Hadron Spectroscopy from Lattice QCD

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For more detailed reviews, please refer to:

- "Hadron spectroscopy"
  Sasa Prelovsek,
  plenary talk given in the Lattice 2014 conference
  https://indico.bnl.gov/contributionListDisplay.py?confId=736

- "Hadron Spectroscopy"
  Sasa Prelovsek, arXiv:1411.0405 (hep-lat)

Outline

I. Introduction

Lattice methods on the spectroscopic study

II. Single-particle treatment of hadron spectrum from lattice QCD

Light hadrons below the threshold

Charmonium spectrum (below open-charm threshold)

III . Resonance and bound state study

Bound state vs. X(3872),Ds*(2317), Ds(2460)

Zc particles

Resonances

IV Summary

I. Introduction

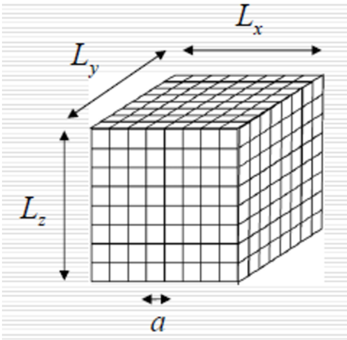
1. The lattice formulation of QCD---Lattice QCD

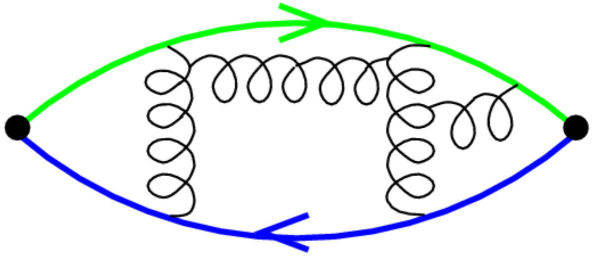
$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, e^{-S}$$

$$S = S_{gauge} + S_{quarks} = \int d^4x \, \left(\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}\right) - \sum_i \log(\mathrm{Det}\, M_i)$$

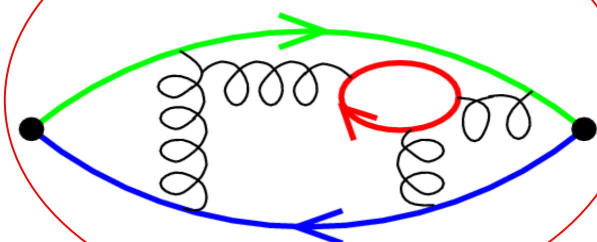
$$Z = \int \mathcal{D}A_\mu \, \det M \, e^{\int d^4x \, (-\frac{1}{4} F_{\mu\nu} F^{\mu\nu})} .$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \, \mathcal{O} \, e^{-S} .$$





(A) Quenched QCD: quark loops neglected



(B) Full QCD

Dominated in the present era

2. The methods for the hadron spectroscoapy in lattice QCD

- Interpolation field operators --- starting point
  for a meson (-like) system with given J^{PC} and flavor quantrum numbers:

$$\mathcal{O}_i: \quad \bar{q}_1\Gamma q_2 \quad [\bar{q}_1\Gamma_1 q\,][\bar{q}\Gamma_2 q_2\,] \quad [q_1^T\Gamma_1 q][\bar{q}\Gamma_2\bar{q}_2^T\,], \ldots\ldots$$

- Two-point functions --- Observables

$$\mathcal{C}_{ij}(t) = \left\langle 0\middle|\mathcal{O}_i(t)\mathcal{O}_j^+(0)\middle|0\right\rangle$$
$$= \sum_n \langle 0|\mathcal{O}_i|n\rangle\left\langle n\middle|\mathcal{O}_j^+\middle|0\right\rangle e^{-E_n t}$$

In principle, all the physical states with the same quantum numbers $|n\rangle$ contribute to the two point functions $\mathcal{C}_{ij}(t)$ as the eigenstates of the QCD Hamiltonian with the energy eigenvalue E_n:

- " one-particle state" : $E_n = m_n$
- " two-particle state" : $E_n = \sqrt{m_1^2+\vec{p}^2} + \sqrt{m_2^2+\vec{p}^2} + \Delta E,\ \vec{p} = \frac{2\pi}{L}\vec{n}$
- ……

Comparison of the hadron spectra

Euclidean spacetime lattice

One particle states

Multiple particle states with discrete relative spatial Momentum (scattering States in a finite volume

All the energies are Discretized.

Minkowski continuum spacetime

Stable particles

Bound states of hadrons

Resonances

Continuum scattering states

Luescher's Relation:

$$E_n = (m_1^2 + p^2)^{1/2} + (m_2^2 + p^2)^{1/2}$$

$$\tan\delta(p) = \frac{\sqrt{\pi}\, p\, L}{2\, \mathcal{Z}_{00}\left(1; (\frac{pL}{2\pi})^2\right)}$$

Resonances

$$T(p) = \frac{-\sqrt{s}\,\Gamma(p)}{s - m_R^2 + i\sqrt{s}\,\Gamma(p)} = \frac{1}{\cot\delta(p) - i}$$

$$\Gamma(p) = g^2 \frac{p^{2l+1}}{s}, \quad \frac{p^{2l+1}}{\sqrt{s}} \cot\delta(p) = \frac{1}{g^2}(m_R^2 - s)$$

