Physics Beyond Standard Model

- Standard Model
- Neutrino Search
- Big Bang Dark Matter
- Big Bang Dark Energy
- Quantum entanglement (量子纠缠)

J. Ulbricht ETHZ Lecture USTC 2017

History of Big Bang and Dark Energy observations

- 1915-1916 Einstein construct General Relativity.
- 1917 Einstein introduced the Cosmological Constant to form a static Universe.
- 1920 Slipher and Hubble discovered Red Shift.
- 1927 G. Lemaître derived the Hubble Law.
- 1970 A. Guth proposed that a negative pressure field, similar in concept to dark energy, could drive cosmic inflation in the very early universe.
- 1998 M. Turner used first the term "dark energy", echoing F. Zwickys "dark matter" from the 1930.
- 1998-1999 Riess and Perlmutter observe first direct evidence for dark energy came from supernova observations of accelerated expansion.
- 1998 The **ΛCDM** or **Lambda-CDM** model became the standard model following the observations of accelerating expansion of the universe.

Observational evidence of Dark Energy

- The age of the Universe compared to old stars.
- Supernovae observations (SN)
- Cosmic Microwave Background (CBM).
- Baryon acoustic oscillations (BAO).
- Large-scale structures (LSS)

The references are found in" Dark Energy L. Amendola and S. Tsujikawa Cambridge University Press ISBN 978-0-521-51600-6 (2010)"
(DE-AT) unless the are given in the talk.

The age of the Universe compared to old stars

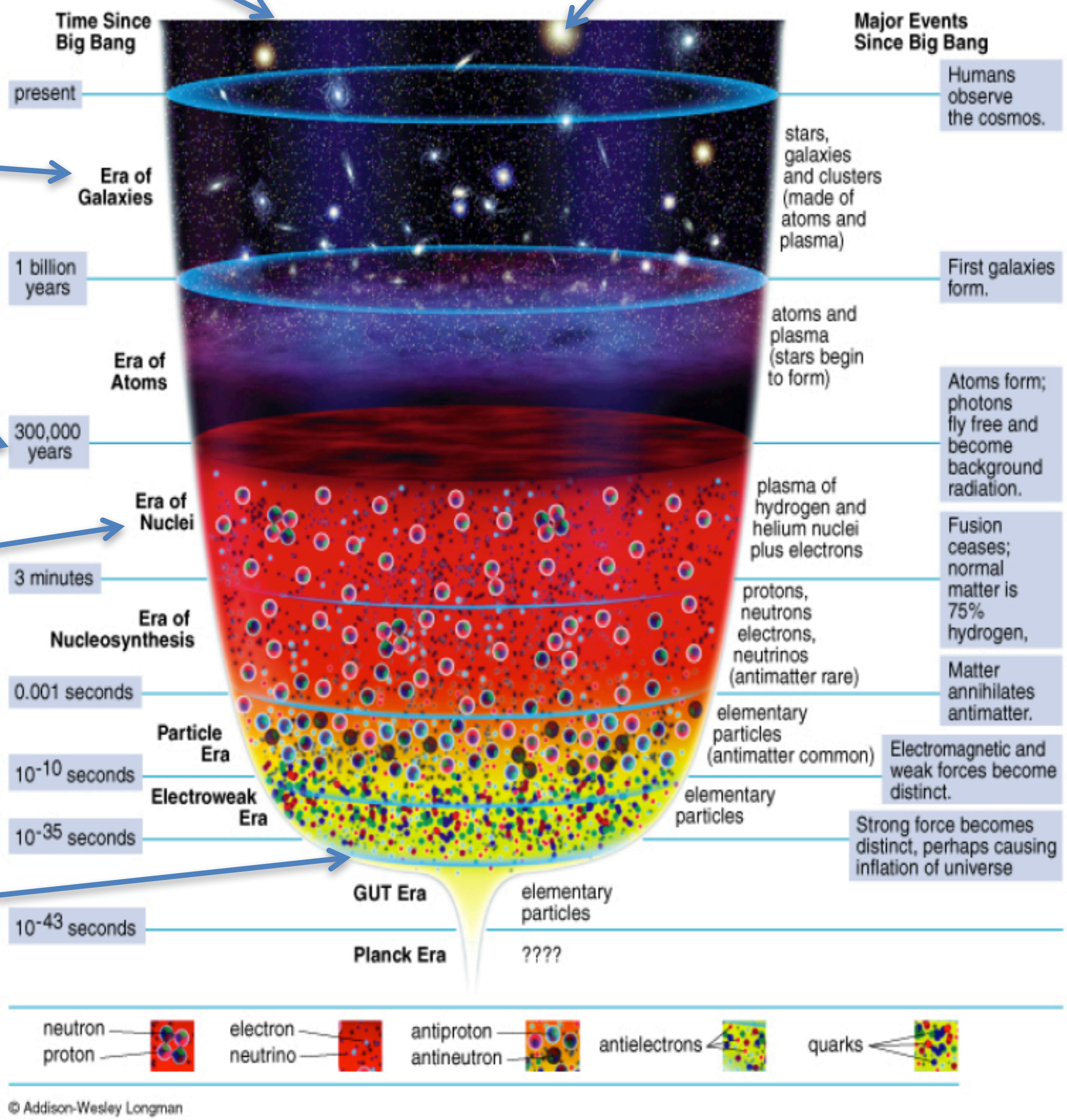
Supernovae observations (SN)

Large-scale structures (LSS)

Cosmic Microwave Background(CBM)

Baryon acoustic oscillations (BAO)

Generation Large-scale structures (LSS)



The age of the Universe compared to old stars

As already discussed in the Dark Matter talk is the age of the universe $t_0 = 1/ H_0$ an approximate number for the age of the universe. To compare this age with the measurement of the oldest stars a more detailed calculation is necessary.

It is first necessary to introduce an equation of state for dark energy equ. 1

$$\rho_{DE} = \rho_{DE}^{(0)}(1+z)^{3(1+\omega_{DE})} \qquad (1)$$

ρ_{DE} is the density at the red shift z, $\rho_{DE}^{(0)}$ the DE density today and ω_{DE} a parameter. The age of the universe could be expressed in equ. 2

$$t_0 = H_0^{-1} \int_1^{\infty} \frac{dz}{E(z)(1+z)} \qquad (2)$$

With E (z) the distribution of densities in the universe.

The density distribution in the universe is the key parameter to calculate the age of the universe shown in the equ. 3.

$$E(z) = \left[\Omega_r^{(0)}(1+z)^4 + \Omega_m^{(0)}(1+z)^3 + \Omega_{DE}^{(0)}(1+z)^{3(1+\omega_{DE})} + \Omega_K^{(0)}(1+z)^2\right]^{1/2} \quad (3)$$

The different densities of the radiation Ω_r, matter Ω_m, Dark Energy Ω_{DE} and curvature Ω_K are shown in equ. 4.

$$\Omega_r^{(0)} = \frac{8\pi G\rho_r^{(0)}}{3H_0^2} \quad \Omega_m^{(0)} = \Omega_{DM}^{(0)} + \Omega_b^{(0)} = \frac{8\pi G\rho_m^{(0)}}{3H_0^2} \quad \Omega_{DE} = \frac{8\pi G\rho_{DE}^{(0)}}{3H_0^2} \quad \Omega_K^{(0)} = -\frac{K}{(a_0H_0)^2} \quad (4)$$

To insert equ. 3 in equ. 2 and neglect the small amount of Ωr for low red shift on get for the age of the universe equ. 5

$$t_0 = H_0^{-1}\int_1^\infty \frac{dx}{x\left[\Omega_m^{(0)}x^3 + \Omega_{DE}^{(0)} + \Omega_K^{(0)}x^2\right]^{1/2}} \quad (5)$$

In equ. 5 is x = 1 + Z and $\Omega_m^{(0)} + \Omega_{DE}^{(0)} + \Omega_K^{(0)} = 1$.

In a flat universe with $\Omega_K^{(0)} = 0$ is it easy possible to integrate equ. 5 to equ. 6

$$\boxed{t_0 = \frac{H_0^{-1}}{\sqrt[3]{(1-\Omega_m^{(0)})}} \ln\left(\frac{1+\sqrt{1-\Omega_m^{(0)}}}{1-\sqrt{1-\Omega_m^{(0)}}}\right)} \qquad (6)$$

The integration is done in the limit $\Omega_m^{(0)} + \Omega_{DE}^{(0)} = 1$. For $\Omega_{DE}^{(0)} \to 0$ equ. 6 give the age of the universe in equ. 7

$$\boxed{t_0 = \frac{2}{3} H_0^{-1}} \qquad (7)$$

With h = 0.72 ± 0.08 the equ. 7 the age of the universe without Dark Energy is in the range of

8.2 Gyr < t_0 < 10.2 Gyr

Carretta et. al.(Astro. Phys. J. 533 (2000) 215) estimated age of globular cluster in the Milky Way

12.9 ± 2.9 Gyr

Jimenez et. al. (Mon. Not. Roy. Astron. Soc. 282 (1996) 926) obtained a value of

13.5 ± 2 Gyr

AGE of UNIVERSE from Planck+WP+highL+BAO (arXiv:1303.5062v1 [astro-ph.CO] 20 Mar 2013) is

13.798 ± 0.037 Gyr

It is clear visible from the age discussion without Dark Energy it is not possible to get the age of the universe $t_0 > 11$ Gyr.

To increase the age of the universe in equ. 6 it is possible to decrease $\Omega_m^{(0)} \rightarrow 0$ what will push the time $t_0 \rightarrow \infty$. The correct contribution of $\Omega_m^{(0)}$ and $\Omega_{DE}^{(0)}$ under the condition $\Omega_m^{(0)} + \Omega_{DE}^{(0)} = 1$ allows, if the age of universe $t_0 = 13.73 \pm 0.12$ Gys is known, to define the contributions from $\Omega_m^{(0)}$ and $\Omega_{DE}^{(0)}$.

In fig. 1 (ref. (DE-AT)) a plot of t_0 as function $\Omega_m^{(0)}$ is shown under for the condition t_0 - > 11 Gys,

$0 < \Omega_m^{(0)} < 0.55$ ·

The WMAP 5 bound $\boxed{t_0^{WMAP} = 13.73 Gys > t_0^{OLD} \approx 13 Gys}$

for a flat universe k = 0, but with $\Omega_{DE}^{(0)} > 0$ equivalent Λ ≠ 0 leads to

$$\boxed{0.245 < \Omega_m^{(0)} < 0.261}$$

Under the condition $\Omega_m^{(0)} + \Omega_{DE}^{(0)} = 1$ sets this a limit for

$$\boxed{0.739 < \Omega_{DE}^{(0)} < 0.755}$$

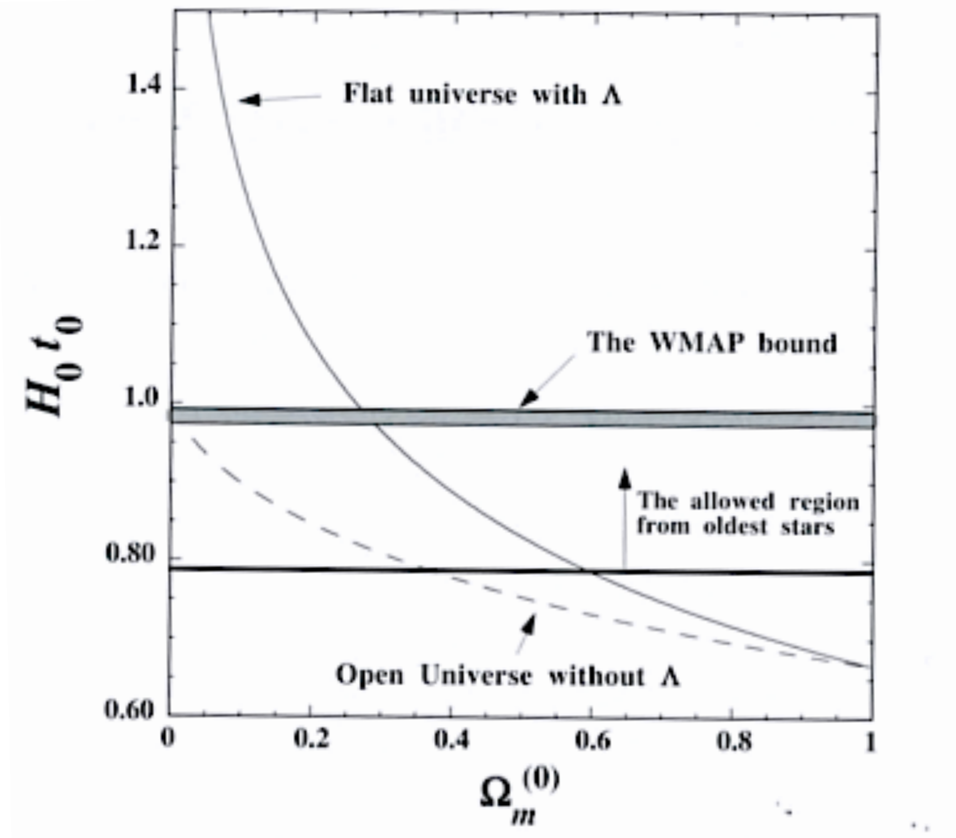


Fig. 1 The age of the universe t_0 normalized to H_0 as function of $\Omega_m^{(0)}$ (ref. DE-AT) .

The age of the Universe compared to old stars

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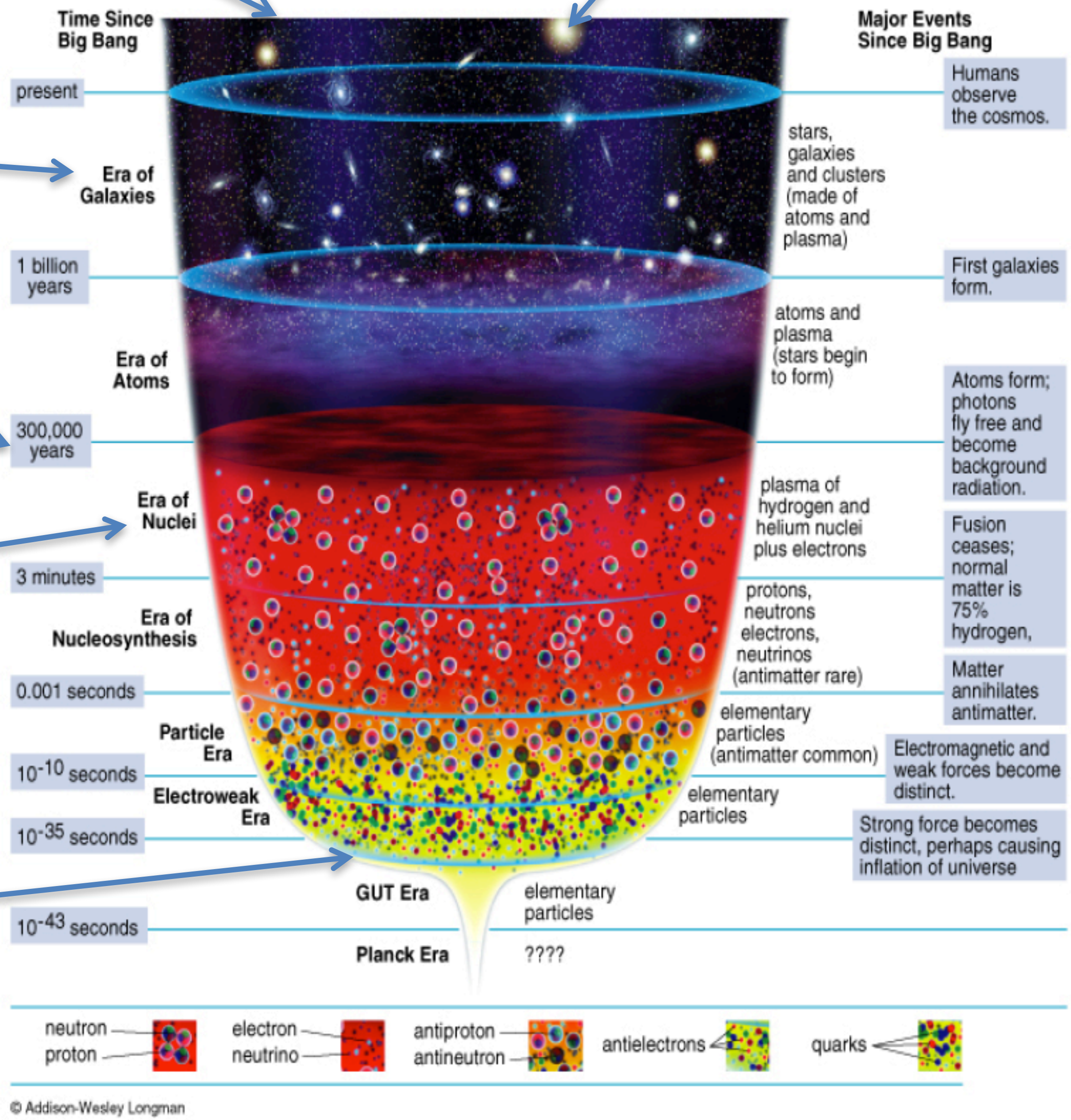


Supernovae observations (SN)

The explosion of supernovae is extremely luminous and causes a burst of radiation. The supernovae are classified according to the absorption lines of chemical elements:

- TYP II If the spectrum of the supernovae includes spectral lines of hydrogen
- TYP I If the spectrum of the supernovae includes NO spectral lines of hydrogen
- TYP Ia If the spectrum of the supernovae includes NO spectral lines of hydrogen but in addition absorption lines of single ionized silicon.

