Dalitz analysis of $D^0 \to K^- \pi^+ \eta$

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- Dalitz analysis
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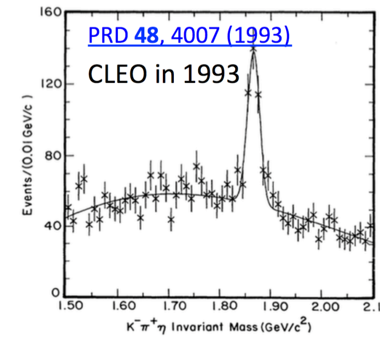
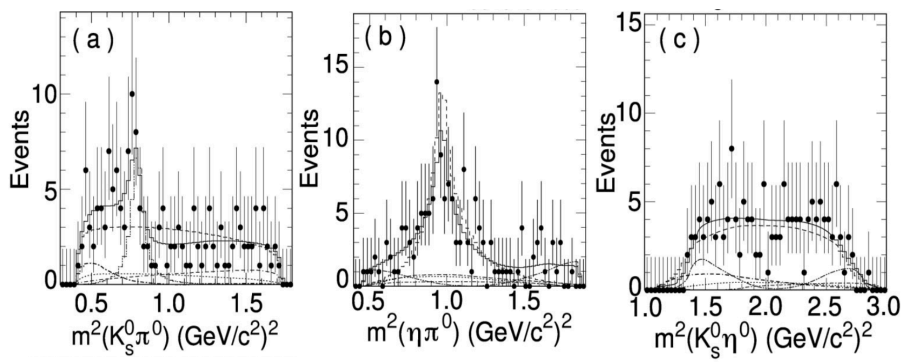
Motivation



- CLEO studied $D^0 \to \overline{K}^{*0}\eta$, presented signal of $D^0 \to K^-\pi^+\eta$
- Dalitz analysis: $D^0 \to K_S\pi^0\eta$ PRL **93**, 111801 (2004)



- dominant from $a_0(980)^0$, $K^*(892)^0$
- not sufficient to describe data

$\Longrightarrow$ unknown intermediate resonances

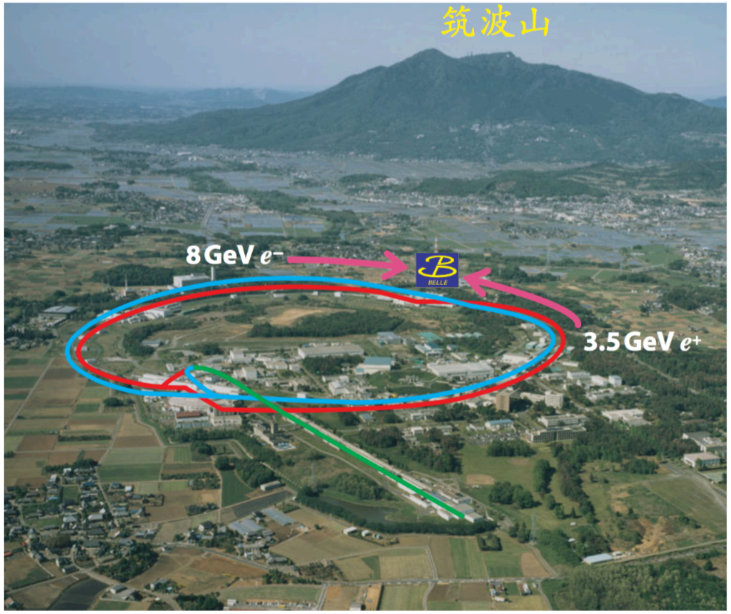
- Light scalar meson $a_0(980)^\pm$ nature unclear

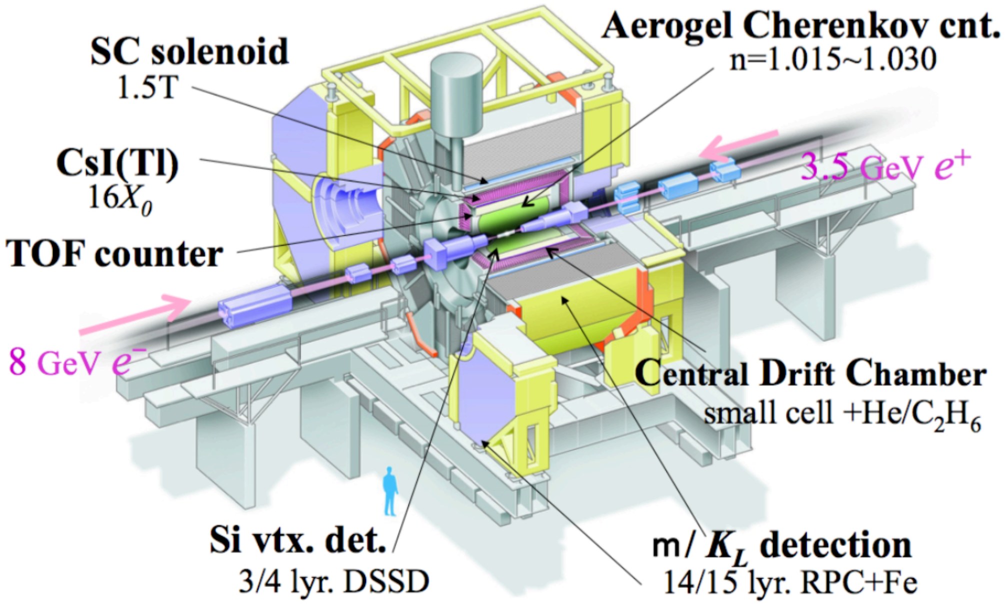
$\Longrightarrow$ measure Flatte parameter $g_{\eta\pi^\pm}$ in $D^0 \to K^-\pi^+\eta$

$$\mathcal{A}_0(K^\mp\pi^\pm\eta|a_0(980)^\pm) = \frac{1}{M^2_{a_0(980)^\pm} - m^2_\pm - i(g^2_{\eta\pi^\pm}\rho_{\eta\pi^\pm} + g^2_{K^0K^\pm}\rho_{K^0K^\pm})}$$

Belle Detector at KEKB

- KEKB: asymmetry electron-positron collider at KEK
- Word record: highest peak luminosity 2.1×10^{34} $cm^{-2}s^{-1}$
- √s = 10.58 GeV ⟹ B-Factory, charm Factory $\sigma_{b\bar{b}}$=1.1 nb; $\sigma_{c\bar{c}}$=1.3 nb



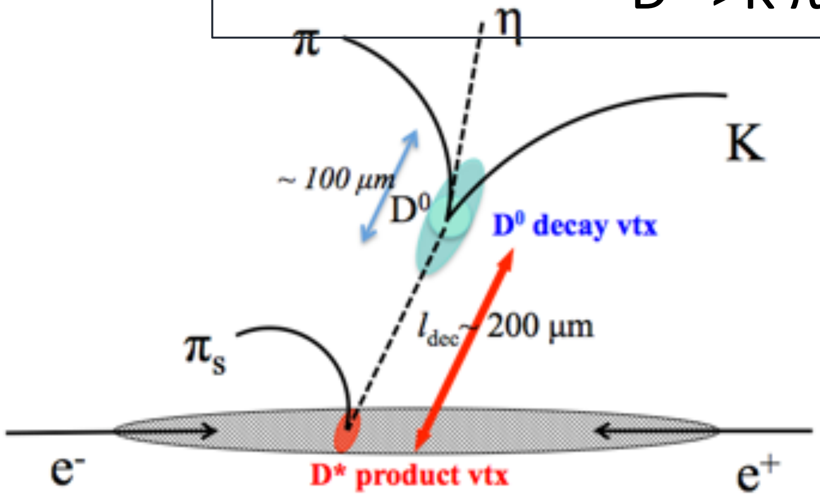


Selection & Reconstruction

Data sample($953\ fb^{-1}$)
Υ(nS)(n=1, 2, 3),
continuum, Υ(4S), Υ(5S)

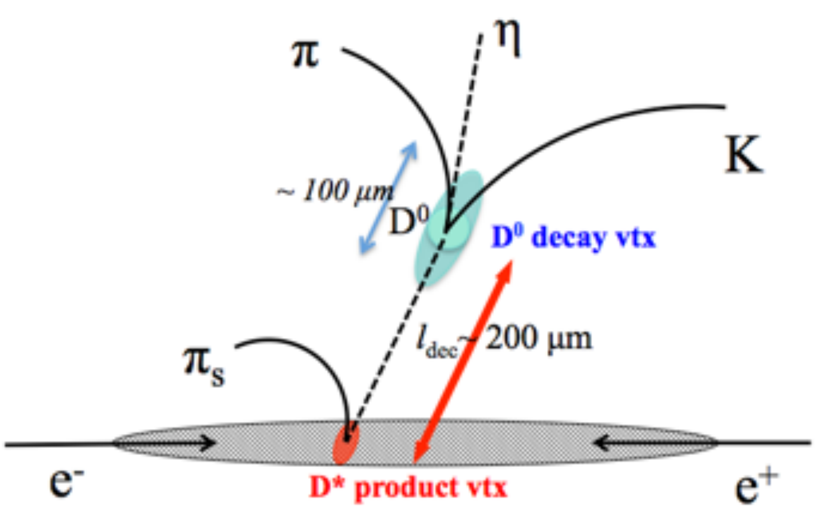
- charged track:
  - PID:LK=p(K)/(p(K)+p(π))>0.7 for K;others for π
    μId < 0.95 and eId < 0.95
  - ≥ 2 SVD hit(charged particle of D^0 in rϕ and z planes)
  - Nchg ≥ 3; N(π) ≥ 2, N(K) ≥ 1
- gamma candidates
  - E(γ) > 60/120 MeV for barrel/end-cup
  - e9/e25 > 0.8, N(γ) ≥ 2
- η candidates
  - $-60 < M_{\gamma\gamma} - M_{\eta} < 50$ MeV
  - mass constraint fit with $\chi^2 < 50$
  - $|\cos\theta_{\eta}(\text{helicity})| < 0.8$
  - $p_{\eta} > 1.0$ GeV/c

$e^+e^- \to \gamma^* \to D^{*+}$+Anything
$D^{*+} \to D^0 \pi_s^+$
$D^0 \to K^-\pi^+\eta$



Selection & Reconstruction

- ➢ D^0/D^* candidates
  - ✧ $1.72 < M(M_{D^0}) < 2.0$ GeV/c^2
  - ✧ $-3 < Q(M_{D^*}-M_{D^0}-m_{\pi s}) < 21$ MeV/c^2
  - ✧ D^0 flavour tagged by π_s charge
  - ✧ D^0 decay vertex fit by $K\pi$, add P_η to D^0 for right momentum
  - ✧ D^0 product vertex (D^*) fit by D^0 and IP
  - ✧ refit D^* vertex with D^0 product vertex and π_s
  - ✧ sum three vertex fit qualities $\Sigma\chi^2/\Sigma ndf < 20$
  - ✧ D^0 lifetime error $\sigma_t < 800$ fs
  - ✧ $p^*(D^*) > 2.4/2.5/3.1$ GeV/c for Υ(nS)+continuum/ Υ(4S)/ Υ(5S) to suppress combinatorial background or veto D^* signals from B decay



- ➢ multi-candidates:
  - ✧ Best Candidate Selection(BCS) by smallest vertex fitting quality $\sum\chi^2/\sum ndf$
- ➢ Dalitz plot variables
  - ✧ After D^0 mass constraint fit

