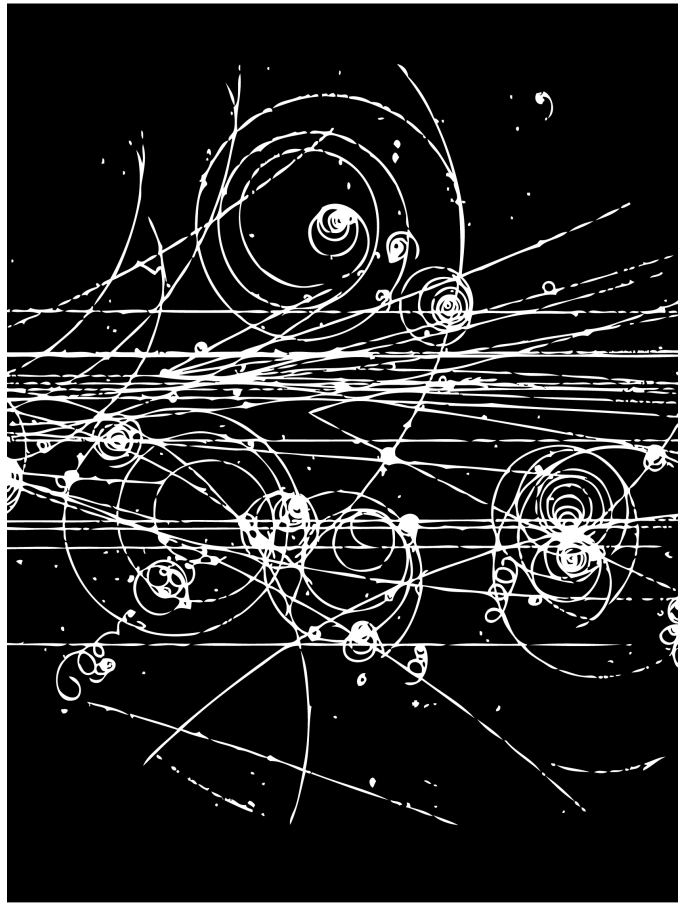
Hadron Spectroscopy



Stephen Lars Olsen
Institute for Basic Science (KOREA)
University of the Chinese Academy of Science

UCAS Physics-department, June 20 – July 6, 2014

Lecture 1: What are hadrons?


Lev Okun 1929-2015

What is a hadron?

Lepton (pre-euro Greek coin)

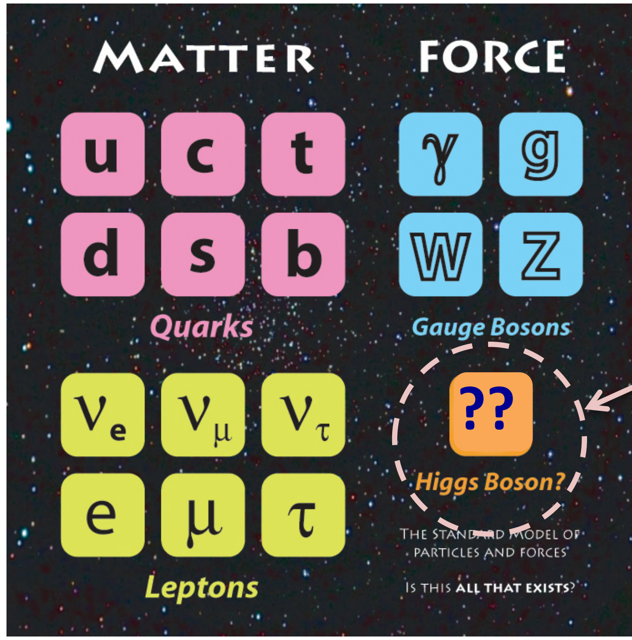
ICHEP-XI   Geneva 1962

Notwithstanding the fact that this report deals with weak interactions, we shall frequently have to speak of strongly interacting particles. These particles pose not only numerous scientific problems, but also a terminological problem. The point is that "strongly interacting particles" is a very clumsy term which does not yield itself to the formation of an adjective. For this reason, to take but one instance, decays into strongly interacting particles are called non-leptonic. This definition is not exact because "non-leptonic" may also signify "photonic". In this report I shall call strongly interacting particles "hadrons", and the corresponding decays "hadronic" (the Greek *ἁδρός* signifies "large", "massive", in contrast to *λεπτός* which means "small", "light"). I hope that this terminology will prove to be convenient. — Lev B. Okun, 1962

Outline

- Main Lecture
  - a little bit of the history of particle physics
    discover of the π and K mesons and strange baryons
    units, symmetries, commonly used jargon, etc
  - meson and baryon octets and baryon decuplet
  - SU(3) flavor-symmetry
  - discovery of the Ω^- and the $\phi(1020)$
  - OZI rule and the need for something new
- Special topic
  - Dalitz plots
  - $\omega \rightarrow \pi^+\pi^-\pi^0$ Dalitz plot
  - $D \rightarrow K\rho^0 \rightarrow K\pi^+\pi^-$ Dalitz plot
  - $D \rightarrow K^{*-}\pi^+ \rightarrow K\pi^+\pi^-$

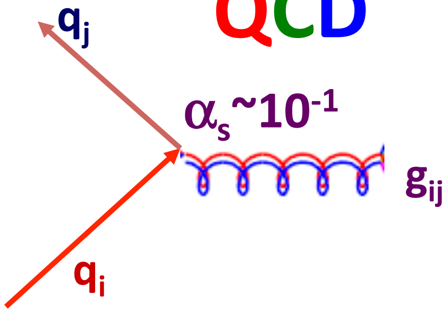
Standard Model of Particle Physics pre 2012



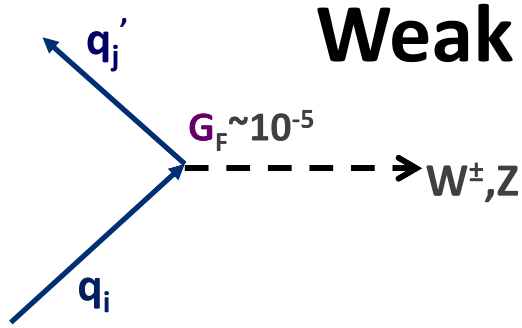
Higgs?: the only element not experimentally confirmed

$$SU(3)_{color} \otimes SU(2)_{weak} \otimes U(1)_{EM}$$

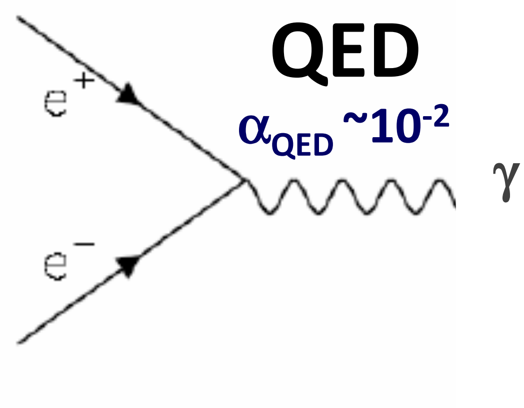
QCD



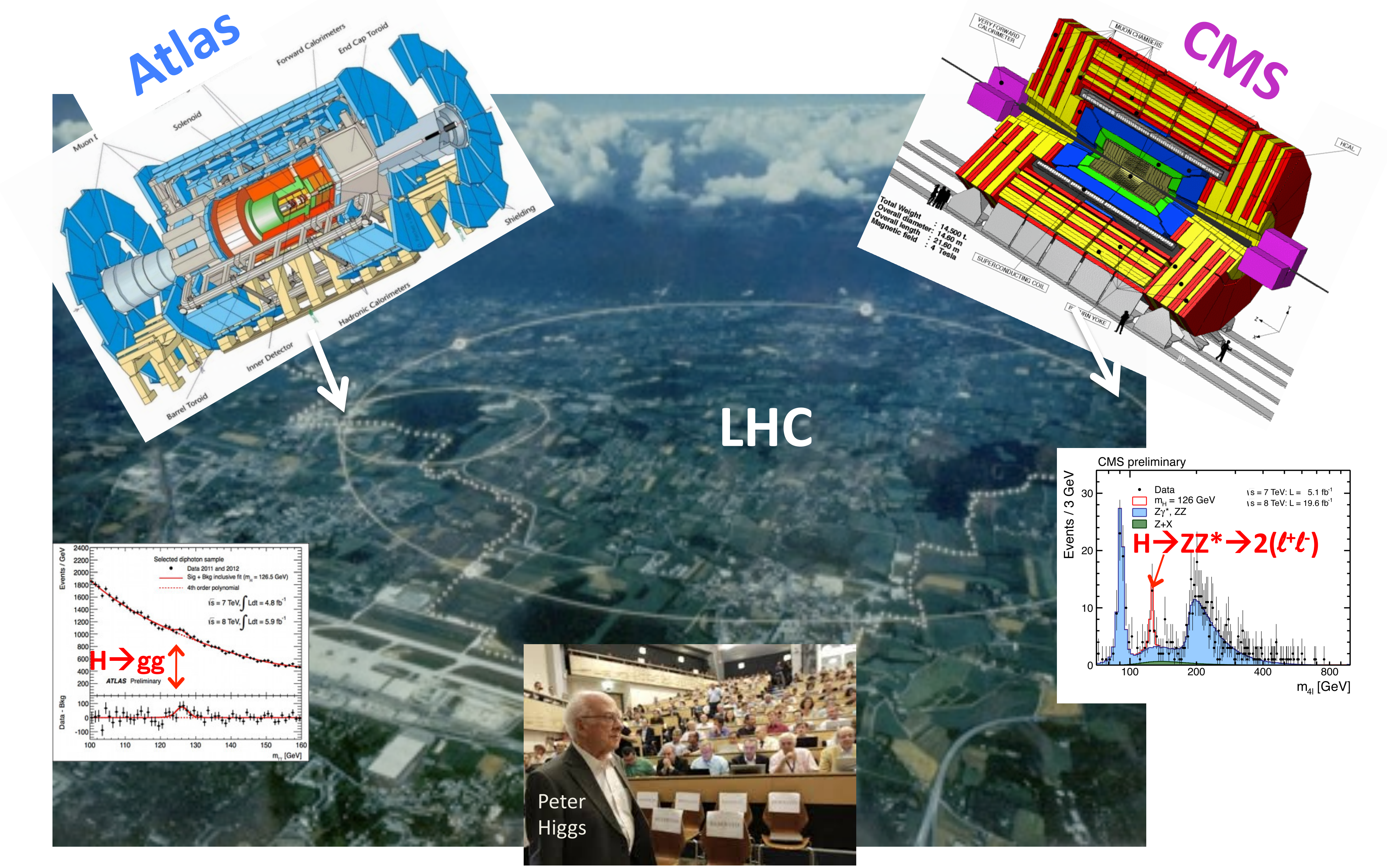
Weak



QED



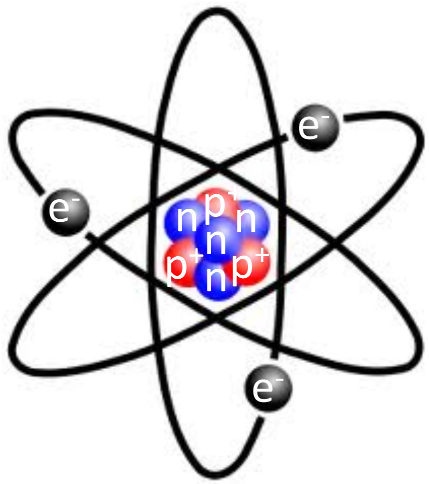
July 4th, 2012: Higgs is discovered at CERN



We have been there before

1935: particle physics

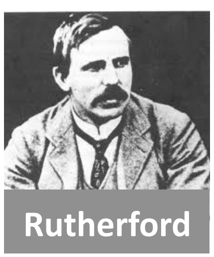
Matter

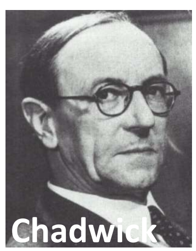


e^- electron

p^+ proton

n^0 neutron

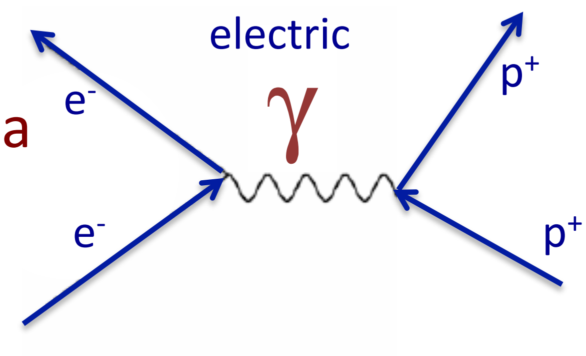




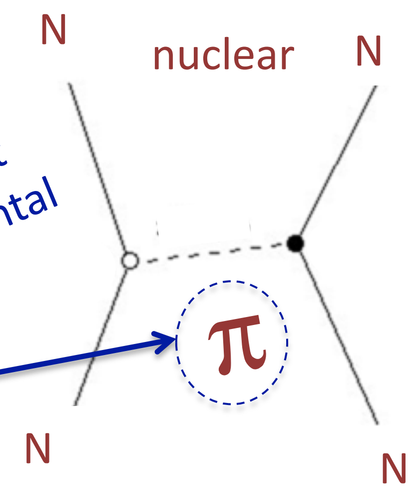
Forces

γ gamma

$\pi^\pm$ pion



pion: only element lacking experimental confirmation



prediction:

$m_p \approx 1/7\ m_{proton}$

Search for Yukawa's pion

Cosmic rays: Nature's accelerator lab

High energy proton from outer space

~30km

1936 μ, not π is discovered

Phys Rev 50, 263 (1936)

Cloud Chamber Observations of Cosmic Rays at 4300 Meters Elevation and Near Sea-Level

CARL D. ANDERSON AND SETH H. NEDDERMEYER, *Norman Bridge Laboratory of Physics, California Institute of Technology*

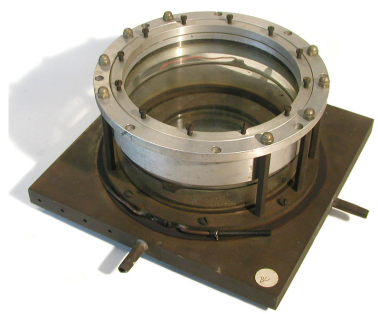
(Received June 9, 1936)

Phys Rev 51, 884 (1937)

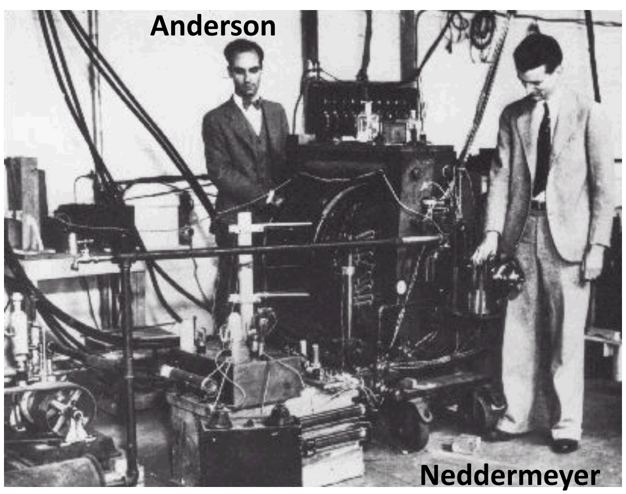
Note on the Nature of Cosmic-Ray Particles

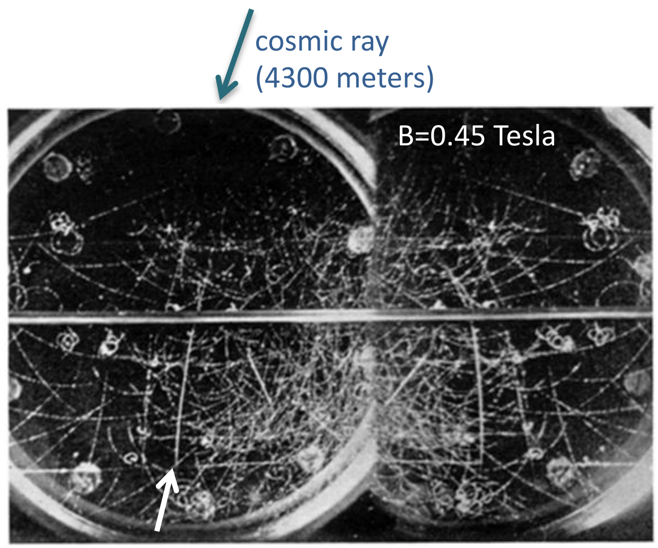
SETH H. NEDDERMEYER AND CARL D. ANDERSON
California Institute of Technology, Pasadena, California