Bound states

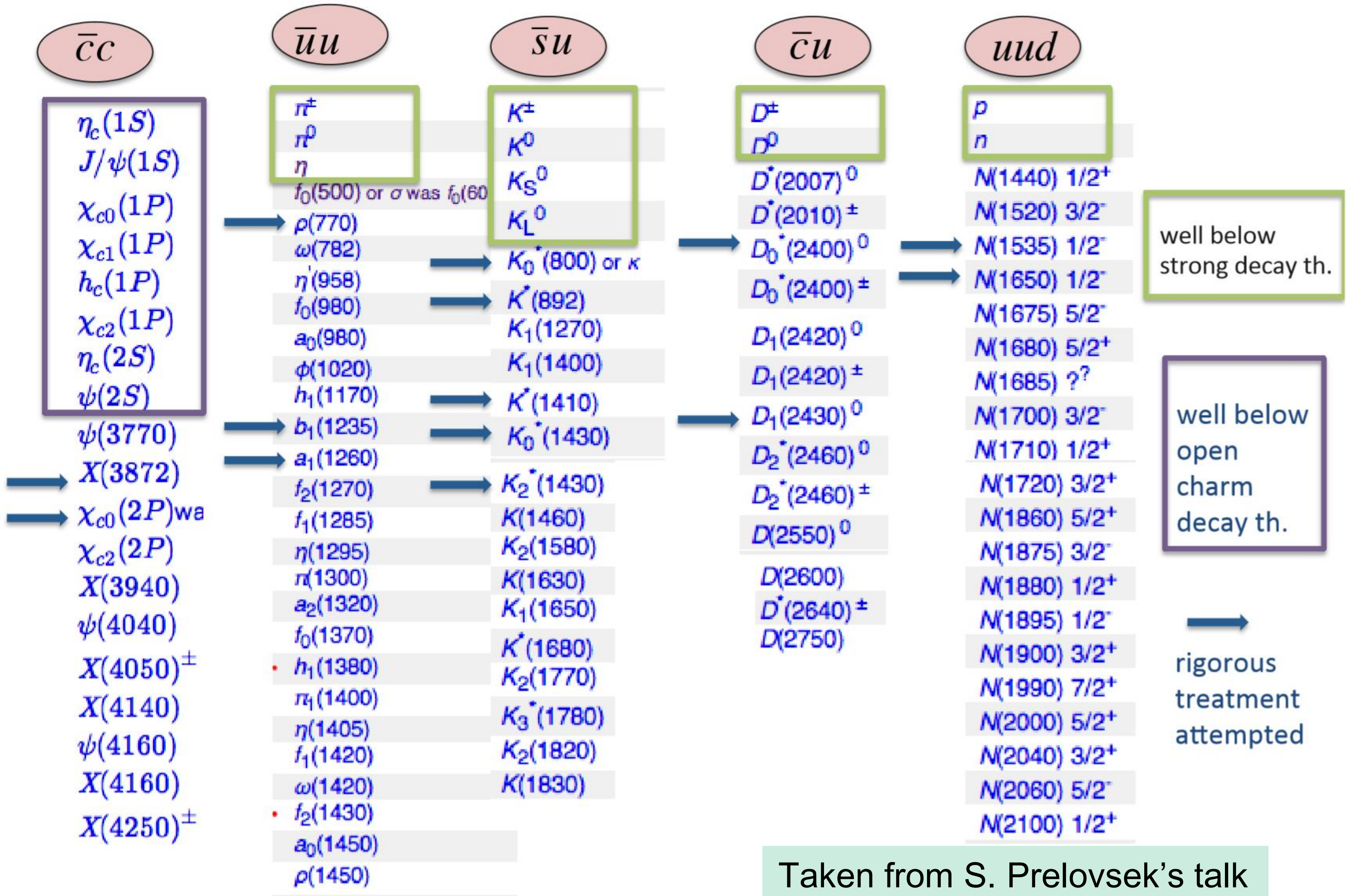
$$p\cot(\delta_0(p)) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2, \quad -|p_B| = \frac{1}{a_0} - \frac{1}{2} r_0 |p_B|^2$$

$$T = \frac{}{\cot(\delta_l(p_B)) - i} = \infty$$

$$m_B = E_{H_1}(p_B) + E_{H_2}(p_B), \; p_B = i|p_B|$$

3. Present status

- Experimental hadron spectroscopy --- most are resonances



Taken from S. Prelovsek's talk

- Experimental hadron spectroscopy --- XYZ particles

TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle [772, 992] (>10), BaBar [993] (8.6)	2003	Ok
				$p\bar{p} \to (\pi^+\pi^- J/\psi)\ldots$	CDF [994, 995] (11.6), D0 [996] (5.2)	2003	Ok
				$pp \to (\pi^+\pi^- J/\psi)\ldots$	LHCb [997, 998] (np)	2012	Ok
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle [999] (4.3), BaBar [1000] (4.0)	2005	Ok
				$B \to K(\gamma\, J/\psi)$	Belle [1001] (5.5), BaBar [1002] (3.5) LHCb [1003] (> 10)	2005	Ok
				$B \to K(\gamma\, \psi(2S))$	BaBar [1002] (3.6), Belle [1001] (0.2) LHCb [1003] (4.4)	2008	NC!
				$B \to K(D\bar{D}^*)$	Belle [1004] (6.4), BaBar [1005] (4.9)	2006	Ok
$Z_c(3885)^+$	3883.9 ± 4.5	25 ± 12	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III [1006] (np)	2013	NC!
$Z_c(3900)^+$	3891.2 ± 3.3	40 ± 8	$?^{?-}$	$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III [1007] (8), Belle [1008] (5.2) T. Xiao *et al.* [CLEO data] [1009] (>5)	2013	Ok
$Z_c(4020)^+$	4022.9 ± 2.8	7.9 ± 3.7	$?^{?-}$	$Y(4260, 4360) \to \pi^-(\pi^+ h_c)$	BES III [1010] (8.9)	2013	NC!
$Z_c(4025)^+$	4026.3 ± 4.5	24.8 ± 9.5	$?^{?-}$	$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III [1011] (10)	2013	NC!
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(10860) \to \pi(\pi\Upsilon(1S, 2S, 3S))$	Belle [1012–1014] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B\bar{B}^*)^+$	Belle [1015] (8)	2012	NC!
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(10860) \to \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$	Belle [1012, 1013] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B^*\bar{B}^*)^+$	Belle [1015] (6.8)	2012	NC!

