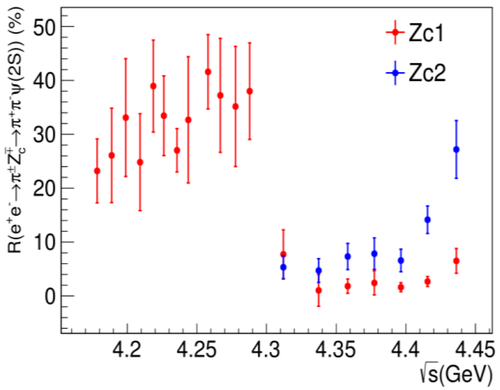
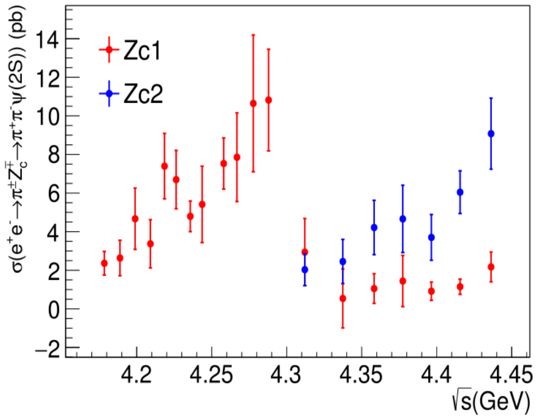
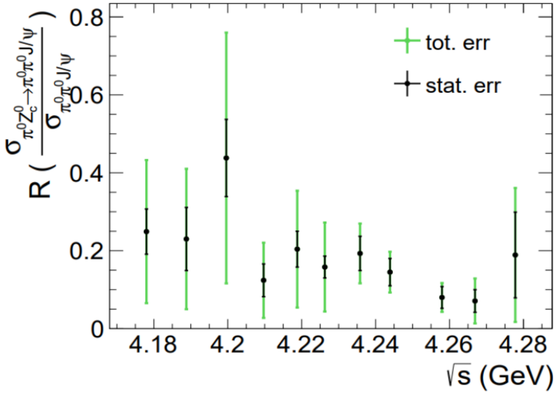
Motivation

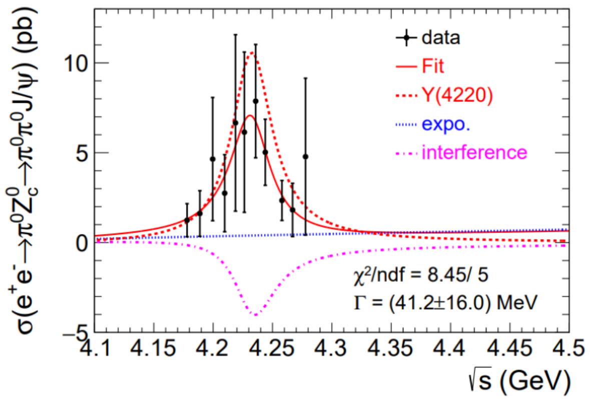
Covariant tensor formalism



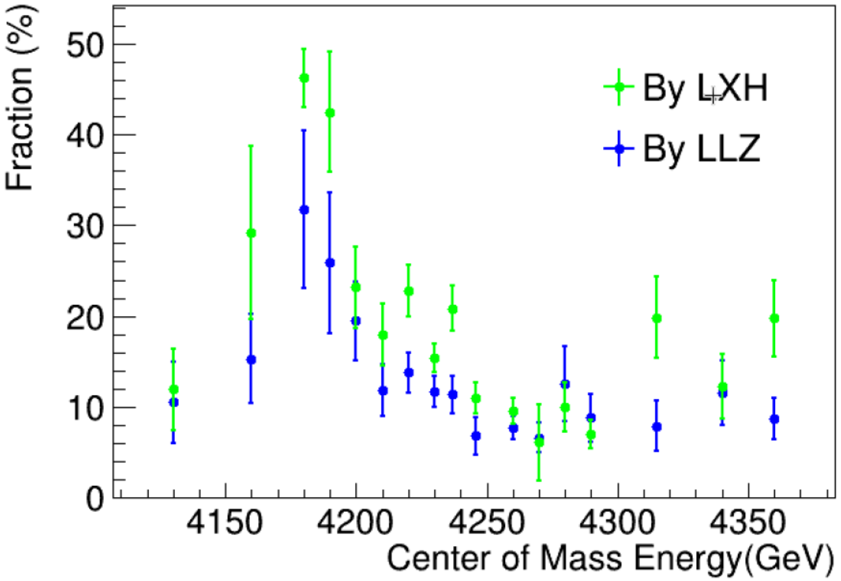


Helicity formalism





- The $Z_c(3900)$ cross section line shapes from $e^+e^- \to \pi^{+/0}\pi^{-/0}J/\psi$ and $e^+e^- \to \pi^+\pi^-\psi(3686)$ are different
- To validate the analysis of $e^+e^- \to \pi^+\pi^-\psi(3686)$, checks on amplitude and program are needed



Helicity amplitude construction

- Two body decay

$$a(J_a,\eta_a) \to b(J_b,\eta_b) + c(J_c,\eta_c)$$

$$A^{J_a}_{\lambda_b,\lambda_c}(\theta,\phi;M) = N_{J_a} F^{J_a}_{\lambda_b,\lambda_c} D^{J_a *}_{M,\lambda}(\phi,\theta,0), (\lambda = \lambda_b - \lambda c)$$

- $F^{J}_{\lambda_b,\lambda_c}$ is helicity decay amplitude

$$F^{J_a}_{\lambda_b,\lambda_c} = \sum_{ls} (\frac{2l+1}{2J_a+1})^{1/2} < l0s\lambda|J_a\lambda >< s_b\lambda_b s_c - \lambda_c|s\lambda > G^{J_a}_{ls} r^l B_l(r)$$

$$G^J_{ls} = 4\pi \left(\frac{w}{p}\right)^{\frac{1}{2}} \langle JMls|\mathcal{M}|JM\rangle$$

- ➢ G_{LS} is LS coupling partial wave amplitude
- ➢ With a definite set of helicity of (b,c), G_{LS} should be same
- ➢ In fit, G_{LS} is a float complex parameter

- Helicity coupling amplitudes depend on the Lorentz factor for particles with spin 1 or higher

$$\xi_s(\lambda) \equiv f^{(1)}_{\lambda}(\gamma_s) = \begin{cases} [\chi^{(1)*}(\lambda)\cdot\omega(\lambda)] \\ 1 \quad \text{for } \lambda = \pm 1 \\ \gamma_s \quad \text{for } \lambda = 0, \end{cases}$$

$$f^{(2)}_{\lambda}(\gamma_s) = \begin{cases} 1 & \text{for } \lambda = \pm 2 \\ \gamma_s & \text{for } \lambda = \pm 1 \\ \frac{2}{3}\gamma_s^2 + \frac{1}{3} & \text{for } \lambda = 0. \end{cases}$$

Covariant helicity coupling amplitude

$$F^{J}_{\lambda\nu} = \sum_{\ell S} g_{\ell S} A_{\ell S}(\lambda\nu),$$

where

$$A_{\ell S}(\lambda\nu) = \left(\frac{2\ell+1}{2J+1}\right)^{1/2} (\ell 0 S\delta|J\delta)(s\lambda\sigma-\nu|S\delta) \times W^n r^{\ell} f^s_{\lambda}(\gamma_s) f^{\sigma}_{\nu}(\gamma_\sigma),$$

PhysRevD.57.431 (1998) by S. U. Chung

Helicity amplitude construction

Sequential decays: $Y \to \pi^+\pi^- J/\psi, J/\psi \to l^+l^-$

- Helicity formalism

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i}} A_{R_i} \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} \boxed{F_1^J D_1^J} \cdot \boxed{F_2^J D_2^J} \cdot \boxed{F_3^J D_3^J} \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} (\sum_{LS} G_{LS} \cdot CG_1 \cdot B_l) D_1^J \cdot (\sum_{ls} G_{ls} \cdot CG_2 \cdot B_l) D_2^J \cdot F_3^J D_3^J \right|^2$$

- $J/\psi \to l^+l^-$ is included in helicity formalism

For the last step $J/\psi \to \ell^+\ell^-$, at the relativistic limit, by QED calculation, $F^{J_{J/\psi}}_{1/2,1/2} = F^{J_{J/\psi}}_{-1/2.-1/2} \approx 0$. Here we define $\Delta\lambda_\ell = \lambda_{\ell^+} - \lambda_{\ell^-}$, we can see only $\Delta\lambda_\ell = \pm 1$ is allowed.