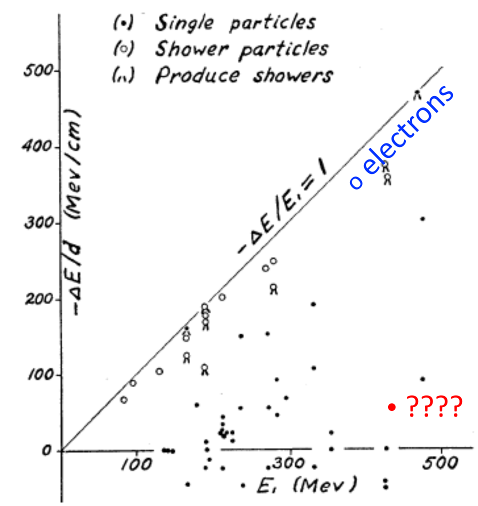
(Received March 30, 1937)



Anderson-Neddermeyer cloud chamber



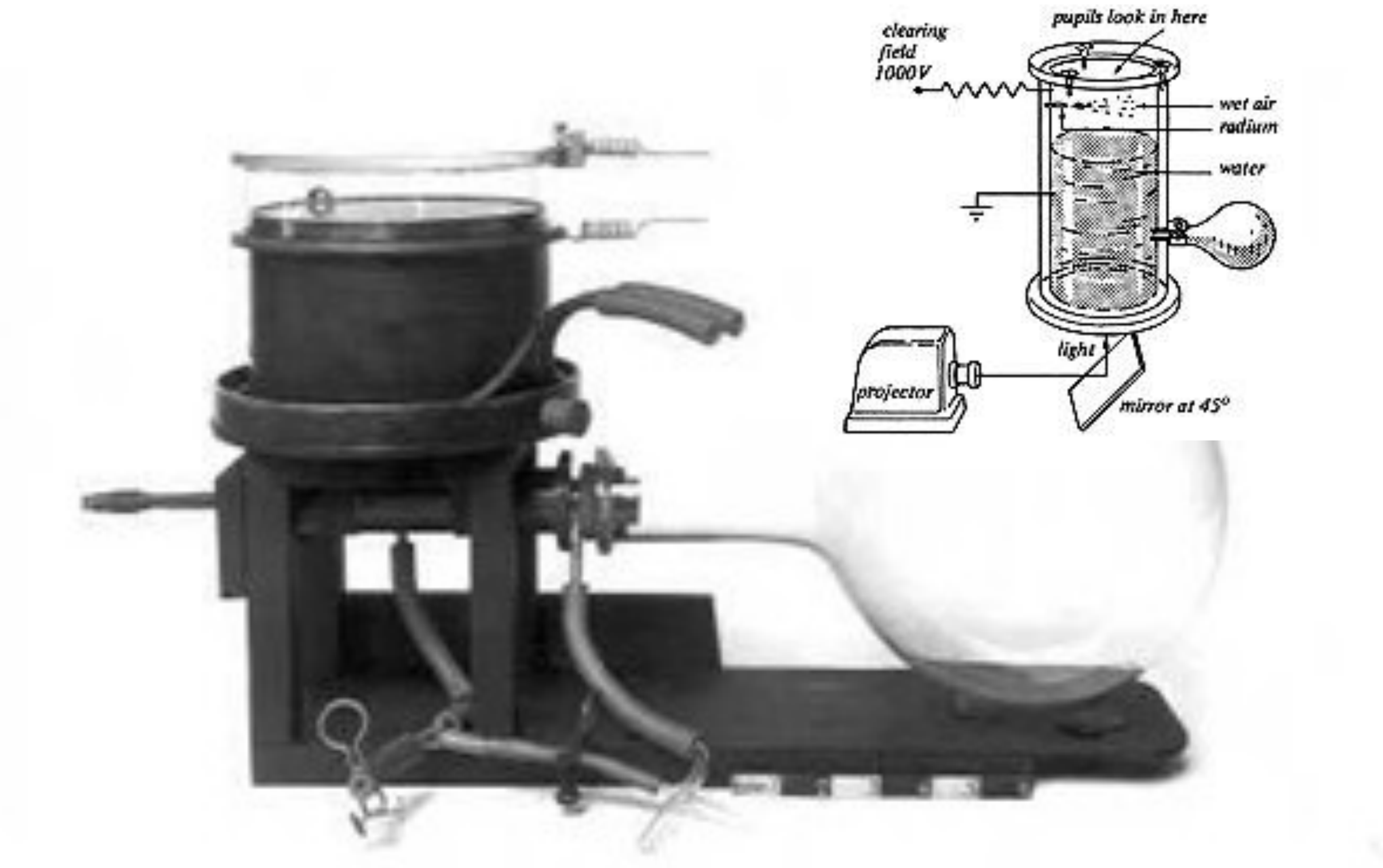




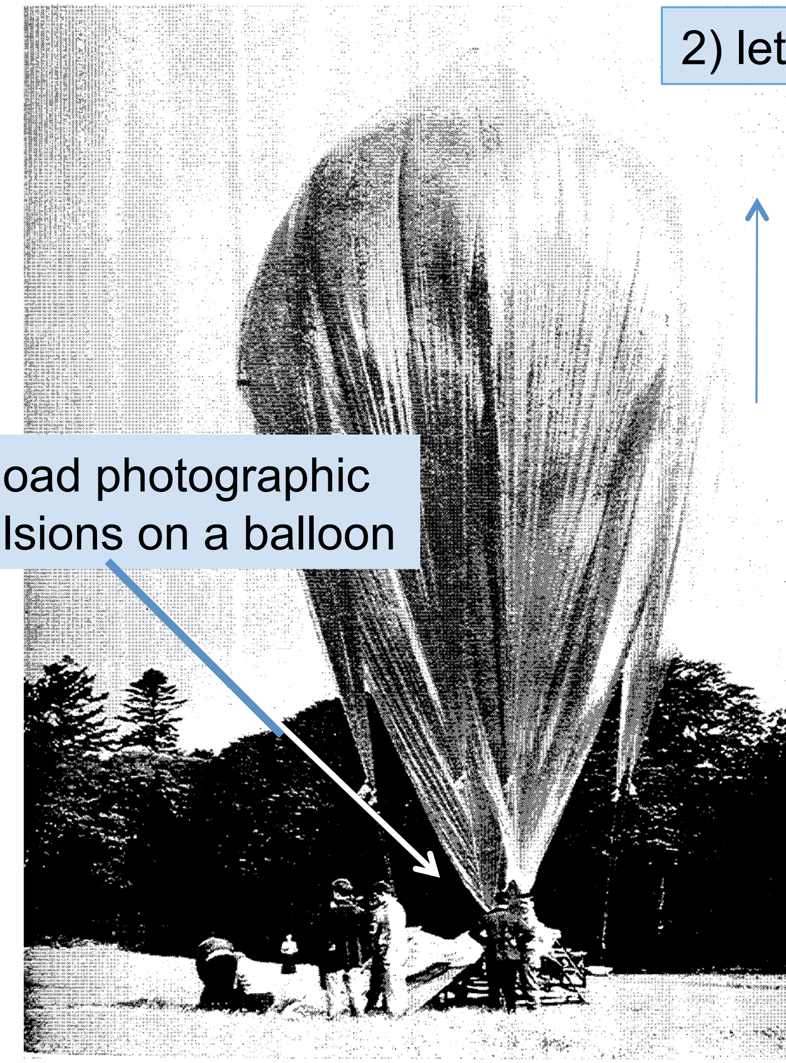
Energy loss in 1 cm of platinum.

from (anomalous) ionization density
range & momentum: $\mu \approx 130 m_e$

Cloud chambers

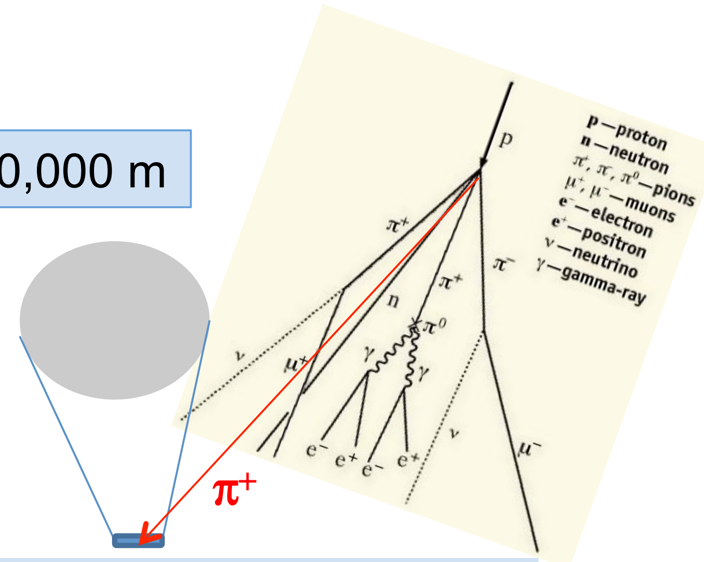


“improved” π-meson search technique



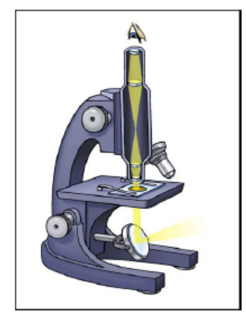
1) load photographic emulsions on a balloon

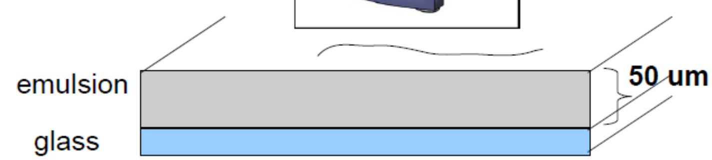
2) let it fly to ~ 30,000 m



3) hope that some π+ mesons stop in the emulsion & decay

4) get some people to scan the emulsion under a microscope

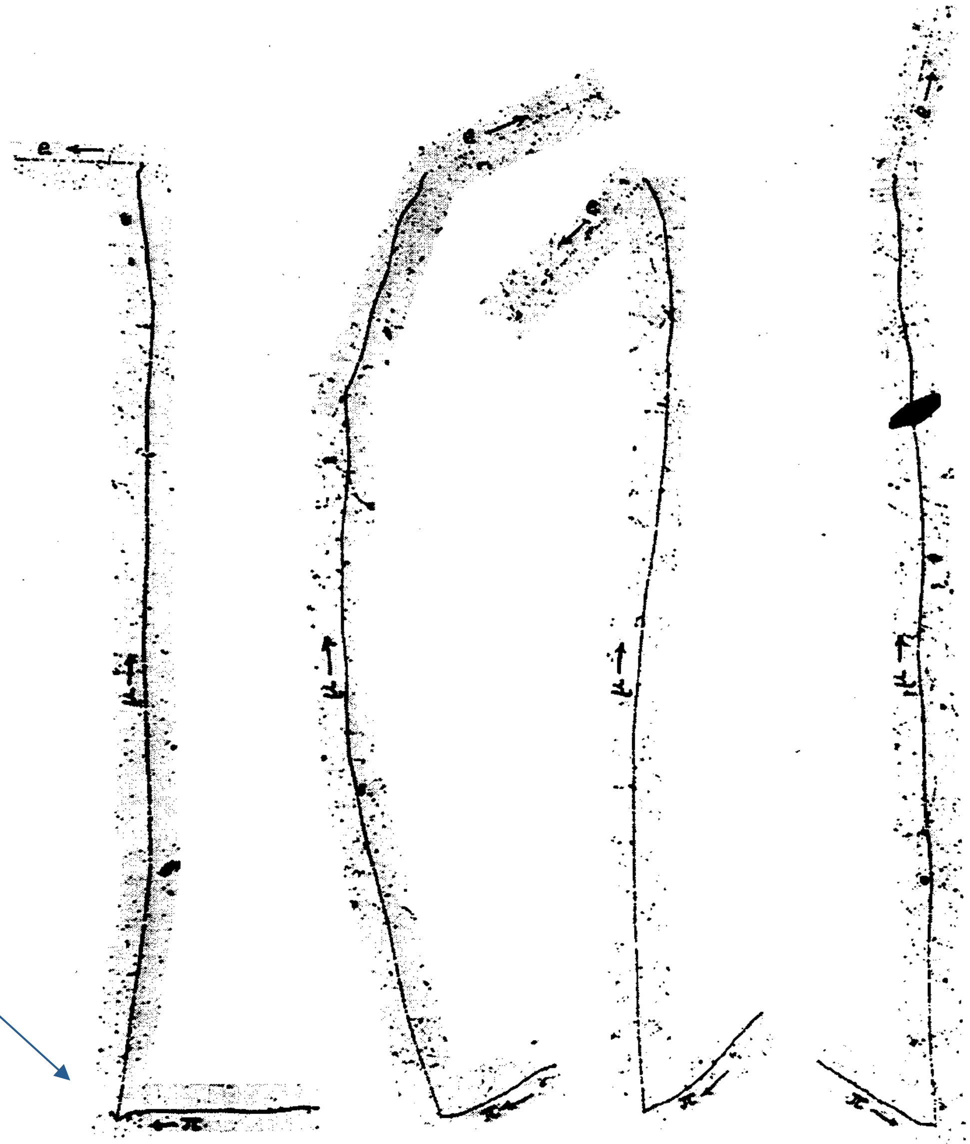




Yukawa's π finally discovered in 1947

C. Powell
1950 Nobel Prize

π→μ→e decays registered in balloon-borne photographic emulsion

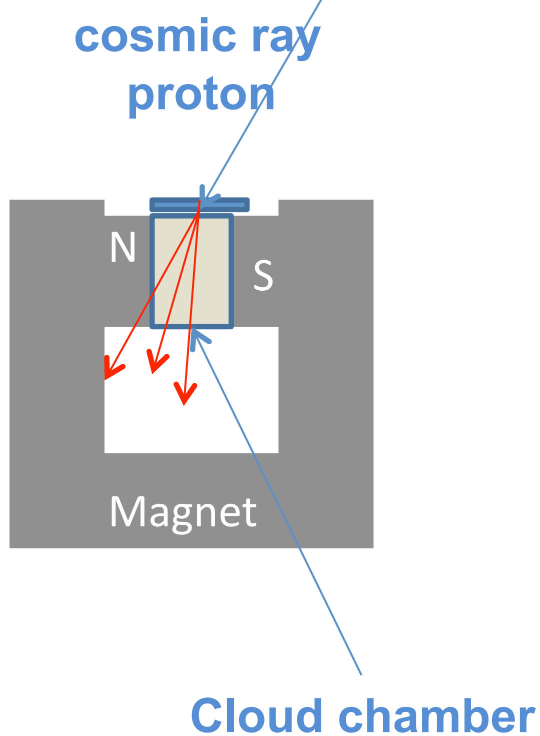


Nature 159, 4047 (1947)

May 24, 1947

But, at the same time (1947).....

