Helicity formalism for semileptonic-hadronic decays

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Semileptonic meeting
15 December 2020

Introduction

- Helicity method allows:
  - Compact calculations of the angular decay distributions
  - Analyze the semileptonic decays of polarized hyperon
  - Take into account the lepton mass effects

    $\Rightarrow$ vector and axial-vector currents
- **Aim**: a general modular method for the semileptonic hyperon decays
- Analysis is based on:
  - Helicity analysis for $\Xi^0 \to \Sigma^+(\to p\pi^0)l^-\bar{\nu}_l$ $(l = e^-, \mu^-)$ [EPJ C59 (2009) 27]
  - Polarization observables in e^+e^- annihilation to a $B\bar{B}$ pair [PRD 99 (2019) 056008]

Spin density matrix and decay matrix

- $e^+e^- \to \Lambda\bar{\Lambda}$
- Spin density matrix for a two particle $\frac{1}{2} + \frac{\bar{1}}{2}$ system:

$$\rho_{\frac{1}{2},\frac{\bar{1}}{2}} = \frac{1}{4}\sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}}\sigma_{\mu}^{\Lambda} \otimes \sigma_{\bar{\nu}}^{\bar{\Lambda}}$$

- Spin density matrix of production process [PRD99 (2019) 056008]
- Main parameters: $\theta_1 \equiv \theta_\Lambda$, α_ψ, $\Delta\Phi$

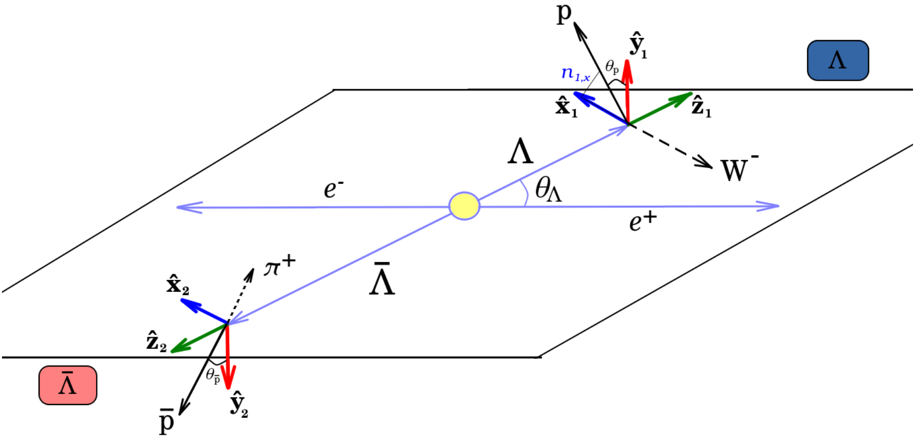
$$C_{\mu\bar{\nu}} = \begin{pmatrix} 1+\alpha_\psi\cos^2\theta_1 & 0 & \beta_\psi\sin\theta_1\cos\theta_1 & 0 \\ 0 & \sin^2\theta_1 & 0 & \gamma_\psi\sin\theta_1\cos\theta_1 \\ -\beta_\psi\sin\theta_1\cos\theta_1 & 0 & \alpha_\psi\sin^2\theta_1 & 0 \\ 0 & -\gamma_\psi\sin\theta_1\cos\theta_1 & 0 & -\alpha_\psi-\cos^2\theta_1 \end{pmatrix}$$

$$\beta_\psi = \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi) \qquad \gamma_\psi = \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)$$

- Apply decay matrix for $\Lambda \to p(W^- \to e^-\bar{\nu}_e)$ and $\bar{\Lambda} \to \bar{p}\pi^+$:

$$\sigma_{\mu}^{\Lambda} \to \sum_{\mu'=0}^{3} b_{\mu\mu'}^{\Lambda}\sigma_{\mu'}^{p-W^-} \qquad \sigma_{\bar{\nu}}^{\bar{\Lambda}} \to \sum_{\bar{\nu}'=0}^{3} a_{\bar{\nu}\bar{\nu}'}^{\bar{\Lambda}}\sigma_{\bar{\nu}'}^{\bar{p}} \implies \mathrm{Tr}\rho_{pW\bar{p}} \propto \mathcal{W}(\boldsymbol{\xi};\boldsymbol{\omega}) = \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}}b_{\mu0}^{\Lambda}a_{\bar{\nu}0}^{\bar{\Lambda}}$$

Exclusive joint angular distribution (1)



- $\Lambda \to pW^-$: $\hat{\mathbf{n}}_1 \to (\cos\theta_p, \phi_p)$: $\alpha_\Lambda^{sl}(W^- \to e^-\bar{\nu}_e) \equiv \alpha_\Lambda^{sl}$
  - $W^- \to e^-\bar{\nu}_e$: (θ_e, χ, q^2) : $g_{av}^\Lambda, g_w^\Lambda$
- $\bar{\Lambda} \to \bar{p}\pi^+$: $\hat{\mathbf{n}}_2 \to (\cos\theta_{\bar{p}}, \phi_{\bar{p}})$: $\alpha_{\bar{\Lambda}}$

Exclusive joint angular distribution (2)

- $\Lambda \to pW^-$: $\hat{\mathbf{n}}_1 \to (\cos\theta_p, \phi_p)$: $\alpha_\Lambda^{sl}(W^- \to e^-\bar{\nu}_e) \equiv \alpha_\Lambda^{sl}$
- $\bar{\Lambda} \to \bar{p}\pi^+$: $\hat{\mathbf{n}}_2 \to (\cos\theta_{\bar{p}}, \phi_{\bar{p}})$: $\alpha_{\bar{\Lambda}}$
- $\boldsymbol{\xi}'$: $(\cos\theta_\Lambda, \hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) \leftarrow$ 5D PhSp

$$\mathrm{d}\Gamma \propto \mathcal{W}(\boldsymbol{\xi}'; \boldsymbol{\alpha}_\psi, \boldsymbol{\Delta}\Phi, \boldsymbol{\alpha}_\Lambda^{sl}, \boldsymbol{\alpha}_{\bar{\Lambda}}) =$$

$$\boxed{1 + \boldsymbol{\alpha}_\psi \cos^2\theta_\Lambda}$$

Cross section

$$\boxed{\begin{aligned} &+ \boldsymbol{\alpha}_\Lambda^{sl}\boldsymbol{\alpha}_{\bar{\Lambda}} \left(\sin^2\theta_\Lambda (n_{1,x}n_{2,x} - \boldsymbol{\alpha}_\psi n_{1,y}n_{2,y}) + (\cos^2\theta_\Lambda + \boldsymbol{\alpha}_\psi) n_{1,z}n_{2,z}\right) \\ &+ \boldsymbol{\alpha}_\Lambda^{sl}\boldsymbol{\alpha}_{\bar{\Lambda}} \sqrt{1 - \boldsymbol{\alpha}_\psi{}^2} \cos(\boldsymbol{\Delta}\Phi) \sin\theta_\Lambda \cos\theta_\Lambda (n_{1,x}n_{2,z} + n_{1,z}n_{2,x}) \end{aligned}}$$