$$\begin{array}{llll} Decay: Y & \to Z_c & \pi & \qquad Z_c \to \psi \quad \pi \\ J^{PC}: 1^{--} & \to 1^+ & 0^- & \qquad 1^+ \to 1^{--} 0^- \end{array}$$

$$A_{Z_c}(\lambda_Y, \lambda_{Z_c}, \lambda_{\ell^+}, \lambda_{\ell^-}) = F^{J_Y}_{\lambda_{Z_c},0} D^{J_Y}_{\lambda_Y,\lambda_{Z_c}}(\theta_{Z_c},\phi_{Z_c}) \cdot BW(Z_c) \cdot F^{J_{Z_c}}_{\lambda_{J/\psi},0} D^{J_{Z_c}}_{\lambda_{Z_c},\lambda_{J/\psi}}(\theta_{J/\psi},\phi_{J/\psi}) \cdot F^{J_{J/\psi}}_{\lambda_{\ell^+},\lambda_{\ell^-}} D^{J_{J/\psi}}_{\lambda_{J/\psi},\lambda_{\ell^+}-\lambda_{\ell^-}}(\theta_{\ell^+},\phi_{\ell^+}),$$

$$\begin{array}{llll} Decay: Y & \to \psi & f_0 & \qquad f_0 \to \pi^+ \quad \pi^- \\ J^{PC}: 1^{--} & \to 1^{--} & 0^+ & \qquad 0^+ \to 0^- \quad 0^- \end{array}$$

$$A_{R_f}(\lambda_Y, \lambda_{R_f}, \lambda_{\ell^+}, \lambda_{\ell^-}) = F^{J_Y}_{\lambda_{R_f},\lambda_{J/\psi}} D^{J_Y}_{\lambda_Y,\lambda_{R_f}-\lambda_{J/\psi}}(\theta_{R_f},\phi_{R_f}) \cdot BW(R_f) \cdot F^{J_{R_f}}_{0,0} D^{J_{R_f}}_{\lambda_{R_f},0}(\theta_{\pi^+},\phi_{\pi^+}) \cdot F^{J_{J/\psi}}_{\lambda_{\ell^+},\lambda_{\ell^-}} D^{J_{J/\psi}}_{\lambda_{J/\psi},\lambda_{\ell^+}-\lambda_{\ell^-}}(\theta_{\ell^+},\phi_{\ell^+}),$$

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y,\Delta\lambda_l} | \sum_{\lambda_{Z_c},\lambda_{R_f}} (A_{R_f} + e^{i\Delta\lambda_\ell \alpha_\ell(Z_c^+)} A_{Z_c^+} + e^{i\Delta\lambda_\ell \alpha_\ell(Z_c^-)} A_{Z_c^-})|^2$$

Helicity amplitude construction

- Sequential decay: $Y \to \pi^+\pi^- J/\psi$, $J/\psi \to l^+ l^-$

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i}} A_{R_i} \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} \boxed{F_1^J D_1^J} \cdot \boxed{F_2^J D_2^J} \cdot \boxed{F_3^J D_3^J} \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} \left(\sum_{LS} G_{LS} \cdot CG_1 \cdot B_l\right) D_1^J \cdot \left(\sum_{ls} G_{ls} \cdot CG_2 \cdot B_l\right) D_2^J \cdot F_3^J D_3^J \right|^2$$

For specific (LS, ls) wave component

$$\sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} (G_{LS} \cdot CG_1 \cdot B_l) D_1^J \cdot (G_{ls} \cdot CG_2 \cdot B_l) D_2^J \cdot F_3^J D_3^J \right|^2$$

$$\begin{array}{llll} Decay: Y & \to Z_c & \pi & L = 0(S-wave) \\ J^{PC}: 1^{--} & \to 1^{+} & 0^{-} & L = 2(D-wave) \end{array}$$

$$F_{1,0}^1 = \boxed{+g_{01}\sqrt{\frac{1}{3}}r^0} + \boxed{g_{21}\sqrt{\frac{1}{6}}r^2}$$

$$F_{0,0}^1 = \boxed{+g_{01}\sqrt{\frac{1}{3}}r^0\gamma_s} - \boxed{g_{21}\sqrt{\frac{2}{3}}r^2\gamma_s}$$

$$\begin{array}{lll} Z_c \to \psi & \pi & L = 0(S-wave) \\ 1^{+} \to 1^{--} & 0^{-} & L = 2(D-wave) \end{array}$$

$$F_{1,0}^1 = \boxed{+g_{01}\sqrt{\frac{1}{3}}r^0} + \boxed{g_{21}\sqrt{\frac{1}{6}}r^2}$$

$$F_{0,0}^1 = \boxed{+g_{01}\sqrt{\frac{1}{3}}r^0\gamma_s} - \boxed{g_{21}\sqrt{\frac{2}{3}}r^2\gamma_s}$$

- Four components: SS, SD, DS and DD

Amplitude construction

Sequential decays: $Y \to \pi^+\pi^- J/\psi$

- Covariant tensor formalism

$$A = \phi_\mu(m_1)\omega_\nu^*(m_2)A^{\mu\nu} = \phi_\mu(m_1)\omega_\nu^*(m_2)\sum_i \Lambda_i U_i^{\mu\nu}$$

$$\sum_{m_1}^{2} \phi_\mu(m_1)\phi_{\mu'}^*(m_1) = \delta_{\mu\mu'}(\delta_{\mu 1} + \delta_{\mu 2})$$

$$\sum_{m_2=1}^{3} \omega_\nu(m_2)\omega_{\nu'}^*(m_2) = -g_{\nu\nu'} + \frac{p_{(\psi)\nu}p_{(\psi)\nu'}}{p_\psi^2} \equiv -\tilde{g}_{\nu\nu'}(p_{(\psi)})$$

$$\frac{d\sigma}{d\Phi_n} \propto \frac{1}{2}\sum_{m_1}^{2}\sum_{m_2}^{3} \phi_\mu(m_1)\omega_\nu^*(m_2)A^{\mu\nu}\phi_{\mu'}^*(m_1)\omega_{\nu'}(m_2)A^{*\mu'\nu'}$$

$$\Downarrow$$

$$\frac{d\sigma}{d\Phi_n} \propto -\frac{1}{2}\sum_{\mu=1}^{2} \tilde{g}_{\nu\nu'}(p_{(\psi)})A^{\mu\nu}A^{*\mu\nu'}$$

$$= -\frac{1}{2}\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2} U_i^{\mu\nu}\tilde{g}_{\nu\nu'}(p_{(\psi)})U_j^{*\mu\nu'}$$

- $U^{\mu\nu}$ is the partial wave amplitude constructed according to LS coupling

$$U^{\mu\nu} = (A_{LS})(A_{ls})$$

$$\begin{array}{llll} Decay: Y & \to Z_c & \pi & \qquad Z_c \to \psi \quad \pi \\ J^{PC}: 1^{--} & \to 1^{+} & 0^{-} & \qquad 1^{+} \to 1^{--} 0^{-} \end{array}$$

$$U^{\mu\nu}_{(Y\to Z_c^{\pm}\pi^{\mp})SS} = \tilde{g}^{\mu\nu}_{(Z_c^+)} f^{(Z_c^+)}_{(01)} + \tilde{g}^{\mu\nu}_{(Z_c^-)} f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^{\pm}\pi^{\mp})SD} = \tilde{t}^{(2)\mu\nu}_{(\psi\pi^+)} f^{(Z_c^+)}_{(01)} + \tilde{t}^{(2)\mu\nu}_{(\psi\pi^-)} f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^{\pm}\pi^{\mp})DS} = \tilde{T}^{(2)\mu\lambda}_{(Z_c^+\pi^-)}\tilde{g}_{(Z_c^+)\lambda\sigma}g^{\sigma\nu} f^{(Z_c^+)}_{(01)} + \tilde{T}^{(2)\mu\lambda}_{(Z_c^-\pi^+)}\tilde{g}_{(Z_c^-)\lambda\sigma}g^{\sigma\nu} f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^{\pm}\pi^{\mp})DD} = \tilde{T}^{(2)\mu\lambda}_{(Z_c^+\pi^-)}\tilde{t}^{(2)}_{(\psi\pi^+)\lambda\sigma}g^{\sigma\nu} f^{(Z_c^+)}_{(01)} + \tilde{T}^{(2)\mu\lambda}_{(Z_c^-\pi^+)}\tilde{t}^{(2)}_{(\psi\pi^-)\lambda\sigma}g^{\sigma\nu} f^{(Z_c^-)}_{(02)}$$

$$\begin{array}{llll} Decay: Y & \to \psi & f_0 & \qquad f_0 \to \pi^+ \quad \pi^- \\ J^{PC}: 1^{--} & \to 1^{--} & 0^{+} & \qquad 0^{+} \to 0^{-} \quad 0^{-} \end{array}$$

$$U^{\mu\nu}_{(Y\to\psi(2S)f_0)SS} = \langle\psi f_0|01\rangle = g^{\mu\nu} f^{(f_0)}_{(12)}$$

$$U^{\mu\nu}_{(Y\to\psi(2S)f_0)DS} = \langle\psi f_0|21\rangle = \tilde{T}^{(2)\mu\nu}_{(\psi f_0)} f^{(f_0)}_{(12)}$$

Amplitude construction

Two points:

- $J/\psi \to l^+ l^-$ is included in the helicity formalism and not in covariant tensor formalism
- Lorentz factor is considered in covariant helicity formalism

$$F^{J_a}_{\lambda_b,\lambda_c} = \sum_{ls} \left(\frac{2l+1}{2J_a+1}\right)^{1/2} < l0s\lambda | J_a\lambda > < s_b\lambda_b s_c - \lambda_c | s\lambda > G^{J_a}_{ls} r^l B_l(r)$$

$$F^J_{\lambda\nu} = \sum_{\ell S} g_{\ell S} A_{\ell S}(\lambda\nu),$$

where

$$A_{\ell S}(\lambda\nu) = \left(\frac{2\ell+1}{2J+1}\right)^{1/2} (\ell 0 S\delta | J\delta)(s\lambda\sigma - \nu | S\delta) \times W^n r^\ell f^s_\lambda(\gamma_s) f^\sigma_\nu(\gamma_\sigma),$$

