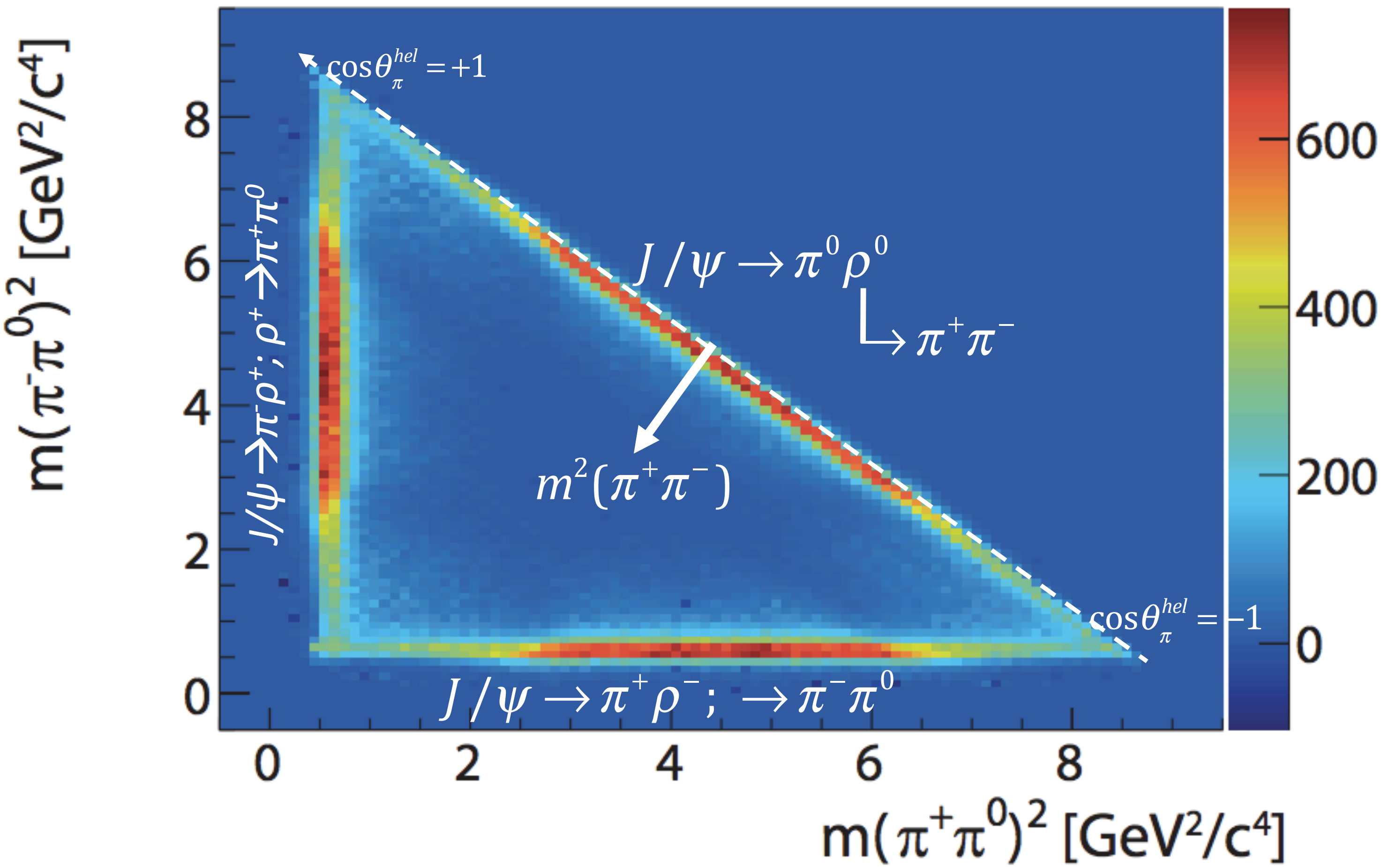
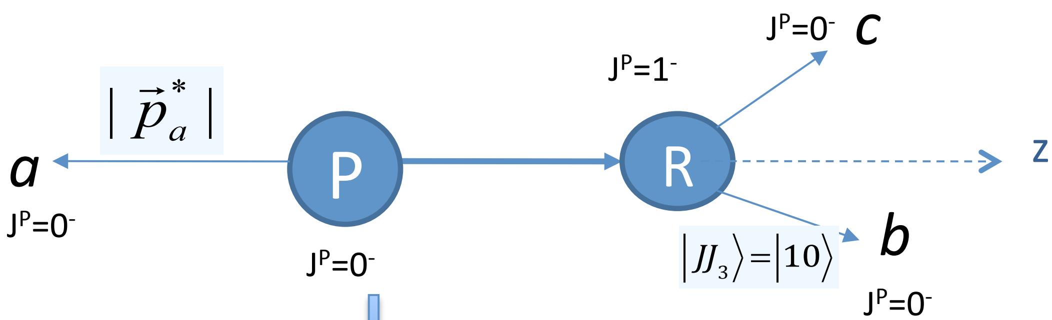
Dalitz plots II



Stephen Lars Olsen

3-body decay $P \to aR;\ R \to b+c$

where P has $J^P=0^-$ and a, b & c are pions or kaons



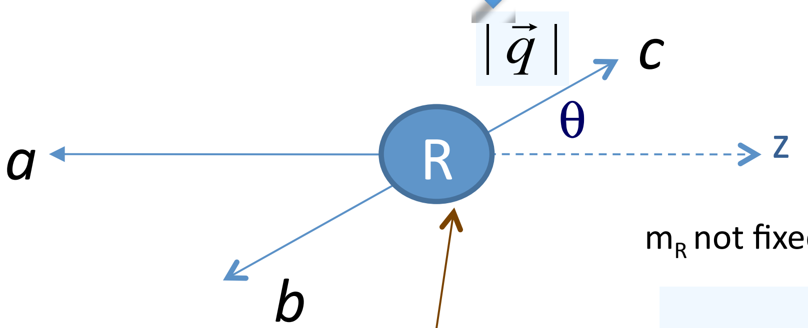
Energy-momentum conservation

$$p_P = p_a + p_R$$

$$p_R = p_b + p_c$$

4-vectors

Transform to the R rest frame:



$$|JJ_3\rangle = |10\rangle = Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