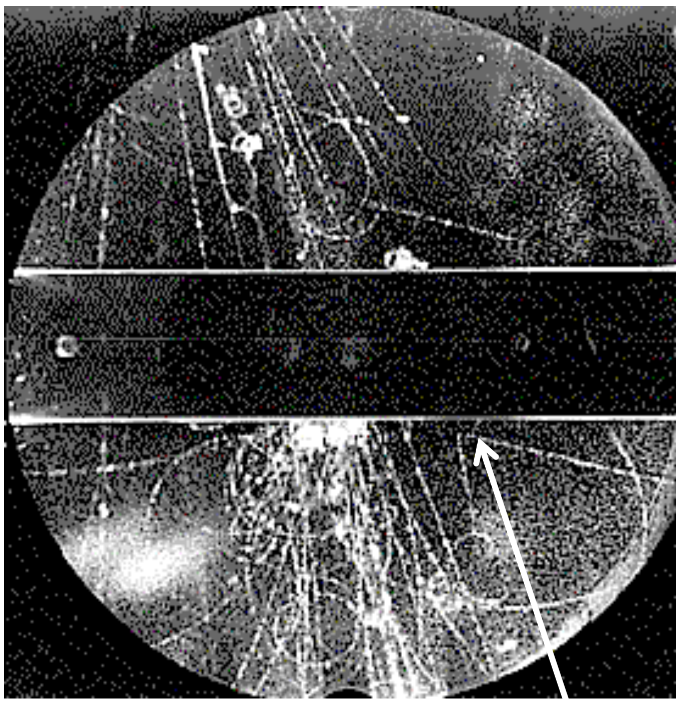
Mt. Pic Du Midi, 3000 m



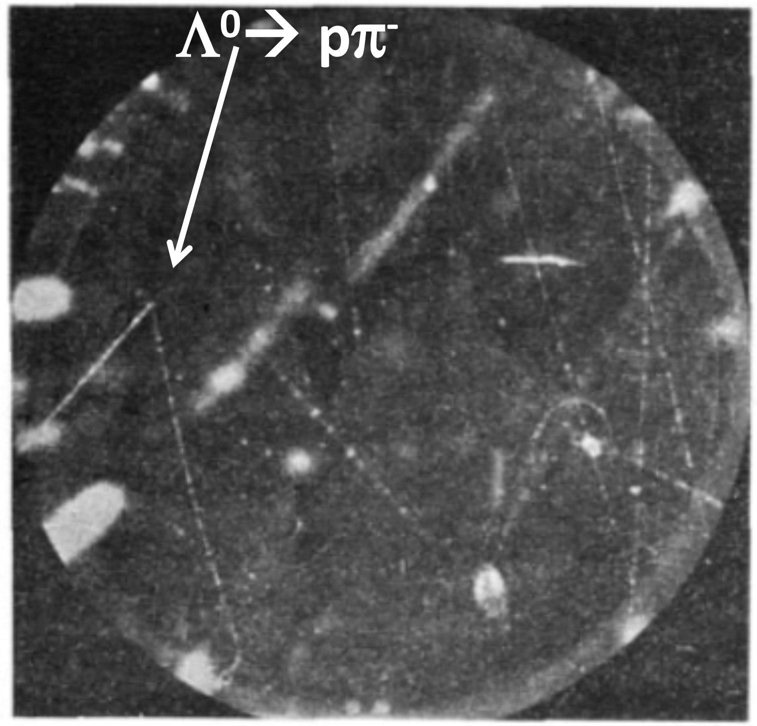
George Rochester

Clifford Butler

“V” Particles $\leftarrow$ totally unexpected



$\theta^0 \rightarrow \pi^+\pi^-$

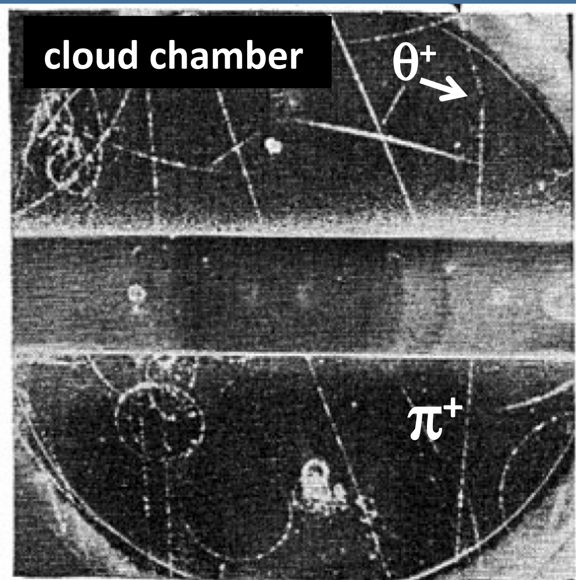


Nature 160, 855 (1947)

Dec 20, 1947

& K^+ mesons (also not expected)

1947



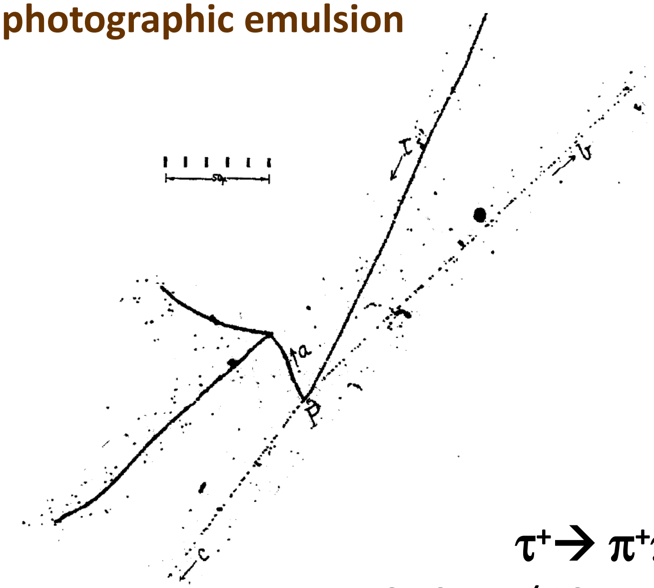
$\theta^+ \rightarrow \pi^+\pi^0$

$m_\theta \approx m_p/2$

G.D. Rochester & C.C.Butler, Nature 160, 855 (1947)

1949

photographic emulsion



$\tau^+ \rightarrow \pi^+\pi^+\pi^-$

m_τ = 970 m_e (495 MeV)

R.Brown et al., Nature 163, 47,82 (1949)

$P(\theta^+)=(-1)(-1) = +\ \theta^+$

q has even parity

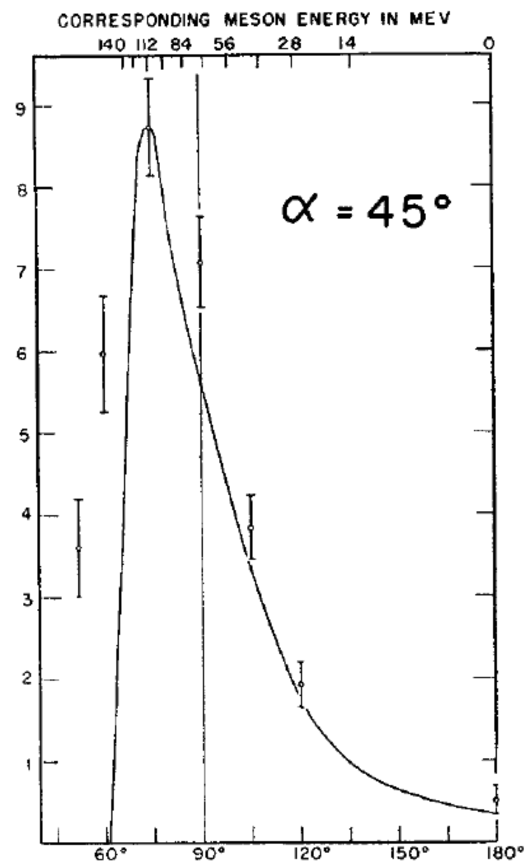
same mass,
same lifetime,
opposite P

$P(\tau^+)=(-1)^3 = -\tau^+$

t has odd parity

T.D. Lee & C.N. Yang PRL 104, 254 (1956) $\theta^+ = \tau^+$ (now = K^+ meson) & parity is violated in weak decays

π^0 →YY discovery



PHYSICAL REVIEW VOLUME 78, NUMBER 6 JUNE 15, 1950

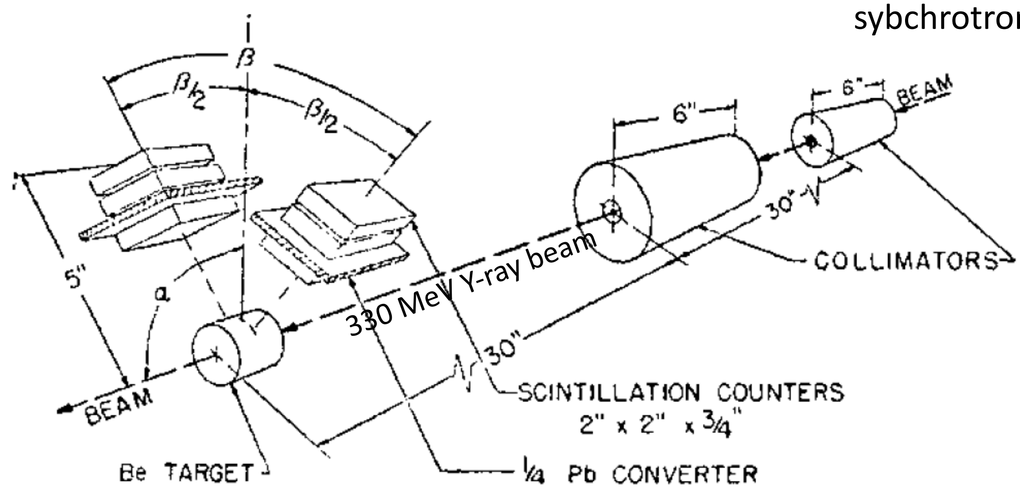
Evidence for the Production of Neutral Mesons by Photons*

J. Steinberger, W. K. H. Panofsky, and J. Steller
Radiation Laboratory, Department of Physics, University of California, Berkeley, California

(Received April 28, 1950)

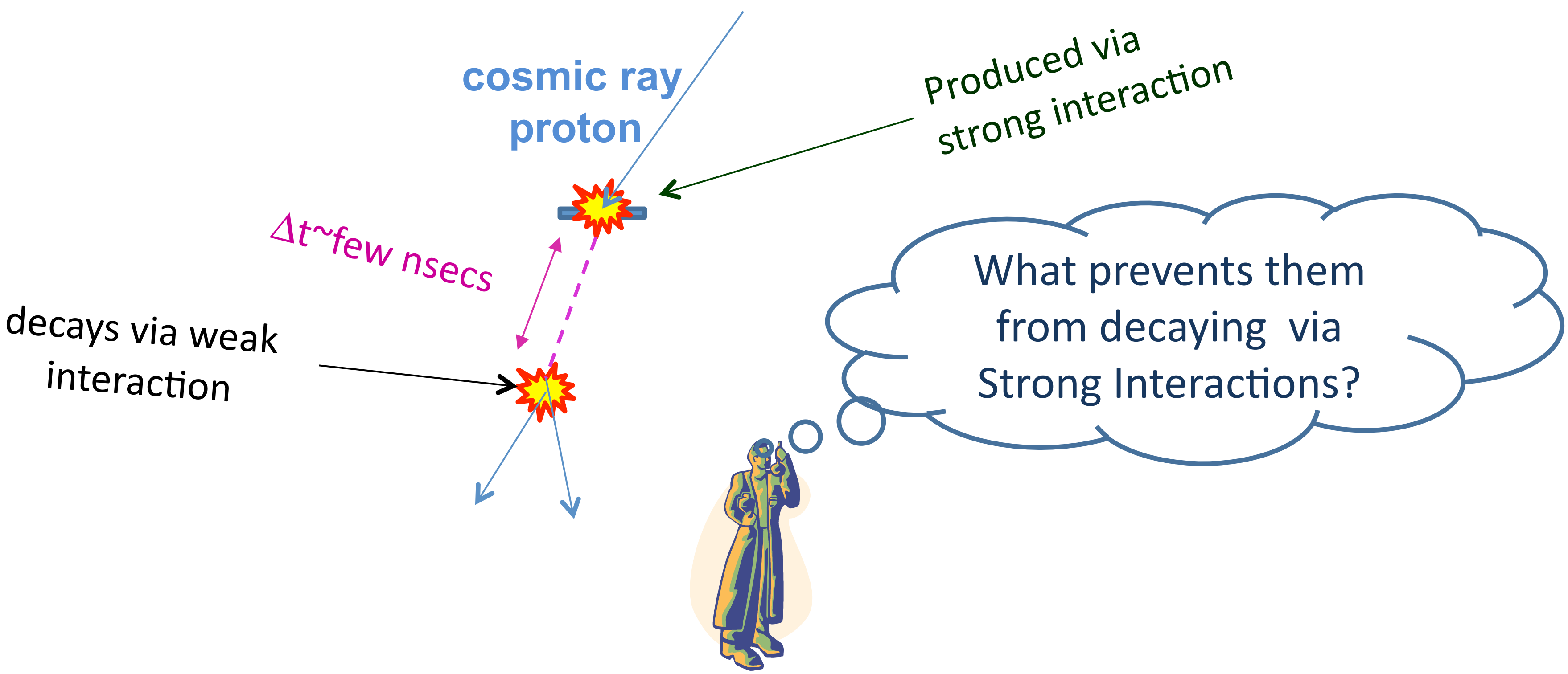
$$\gamma + N \rightarrow \pi^0 + N$$

Berkeley e$^-$ sybchrotron

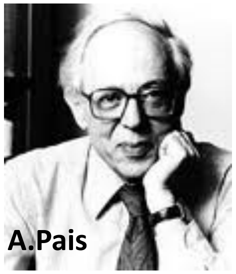


Major puzzle

The V^0, $\theta^{\pm}$ and $\tau^{\pm}$ particles are made by strong interactions but live a long time and, so, must decay by weak interactions.



1952: Pais proposes a new Quantum Number



→for neutron/proton/π ("old particles") this new QN = 0

→for Λ^0/K^0… ("new particles") the new QN = +1

Pais' principle of "**evenness:**"

-Strong interactions: only *even* changes in the new QN are allowed

-Weak Interactions: odd changes in the new QN are okay

Unambiguous prediction: the new particles are *always produced in pairs*

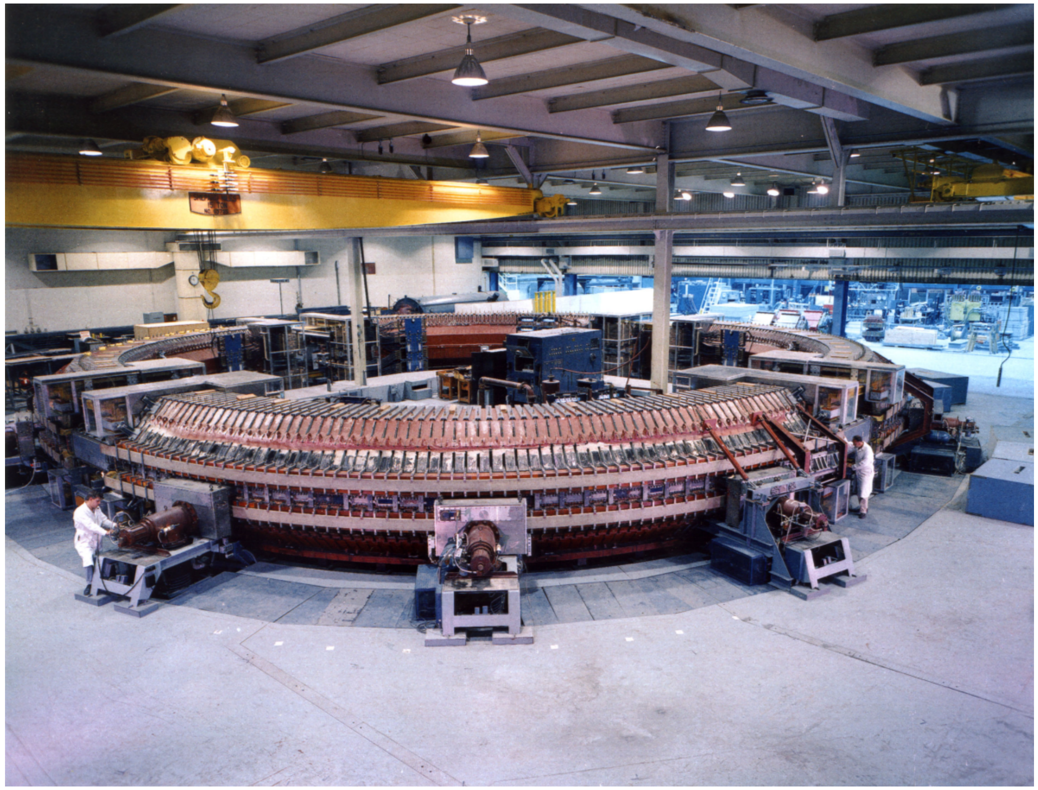
e.g.: $\pi^- p \rightarrow K^0\Lambda^0$ Δ(new QN) =2 ← allowed

$\pi^- p \rightarrow \pi^0\Lambda^0$ Δ(new QN) = 1 ← not allowed

(Phys Rev 86, 663 (1952)):

1953: The Cosmotron

3.3 GeV proton accelerator at Brookhaven Lab (near NYC)



This produced the new particles under controlled conditions

"New" particles are produced in pairs

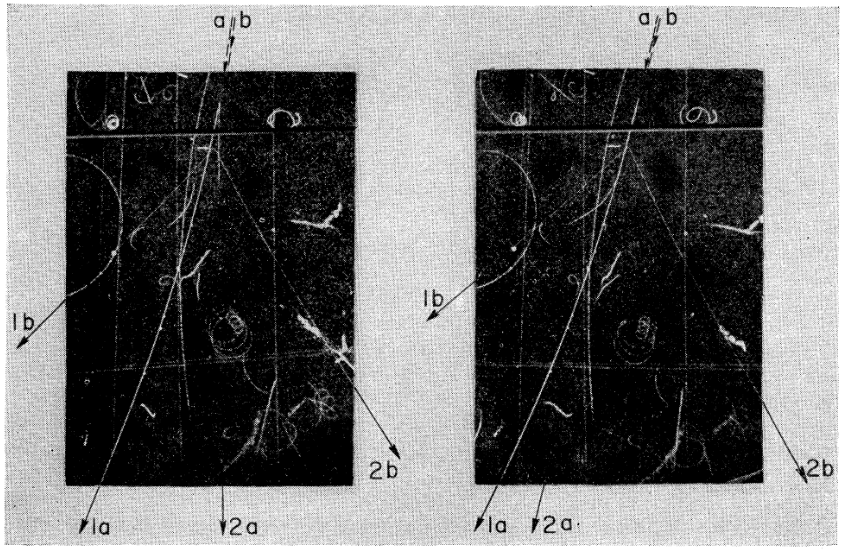


FIG. 1. Case C. Diffusion cloud-chamber photograph of two neutral V particles (a) and (b), whose lines of flight are almost colinear. (a) is believed to be a Λ^0 decaying into a proton (1a) and a negative π meson (2a). Tracks $1a$ and $2a$ practically coincide in the right view. (b) is probably a ϑ^0 decaying into π^+ (1b) and π^- (2b).