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y, \Delta\lambda_l} \left| \sum_{\lambda_{Z_c}, \lambda_{R_f}} (A_{R_f} + e^{i\Delta\lambda_\ell \alpha_\ell(Z_c^+)} A_{Z_c^+} + e^{i\Delta\lambda_\ell \alpha_\ell(Z_c^-)} A_{Z_c^-}) \right|^2$$

$$\frac{d\sigma}{d\Phi_n} \propto -\frac{1}{2} \sum_{\mu=1}^{2} \tilde{g}_{\nu\nu'}(p_{(\psi)}) A^{\mu\nu} A^{*\mu\nu'}$$
$$= -\frac{1}{2} \sum_{i,j} \Lambda_i \Lambda_j^* \sum_{\mu=1}^{2} U_i^{\mu\nu} \tilde{g}_{\nu\nu'}(p_{(\psi)}) U_j^{*\mu\nu'}$$

Amplitude construction

$$A = \phi_\mu(m_1)\omega_\nu^*(m_2)A^{\mu\nu} = \phi_\mu(m_1)\omega_\nu^*(m_2)\sum_i \Lambda_i U_i^{\mu\nu}$$

$$\mathcal{B} = ie\omega_\beta(m_2)\bar{u}_{e^-}\gamma^\beta \nu_{e^+}\frac{em_\psi}{f_\psi}$$

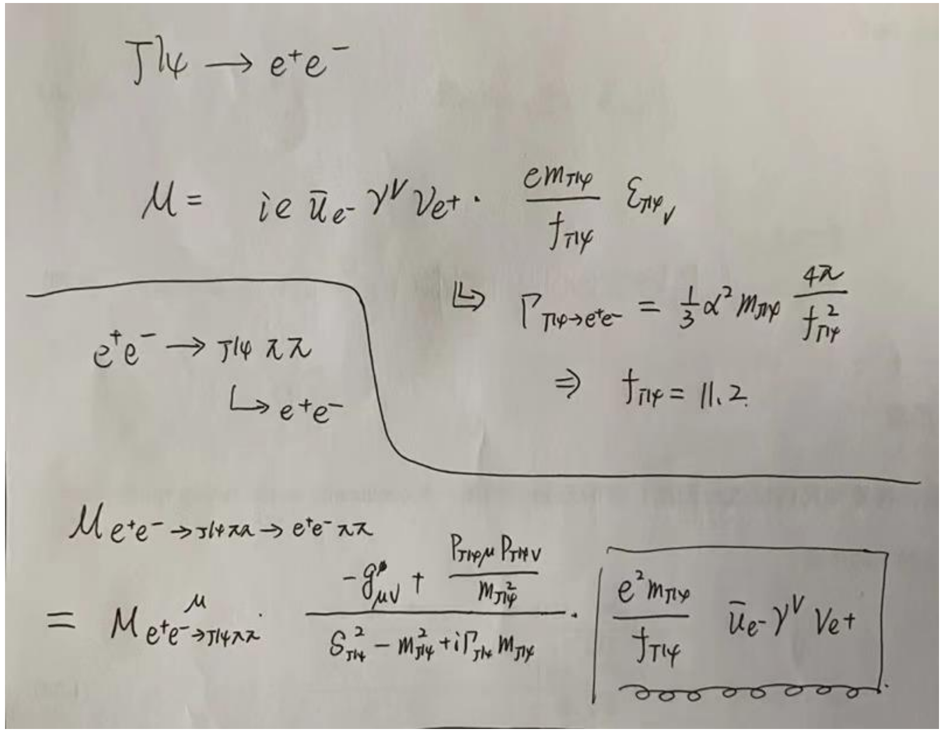
- $\omega_\beta(m_2)$是ψ的极化矢量
- $f_\psi = 11.2$是常数
- $\bar{u}_{e^-}$ (ν_{e^+})是电子的极化矢量

$$\frac{d\sigma}{d\Phi_n} \propto -\frac{1}{2}\sum_{\mu=1}^{2}\tilde{g}_{\nu\nu'}(p_{(\psi)})A^{\mu\nu}A^{*\mu\nu'}$$

$$= -\frac{1}{2}\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2}U_i^{\mu\nu}\boxed{\tilde{g}_{\nu\nu'}}(p_{(\psi)})U_j^{*\mu\nu'}$$

$$\frac{d\sigma}{d\Phi_n} \propto 2|ie\frac{em_\psi}{f_\psi}|^2\sum_{\mu=1}^{2}\tilde{g}_{\nu\nu'}^{\psi\to ll}A^{\mu\nu}A^{*\mu\nu'}$$

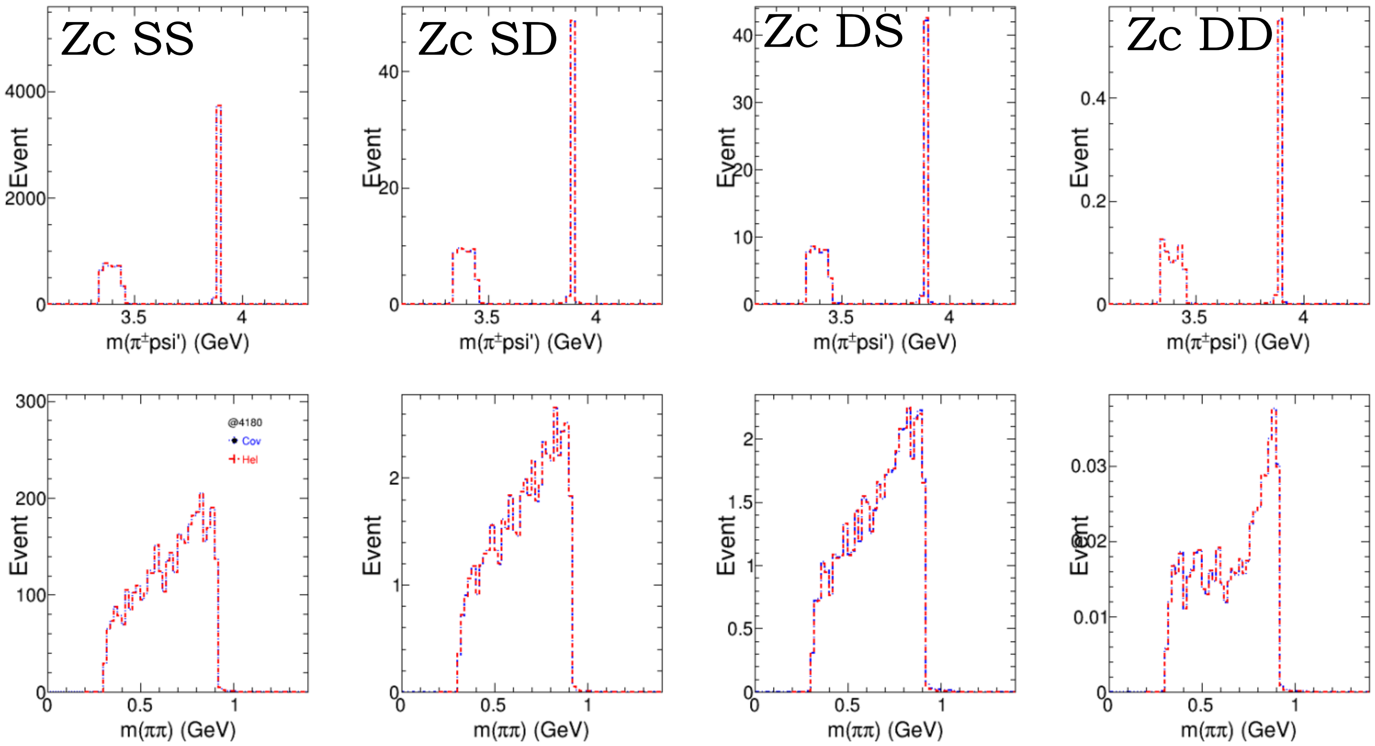
$$= 2|ie\frac{em_\psi}{f_\psi}|^2\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2}U_i^{\mu\nu}\boxed{\tilde{g}_{\nu\nu'}^{\psi\to ll}}U_j^{*\mu\nu'}$$



$$\sum_{m_2=1}^{3}\omega_\nu(m_2)\omega_{\nu'}^*(m_2) = -g_{\nu\nu'} + \frac{p_{(\psi)\nu}p_{(\psi)\nu'}}{p_\psi^2} \equiv -\tilde{g}_{\nu\nu'}(p_{(\psi)})$$

$$\tilde{g}_{\nu\nu'}^{\psi\to ll} = \tilde{g}_{\nu\beta}(p_{(\psi)})\tilde{g}_{\nu'\beta'}(p_{(\psi)})\left[p^\beta p'^{\beta'} + p'^\beta p^{\beta'} - g^{\beta\beta'}\left(p\cdot p' + m_l^2\right)\right]$$

Test with MC



- The simplest MC sample:
  - Only Zc SS component
  - No BW width
  - Generated by helicity formalism
- Fit in two formalisms

With $J/\psi \to l^+l^-$

```
Cov: SS wave fraction = 0.976046      Hel: SS wave fraction = 0.975296
Cov: SD wave fraction = 0.0127248     Hel: SD wave fraction = 0.0130789
Cov: DS wave fraction = 0.0110846     Hel: DS wave fraction = 0.0114711
Cov: DD wave fraction = 0.000144749   Hel: DD wave fraction = 0.000154264
```

Without $J/\psi \to l^+l^-$

```
Cov: SS wave fraction = 0.151297
Cov: SD wave fraction = 0.492753
Cov: DS wave fraction = 0.0834154
Cov: DD wave fraction = 0.272534
```

The test result supports the necessity of $J/\psi \to l^+l^-$

Test with MC

- The fractions are consistent between covariant tensor formalism and helicity formalism when $J/\psi \to l^+l^-$ is included
- Different (LS) wave components for two formalisms are in match
- $J/\psi \to l^+l^-$ part is necessary