Brambilla et al., arXiv:1404.2723

TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

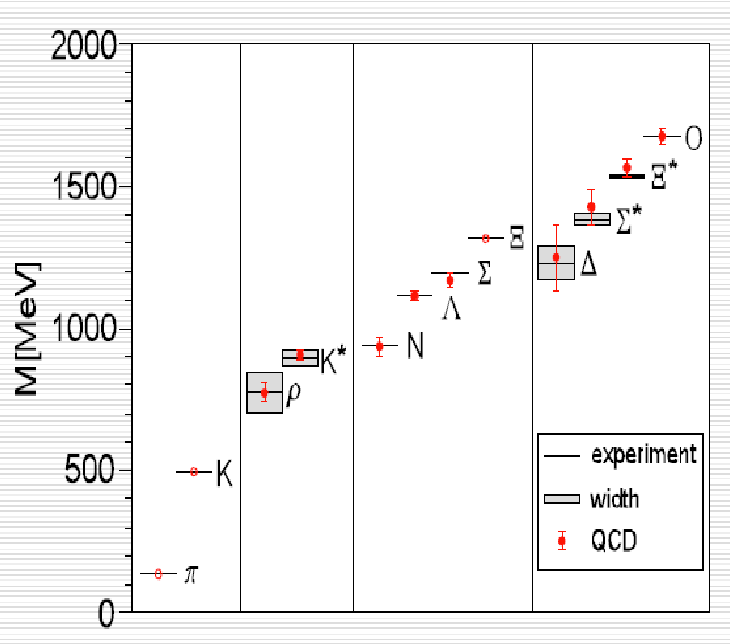
State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$Y(3915)$	3918.4 ± 1.9	20 ± 5	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [1050] (8), BaBar [1000, 1051] (19)	2004	Ok
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [1052] (7.7), BaBar [1053] (7.6)	2009	Ok
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle [1054] (5.3), BaBar [1055] (5.8)	2005	Ok
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	Belle [1048, 1049] (6)	2005	NC!
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle [1008, 1056] (7.4)	2007	NC!
$\psi(4040)$	4039 ± 1	80 ± 10	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)}(\pi))$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.0)	2013	NC!
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (1.1)	2008	NC!
$Y(4140)$	4145.8 ± 2.6	18 ± 8	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (5.0), Belle [1061] (1.9),	2009	NC!
					LHCb [1062] (1.4), CMS [1063] (>5)		
					D0 [1064] (3.1)		
$\psi(4160)$	4153 ± 3	103 ± 8	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)})$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.5)	2013	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\,(D^*\bar{D}^*)$	Belle [1049] (5.5)	2007	NC!
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (7.2)	2014	NC!
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (2.0)	2008	NC!
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	BaBar [1066, 1067] (8), CLEO [1068, 1069] (11)	2005	Ok
					Belle [1008, 1056] (15), BES III [1007] (np)		
				$e^+e^- \to (f_0(980) J/\psi)$	BaBar [1067] (np), Belle [1008] (np)	2012	Ok
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
				$e^+e^- \to (\gamma\, X(3872))$	BES III [1070] (5.3)	2013	NC!
$Y(4274)$	4293 ± 20	35 ± 16	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (3.1), LHCb [1062] (1.0),	2011	NC!
					CMS [1063] (>3), D0 [1064] (np)		
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [1071] (3.2)	2009	NC!
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \to (\pi^+\pi^- \psi(2S))$	Belle [1072] (8), BaBar [1073] (np)	2007	Ok
$Z(4430)^+$	4458 ± 15	166^{+37}_{-32}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle [1074, 1075] (6.4), BaBar [1076] (2.4)	2007	Ok
					LHCb [1077] (13.9)		
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (4.0)	2014	NC!
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle [1078] (8.2)	2007	NC!
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \to (\pi^+\pi^- \psi(2S))$	Belle [1072] (5.8), BaBar [1073] (5)	2007	Ok
$\Upsilon(10860)$	10876 ± 11	55 ± 28	1^{--}	$e^+e^- \to (B^{(*)}_{(s)}\bar{B}^{(*)}_{(s)}(\pi))$	PDG [1]	1985	Ok
				$e^+e^- \to (\pi\pi\Upsilon(1S, 2S, 3S))$	Belle [1013, 1014, 1079] (>10)	2007	Ok
				$e^+e^- \to (f_0(980)\Upsilon(1S))$	Belle [1013, 1014] (>5)	2011	Ok
				$e^+e^- \to (\pi Z_b(10610, 10650))$	Belle [1013, 1014] (>10)	2011	Ok
				$e^+e^- \to (\eta\Upsilon(1S, 2S))$	Belle [948] (10)	2012	Ok
				$e^+e^- \to (\pi^+\pi^-\Upsilon(1D))$	Belle [948] (9)	2012	Ok
$Y_b(10888)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$	Belle [1080] (2.3)	2008	NC!

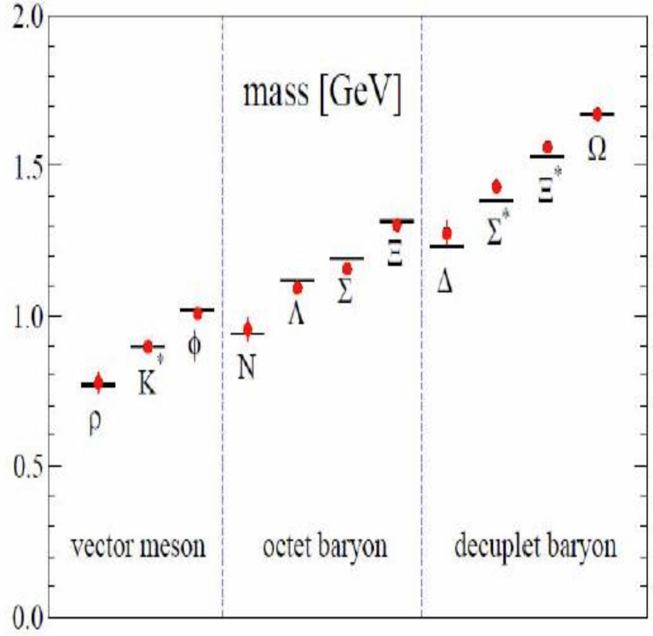
II. Single-particle Treatment of the Hadron Spectrum from Lattice QCD

- Just quark bilinear (for mesons) operators
- All the hadrons are taken as stable states
- QCD ground states are reproduced very well
- This treatment is going to the history for the near-threshold hadrons and those above the threshold.

1. Light hadron spectrum

Expemerimental spectrum is well reproduced by (lattice) QCD
Single-particle treatment, all systematic uncertainties are under control..





Butapest-Marseille-Wuppertal Collaboration @ Lattice08

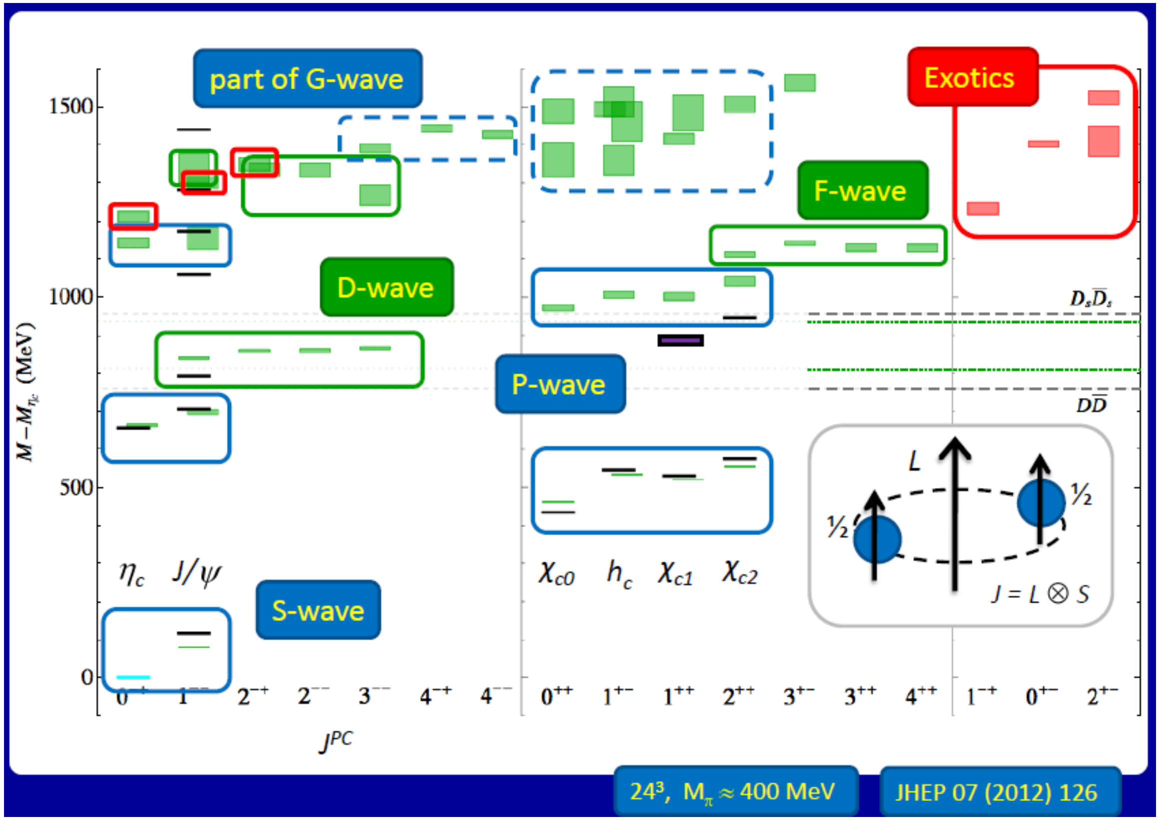
Wilson-clover action
Nf=2+1 full QCD
3 lattice spacing and
continuum
extrapolated