Figure of Merit

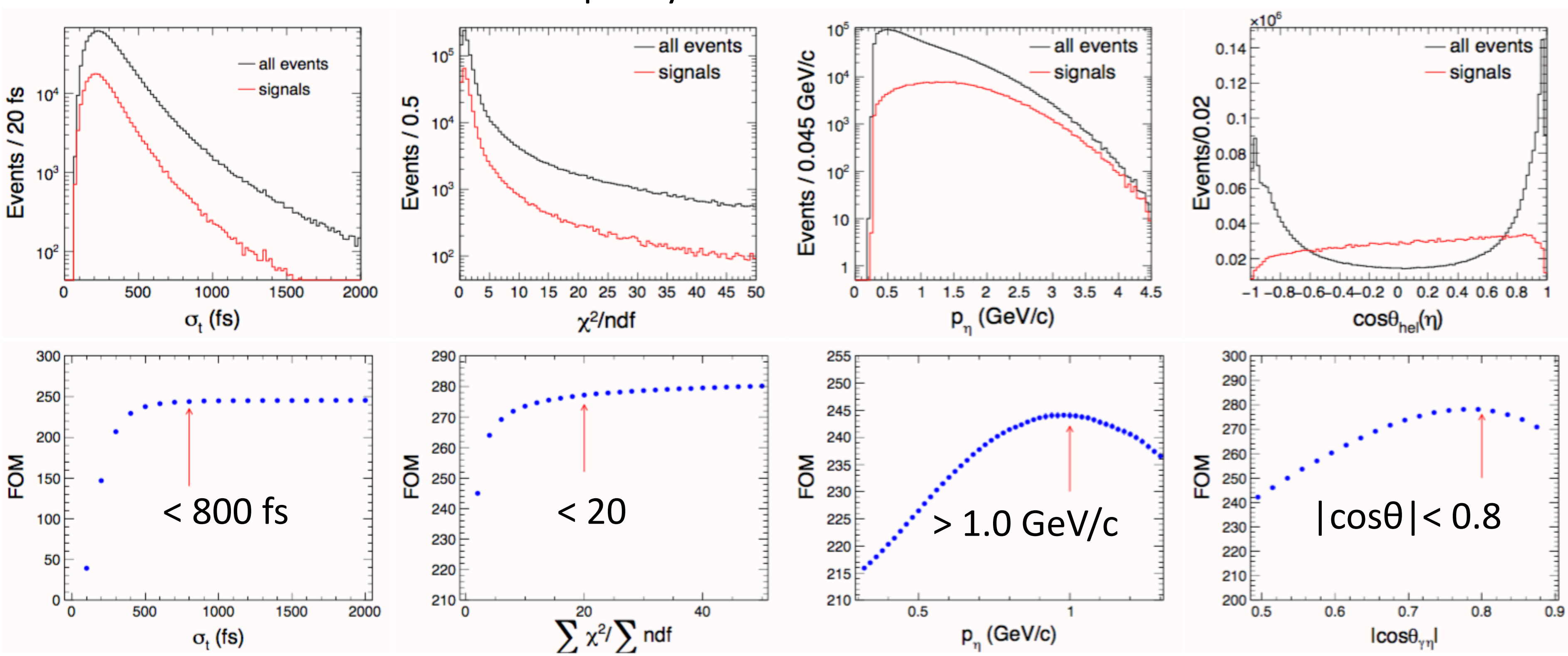
$$FOM = \frac{N_s}{\sqrt{N_s + N_{bg}}}$$

lifetime error of D^0

sum three vertex fit quality

momentum of η

cosine of helicity angle

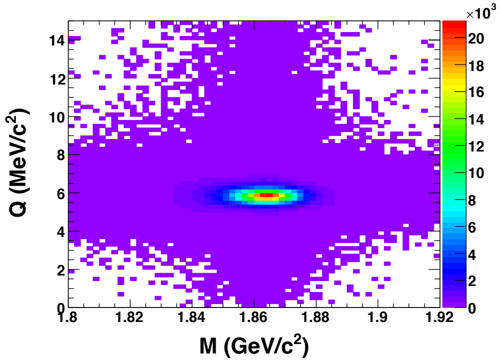


M-Q distribution

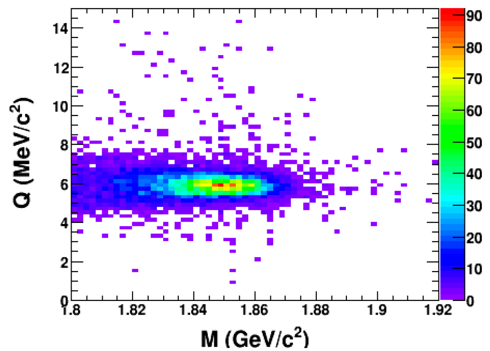
M: $M_{K\pi\eta}$

Q: $M_{k\pi\eta\pi s}-M_{k\pi\eta}-m_{\pi s}$

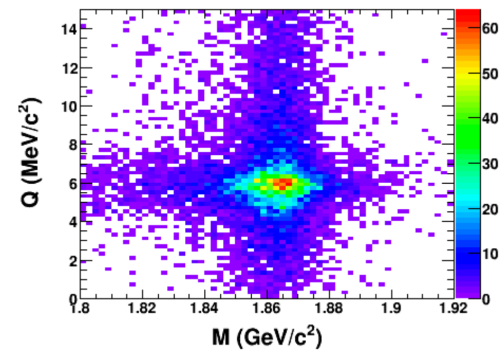
- Signal categories of generic MC



All signals

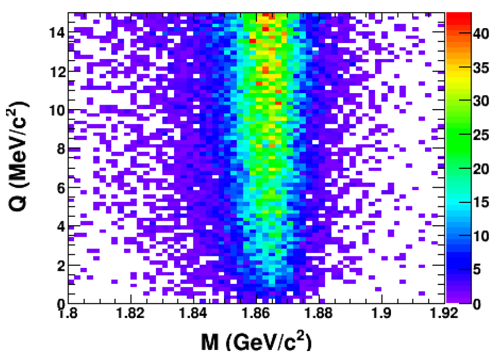


FSR

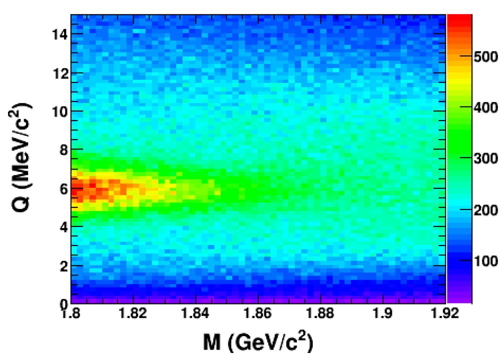


π-> $\mu\nu$

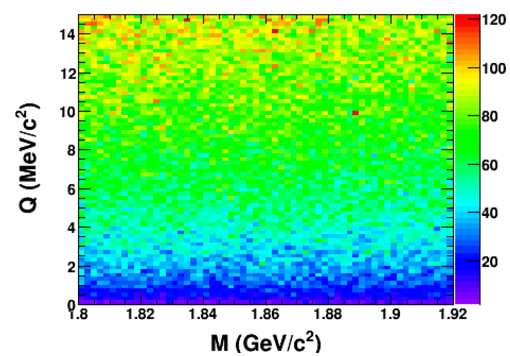
- Background categories of generic MC



random π_s: D^0 √, π_s ×



correlated combinatorial: D^0 ×, π_s √



other combinatorial D^0 ×, π_s ×

M-Q projection

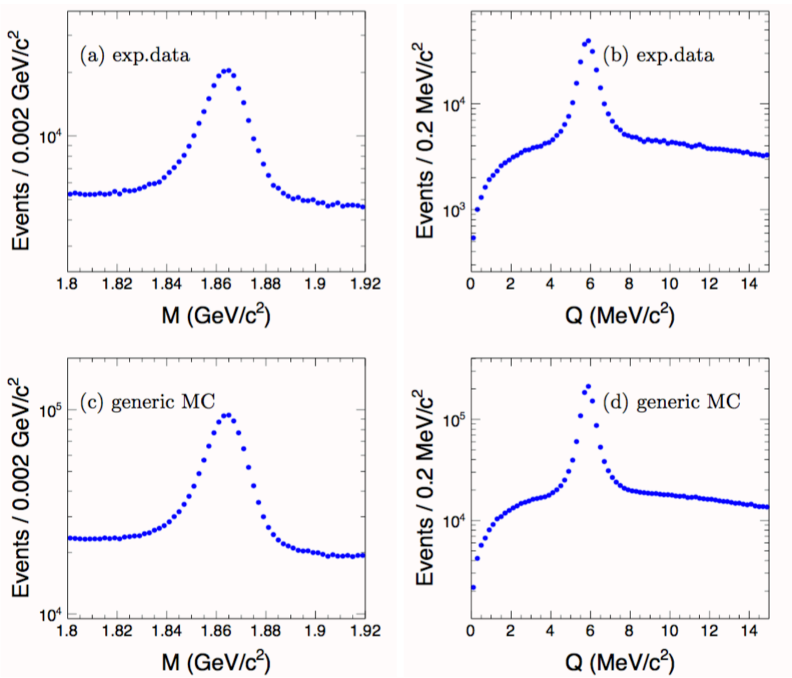


Fig. 6. The projections of M (left) and Q (right) of real data (a-b) and of four streams generic MC (c-d) in fit region $1.80 < M < 1.92$ GeV/c^2 and $0 < Q < 15$ MeV/c^2.

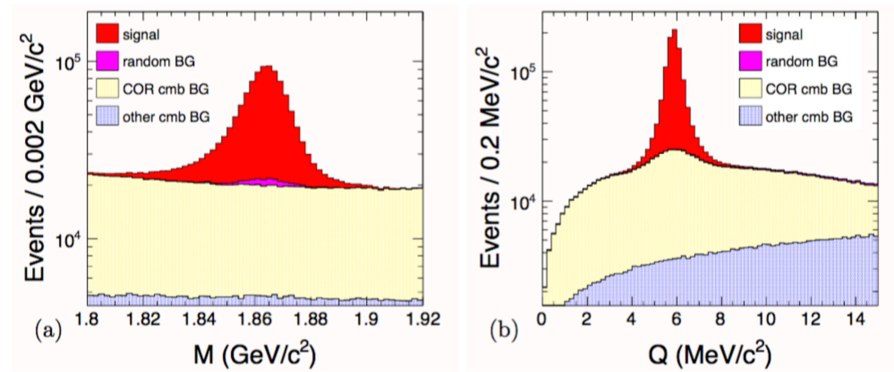


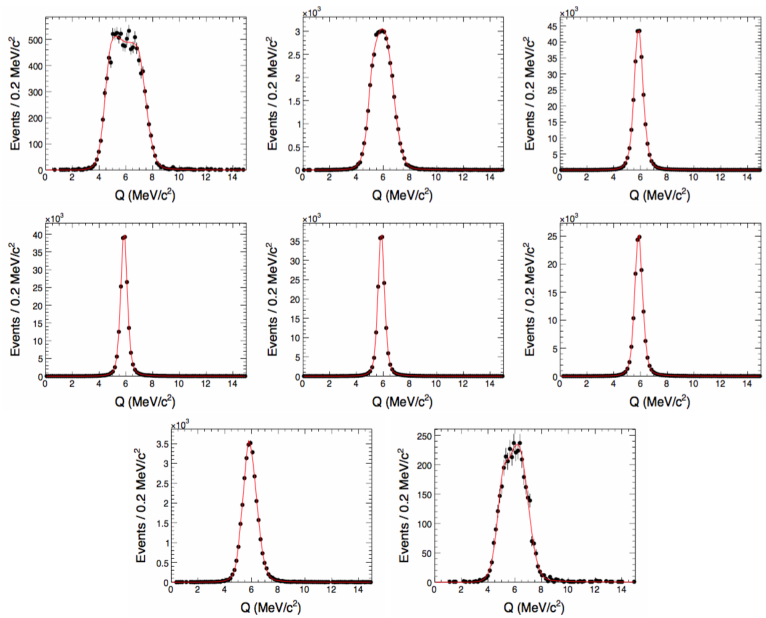
Fig. 9. The projections of M and Q of four streams generic MC with different processed components in fit region $1.80 < M < 1.92$ GeV/c^2 and $0 < Q < 15$ MeV/c^2.