The absolute luminosity of TYP Ia is almost constant at the peak of brightness. The TYP Ia is used as a " STANDARD CANDLE ".

The apparent magnitude m is used to measure the brightness of a star what is a function of the distance d_L to the star, at the same time is the distance to the star a function of z and the density Ω_{DE}.

$$m = f (d_L)$$

$$d_L = f (z, \Omega_{DE})$$

This correlation can be used to limit Ω_{DE} via the observation of m.

In the Dark Matter $d_L = f(L_s)$ was already discussed. For the SN Ia the relation is shown in equ. 8

$$d_L^2 = \frac{L_s}{4\pi F} \qquad (8)$$

d_L is the luminosity distance, L_s the absolut luminosity of the source and F its observed flux. Equ. 8 can be developed to equ. 9.

$$\boxed{m - M = 5\log_{10} d_L + 25} \qquad \boxed{m_1 - m_2 = 5\log_{10}\left(\frac{d_{L_1}}{d_{L_2}}\right)} \qquad (9)$$

In equ. 9 M is the absolute magnitude of one SN Ia source and m_1, m_2 the apparent magnitude from two SN Ia sources at the distance d_{L1} and d_{L2}. With $d = v / H$ and $v \sim c \cdot z$ and equ. 9 it is possible to write d as power low of z what is shown in equ. 10 for low red shift.

$$\boxed{d_L(z) = \frac{c}{H_0}\left[z + \frac{1}{4}(1 - 3\omega_{DE}\Omega_{DE}^{(0)} + \Omega_K^{(0)})z^2 + \Re(z^3)\right]} \qquad (10)$$

In equ. 10 is z the red shift, $\omega_{DE} = P_{DE}/\rho_{DE}$ and $\Re(z^3)$ higher polynomials.

Fig. 2 shows a calculation from equ. 10 for $d_L = f(z)$ for the following conditions :

- A flat universe K = 0 without DE.
- An open universe $\Omega_K^{(0)} = 0.0085$ without DE.
- A flat universe with $\Omega_{DE}^{(0)} = 0.7$ and $\omega_{DE} = -1$

From fig. 2 it is visible that the dark energy together with open universe leads to an increase in d_L with increasing z.

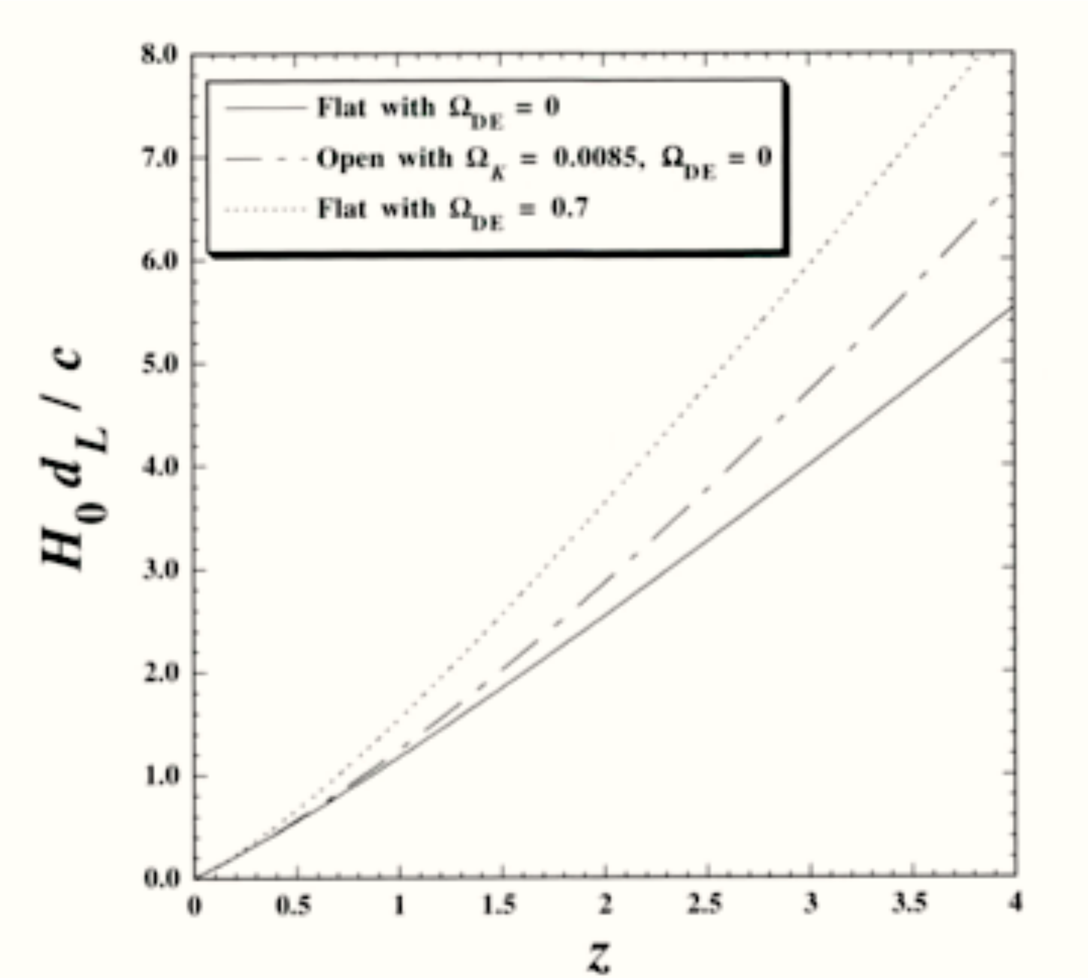


Fig. 2 d_L as function of z for 3 cases (ref. (DE-AT) .

To compare with the observations it is necessary to combine low red shift data described in equ. 10 and high red shift date calculated in equ. 11 to get enough statistic for a signal of dark matter.

$$d_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{d\tilde{z}}{\left[(1-\Omega_{DE}^{(0)})(1+\tilde{z})^3 + \Omega_{DE}^{(0)}\right]^{1/2}} \qquad (11)$$

At low red shifts is d_L not very sensitive to DE. At the high red shift regime of equ. 11 is more sensitivity to the observation possible. As example ref. (DE-AT) uses the data from Perlmutter et. al. (Astrophys. J. 517 (1999) 565) :

- SN Ia 1997R with magnitude m = 23.83 (z = 0.657)
- SN Ia 1995ck with magnitude m = 23.57 (z = 0.656)

With the known absolute magnitude of SN Ia M = - 19.15 it is possible to calculate for

- SN Ia 1997R $H_0 \, d_L / c = 0.920$
- SN Ia 1995ck $H_0 \, d_L / c = 0.817$

To satisfy equ. 11 is is necessary to insert for

- SN Ia 1997R $\Omega_{DE} = 0.7$
- SN Ia 1997R $\Omega_{DE} = 0.38$

→ From this data it is visible that Dark Energy is needed to confirm the observations.

Combining 42 high-redshift SN Ia data 0.18 < z < 0.83 and 18 low redshift data from the SN Ia Calan/Tololo Supernova Survey shows that the cosmological constant Λ is present at the 99 % CL. Fig. 3 shows m_B as function of z (Perlmutter et. al. (Astrophys. J. 517 (1999) 565) for various theoretical predictions: (Perlmutter Nobel Prize 2011)

- Cosmological models with Λ = 0 three cases $(\Omega_M,\Omega_\Lambda)=(0,0;1,0;2,0)$ (Solid lines)
- Cosmological models including Λ four cases $(\Omega_M,\Omega_\Lambda)=(0,1;0.5,0.5; 1,0;1.5,-0.5)$ (broken lines)

From fig. 3 it is visible high redshift, the data prefer calculation higher as CDM model $(\Omega_M,\Omega_\Lambda)=(1,0)$.

A best fit for Cold Dark Matter density gives $\Omega_m^{(0)} = 0.28^{+0.09}_{-0.08}$ leading to $\Omega_{DE} \approx 0.72$ for

. $\Omega_m^{(0)} + \Omega_{DE}^{(0)} = 1$

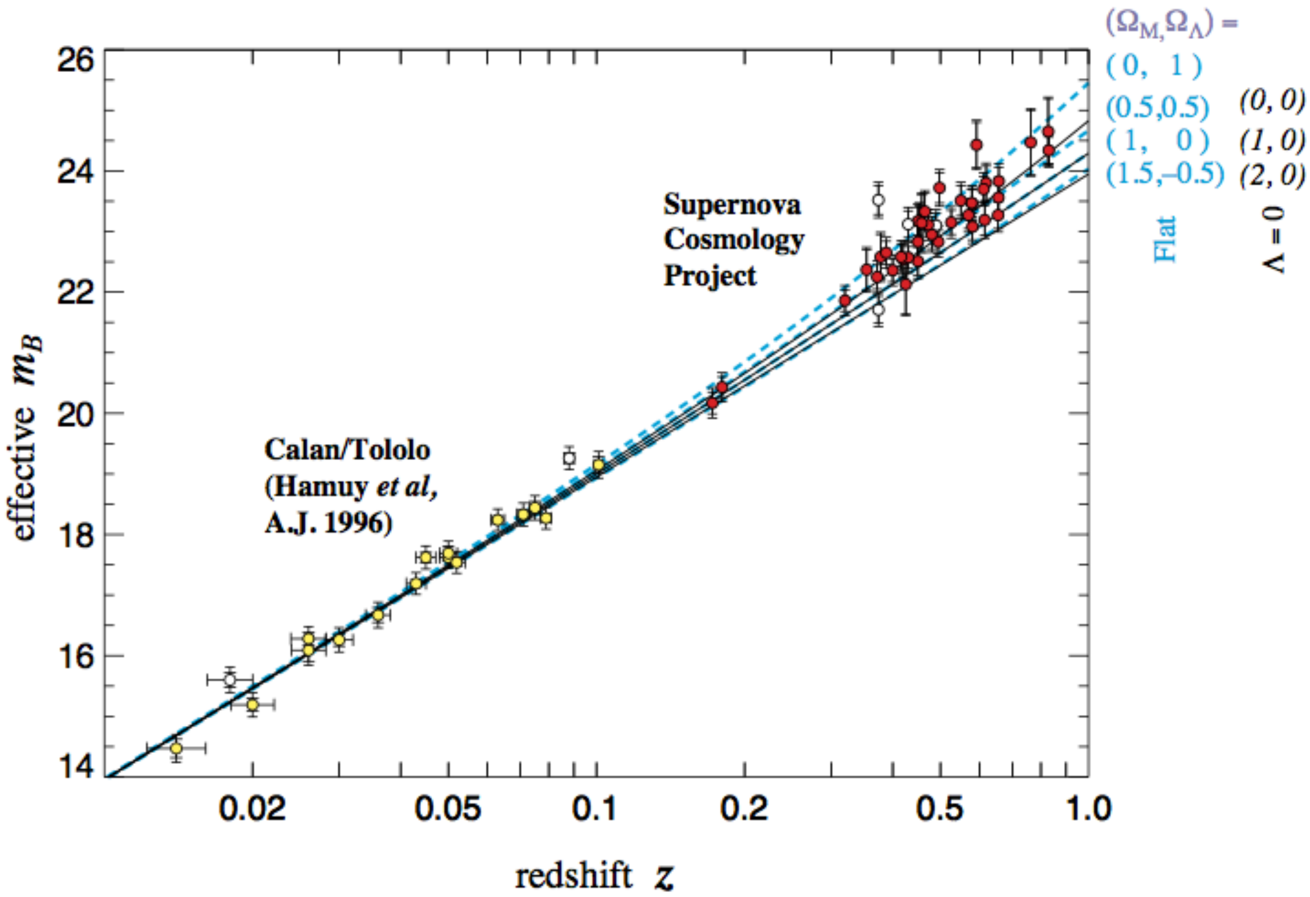


Fig. 3 The magnitude m_B as function of z for various theories. (See also arXiv:astro-ph/0303428v1 18 Mar 2003)

After 1998 more SN Ia data get collected from:

- Super Nova Legacy Survey (SNLS). Canada-France-Hawaii Telescope detect 2000 SUPERNOVA.
- Hubble Space Telescope (HST) search for $z > 1$ “GOLD” data.
- Equation of State: SupErNovae trace Cosmic Expansion (ESSENCE) search for $0.2 < z < 0.8$ detect 200 SUPERNOVA.

Fig. 4 shows a plot of $\Omega_m^{(0)}$ as function of ω_{DE} , summarizing the mean results of these of the SNLS, HST and ESSENCE observations.

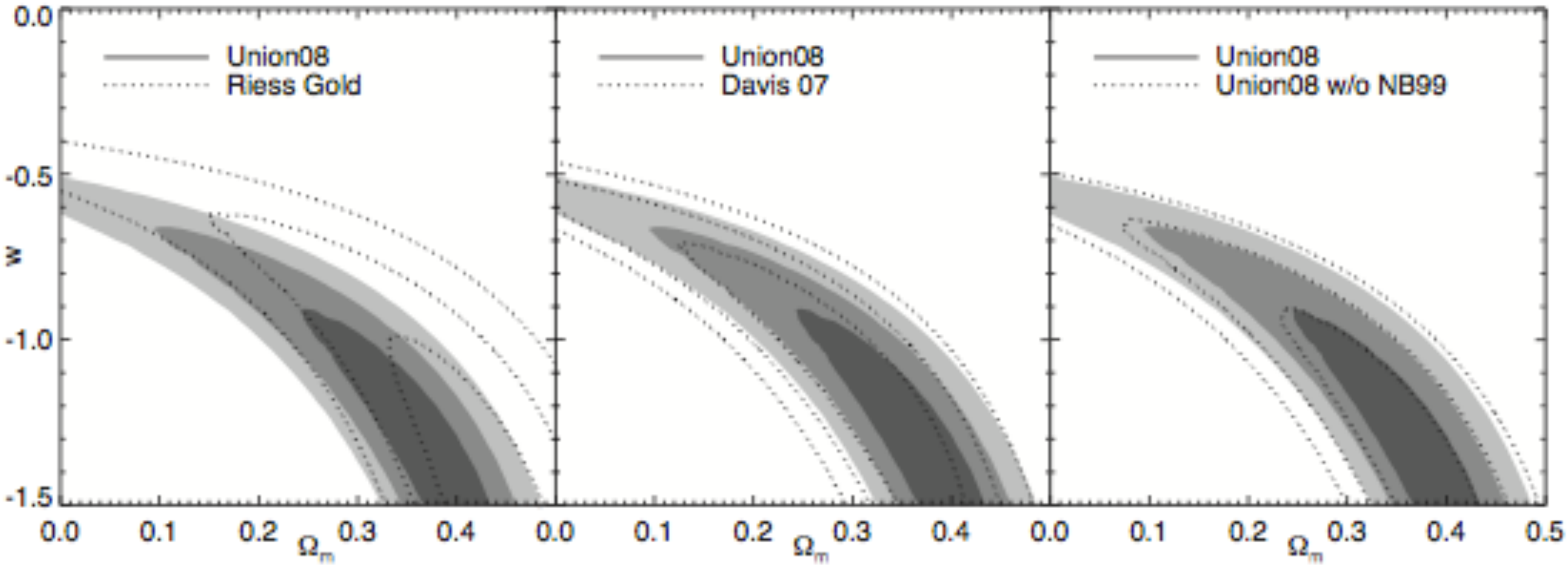


Fig. 4 The $\Omega_m^{(0)}$ as function of ω_{DE} at the 68.3 %, 95.4 % and 99.7 % Cl for constant ω_{DE} .

- In each plot the filled contours correspond to the results from the Union data by Kowalski et. al. (Astrophys. J. 686 (2008) 749)
- The empty contours in the left and middle plot are the constrains from the HST and ESSENCE data.
  The right plot shows the impact of SCP 1999 data from Kowalski et . al..

In 2014 the latest data of w as function of Ω_m are published from M. Betoule et al.: arXiv:1401.4064v1 [astro-ph.CO] 16 Jan 2014.