MC sample:

Four components: SS+SD+DS+DD

BW: different Zc mass and 1 MeV width

	With $J/\psi \to l^+l^-$		Without $J/\psi \to l^+l^-$
Zc mass=3880 MeV Width = 1 MeV	SS wave: Cov = 0.11943	Hel = 0.117659	Cov = 0.108338
	SD wave: Cov = 0.0223932	Hel = 0.0220939	Cov = 0.143104
	DS wave: Cov = 0.723714	Hel = 0.725169	Cov = 0.321651
	DD wave: Cov = 0.134462	Hel = 0.135078	Cov = 0.426907
Zc mass=3885 MeV Width = 1 MeV	SS wave: Cov = 0.140743	Hel = 0.137783	Cov = 0.0721274
	SD wave: Cov = 0.0218467	Hel = 0.0217328	Cov = 0.283996
	DS wave: Cov = 0.724729	Hel = 0.725696	Cov = 0.130078
	DD wave: Cov = 0.112681	Hel = 0.114788	Cov = 0.513799
Zc mass=3890 MeV Width = 1 MeV	SS wave: Cov = 0.126908	Hel = 0.122224	Cov = 0.0780388
	SD wave: Cov = 0.0227674	Hel = 0.0222862	Cov = 0.164744
	DS wave: Cov = 0.722514	Hel = 0.724973	Cov = 0.243955
	DD wave: Cov = 0.127811	Hel = 0.130517	Cov = 0.513263

Test with MC

Zc3900 MC sample: only SS component, BW function has width

Without $J/\psi \to l^+l^-$ in covariant tensor formalism

Cov: SS wave fraction = $\boxed{0.292975}$	Hel: SS wave fraction = 0.905202
Cov: SD wave fraction = 0.500984	Hel: SD wave fraction = 0.00800091
Cov: DS wave fraction = $\boxed{0.0744949}$	Hel: DS wave fraction = 0.0859795
Cov: DD wave fraction = 0.131546	Hel: DD wave fraction = 0.000817607

With $J/\psi \to l^+l^-$ in covariant tensor formalism

Cov: SS wave fraction = $\boxed{0.930268}$	Hel: SS wave fraction = $\boxed{0.905202}$
Cov: SD wave fraction = 0.00704404	Hel: SD wave fraction = 0.00800091
Cov: DS wave fraction = $\boxed{0.062201}$	Hel: DS wave fraction = $\boxed{0.0859795}$
Cov: DD wave fraction = 0.000487272	Hel: DD wave fraction = 0.000817607

- The test result supports the necessity of $J/\psi \to l^+l^-$
- Differences appear in fractions of two formalisms
- Invariant scattering amplitude has mass-dependent term? This term is treated as a constant?

$$G^J_{ls} = 4\pi \left(\frac{w}{p}\right)^{\frac{1}{2}} \langle JMls|\mathcal{M}|JM\rangle$$

Consistency between two formalisms

5.3.1 $\psi \to \rho^0\pi^0 \to \pi^+\pi^-\pi^0$

第一个例子，我们采用相空间产生子，产生了$\psi \to \rho^0\pi^0 \to \pi^+\pi^-\pi^0$ 过程的相空间事例，这时Breit-Wigner分布已经包含在内了，但是不包含势垒因子和角分布。

5.3.1.0.1 截面 我们先把每个事例的截面计算出来（去掉Breit-Wigner），看看两种理论描述的计算结果，并求它们的比值，下面是打印出的数值信息：Helicity计算的截面、张量方法计算的截面和它们的比值。它们在计算精度内完全一致。

Helicity: 0.353986	L-S: 0.353986	Ratio: 1.000000
Helicity: 0.795525	L-S: 0.795525	Ratio: 1.000000
Helicity: 0.410571	L-S: 0.410571	Ratio: 1.000000
Helicity: 0.004812	L-S: 0.004812	Ratio: 1.000000
Helicity: 0.704383	L-S: 0.704383	Ratio: 1.000000
Helicity: 0.480020	L-S: 0.480020	Ratio: 1.000000

不考虑能量依赖,$\psi \to \rho^0\pi^0$ 道的完整角分布表达式为

$$
\begin{aligned}
\left(\frac{d\sigma}{d\Phi}\right)_H &\propto |\mathcal{C}_1|^2 \sum_M \left| D^{1\star}_{M1}(\theta\phi) D^{1\star}_{10}(\theta'\phi') - D^{1\star}_{M-1}(\theta\phi) D^{1\star}_{-10}(\theta'\phi') \right|^2 \\
&\propto \frac{1}{2}[(1+\cos^2\theta)\sin^2\theta' + \sin^2\theta \sin^2\theta' \cos 2\phi']
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\sigma}{d\Phi} &\propto \frac{|\mathcal{C}_2|^2}{M_\psi^2} \sum_{\mu=1}^{2} U_\rho^\mu U_\rho^{\star\mu} \\
&= |\mathcal{C}_2|^2 \epsilon_{ijk} p_1^j p_2^k \epsilon^{imn} p_{1m} p_{2n} \\
&= |\mathcal{C}_2|^2 (\vec{p}_1 \times \vec{p}_2)^2_{xy} \, .
\end{aligned}
$$

Consistency between two formalisms

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UDC ______________ 编号 ______________

中国高等科技中心
博士后研究报告

分波分析方法

李　刚

合作导师　叶铭汉 院士，朱永生 研究员

工作完成日期　2004年7月— 2006年8月

报告提交日期　2006年8月

中国高等科技中心（北京）
2006年8月

$D \to \rho^0\pi^0 \to \pi^+\pi^-\pi^0,\ \phi\pi^0 \to K^+K^-\pi^0$

Both P wave for two steps

$D \to f_2\pi^0 \to \pi^+\pi^-\pi^0$

Both D wave for two steps

Helicity formalism:

$$\begin{aligned} Z &= D^0_{0\lambda}(0,\theta_0\phi_0)f_\lambda(\gamma)D^{*1}_{\lambda 0}(0\theta\phi) \\ &= \gamma d^1_{00}(\theta) \qquad \lambda = 0 \\ &= \gamma\cos\theta\ , \end{aligned}$$

$$\begin{aligned} Z &= D^0_{0\lambda}(0,\theta_0\phi_0)f_\lambda(\gamma)D^{*2}_{\lambda 0}(0\theta\phi \\ &= (\frac{2}{3}\gamma^2+\frac{1}{3})d^2_{00}(\theta) \\ &= (\frac{2}{3}\gamma^2+\frac{1}{3})(\frac{3\cos^2\theta-1}{2})\ , \end{aligned}$$

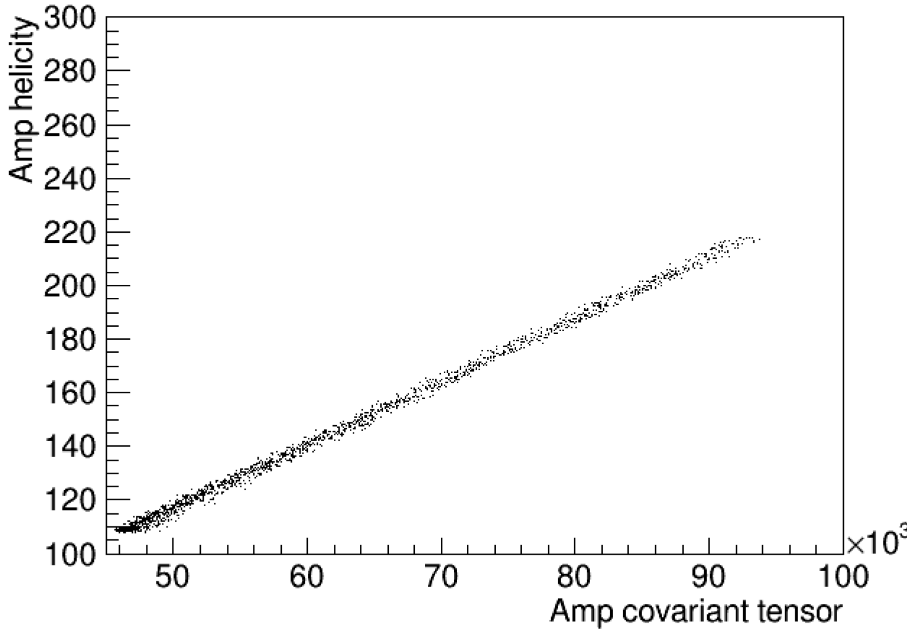
Covariant formalism:

$$\begin{aligned} Z &= -2p(p_a-p_b)^3 \\ &= -2p[(\gamma\beta E^*-\gamma q_3)-(\gamma\beta E^*+\gamma q_3)] \\ &= 4p\gamma(q\cos\theta). \\ &= 4pq\gamma\cos\theta\ , \end{aligned}$$