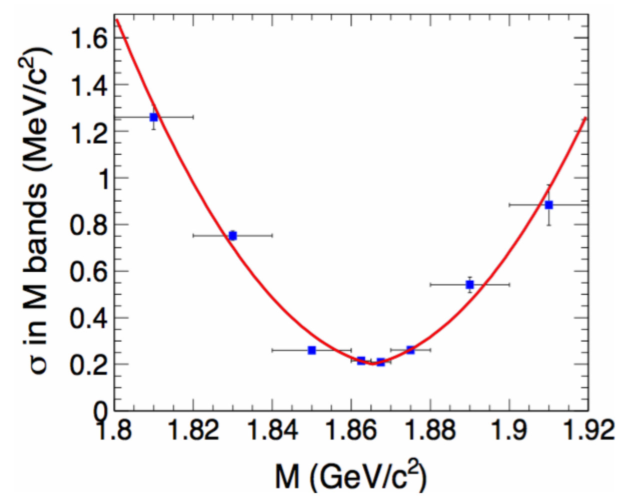
category	fit region	signal region
total signal	38.44%	95.00%
random π_s BG	0.93%	0.18%
all combinatorial BG	60.63%	4.82%
signal pure / All signal	97.87%	99.25%
signal w/ FSR / All signal	0.93%	0.33%
signal w/ π_s decay/ All signal	1.20%	0.42%
rnd BG / All signal	2.362%	0.187%
COR cmb BG / All cmb BG	76.97%	83.76%
other cmb BG / All cmb BG	23.03%	16.24%

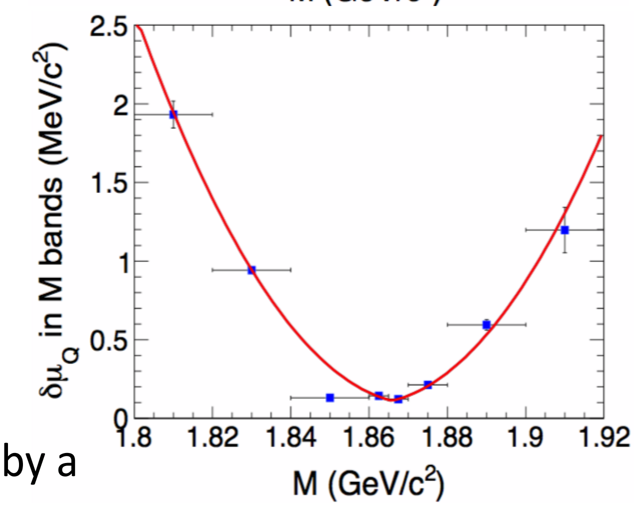
M-Q correlation of signal

- fit Q in different M slices

bifurcated Gaussian+bifurcated Student







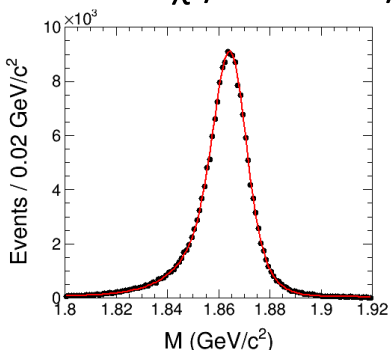
Correlation btw. $\sigma_{effective}$ and M, (μ_G-μ_{Stu}) and M by a 2-order polynomial function of $|M-\mu|$

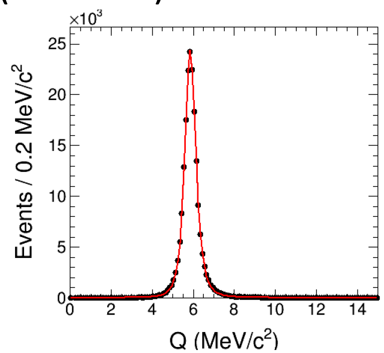
M-Q fit for signal

- M: doubly Gaussian + doubly bifurcated Gaussian
  Q: bifurcated Student+Gaussian+Cruijff

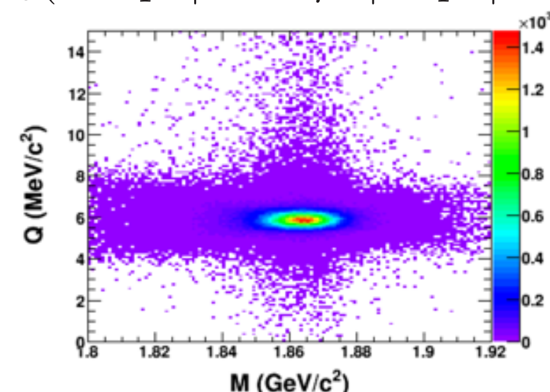
fit region(F.R.): M:1.80-1.92GeV/c^2, Q: 0-15 MeV/c^2

χ^2/ndf=4538/(5823-30)=0.78

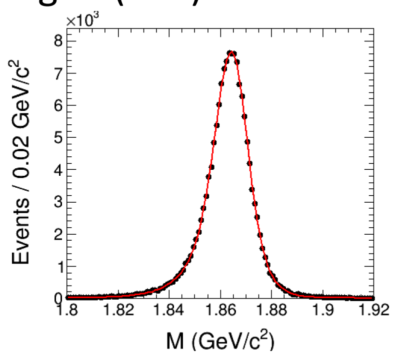


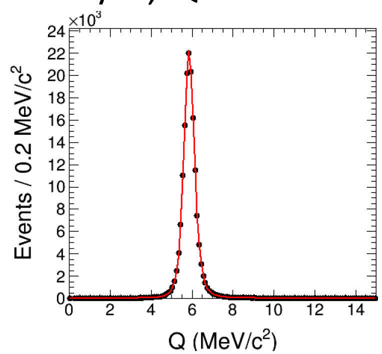


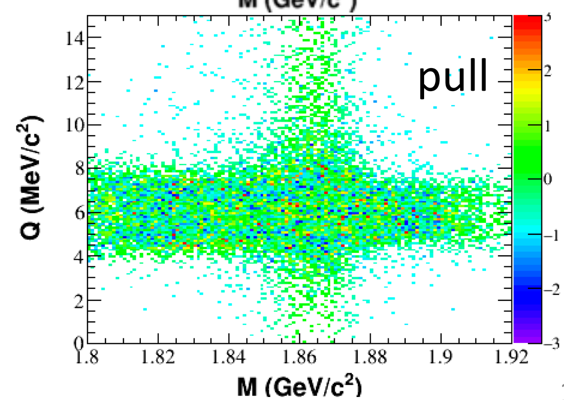
$$\mu_{cf/s} = \mu_g \pm (c_1|M-\mu_1| + c_2|M-\mu_1|^2)$$
$$\sigma_s = \sigma_{s0}(1 + p_1|M-\mu_2| + p_2|M-\mu_2|^2)$$



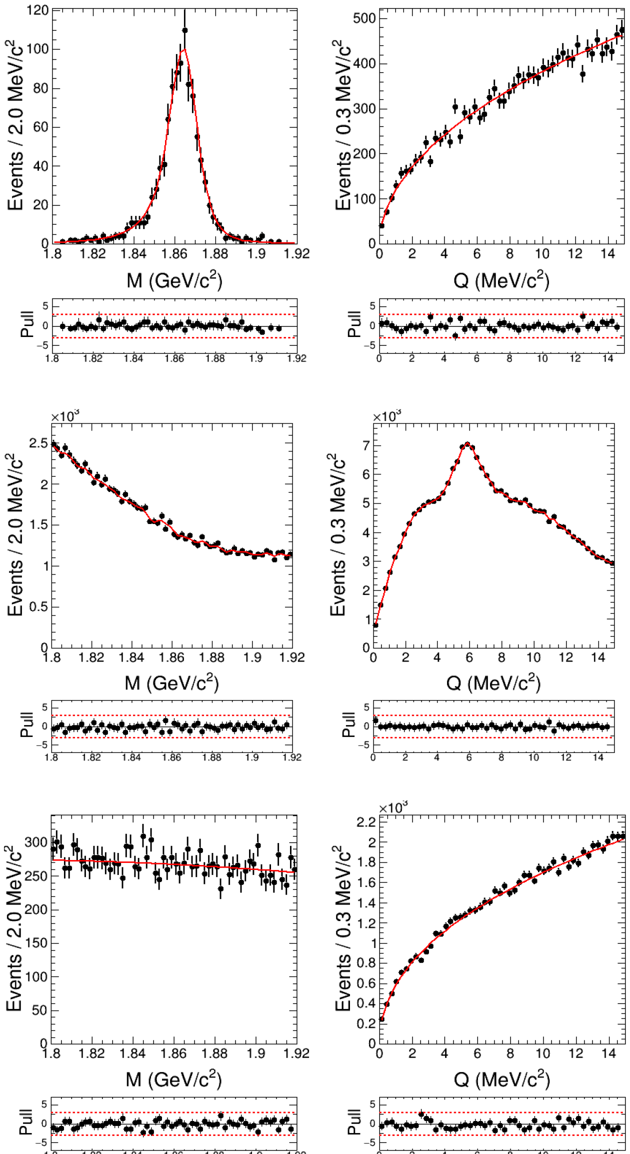
signal region(S.R.): M:1.85-1.88GeV/c^2, Q: 5.35-6.35 MeV/c^2







M-Q fit for bkg in S.R.



- random π_s bkg: M function used from signal

$$F_{rnd}(M,Q) = F_{sig}(M) \times (Q^{\alpha} \cdot e^{-\beta \cdot Q})$$

- correlated combinatorial bkg: bilinear interpolation

- other combinatorial bkg

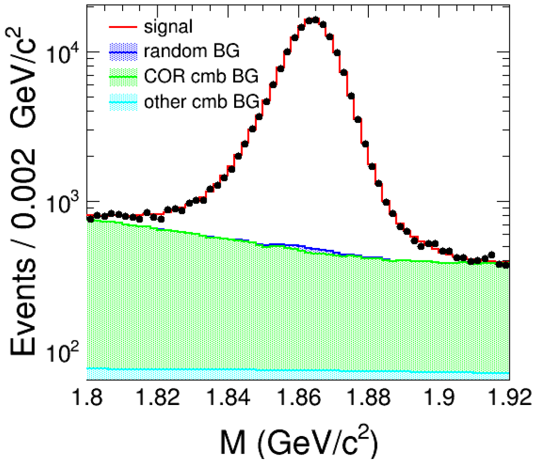
$$F_{cmb}(M,Q) = P_3(m;a,b,c) \times (Q^{\alpha} e^{-\beta \cdot Q})$$

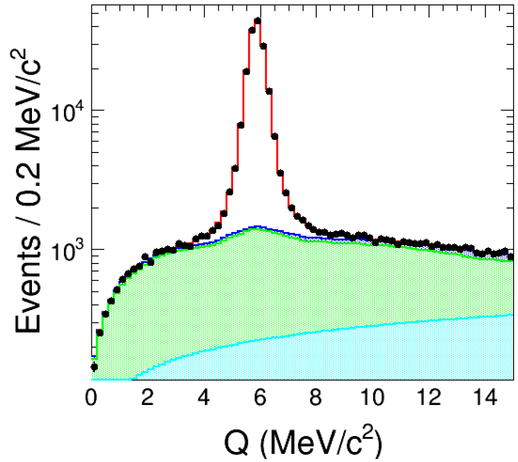
I/O check for M-Q fit

- generic MC samples

SR	sig+rnd count(purity)	sig+rnd fit(purity)	cmb Count	cmb fit
stream 0	136118(95.18%)	136619±504(95.54%)	6892	6828±108
stream 1	136829(95.21%)	137199±524(95.47%)	6878	6833±108
stream 2	135781(95.17%)	136238±533(95.49%)	6895	6810±109
stream 3	136981(95.18%)	137435±506(95.49%)	6944	6815±104