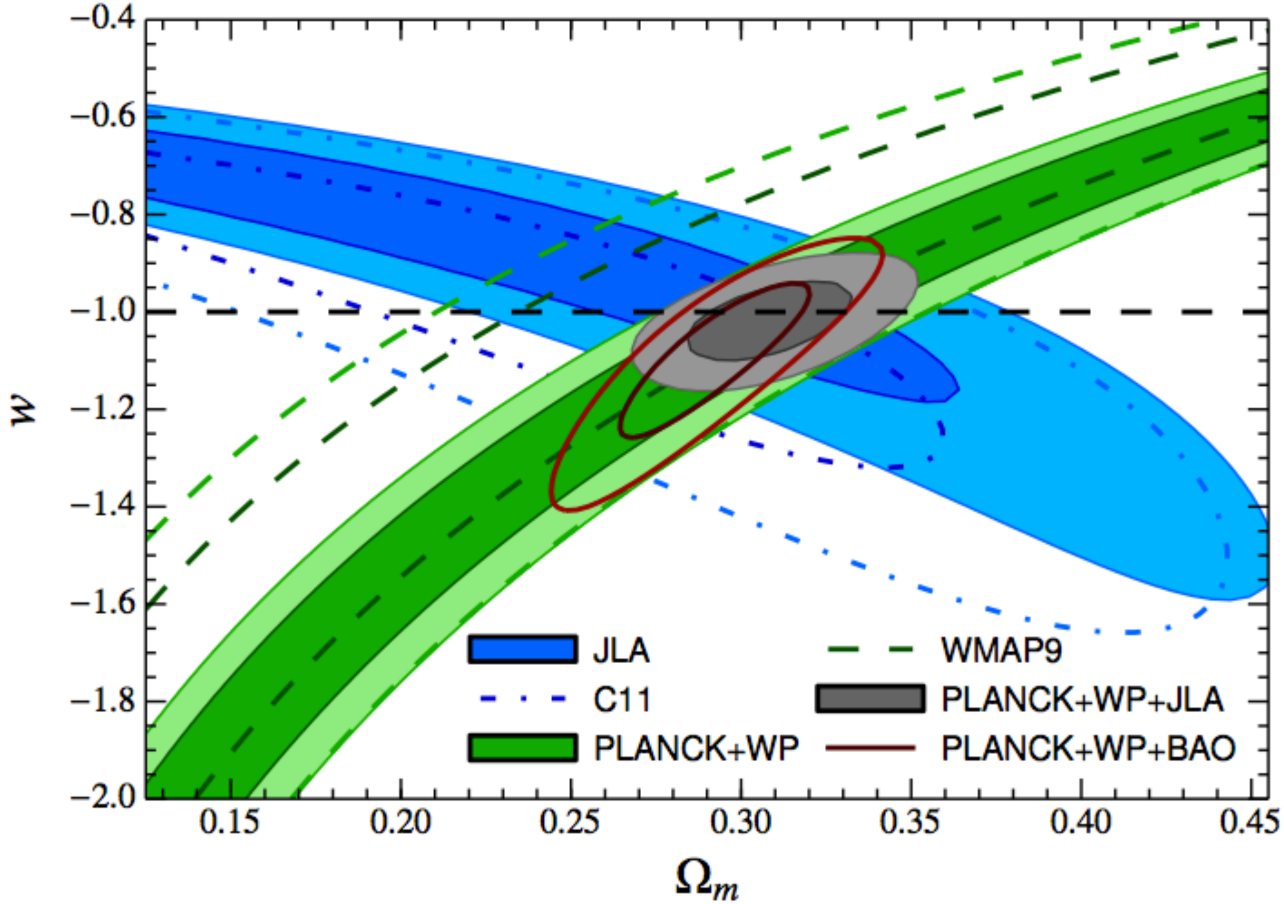


Fig. 5 Confidence contours at 68% and 95% (including systematic uncertainty) for the w and Ω_m cosmological parameters for the flat w-CDM model. The black dashed line corresponds to the cosmological constant hypothesis. (Ref. M. Betoule et. al.)

Best FIT data of w and Ω_m

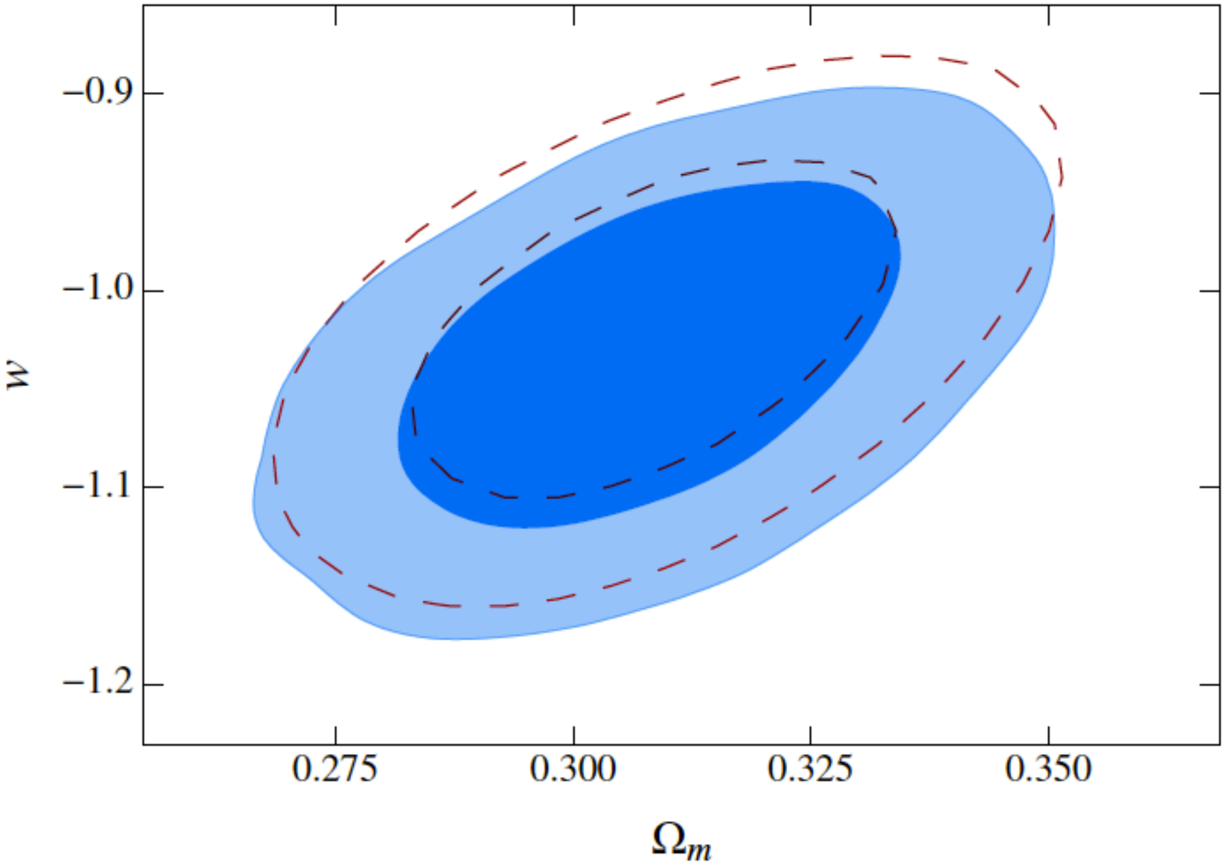


Fig. 6 Comparison of two derivations of the 68 and 95% confidence contours in the Ω_m and w parameters for a flat w-CDM cosmology. In one case, constraints are derived from the exploration of the full Planck+WP+JLA likelihood (Blue). In the other case CMB constraints are summarized by the geometric distance (dashed red). (Ref. M. Betoule et. al.)

Best FIT data of w and Ω_m

Parameter	Planck+WP+JLA	DP+JLA
$\Omega_b h^2$	$0.02201^{+0.00028}_{-0.00028}$	0.02208 ± 0.00028
Ω_m	$0.308^{+0.017}_{-0.017}$	0.307 ± 0.017
H_0	$68.1^{+1.6}_{-1.6}$	68.1 ± 1.6
τ	$0.089^{+0.012}_{-0.015}$	–
n_s	$0.9590^{+0.0071}_{-0.0071}$	–
$\ln(10^{10} A_s)$	$3.088^{+0.024}_{-0.027}$	–
w	$-1.034^{+0.059}_{-0.055}$	-1.018 ± 0.057

Table 1. Best-fit parameters of the w-CDM fit for the full PLANCK+WP+JLA likelihood, and for the distance prior (DP+JLA). (Ref. M. Betoule et. al.)

Fig. 4, 5 and 6 clearly shows the presence of DARK ENERGY responsible for the late-time cosmic acceleration.

$$\boxed{\omega_{DE} < -1/3}$$

Baryon acoustic oscillations and Cosmic Microwave Background

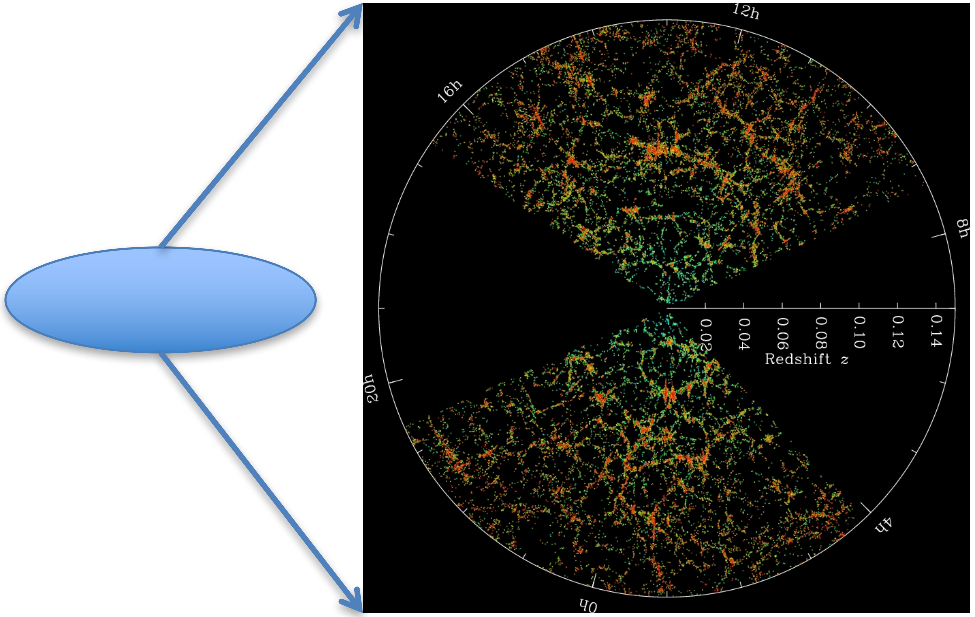
The universe is after a volume explosion at the time ZERO very homogenous. But all observations so far support the assumption that this volume explosion is superimposed by a very small density fluctuation, may be caused be quantum fluctuation before the Planck time where gravity was dominant.

In the evolution of the universe are two important time scales to observe a signal from these early fluctuations of density.

First the Baryon acoustic oscillations in the time scale where the temperature of the universe allows a superfluid of baryons, electrons and light. This period is dominated be the Compton Scattering between light and electrons. Usually it is called the DRAG EPOCH The important parameters to describe this period is the sound horizon r_{drac} at the redshift z_{drag}. In these parameters the universe send us an imprint of the fluctuations.

The second is the Cosmic Microwave Background in the time scale where the temperature of the universe is cold enough that the electrons get bound in atoms. In this moment the universe get transparent for light and only elastic scattering of light on rest charged particle the Thomson scattering is important. The period is called DECOUPLING or RECOMBINATION. The leading parameter is $z \approx 1090$.

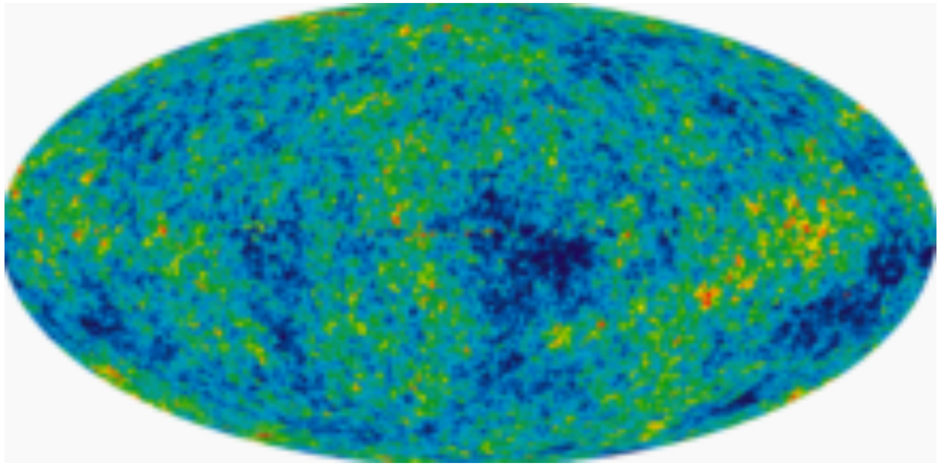
Baryon Acoustic Oscillation



SIGNAL:

- sound horizon r_{drac}
- redshift z_{drac}

Cosmic Microwave Background



SIGNAL:

- Temperature fluctuations
- Redshift z_{dec}

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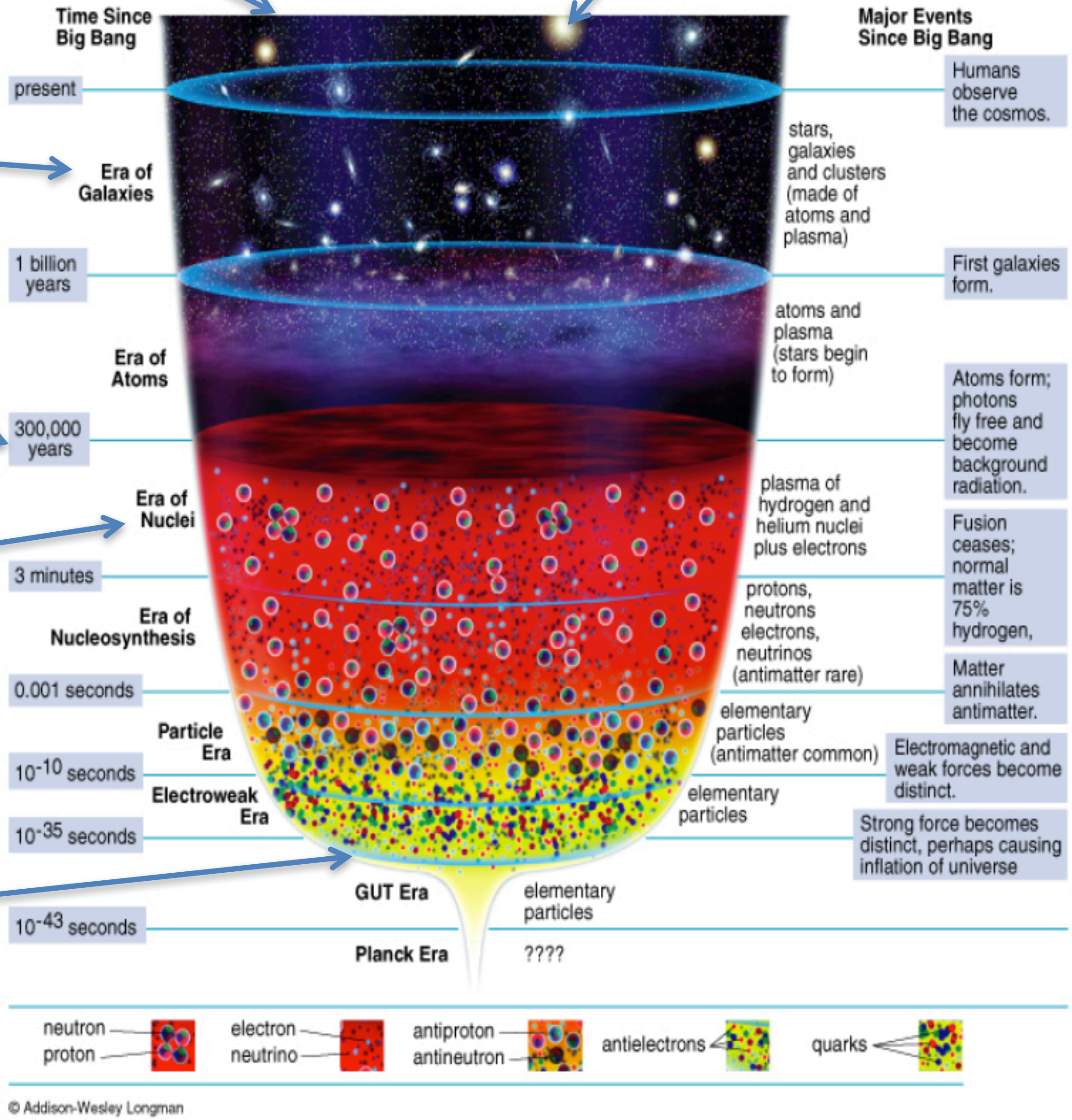
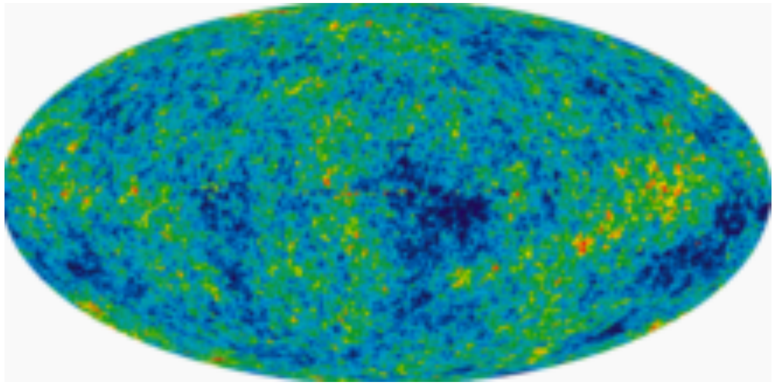


Cosmic Microwave Background (CMB)

Two effects are important in the CMB to search for Dark Energy.

- First is the change of the position of the acoustic peak coming from the modification of the angular diameter distance.
- The second effect (Sachse-Wolfe) causes by the variation of the gravitational potential.