Dalitz plot quantities

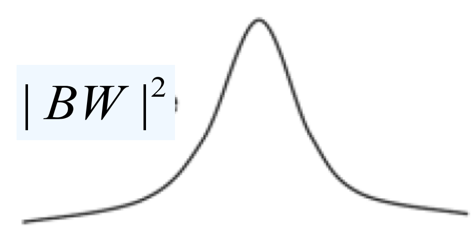
$$\cos\theta = \frac{m_R}{4|\vec{p}_a^{\,*}||\vec{q}\,|m_P}\left[(m_{ac}^2 - m_{ab}^2) + \frac{(m_P^2-m_a^2)(m_b^2-m_c^2)}{m_R^2}\right]$$

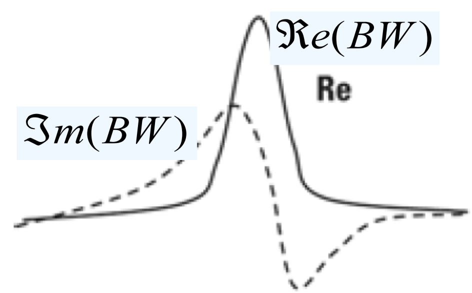
m_R not fixed; has a BW resonance form:

$$BW = \frac{i\sqrt{m_{bc}\Gamma_0}}{(m_0^2 - m_{bd}^2) - im_0\Gamma_0}$$

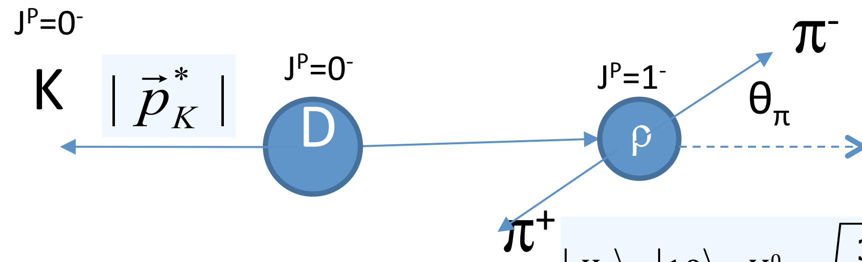
$$\Re e(BW) = \frac{(m_0\Gamma_0)^{3/2}}{(m_0^2-m_{bc}^2)^2 + m_0^2\Gamma_0^2}$$

$$\Im m(BW) = \frac{\sqrt{m_0\Gamma_0}(m_0^2-m_{bc}^2)}{(m_0^2-m_{bc}^2)^2 + m_0^2\Gamma_0^2}$$





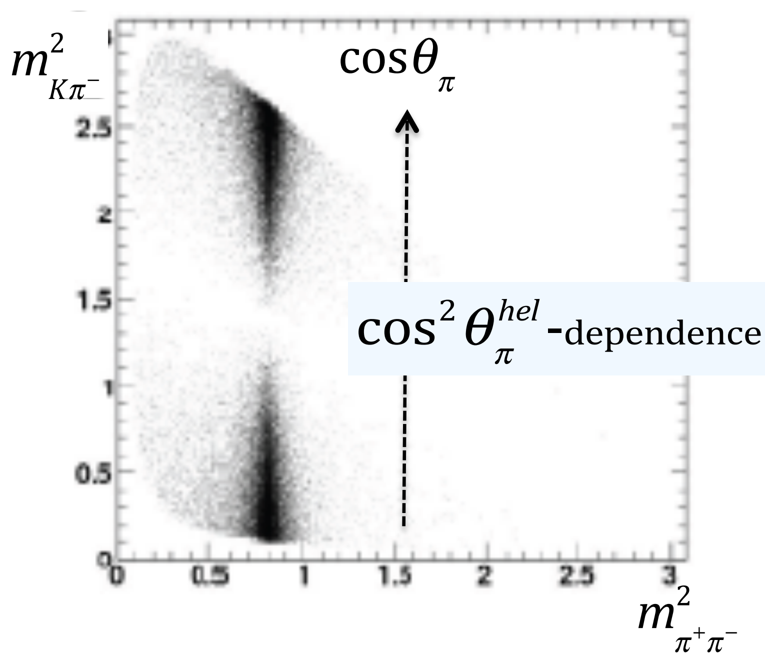
$$D \to K\rho^0$$
$$\quad\hookrightarrow \pi^+\pi^-$$



$$|JJ_3\rangle = |10\rangle = Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta_\pi$$

$$|\mathcal{M}|^2 = \left|BW \times |10\rangle\right|^2 = \left|\frac{i\sqrt{m_{\pi^+\pi^-}\Gamma_0}}{(m_0^2 - m_{\pi^+\pi^-}^2) - im_0\Gamma_0}\sqrt{\frac{3}{4\pi}}\cos\theta_\pi\right|^2$$

$$\cos\theta_\pi = \frac{m_{\pi^+\pi^-}(m_{K\pi^-}^2 - m_{K\pi^+}^2)}{4|\vec{q}_{\pi\pi}||\vec{p}_K^{\,*}|m_D}$$



I used the simplest BW factor

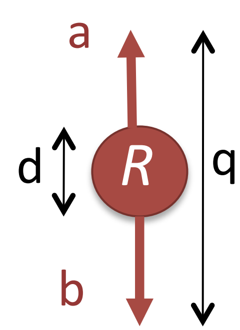
$$BW = \frac{i\sqrt{m_{ab}\Gamma_0}}{(m_0^2 - m_{ab}^2) - i m_0 \Gamma_0}$$

Usually BW is multiplied by a "Barrier Factor" to account for the finite size of the resonance

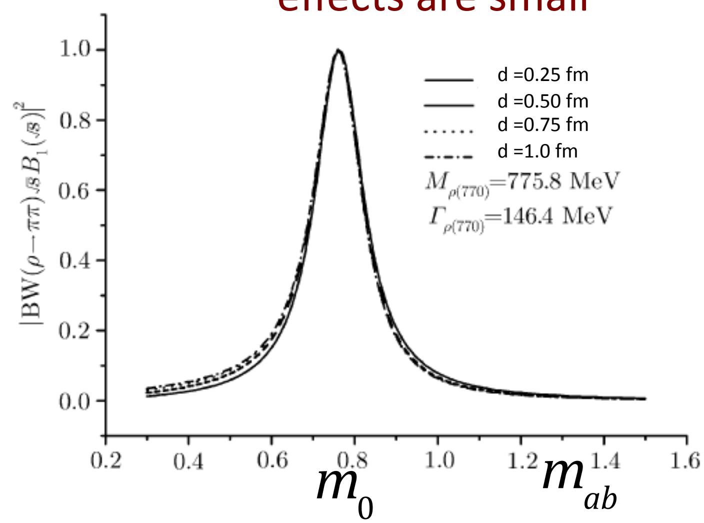
L	$B_L(q)$	$B'_L(q, q_0)$
0	1	1
1	$\sqrt{\frac{2z}{1+z}}$	$\sqrt{\frac{1+z_0}{1+z}}$
2	$\sqrt{\frac{13z^2}{(z-3)^2+9z}}$	$\sqrt{\frac{(z_0-3)^2+9z_0}{(z-3)^2+9z}}$