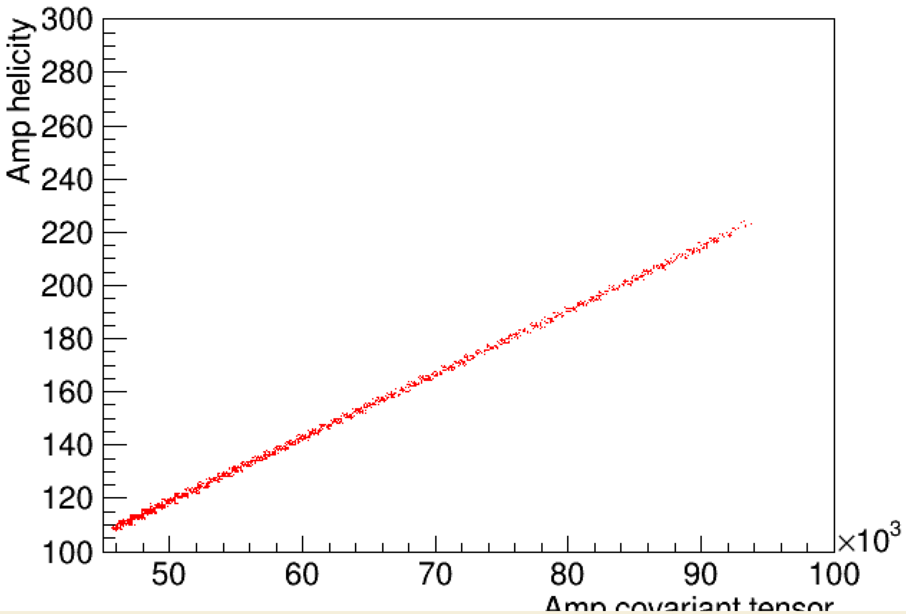
$$\begin{aligned} Z &= \tilde{T}^{(2)}_{\mu\nu}(p_a+p_b+p_c)\tilde{t}^{\mu\nu(2)}(p_a+p_b) \\ &= \left[(p_a+p_b-p_c)_i(p_a+p_b-p_c)_j-\frac{1}{3}\delta_{ij}(p_a+p_b-p_c)^2\right] \\ &\quad \left[(p_a-p_b)^i(p_a-p_b)^j-\frac{1}{3}\delta^{ij}(p_a-p_b)^2\right] \\ &= [(p_a+p_b-p_c)\cdot(p_a-p_b)]^2+\frac{1}{3}(p_a+p_b-p_c)^2(p_a-p_b)^2 \\ &\quad -\frac{1}{3}(p_a+p_b-p_c)^2(p_a-p_b)^2-\frac{1}{3}(p_a+p_b-p_c)^2(p_a-p_b)^2 \\ &= [(p_a+p_b-p_c)\cdot(p_a-p_b)]^2-\frac{1}{3}(p_a+p_b-p_c)^2(p_a-p_b)^2 \\ &= 16p^2q^2\gamma^2\cos^2\theta-\frac{1}{3}16p^2q^2(\sin^2\theta+\gamma^2\cos^2\theta) \\ &= \boxed{\frac{64}{3}\times p^2q^2\left[(\frac{2}{3}\gamma^2+\frac{1}{3})(\frac{3\cos^2\theta}{2})-\frac{1}{2}\right]} \end{aligned}$$

Consistency between two formalisms

MC: $Y \to \pi^+\pi^- J/\psi$, $J/\psi \to l^+l^-$ phsp MC
Amplitude: only Zc SS component



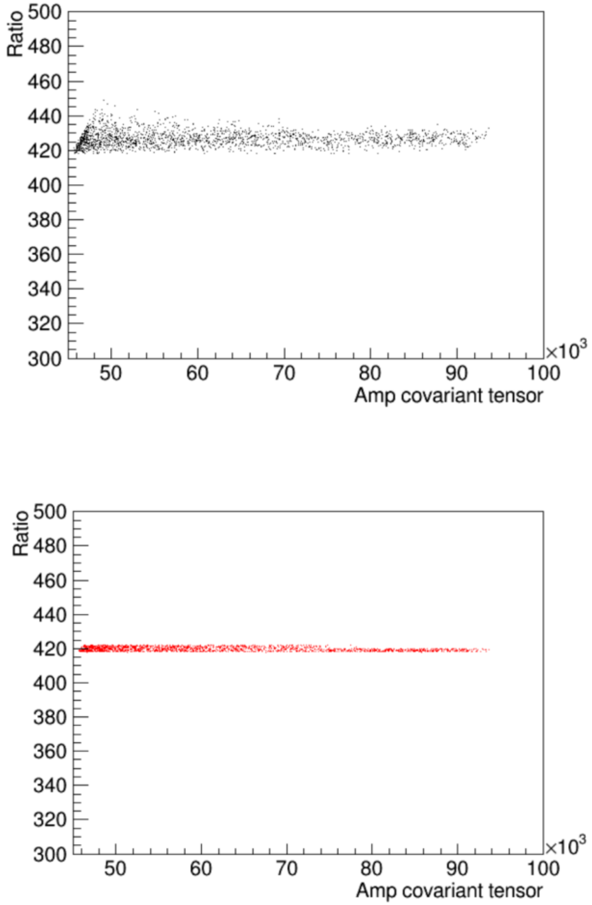
xsec_cov=69738.5	xsec_hel=161.882	R=430.797
xsec_cov=56658.7	xsec_hel=134.551	R=421.094
xsec_cov=47518.5	xsec_hel=110.925	R=428.384
xsec_cov=91253.7	xsec_hel=212.259	R=429.916
xsec_cov=81966.6	xsec_hel=190.662	R=429.906



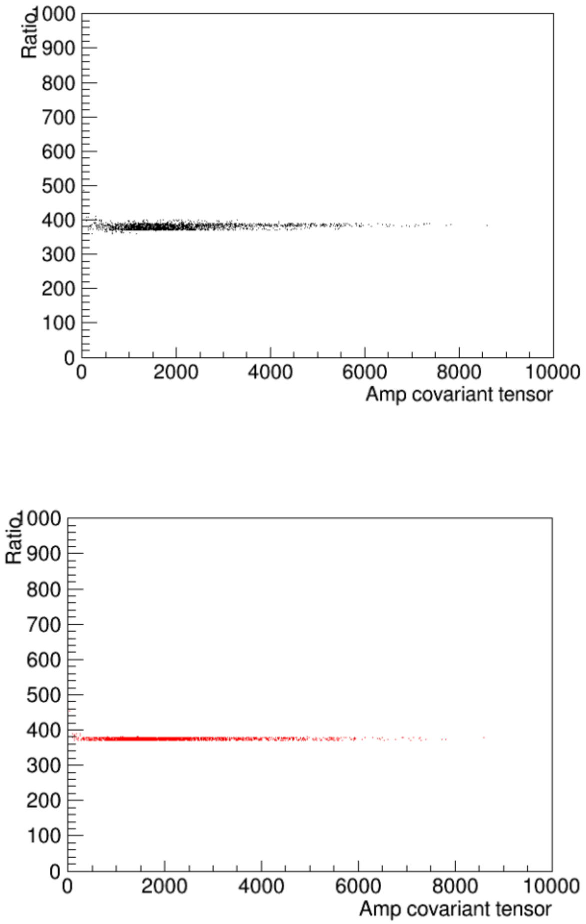
xsec_cov=69738.5	xsec_hel_RF=165.977	R=420.17
xsec_cov=56658.7	xsec_hel_RF=135.051	R=419.537
xsec_cov=47518.5	xsec_hel_RF=112.773	R=421.365
xsec_cov=91253.7	xsec_hel_RF=217.526	R=419.508
xsec_cov=81966.6	xsec_hel_RF=195.262	R=419.777

- The ratio is close to a constant as well
- The relativistic factor also makes the ratio more stable