Change of the position of the acoustic peak



The temperature fluctuations δT in the CMB get developed in a perturbation Θ in terms of spherical harmonics equ. 12 and equ. 13.

$$\Theta = \frac{\delta T}{T} \qquad \Theta(x,\eta) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm}(x,\eta) Y_{lm}(\hat{n}) \qquad (12)$$

$$C_l \equiv \left\langle \left| a_{lm} \right|^2 \right\rangle \qquad C_l = \frac{2}{\pi} \int_0^{\infty} dk \cdot k^2 \left| \Theta_l(k) \right|^2 \qquad (13)$$

In fig. 7 are plotted the temperature anisotropies $l(l+1)C_l\,/\,2\pi$ as function of the multipole moment l

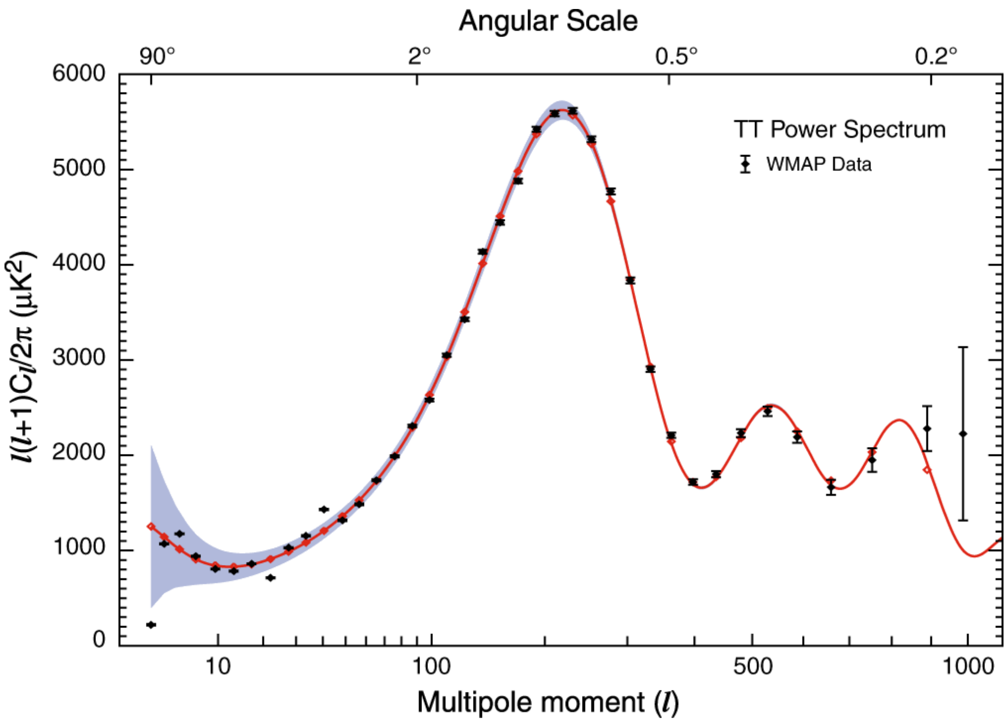


Fig. 7 The temperature anisotropies $l(l+1)C_l\,/\,2\pi$ as function of the l .

The points are the WMAP5-year observational data and flying colours the theoretical power spectrum. The position of the first peak contains the information about the Dark Energy.

Latest results from **Planck 2013**

arXiv:1303.5062v1 [astro-ph.CO] 20 Mar 2013

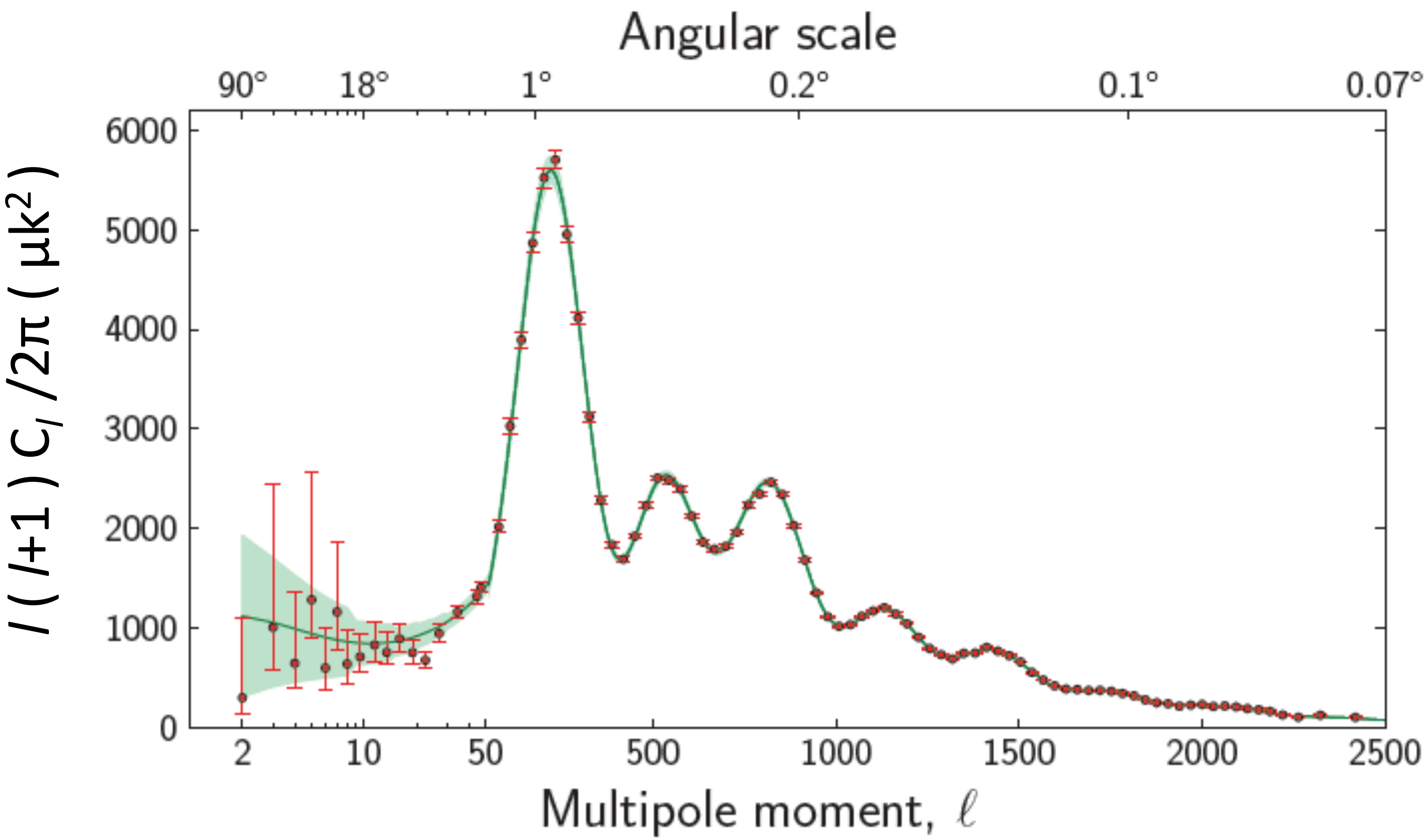


Fig. 8 The temperature anisotropies $l(l+1)C_l\,/\,2\pi$ as function of the l .

The points are the Planck 2013 observational data and flying colours the theoretical power spectrum. The position of the first peak contains the information about the Dark Energy.

It is interesting to calculate the position of the first peak $\boxed{l_a}$. In the developed of the perturbation of Θ in terms of spherical harmonics, it is possible to show that the first peak of the temperature fluctuations can be expressed as a function of the sound horizon $r_S(z_{dec})$ at the redshift of the decoupling z_{dec}.

$$k \cdot r_S(z_{dec}) = n \cdot \pi \qquad (14)$$

With k as wave number and n as integer. It is than possible to define a characteristic angle for the location of the peaks.

$$\boxed{\theta_A \equiv \frac{r_s(z_{dec})}{d_A^{(c)}(z_{dec})}} \qquad (15)$$

The term $d_A^{(c)}(z_{dec})$ angular diameter distance defined by

$$\boxed{d_A^{(c)}(z_{dec}) = \frac{d_A(z)}{a} = (1+z)d_A(z)} \qquad a = 1/(1+z) \qquad (16)$$

The physical diameter distance is

$$d_A \equiv \frac{\Delta x}{\Delta \theta} \qquad (17)$$

With Δx as size of the object orthogonal to the line of sight and ΔΘ the angle.

The multipole l_A corresponding to the angle of equ. 15 is than shown in equ. 18.

$$\boxed{l_A = \frac{\pi}{\theta_A} = \pi \frac{d_A^{(c)}(z_{dec})}{r_s(z_{dec})}} \tag{18}$$

Including the Friedmann-Lemaitre-Robertson Walker metric it is possible to develop $d_A^{(c)}(z_{dec})$ as function of H_0, Ω_m and the shift parameter $\Re$, shown in equ. 19 and equ. 20.

$$\boxed{d_A^{(c)} = \frac{c}{H_0} \frac{1}{\sqrt{\Omega_m^{(0)}}} \Re} \tag{19}$$

$$\boxed{\Re = \sqrt{\frac{\Omega_m^{(0)}}{\Omega_K^{(0)}}} \sinh\left(\sqrt{\Omega_K^{(0)}} \int_0^{z_{dec}} \frac{dz}{E(z)} \right)} \tag{20}$$

Including the cosmological perturbation theory it is possible to develop the sound horizon at the decoupling as function of the energy E(z) (equ. 3) in equ. 21.

$$\boxed{r_s(z_{dec}) = \frac{c}{\sqrt{3a_0}H_0}\int_{z_{dec}}^{\infty}\frac{dz}{\sqrt{1+R_s}E(z)}} \qquad (21)$$

With neglecting the Dark Energy for z > zdec in E (z) the energy E is shown in equ. 22

$$E = (\sqrt{a+a_{eq}}\,/\,a^2)\sqrt{\Omega_m^{(0)}} \qquad (22)$$

With the scale factors $a = 1/(1+z)$ and $a_{eq} = 1/(1+z_{eq})$, a_{eq} stands for the border of the large scale and small scale modes.

Inserting equ. 22 in equ. 21 leads to equ. 23

$$\boxed{r_s(z_{dec}) = \frac{4}{3}\cdot\frac{c\cdot h}{H_0}\sqrt{\frac{\omega_\gamma}{\omega_m\omega_b}}\ln\left(\frac{\sqrt{R_s^{(dec)}+R^{(eq)}}+\sqrt{1+R_s^{(dec)}}}{1+\sqrt{R_s^{(eq)}}}\right)} \qquad (23)$$

With $\omega_i = \Omega_i \cdot h^2$, $R_s = (3\omega_b / 4\omega_\gamma)a$, $R_s^{(dec)} \equiv R_s(a_{dec})$ and $R_s^{(eq)} \equiv R(a_{eq})$.

Taking all the information from equ. 14 to equ. 23 together it is possible to calculate the peak position l_A in equ. 24.

$$\boxed{l_A = \frac{3\pi}{4}\sqrt{\frac{\omega_b}{\omega_\gamma}}\Re\left[\ln\left(\frac{\sqrt{R_s^{(dec)}+R_s^{(eq)}}+\sqrt{1+R_s^{(dec)}}}{1+\sqrt{R_s^{(eq)}}}\right)\right]^{-1}} \qquad (24)$$

From equ. 24 is possible to calculate the peak position l_A of the first peak with the following parameter input:

- From the WMAP-5 year data $\Re = 1.710 \pm 0.019$
- ω_b = 0.02265 ω_m = 0.1369 $\omega_\gamma = 2.469 \times 10^{-5}$

The result is $\boxed{l_A = 299}$

This result is compared to the observations in fig. 7 with $l_1 = 220$ by 79 units off. The shift is originated be neglecting in equ. 24, Dark Energy, free streaming photons and dipole contributions. Doran and Lilley (Mon. Not. Roy. Astron. Soc. 330 (2002) 965) used a shift parameter to calculate the peak position equ. 25

$$l_m = l_A(m - \phi_m) \qquad (25)$$

For m = 1 for the first peak and Φ_m for the shift of multipoles. According the fit of these authors ($\Omega_m \approx 0.4$ and $\Omega_\Lambda \approx 0.6$) they found

$$\boxed{\phi_1 = 0.265}$$

This leads to the observed value of including $l_A = 299$

$$\boxed{l_m = 220}$$

The test of Dark Energy so far is not straight forward because many contributions contribute to the discussed shift.

An other possibility is to recalculate $\Re$ in a flat universe. The shift parameter is under this circumstances in equ. 26.

$$\Re = \sqrt{\Omega_m^{(0)}} \int_0^{z_{dec}} \frac{dz}{E(z)} \qquad (26)$$

Important is in equ. 26 that the term E (z) shown in equ. 27, depends about the density of Dark Energy $\Omega_{DE}^{(0)}$ and the parameter ω_{DE} of the equation of state of the Dark Energy.

$$E(z) = \left[\Omega_m^{(0)}(1+z)^3 + \Omega_r^{(0)}(1+z)^4 + \Omega_{DE}^{(0)}(1+z)^{3(1+\omega_{DE})}\right]^{1/2} \qquad (27)$$

Using equ. 26 and 27 it is possible to calculate the shift parameter $\Re$ as a function of $\Omega_{DE}^{(0)}$ and ω_{DE} what is shown in fig. 9. (ref. (DE-AT)). For larger ω_{DE} the shift parameter decreases.

For $\omega_{DE} = -1$ and using the WMAP bound $\Re = 1.710 \pm 0.019$ the constrain for $\Omega_{DE}^{(0)}$ is

$$\boxed{0.72 < \Omega_{DE}^{(0)} < 0.77}$$

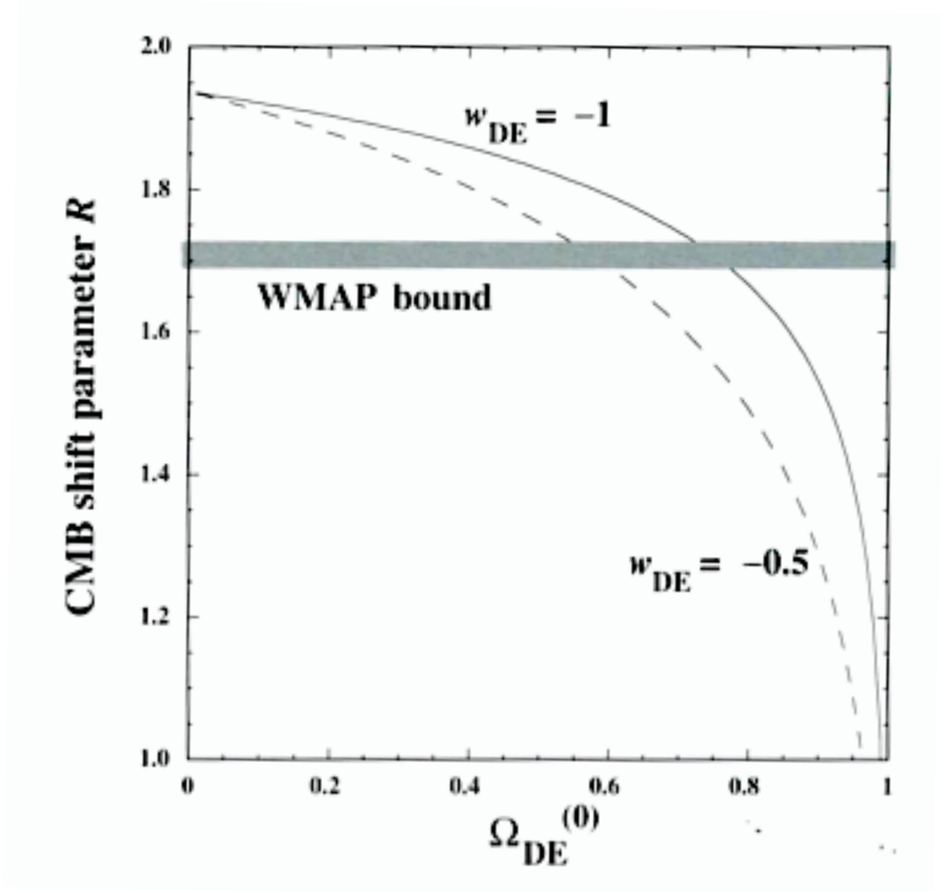


Fig. 9 The shift parameter $\Re$ as function of $\Omega_{DE}^{(0)}$.

To restrict the range of Dark Energy as a function of ω_{DE} it is not sufficient to use only ONE type of observations. It is necessary to combine some data sets of different observation techniques. The combination of 5 sets of data restrict ω_{DE} and $\Omega_{DE}^{(0)}$ very efficient. The following sets are used in the fig. 10.

- WMAP-5 year — Wilkinson Microwave Anisotropy Probe, (arXiv:0803.0547v2 [astro-ph] 17 Oct 2008)
- WMAP+HST — HST - Hubble Space Telescope search for z > 1 “GOLD” data.
- WMAP+BAO — BAO - The baryon acoustic oscillations search. (Astrophys. J.633 (2005) 560)
- WMAP+SN — SN – Supernovae Ia data.
- WMAP+BAO+SN — The baryon acoustic oscillations plus SN – Supernovae Ia data.