- one stream generic MC combined fit in signal region



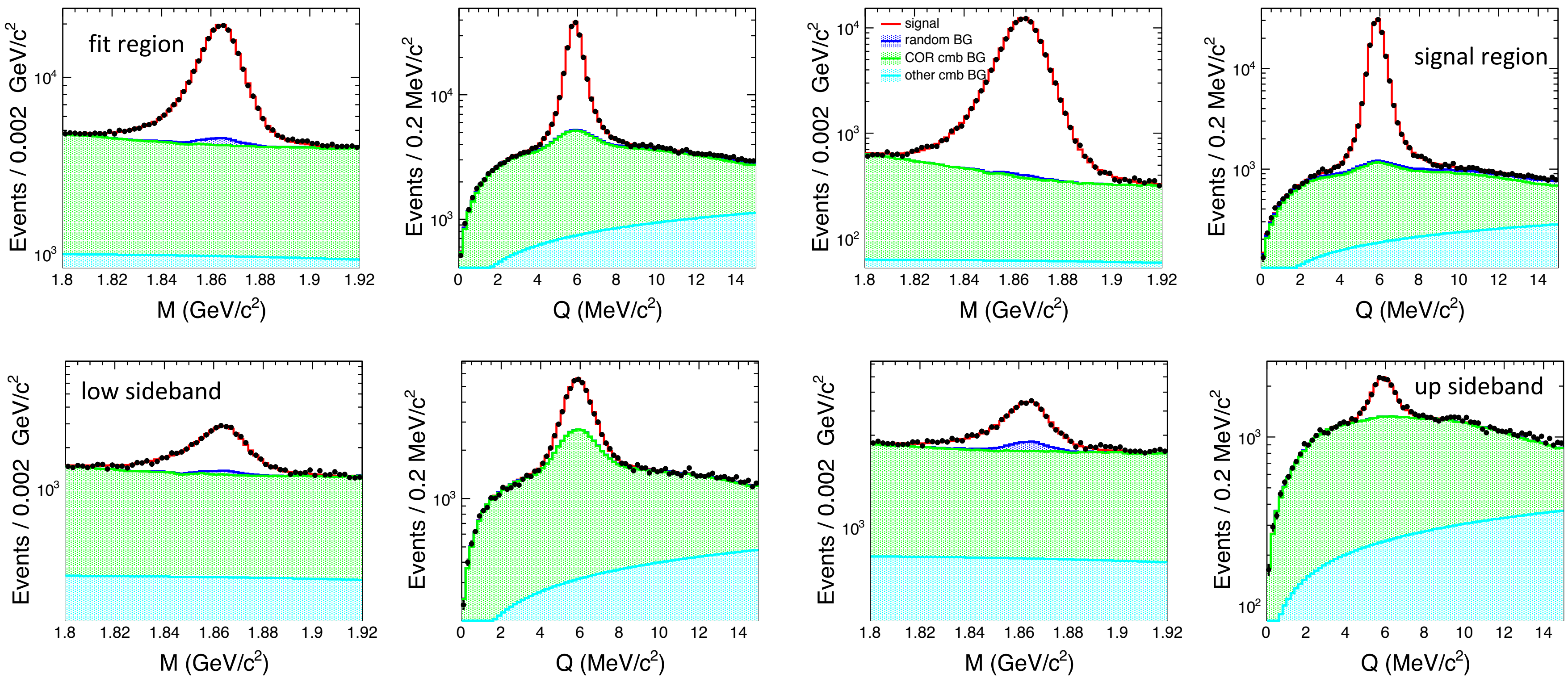


M-Q fit and purity

- M-Q fit for experimental data with high purity 95.32% in signal region

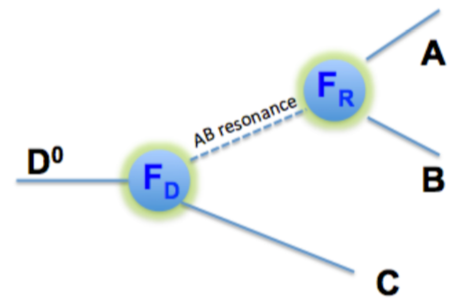
S.R.	Fit
purity	95.32%
signal+rnd	106312±914
cmb bkg	5714±102

$\chi^2/ndf=4668/(4500-32)=1.0$



Dalitz analysis formalism

- Dalitz standard form: $d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{M}|^2 dm_{AB}^2 dm_{BC}^2$
- Isobar model: $\mathcal{M}(m_{AB}^2, m_{BC}^2) = a_{NR}e^{i\phi_{NR}} + \sum_r a_r e^{i\phi_r}\mathcal{A}_r(m_{AB}^2, m_{BC}^2)$
- A_r: $\mathcal{A}(ABC|r) = F_D \times F_r \times T_r \times \Omega_J$
  - form factor: F_r, F_D
  - angular distribution function Ω_J
  - dynamical function T_r



$$\begin{cases} J=0 \implies & F = 1; \\ J=1 \implies & F = \frac{\sqrt{1+R^2p_r^2}}{\sqrt{1+R^2p_{AB}^2}}; \\ J=2 \implies & F = \frac{\sqrt{9+3R^2p_r^2+R^4p_r^4}}{\sqrt{9+3R^2p_{AB}^2+R^4p_{AB}^4}} \end{cases}$$

$$\begin{cases} \Omega_0(ABC|r) = 1. \\ \Omega_1(ABC|r) = m_{AC}^2 - m_{BC}^2 + \frac{(M_D^2-M_C^2)(M_B^2-M_A^2)}{M_r^2}. \\ \Omega_2(ABC|r) = \left(m_{BC}^2 - m_{AC}^2 + \frac{(M_D^2-M_C^2)(M_A^2-M_B^2)}{M_r^2}\right)^2 \end{cases}$$
$$-\frac{1}{3}\left(m_{AB}^2 - 2M_D^2 - 2M_C^2 + \frac{(M_D^2-M_C^2)^2}{M_r^2}\right) \times \left(m_{AB}^2 - 2M_A^2 - 2M_B^2 + \frac{(M_A^2-M_B^2)^2}{M_r^2}\right)$$

$$\begin{cases} T_r = \frac{1}{M_r^2 - m_{AB}^2 - iM_r\Gamma_{AB}} \quad \Gamma_{AB} = \Gamma_0^r\left(\frac{p_{AB}}{p_r}\right)^{2J+1}\left(\frac{M_r}{m_{AB}}\right)F_r^2 \\ \mathcal{A}_0(K^{\mp}\pi^{\pm}\eta|a_0(980)^{\pm}) = \frac{1}{M_{a_0(980)^{\pm}}^2 - m_{\pm}^2 - i(g_{\eta\pi^{\pm}}^2\rho_{\eta\pi^{\pm}} + g_{K^0K^{\pm}}^2\rho_{K^0K^{\pm}})} \\ \mathcal{A}_{K\pi,L=0}(m_{ab}^2, m_{bc}^2) = \frac{B\cdot\sin(\delta_B+\phi_B)e^{i(\delta_B+\phi_B)} + R\cdot\sin\delta_R e^{i(\delta_R+\phi_R)}e^{i\cdot 2(\delta_B+\phi_B)}}{\rho(m_{K\pi}^2)} \end{cases}$$

Dalitz analysis formalism

- Isobar model:

$$\mathcal{M}(m^2_{AB}, m^2_{BC}) = a_{NR}e^{i\phi_{NR}} + \sum_r a_r e^{i\phi_r}\mathcal{A}_r(m^2_{AB}, m^2_{BC})$$

- Signal p.d.f of DP (corrected by efficiency plane)

$$p_{sig}(m^2_{K\pi,i}, m^2_{\pi\eta,i}) = \frac{\sum_{j=con,4S,5S}|\mathcal{M}(m^2_{K\pi,i}, m^2_{\pi\eta,i})|^2\epsilon_j(m^2_{K\pi,i}, m^2_{\pi\eta,i})}{\sum_{j=con,4S,5S}\iint_{DP} dm^2_{K\pi}dm^2_{\pi\eta}|\mathcal{M}(m^2_{K\pi}, m^2_{\pi\eta})|^2\epsilon_j(m^2_{K\pi}, m^2_{\pi\eta})}$$

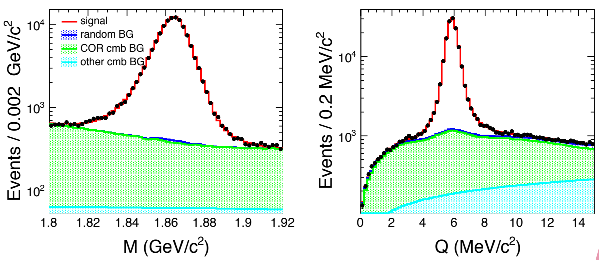
- An un-binned maximum likelihood:
  (fraction extracted from M-Q fit)

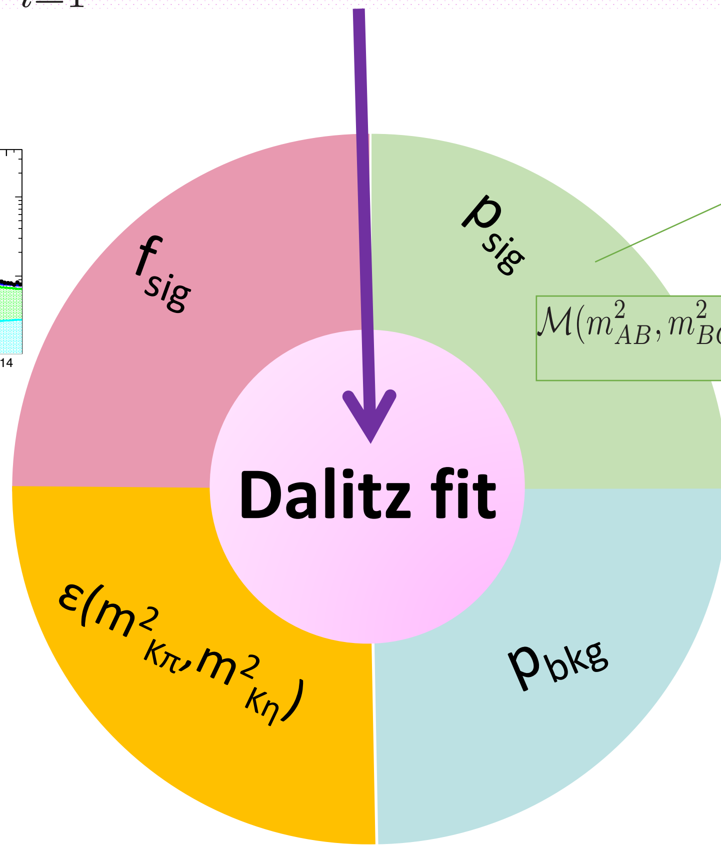
$$-2ln\mathcal{L}(m^2_{AB}, m^2_{BC}) = -2\sum_{i=1}^{n} ln[f^i_{sig}p_{sig}(m^2_{AB,i}, m^2_{BC,i}) + f^i_{bkg}p_{bkg}(m^2_{AB,i}, m^2_{BC,i})]$$

Generalization of Dalitz fit

$$-2ln\mathcal{L}(m^2_{AB},m^2_{BC}) = -2\sum_{i=1}^{n} ln[f^i_{sig}p_{sig}(m^2_{AB,i},m^2_{BC,i}) + f^i_{bkg}p_{bkg}(m^2_{AB,i},m^2_{BC,i})]$$

M-Q 2D fit

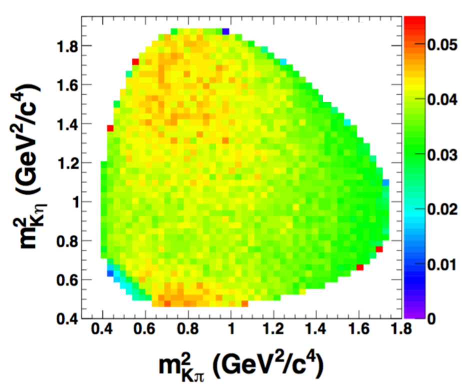




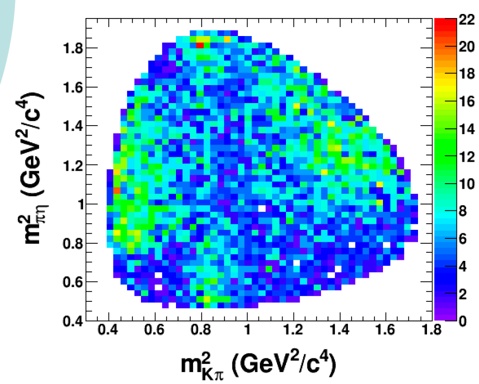
Isobar Model

$$\mathcal{M}(m^2_{AB},m^2_{BC}) = a_{NR}e^{i\phi_{NR}} + \sum_r a_r e^{i\phi_r}\mathcal{A}_r(m^2_{AB},m^2_{BC})$$