Spin correlations

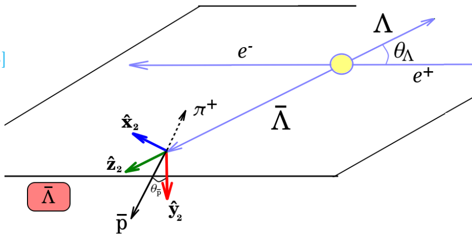
$$\boxed{+ \sqrt{1 - \boldsymbol{\alpha}_\psi{}^2} \sin(\boldsymbol{\Delta}\Phi) \sin\theta_\Lambda \cos\theta_\Lambda \left(\boldsymbol{\alpha}_\Lambda^{sl} n_{1,y} + \boldsymbol{\alpha}_{\bar{\Lambda}} n_{2,y}\right)}$$

Polarization

- $\Delta\Phi \neq 0 \Rightarrow$ independent determination of α_Λ^{sl} and $\alpha_{\bar{\Lambda}}$
- Same expression for $e^+e^- \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$ [PLB772 (2017) 16]
  - $\alpha_\Lambda^{sl} \Leftrightarrow \alpha_\Lambda$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + 0$ (1)

- Full determination in [PRD99 (2019) 056008]
- Decay matrix or transition matrix $a_{\mu\nu}$



$$\sigma_\mu \to \sum_{\nu=0}^{3} a_{\mu\nu}\sigma_\nu^d$$

- $J = \frac{1}{2}$ hyperon ($\bar{\Lambda}$) decays into a $J = \frac{1}{2}$ baryon ($\bar{p}$) and a $J = 0$ pseudoscalar (π^+)

$$a_{\mu\nu} = \frac{1}{4\pi} \sum_{\lambda,\lambda'=-1/2}^{1/2} B_\lambda B^*_{\lambda'} \sum_{\kappa,\kappa'=-1/2}^{1/2} (\sigma_\mu)^{\kappa,\kappa'}(\sigma_\nu)^{\lambda',\lambda}\mathcal{D}^{1/2*}_{\kappa,\lambda}(\Omega)\mathcal{D}^{1/2}_{\kappa',\lambda'}(\Omega)$$

- Two helicity amplitudes: $\{B_{\frac{1}{2}}, B_{-\frac{1}{2}}\}$
- κ, κ' - index of mother hyperon ($\bar{\Lambda}$); λ, λ' - index of daughter baryon ($\bar{p}$)
- $\Omega = \{\varphi, \theta, 0\}$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + 0$ (2)

[PRD99 (2019) 056008]

- Relation between helicity amplitudes and decay parameters

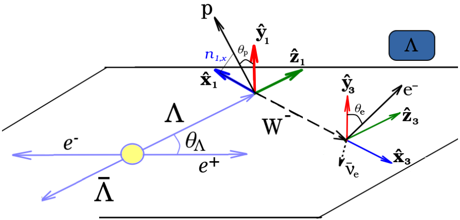
$$\text{normalization } |A_S|^2 + |A_P|^2 = |B_{-\frac{1}{2}}|^2 + |B_{\frac{1}{2}}|^2 = 1,$$
$$\alpha_D = -2\Re(A_S^* A_P) = |B_{1/2}|^2 - |B_{-1/2}|^2,$$
$$\beta_D = -2\Im(A_S^* A_P) = 2\Im(B_{1/2} B^*_{-1/2}),$$
$$\gamma_D = |A_S^*|^2 - |A_P|^2 = 2\Re(B_{1/2} B^*_{-1/2}),$$
$$\text{where } \beta_D = \sqrt{1-\alpha_D^2}\sin\varphi_D \text{ and } \gamma_D = \sqrt{1-\alpha_D^2}\cos\varphi_D$$

- Non-zero elements of the decay matrix $a_{\mu\nu}$:

$$\boxed{a_{00} = 1,} \qquad a_{21} = \beta_D \cos\varphi + \gamma_D \cos\theta \sin\varphi,$$
$$a_{03} = \alpha_D, \qquad a_{22} = \gamma_D \cos\varphi - \beta_D \cos\theta \sin\varphi,$$
$$\boxed{a_{10} = \alpha_D \cos\varphi \sin\theta,} \qquad a_{23} = \sin\theta \sin\varphi,$$
$$a_{11} = \gamma_D \cos\theta \cos\varphi - \beta_D \sin\varphi, \qquad \boxed{a_{30} = \alpha_D \cos\theta,}$$
$$a_{12} = -\beta_D \cos\theta \cos\varphi - \gamma_D \sin\varphi, \qquad a_{31} = -\gamma_D \sin\theta,$$
$$a_{13} = \sin\theta \cos\varphi, \qquad a_{32} = \beta_D \sin\theta,$$
$$\boxed{a_{20} = \alpha_D \sin\theta \sin\varphi,} \qquad a_{33} = \cos\theta$$

- Main parameters: $\theta \equiv \theta_{\bar{p}}$, $\varphi \equiv \varphi_{\bar{p}}$, $\alpha_D \equiv \alpha_{\bar{\Lambda}}$, $\varphi_D \equiv \varphi_{\bar{\Lambda}}$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}$



- $\Lambda \to p e^- \bar{\nu}_e \Longrightarrow \Lambda \to p(W^- \to e^- \bar{\nu}_e)$
- Decay matrix or transition matrix $b_{\mu\nu}$

$$\sigma_\mu \to \sum_{\nu=0}^{3} b_{\mu\nu} \sigma_\nu^{d-W}$$

- $J = \frac{1}{2}$ hyperon (Λ) decays into a $J = \frac{1}{2}$ baryon (p) and a $J = \{0, \pm 1\}$ W^--boson

$$b_{\mu\nu} = \frac{1}{4\pi} \sum_{\lambda_2=-1/2}^{1/2} \sum_{\lambda_W, \lambda'_W=-1}^{1} H_{\lambda_2\lambda_W} H^*_{\lambda_2\lambda'_W} \sum_{\kappa,\kappa'=-1/2}^{1/2} (\sigma_\mu)^{\kappa,\kappa'} (\sigma_\nu)^{\lambda_2-\lambda'_W, \lambda_2-\lambda_W} \mathcal{D}^{1/2*}_{\kappa,\lambda_2-\lambda_W}(\Omega) \mathcal{D}^{1/2}_{\kappa',\lambda_2-\lambda'_W}(\Omega) \times$$

$$\sum_{\lambda_l,\lambda_\nu=-1/2}^{1/2} |h^l_{\lambda_l\lambda_\nu=\pm 1/2}|^2 \mathcal{D}^{1*}_{\lambda_W,\lambda_l-\lambda_\nu}(\Omega') \mathcal{D}^{1}_{\lambda'_W,\lambda_l-\lambda_\nu}(\Omega'),$$

- Four helicity amplitudes: $\{H_{\frac{1}{2}0}, H_{-\frac{1}{2}0}, H_{\frac{1}{2}1}, H_{-\frac{1}{2}-1}\}$
- κ, κ' - index of mother hyperon (Λ); λ_2 - index of daughter baryon (p)
- λ_W, λ'_W - index of W^--boson; λ_l, λ_ν - index of lepton and neutrino
- $\Omega = \{\varphi, \theta, 0\}$, $\Omega' = \{\chi, \theta_l, 0\}$

Decay chain: $\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}$ (2)