PACS-CS Collaboration
Wilson-clover action
Nf=2+1 full QCD
32^3x64
a=0.09fm

hep-lat arXiv:0807.1661v1

2. Charmonium spectrum

- Latest charmonium spectrum from lattice QCD
- Single-particle threatment
- Spectrum compatible with the $n\,^{2S+1}L_J$ multiplet assignment in QM



Liu et al. [HSC], JHEP 07 (2012)126, arXiv:1204.5425

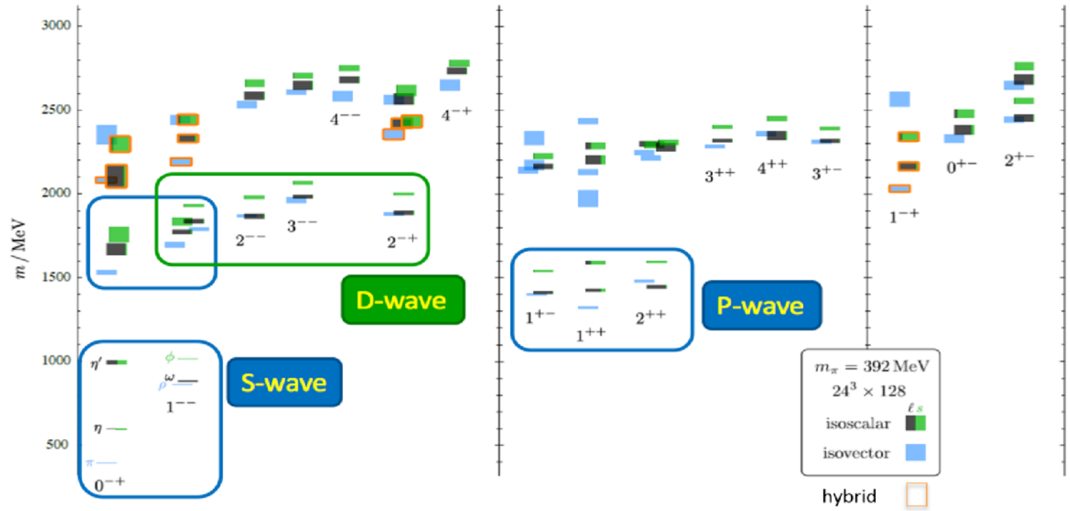
3. Isocalar meson spectrum

- Similar to charmonium
- In addition,

$$\begin{pmatrix}|\mathfrak{a}\rangle\\|\mathfrak{b}\rangle\end{pmatrix}=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}|\ell\rangle\\|s\rangle\end{pmatrix}$$

$$|\ell\rangle \equiv \tfrac{1}{\sqrt{2}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle\right)$$

$$|s\rangle \equiv |s\bar{s}\rangle$$



Dudek et al. [Hadron Spectrum Coll.]
PRD88(2013)094505,arXiv:1309.2608

4. Baryon spectrum

Bulava et al. [Hadron Spectrum Coll.]
PRD82(2010)014507, arXiv:1004.5072

Lin, CJP49(2011), arXiv:1106.1608 (a review article)

III. Bound State and Resonance Study

- Near and above threshold mesons
- Possible multi-quark states
- More operators (including multi-hadron operators)
- Hadron-hadron scattering
- Bound state identification and Resonance parameters

1. X(3872)

Experimental status:

- $I^G J^{PC} = 0^+ 1^{++}$, $M_X = 3871.68 \pm 0.17$MeV, $\Gamma_X < 1.2$ MeV
- Below but very close to the $D\bar{D}^*$ threshold
- Isospin violation suggests molecular picture of X(3872)

$$\frac{B(X \to J/\psi\omega)}{B(X \to J/\psi\rho)} = 0.8 \pm 0.3$$

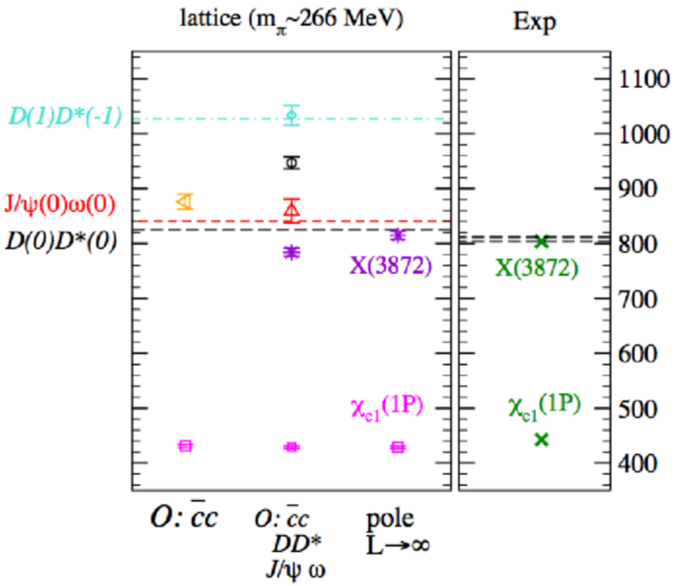
- Radiative decays are compatible with the $c\bar{c}$ assignment

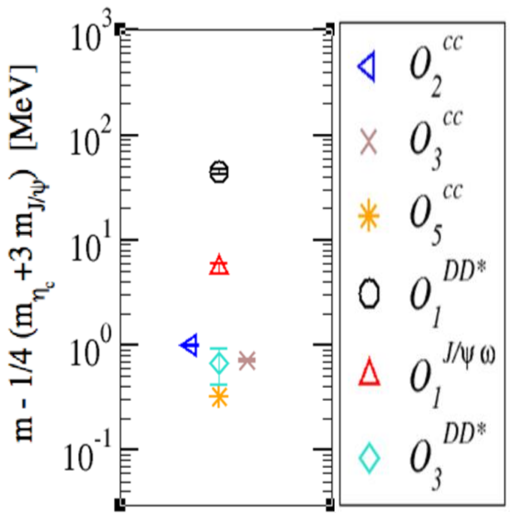
$$\frac{B(X \to \psi' \gamma)}{B(\to J/\psi\ \gamma)} \backsim 2 - 3$$

Latest lattice studies:

- Lattice 1: operators $c\bar{c}$, $D\bar{D}^*$, $J\psi\omega$
  [S. Prelovsek and L. Leskovec, PRL111(2013)192001]
- Lattice 2: operators $c\bar{c}$, $D\bar{D}^*$
  [C. DeTar and S. Lee, poster in Lattice 2014]