Efficiency plane



Combinatorial BG

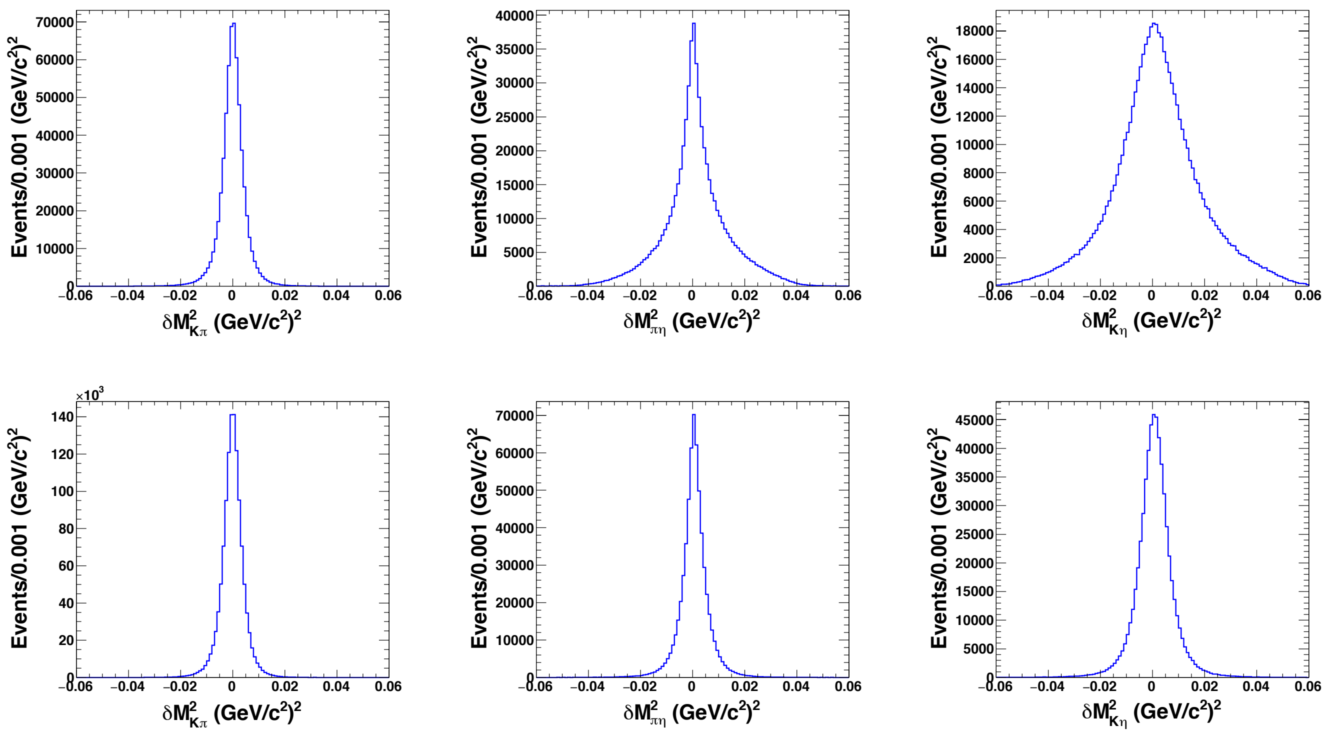


Mass resolution

- Dalitz variables with D^0 mass constraint fit in M-Q S.R.

before

after

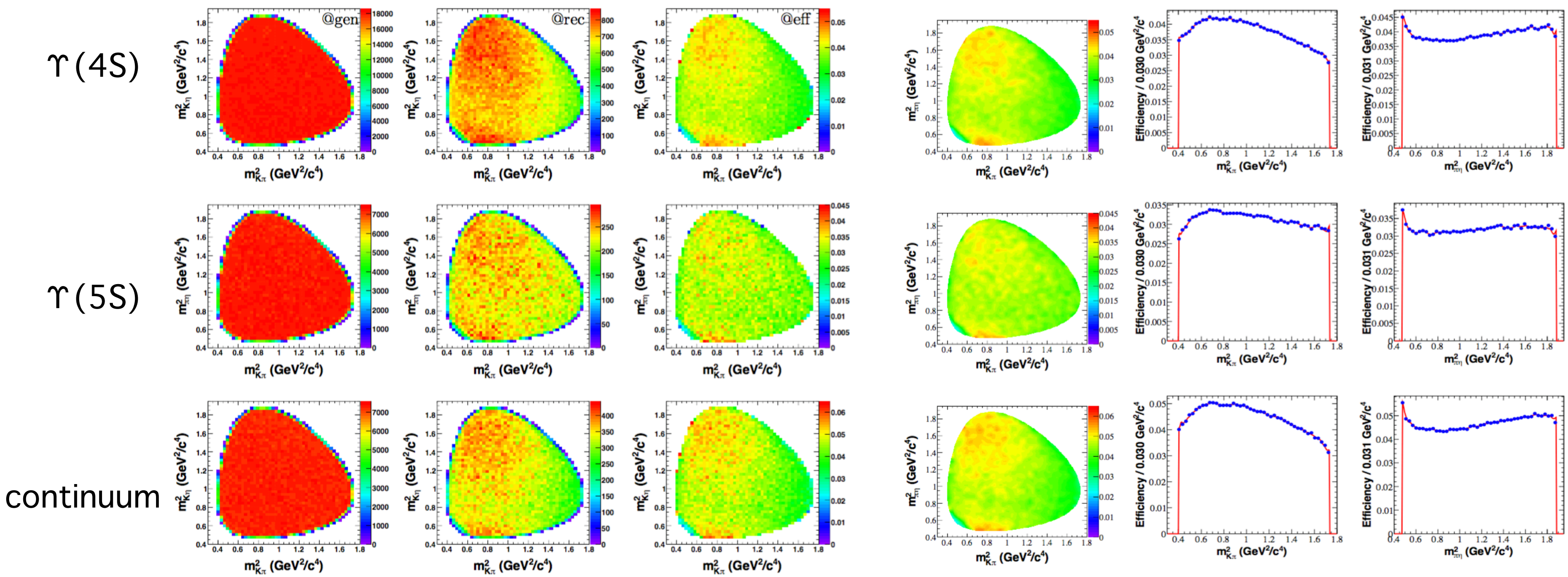


Res=0.005 << $2m\Gamma(K^*(892))$=0.084 $(GeV/c^2)^2$

So don't consider mass resolution effect.

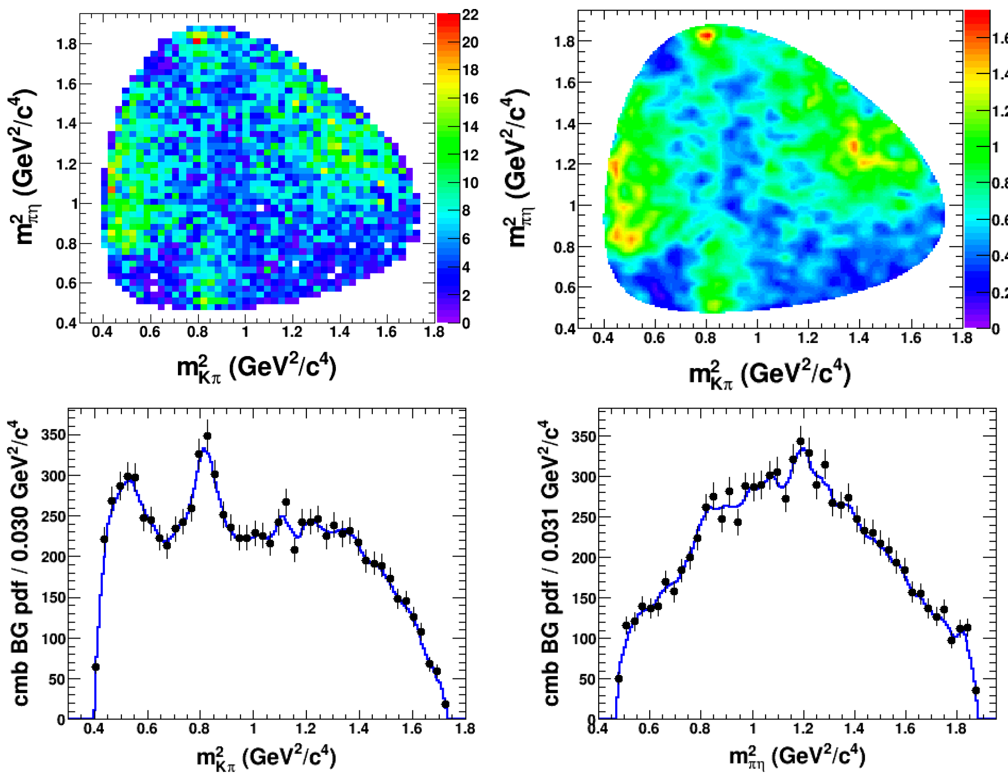
Efficiency plane

- Signal MC samples produced at PHSP by Evtgen and Gsim
- obtain efficiency plane via bilinear interpolation



Combinatorial BG of DP

- S.B.: M sideband (1.755<M<1.775 or 1.935<M<1.955 GeV/c^2) with Q signal region (5.35<Q<6.35 MeV/c^2)
- directly use DP background of real data in S.B. to estimate background of real data in S.R.

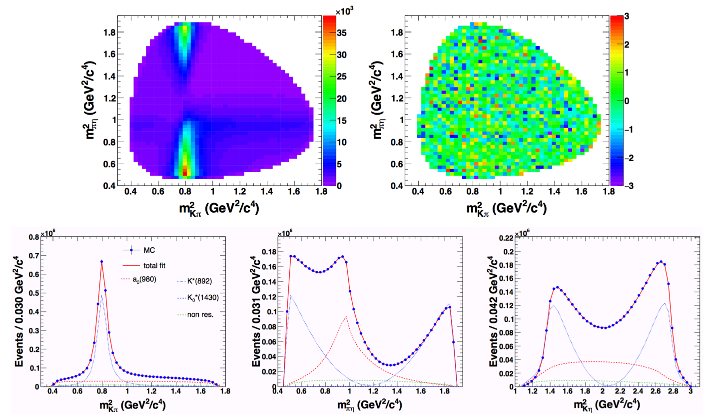


MC test on 3 res

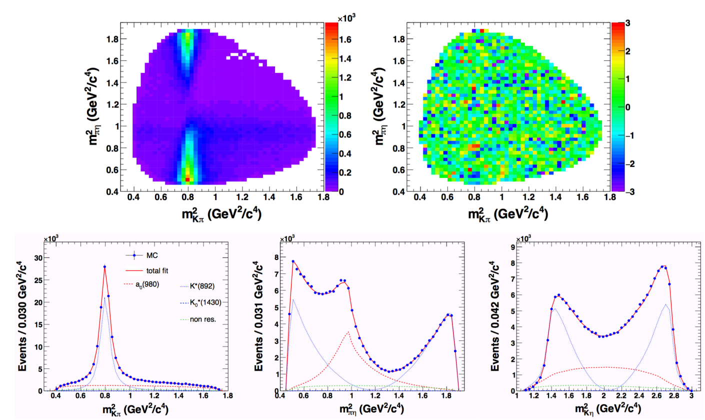
- Multiparameter fit, check unbiasedness
- MC test at generator level and reconstruction level