- Relation between helicity amplitudes and decay parameters

$$\text{normalization}\ \ \frac{1}{4}(1-\cos\theta_l)^2|H_{\frac{1}{2}1}|^2 + \frac{1}{4}(1+\cos\theta_l)^2|H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2) = 1,$$

$$\alpha_D^{sl} = \frac{1}{4}(1-\cos\theta_l)^2|H_{\frac{1}{2}1}|^2 - \frac{1}{4}(1+\cos\theta_l)^2|H_{-\frac{1}{2}-1}|^2 + \frac{1}{2}\sin^2\theta_l(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),$$

$$\beta_D^{sl} = \frac{1}{\sqrt{2}}\sin\theta_l\sin\chi((1+\cos\theta_l)\Im(H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}-1}) + (1-\cos\theta_l)\Im(H^*_{\frac{1}{2}1}H_{\frac{1}{2}0})),$$

$$\gamma_D^{sl} = \frac{1}{\sqrt{2}}\sin\theta_l\cos\chi((1+\cos\theta_l)\Re(H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}-1}) + (1-\cos\theta_l)\Re(H^*_{\frac{1}{2}1}H_{\frac{1}{2}0}))$$

$$\text{where}\ \beta_D^{sl} = \sqrt{1-(\alpha_D^{sl})^2}\sin\phi_D^{sl}\ \text{and}\ \gamma_D^{sl} = \sqrt{1-(\alpha_D^{sl})^2}\cos\phi_D^{sl}$$

- Non-zero elements of the decay matrix $b_{\mu\nu}$:

$$\begin{aligned}
b_{00} &= 1, & b_{21} &= -\gamma_D^{sl}\cos\theta\sin\phi - \beta_D^{sl}\cos\phi,\\
b_{03} &= \alpha_D^{sl}, & b_{22} &= \beta_D^{sl}\cos\theta\sin\phi - \gamma_D^{sl}\cos\phi,\\
b_{10} &= \alpha_D^{sl}\cos\phi\sin\theta, & b_{23} &= \sin\theta\sin\phi,\\
b_{11} &= -\gamma_D^{sl}\cos\theta\cos\phi + \beta_D^{sl}\sin\phi, & b_{30} &= \alpha_D^{sl}\cos\theta,\\
b_{12} &= \beta_D^{sl}\cos\theta\cos\phi + \gamma_D^{sl}\sin\phi, & b_{31} &= \gamma_D^{sl}\sin\theta,\\
b_{13} &= \sin\theta\cos\phi, & b_{32} &= -\beta_D^{sl}\sin\theta,\\
b_{20} &= \alpha_D^{sl}\sin\theta\sin\phi, & b_{33} &= \cos\theta
\end{aligned}$$

- Main parameters: $\theta \equiv \theta_p$, $\phi \equiv \phi_p$, $\alpha_D^{sl} = \alpha_\Lambda^{sl}(\theta_l, \chi, q^2)$, $\phi_D^{sl} = \phi_D^{sl}(\theta_l, \chi, q^2)$
- Each element of $b_{\mu\nu}$ is multiplied by $q^2 p$ where $p = \sqrt{M_+(q^2)M_-(q^2)}/(2M_1)$

Semileptonic Λ decay

- Momenta and masses: $\Lambda(p_1, M_1) \to p(p_2, M_2) + e^-(p_e, m_e) + \bar{\nu}_e(p_{\bar{\nu}_e}, 0)$
- Standard set of invariant FF for weak current-induced baryonic $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions:

$$M_\mu = M^V_\mu + M^A_\mu = \langle p(p_2)|J^{V+A}_\mu|\Lambda(p_1)\rangle =$$

$$= \bar{u}(p_2)\left[\gamma_\mu\left(F^V_1(q^2) + F^A_1(q^2)\gamma_5\right) + \frac{i\sigma_{\mu\nu}q^\nu}{M_1}\left(F^V_2(q^2) + F^A_2(q^2)\gamma_5\right) + \frac{q_\mu}{M_1}\left(F^V_3(q^2) + F^A_3(q^2)\gamma_5\right)\right]u(p_1)$$

where $q_\mu = (p_1 - p_2)_\mu$

- For $\Lambda \to p e^- \bar{\nu}_e$ at $\mathcal{O}(\frac{m_e^2}{2q^2}) \to 0 \Rightarrow F_3^{V,A} \to 0$

- Helicity amplitude is $H_{\lambda_2\lambda_W} = (H^V_{\lambda_2\lambda_W} + H^A_{\lambda_2\lambda_W})$ with ($\lambda_2 = \pm\frac{1}{2}$; $\lambda_W = 0, \pm 1$):

vector

$$H^V_{\frac{1}{2}1} = \sqrt{2M_-}\left(-F^V_1 - \frac{M_1 + M_2}{M_1}F^V_2\right),$$

$$H^V_{\frac{1}{2}0} = \frac{\sqrt{M_-}}{\sqrt{q^2}}\left((M_1 + M_2)F^V_1 + \frac{q^2}{M_1}F^V_2\right),$$

axial-vector

$$H^A_{\frac{1}{2}1} = \sqrt{2M_+}\left(F^A_1 - \frac{M_1 - M_2}{M_1}F^A_2\right),$$

$$H^A_{\frac{1}{2}0} = \frac{\sqrt{M_+}}{\sqrt{q^2}}\left(-(M_1 - M_2)F^A_1 + \frac{q^2}{M_1}F^A_2\right).$$

where $M_\pm(q^2) = (M_1 \pm M_2)^2 - q^2$; $H^{V,A}_{-\lambda_2,-\lambda_W} = \pm H^{V,A}_{\lambda_2\lambda_W}$

Form factors

$$F_i^{V,A}(q^2) = F_i^{V,A}(0)\prod_{n=0}^{n_i}\frac{1}{1-\frac{q^2}{m_{V,A}^2+n\alpha'^{-1}}} \approx F_i^{V,A}(0)\left(1+q^2\sum_{n=0}^{n_i}\frac{1}{m_{V,A}^2+n\alpha'^{-1}}\right) \equiv F_i^{V,A}(0)c_i^{V,A}(q^2)$$

	$F_i^{V,A}(0)(\Lambda \to p)$	$m_{V,A}$	α' [GeV^{-2}]	n_i
$F_1^V(q^2)$	$-\sqrt{\frac{3}{2}}$ [1]	$m_{K^*(892)^0}$ ($J^P=1^-$)	0.9	$n_1=1$
$F_2^V(q^2)$	$\frac{M_\Lambda \mu_p}{2M_p}F_1^V(0)$ [2]			$n_2=2$
$F_3^V(q^2)$	0 [4]	-		$n_3=2$
$F_1^A(q^2)$	$0.719F_1^V(0)$ [3]	$m_{K^*(1270)^0}$ ($J^P=1^+$)		$n_1=1$
$F_2^A(q^2)$	0 [4]	–		$n_2=2$
$F_3^A(q^2)$	$\frac{M_\Lambda(M_\Lambda+M_p)}{(m_{K^-})^2}F_1^A(0)$ [4]	m_K ($J^P=0^-$)		$n_3=2$

- [1] [PR135(1964)B1483], [PRL13(1964)264]
- [2] $\mu_p = 1.793$ [Lect.NotesPhys.222(1985)1], [Ann.Rev.Nucl.Part.Sci.53(2003)39], [JHEP0807(2008)132]
- [3] [PRD41(1990)780]
- [4] Vanish in the $SU(3)$ symmetry limit; Goldberger-Treiman relation [PR110(1958)1178], [PR111(1958)354]