The combination of these five data sets are shown in fig. 10
(arXiv:0803.0547v2 [astro-ph] 17 Oct 2008)

The combination of these 5 data sets give the best restriction of ω_{DE} so far:

$$-1.098 < \omega_{DE} < -0.841$$

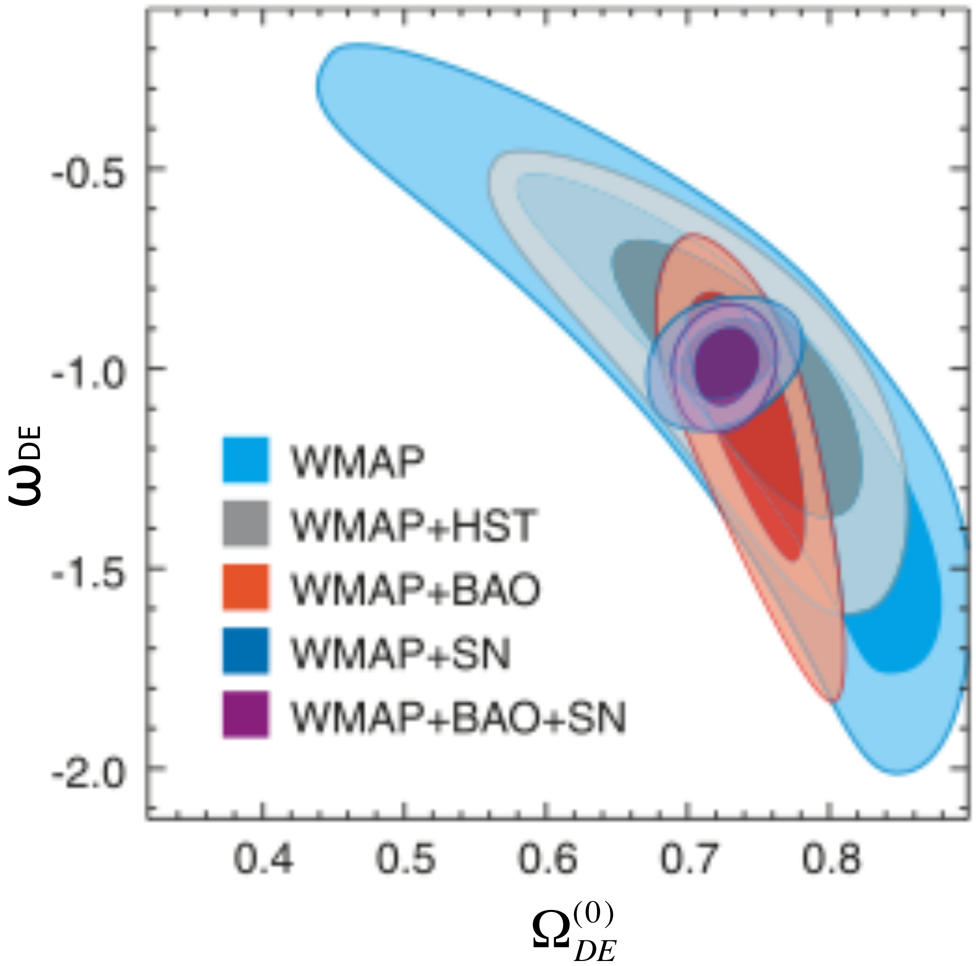


Fig. 10 Observational contours of 68 % and 95 % CL on 5 data sets for ω_{DE} as a function of $\Omega_{DE}^{(0)}$

A.G. Sanchez from the Max Planck Institute reports 2013 the latest values of w_{DE} as function from $\Omega_{DE}^{(0)}$ with the BOSS analysis fig. 11.

The best fit value including the BOSS analysis is

$$\omega_{DE} = -1.03 \pm 0.07$$

This value is perfect in agreement with the ΛCDM.

$$\omega_{DE}(\Lambda CDM) = -1.00$$

nur MWH -> Micro Wave only
BOSS -> Precise cosmology on the large scale structure of the universe.
MHW+BOSS -> Micro Wave + BOSS
MHW+BOSS+SN+BAO -> Micro Wave + BOSS + Super Nova + Baryon Oscillations

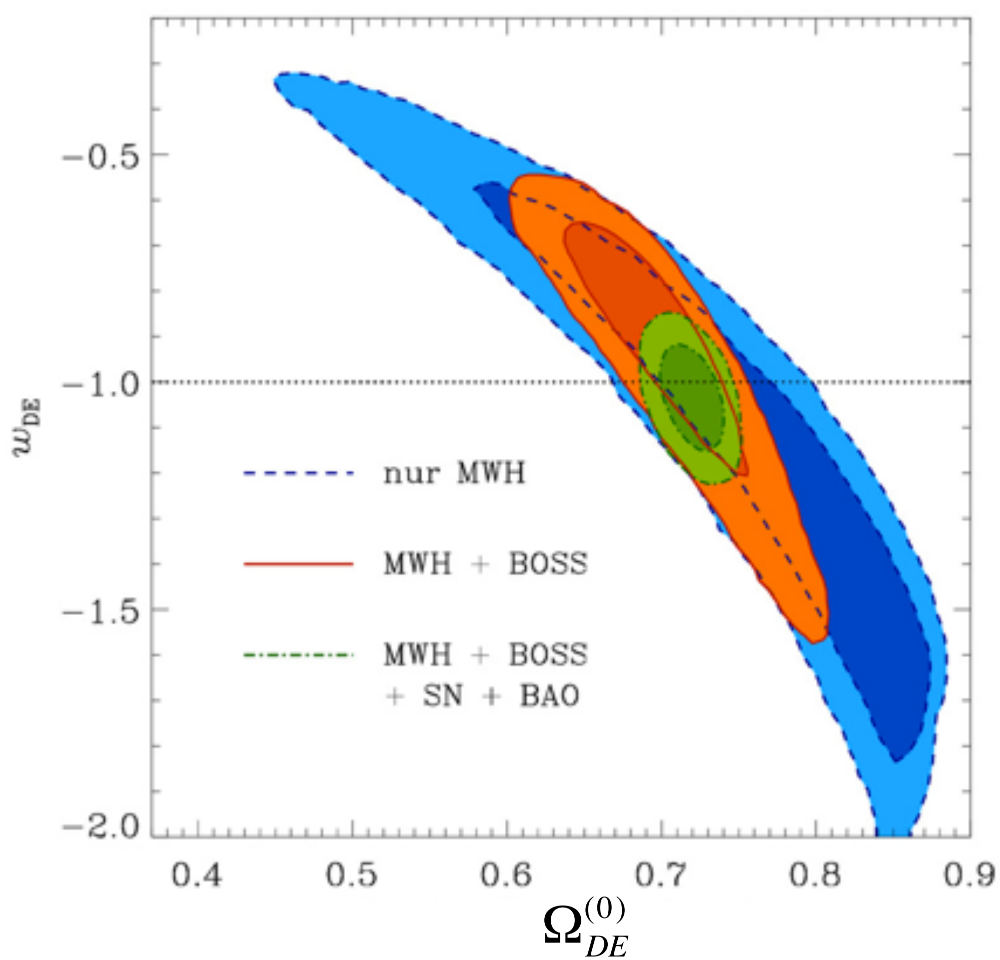


Fig. 11 Observational contours of 68 % and 95 % CL for ω_{DE} as a function of $\Omega_{DE}^{(0)}$.

(Forschungs Bericht 2013 of the Max Planck Institut; www.mpg.de/6730536/MPE_JB_2013?c=7291742)

The age of the Universe compared to old stars

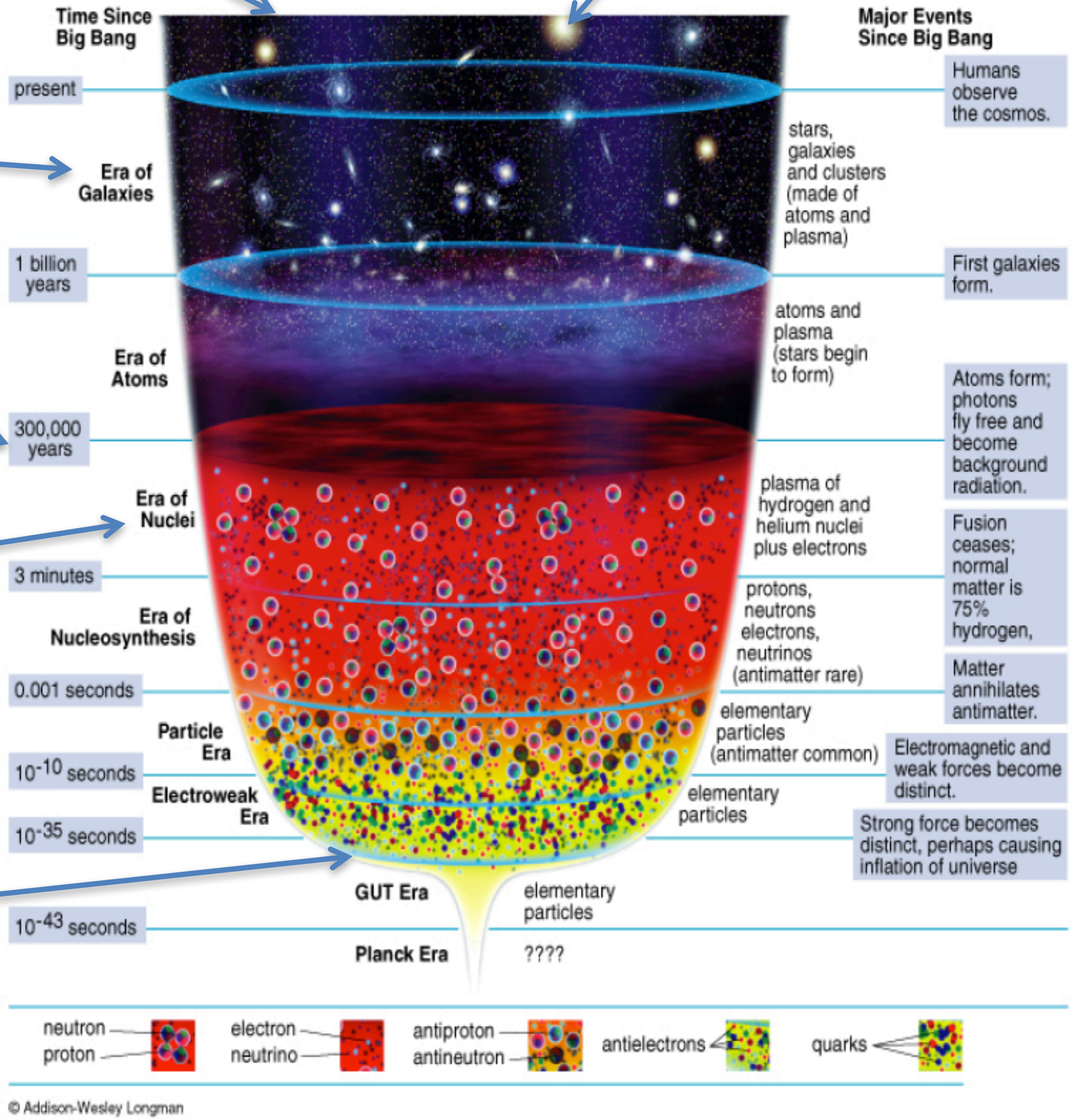
Supernovae observations (SN)

Large-scale structures (LSS)

Cosmic Microwave Background(CBM)

Baryon acoustic oscillations (BAO)

Generation Large-scale structures (LSS)

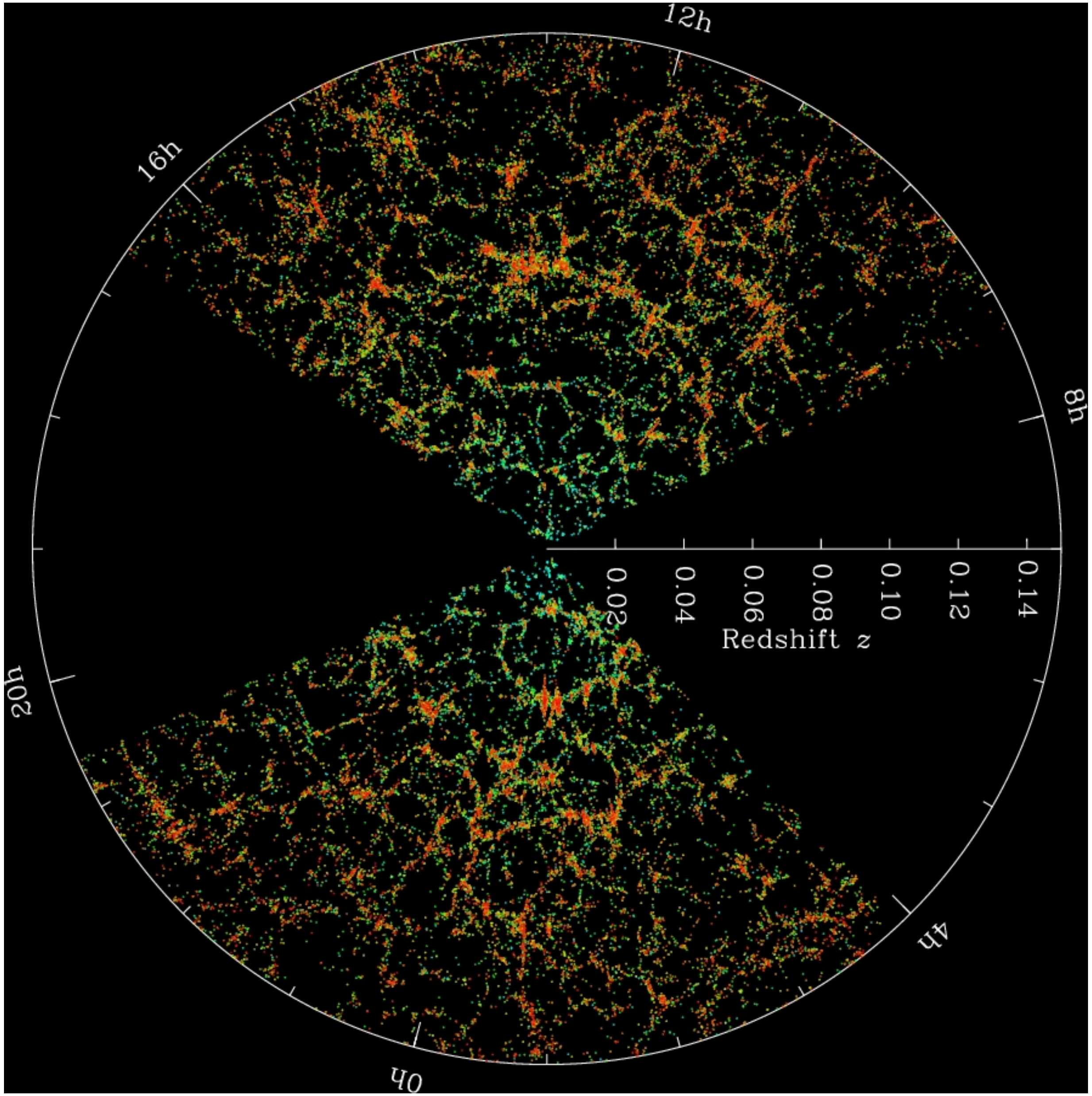


Baryon acoustic oscillations (BAO)

Since baryons are tightly coupled to photons before the recombination epoch, the oscillation of sound waves should be imprinted in the baryon perturbations as well as the temperature anisotropies. The signal is available in the large-scale redshift correlation function $\zeta(s)$ as function of the Comoving separation.

$$\Delta r = (r_1 - r_2) = a (t) (x_1 - x_2)$$

SDSS MAP



Slices through the SDSS 3-dimensional map of the distribution of galaxies. Earth is at the centre, and each point represents a galaxy, containing about 100 billion stars. Galaxies are colored according to the age of their stars, with redder, more strongly clustered points showing galaxies that are made of older stars. The outer circle is at a distance of two billion light years. The region between the wedges was not mapped by SDSS because dust in our Galaxy obscure the view of the distance universe in this direction.

M. Blanton and SDSS
http://www.sdss.org/

According to Peebles the **two-point correlation function** $\xi(r)$ determines a probability dP to find simultaneously two objects on a distance r from each other within two volume elements $\delta V1$ and $\delta V2$ in a sample with number density n as in equ. 28 a.

$$\boxed{dP = n^2 \left[1 + \zeta(r) \right] \delta V_1 \delta V_2} \quad (28\ a)$$

Peebles P. J. E. *The Large-Scale Structure of the Universe*, Princeton University Press, Princeton, New Jersey, 1980

To calculate the large-scale redshift correlation function ζ (s) it is necessary, similar to the cosmic ray back ground, to develop the fluctuations in a power spectrum. In the first step the power spectrum (With power k) in real space $P_r(k)$ get connected to the spectrum in the redshift space $P_s(k)$ by the equ. 28 b

$$P_s(k) = P_r(k)(1+\beta\mu^2)^2 \qquad (28\ b)$$

The large-scale redshift correlation function ζ (s) as function of Δr , the probability that one galaxy will be found within a given distance pin of another and integrated over all powers k of the power spectrum is shown in equ. 29

$$\boxed{\xi_s(r) = \int P_r(k)(1+\beta\mu^2)^2 e^{ikr} d^3k} \qquad (29)$$

$$\beta = f/b \qquad f = \Omega_m^\gamma = \Omega_{DM} + \Omega_b \qquad b = \frac{\delta_g}{\delta_m} \qquad \mu^2 = \frac{k_\pi^2}{k^2}$$

With δ_g as galaxy density contrast and δ_m as total density contrast.