W.D.Fowler, R.P.Shutt, A.M.Thorndike & W.L.Whittemore

Phys. Rev. 93, 861 (1953)

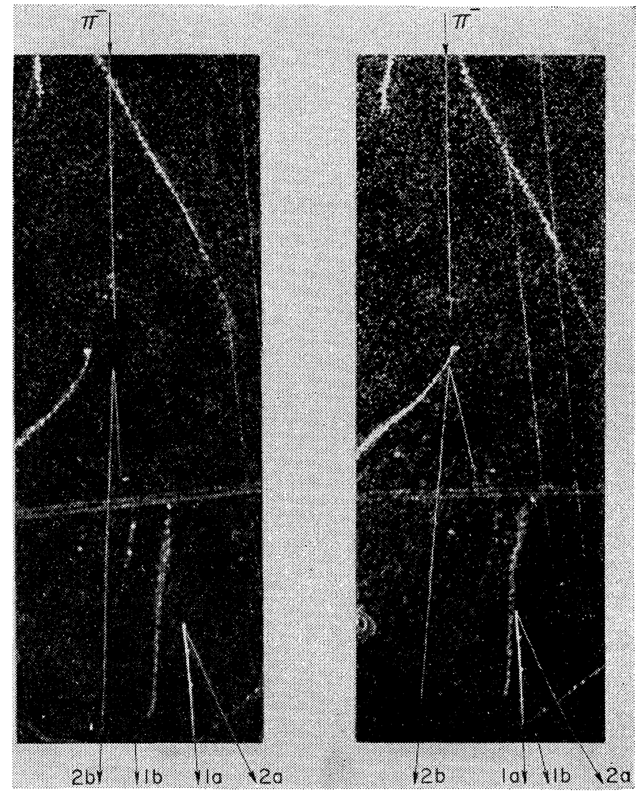
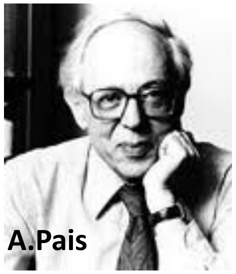


FIG. 2. Case D. Photograph of a 1.5-Bev π^- producing two neutral V particles in a collision with a proton. Tracks 1a and 2a, believed to be proton and π^-, respectively, are the decay products of a Λ^0. A ϑ^0 is probably seen to decay into π^+ (1b) and π^- (2b). Because of the rather "foggy" quality of this picture tracks 1b, 2a, and 2b have been retouched for better reproduction.

1955: Pais & Gell-Mann



Revised Pais' original idea:

→for neutron/proton/π the new QN = 0

→for K^0/K^+... the new QN = **+1**

→for $\Lambda^0/\Sigma^{+/-/0}$... the new QN =-1

1st example of particle "flavor"

Gave the new QN a name: **Strangeness**

-Strong interactions conserve strangeness

-Weak Interactions do not conserve strangeness

Birth of High Energy Physics

High energy: Sufficient energy to produce Strange particles

Units, notation, important quantum numbers

Units

In this class: $\hbar = c = 1$

All quantities are expressed in MeV (or GeV)

[distance] unit $\Rightarrow$ MeV^{-1}

[time] unit $\Rightarrow$ MeV^{-1}

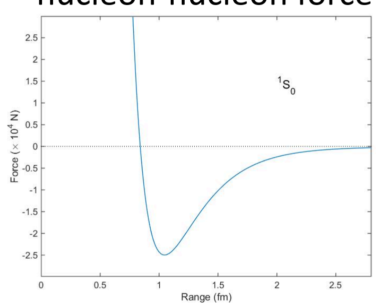
to convert back to ordinary units:

$$[\text{distance}] = \frac{\hbar c}{1\text{ MeV}} = 197\text{ fm} = 1.97\times10^{-13}\text{m}$$

$$[\text{time}] = \frac{[\text{distance}]}{c} = \frac{\hbar}{1\text{ MeV}} = 6.58\times10^{-22}\text{sec}$$

Example:

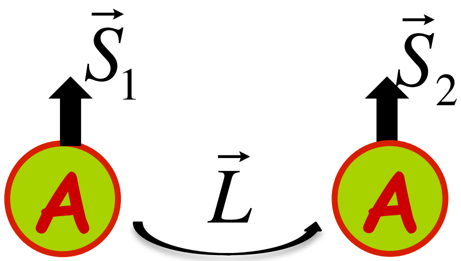
nucleon-nucleon force



nuclear force range $\approx$ 1.4 fm

$$= \frac{1.4}{197\text{ MeV}} = \frac{1}{140\text{ MeV}} \approx 1/7\ m_{proton}$$

Spectroscopic Notation



J^{PC} quantum numbers

$$\vec{S} = \vec{S}_1 + \vec{S}_2$$

$$\vec{J} = \vec{L} + \vec{S}$$

S=1 ➔ triplet of state
S=0 ➔ singlet

radial q.n.

$$\boxed{n_r\,{}^{2S+1}L_J}$$

$L=0: \quad \mathrm{S}$

$L=1: \quad \mathrm{P}$

$L=2: \quad \mathrm{D}$

$\ldots$

Important Quantum numbers

Important Quantum numbers

Parity

- A parity transformation, P, inverts every spatial coordinate: $P(t, \mathbf{x}) = (t, -\mathbf{x})$
  $P^2 = I$, and therefore the eigenvalues of P are ± 1.
- Ordinary vector $\mathbf{v}$. $P(\mathbf{v}) = -\mathbf{v}$.
- Scalar from $\mathbf{v}$: $s = \mathbf{v} \cdot \mathbf{v}$
  $P(s) = P(\mathbf{v} \cdot \mathbf{v}) = (-\mathbf{v}) \cdot (-\mathbf{v}) = \mathbf{v} \cdot \mathbf{v} = +s$
- Cross product of two vectors: $\mathbf{a} = \mathbf{v} \times \mathbf{w}$
  $P(\mathbf{a}) = P(\mathbf{v} \times \mathbf{w}) = (-\mathbf{v}) \times (-\mathbf{w}) = \mathbf{v} \times \mathbf{w} = +\mathbf{a}$ called an "axial" vector
- Scalar from $\mathbf{a}$ and $\mathbf{v}$: $p = \mathbf{a} \cdot \mathbf{v}$
  $P(p) = P(\mathbf{a} \cdot \mathbf{v}) = (+\mathbf{a}) \cdot (-\mathbf{v}) = -\mathbf{a} \cdot \mathbf{v} = -p$

Scalar	$P(s) = +s$
Pseudoscalar	$P(p) = -p$
Vector	$P(\mathbf{v}) = -\mathbf{v}$
Pseudovector	$P(\mathbf{a}) = +\mathbf{a}$

Slide copied from Diego Bettoni

Parity in Physical Systems

- Two-body systems have parity $p_A p_B (-1)^\ell$
  $P\phi(12) = p_1 p_2 (-1)^\ell \phi(12)$
- Intrinsically fermions and antifermions have opposite parity
  Bound states like positronium $e^+ \; e^-$ and mesons $q\overline{q}$ have parity of $(-1)^{\ell+1}$.
- Photons have a parity of (-1), and this underlies the $\Delta\ell = \pm 1$ selection rule in atomic transitions.
- Note that parity is a *multiplicative* quantum number.
  This is true for all discrete symmetries.
  Continuous symmetries have *additive* quantum numbers.

Slide copied from Diego Bettoni

Charge Conjugation I

- The charge conjugation operator, C, converts a particle to i ts antiparticle.
  $C\,|p\rangle = |\overline{p}\rangle$
- In particular, C reverses *every* internal quantum number (e.g. charge, baryon/lepton number, strangeness, etc.).
- $C^2 = I$ implies that the only allowed eigenvalues of C are ± 1.
- Unlike parity, very few particles are C eigenstates.
  Only particles that are their own antiparticles (π^0, η, γ) are C eigenstates.

  For example,
  $C\,|\pi^+\rangle = |\pi^-\rangle$
  $C\,|\gamma\rangle = -\,|\gamma\rangle$

Slide copied from Diego Bettoni

Charge Conjugation II

- The photon has a $C = -1$ $(A_\mu \leftrightarrow -A_\mu$ when $q \leftrightarrow -q)$
- $f\overline{f}$ bound states have $C = (-1)^{\ell+s}$
- Charge conjugation is respected by both the strong and electromagnetic interactions.
- Example: the π^0 $(\ell = s = 0 \Rightarrow C = +1)$ can decay into 2γ but not 3γ

$$C\,|n\gamma\rangle = (-1)^n\,|\gamma\rangle$$
$$C\,|\pi^0\rangle = |\pi^0\rangle$$

$\pi^0 \rightarrow 2\gamma$ is allowed (and observed)
$\pi^0 \rightarrow 2\gamma$ is not allowed (and not observed $< 3.1 \times 10^{-8}$)

Slide copied from Diego Bettoni

Isospin

$$m_p = 938.27\,MeV \quad m_n = 939.57\,MeV$$

$$m_p \approx m_n$$

Heisenberg (1932):

*Proton and neutron considered as different charge substates of one particle, the **Nucleon**.*

A nucleon is ascribed a quantum number, **isospin**, conserved in the strong interaction, not conserved in electromagnetic interactions.

Nucleon is assigned isospin $I = \frac{1}{2}$

$$I_3 = +\tfrac{1}{2} \quad p$$

$$I_3 = -\tfrac{1}{2} \quad n$$

$$\frac{Q}{e} = \frac{1}{2} + I_3$$

Slide copied from Diego Bettoni

The nucleon has an internal degree of freedom with two allowed states (the proton and the neutron) which are not distinguished by the nuclear force.

Let us write the nucleon states as $\left| I, I_3 \right\rangle$

$$\left| p \right\rangle = \left| \tfrac{1}{2}, \tfrac{1}{2} \right\rangle \quad \left| n \right\rangle = \left| \tfrac{1}{2}, -\tfrac{1}{2} \right\rangle$$

For a two-nucleon system we have therefore:

Triplet (symmetric)

$$\begin{cases} \chi(1,1) = \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \\ \chi(1,0) = \tfrac{1}{\sqrt{2}}\left(\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle + \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \right) \\ \chi(1,-1) = \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \end{cases}$$

Singlet (antisymmetric)

$$\left\{ \chi(0,0) = \tfrac{1}{\sqrt{2}}\left(\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle - \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \right) \right.$$

Slide copied from Diego Bettoni

Example: deuteron (S-wave *pn* bound state)

$$\psi = \phi(spazio) \times \alpha(spin) \times \chi(isospin)$$

$$\underbrace{\begin{matrix}(-1)^l = +1 & (-1)^{S+1} = +1 \\ (l = 0) & (S = 1)\end{matrix}}_{\boxed{(-1)^{I+1} = -1 \quad \Rightarrow \quad I = 0}} \qquad (-1)^{I+1}$$

ψ is the wave function for *two identical fermions* (two nucleons), hence it must be *globally antisymmetric*. This implies that the deuteron must have **zero isospin**:

$$I_d = 0$$

Slide copied from Diego Bettoni

As an example let us consider the two reactions

$$p + p \rightarrow \pi^+ + d \qquad\qquad (I_\pi = 1)$$
$$p + n \rightarrow \pi^0 + d$$

Since I_d=0 in each case the final state has isospin 1.

Let us now consider the initial states:

$$pp = |1{,}1\rangle$$
$$np = \tfrac{1}{\sqrt{2}}\left(|1{,}0\rangle - |0{,}0\rangle\right)$$

The cross section

$$\sigma \propto |ampiezza|^2 \approx \sum_I \left|\langle I', I_3' | A | I, I_3\rangle\right|^2$$

Isospin conservation implies

$$I = I' = 1 \quad I_3 = I_3'$$

The reaction $np \rightarrow \pi^0 d$ proceeds with probability $\left(\frac{1}{\sqrt{2}}\right)^2$ with respect to $pp \rightarrow \pi^+ d$ hence:

$$\frac{\sigma\left(pp \rightarrow \pi^+ d\right)}{\sigma\left(np \rightarrow \pi^0 d\right)} = 2$$

Slide copied from Diego Bettoni

Isospin in the πN System

The π meson exists in three charge states of roughly the same mass:

$$m_{\pi^\pm} = 139.57\, MeV$$

$$m_{\pi^0} = 134.98\, MeV$$

Consequently it is assigned $I_\pi=1$, with the charge given by $Q/e=I_3$.

$$|\pi^+\rangle = |1,1\rangle \quad |\pi^0\rangle = |1,0\rangle \quad |\pi^-\rangle = |1,-1\rangle$$

For the π $B=0$:

$$\boxed{\frac{Q}{e} = I_3 + \frac{B}{2}}$$

Slide copied from Diego Bettoni

For the πN system the total isospin can be either I=1/2 or I=3/2

$$\left.\begin{array}{l}\pi^+ p \to \pi^+ p \\ \pi^- n \to \pi^- n\end{array}\right\}$$

pure
I=3/2

$$\left.\begin{array}{l}\pi^- p \to \pi^- p \\ \pi^- p \to \pi^0 n \\ \pi^+ n \to \pi^+ n \\ \pi^+ n \to \pi^0 p\end{array}\right\}$$

combination of
I=1/2 and I=3/2

		$I=\frac{3}{2}$			$I=\frac{1}{2}$	
$1\times\frac{1}{2}$ $\quad I_3$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
$\pi^+ p$	1					
$\pi^+ n$		$\sqrt{\frac{1}{3}}$			$\sqrt{\frac{2}{3}}$	
$\pi^0 p$		$\sqrt{\frac{2}{3}}$			$-\sqrt{\frac{1}{3}}$	
$\pi^0 n$			$\sqrt{\frac{2}{3}}$			$\sqrt{\frac{1}{3}}$
$\pi^- p$			$\sqrt{\frac{1}{3}}$			$-\sqrt{\frac{2}{3}}$
$\pi^- n$				1		

The coefficients in the linear combinations, i.e. the relative weights of the 1/2 and 3/2 amplitudes, are given by *Clebsch-Gordan coefficients*

$$\left|\pi^+ n\right\rangle = \left|1,1\right\rangle \times \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$$
$$= \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{3}}\left|\pi^+ n\right\rangle + \sqrt{\tfrac{2}{3}}\left|\pi^0 p\right\rangle$$
$$= \sqrt{\tfrac{1}{3}}\left|1,1\right\rangle \times \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\left|1,0\right\rangle \times \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$$