Intermediate step (1)

- $\alpha_\Lambda^{sl}(\theta_l, \chi, q^2, g_{av}^\Lambda, g_w^\Lambda) \Rightarrow \{n, \alpha, \alpha', \alpha'', \beta_{1,2}, \gamma_{1,2}\}(q^2)$: g_{av}^Λ, g_w^Λ
- Introduce the intermediate parameters:

$$\begin{aligned} n &= |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 + 2(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2), \\ \alpha &= |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2 + 2(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2), \\ \alpha' &= |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2 - 2(|H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}0}|^2), \\ \alpha'' &= |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2 - 2(|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2), \\ \beta_{1,2} &= 2(\Im(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) \pm \Im(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0})), \\ \gamma_{1,2} &= 2(\Re(H^*_{-\frac{1}{2}0} H_{-\frac{1}{2}-1}) \pm \Re(H^*_{\frac{1}{2}1} H_{\frac{1}{2}0})), \end{aligned}$$

where $\beta_{1,2} = \frac{1}{2}\sqrt{n^2 - \alpha^2 - (\alpha')^2 + (\alpha'')^2}\sin\phi_{1,2}$ and $\gamma_{1,2} = \frac{1}{2}\sqrt{n^2 - \alpha^2 - (\alpha')^2 + (\alpha'')^2}\cos\phi_{1,2}$

- Non-zero elements of the decay matrix $b_{\mu\nu}$:

$$\begin{aligned} &\boxed{b_{00} = n + \alpha'\cos^2\theta_l - (\alpha + \alpha'')\cos\theta_l,} && b_{21} = \sqrt{2}\sin\theta_l(-A\cos\theta_p\sin\phi_p - B\cos\phi_p), \\ &b_{03} = \alpha + \alpha''\cos^2\theta_l - (n + \alpha')\cos\theta_l, && b_{22} = \sqrt{2}\sin\theta_l(B\cos\theta_p\sin\phi_p - A\cos\phi_p), \\ &\boxed{b_{10} = b_{03}\cos\phi_p\sin\theta_p,} && b_{23} = b_{00}\sin\theta_p\sin\phi_p, \\ &b_{11} = \sqrt{2}\sin\theta_l(-A\cos\theta_p\cos\phi_p + B\sin\phi_p), && \boxed{b_{30} = b_{03}\cos\theta_p,} \\ &b_{12} = \sqrt{2}\sin\theta_l(B\cos\theta_p\cos\phi_p + A\sin\phi_p), && b_{31} = \sqrt{2}A\sin\theta_l\sin\theta_p, \\ &b_{13} = b_{00}\sin\theta_p\cos\phi_p, && b_{32} = -\sqrt{2}B\sin\theta_l\sin\theta_p, \\ &\boxed{b_{20} = b_{03}\sin\theta_p\sin\phi_p,} && b_{33} = b_{00}\cos\theta_p, \end{aligned}$$

where $A = \cos\chi(\gamma_1 + \cos\theta_l\gamma_2) + \sin\chi(\beta_1 + \cos\theta_l\beta_2)$ and $B = \sin\chi(\gamma_1 + \cos\theta_l\gamma_2) - \cos\chi(\beta_1 + \cos\theta_l\beta_2)$

- $\alpha_\Lambda^{sl} = \alpha + \alpha''\cos^2\theta_l - (n + \alpha')\cos\theta_l$

Intermediate step (2)

- Using the definition of helicity amplitudes the main parameters to describe semileptonic hyperon decays are:

  - $F_1^V(0)$, $F_2^V(0)$, $F_1^A(0)$

  or

  - $g_{av}^D(0) = \dfrac{F_1^A(0)}{F_1^V(0)}$, $\quad g_w^D(0) = \dfrac{F_2^V(0)}{F_1^V(0)}$

- Relations between intermediate and decay parameters:

$$n = 4(F_1^V(q^2))^2 \left[((M_-M_+)^2 - q^4)(1 + (g_{av}^D(q^2))^2)\right.$$
$$\left. + q^2 \left(4M_1M_2((g_{av}^D(q^2))^2 - 1) + (g_w^D(q^2))^2 \frac{q^2}{M_1^2}(M_-^2 - q^2)(M_+^2 + q^2) + 4g_w^D(q^2)\frac{q^2}{M_1^2}(M_-^2 - q^2)M_+\right)\right]$$

$$\alpha = 8(F_1^V(q^2))^2 \sqrt{(M_-M_+)^2 - 2q^2(M_1^2 + M_2^2) + q^4}\left[g_{av}^D(q^2)(q^2 - M_-M_+) + 2g_{av}^D(q^2)g_w^D(q^2)q^2\frac{M_2}{M_1}\right]$$

$$\alpha' = 4(F_1^V(q^2))^2(M_-^2 - q^2)(M_+^2 - q^2)\left[-(1 + (g_{av}^D(q^2))^2) + (g_w^D(q^2))^2\frac{q^2}{M_1^2}\right]$$

$$\alpha'' = 8(F_1^V(q^2))^2 \sqrt{(M_-M_+)^2 - 2q^2(M_1^2 + M_2^2) + q^4}\left[g_{av}^D(q^2)(q^2 + M_-M_+) + 2g_{av}^D(q^2)g_w^D(q^2)q^2\right]$$

where $M_- = M_1 - M_2$ and $M_+ = M_1 + M_2$

- $q^2 \in (m_e^2, (M_1 - M_2)^2)$ and $F_1^V(q^2) = F_1^V(0)c_1^V(q^2)$

Joint angular distribution

- Process $e^+e^- \to (\Lambda \to pe^-\bar{\nu}_e)(\bar{\Lambda} \to \bar{p}\pi^+)$

$$\mathrm{Tr}\rho_{pW\bar{p}} \propto \mathcal{W}(\boldsymbol{\xi};\boldsymbol{\omega}) = \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}} b^{\Lambda}_{\mu 0} a^{\bar{\Lambda}}_{\bar{\nu}0}$$

- $C_{\mu\bar{\nu}}(\theta_\Lambda;\alpha_\psi,\Delta\Phi)$
- $b_{\mu 0}$ matrices for $1/2 \to 1/2 + \{0,\pm 1\}$ decays $\Leftrightarrow b^{\Lambda}_{\mu 0} \equiv b_{\mu 0}(\theta_p,\varphi_p,\theta_e,\chi,q^2;\boxed{g^{\Lambda}_{av},g^{\Lambda}_{w}})$ $\quad \boxed{F^V_1, F^V_2, F^A_1} \Updownarrow$
- $a_{\bar{\nu}0}$ matrices for $1/2 \to 1/2 + 0$ decays $\Leftrightarrow a^{\bar{\Lambda}}_{\bar{\nu}0} \equiv a_{\bar{\nu}0}(\theta_{\bar{p}},\varphi_{\bar{p}};\boxed{\alpha_{\bar{\Lambda}}})$
  - $\boldsymbol{\xi} \equiv (\theta_\Lambda,\theta_p,\varphi_p,\theta_e,\chi,q^2,\theta_{\bar{p}},\varphi_{\bar{p}})$
  - $\boldsymbol{\omega} \equiv (\alpha_\psi,\Delta\Phi,\boxed{g^{\Lambda}_{av},g^{\Lambda}_{w}},\boxed{\alpha_{\bar{\Lambda}}})$