After some development it is possible to write down large-scale redshift correlation function ξ as function from the galaxy velocity σ_v , the observed line-of-sight separation π and integrated over the peculiar velocities v in equ. 30.

$$\boxed{\xi_s(\sigma,\pi) = \int_0^\infty \xi_s(r)\left[\sigma \cdot \pi - \frac{\mathrm{v}(1+z)}{H(z)}\right] f(\mathrm{v})\,d\,\mathrm{v}} \qquad (30)$$

In equ. 30 is v the peculiar velocity and π with f (v) is shown below.

$$\pi = \pi_t + \frac{\mathrm{v}(1+z)}{H(z)} \qquad f(\mathrm{v}) = \frac{1}{\sqrt{2}\sigma_{\mathrm{v}}} e^{-\sqrt{2}|\mathrm{v}|/\sigma_{\mathrm{v}}}$$

Important is also the Comoving separation, what is the distance Δr between two objects connected to the comoving distance x by the relation.

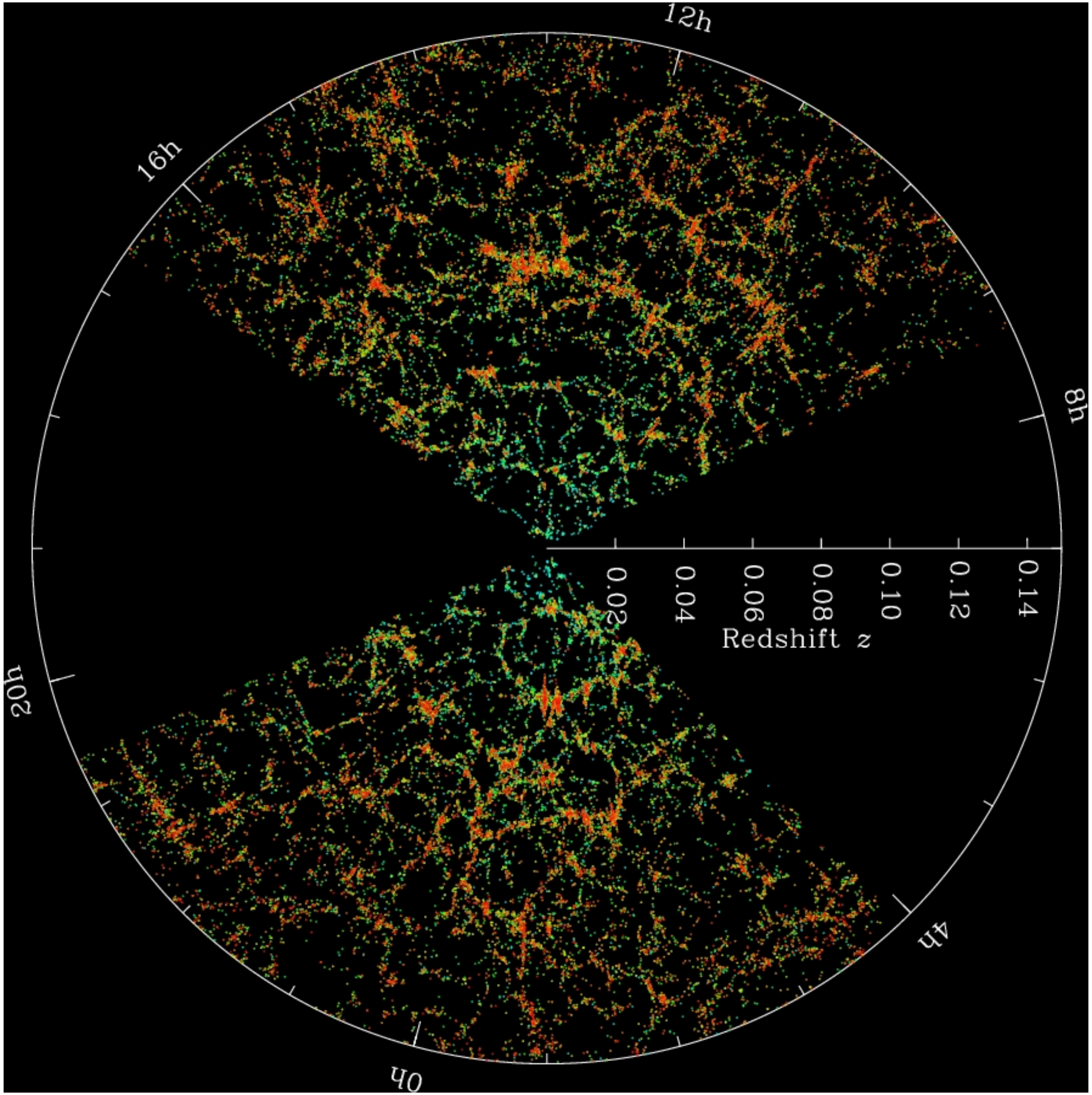
$$\Delta r = (r_1 - r_2) = a (t) (x_1 - x_2)$$

SDSS MAP

in equ. 30

Visible are all parameters of equ. 30 and 29:

- Volume:
- δV_1 ; δV_2
- Contrast:
- δ_g ; δ_m
- Velocity:
- σ_V ; v
- Redshift
- z
- Fit-PARAMETERS
- Distance:
- Δr
- Densities:
- $\Omega_m = \Omega_{DM} + \Omega_b$



Eisenstein et. al. (arXiv:astro-ph/0501171v1 10 Jan 2005) found a peak of baryon acoustic oscillation in the large-scale correlation function ξ at 100 Mpc / h separation measured from a spectroscopic sample of 46 748 luminous red galaxies from the Slon Digital Sky Suvey (SDSS) shown in fig. 12.

$$\Omega_m^{(0)} = \Omega_{DM} + \Omega_b$$

Shown are 3 cases with $\Omega_b^{(0)} h^2 = 0.024$ of the SDSS –model :

- $\Omega_m^{(0)} h^2$ = 0.12 (green)
- $\Omega_m^{(0)} h^2$ = 0.13 (red)
- $\Omega_m^{(0)} h^2$ = 0.14 (blue)

and one case with no baryons, of the pure COLD DARK MATTER CDM model:

- $\Omega_m^{(0)} h^2$ = 0.105 (purple)

A clear signal is visible at 100 Mpc / h for the SDSS – model including BARYIONS.

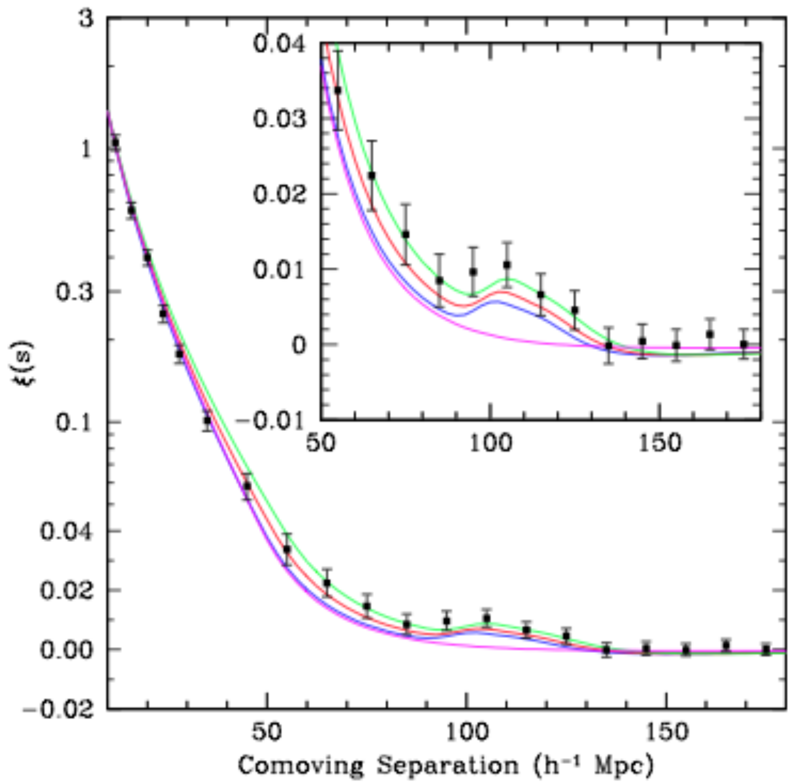


fig. 12 large-scale correlation function ξ as function of comoving separation.

At 2006 C. L. Bennet published an extended analysis (Nature 440 (2006) 1126) shown in fig. 13.

The data are from red luminous galaxies in the 3816 square degree Sloan Digital Sky Survey (SDSS) (*Nature* 386 (1997) 37). The baryon oscillation signal visible at ~100 h^{-1} Mpc is detected at 3.4*σ.*

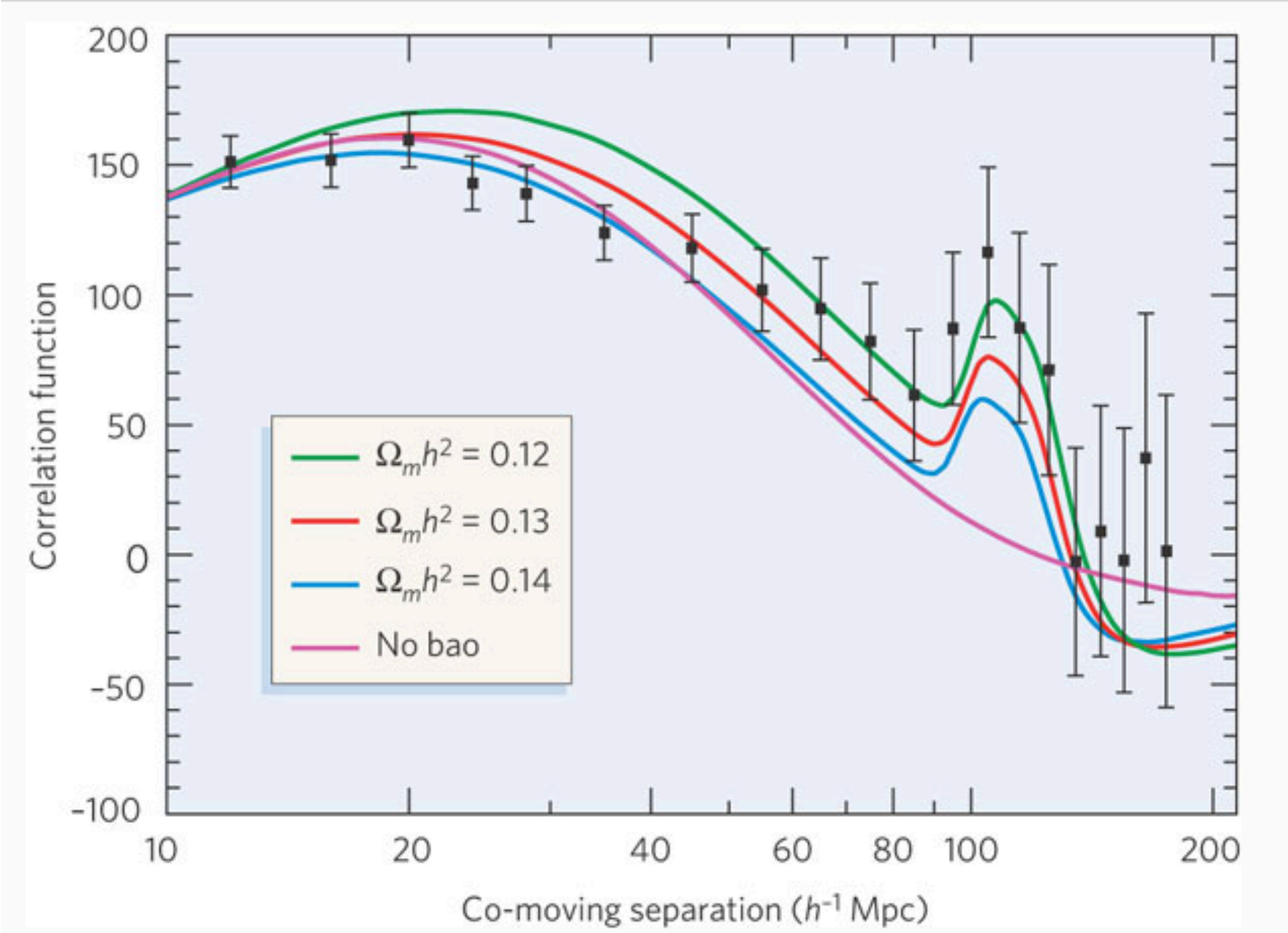


fig. 13 Galaxy redshift–space correlation function as function of comoving separation. The co-moving sound horizon scale is ~150 *h*–1 Mpc.

The impact of Dark Matter on the baryon acoustic oscillations is in fig. 12 and 13 clear demonstrated.

The impact of Dark Energy is visible in the red shift z_{drac} and the radius r_{bao} where the oscillation (Compton d_{drac}) stops.

Impact of the dark energy on the baryon acoustic oscillations

As discussed in the beginning two parameters are important to search for the impact for DARK ENERGY on the BARYONIC OSCILLATIONS.

First the REDSHIFT z_{drac} where the Compton drac stops. It is possible after D. J. Eisenstein and W. Hu Astrophys. J. 496 (1998) 605 to calculate z_{drac} with the equation. .

$$z_{drac} = \frac{1291\omega_m^{0.251}}{1+0.659\omega_m^{0.828}}(1+b_1\omega_b^{b_2})$$

$$\omega_i = \Omega_i^{(0)} \cdot h^2 \qquad b_1 = 0.313\omega_m^{-0.419}(1+0.607\omega_m^{0.674}) \qquad b_2 = 0.238\omega_m^{0.233}$$

Second the sound horizon r_s (z_{drac}) at which baryons were released from the Compton drag of photons plays a crucial role to determine the location of baryon acoustic oscillations at z = zdrac . This parameter is the related to the Baryon Acustic Distance BAO r_{BAO}(z) as function of the redshift z and Energy E (z) shown in equ. 31

$$\boxed{r_{BAO}(z) \equiv r_s(z_{drac}) / D_V(z, E_z)} \qquad (31)$$

The sound horizon r_s (z_{drac}) is

$$\boxed{r_s(z_{drac}) = \int_0^{\eta_{drac}} d\eta \cdot c_s(\eta)}$$

$$c_s^2 \equiv \frac{1}{3(1+R_s)} \qquad R_s \equiv \frac{3}{4}\frac{\rho_b}{\rho_\gamma}$$

The comoving diameter distance is as before $d_A^{(c)}(z_{dec}) = (1+z)d_A(z)$ leading to the effective distance $D_V(z)$ as a function the spatial dimensions orthogonal and along the direction of sight.

$$\boxed{D_V(z) = \left[(1+z)^2 d_A^2(z, E(z)) \frac{cz}{H(z)}\right]^{1/3}}$$

Including the definition:

- Δx Size of the object orthogonal to the line of sight.
- $\Delta\theta$ angle to the object orthogonal to the line of sight.

$$\boxed{d_A \equiv \frac{\Delta x}{\Delta \theta}}$$

$$d_A(E) = \frac{1}{1+z} \frac{c}{H_0\sqrt{\Omega_K^{(0)}}} \sinh\left(\sqrt{\Omega_K^{(0)}} \int_0^{\tilde{z}} \frac{d\tilde{z}}{E(\tilde{z})}\right)$$

Including the details of equ. 31, it is possible to write down the equation for r_{BAO} equ. 32

$$r_{BAO}(z) = \frac{4}{3}\sqrt{\frac{\omega_\gamma}{\Omega_m^{(0)}\omega_b}}\left[\frac{z}{E(z)}\right]^{-1/3}\left[\frac{1}{\sqrt{\Omega_K^{(0)}}}\sinh\left(\sqrt{\Omega_K^{(0)}}\int_0^z \frac{d\tilde{z}}{E(\tilde{z})}\right)\right]^{-2/3} \times \ln\left(\frac{\sqrt{R_s^{(drac)}+R_s^{(eq)}}+\sqrt{1+R_s^{(drac)}}}{1+\sqrt{R_s^{(eq)}}}\right) \qquad (32)$$

The functions ω and R_s are shown in equ. 33.

$$\omega_i = \Omega_i^{(0)} \cdot h^2 \qquad R_s^{(drag)} \equiv R_s(z_{drac}) = (3\omega_b/\omega_\gamma)a_{drac} \qquad R_s^{(eq)} \equiv R_s(z_{eq}) = (3\omega_b/\omega_\gamma)a_{eq} \qquad (33)$$

Including the scale-factor at radiation-matter equality

$$a_{eq} = (1+z_{eq})^{-1}$$

Including the scale-factor were baryons get released from the Compton drag of photons

$$a_{drac} = (1+z_{drac})^{-1}$$

With equ. 32 it is possible to calculate rBAO in fig. 14 for three cases including the observational values from the