Slide copied from Diego Bettoni

$$(1) \quad \pi^+ p \to \pi^+ p$$
$$(2) \quad \pi^- p \to \pi^- p$$

Elastic scattering

$$(3) \quad \pi^- p \to \pi^0 n$$

Charge exchange

$$\sigma \propto \left|\langle f | H | i \rangle\right|^2 = \left|M_{if}\right|^2$$

H = { H_1 if it acts between states of *I=1/2*; H_3 if it acts between states of *I=3/2* }

let $M_1 = \langle I = \tfrac{1}{2} | H_1 | I = \tfrac{1}{2} \rangle$

$M_3 = \langle I = \tfrac{3}{2} | H_3 | I = \tfrac{3}{2} \rangle$

(1) $\sigma_1 = K|M_3|^2$

$$|i\rangle = |f\rangle = \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

(2) $\sigma_2 = K\left|\langle f | (H_1 + H_3) | i \rangle\right|^2$

$$\sigma_2 = K\left|\tfrac{1}{3}M_3 + \tfrac{2}{3}M_1\right|^2$$

(3)
$$|i\rangle = \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$
$$|f\rangle = \sqrt{\tfrac{2}{3}}\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$
$$\sigma_3 = K\left|\sqrt{\tfrac{2}{9}}M_3 - \sqrt{\tfrac{2}{9}}M_1\right|^2$$

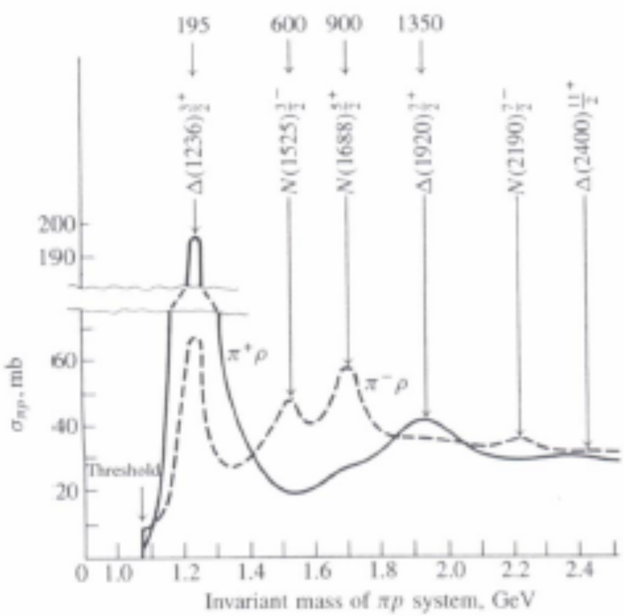
$$\boxed{\sigma_1 : \sigma_2 : \sigma_3 = |M_3|^2 : \tfrac{1}{9}|M_3 + 2M_1|^2 : \tfrac{2}{9}|M_3 - M_1|^2}$$

$M_3 >> M_1 \quad \sigma_1 : \sigma_2 : \sigma_3 = 9 : 1 : 2$

$M_1 >> M_3 \quad \sigma_1 : \sigma_2 : \sigma_3 = 0 : 2 : 1$

Slide copied from Diego Bettoni

$\pi^{\pm}p$ Total Cross Section



$$\Delta(1236) \quad \Gamma = 120\,MeV$$

$$J^P = \frac{3}{2}^+ \quad I = \frac{3}{2} \quad (3,3)$$

$$\sigma(E) = \frac{2J+1}{(2s_1+1)(2s_2+1)} \frac{\pi}{k^2} \frac{\Gamma_{ab}\Gamma_{cd}}{(E-M_R)^2 + \frac{\Gamma^2}{4}}$$

$a+b \rightarrow R \rightarrow c+d$

Slide copied from Diego Bettoni

Strangeness S

Strange particles are copiously produced in strong interactions

They have a long lifetime, typical of a weak decay.

S quantum number: strangeness *conserved in strong and electromagnetic interactions, not conserved in weak interactions*.

Example: $\pi^- + p \rightarrow \Lambda + K^0$

$\hookrightarrow p + \pi^-$ $\quad \tau = 2.6 \times 10^{-10}\, s$

$$\begin{array}{cccc} & \Lambda \rightarrow & p + & \pi^- \\ I & 0 & \tfrac{1}{2} & 1 \\ I_3 & 0 & \tfrac{1}{2} & -1 \end{array}$$

I = 0, because the Λ has no charged counterparts

$$\begin{array}{ccccc} & \pi^- + & p \rightarrow & \Lambda + & K^0 \\ I & 1 & \tfrac{1}{2} & 0 & \tfrac{1}{2} \\ I_3 & -1 & \tfrac{1}{2} & 0 & -\tfrac{1}{2} \end{array}$$

Slide copied from Diego Bettoni

$$\left.\begin{array}{ll} K^0, K^+ & \dfrac{Q}{e} = I_3 + \dfrac{1}{2} \\ \overline{K}^0, K^- & \dfrac{Q}{e} = I_3 - \dfrac{1}{2} \end{array}\right\} \quad \frac{Q}{e} = I_3 + \frac{B+S}{2}$$

(Gell-Mann Nishijima)

Y = B+S hypercharge

Using the Gell-Mann Nishijima formula strangeness is assigned together with isospin.

Example.:

n, p	$S = 0$	$I = \frac{1}{2}$
Λ	$S = -1$	$I = 0$
K^0, K^+	$S = 1$	$I = \frac{1}{2}$
$K^-, \overline{K}^0$	$S = -1$	$I = \frac{1}{2}$

Example of strangeness conservation:

$$K^- + p \to \Lambda + \pi^0$$

S	-1	0	-1	0
I_3	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0

$$\pi^\pm + p \to \Sigma^\pm + K^+$$

S	0	0	-1	$+1$
I	1	$\frac{1}{2}$	1	$\frac{1}{2}$
I_3	± 1	$\frac{1}{2}$	± 1	$\frac{1}{2}$

$\Sigma^0 \to \Lambda + \gamma$ *e.m.*

S	-1	-1	0

$\Sigma^+ \to n + \pi^+$ *weak*

S	-1	0	0

$\Xi^- \to \Lambda + \pi^-$ *weak*

S	-2	-1	0

Slide copied from Diego Bettoni

G-parity G

$$G = Ce^{i\pi I_2}$$

Rotation of π around the 2 axis in isospin space followed by charge conjugation.

$$I_3 \xrightarrow{e^{i\pi I_2}} -I_3 \xrightarrow{C} I_3$$

Consider an isospin state $\chi(I, I_3=0)$: under isospin rotations this state behaves like $Y^0_l(\theta,\varphi)$ (under rotations in ordinary space)

The rotation around the 2 axis implies:

$$\vartheta \to \pi - \vartheta \quad \varphi \to \pi - \varphi$$

$$Y_l^0 \to (-1)^l Y_l^0$$

therefore

$$\chi(I,0) \to (-1)^I \chi(I,0)$$

Slide copied from Diego Bettoni

Example: for a nucleon-antinucleon state the effect of C is to give a factor $(-1)^{l+s}$ (just is in the case of positronium). Therefore:

$$G|\psi(N\bar{N})\rangle = (-1)^{l+s+I}|\psi(N\bar{N})\rangle$$

This formula has **general validity**, not limited to the $I_3=0$ case.

For the π

$$G|\pi^+\rangle = \pm|\pi^+\rangle$$
$$G|\pi^-\rangle = \pm|\pi^-\rangle$$
$$G|\pi^0\rangle = \pm|\pi^0\rangle$$

For the π^0 C=+1 ($\pi^0\to\gamma\gamma$), the rotation gives $(-1)^I=-1$ ($I=1$) so that G= -1.

$$G_{\pi^0} = -1$$

It is the practice to assign the phases so that *all members of an isospin triplet have the same G-parity as the neutral member.*

$$G|\pi^\pm\rangle = -|\pi^\mp\rangle \qquad \text{with} \quad C|\pi^\pm\rangle = -|\pi^\mp\rangle$$

Slide copied from Diego Bettoni

Since the C operation reverses the sign of the baryon number B, **the eigenstates of G-parity must have baryon number zero B=0**.

G is a multiplicative quantum number, so for a system of n π

$$G=(-1)^n$$

$$\rho \to \pi\pi \qquad G_\rho = +1$$

$$\omega \to \pi\pi\pi \qquad G_\omega = -1 \qquad B.R. = 89\%$$

$$\omega \to \pi\pi \qquad G_f = +1 \qquad B.R. = 2.2\%$$

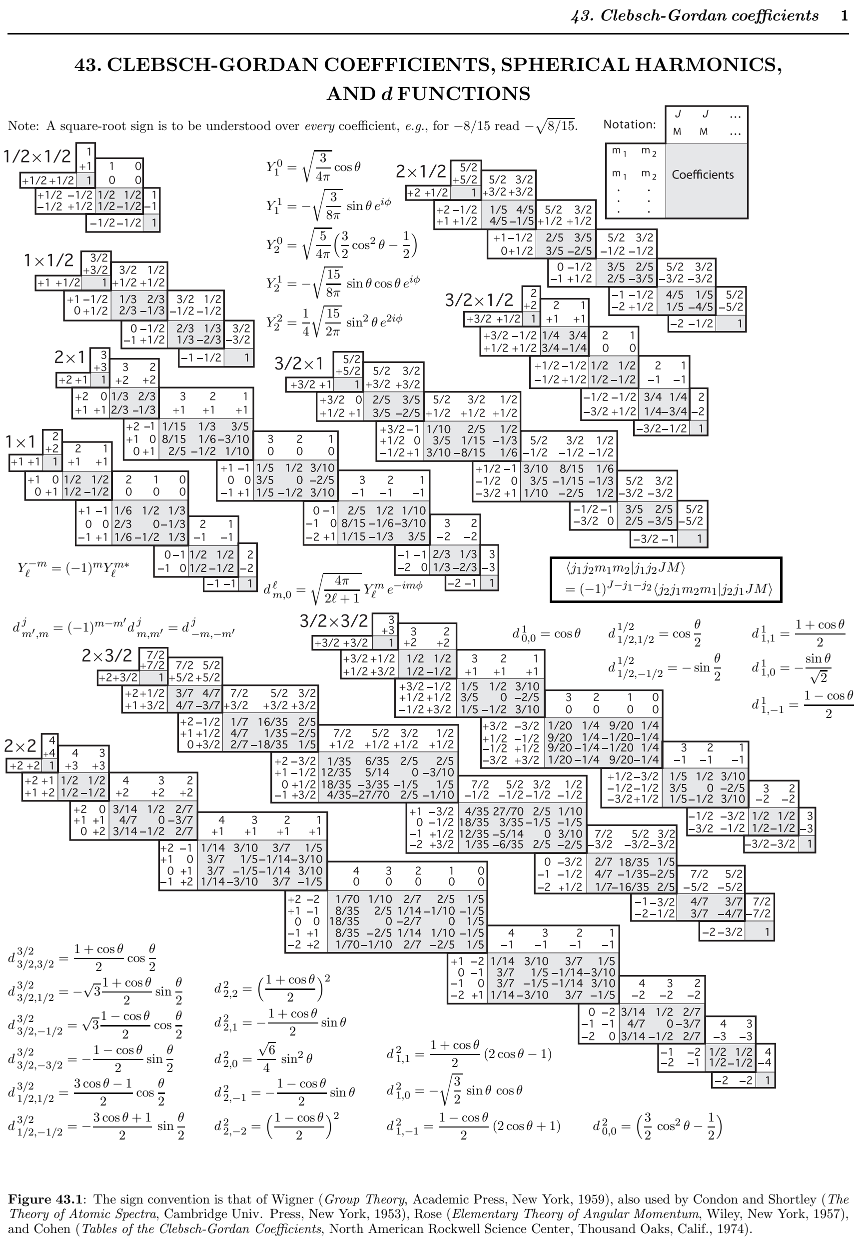
$\eta \to \gamma\gamma$ C=+1 which, with I=0, yields G=+1.

$$\eta \not\to \pi\pi \qquad viola\ P$$

$$\eta \to \pi\pi\pi \qquad viola\ G \Rightarrow e.m.$$

Slide copied from Diego Bettoni

important page in PDG for this class



43. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^1_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta - 1)$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta - 1}{2}\cos\frac{\theta}{2} \qquad d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta + 1}{2}\sin\frac{\theta}{2} \qquad d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2 \qquad d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta + 1) \qquad d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

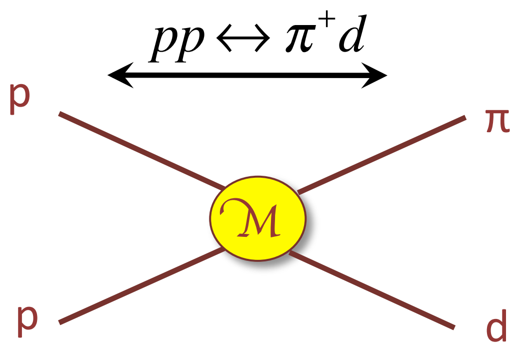
Figure 43.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).

properties of π- and K-mesons

"Particles, particles, particles."

Spin of the π^+

$$\frac{d\sigma}{d\Omega}\left(pp \to \pi^+ d\right) \leftarrow \text{compare} \to \frac{d\sigma}{d\Omega}\left(\pi^+ d \to pp\right)$$

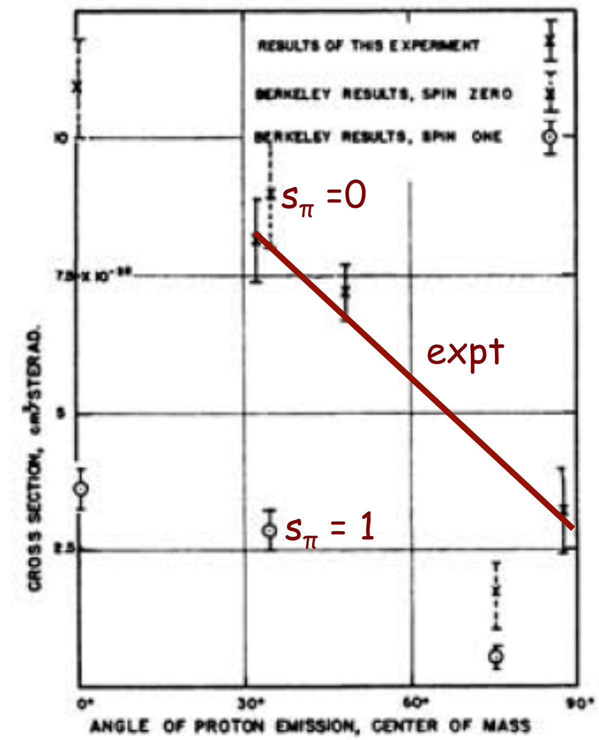


$$\frac{d\sigma}{d\Omega} \propto \left|\mathcal{M}\right|^2 \times ps \times (\#\, spin\ states)$$

same both ways

$$\frac{d\sigma(\pi^+ d \to pp)/d\Omega}{d\sigma(pp \to \pi^+ d)/d\Omega} = \frac{(2s_p+1)^2}{(2s_d+1)(2s_\pi+1)} \frac{p_{pp}^2}{p_{\pi d}^2}$$

$=4/3$ for $s_\pi = 0$
$=4/9$ for $s_\pi = 1$



D. L. Clark, A. Roberts, and R. Wilson, "Cross section for the reaction $\pi^+ d \to pp$ and the spin of the π^+ meson." *Phys. Rev.*, **83**, 649 (1951).
R. Durbin, H. Loar, and J. Steinberger, "The Spin of the Pion via the Reaction $\pi^+ + d \leftrightarrows p + p$." *Phys. Rev.*, **83**, 646 (1951).
R. Plano *et al.*, "Parity of the Neutral Pion." *Phys. Rev. Lett.*, **3**, 525 (1959).