Evidence for X(3872)





$$p \cdot \cot\delta(p) = \frac{2Z_{00}(1;q^2)}{\sqrt{\pi}L}, \qquad q^2 \equiv \left(\frac{L}{2\pi}\right)^2 p^2$$

$$p\cot\delta(p) = \frac{1}{a_0^{DD*}} + \frac{1}{2} r_0^{DD*} p^2$$

$$a_0^{DD^*} = -1.7 \pm 0.4 \text{ fm},$$

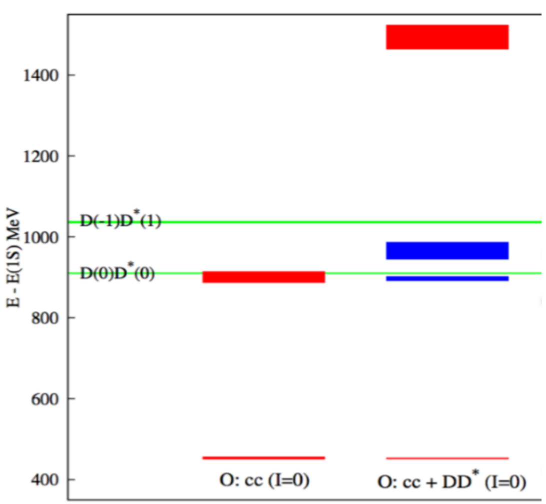
$$r_0^{DD^*} = 0.5 \pm 0.1 \text{ fm}$$

$$S \propto (\cot\delta(p) - i)^{-1},$$

$$p_{\mathrm{BS}}^2 = -0.020(13) \text{ GeV}^2$$

$$m_X^{\mathrm{lat}}(L \to \infty) = E_D(p_{\mathrm{BS}}) + E_{D^*}(p_{\mathrm{BS}})$$

Prelovsek&Leskovec, PRL111(2013)192001



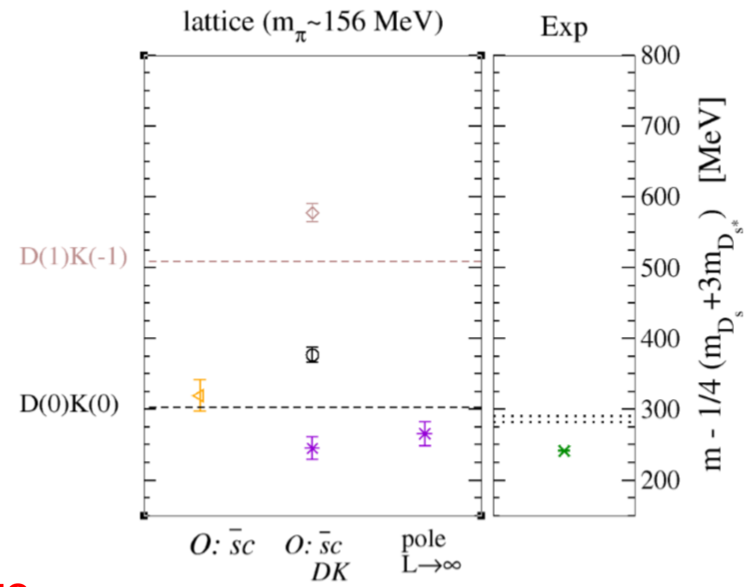
DeTar&Lee. Lattice 2014

X(3872)	$m_X - \left(m_{D^0} + m_{D_0^*}\right)$
Lattice 1	-11 ± 7 MeV
Lattice 2	-13 ± 6 MeV
Exp.	-0.003 ± 0.192 MeV

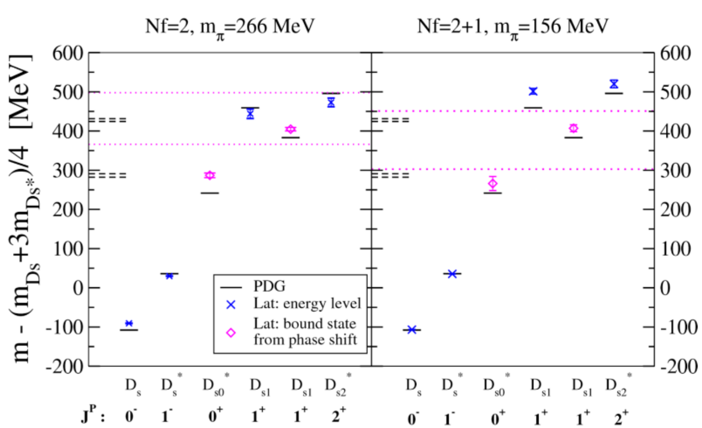
Exp. : A. Tomaradze et al., arXiv:1501.01658 (hep-ex)

2. Ds*(2317) and Ds*(2460)

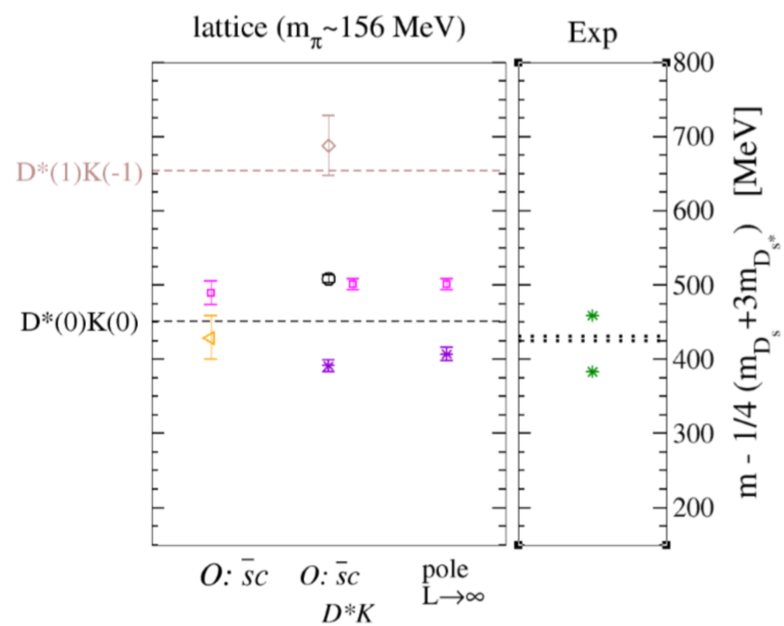
- Ds*(2317) and Ds*(2460) are slightly below the DK and D*K thresholds, respectively.
- QM predicts the masses of 0+ and 1+ Ds mesons are higher than these thresholds.
- They might be shallow bound states of DK and D*K in the s-wave.
- A large and negative scattering length a0 can be an indication of a shallow bound state.



Mohler et al. PRL'13, 1308.3175



	$D_{s0}^*(2317)$	$D_{s1}^*(2460)$
a_0	$-1.33 \pm 0.20 fm$	$-1.11 \pm 0.11 fm$
r_0	$0.27 \pm 0.17 fm$	$0.10 \pm 0.10 fm$



Lang et al. arXiv: 1403.8103

3. Z_c^+ particles

- These are charged particles with closed charm.
- If exist, their minimal quark configuration should be $c\bar{c}u\bar{d}$
- Can be the good multiquark candidates.
- Near $D\bar{D}$ threshold states.