Resonance	data	Amplitude	Phase(°)	Fraction(%)
$K^*(892)^0$	Input	1.0(fix)	0.0(fix)	55.34
	Gen.ed	1.0(fix)	0.0(fix)	55.20 ± 0.15
	Rec.ed	1.0(fix)	0.0(fix)	54.87 ± 0.79
$a_0(980)^\pm$	Input	2.000	80.0	30.90
	Gen.ed	2.000 ± 0.005	80.2 ± 0.1	30.90 ± 0.15
	Rec.ed	2.032 ± 0.025	80.6 ± 0.5	31.46 ± 0.78
	$g_{\pi\eta}$ (GeV/c^2)	0.6000 0.5995 ± 0.0007 0.6015 ± 0.0033		
$K_0^*(1430)^0$	Input	0.500	160.0	0.35
	Gen.ed	0.488 ± 0.011	160.8 ± 1.1	0.34 ± 0.02
	Rec.ed	0.493 ± 0.060	165.2 ± 5.8	0.34 ± 0.08
non-res.	Input	2.500	130.0	7.78
	Gen.ed	2.529 ± 0.010	130.2 ± 0.3	7.94 ± 0.06
	Rec.ed	2.491 ± 0.052	129.6 ± 1.4	7.66 ± 0.32
Total fraction	Input	94.37%		
	Gen.ed	$(94.37 \pm 0.22)\%$		
	Rec.ed	$(94.32 \pm 1.12)\%$		

$\chi^2/ndf=1660/(1621-7)= 1.0$



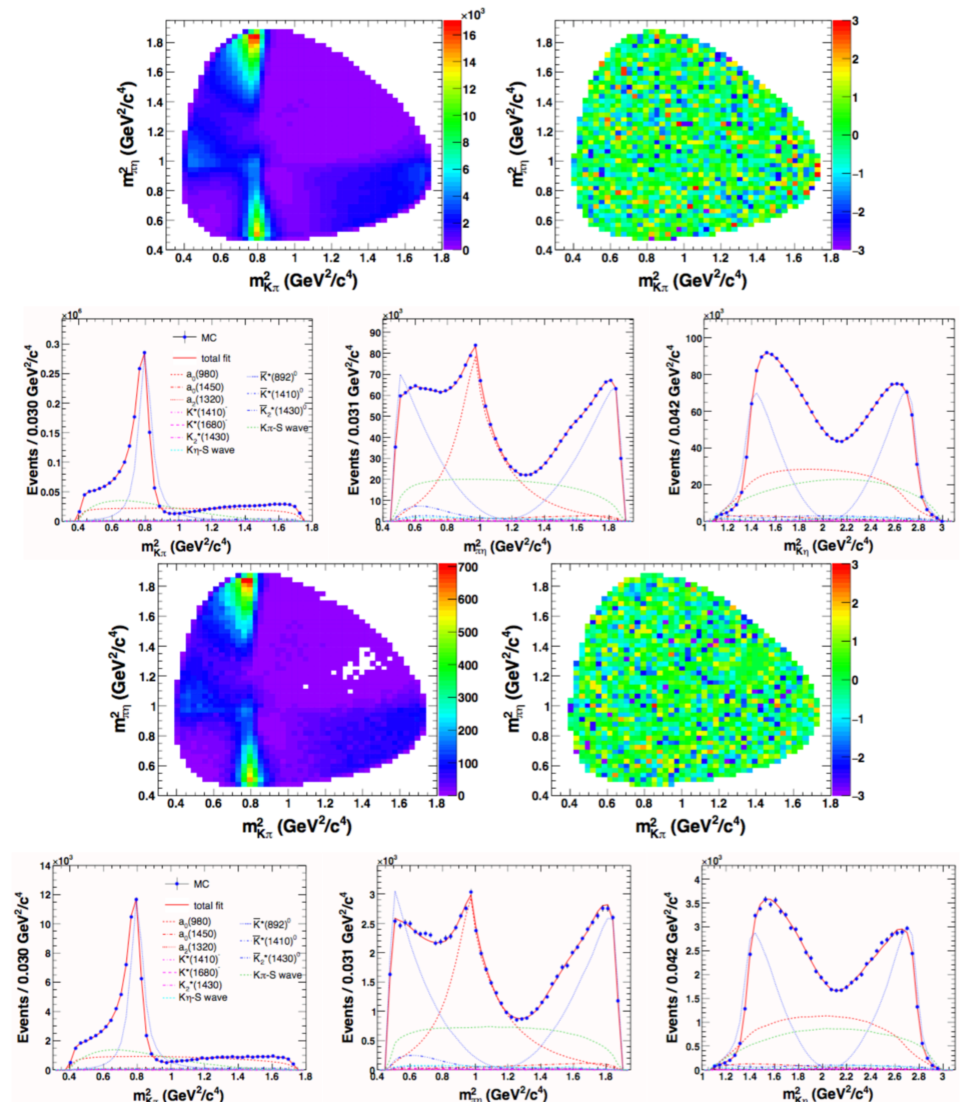
$\chi^2/ndf=19401/(1592-7)= 1.2$



The output values are consistent with input values

MC test on 11 res

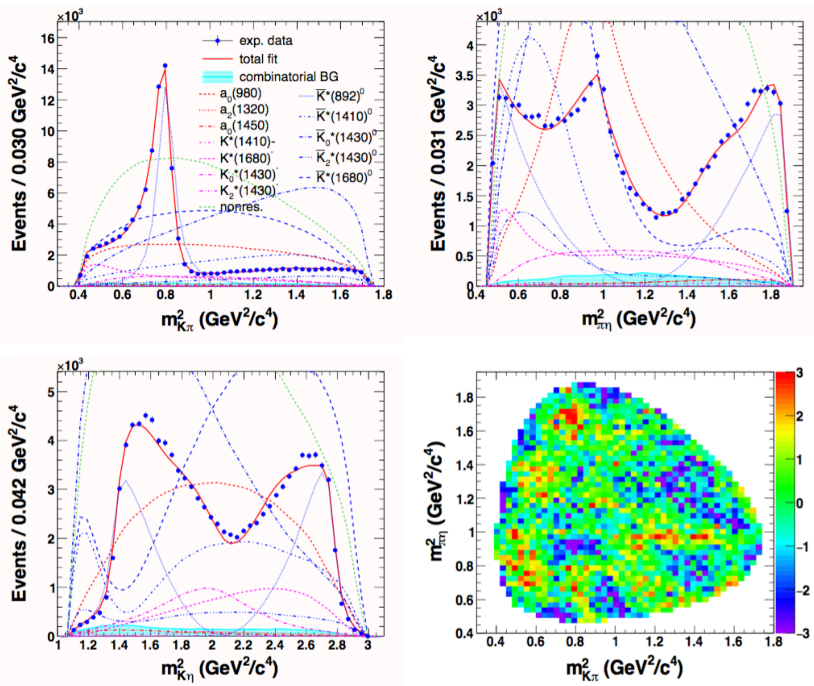
Resonance	data	Amplitude	Phase(°)	Fraction(%)
$\bar{K}^*(892)^0$	Input	1.0(fix)	0.0(fix)	43.49
	Gen.ed	1.0(fix)	0.0(fix)	43.34 ± 0.36
	Rec.ed	1.0(fix)	0.0(fix)	43.54 ± 1.92
$a_0(980)^\pm$	Input	2.160	299.2	32.37
	Gen.ed	2.153 ± 0.010	299.3 ± 0.2	32.06 ± 0.31
	Rec.ed	2.224 ± 0.055	300.2 ± 0.9	34.01 ± 1.67
	$g_{\pi\eta}$ (GeV/c^2)	0.565		
		0.565 ± 0.001		
		0.568 ± 0.006		
$a_2(1320)$	Input	0.929	239.7	0.55
	Gen.ed	0.955 ± 0.014	238.0 ± 0.9	0.58 ± 0.02
	Rec.ed	0.845 ± 0.070	242.3 ± 5.1	0.46 ± 0.08
$a_0(1450)$	Input	1.504	152.0	2.41
	Gen.ed	1.554 ± 0.023	152.9 ± 0.8	2.57 ± 0.08
	Rec.ed	1.591 ± 0.125	161.4 ± 4.1	2.70 ± 0.43
$K^*(1410)^-$	Input	0.356	325.5	0.17
	Gen.ed	0.364 ± 0.008	324.2 ± 1.3	0.17 ± 0.01
	Rec.ed	0.425 ± 0.042	328.5 ± 5.9	0.24 ± 0.05
$\bar{K}^*(1410)^0$	Input	3.068	2.7	1.54
	Gen.ed	3.014 ± 0.026	1.9 ± 0.4	1.48 ± 0.03
	Rec.ed	2.508 ± 0.140	5.0 ± 2.7	1.03 ± 0.11
$K^*(1680)^-$	Input	2.433	255.6	0.49
	Gen.ed	2.534 ± 0.047	249.6 ± 1.3	0.53 ± 0.02
	Rec.ed	2.854 ± 0.241	260.9 ± 6.2	0.67 ± 0.11
$\bar{K}_0^*(1430)^0$	Input	7.032	69.3	27.70
	Gen.ed	6.996 ± 0.018	69.5 ± 0.2	27.32 ± 0.14
	Rec.ed	6.892 ± 0.096	69.2 ± 0.9	26.64 ± 0.74
$K_0^*(1430)^-$	Input	1.494	4.5	2.09
	Gen.ed	1.497 ± 0.006	4.7 ± 0.2	2.09 ± 0.02
	Rec.ed	1.530 ± 0.033	6.5 ± 1.3	2.19 ± 0.09
$\bar{K}_2^*(1430)^0$	Input	3.403	281.7	3.16
	Gen.ed	3.761 ± 0.034	284.6 ± 0.7	3.85 ± 0.07
	Rec.ed	3.686 ± 0.186	281.7 ± 3.8	3.71 ± 0.37
$K_2^*(1430)^-$	Input	0.320	96.1	0.14
	Gen.ed	0.331 ± 0.006	97.4 ± 1.0	0.15 ± 0.01
	Rec.ed	0.321 ± 0.031	90.9 ± 5.4	0.14 ± 0.03
Total fraction	Input	114.1%		
	Gen.ed	(114.1 ± 0.5)%		
	Rec.ed	(115.3 ± 2.7)%		



Dalitz fit on exp. data

- **12 resonances + NR (BW+Flatte):**
  - All possible component

$a_0(980)$, $a_0(1320)$, $a_0(1450)$, $K^*(892)^0$, $K^*(1410)^{0,-}$, $K^*_0(1430)^{0,-}$, $K^*_2(1430)^{0,-}$, $K^*(1680)^{0,-}$, non-res



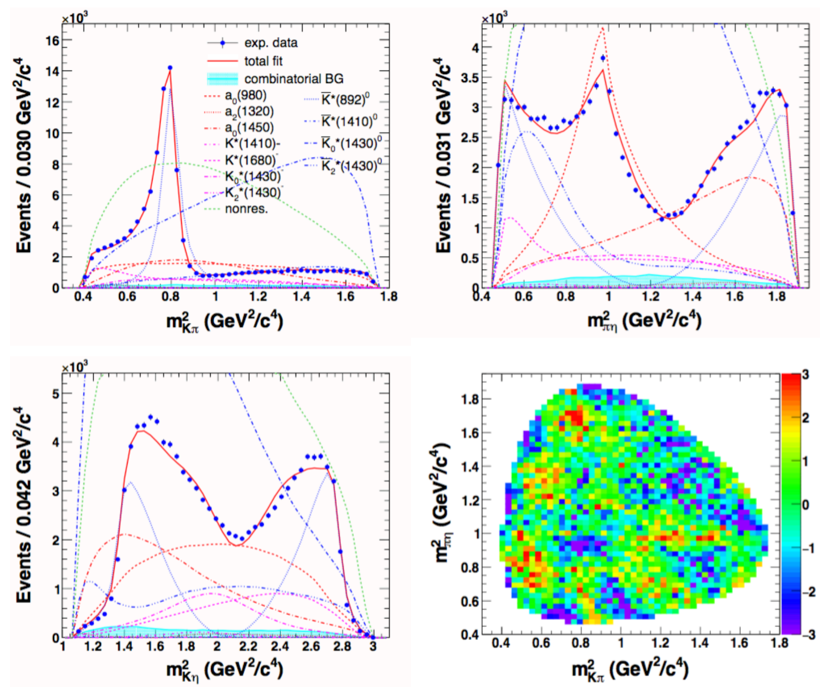
$$\chi^2/ndf = 3654/(1607-25) = 2.3$$

- **11 resonances + NR (BW+Flatte):**
  - removing $K^*(1680)^0$

$K^*(1410)^0$ VS. $K^*(1680)^0$

similar contribution,

$\Delta\phi \sim 180^\circ$ destructive interf.