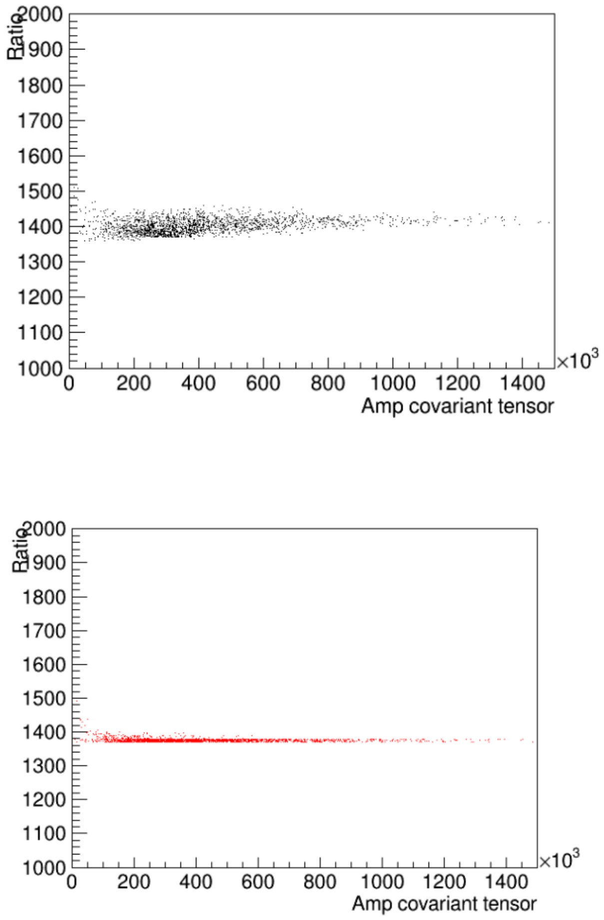
Consistency between two formalisms



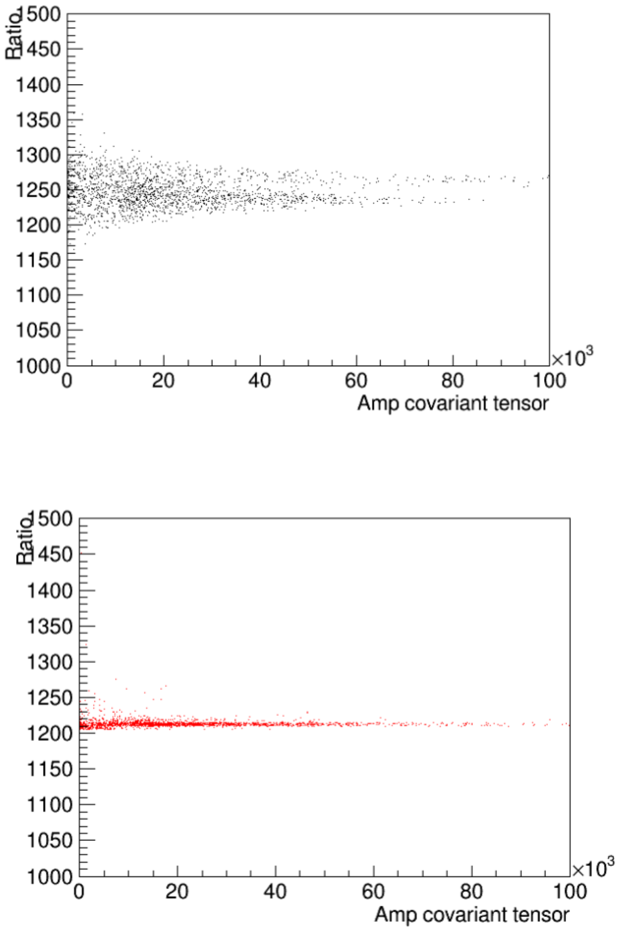
Zc SS



Zc SD



Zc DS



Zc DD

Consistency between two formalisms

其中$z=[qd]^2$，$z_0=[q_0d]^2$，d 是相互作用的典型力程。

L	$B_L(q)$
0	1
1	$\sqrt{\frac{2z}{1+z}}$
2	$\sqrt{\frac{13z^2}{(z-3)^2+9z}}$
3	$\sqrt{\frac{277z^3}{z^3+6z^2+45z+225}}$
4	$\sqrt{\frac{12746z^4}{z^4+10z^3+135z^2+1575z+11025}}$

```
);

)*BarrierF(2,rhoJpsi_zcp*2.0);

)*BarrierF(2,rhoZcp*2.0);
```

$$p^\alpha=(W;0,0,0),$$

$$q^\alpha=(q_0;0,0,q)=(\gamma_s m;0,0,\gamma_s\beta_s m),$$

$$k^\alpha=(k_0;0,0,-q)=(\gamma_\sigma\mu;0,0,-\gamma_\sigma\beta_\sigma\mu), \quad (17)$$

$$r^\alpha=(q_0-k_0;0,0,2q),$$

where $W=q_0+k_0$, $q_0=\sqrt{m^2+q^2}$, $k_0=\sqrt{\mu^2+q^2}$, and $r=q-k$, the wave functions in the J rest frame are given by

$$A_{\ell S}(\lambda\nu)=\left(\frac{2\ell+1}{2J+1}\right)^{1/2}(\ell 0S\delta|J\delta)(s\lambda\sigma-\nu|S\delta)\times W^{\prime}\boxed{r^{\ell}}f^s_\lambda(\gamma_s)f^\sigma_\nu(\gamma_\sigma),$$

$$Q^2_{abc}=\frac{(s_a+s_b-s_c)^2}{4s_a}-s_b$$

Here Q_0 is a hadron "scale" parameter $Q_0=0.197321/R$ GeV/c, where R is the radius of the centrifugal barrier in fm. We remark that in these Blatt-Weisskopf factors, the approximation is made that the centrifugal barrier may be replaced by a square well of radius R.

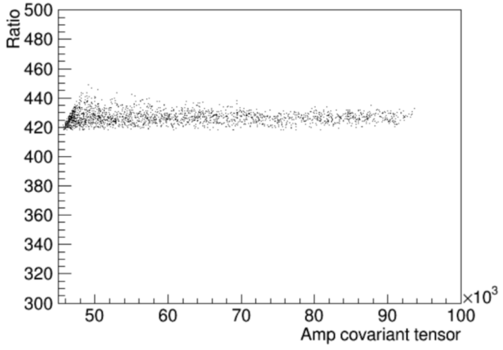
$$B_1(Q_{abc})=\sqrt{\frac{2}{Q^2_{abc}+Q^2_0}},$$

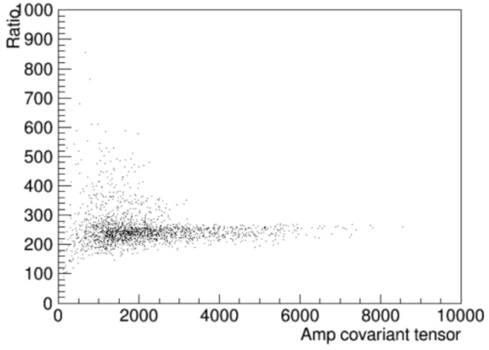
$$B_2(Q_{abc})=\sqrt{\frac{13}{Q^4_{abc}+3Q^2_{abc}Q^2_0+9Q^4_0}},$$

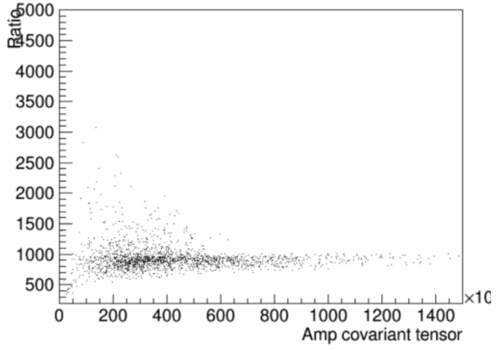
$$B_3(Q_{abc})=\sqrt{\frac{277}{Q^6_{abc}+6Q^4_{abc}Q^2_0+45Q^2_{abc}Q^4_0+225Q^6_0}},$$

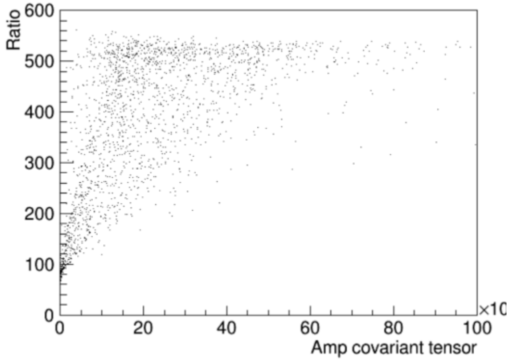
$$B_4(Q_{abc})=\sqrt{\frac{12746}{Q^8_{abc}+10Q^6_{abc}Q^2_0+135Q^4_{abc}Q^4_0+1575Q^2_{abc}Q^6_0+11025Q^8_0}}.$$