$\bar{c}\ u$
$c\ \bar{d}$

particle	C	J^P	decay	year	coll
$Z^+(4430)$	-	1+	$\psi(2S)\ \pi^+$	2008	Belle, ~~BABAR~~, LHCb
$Z_c^+(3900)$	-	?	$J/\psi\ \pi^+$	2013	BESIII, Belle, CLEOc
$Z_c^+(3885)$	-	1+	$(DD^*)^+$	2013	BESIII
$Z_c^+(4020)$	-	?	$h_c(1P)\ \pi^+$	2013	BESIII
$Z_c^+(4025)$	-	?	$(D^*D^*)^+$	2013	BES III
$Z^+(4200)$	-	1+	$J/\psi\ \pi^+$	2014	Belle
$Z^+(4050)$	+	?	$\chi_{c1}\ \pi^+$	2008	Belle
$Z^+(4250)$	+	?	$\chi_{c1}\ \pi^+$	2008	Belle

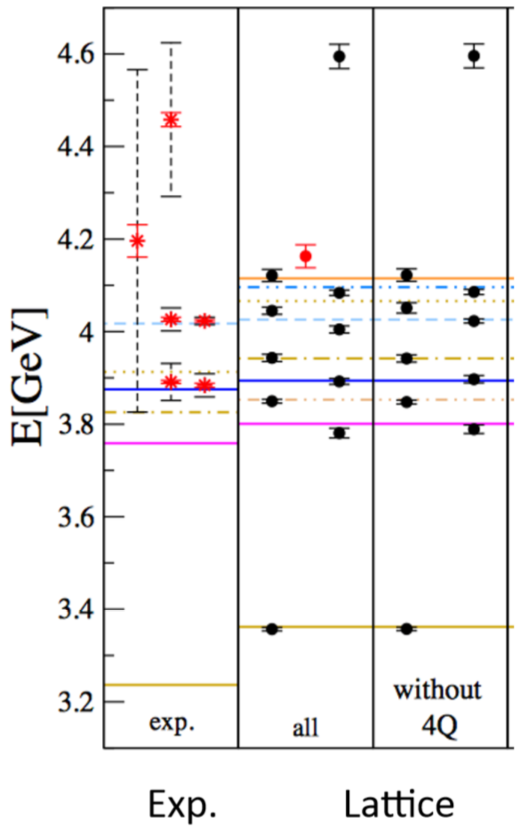
Taken from S. Prelovsek's talk

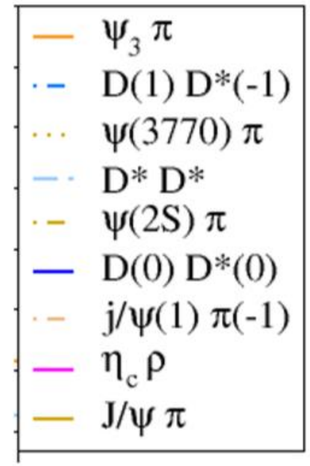
- $Z_c^+(4430)$ and $D\bar{D}_1$ scattering $\left(J^P = (0,1,2)^-\right)$
  in quenched approximation
  the scattering length a0 indicates a weak attractive interaction. No bound state observed.
  [Meng et al., (CLQCD Collab.), PRD 09, 0905.0752]

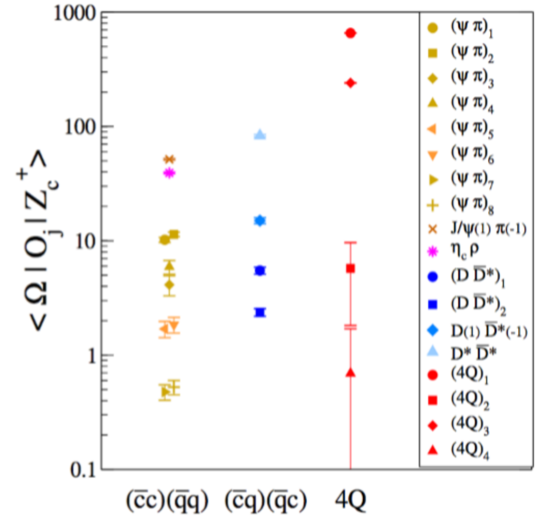
- Search for $Z_c^+(3900)$ with $\psi\pi$ and $D\bar{D}^*$ operators
  $N_f = 2$, m_π=266 MeV. Only two-particle states found
  [Prelovsek and Leskovec, PLB 13, 1308.2097]

- $Z_c^+(3900)$ and $D\bar{D}^*$ scattering $\left(I^G J^P = 1^+1^+\right)$
  $N_f = 2$, three sea quark masses.
  the scattering length a0 indicates a weak repulsive interaction.
  [Chen et al., (CLQCD Collab.), PRD 14, 1403.1318]
- These are negative results for Z_c states.

- **Search for Z_c^+ states in the energy $E < 4.3$ GeV with more operators**
  $N_f = 2$, m_π=266 MeV.
  [Prelovsek et al., arXiv:1405.7623]
  tetraquark operators are added







$$\mathcal{O}_1^{\psi(0)\pi(0)} = \bar{c}\gamma_i c(0)\ \bar{d}\gamma_5 u(0)\,,$$

$$\mathcal{O}^{\psi(1)\pi(-1)} = \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k)\ \bar{d}\gamma_5 u(-e_k)\,,$$

$$\mathcal{O}^{\eta_c(0)\rho(0)} = \bar{c}\gamma_5 c(0)\ \bar{d}\gamma_i u(0)\,,$$

$$\mathcal{O}_1^{D(0)D^*(0)} = \bar{c}\gamma_5 u(0)\ \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,$$

$$\mathcal{O}^{D^*(0)D^*(0)} = \epsilon_{ijk}\ \bar{c}\gamma_j u(0)\ \bar{d}\gamma_k c(0)\,,$$

$$O_1^{4q} \approx [\bar{c}\ C\gamma_5\ \bar{d}]_{\bar{3}_c}[c\ \gamma_i C\ u]_{3c}$$

$$O_2^{4q} \approx [\bar{c}\ C\ \bar{d}]_{\bar{3}_c}[c\ \gamma_i\gamma_5 C\ u]_{3c}$$

Nearby experimental candidates:

$Z_c^+(4020)$, $\Gamma=7.9 \pm 3.7$ MeV BESIII 2013

$Z_c^+(4025)$, $\Gamma=24.8 \pm 9.5$ MeV BESIII 2013

$Z_c^+(4200)$, $\Gamma=370 \pm 110$ MeV Belle, Moriond 2014

Lattice (m_π =266 MeV, Nf=2) :

$m(Z_c^+)$ = 4.16 GeV

$\pm$ 0.163 GeV $\pm$ O(Γ)