Parity of the π⁻

stopped

$$\pi^- d \rightarrow nn \qquad \Leftarrow \text{observed}$$

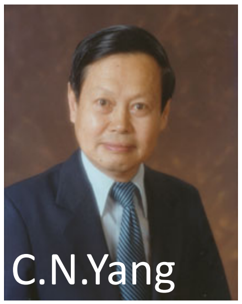
capture occurs in an S-wave $J=S_d=1$

only Pauli-allowed' $J=1$ nn state is $^3P_1$

$$\Rightarrow P_\pi P_d = -1; \quad \text{since } P_d = 1,\ P_\pi = -1$$

W. K. H. Panofsky, R. L. Aamodt, and J. Hadley, "The Gamma-Ray Spectrum Resulting from Capture of Negative π-Mesons in Hydrogen and Deuterium." *Phys. Rev.*, **81**, 565 (1951).

Spin of π^0 ➔ Yang's theorem



Phys. Rev. 77, 242 (1950)

C.N. Yang: Spin=1 particle cannot decay to 2 photons

$$\pi^0 \to \gamma\gamma \Leftarrow \text{cannot be } S = 1$$

Parity of π^0 ← use double "Dalitz" decays

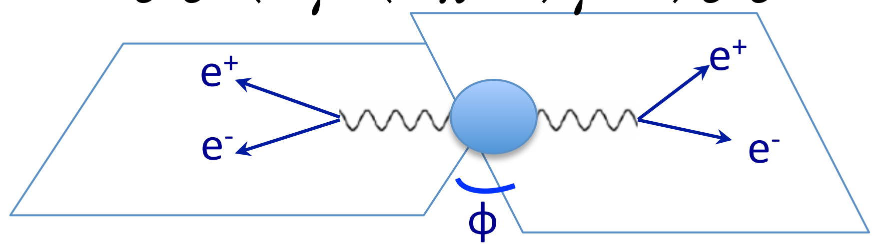
π^0 decay modes from PDG

Γ_1	2γ		(98.823 ± 0.034) %
Γ_2	$e^+ e^- \gamma$	← "Dalitz" decays →	(1.174 ± 0.035) %
Γ_3	γ positronium		$(1.82 \pm 0.29) \times 10^{-9}$
Γ_4	$e^+ e^+ e^- e^-$	← 2x"Dalitz" decays →	$(3.34 \pm 0.16) \times 10^{-5}$

Phys. Rev. 98, 1355 (1955)

$$\boxed{\frac{dN}{d\phi} \propto 1 \pm \alpha \cos 2\phi \qquad + \Rightarrow 0^+;\ - \Rightarrow 0^-}$$

$$e^+e^- \leftarrow \gamma^* \leftarrow \pi^0 \rightarrow \gamma^* \rightarrow e^+e^-$$



$\mu^+\mu^- \leftarrow Z^0 \leftarrow H \rightarrow Z^{*0} \rightarrow \mu^+\mu^-$ used to determine parity of the Higgs

$$e^+e^- \leftarrow \gamma^* \leftarrow \pi^0 \rightarrow \gamma^* \rightarrow e^+e^-$$

Phys. Rev. Lett. 3, 525 (1959)

for this expt's acceptance: $\alpha_{theory} = 0.48$

"typical" event

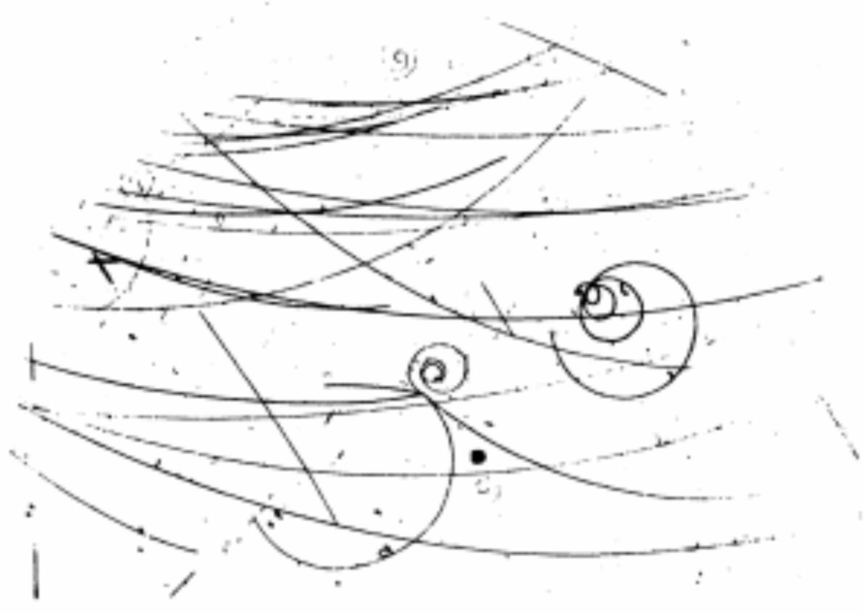


FIG. 1. A photograph of a typical double internal conversion.

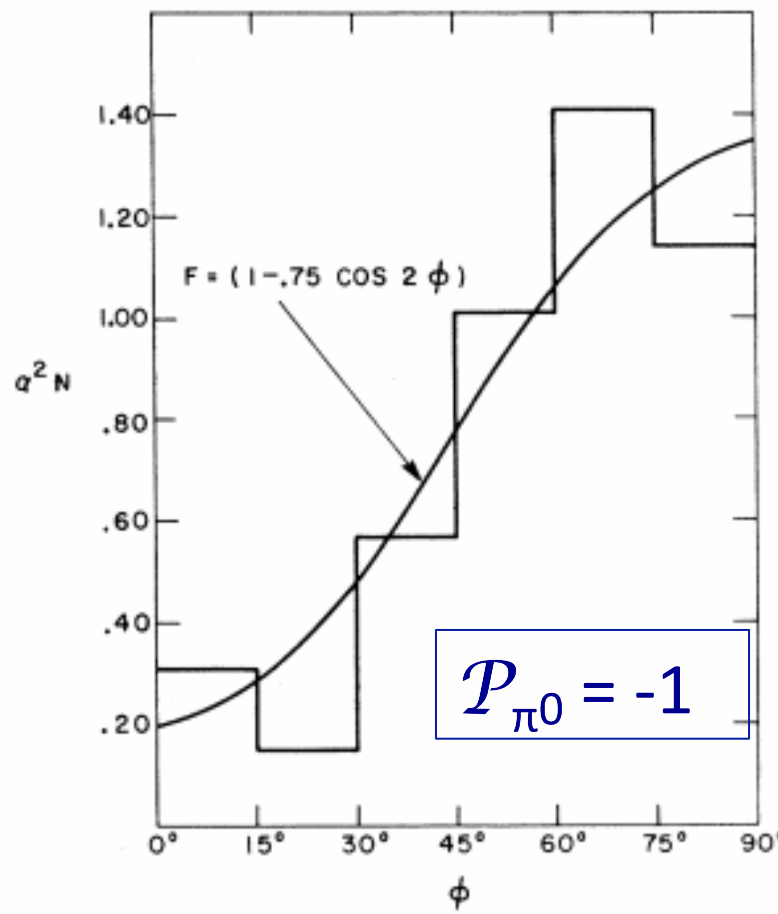


FIG. 2. Plot of weighted frequency distribution of angle between planes of polarization.

$\boxed{\mathcal{P}_{\pi 0} = -1}$

Parity of the K^-

strong interaction

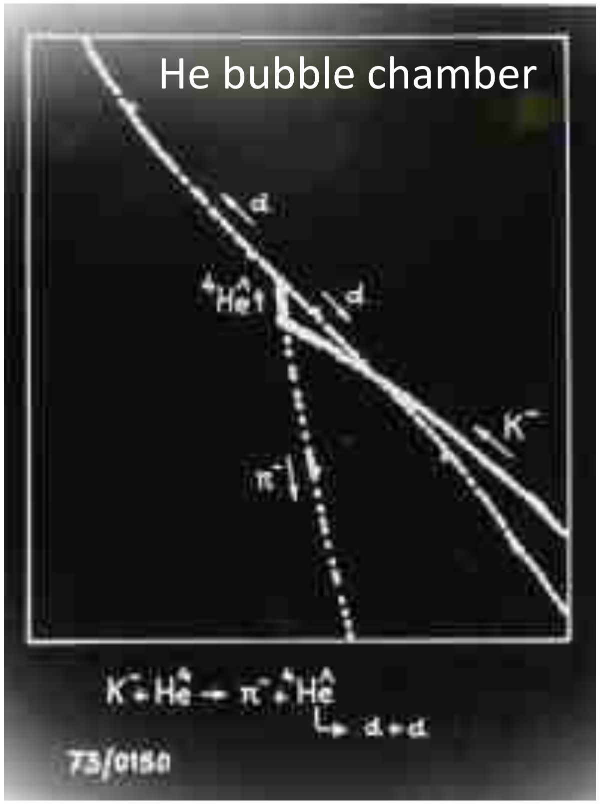
stopped

^{4}He nucleus
with n→Λ

$$K^- \ {}^4He \rightarrow \pi^- \ {}^4_\Lambda He$$

$$J^P : \ 0^? \quad 0^+ \quad 0^- \quad 0^+$$

capture occurs
in an S-wave
J=0



$$\Rightarrow P_K = P_\pi = -1$$

Resonances

The Fermi π-Nucleon resonance: Δ(1236)

Total Cross Sections of Positive Pions in Hydrogen*

H. L. Anderson, E. Fermi, E. A. Long,† and D. E. Nagle
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received January 21, 1952)

In a previous letter,[1] measurements of the total cross sections of negative pions in hydrogen were reported. In the present letter, we report on similar experiments with positive pions.

$$E_{cm}^{max}(\pi^+ p) = 1.19\ \text{GeV}$$

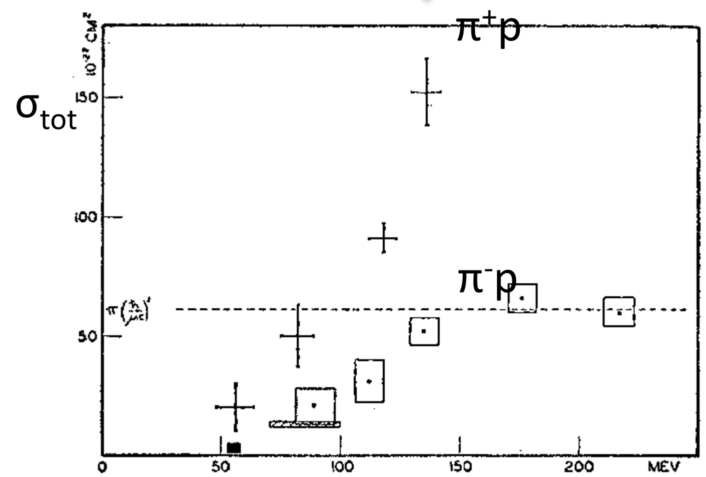
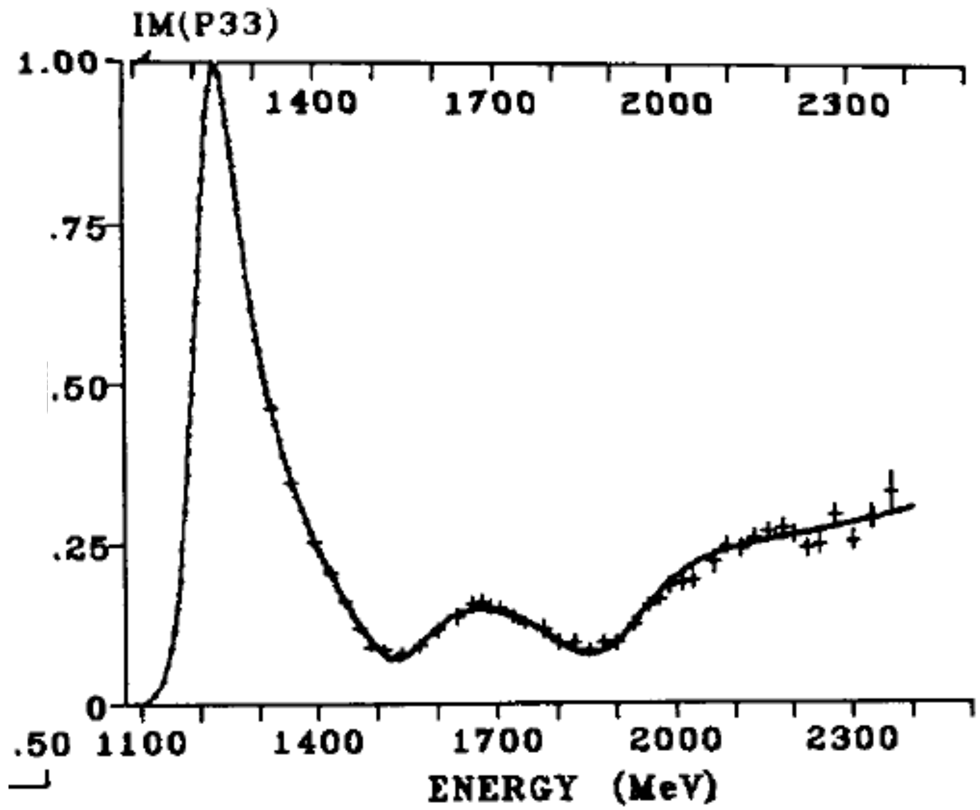


FIG. 1. Total cross sections of negative pions in hydrogen (sides of the angle represent the error) and positive pions in hydrogen (arms of the s represent the error). The cross-hatched rectangle is the Columbia lt. The black square is the Brookhaven result and does not include the rge exchange contribution.

if there is only one dominant intermediate state of spin 3/2 and isotopic spin 3/2, the total cross section of negative pions should at all points be less than $(8/3)\pi\lambda\!\!\!^{-2}$. Apparently, the experimental cross section above 150 Mev is larger than this limit, which indicates that other states contribute appreciably at these energies. Naturally, if a single state were dominant, one could expect that the cross sections would go through a maximum at an energy not far from the energy of the state involved. Unfortunately, we have not been able to push our measurements to sufficiently high energies to check on this point.

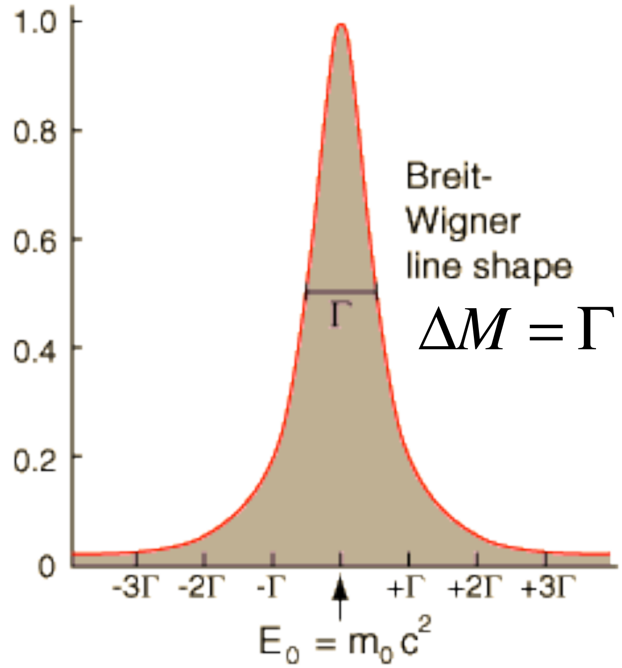
Based on these data, Fermi predicted a J=3/2; I=3/2 resonance with mass just above 1200 MeV