where $z = (\lvert q\rvert\, d)^2$ and $z_0 = (\lvert q_0\rvert\, d)^2$

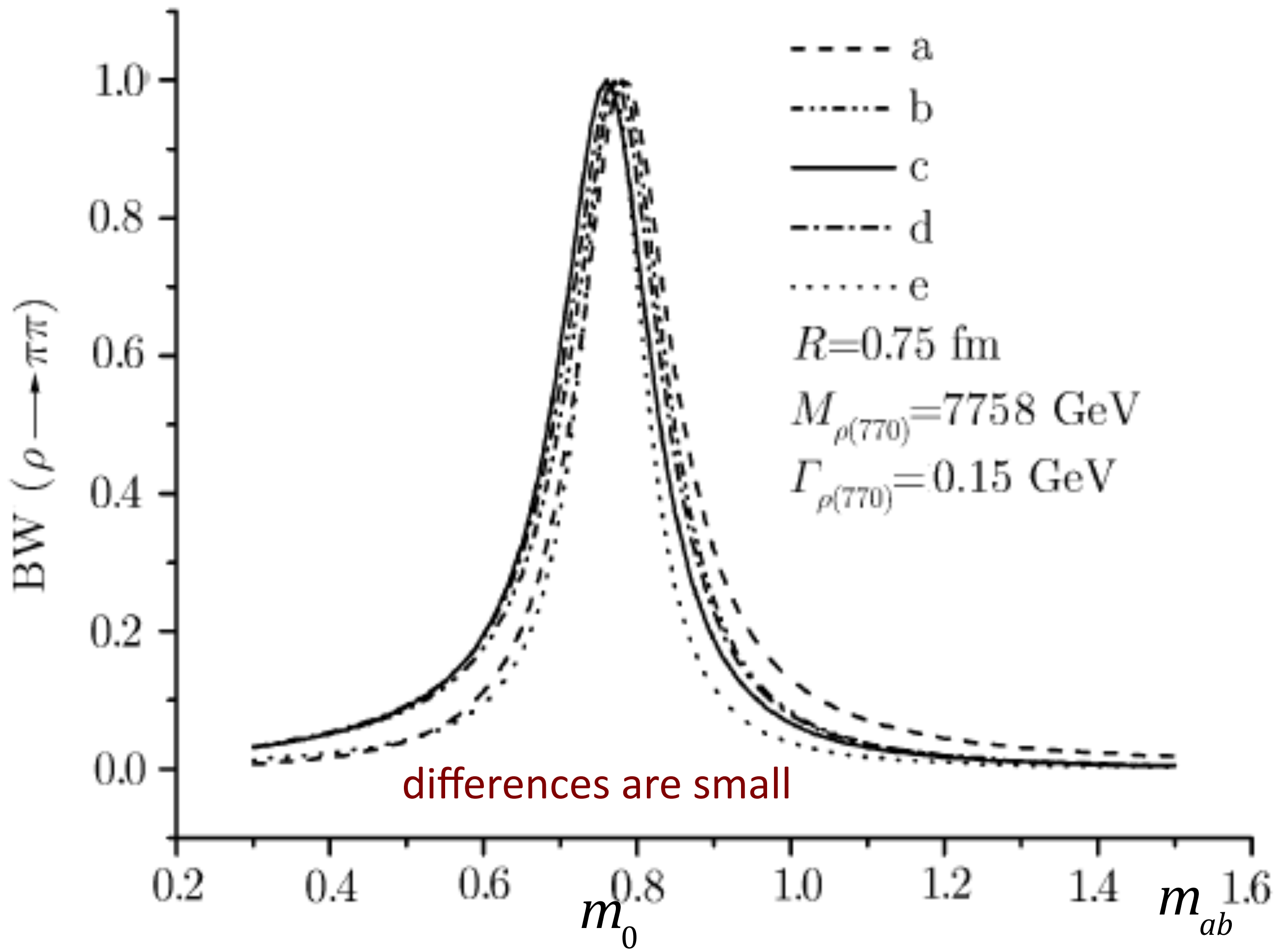
$q_0 = q(m_0)$



if the resonance is not too wide, BF effects are small



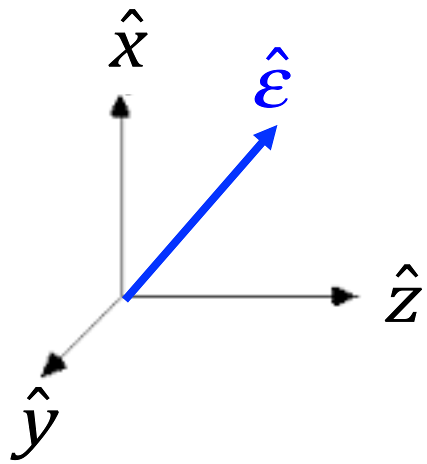
Many different forms for BW



vector meson "polarizations"

A spin=1 particle in a definite angular momentum state is "polarized."