2 – degree Field (2df) Galaxy Redshift Survey z = 0.2 and z = 0.35

- rBAO (z = 0.2) = 0.1980±0.0058
- rBAO (z = 0.35) = 0.1094±0.0033

For the tree cases:

$$\boxed{(i) \qquad \Omega_{DE}^{(0)} = 0.75}$$

$$(ii) \qquad \Omega_{DE}^{(0)} = 0.0$$

$$(iii) \qquad \Omega_{DE}^{(0)} = 0.95$$

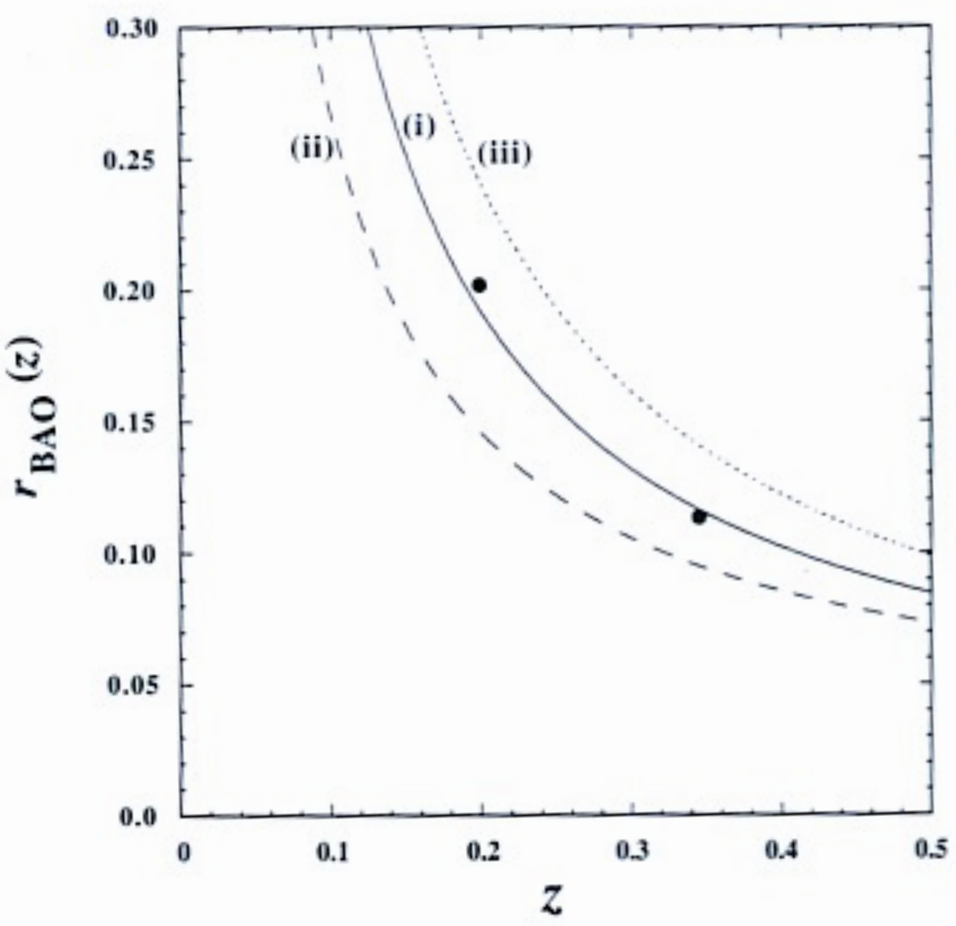


Fig. 14 rBAO is function of z for K = 0 ω_{DE} = -1 and $\omega_\gamma = 2.469 \times 10^{-5}$ $\omega_b = 0.02265$

(DE-AT)

The age of the Universe compared to old stars

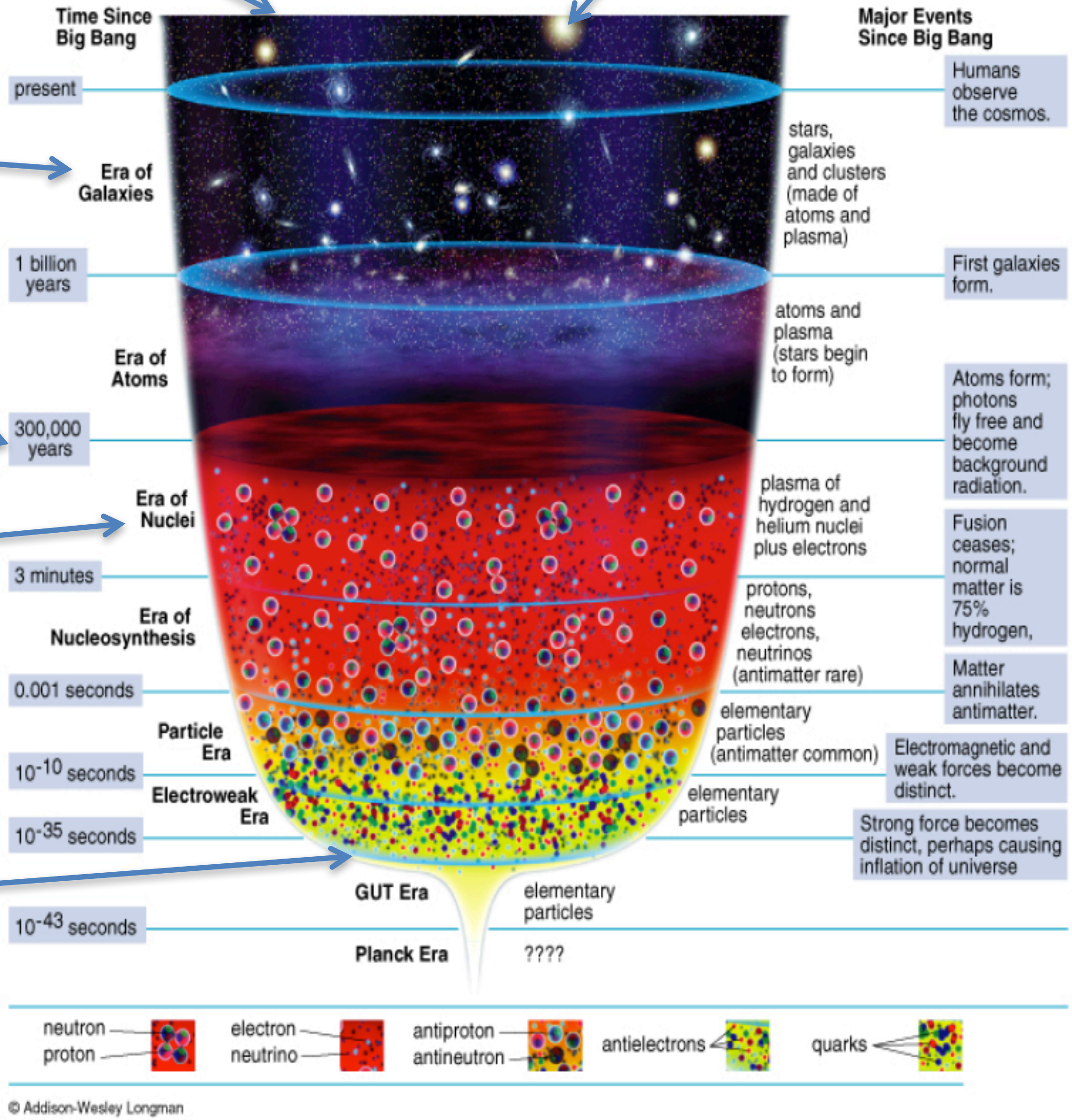
Supernovae observations (SN)

Large-scale structures (LSS)

Cosmic Microwave Background(CBM)

Baryon acoustic oscillations (BAO)

Generation Large-scale structures (LSS)



Large-scale structures (LSS)

The large-scale structures such as the galaxy clustering provides on more test of the existence of Dark Energy.

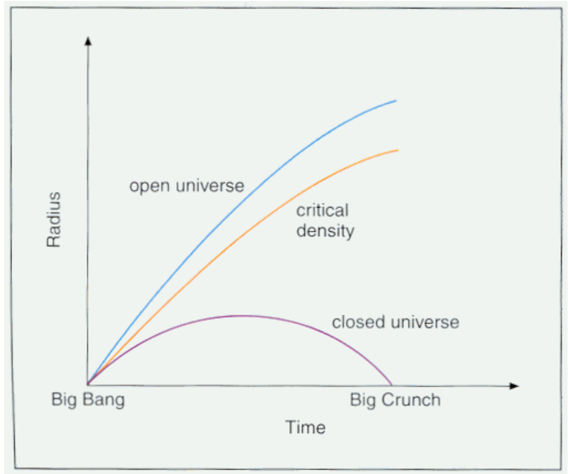
To investigate these structures it is necessary to introduce first the according metric. As fare we used the Robertson – Walker metric:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

(x_i, t_i)

ds 4-dim

($x_{(i+1)}$, $t_{(i+1)}$)



In this metric is the parameter to describe the universe a and k. For example k = -1 (open), k = 0 (critical) and k = 1 (closed)

For the LSS structure it is usefully to use the Newtonian metric:

$$\boxed{ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1+2\Phi)\delta_{ij}dx^i dx^j\right]}$$

With $\eta = \int a^{-1}dt$, Ψ and Φ as gravitational potentials. In this case are the both potentials Ψ and Φ the important parameters to describe the universe.

Again an initial power spectrum $P_{\Phi}^{(i)}$ power k of Φ is introduced. It is generated during inflation at the present epoch (inflation), shown in equ. 34.

$$\boxed{P_{\delta_m} \equiv \left\langle \left| \delta_m(k,a_0)^2 \right| \right\rangle = \frac{2\pi^2\delta_H^2}{(\Omega_m^{(0)})^2}\left(\frac{k}{H_0}\right)^{n_s} T^2(k)D^2(a_0)H_0^{-3}} \qquad (34)$$

The parameters are:

- n_s the spectral index for $\delta_m(k,a) = \frac{2k^2 a}{3\Omega_m^{(0)}H_0^2}\Phi(k,a)$
- δ_H^2 the amplitude of the gravitational potential
- $k^3\left\langle \left|\Phi(k,a_i)\right|^2 \right\rangle = c$ is constant.
- the transfer function T (k) from radiation (ρ_γ) to matter (ρ_ν) epoch.

$$T(k) \equiv \frac{\Phi(k,a_T)}{\Phi_{LS}(k,a_T)} \qquad \Omega_{LS}(k,a_i) = \frac{9}{10}\Omega(k,a_i)$$

- $\frac{\Phi(a)}{\Phi(a_T)} = \frac{D(a)}{a} \quad a > a_T$

and the relation between Φ and Ψ $\quad \Phi = -\Psi\left(1 + \frac{2}{5}f_\nu\right) \qquad f_\nu = \frac{\rho_\nu}{\rho_\nu + \rho_\gamma}$

The gravitational potential today Φ (k, t) from the beginning of the radiation-dominated epoch to the present, depends about the mode k. For this reason it is necessary to know k_{eq} that characterizes the border of Large-Scale and Small-Scale modes. This is shown in equ. 35.

$$\boxed{k_{eq} = H_0 \sqrt{\frac{2\Omega_m^{(0)}}{a_{eq}}}} \qquad (35)$$

Fig. 15 shows a calculation of $P_{\delta m}$ as function of k_{eq}. with the following parameters:

- ΛCDM model including DE $\Omega_m^{(0)} = 0.28$
- CDM model excluding DE $\Omega_m^{(0)} = 1.0$
- $\delta_H^2 = 3.2 \times 10^{-10}$

The peak of the CDM model shift from

$$\boxed{k_{eq} = 0.051 \cdot h \; \left[Mpc^{-1}\right]}$$

to lower values for the ΛCDM model

$$\boxed{k_{eq} = 0.014 \cdot h \; \left[Mpc^{-1}\right]}$$

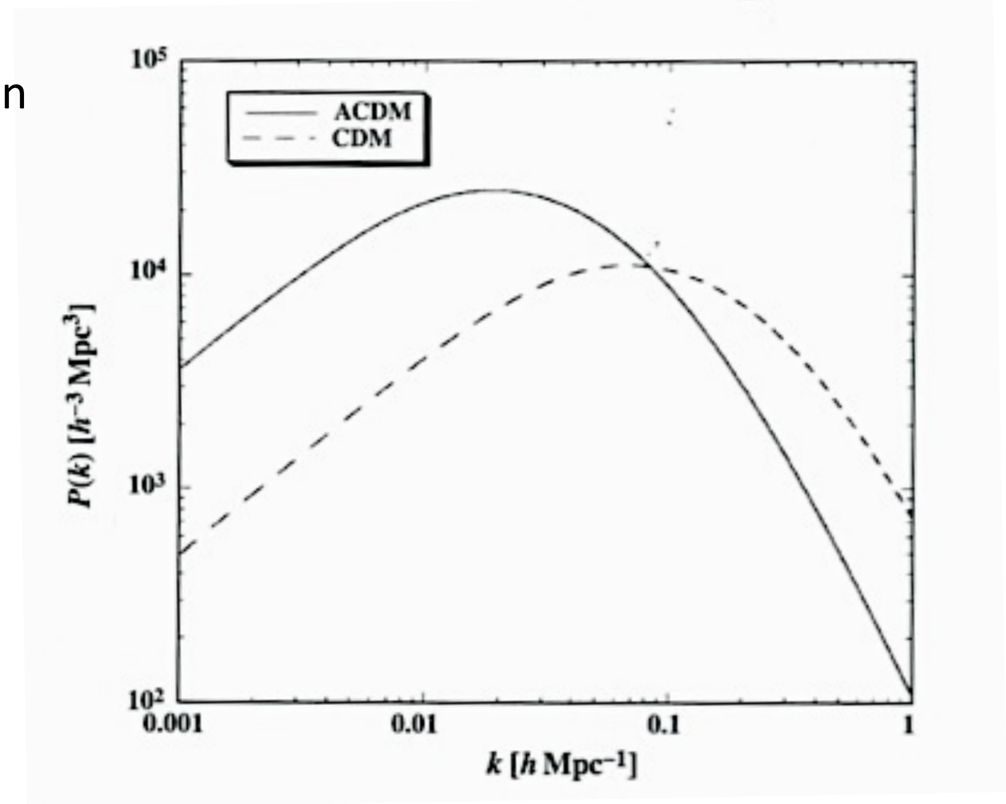


Fig. 15 $P(k) \equiv P_{\delta_m}$ as function of k_{eq} = k
ref. (DE-AT)

The matter spectrum of the total density contrast P_{δ_m} is related to the observed galaxy power spectrum P_{δ_g} by the equ. 36.

$$\boxed{P_{\delta_g} = b^2 P_{\delta_m}} \qquad (36)$$

The scale factor or bias factor $b \equiv \dfrac{\delta_g}{\delta_m}$

→ δ_g is the observed galaxy density contrast

→ δ_m the total density contrast of all galaxies.

Both parameters are depending about the observation and of WMAP data, the SDSS luminous Red Galaxies data and different theories. For example in ref. arXiv:astro-ph/0608632v2 30 Oct 200 Table 3

$$\boxed{1.853^{+0.081}_{-0.077} < b < 2.03^{+0.11}_{-0.10}}$$

In fig. 16 the measured galaxy power spectra $P_{\delta_g} \equiv P(k)$ of LUMINOS RED GALAXIES (LRG) and main (Main)galaxy samples of the SDAA are plotted via the k – mode $k_{eq} \equiv k$ of equ. 35 (ref. arXiv:astro-ph/ 0608632v2 30 Oct 200) .

The used parameters are:

- The theoretical spectra for the ΛCDM model for the LRG data with galaxy bias b = 1.9 (solid red)
- The theoretical spectra for the ΛCDM model for the Main data with galaxy bias b = 1.1 (solid red)
- Non-linear corrections by Cole et. al. (Not. Roy. Astron. Soc. 362 2005 505) (red broken lines)

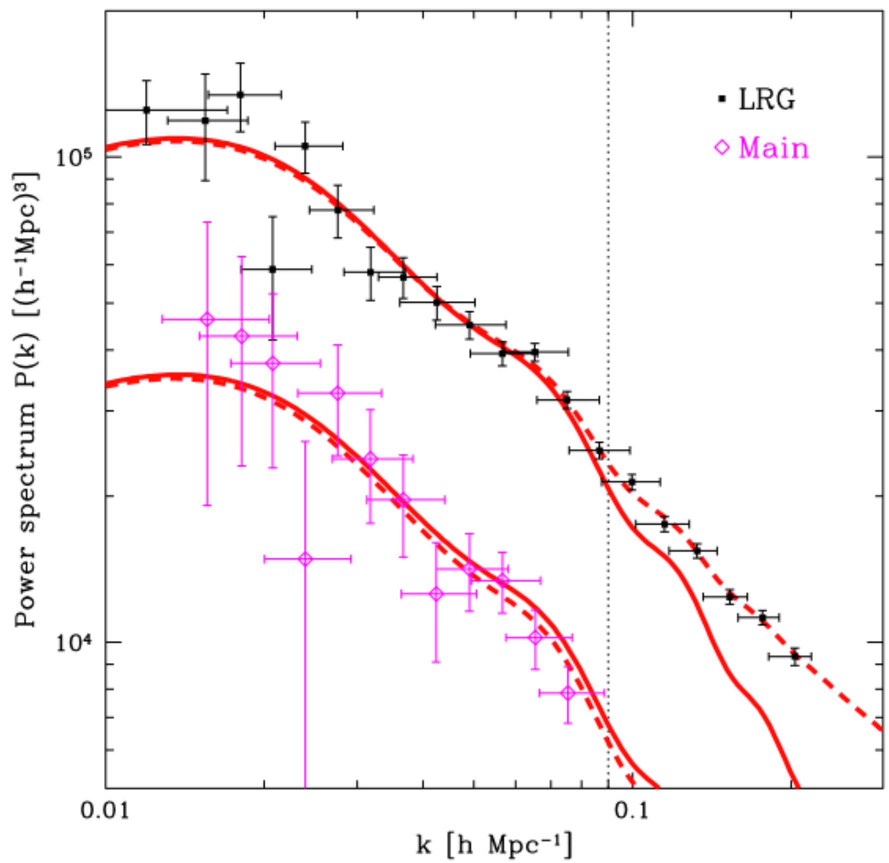


Fig. 16 Power spectrum $P_{\delta_g} \equiv P(k)$ via $k_{eq} \equiv k$.