$$\Updownarrow$$

$$\boxed{F^V_1, F^V_2, F^A_1}$$

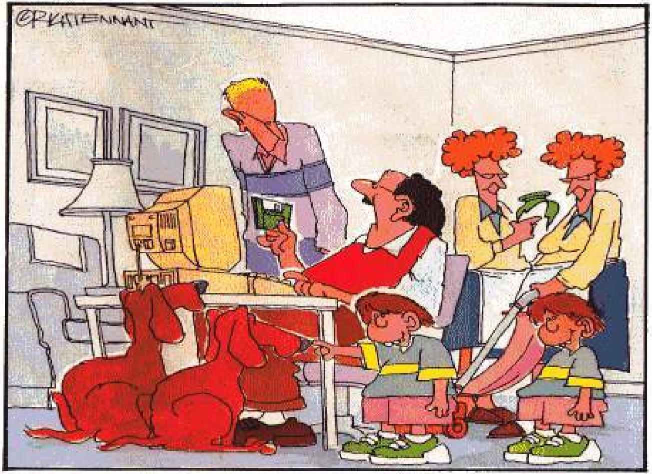
- Range of $q^2 \in (m_l^2,(M_1-M_2)^2)$ is specific for each decay

Next steps

- Test formalism using
  1. Production of `mDIY` and `MC PhSp` for $e^+e^- \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$
     ◦ true and reco `mDIY` and `MC PhSp`
     ◦ allow to extract true and reco values of α_Λ and $\alpha_{\bar{\Lambda}}$ decay parameters
  2. Modification of `mDIY` to include the semileptonic decay formalism
  3. Production of true and reco `mDIY` and `MC PhSp` for $\Lambda \to pe^-\bar{\nu}_e$
     ◦ extraction of the g_{av}^Λ and g_w^Λ decay parameters
  4. If all steps work, consider more difficult scenario, mixed `MC` samples
  5. If previous step works, move to the real data

- Independent step:
  - Expand the modular method for the cascade semileptonic decays
    ◦ $B_1 \to (B_2 \to p\pi) + l + \nu_l$
  - Test the same steps as for semileptonic decay

Backups



" I ALWAYS BACK UP EVERYTHING."

Helicity amplitudes of the lepton pair $h^l_{\lambda_l \lambda_\nu}$

- Lepton and antineutrino spinors

$$\bar{u}_{l^-}(\mp\frac{1}{2}, p_{l^-}) = \sqrt{E_l + m_l}\left(\chi^\dagger_\mp, \frac{\pm|\vec{p}_{l^-}|}{E_l + m_l}\chi^\dagger_\mp\right), \qquad v_{\bar{\nu}}(\frac{1}{2}) = \sqrt{E_\nu}\begin{pmatrix}\chi_+ \\ -\chi_+\end{pmatrix},$$

where $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$ and $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$ are Pauli two-spinors

- SM form of the lepton current ($\lambda_W = \lambda_{l^-} - \lambda_{\bar{\nu}}$)

$$h^l_{\lambda_{l^-}=\mp 1/2, \lambda_{\bar{\nu}}=1/2} = \bar{u}_{l^-}(\mp\frac{1}{2})\gamma^\mu(1+\gamma_5)v_{\bar{\nu}}(\frac{1}{2})\begin{Bmatrix}\epsilon_\mu(-1)\\ \epsilon_\mu(0)\end{Bmatrix}$$

where $\epsilon^\mu(0) = (0;0,0,1)$ and $\epsilon^\mu(\mp 1) = (0;\mp 1, -i, 0)/\sqrt{2}$

- Moduli squared of the helicity amplitudes [EPJ C59 (2009) 27]

$$\text{nonflip}(\lambda_W = \mp 1) : |h^l_{\lambda_l=\mp\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8(q^2 - m_l^2),$$

$$\text{flip}(\lambda_W = 0) : |h^l_{\lambda_l=\pm\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}|^2 = 8\frac{m_l^2}{2q^2}(q^2 - m_l^2)$$

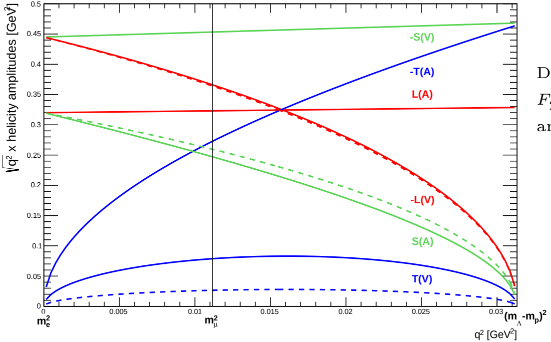
- Upper and lower signs refer to the configurations $(l^-, \bar{\nu}_l)$ ($\lambda_\nu = 1/2$) and (l^+, ν_l) ($\lambda_\nu = -1/2$), respectively
- In case of the e-mode only nonflip transition remains under assumption $\frac{m_e^2}{2q^2} \to 0$

Size estimations of helicity amplitudes

$\mathrm{T(V,A)} := \sqrt{q^2} H^{V,A}_{\frac{1}{2}1}$

$\mathrm{L(V,A)} := \sqrt{q^2} H^{V,A}_{\frac{1}{2}0}$

$\mathrm{S(V,A)} := \sqrt{q^2} H^{V,A}_{\frac{1}{2}t}$



Dashed lines: $F_2^V(q^2)$ and $F_3^A(q^2)$ are switched off

- If $q^2 = m_e^2 \Rightarrow H^V_{\frac{1}{2}0}$ and $H^A_{\frac{1}{2}0}$ are dominated
- If $q^2 = (M_\Lambda - M_p)^2 \Rightarrow H^A_{\frac{1}{2}1} = -\sqrt{2} H^A_{\frac{1}{2}0}$ are dominated
- Using data of the E-555 experiment (Fermilab) [PRD41 (1990) 780]
  $\Rightarrow \frac{F_1^A(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.731 \pm 0.016$ and $\frac{F_2^V(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.15 \pm 0.30$
  $\Rightarrow \frac{F_1^A(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) = 0.719 \pm 0.016$ with constraint $\frac{F_2^V(0)}{F_1^V(0)}(\Lambda \to p e^- \bar{\nu}_e) \to 0.97$ (CVC)

Two boundary cases: $q^2_{\min}$ and $q^2_{\max}$

- $q^2_{\min} = m_e^2 \longrightarrow 0$:

$$
\begin{aligned}
b_{00} &= (1 + (g^D_{av}(0))^2)\sin^2\theta_l \Leftrightarrow ((F^V_1(0))^2 + (F^A_1(0))^2)\sin^2\theta_l \Longrightarrow 1,\\
b_{03} &= -2g^D_{av}(0)\sin^2\theta_l \Leftrightarrow -2F^V_1(0)F^A_1(0)\sin^2\theta_l,\\
b_{10} &= b_{03}\sin\theta_p\cos\phi_p,\\
b_{13} &= b_{00}\sin\theta_p\cos\phi_p,\\
b_{20} &= b_{03}\sin\theta_p\sin\phi_p,\\
b_{23} &= b_{00}\sin\theta_p\sin\phi_p,\\
b_{30} &= b_{03}\cos\theta_p,\\
b_{33} &= b_{00}\cos\theta_p.
\end{aligned}
$$