$\hat{\varepsilon}$ is a unit vector along the polarization direction

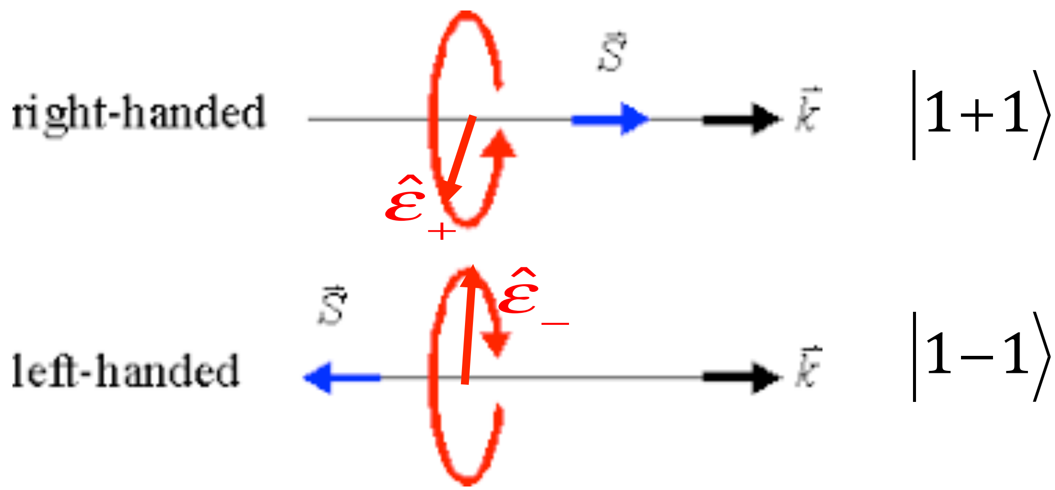


$$\left|J=1; J_z=m\right\rangle = \begin{cases} \left|1+1\right\rangle \Rightarrow \hat{\varepsilon}_{+} = \frac{1}{\sqrt{2}}\left(\hat{x}+i\hat{y}\right) \\ \left|10\right\rangle \Rightarrow \hat{\varepsilon}_{0} = \hat{z} \\ \left|1-1\right\rangle \Rightarrow \hat{\varepsilon}_{-} = \frac{1}{\sqrt{2}}\left(\hat{x}-\hat{y}\right) \end{cases}$$

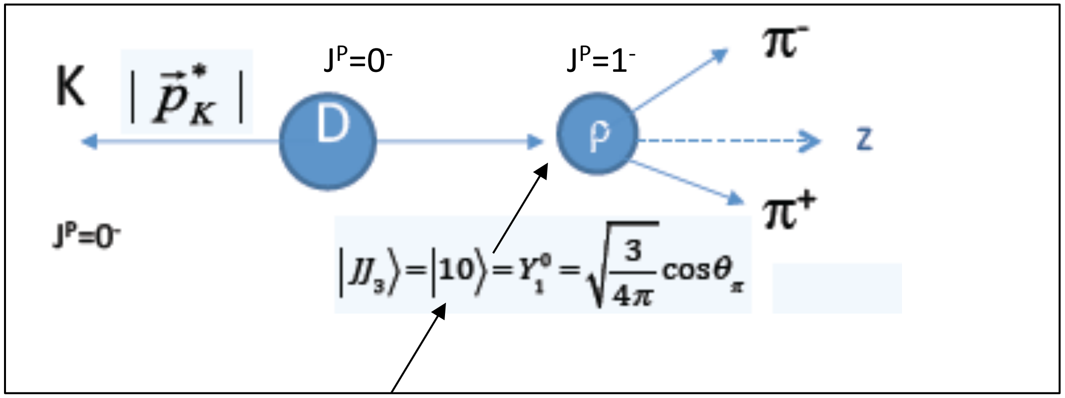
$\hat{\varepsilon}$ is a an axial vector: $\mathcal{P}(\hat{\varepsilon}) = +\hat{\varepsilon} \quad \Longrightarrow J^P = \mathbf{1}^+$

A photon can only be in $|1+1\rangle$ or $|1-1\rangle$ spin states; & not $|10\rangle$

$$\Rightarrow \hat{\varepsilon}_{\gamma} \perp \vec{p}_{\gamma}$$ photons are "transversely" polarized



ρ-meson "polarization" in $D \rightarrow K\rho$ decays



z-axis in the ρ restframe is opposite the K-meson direction

-Initial angular momentum =0 (D is a 0- meson)
-K-meson is also 0- ➔ no angular momentum
-orbital angular momentum: $\vec{L} = \vec{r} \times \vec{p}$ ➔ no component along $\vec{p}$ direction

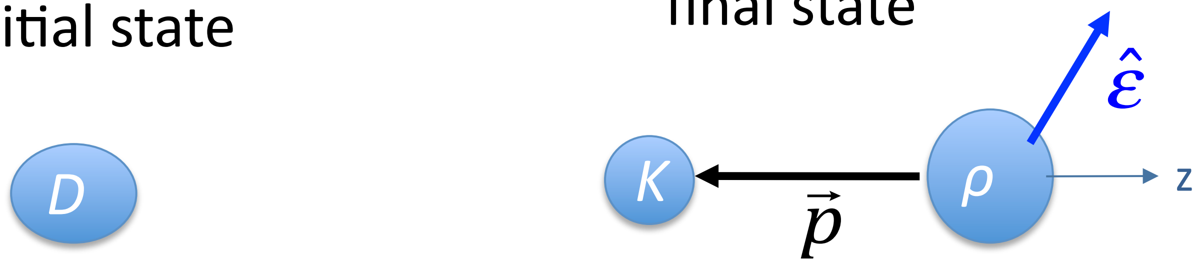
➔ the z-component of the ρ-mesons angular momentum = 0

$\boxed{\text{ρ-meson from } D \rightarrow K\rho \text{ is polarized: } \hat{\varepsilon}_\rho = \hat{z}}$

Another way to look at this

initial state

final state



$\hat{\varepsilon}_\rho$ & $\vec{p}$ are the only vectors

$$J^P = 0^-$$

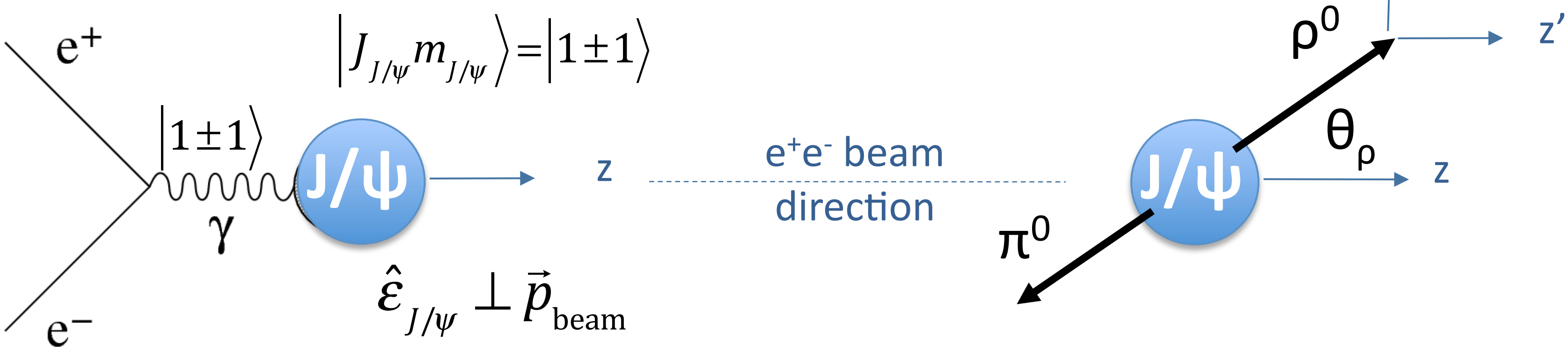
J^P must also be 0^-

$$\Rightarrow \propto \hat{\varepsilon}_\rho \cdot \vec{p} \;\Rightarrow \hat{\varepsilon}_\rho \,||\, \vec{p}$$

$$\Rightarrow \left| J_\rho m_\rho \right\rangle = \left| 10 \right\rangle$$

$e^+e^- \rightarrow J/\psi \rightarrow \pi^0\rho^0$

"partial-wave" basis



these are different final states
➔ they do not interfere

consider $|J_{J/\psi} m_{J/\psi}\rangle = |1+1\rangle$:

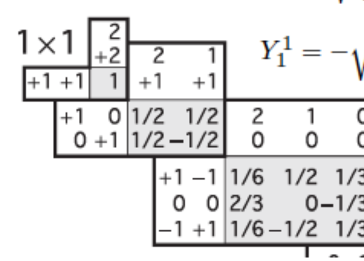
$$|1+1\rangle_{J/\psi} = \tfrac{1}{\sqrt{2}}\left(Y_1^0(\theta_\rho,\phi_\rho)|1+1\rangle_\rho - Y_1^{+1}(\theta_\rho,\phi_\rho)|10\rangle_\rho\right)$$

$$\left.\frac{d\Gamma}{d\theta_\rho d\phi_\rho}\right|_{m_{J/\psi}=+1;m_\rho=0} \propto \left|{}_\rho\langle 10\|11\rangle_{J/\psi}\right|^2 \propto \left|Y_1^1(\theta_\rho,\phi_\rho)\right|^2 = \tfrac{1}{2}\sin^2\theta_\rho$$

$$\left.\frac{d\Gamma}{d\theta_\rho d\phi_\rho}\right|_{m_{J/\psi}=+1;m_\rho=+1} \propto \left|{}_\rho\langle 11\|11\rangle_{J/\psi}\right|^2 \propto \left|Y_1^0(\theta_\rho,\phi_\rho)\right|^2 = \cos^2\theta_\rho$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

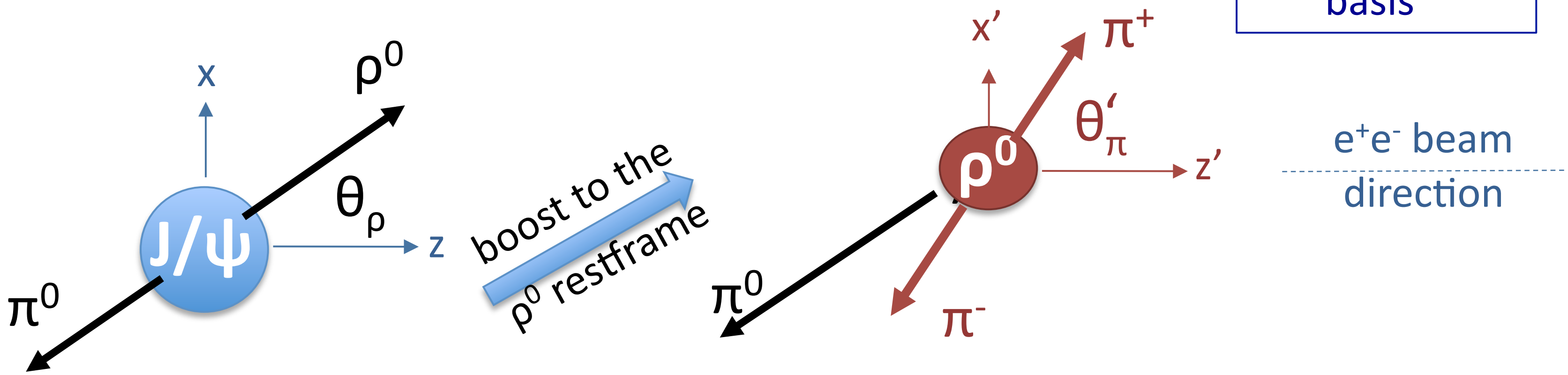
$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$



incoherent sum: $$\left.\frac{d\Gamma}{d\theta_\rho d\phi_\rho}\right|_{m_{J/\psi}=+1;m_\rho=+1\&m_\rho=0} \propto 1+\cos^2\theta_\rho$$

add $\rho^0 \to \pi^+\pi^-$

"partial-wave" basis



these are the same final states

➔ coherent sum

$$\langle \pi\pi | 1\,{+1}\rangle_{J/\psi} = \tfrac{1}{\sqrt{2}}\Big(Y_1^0(\theta_\rho,\phi_\rho)\langle \pi\pi | 1\,{+1}\rangle_\rho - Y_1^{+1}(\theta_\rho,\phi_\rho)\langle \pi\pi | 1\,0\rangle_\rho \Big)$$

$$= \tfrac{1}{\sqrt{2}}\Big(Y_1^0(\theta_\rho,\phi_\rho) Y_1^{+1}(\theta'_\pi,\phi'_\pi) - Y_1^{+1}(\theta_\rho,\phi_\rho) Y_1^0(\theta'_\pi,\phi'_\pi) \Big)$$