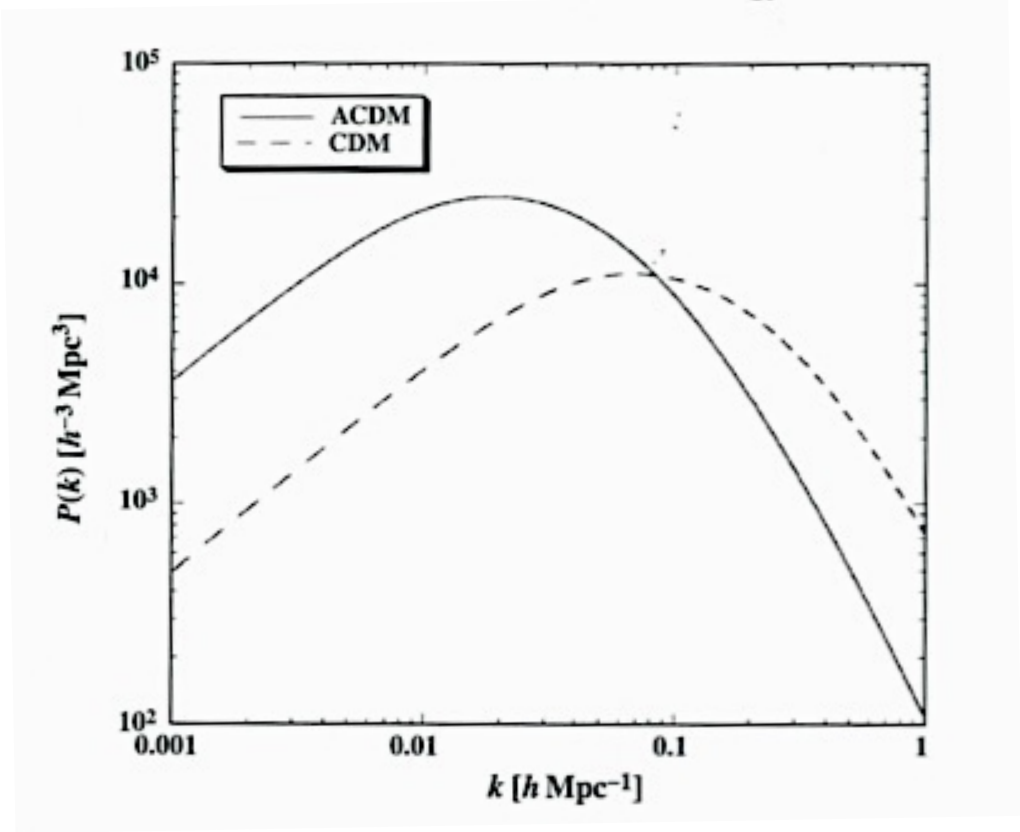


Fig. 15 $P(k) \equiv P_{\delta_m}$ as function of keq = k
ref. (DE-AT)

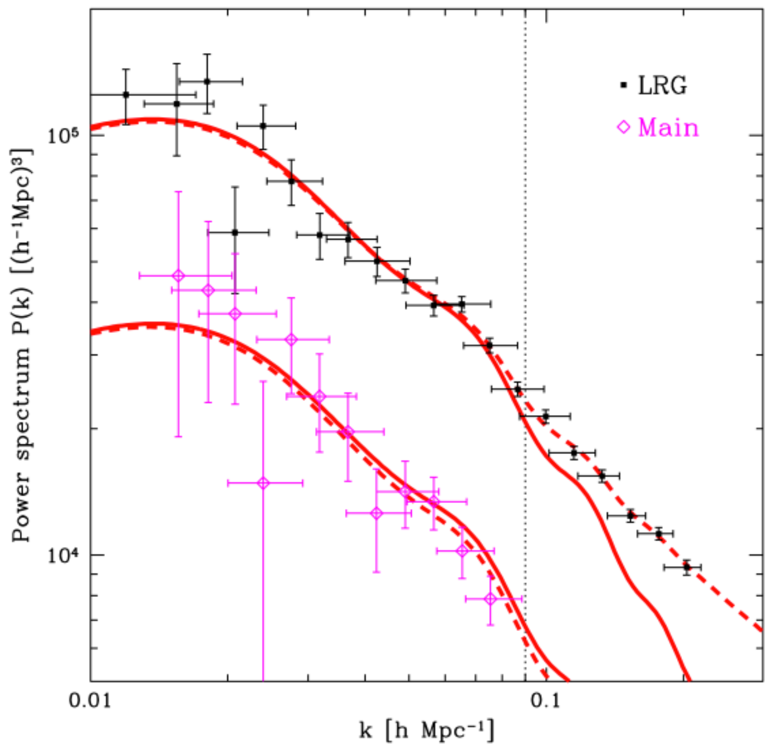


Fig. 16 Power spectrum $P_{\delta_g} \equiv P(k)$ via $k_{eq} \equiv k$.

The position of the peak is in fig. 15

$$\boxed{0.01 hMpc^{-1} < k < 0.02 hMpc^{-1}}$$

is in agreement with the peak position of fig. 16

This favours the ΛDCM – model what includes DARK ENERGY .

Non linear effects get important for $\boxed{k \geq 0.09 hMpc^{-1}}$

The data from the, Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results

C. L. Bennett et. al. arXiv:1212.5225v3 [astro-ph.CO] 4 Jun 2013, give a very precise overview about the correlation between Dark Matter and Dark Energy. The data shown in fig. 19 are displayed under the following conditions:

- Flat universes fall on the $\Omega_m + \Omega_\Lambda = 1$ line.
- Allowed regions are shown for WMAP, CMB, and CMB combined with BAO and H_0 data, all with a hard prior of $H_0 < 100\ \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$.
- WMAP data is represented by 290,000 Markov chain points, colored by their value of H_0.
- The WMAP data follow a geometric degeneracy ridge represented by the slightly curved line, a parabola with equation $\Omega_\Lambda = 0.06202\ \Omega_\Lambda^2 - 0.825\ \Omega_m + 0.947$

The best fit values are:

$\Omega_\Lambda = 0.721$

$\Omega_m = 0.279$

Constraints are consistent with:

$\Omega_{tot} = 1.0027^{+0.0038}_{-0.0039}$

The contours show constraints when adding high-l CMB data (blue) and BAO and H_0 data (black).

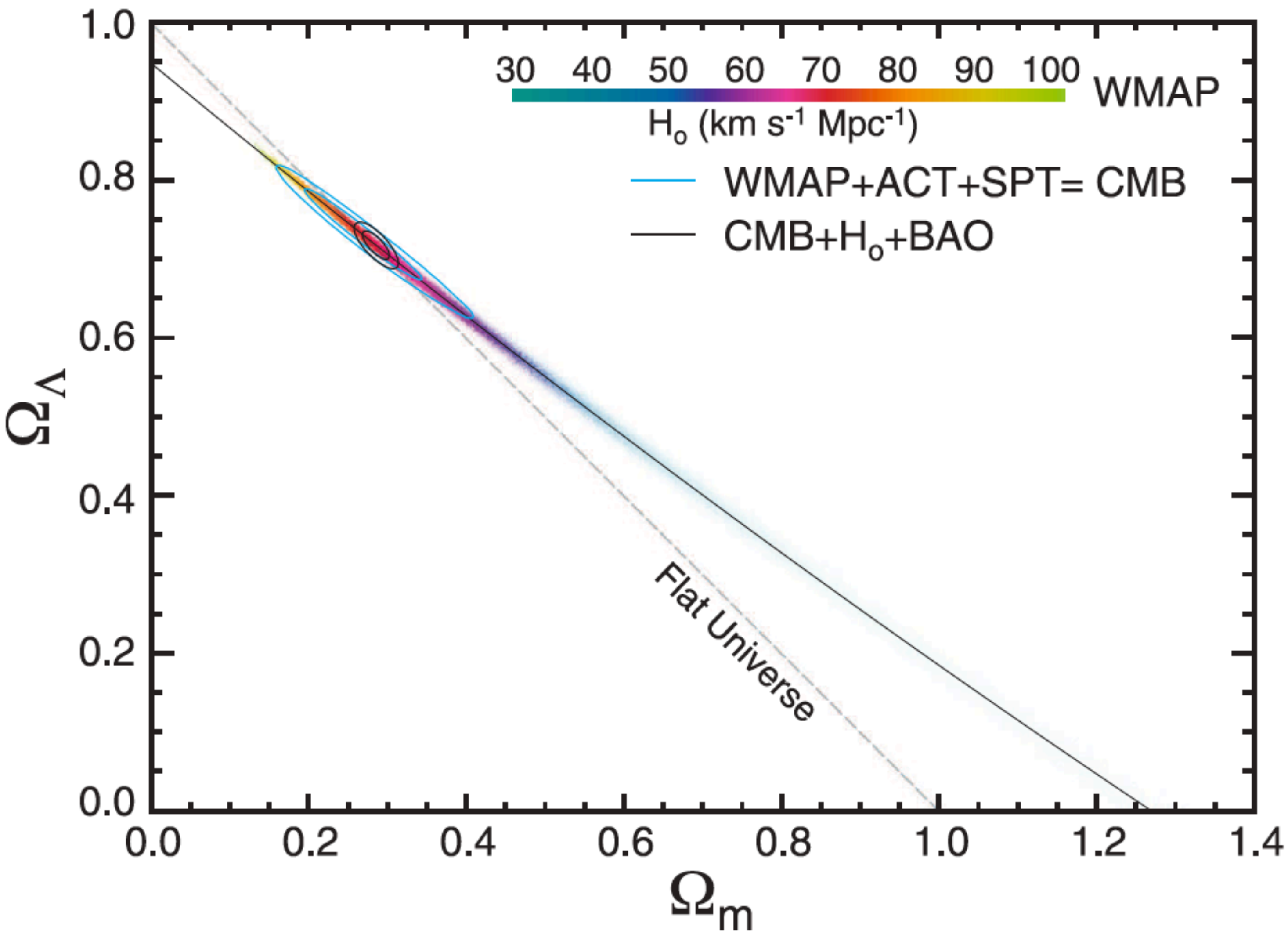


Fig 19 Dark Energy content Ω_Λ as function of Dark Matter content Ω_m .

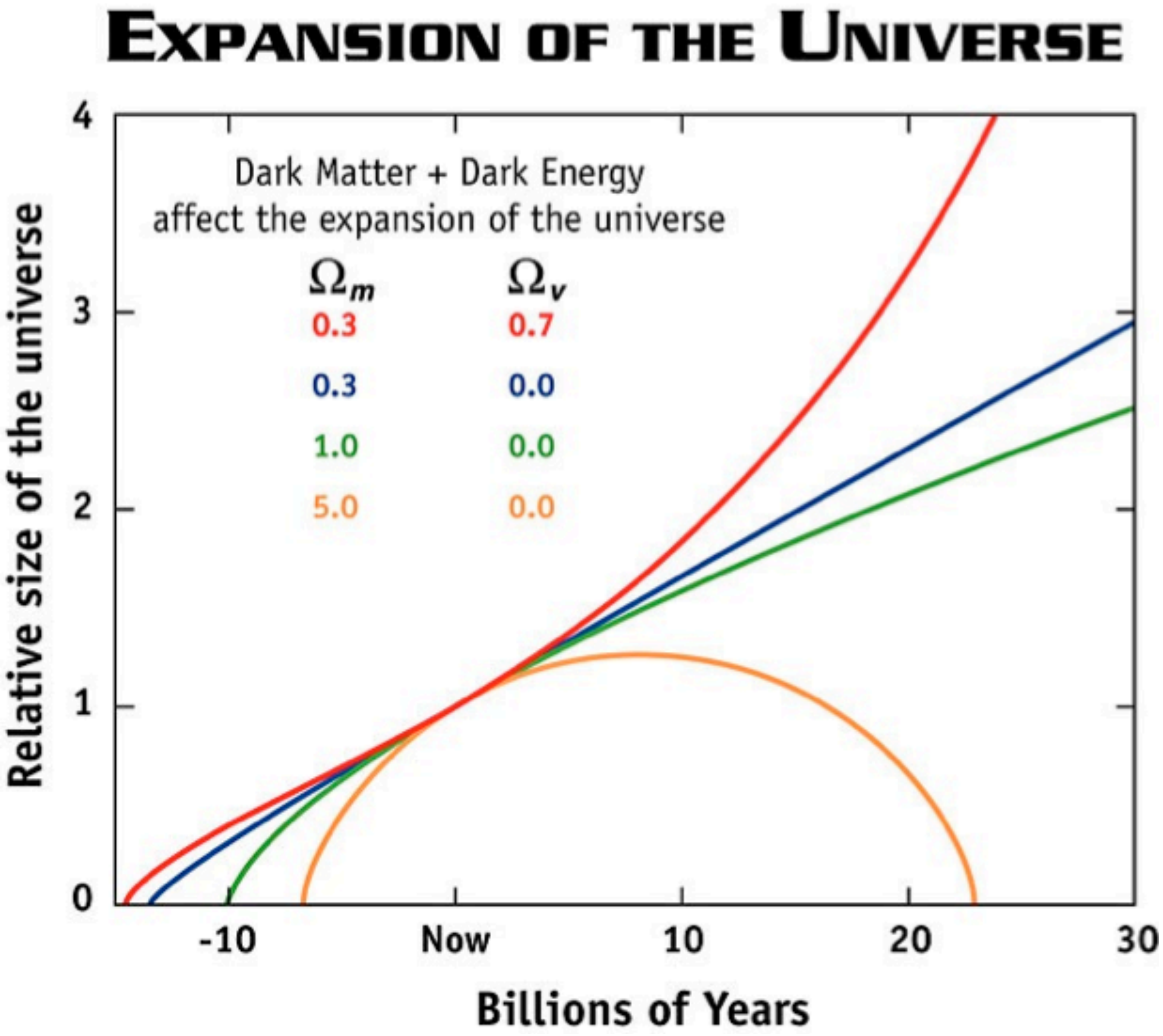
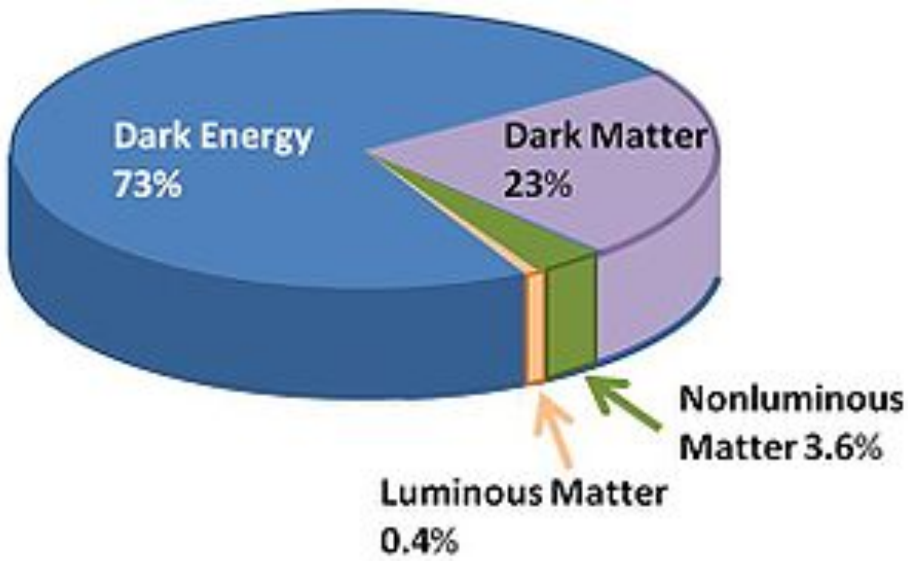


Fig. 18 Expansion of UNIVERSE (Wikipedia)

CONCLUSION

The Einstein equations as source of the Big Bang theory and the ΛCDM model together with observations in the last 15 years have established evidence of DARK ENERGY. But the nature of DARK ENERGY is still under controversy discussion.

The composition of the total energy of the universe with 73 % Dark Energy, 23 % Dark Matter and and 4.0 % Baryonic Energy tells us that we understood so far only some percent of the universe. The physics is still at the beginning.



OUTLOOK

Substantial effort is on the way to study the nature of DARK ENERGY. A collaboration called " THE DARK ENERGY SURVEY (https://www.darkenergysurvey.org) organizes the details of the DE research. A special camera " The Dark Energy Camera " is installed (arXiv:1504.02900v1 [astro-ph.IM] 11 Apr 2015) on the Victor M. Blanco 4-meter telescope on
Cerro Tololo near La Serena, Chile. The camera was designed and constructed by the Dark Energy Survey Collaboration, and meets or exceeds the stringent requirements designed for the wide-field and supernova surveys for which the collaboration uses it.