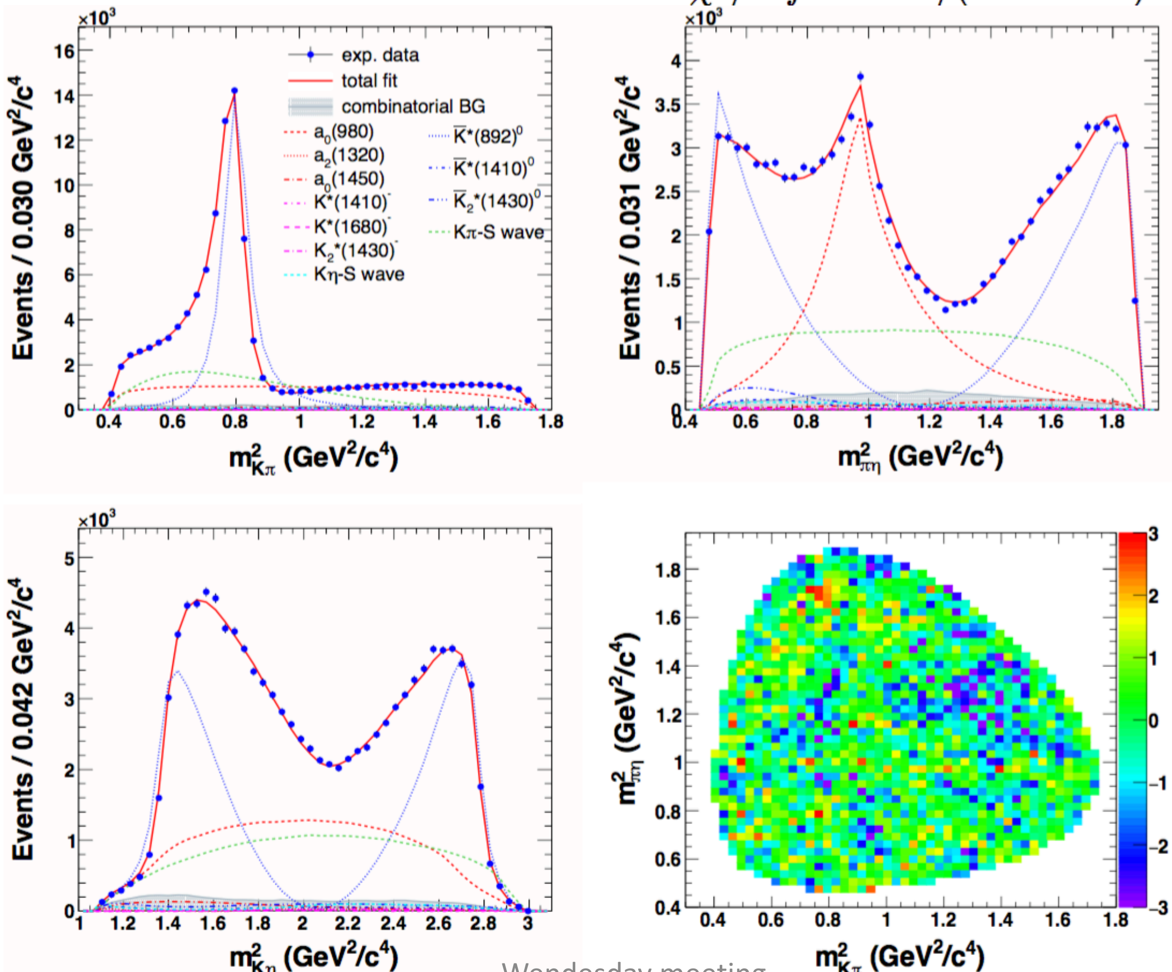


$$\chi^2/ndf = 3902/(1607-23) = 2.5$$

Dalitz fit on exp. data

- 11 resonances (BW+Flatte+**LASS**):

$$\chi^2/ndf = 2537/(1607-31) = 1.61$$



significance test

Model	Likelihood	Δndf	significance
11 res	63164	31	-
-$K^*(1410)^-$	**63109(55)**	**29**	**7.1σ**
-$K^*(1410)^0$	63099(65)	29	7.8σ
-$K^*(1680)^-$	**63113(51)**	**29**	**6.8σ**
-$K^*_0(1430)^0$	59646(3518)	24	>>10σ
-$K^*_0(1430)^-$	61490(1674)	24	>>10σ
-$K^*_2(1430)^0$	63025(139)	29	11.6σ
-$K^*_2(1430)^-$	63077(87)	29	9.1σ
-$a_2(1320)^+$	63017(147)	29	11.9σ
-$a_0(1450)^+$	63115(49)	29	6.7σ

All 11 resonances have significant contributions

fitting results

- 11 resonances (BW+Flatte+**LASS**):

Resonances	Amplitude	Phase(°)	Fraction(%)
$\bar{K}^*(892)^0$	1.0(fix)	0.0(fix)	43.49 ± 2.85
$a_0(980)^+$	2.160 ± 0.058	299.2 ± 1.1	32.37 ± 1.74
$a_2(1320)^+$	0.929 ± 0.064	239.7 ± 4.7	0.55 ± 0.08
$a_0(1450)^+$	1.504 ± 0.143	152.0 ± 5.0	2.41 ± 0.46
$K^*(1410)^0$	3.068 ± 0.285	2.655 ± 3.2	1.54 ± 0.29
$K^*(1410)^-$	0.356 ± 0.039	325.5 ± 6.9	0.17 ± 0.04
$K^*(1680)^-$	2.433 ± 0.209	255.6 ± 7.1	0.49 ± 0.08
$\bar{K}_2^*(1430)^0$	3.403 ± 0.133	281.7 ± 5.2	3.16 ± 0.25
$K_2^*(1430)^-$	0.320 ± 0.031	96.1 ± 5.0	0.14 ± 0.03
$K\pi$ S-wave	7.032 ± 0.276	69.3 ± 2.9	27.70 ± 2.17
$K\eta$ S-wave	1.494 ± 0.050	4.5 ± 9.7	2.09 ± 0.14
$\sum fraction$			114.1 ± 4.0

LASS model parameters	$K\pi$ S-wave	$K\eta$ S-wave	
F	0.549 ± 0.041	0.523 ± 0.083	
$\phi_F(°)$	177.3 ± 0.5	41.3 ± 6.1	
R	1.0(fix)	1.0(fix)	
$\phi_R(°)$	1.9 ± 3.3	69.1 ± 4.6	
a($\mathrm{GeV}^{-1}c$)	4.358 ± 0.114	0.286 ± 0.014	
r($\mathrm{GeV}^{-1}c$)	-3.554 ± 0.066	-16.24 ± 0.82	
$a_0(980)$ parameters	m (GeV/c^2)	$g_{\pi^\pm\eta}$ (GeV/c^2)	$g_{\bar{K}^0K^\pm}$ (GeV/c^2)
$a_0(980)^\pm$	0.999(fix)	0.565 ± 0.007	0.464(fix)

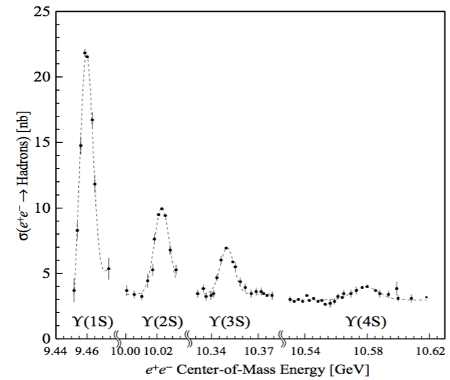
Summary

- Using Belle Υ(nS)(n=1,2,3), near or on Υ(4S) and Υ(5S) data, a Dalitz amplitude analysis for $D^0 \to K^-\pi^+\eta$ is presented.
- After cut criteria optimization, signal & bkg fractions have been extracted via M-Q fit.
- After efficiency plane obtained, combinatorial BG's DP described, we optimize Dalitz model and study the intermediate resonances and give the fit fractions.
- $K^*(1410)^- \to K\eta$, $K^*(1680)^- \to K\eta$ are firstly observed with 5σ.

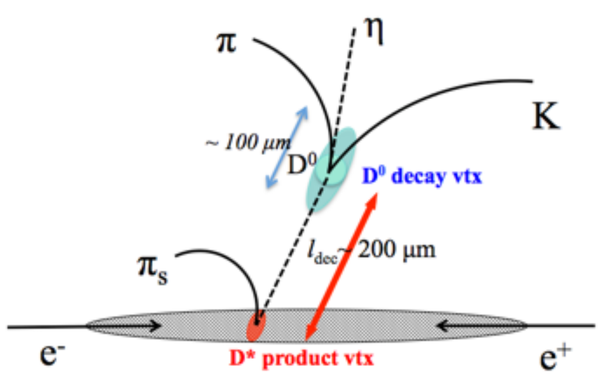
BACK UP

Data sample & Cut criteria

Data sample(953 *fb*$^{-1}$)
Υ(nS)(n=1, 2, 3),
continuum, Υ(4S), Υ(5S)

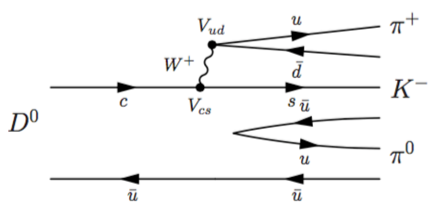


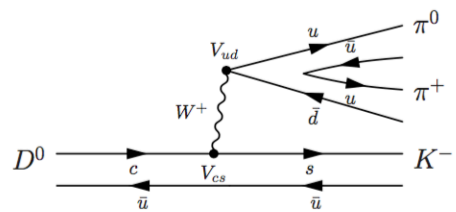
$e^+e^- \to \gamma^* \to D^{*+}$+Anything
$D^{*+} \to D^0 \pi_s^+$
$D^0 \to K^- \pi^+ \eta$



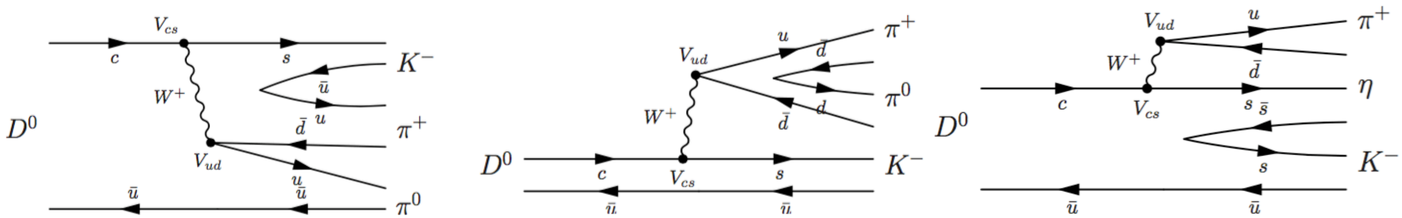
Variable	Requirement
charged tracks	eId< 0.95, muonId< 0.95 SVD hit$\geq$ 2 in $r\phi$ and z plane (except π_s) $\lvert dr \rvert < 1.0$ cm and $\lvert dz \rvert < 3.0$ cm for π_s $R_{(K,\pi)} > 0.7$ for $K^\pm$ $R_{(K,\pi)} < 0.7$ for $\pi^\pm$
$\eta \to \gamma\gamma$	$E_\gamma > 60(120)$ MeV/c^2 for barrel(endcap) good shower with $E9/E25 > 0.8$ (-60, +50) MeV/c^2 within η nominal mass $p_\eta^{lab} > 1.0$ GeV/c $\lvert \cos\theta_{hel} \rvert < 0.8$
D^0 lifetime error	$\sigma_t < 800$ fs
sum vertex fit quality	$\sum\chi^2/\sum ndf < 20$
D^* momentum in C.M.	$p^*(D^*) > 2.4$ GeV/c for Υ(nS) + continuum $p^*(D^*) > 2.5$ GeV/c for Υ(4S) $p^*(D^*) > 3.1$ GeV/c for Υ(5S)
Best Candidate Selection	smallest $\sum\chi^2/\sum ndf$
M-Q fit region	$1.80 < M < 1.92$ GeV/c^2 $0 < Q < 15$ MeV/c^2
Dalitz fit region (signal region)	$\lvert M - 1.865 \rvert < 0.015$ GeV/c^2 $\lvert Q - 5.85 \rvert < 0.5$ MeV/c^2