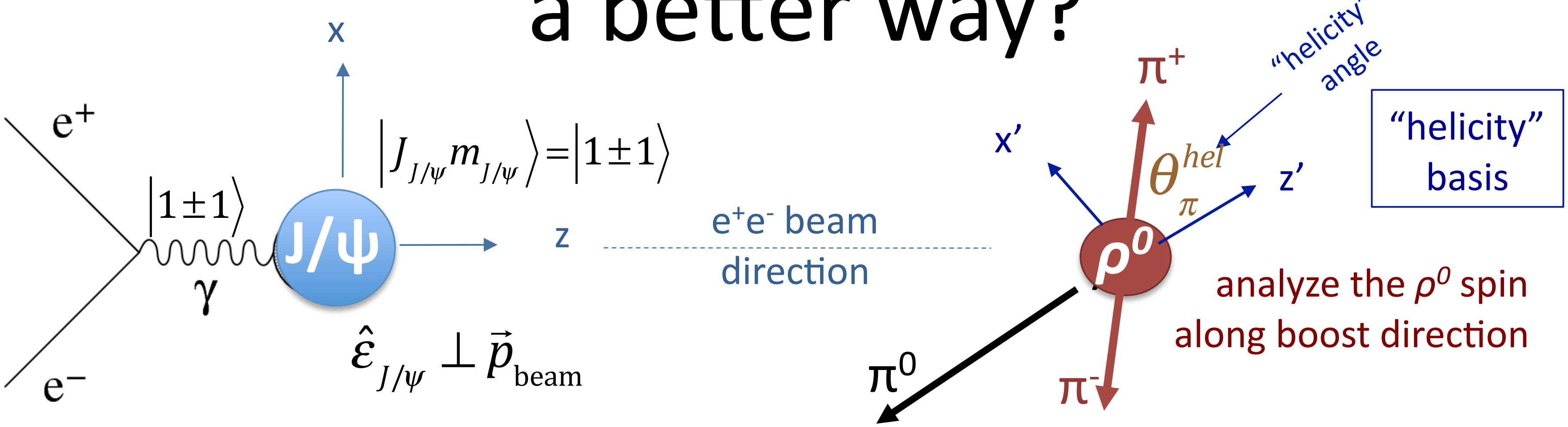
$$\frac{d\Gamma}{d\Omega_\rho\, d\Omega'_\pi\, dm_\rho} \propto \left| BW(m_\rho)\big(Y_1^0(\theta_\rho,\phi_\rho) Y_1^{+1}(\theta'_\pi,\phi'_\pi) - Y_1^{+1}(\theta_\rho,\phi_\rho) Y_1^0(\theta'_\pi,\phi'_\pi)\right|^2$$

Complicated expression, inconvenient variables, not easy to understand

Same problem, different approach

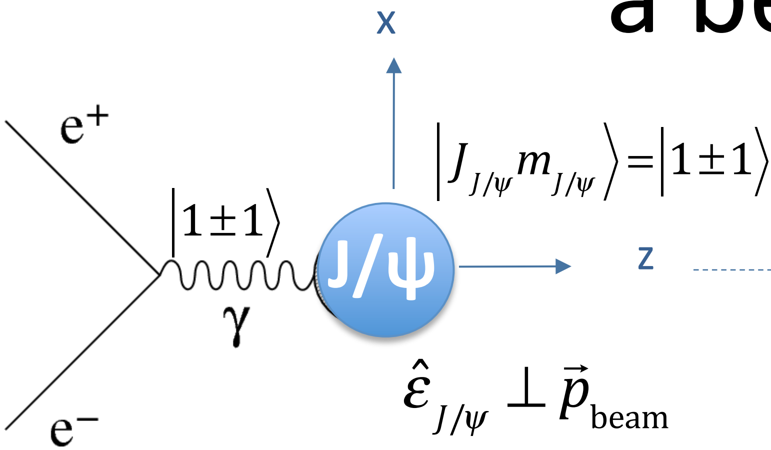
-- work in the “helicity basis’’--

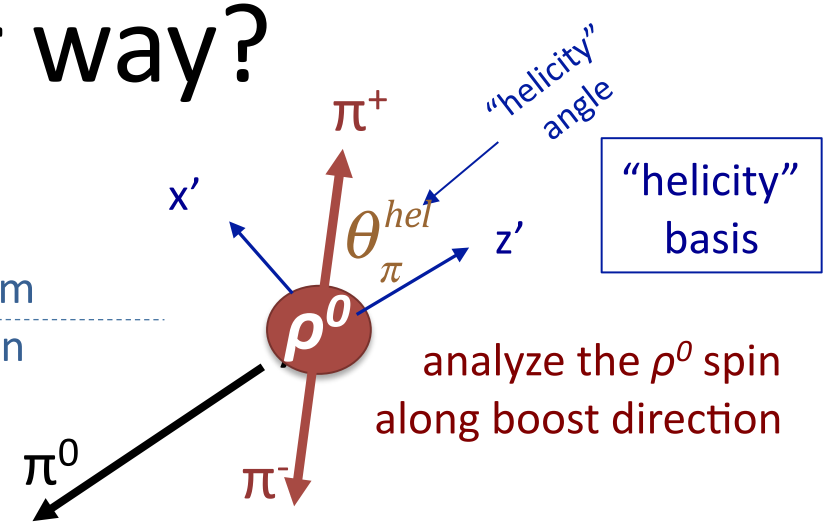
a better way?



e+
e−
$|1\pm1\rangle$
γ
J/ψ
x
z
$|J_{J/\psi} m_{J/\psi}\rangle = |1\pm1\rangle$
$\hat{\varepsilon}_{J/\psi} \perp \vec{p}_{\text{beam}}$
e+e- beam direction
π^+
x'
z'
θ_π^{hel}
"helicity" angle
"helicity" basis
ρ^0
analyze the ρ^0 spin along boost direction
π^0
π^-

a better way?