The Δ(1232) resonance



Breit Wigner line shape

$$BW = |A_{BW}|^2$$

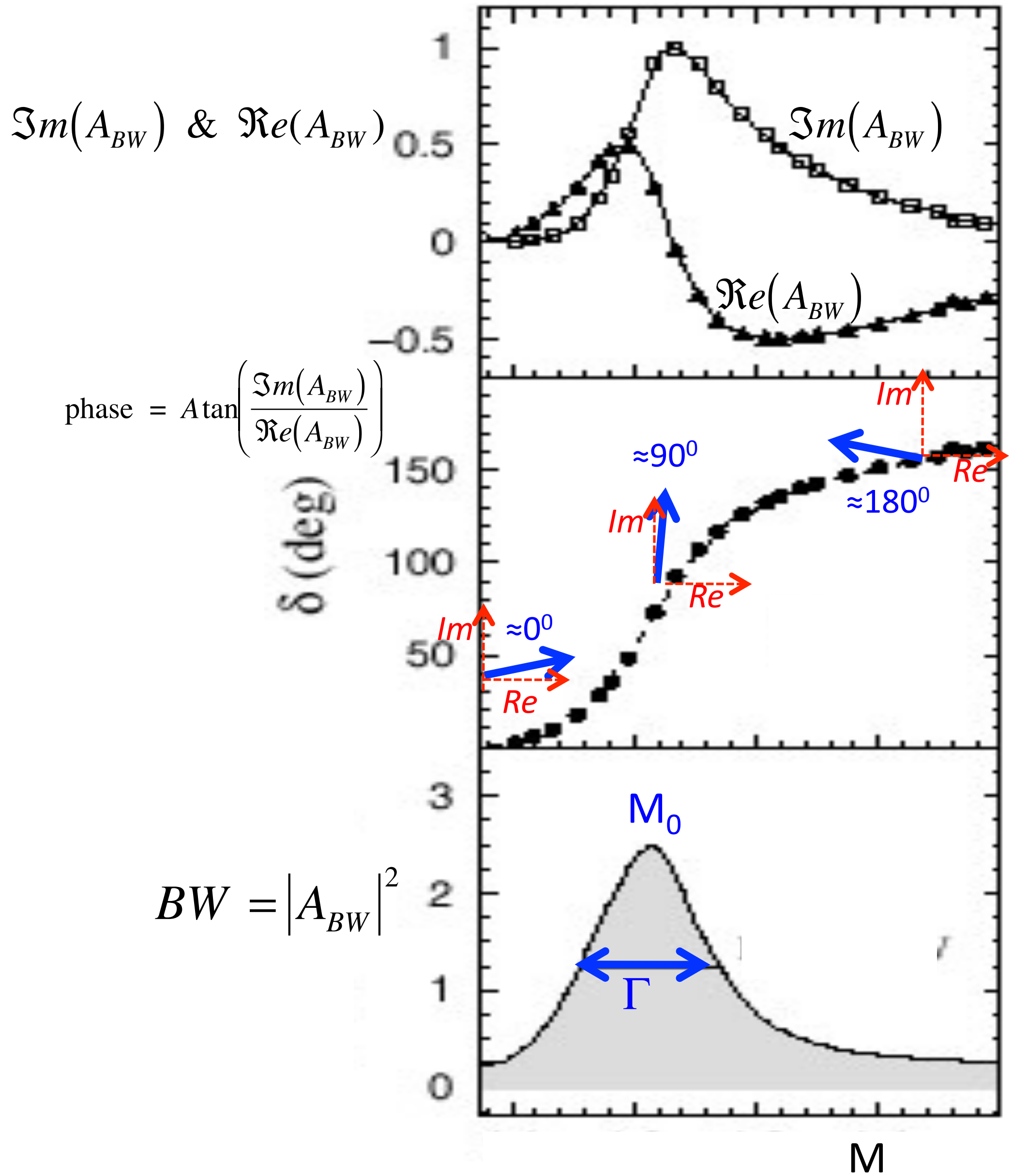


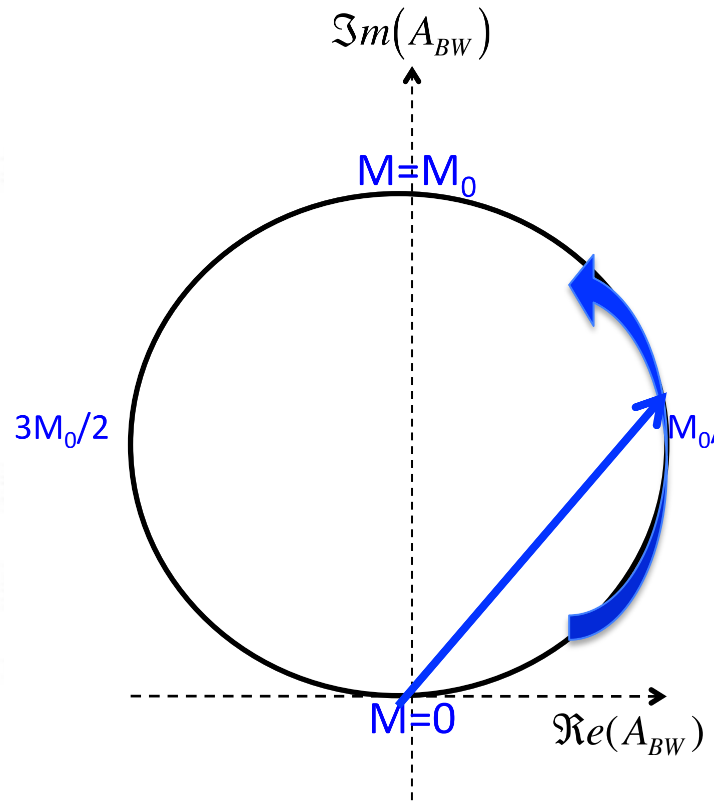
$$\Delta M = \Gamma = FWHM$$

$$A_{BW} = \frac{\Gamma/2}{(M - M_0) - i\Gamma/2} : \quad \Rightarrow \quad BW = |A_{BW}|^2 = \frac{\Gamma^2/4}{(M - M_0)^2 + \Gamma^2/4}$$

Please note: This is the simplest version of many different expressions used to describe resonance line shapes. These general features are common to all forms.

Phase structure of A_{BW}



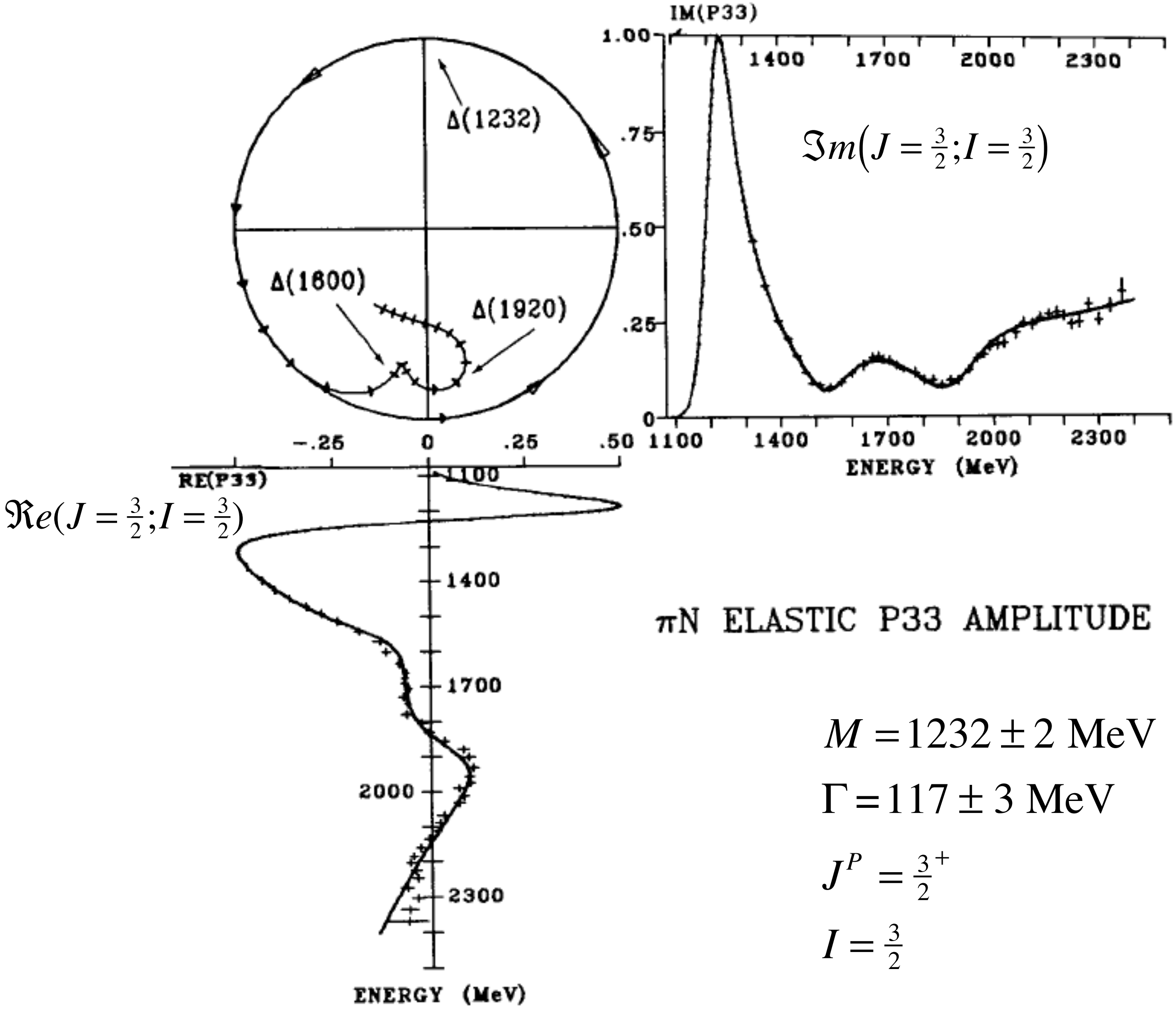


Argand plot

-- A_{BW} moves CCW in a circle

-- rapid "phase motion" where $|A_{BW}|$ is maximum

return to the Fermi resonance: Δ(1232)



πN ELASTIC P33 AMPLITUDE

$M = 1232 \pm 2$ MeV

$\Gamma = 117 \pm 3$ MeV

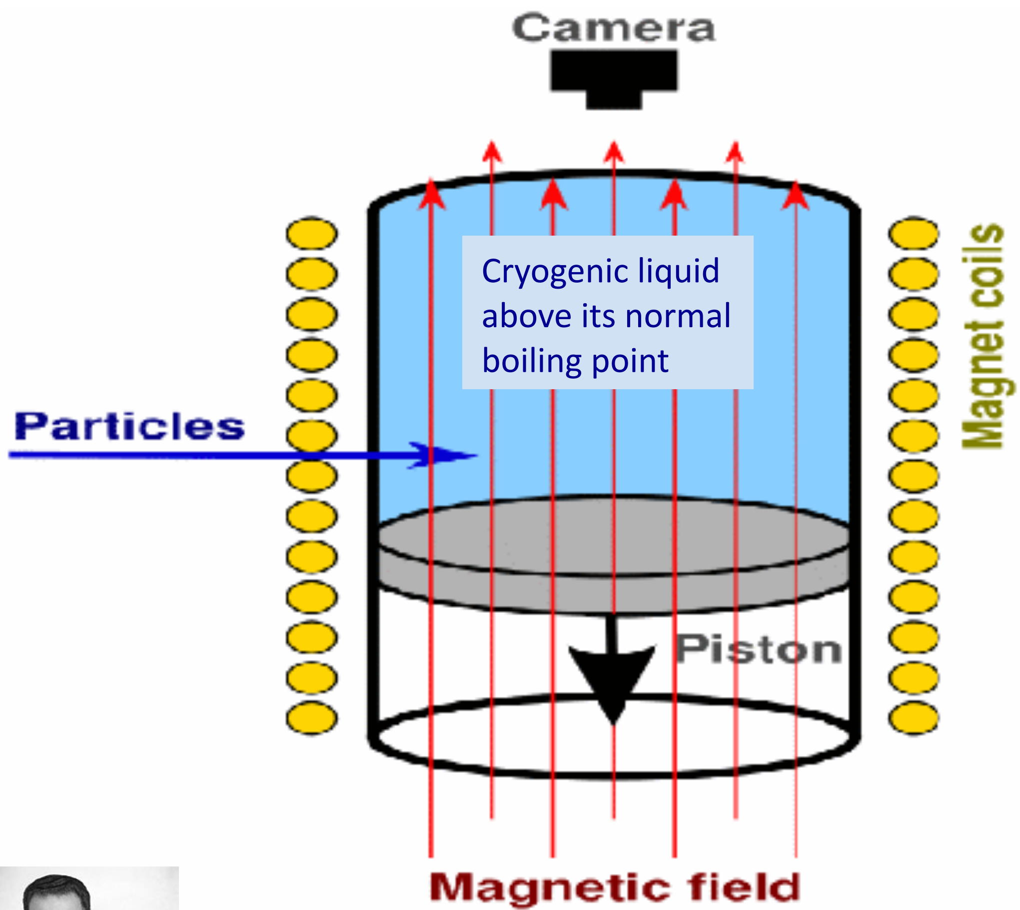
$J^P = \frac{3}{2}^+$

$I = \frac{3}{2}$

Other Resonances

-- The glory days of Bubble Chambers --

Bubble chambers



Donald Glaser
1926-2013

1959 Nobel physics prize

Pressurize the liquid to ~10 Atm to keep it from boiling

Pass particle beam through liquid

Rapidly depressurize the liquid --it starts to boil around the track-induced ions.

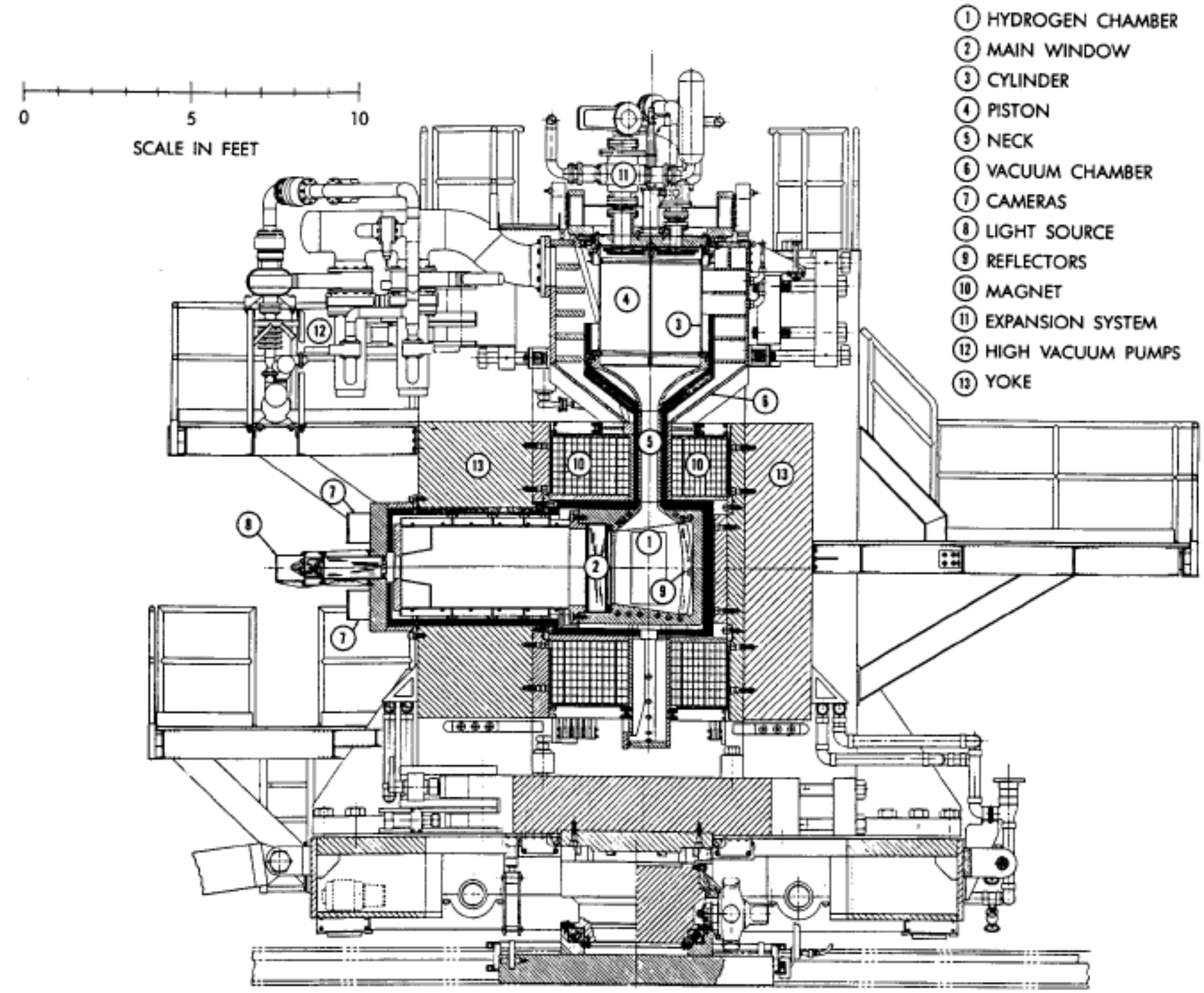
Allow bubbles to grow for ~3 msec

Flash lamps; take photographs

Recompress before general boiling occurs.