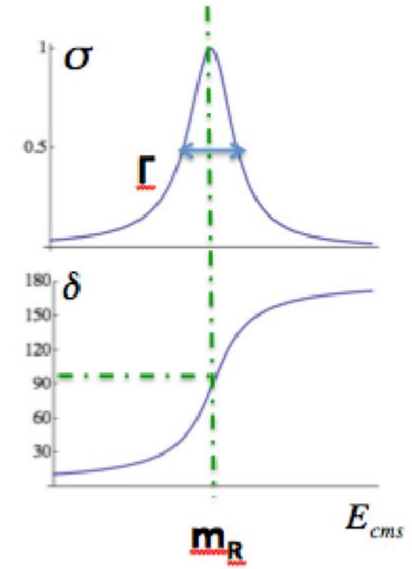
[S.P., Lang, Leskovec, Mohler, 1405.7623]

3. Hadron resonances

Breit-Wigner ansatz

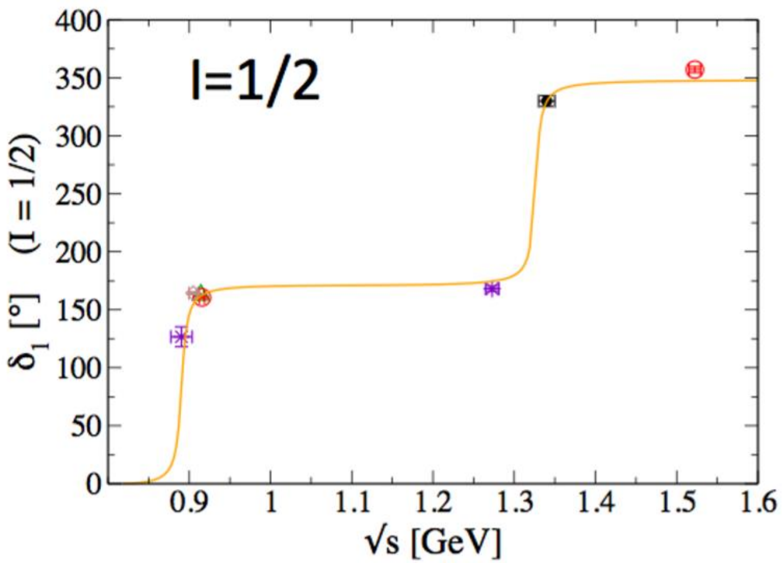
$$T(p) = \frac{-\sqrt{s}\,\Gamma(p)}{s - m_R^2 + i\sqrt{s}\,\Gamma(p)} = \frac{1}{\cot\delta(p) - i}$$

$$\Gamma(p) = g^2 \frac{p^{2l+1}}{s}, \qquad \frac{p^{2l+1}}{\sqrt{s}} \cot\delta(p) = \frac{1}{g^2}(m_R^2 - s)$$



- ρ resonance in the $\pi\pi(I=1)$ scattering
- K^* resonance in the $\pi K(I=1/2)$ scattering

(see the review of T. Yamzaki on Lattice 2014 (plenary talk).



	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6
exp	891.66 ± 0.26	5.72 ± 0.06

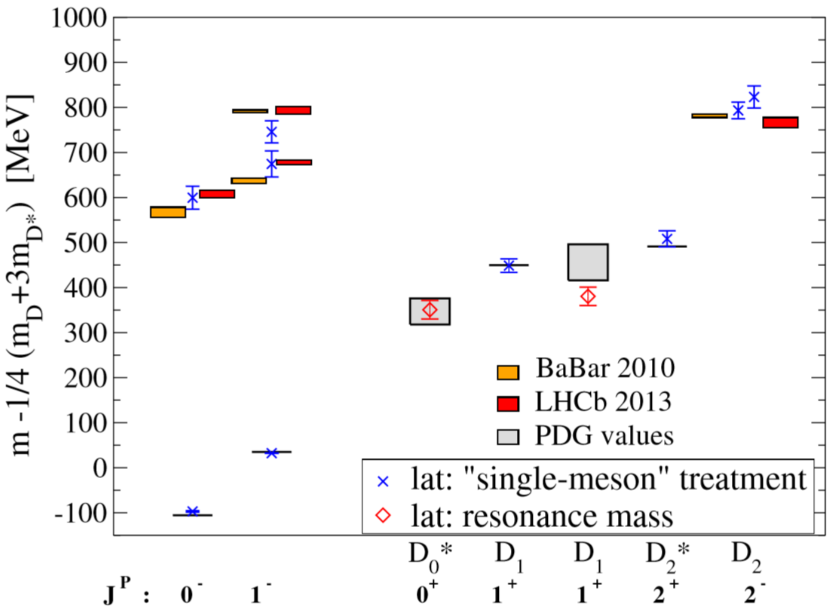
- D resonances in the $D\pi$ and $D^*\pi$ scattering
  Mohler et al., PRD 13, 1208.4059

$J^P=0^+$: D π

$D_0^*(2400)$	m - 1/4(mD+3 mD*)	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

$J^P=1^+$: D* π

$D_1(2430)$	m - 1/4(mD+3 mD*)	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV



- $a_1(1260)$ and $b_1(1235)$
  Lang et al., JHEP 14, 1401.2088

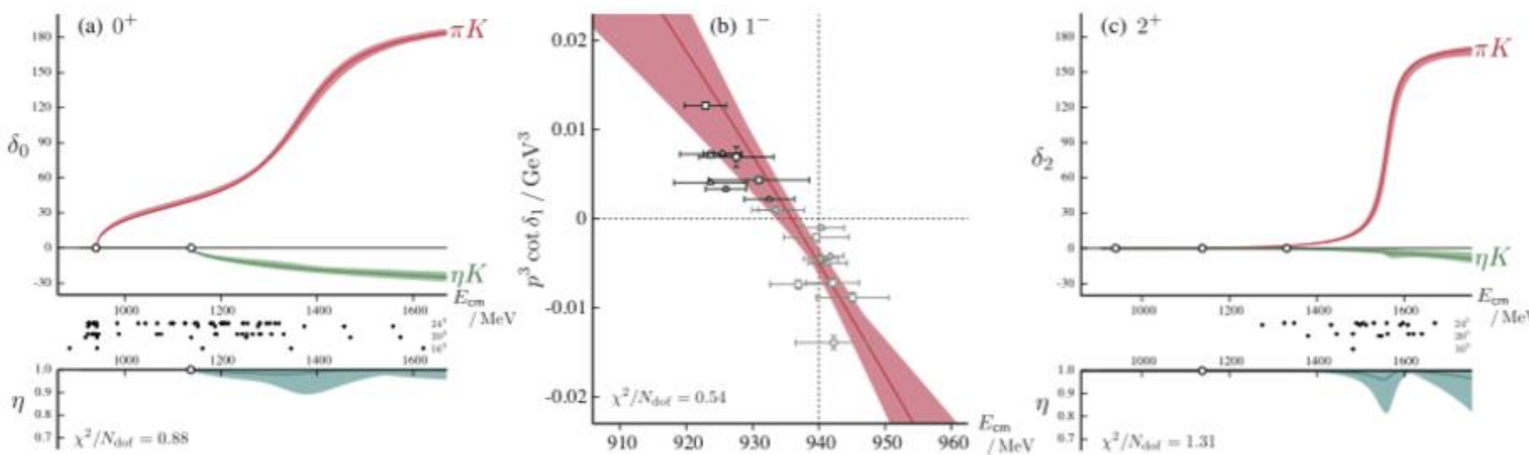
resonance	$a_1(1260)$			$b_1(1235)$	
quantity	$m_{a_1}^{res}$ [GeV]	$g_{a_1\rho\pi}$ [GeV]	$a_{l=0}^{\rho\pi}$ [fm]	$m_{b_1}^{res}$ [GeV]	$g_{b_1\omega\pi}$ [GeV]
lat	$1.435(53)(^{+0}_{-109})$	1.71(39)	0.62(28)	$1.414(36)(^{+0}_{-83})$	input
exp	1.230(40)	1.35(30)	-	1.2295(32)	0.787(25)