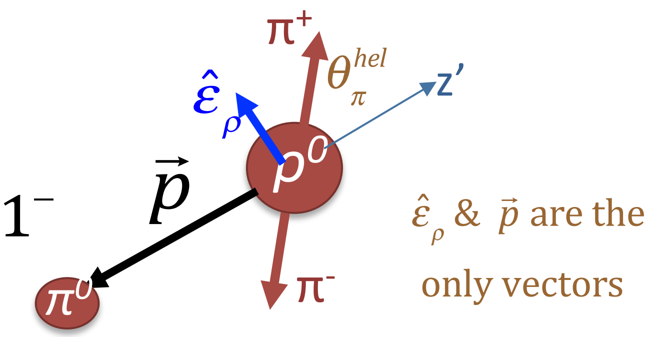


initial state

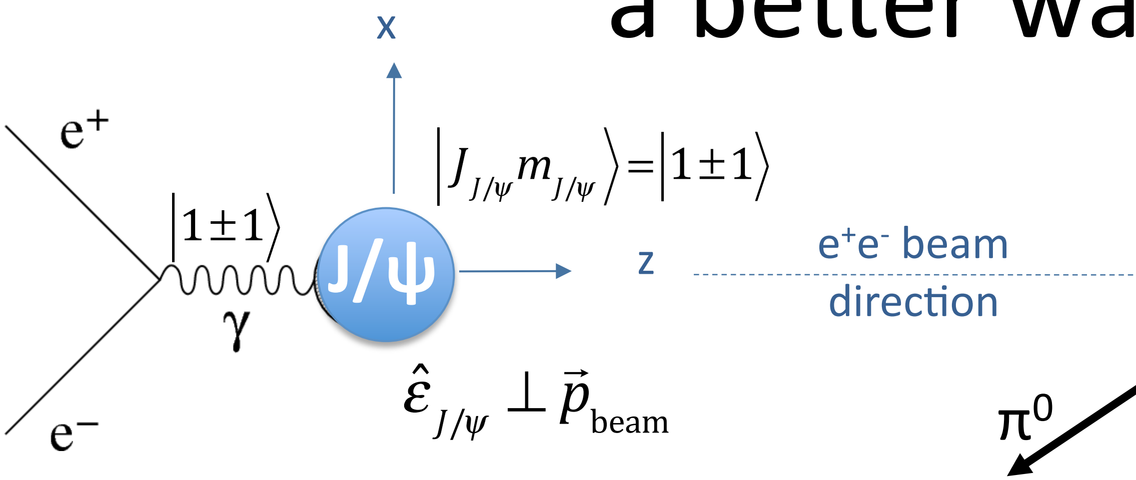
$J^P = 1^-$

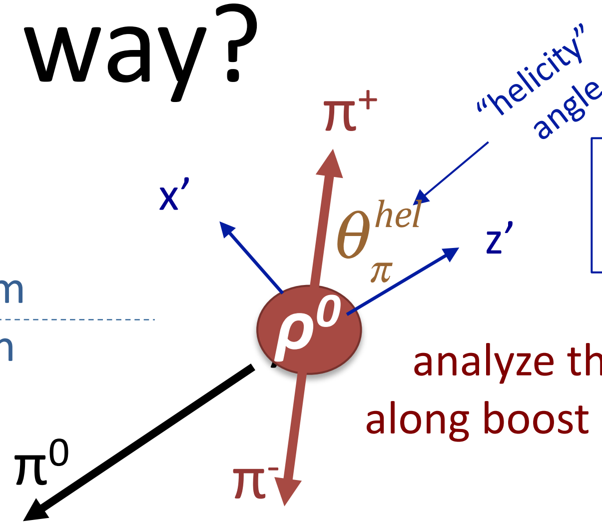
final state

J^P must be 1^-



a better way?





"helicity" basis

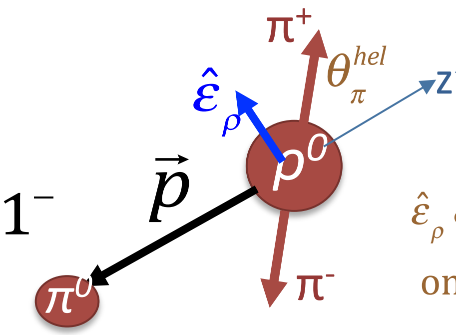
analyze the ρ^0 spin along boost direction

initial state

$J^P = 1^-$

final state

J^P must be 1^-



$\hat{\varepsilon}_\rho$ & $\vec{p}$ are the only vectors

$$1^- \text{ from } \hat{\varepsilon}_\rho \;\&\; \vec{p} \;\Rightarrow\; \hat{\varepsilon}_\rho \times \vec{p}$$

$$\therefore \hat{\varepsilon}_\rho \perp \vec{p} \;\Rightarrow \left|J_\rho m_\rho\right\rangle_{hel} = \left|1\pm1\right\rangle_{hel}$$

$$\left\langle \pi\pi \middle| 1\pm1\right\rangle_{hel} = Y_1^{\pm1}(\theta_\pi^{hel},\phi_\pi) = \tfrac{\mp1}{\sqrt{2}}\sin\theta_\pi^{hel} e^{\pm i\phi_\pi}$$

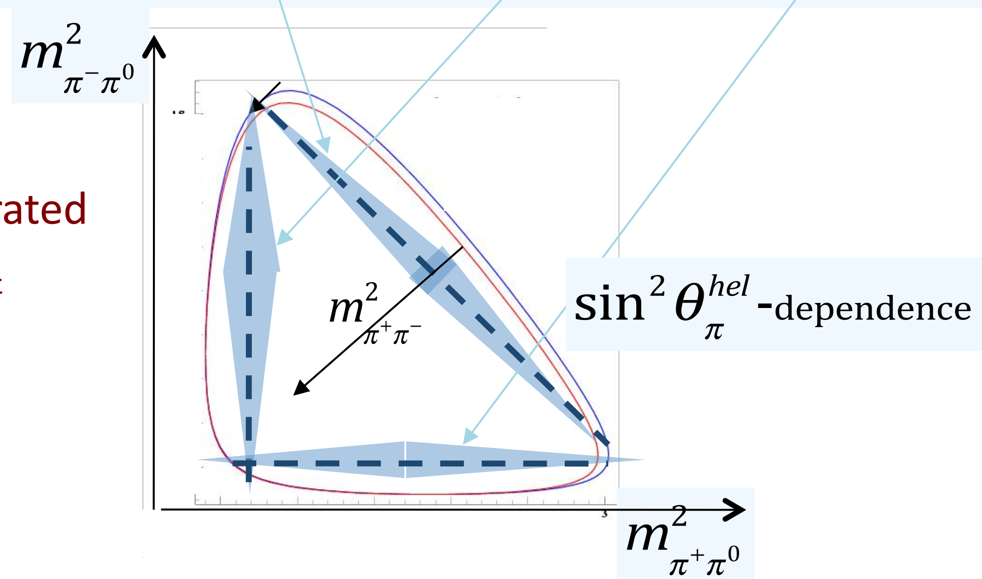
Fermi's Golden Rule

$$d\Gamma(m^2_{\pi^+\pi^0}, m^2_{\pi^-\pi^0}) = \frac{1}{(2\pi)^3 32\sqrt{m^3_{J/\psi}}} \left|\mathcal{M}\right|^2 dm^2_{\pi^+\pi^0} dm^2_{\pi^-\pi^0}$$