CF, DCS. RS, WS





meson SU(3) octet
π^0: dd-uu
η_8: uu+dd-2ss ~η



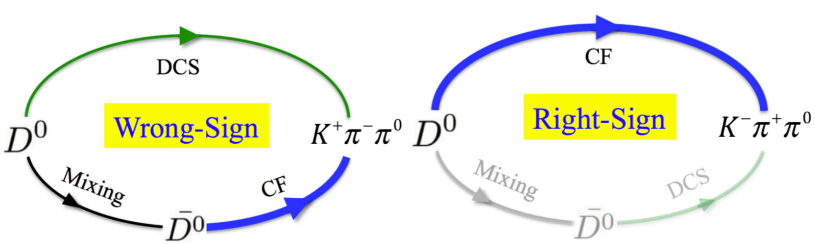


图 **1.15** 错误符号道 $D^0 \to K^+\pi^-\pi^0$ 的两个途经 (左图)。正确符号道 $D^0 \to K^-\pi^+\pi^0$ 的两个途经 (右图)。

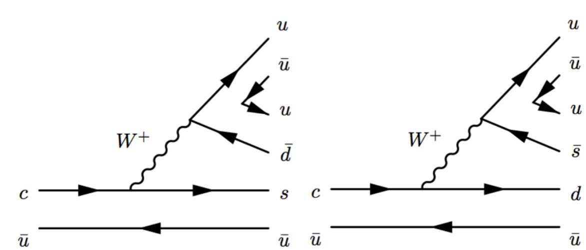


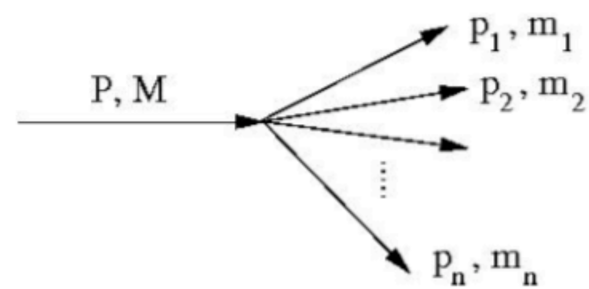
图 **1.16** 正确符号道 $D^0 \to K^-\pi^+\pi^0$ 的卡比玻允许 CF 过程 (左图) 和错误符号道 $D^0 \to K^+\pi^-\pi^0$ 的双卡比玻压低 DCS 过程 (右图) 的最低阶费曼图的外部 W 过程的树图之一。(此 DCS 衰变最低阶树图完整应该包括外部 W 过程，内部 W 过程和 W 交换过程。)

Dalitz analysis formalism

- R. H. Dalitz (1925-2006), Australian Physicsit, To study "$\tau \to 3\pi$"(kaon) decays. [Published: Philosophical Magazine Series 7, V. 44, Issue 357, Oct. 1953, p1068-1080.]

- Lorentz invariant phase space for n-body decay:

$$d\Phi_n(P; p_1, p_2, p_n) = \delta^4(P - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}$$



- Degree of freedom:

Decay types	$P \to PPP$	$P \to PPPP$	$P \to VPP$
Examples	$D^0 \to K^-\pi^+\pi^0$	$D^0 \to 4\pi$	$B^0 \to \psi(2S)K^-\pi^+$
4-vectors	3×4	4×4	3×4
E-p const. laws	-4	-4	-4
final state mass	-3	-4	-3
arbitrary rotations	-3	-3	-1(2 vector helicity)
Total d.o.f	2	5	4

Dalitz analysis formalism

- decays of $P_M \to P_1 P_2 P_3$:

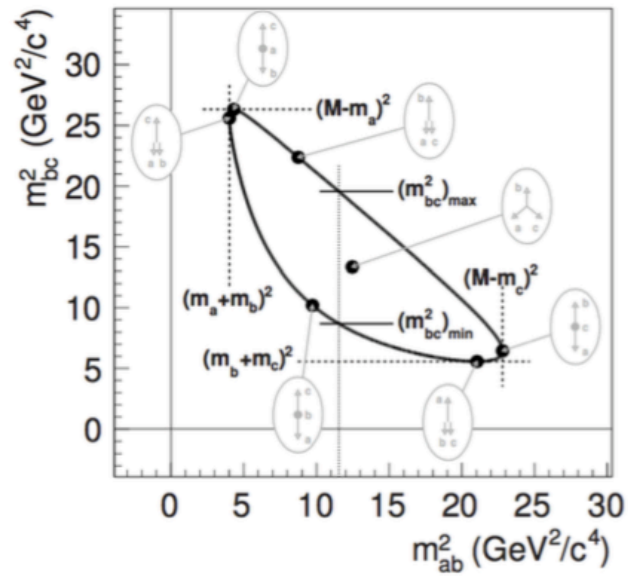
$$m_{12}^2 + m_{13}^2 + m_{23}^2 = M^2 + m_1^2 + m_2^2 + m_3^2 = const.$$

- Standard form of Dalitz plot (DP):

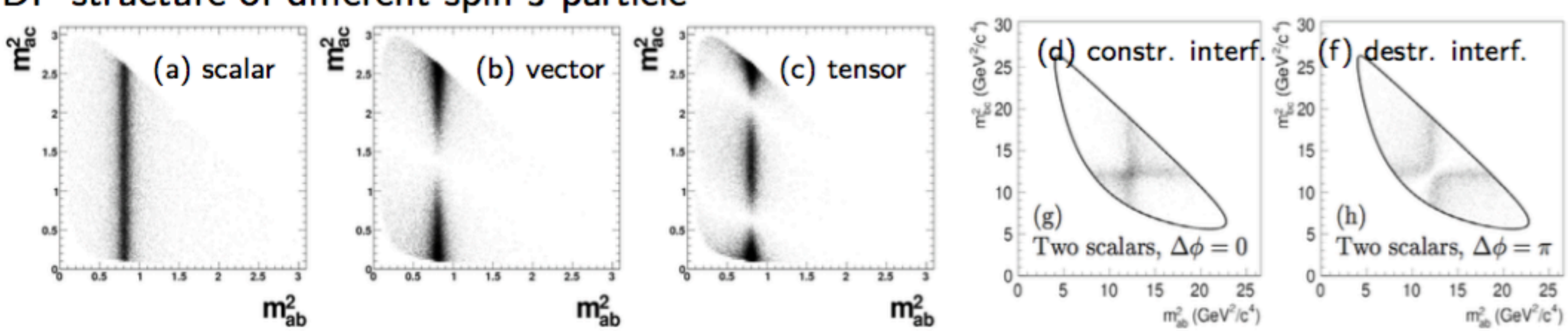
$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}\overline{|\mathcal{M}|^2}dm_{12}^2 dm_{23}^2$$

- DP kinematic limit: (eg: the form related to $\cos\theta_H$)

$$m_{23}^2 = (m_{23}^2)_{min}\frac{1+\cos\theta_{hel}}{2} + (m_{23}^2)_{max}\frac{1-\cos\theta_{hel}}{2}$$
$$\cos\theta_{hel} = -1 \Longrightarrow (m_{23}^2)_{min}$$
$$\cos\theta_{hel} = +1 \Longrightarrow (m_{23}^2)_{max}$$



- DP structure of different spin-J particle



Some functions

- Gaussian function

$$G(M;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(M-\mu)^2}{2\sigma^2}} \tag{A.1}$$

(A.2)

- bifurcated Gaussian function

Two different expression forms of bifurcated Gaussian function:

$$G_{bif}(M;\mu,\sigma,r\sigma) = \begin{cases} \frac{2}{\sqrt{2\pi}(\sigma+r\sigma)} e^{-\frac{(M-\mu)^2}{2\sigma^2}}, & \text{for } M \geq \mu; \\ \frac{2}{\sqrt{2\pi}(\sigma+r\sigma)} e^{-\frac{(M-\mu)^2}{2(r\sigma)^2}}, & \text{for others.} \end{cases} \tag{A.3}$$

$$G_{bif}(Q;\mu_g,\sigma_g,\delta_{\sigma_g}) = \frac{1}{\sqrt{2\pi}\sigma_g} \begin{cases} e^{-\frac{(Q-\mu_g)^2}{2(\sigma_g+\delta_{\sigma_g})^2}}, & \text{for } Q \geq \mu_g; \\ e^{-\frac{(Q-\mu_g)^2}{2(\sigma_g-\delta_{\sigma_g})^2}}, & \text{for others.} \end{cases} \tag{A.4}$$

- bifurcated Student function

$$S_{bif}(Q;\mu_s,\sigma_s,\delta_{\sigma_s},N_h,N_l) = \frac{2P_HP_L}{(P_H+P_L)\sqrt{\pi}} \begin{cases} \left[1+\frac{1}{N_h}\left(\frac{Q-\mu_s}{\sigma_s+\delta_{\sigma_s}}\right)^2\right]^{-\frac{N_h+1}{2}}, & \text{for } Q \geq \mu_s; \\ \left[1+\frac{1}{N_l}\left(\frac{Q-\mu_s}{\sigma_s-\delta_{\sigma_s}}\right)^2\right]^{-\frac{N_l+1}{2}}, & \text{for others.} \end{cases} \tag{A.5}$$

here the factors P_H and P_L in normalization factor are calculated as follows:

$$P_H = \frac{\Gamma(\frac{N_h+1}{2})}{(\sigma_s+\delta_{\sigma_s})\Gamma(\frac{N_h}{2})} \frac{1}{\sqrt{N_h}}; \; P_L = \frac{\Gamma(\frac{N_l+1}{2})}{(\sigma_s-\delta_{\sigma_s})\Gamma(\frac{N_l}{2})} \frac{1}{\sqrt{N_l}}. \tag{A.6}$$

- bifurcated Cruijff function

$$CF_{bif}(Q;\mu_{cf},\sigma_{cf},\delta_{\sigma_{cf}},\alpha_R,\alpha_L) = \frac{1}{\sqrt{2\pi}\sigma_{cf}} \begin{cases} e^{-\frac{(Q-\mu_{cf})^2}{2(\sigma_{cf}+\delta_{\sigma_{cf}})^2+\alpha_R(Q-\mu_{cf})^2}}, & \text{for } Q \geq \mu_{cf}; \\ e^{-\frac{(Q-\mu_{cf})^2}{2(\sigma_{cf}-\delta_{\sigma_{cf}})^2+\alpha_L(Q-\mu_{cf})^2}}, & \text{for others.} \end{cases} \tag{A.7}$$

Model systematic uncertainty

- model uncertainty sources:
  - Form factor R_r choose: 1.5 GeV^{-1} -> 1-2 GeV^{-1}
  - Angular distribution: denominator $M_r^2 \rightarrow M_{ab}^2$
  - LASS parameters: float each parameter within 1σ
  - Add kappa

other systematic uncertainty

- uncertainty sources:
  - M sideband region :
    - M_{low}[1.755-1.775] -> [1.750-1.770], [1.760-1.780]
    - M_{up} [1.935-1.955] -> [1.930-1.950], [1.940-1.960]
  - efficiency plane: -> efficiency = constant
  - Best Candidate Selection: move multiple candidates
  - ratio of signal & bkg in F.R.: change within 1σ
  - mass resolution: float width of resonances