- $q^2_{\max} = (M_1 - M_2)^2$:

$$
\begin{aligned}
b_{00} &= (g^D_{av}(q^2))^2 \equiv (F^A_1(q^2))^2 \Longrightarrow 1 & b_{21} &= \sqrt{2}\sin\theta_l(-A\cos\theta_p\sin\phi_p - B\cos\phi_p),\\
b_{03} &= -b_{00}\cos\theta_l, & b_{22} &= \sqrt{2}\sin\theta_l(B\cos\theta_p\sin\phi_p - A\cos\phi_p),\\
b_{10} &= b_{03}\sin\theta_p\cos\phi_p, & b_{23} &= b_{00}\sin\theta_p\sin\phi_p,\\
b_{11} &= \sqrt{2}\sin\theta_l(-A\cos\theta_p\cos\phi_p + B\sin\phi_p), & b_{30} &= b_{03}\cos\theta_p,\\
b_{12} &= \sqrt{2}\sin\theta_l(B\cos\theta_p\cos\phi_p + A\sin\phi_p), & b_{31} &= \sqrt{2}A\sin\theta_l\sin\theta_p,\\
b_{13} &= b_{00}\sin\theta_p\cos\phi_p, & b_{32} &= -\sqrt{2}B\sin\theta_l\sin\theta_p,\\
b_{20} &= b_{03}\sin\theta_p\sin\phi_p, & b_{33} &= b_{00}\cos\theta_p,
\end{aligned}
$$

$$
\begin{aligned}
\text{where}\quad A &= \tfrac{1}{2}b_{00}[\cos\chi(\cos\phi_1 + \cos\theta_l\cos\phi_2) + \sin\chi(\sin\phi_1 + \cos\theta_l\sin\phi_2)]\\
\text{and}\quad B &= \tfrac{1}{2}b_{00}[\sin\chi(\cos\phi_1 + \cos\theta_l\cos\phi_2) - \cos\chi(\sin\phi_1 + \cos\theta_l\sin\phi_2)]
\end{aligned}
$$

MC generator (step 1)

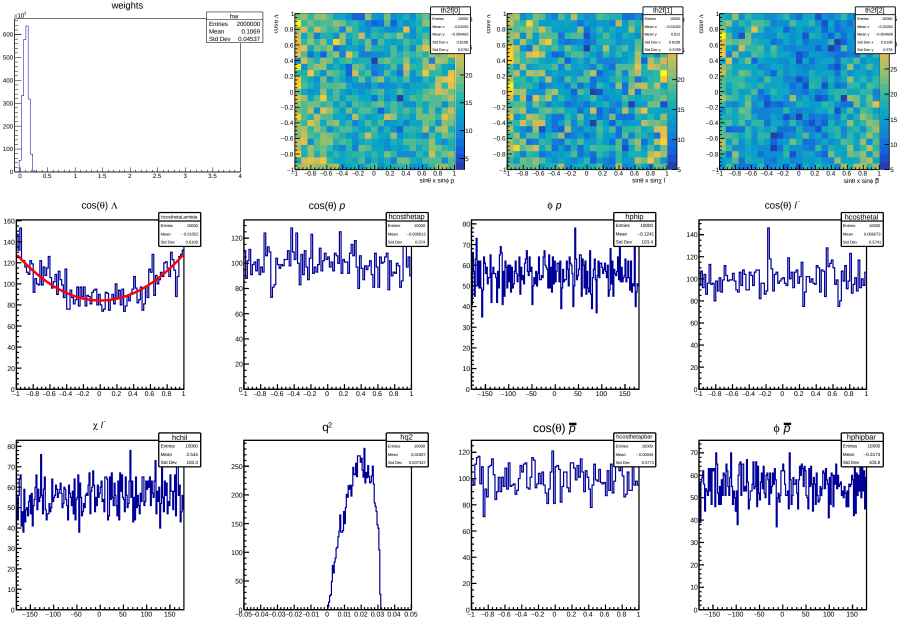
- Using **YYbar_example** package by Patrik, the presented process can be generated
  - MC samples: $N_{\text{evt}}^{\text{sig}}=10^4$ and $N_{\text{evt}}^{\text{phsp}}=10^5$
  - Generate $q^2 \in (m_e^2, (M_\Lambda - M_p)^2)$
  - Two sets of input values:

```
pp[0] =  0.461;      // alpha_J/psi (arxiv:1808.08917)
pp[1] =  0.74;       // Delta_Phi (arxiv:1808.08917, equal 42.4deg)
pp[2] =  0.719;       // gav Lambda->p e- nu_ebar
pp[3] =  1.066;       // gw Lambda->p e- nu_ebar
pp[4] = -0.758;       // alpha_D LambdaBar->pbar pi+ (arxiv:1808.08917)
```

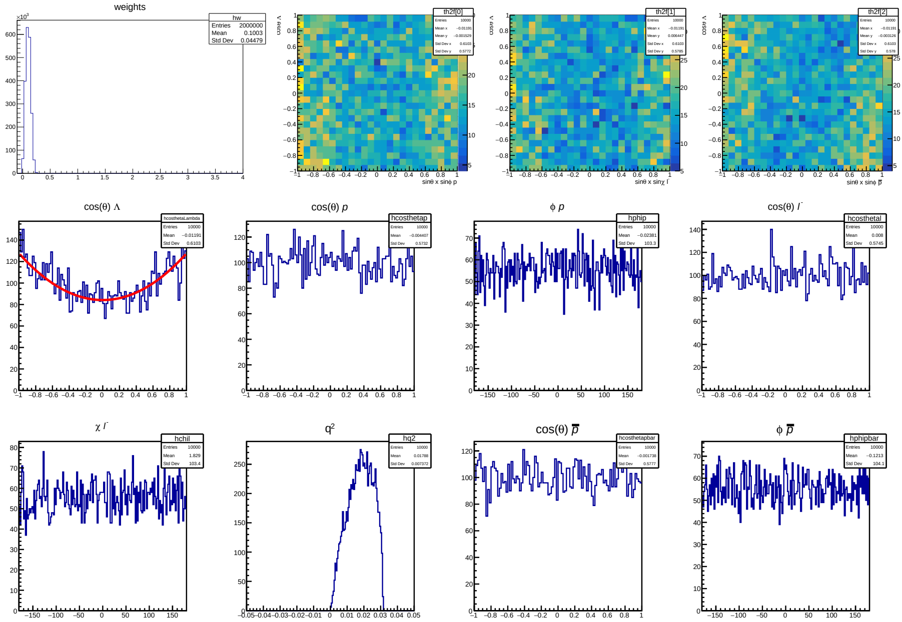
```
pp[0] =  0.461;      // alpha_J/psi (arxiv:1808.08917)
pp[1] =  0.74;       // Delta_Phi (arxiv:1808.08917, equal 42.4deg)
pp[2] = -1.225;       // fv1 Lambda->p e- nu_ebar
pp[3] = -1.306;       // fv2 Lambda->p e- nu_ebar
pp[4] = -0.881;       // fa1 Lambda->p e- nu_ebar
pp[5] = -0.758;       // alpha_D LambdaBar->pbar pi+ (arxiv:1808.08917)
```

- No negative weights are observed
- Maximal weight: $\sim$0.26 for g and $\sim$0.27 for FF