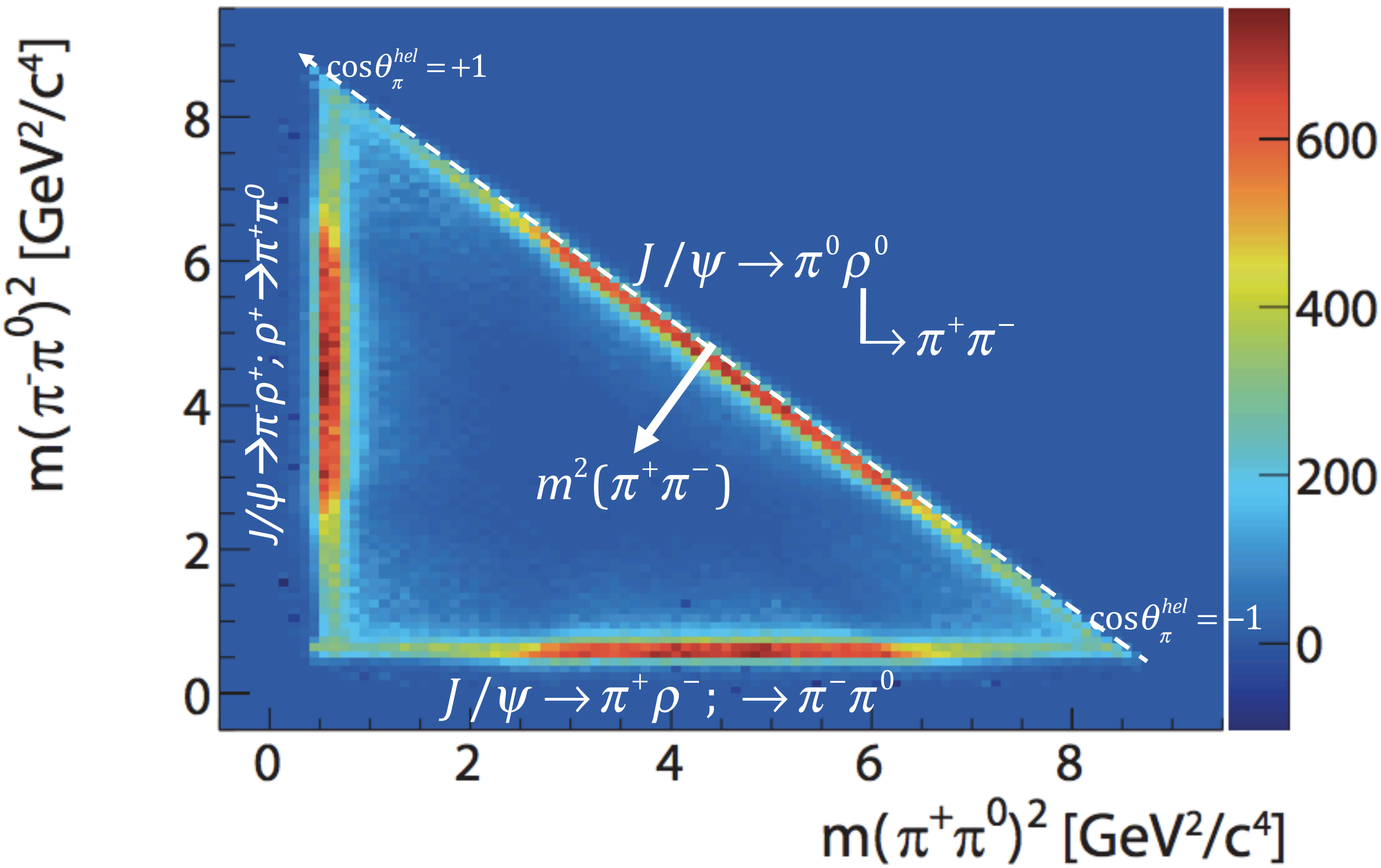
$$\left|\mathcal{M}\right|^2 \propto \left| BW_{\rho^0\to\pi^+\pi^-} \times \sin\theta^{hel}_{\pi^+\pi^-} + BW_{\rho^+\to\pi^+\pi^0} \times \sin\theta^{hel}_{\pi^+\pi^0} + BW_{\rho^-\to\pi^-\pi^0} \times \sin\theta^{hel}_{\pi^-\pi^0} \right|^2$$

$$= \left| \frac{i\sqrt{m_{\pi^+\pi^-}\Gamma_\rho}}{(m^2_\rho - m^2_{\pi^+\pi^-}) - i m_\rho \Gamma_\rho} \sin\theta^{hel}_{\pi^+\pi^-} + \left[\pi^-\rho^+(\to\pi^+\pi^0)\right] + \left[\pi^+\rho^-(\to\pi^-\pi^0)\right] \right|^2$$

here we have integrated over θ_ρ, ϕ_ρ and ϕ_π

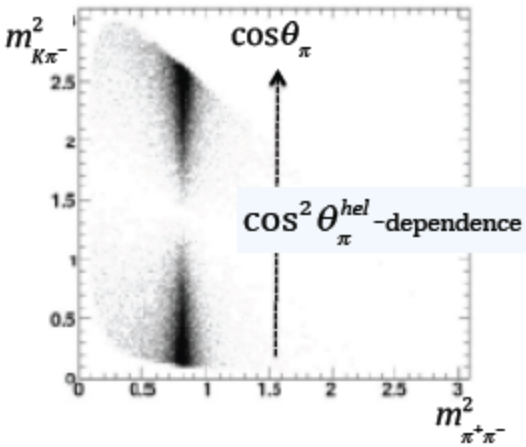


$J/\psi \to \pi^+\pi^-\pi^0$ from BESIII

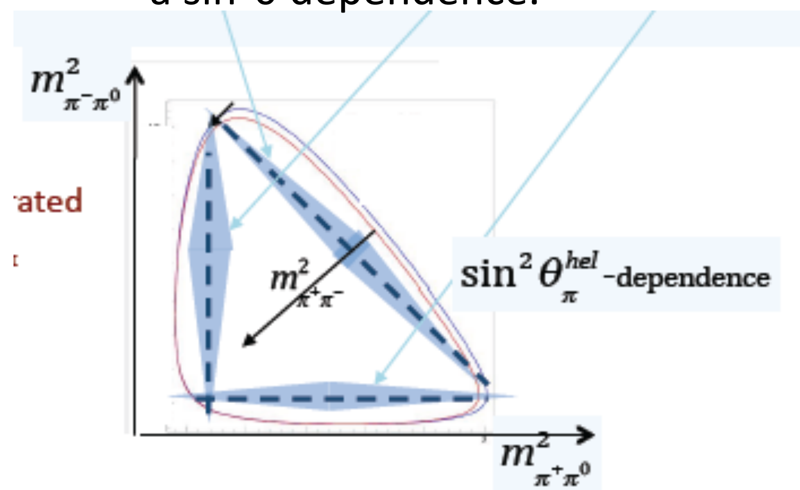


Discussion item

The ρ-band in D→Kρ has a $\cos^2\theta$ dependence.



The ρ-bands in J/ψ→ρ have a $\sin^2\theta$ dependence.



Why are they different?