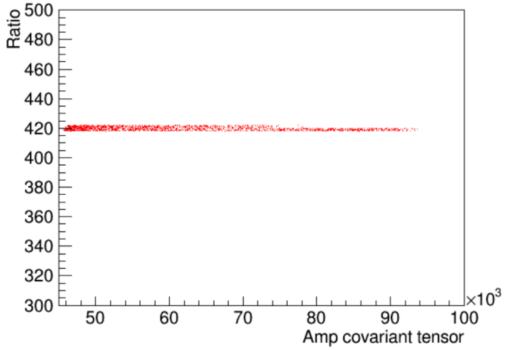
Consistency between two formalisms

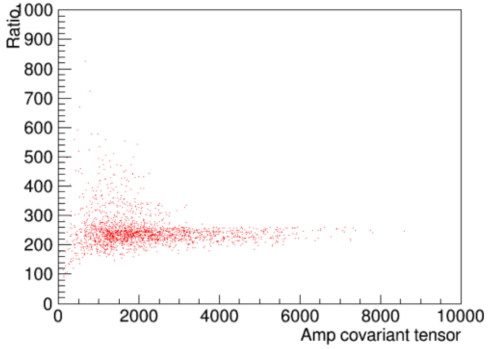


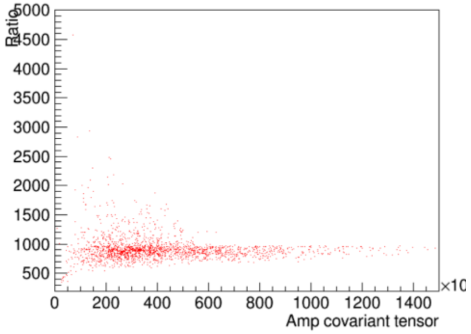


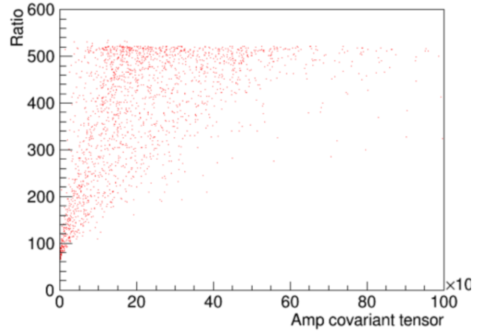












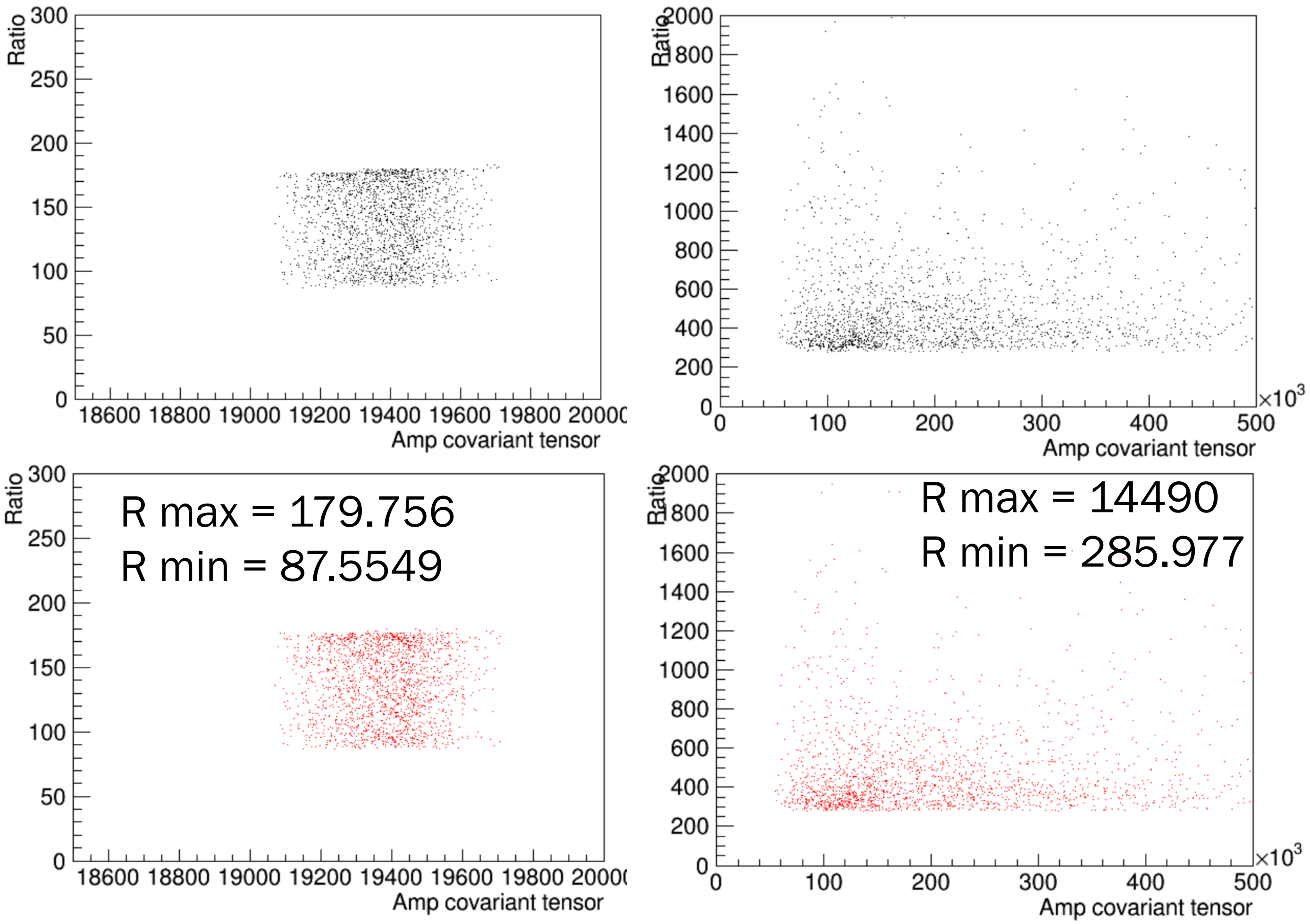
Zc SS:
R max = 421.777
R min = 419.452

Zc SD
R max = 820.681
R min = 75.9445

Zc DS
R max = 4593.4
R min = 368.43

Zc DD:
R max = 533.565
R min = 67.3278

Consistency between two formalisms



Zc SS

Zc DS

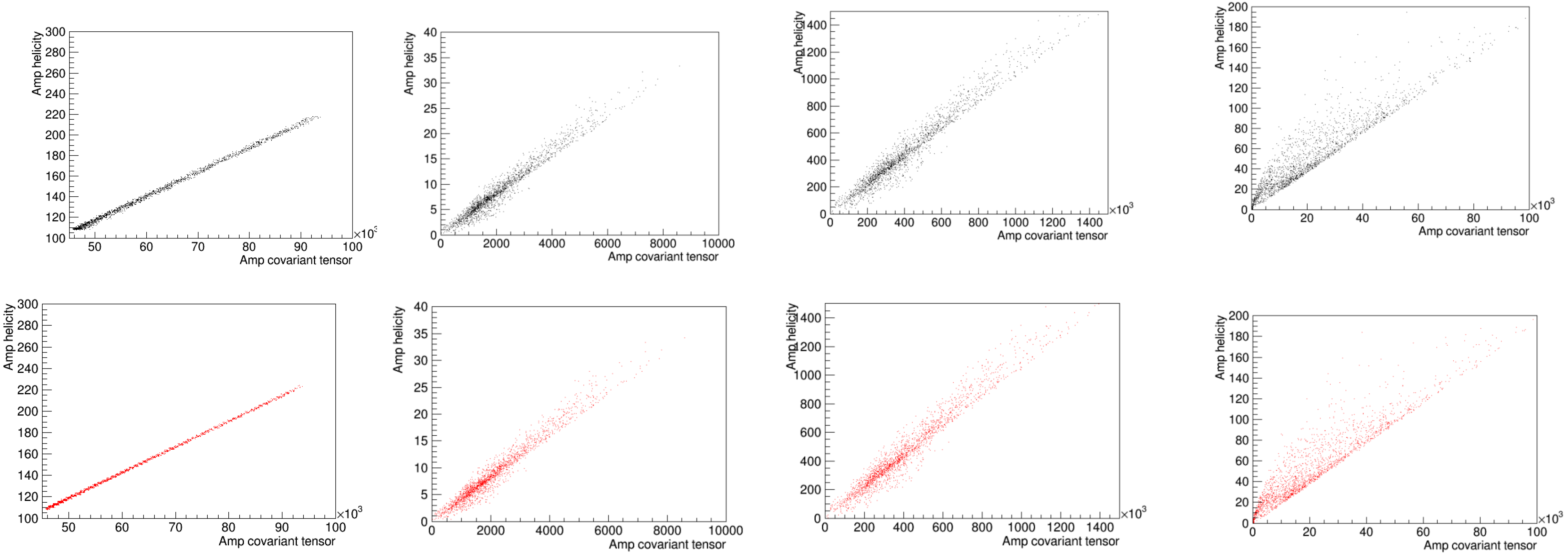
- Without $J/\psi \to l^+ l^-$, even the amplitude ratio of Zc SS component is not a constant

Summary

- In this report, tests are performed to demonstrate the consistency between two commonly used formalisms: helicity formalism and covariant tensor formalism
- For example process $Y \to \pi^+\pi^- J/\psi \to \pi^+\pi^- l^+ l^-$, test results support the necessicity of $J/\psi \to l^+ l^-$
  - How about other case? $\phi \to KK$? $\omega \to \pi^+\pi^-\pi^0$?
  - For $Y \to \pi^+\pi^-\psi(3686)$, only use $\psi(3686) \to \pi^+\pi^- J/\psi \to \pi^+\pi^- l^+ l^-$?
- Even for $\mathrm{L} \geq 2$(D wave or higher), amplitudes constructed from helicity formalism and covariant tensor formalism are consistent
  - Which one describes physical process better? Or Is it possible to search a better amplitude construction method?

BACKUP

Consistency between two formalisms



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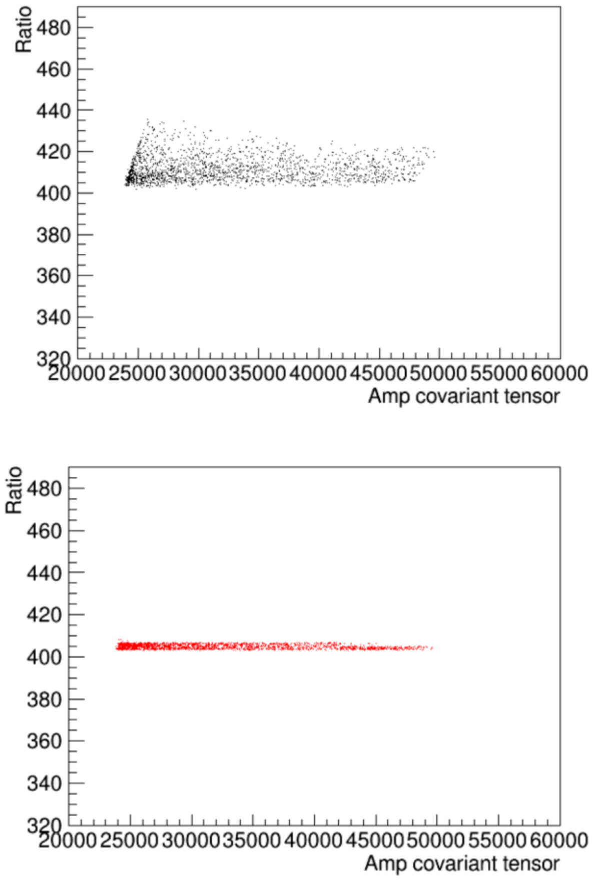
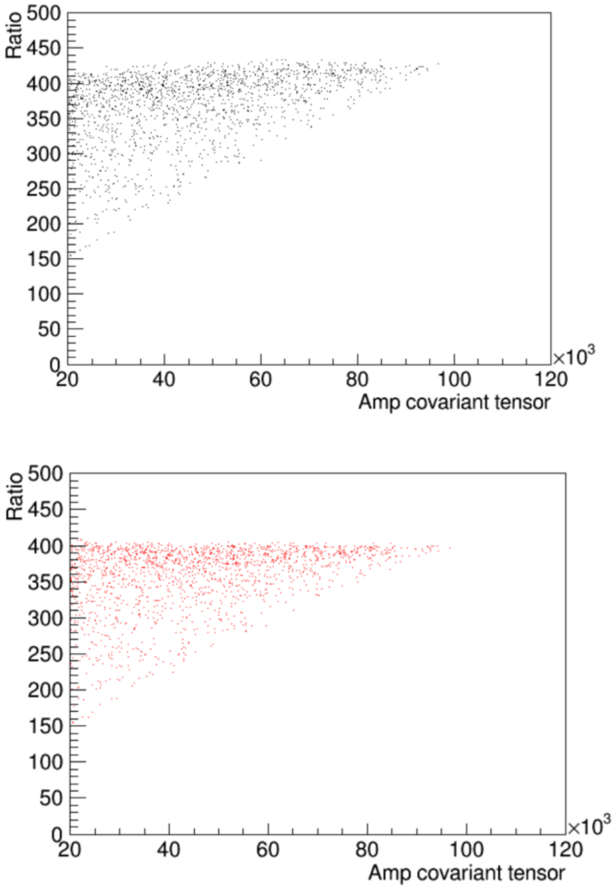
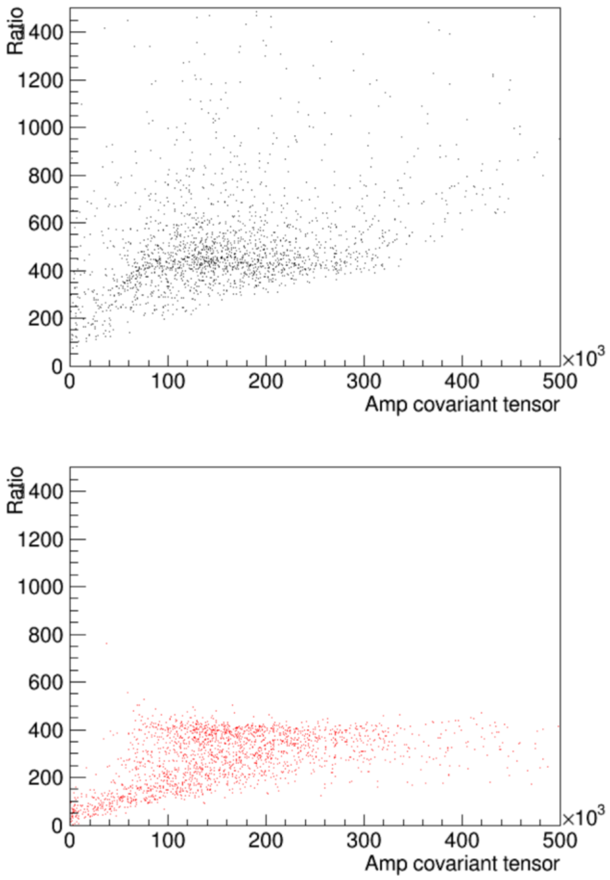
Consistency between two formalisms