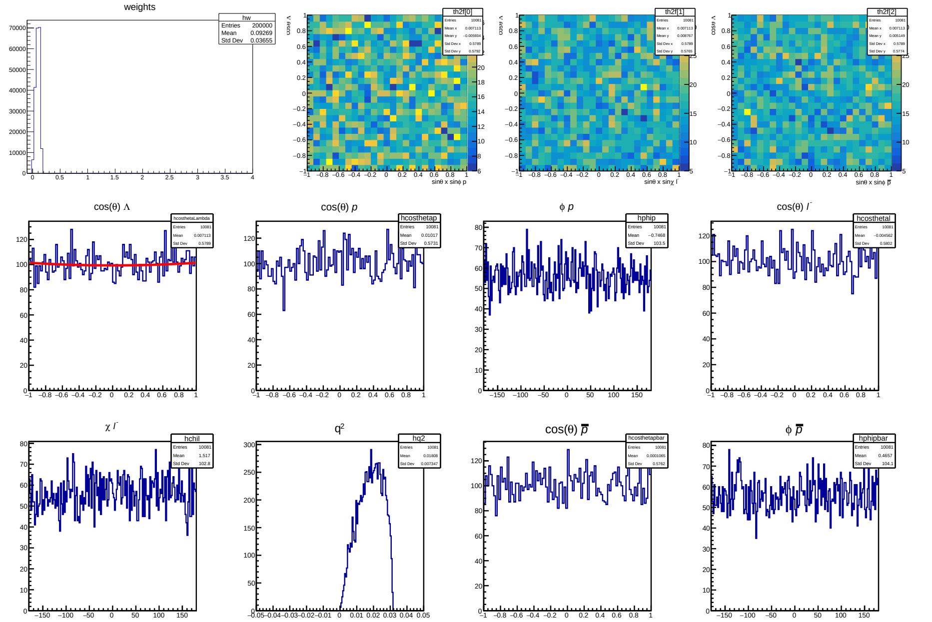
Random samples (g, $N_{sig} = 10^4$) (step 1)



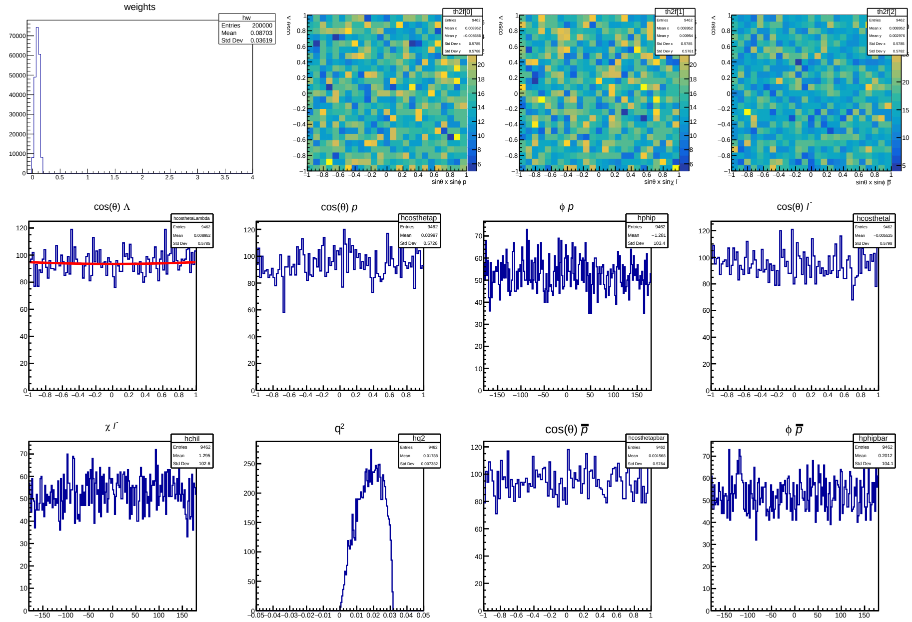
Random samples (ff, $N_{sig} = 10^4$) (step 1)



Random samples (g, $N_{phsp} = 10^5$) (step 1)



Random samples (ff, $N_{phsp} = 10^5$) (step 1)



Run through fit method (step 2)

- Set input values in the fit

```
void mainMLL(){

    // instantiating the values to be measured
    Double_t pp[4];
    for( int i = 0; i < 4; i++ ) pp[i]=0;

    // starting values for fit
    Double_t alpha_jpsi      = 0.461;      // alpha_J/Psi
    Double_t dphi_jpsi       = 0.74;        // relative phase, Dphi_J/Psi
    Double_t gav_lam_plnu    = 0.719;       // gav (Lam->p l nubar_l)
    Double_t gw_lam_plnu     = 1.066;       // gw (Lam->p l nubar_l)
    Double_t alpha_lam_pbarpip  = -0.758;    // alpha (Lambar->pbar pi+)

    alpha_jpsi         = gRandom->Rndm();
    dphi_jpsi          = gRandom->Rndm();
    gav_lam_plnu    = 0.719;
    gw_lam_plnu    = 1.066;
    alpha_lam_pbarpip = -0.758;

    ReadData();
    ReadMC();
```

```
void mainMLL(){

    // instantiating the values to be measured
    Double_t pp[5];
    for( int i = 0; i < 5; i++ ) pp[i]=0;

    // starting values for fit
    Double_t alpha_jpsi      = 0.461;      // alpha_J/Psi
    Double_t dphi_jpsi       = 0.74;        // relative phase, Dphi_J/Psi
    Double_t fv1_lam_plnu    =-1.225;       // fv1 (Lam->p l nubar_l)
    Double_t fv2_lam_plnu    =-1.306;       // fv2 (Lam->p l nubar_l)
    Double_t fa1_lam_plnu    =-0.881;       // fa1 (Lam->p l nubar_l)
    Double_t alpha_lam_pbarpip  = -0.758;    // alpha (Lambar->pbar pi+)

    alpha_jpsi         = gRandom->Rndm();
    dphi_jpsi          = gRandom->Rndm();
    fv1_lam_plnu    = -1.225;
    fv2_lam_plnu   = -1.306;
    fa1_lam_plnu   = -0.881;
    alpha_lam_pbarpip = -0.758;

    ReadData();
    ReadMC();
```

- Output value of fit method

Parameter	$N^{MC}_{sig} = 10^4$ $N^{MC}_{phsp} = 10^5$
α_Ψ	0.4933±0.0462
$\Delta\Phi$	0.7574±0.4150
$\alpha_{\bar{\Lambda}}$ [1]	-0.7082±0.3047
g_{av}	0.8384±0.2792
g_w [2]	-0.3178±2.5996

Parameter	$N^{MC}_{sig} = 10^4$ $N^{MC}_{phsp} = 10^5$
α_Ψ	0.4949±0.0452
$\Delta\Phi$	0.8779±0.2765
$\alpha_{\bar{\Lambda}}$	-0.7083±0.1641
$F_1^V(0)$	-1.0786±0.3622
$F_2^V(0)$ [3]	0.4990±2.1221
$F_1^A(0)$	-1.1567±0.2222

- [1] Strong correlation between $\Delta\Phi$ and $\alpha_{\bar{\Lambda}}$
- [2,3] Value with large uncertainty: g_w and $F_2^V(0)$

Tests: Fit results (step 2)

- Increase statistics:

Parameter	$N_{sig}^{MC} = 10^4$ $N_{phsp}^{MC} = 10^5$	$N_{sig}^{MC} = 10^5$ $N_{phsp}^{MC} = 10^6$
α_Ψ	0.4933±0.0462	0.4534±0.0218
$\Delta\Phi$	0.7574±0.4150	1.4871±0.3084
$\alpha_{\bar{\Lambda}}$	-0.7082±0.3047	-0.5359±0.0334
g_{av}	0.8384±0.2792	0.7998±0.1206
g_w	-0.3178±2.5996	-3.3097±1.7734

Parameter	$N_{sig}^{MC} = 10^4$ $N_{phsp}^{MC} = 10^5$	$N_{sig}^{MC} = 10^5$ $N_{phsp}^{MC} = 10^6$
α_Ψ	0.4949±0.0452	0.4537±0.0217
$\Delta\Phi$	0.8779±0.2765	1.2647±0.1879
$\alpha_{\bar{\Lambda}}$	-0.7083±0.1641	-0.5626±0.0454
$F_1^V(0)$	-1.0786±0.3622	-1.3699±0.2518
$F_2^V(0)$	0.4990±2.1221	1.4957±1.4542
$F_1^A(0)$	-1.1567±0.2222	-0.9571±0.1109

- Fix some parameters to input value:

Parameter	$N_{sig}^{MC} = 10^4$ $N_{phsp}^{MC} = 10^5$			
α_Ψ	0.4933±0.0462	0.4932±0.0461	0.4935±0.0461	0.4988±0.4384
$\Delta\Phi$	0.7574±0.4150	0.74	0.6996±0.1146	0.74
$\alpha_{\bar{\Lambda}}$	-0.7082±0.3047	-0.7210±0.0988	-0.758	-0.758
g_{av}	0.8384±0.2792	0.8307±0.2090	0.8085±0.1959	0.8090±0.1949
g_w	-0.3178±2.5996	-0.2902±2.5216	-0.2092±2.4778	-0.1799±2.4597

Parameter	$N_{sig}^{MC} = 10^4$ $N_{phsp}^{MC} = 10^5$			
α_Ψ	0.4949±0.0452	0.4925±0.0448	0.4955±0.0452	0.4870±0.0424
$\Delta\Phi$	0.8779±0.2765	0.74	0.8121±0.1255	0.74
$\alpha_{\bar{\Lambda}}$	-0.7083±0.1641	-0.7982±0.0988	-0.758	-0.758
$F_1^V(0)$	-1.0786±0.3622	-1.1026±0.3589	-1.0896±0.3578	-1.1143±0.3632
$F_2^V(0)$	0.4990±2.1221	0.4771±2.1023	0.4799±2.1035	0.5194±2.1379
$F_1^A(0)$	-1.1567±0.2222	-1.0722±0.1716	-1.1104±0.1558	-1.1032±0.1579

Output of fit method (step 2)

```
Loglike: 461.054
 MINUIT WARNING IN MIGRAD
 ============== VARIABLE5 IS AT ITS LOWER ALLOWED LIMIT.
 FCN=460.885 FROM MINOS     STATUS=SUCCESSFUL    1253 CALLS        1535 TOTAL
                     EDM=9.92814e-07    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                PARABOLIC         MINOS ERRORS
  NO.   NAME      VALUE          ERROR      NEGATIVE      POSITIVE
   1  alpha_jpsi   4.93279e-01   4.61710e-02  -4.55553e-02   4.68457e-02
   2  dphi_jpsi    7.57375e-01   4.15025e-01  -3.05929e-01   5.16552e-01
   3  gavLam       8.38371e-01   2.79168e-01  -3.13930e-01   2.46018e-01
   4  gwLam       -3.17781e-01   2.59960e+00  -3.22415e+00   2.44056e+00
   5  aLambar     -7.08159e-01   3.04670e-01   at limit      2.06191e-01
                               ERR DEF= 0.5
 EXTERNAL ERROR MATRIX.    NDIM=  25    NPAR=  5    ERR DEF=0.5
  2.134e-03  1.215e-03 -2.005e-04  5.073e-03 -5.173e-04
  1.215e-03  1.733e-01  7.666e-02 -2.584e-01  1.247e-01
 -2.005e-04  7.666e-02  7.818e-02  8.496e-02  6.100e-02
  5.073e-03 -2.584e-01  8.496e-02  7.094e+00 -2.143e-01
 -5.173e-04  1.247e-01  6.100e-02 -2.143e-01  9.924e-02
 PARAMETER  CORRELATION COEFFICIENTS
       NO.  GLOBAL      1      2      3      4      5
        1  0.31718   1.000  0.063 -0.016  0.041 -0.036
        2  0.95619   0.063  1.000  0.659 -0.233  0.951
        3  0.75519  -0.016  0.659  1.000  0.114  0.693
        4  0.47884   0.041 -0.233  0.114  1.000 -0.255
        5  0.96156  -0.036  0.951  0.693 -0.255  1.000
```

```
Loglike: 497.442
 FCN=496.941 FROM MINOS     STATUS=SUCCESSFUL    1146 CALLS        1418 TOTAL
                     EDM=2.73905e-06    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                PARABOLIC         MINOS ERRORS
  NO.   NAME      VALUE          ERROR      NEGATIVE      POSITIVE
   1  alpha_jpsi   4.94922e-01   4.51921e-02  -4.45999e-02   4.58420e-02
   2  dphi_jpsi    8.77898e-01   2.76501e-01  -2.61822e-01   3.39672e-01
   3  fv1Lam      -1.07857e+00   3.62200e-01  -3.97598e-01   3.47342e-01
   4  fv2Lam       4.99047e-01   2.12210e+00  -2.16998e+00   2.21019e+00
   5  fa1Lam      -1.15665e+00   2.22154e-01  -2.06963e-01   2.45773e-01
   6  aLambar     -7.08331e-01   1.64102e-01  -2.37893e-01   1.33985e-01
                               ERR DEF= 0.5
 EXTERNAL ERROR MATRIX.    NDIM=  25    NPAR=  6    ERR DEF=0.5
  2.044e-03  1.717e-03 -1.593e-04 -4.075e-03  5.250e-04 -3.249e-04
  1.717e-03  7.667e-02  1.835e-02  4.087e-03 -3.864e-02  3.930e-02
 -1.593e-04  1.835e-02  1.319e-01  4.515e-01 -3.108e-02  8.134e-03
 -4.075e-03  4.087e-03  4.515e-01  4.649e+00  5.544e-03  1.153e-02
  5.250e-04 -3.864e-02 -3.108e-02  5.544e-03  4.937e-02 -2.597e-02
 -3.249e-04  3.930e-02  8.134e-03  1.153e-02 -2.597e-02  2.743e-02
 PARAMETER  CORRELATION COEFFICIENTS
       NO.  GLOBAL      1      2      3      4      5      6
        1  0.34651   1.000  0.137 -0.010 -0.042  0.052 -0.043
        2  0.87969   0.137  1.000  0.183  0.007 -0.628  0.857
        3  0.74358  -0.010  0.183  1.000  0.576 -0.385  0.135
        4  0.66817  -0.042  0.007  0.576  1.000  0.012  0.032
        5  0.80436   0.052 -0.628 -0.385  0.012  1.000 -0.706
        6  0.90549  -0.043  0.857  0.135  0.032 -0.706  1.000
```