- $K\pi$ and $K\eta$ scattering in coupled channels
  Dudek et al., PRL'14, 1406.4158; 1411.2004(more details)

Couple-channel treatment is used for the first time to the study of hadron-hadron scattering

$$\det\left[\delta_{ij}\delta_{JJ'} + i\rho_i\, t_{ij}^{(J)}(E_{\mathsf{cm}})\left(\delta_{JJ'} + i\mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_i L)\right)\right] = 0$$

$$t_{ii} = \frac{(\eta e^{2i\delta_i} - 1)}{2i\rho_i},\; t_{ij} = \frac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}}$$



IV. Conclusions

Last several years witness a rapid progress in the lattice QCD study on the hadron spectroscopy

- The lowest-lying hadron spectrum is well reproduced with rigorously controlled systematic uncertainties.
  ---testifying QCD?

- Quite a few hadron resonances are investigated through the hadron-hadron scattering study.

- Shed light on the nature of the near-threshold states such as X(3872), Ds*(2317), Ds(2460)

- There are still many challenges:
  scalar mesons
  glueballs and hybrids
  XYZ particles above thresholds

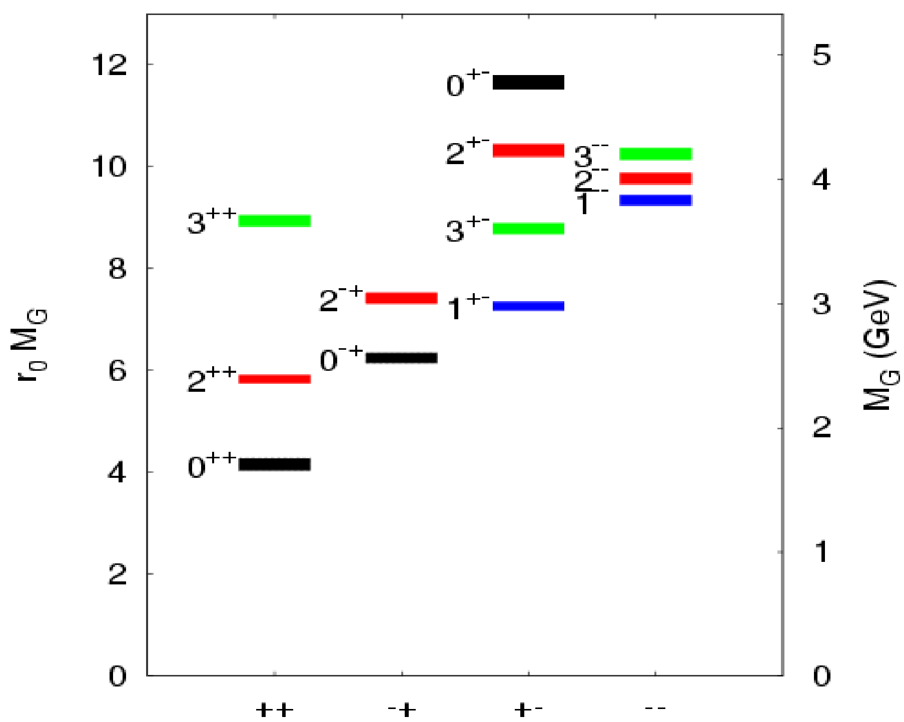
Thanks!

II. Glueballs on the lattice

I). Glueball mass spectrum

- Quenched LQCD predicts glueball spectrum

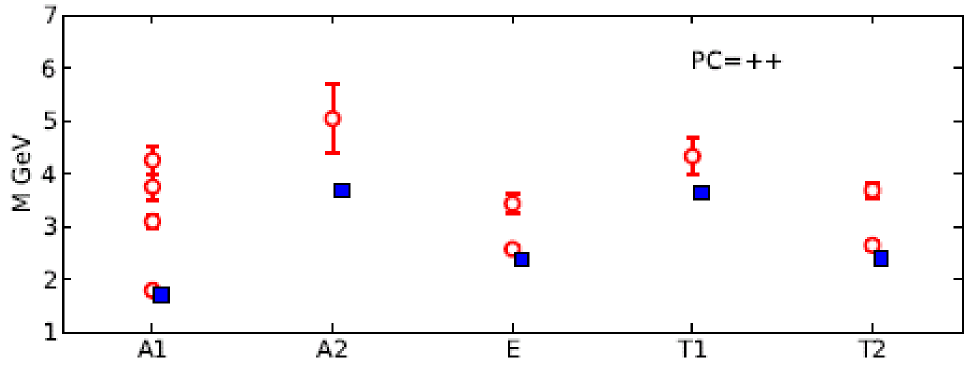
  Lowest-lying glueballs have masses in the range 1~3GeV



J^{PC}	$r_0 M_G$	M_G (MeV)
0^{++}	4.16(11)(4)	1710(50)(80)
2^{++}	5.83(5)(6)	2390(30)(120)
0^{-+}	6.25(6)(6)	2560(35)(120)
1^{+-}	7.27(4)(7)	2980(30)(140)
2^{-+}	7.42(7)(7)	3040(40)(150)
3^{+-}	8.79(3)(9)	3600(40)(170)
3^{++}	8.94(6)(9)	3670(50)(180)
1^{--}	9.34(4)(9)	3830(40)(190)
2^{--}	9.77(4)(10)	4010(45)(200)
3^{--}	10.25(4))(10)	4200(45)(200)
2^{+-}	10.32(7)(10)	4230(50)(200)
0^{+-}	11.66(7)(12)	4780(60)(230)

Y. Chen et al, Phys. Rev. D 73, 014516 (2006)

- Latest results of glueball masses from 2+1 flavor dynamical lattice QCD study, which confirm the prediction of the quenched lattice QCD.
  [E.Gregory et al, JHEP 10 (2012) 170, arXiv:1208.1858(hep-lat)]



Open circles are full-QCD results, and the filled squares are from quenched lattice QCD studies

Linearly combine the correlation functions with different r we can eliminate the conventional vector charmonium and get a relatively clean signal of the exotic vector meson.