F0500 SS

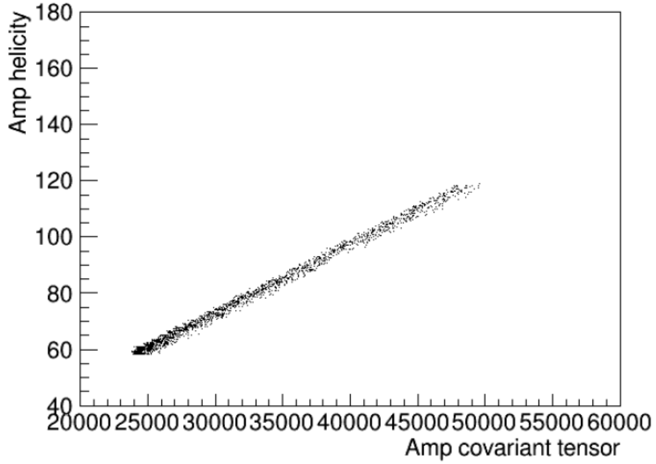


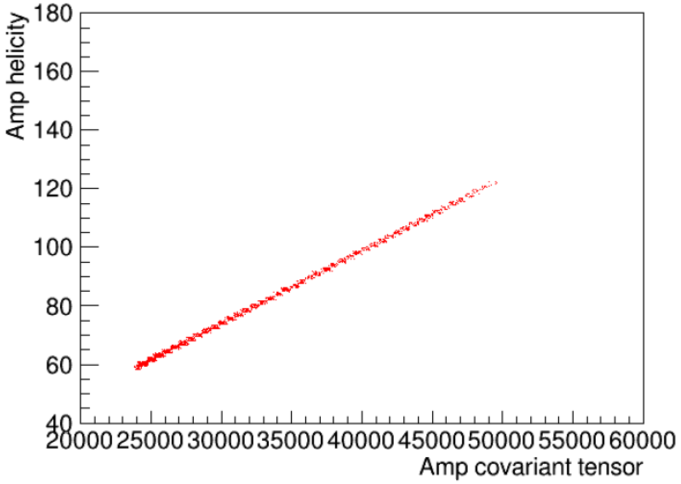
F0500 DS



F21270 SD

Consistency between two formalisms





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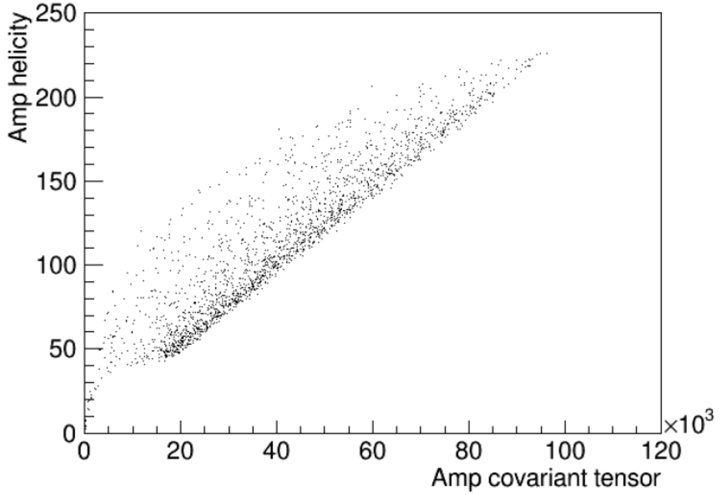

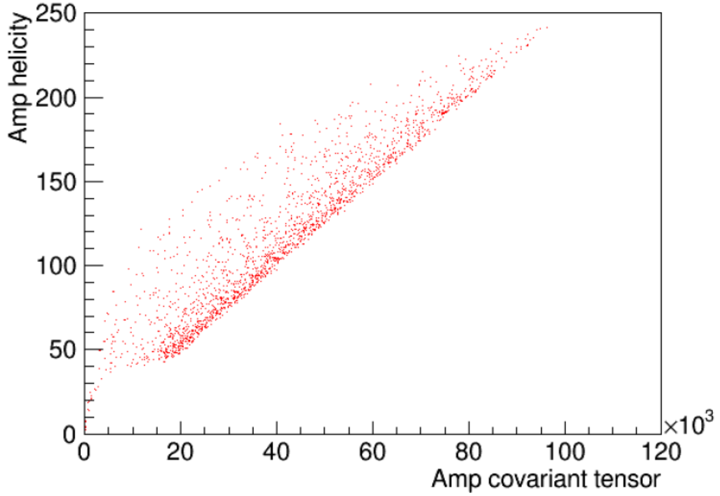

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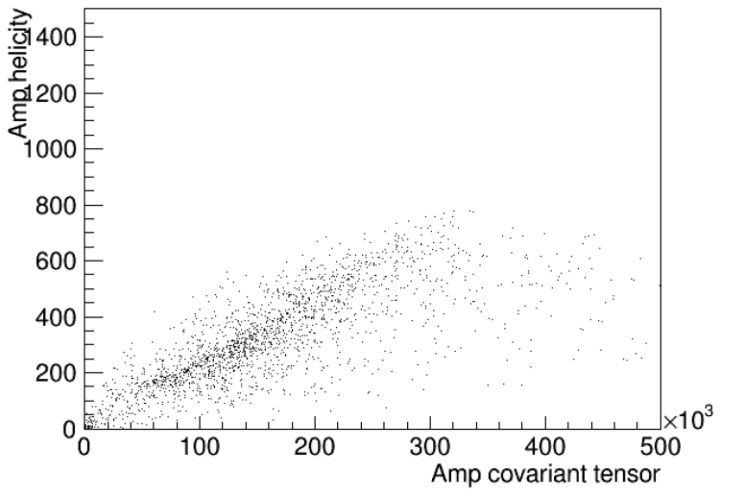

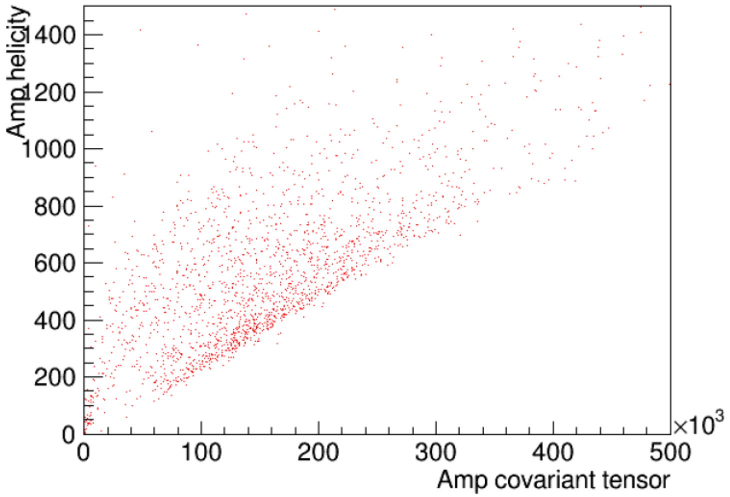

F21270 SD