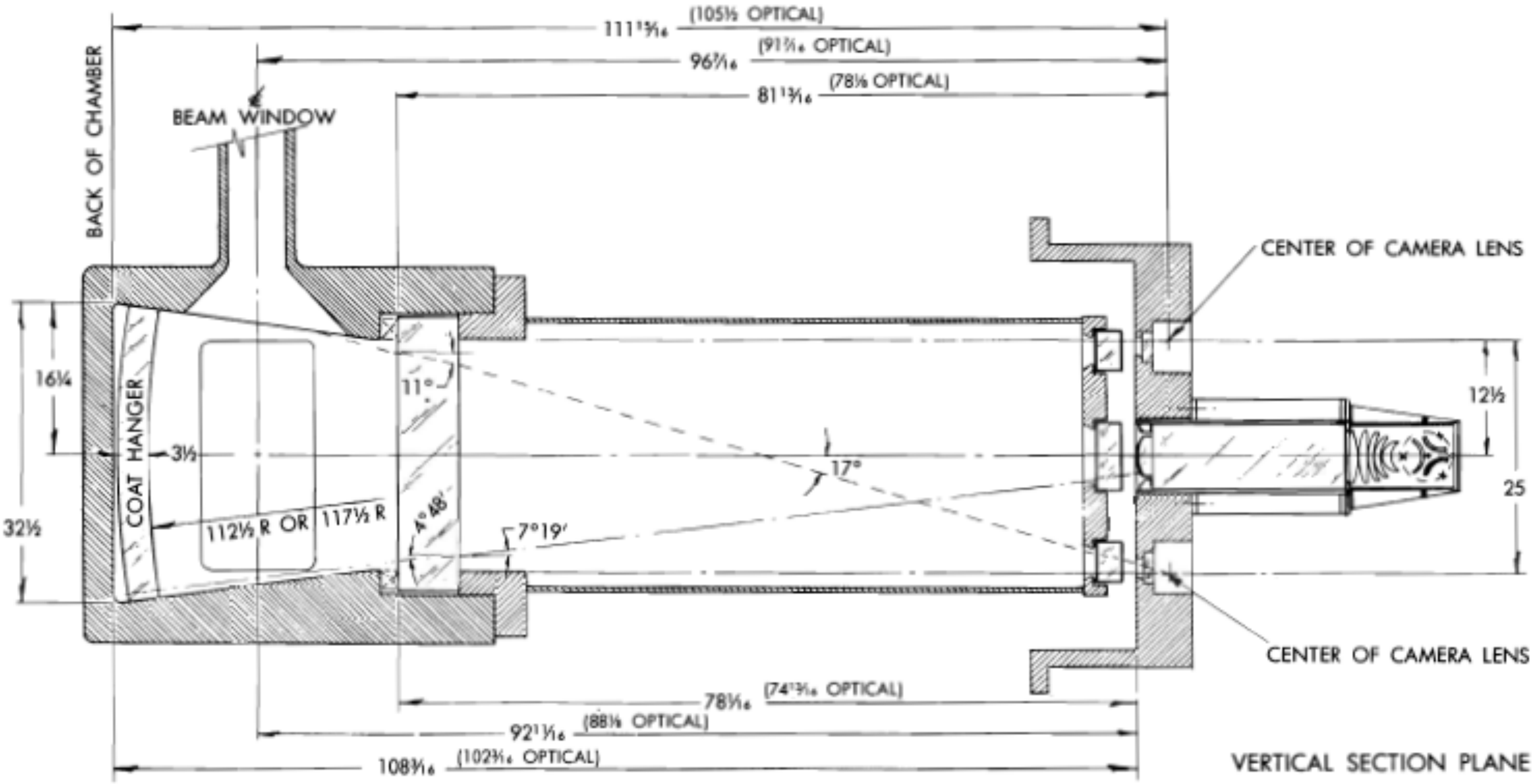
Repeat

80-inch bubble chamber at Brookhaven

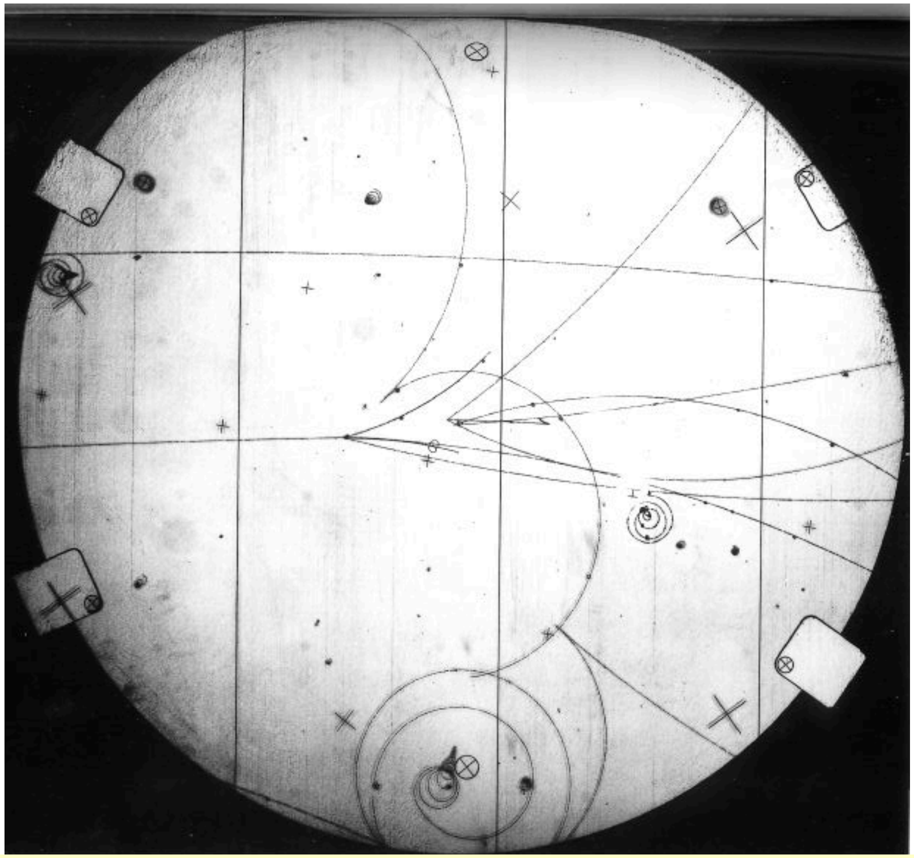


Schematic cross section of the 80-inch liquid hydrogen bubble chamber showing major components.

80-inch bubble chamber optics



Bubble chamber event

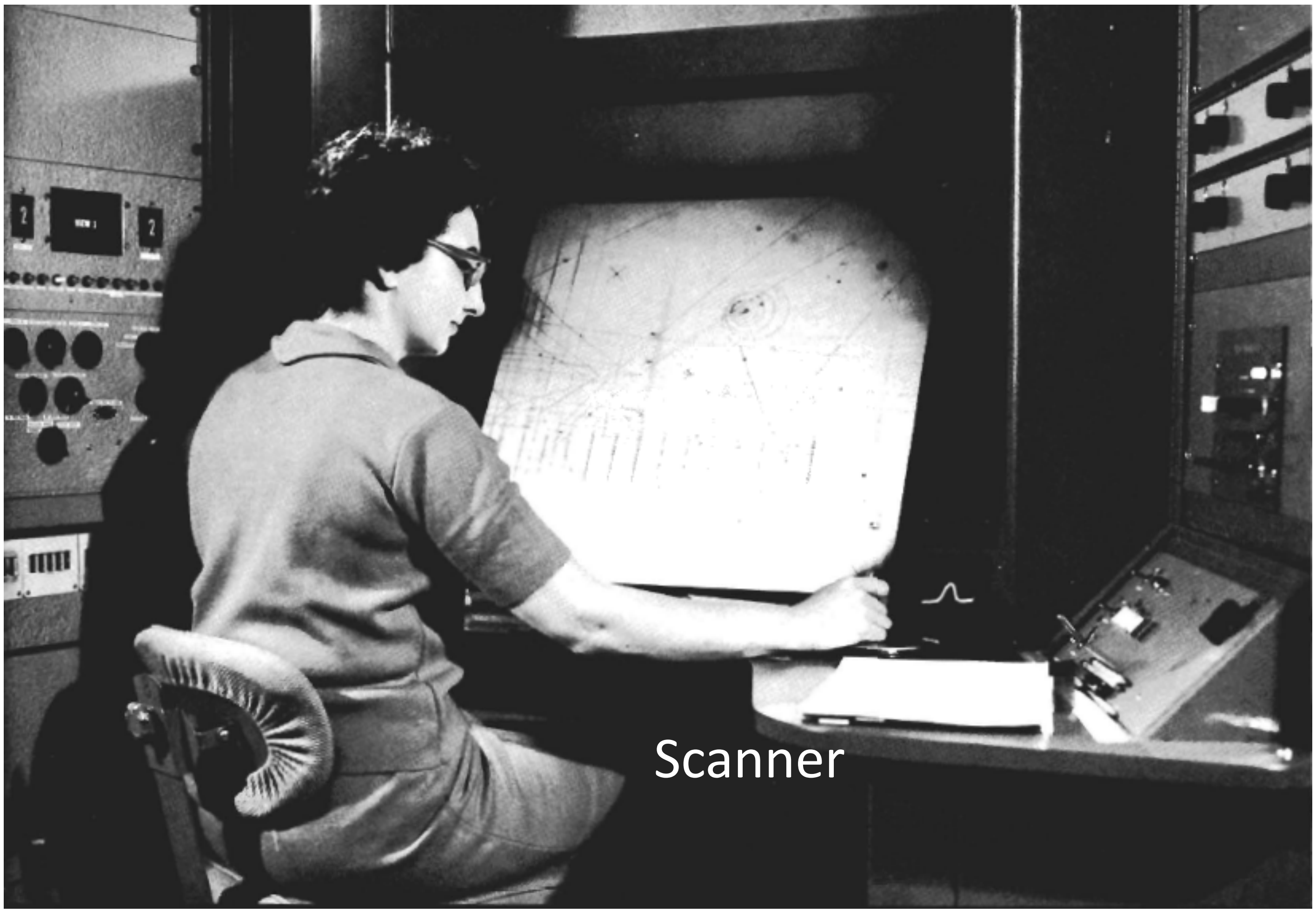


A careful study of this photograph reveals the reaction to be $\bar{p}\,p \rightarrow p\,\pi^+\,K^-\,\pi^-\,\pi^0\,K^0\,\bar{n}$, where

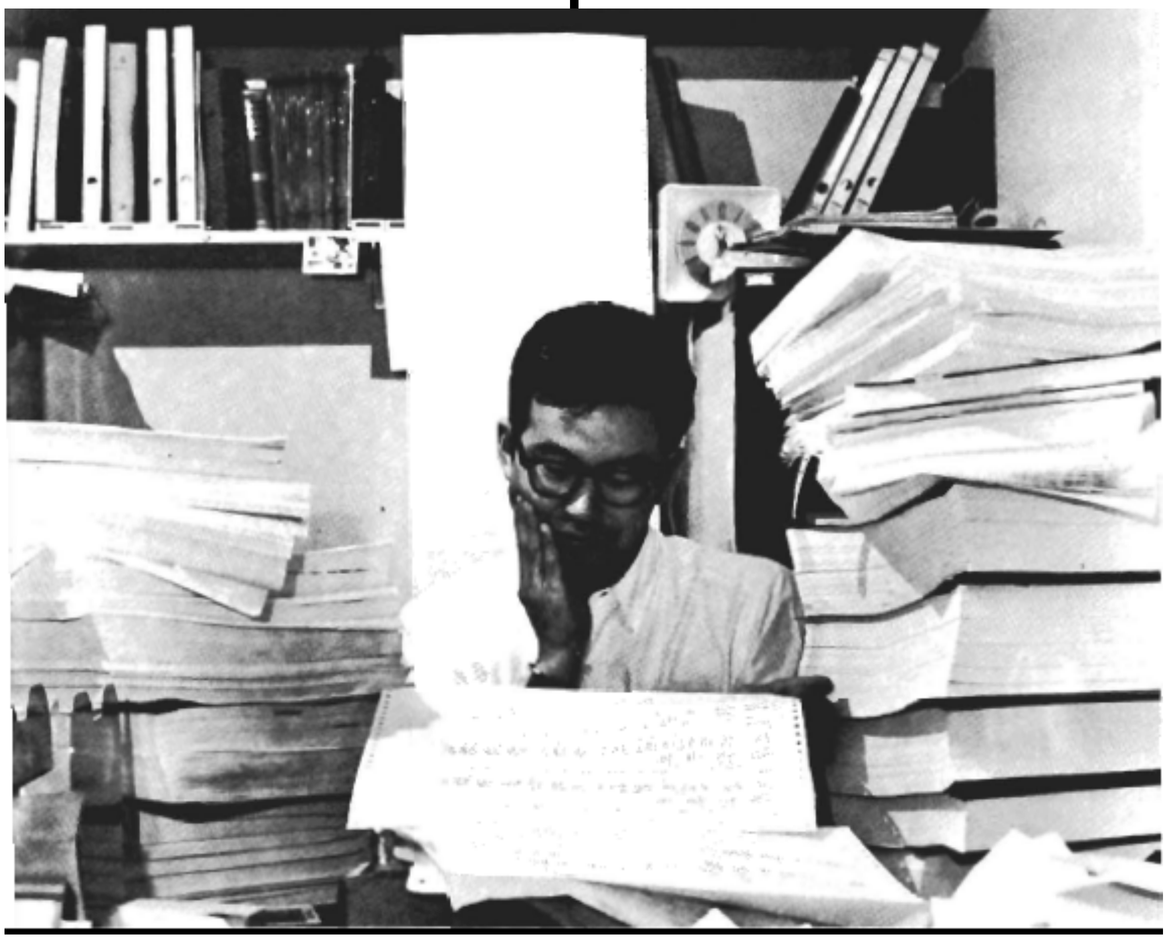
- the slow proton is identifed by its heavier ionisation,
- the K^0 subsequently decays into a pair of charged pions,
- the antineutron annihilates with a proton a short distance downstream from the primary interaction, to produce three charged pions,
- the neutral pion decays into two photons, which (unusually for a hydrogen chamber) both convert into e^+e^- pairs,
- (external particle detectors were used to identify the charged kaon).

Lots and lots of pictures

-- most of them uninteresting --

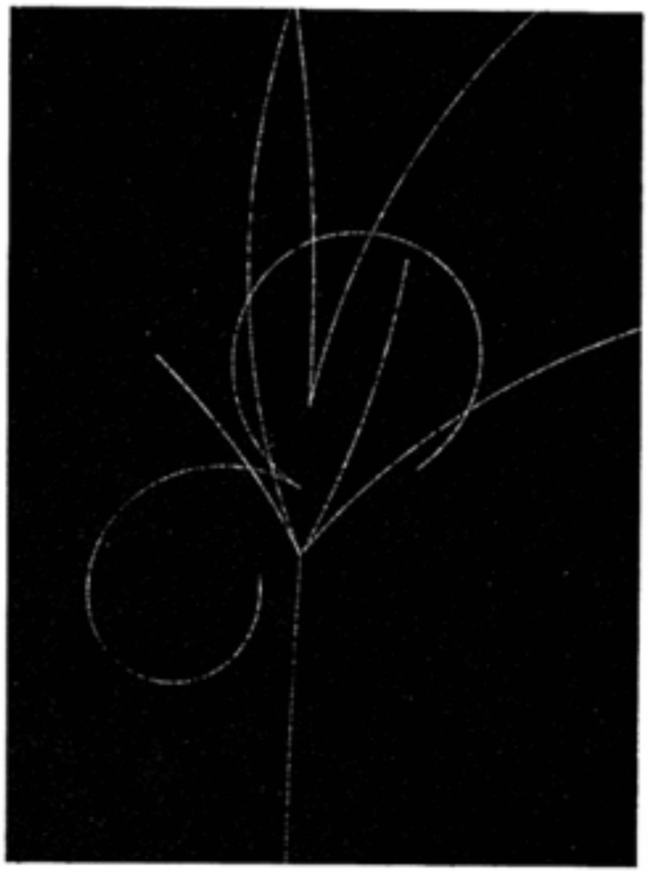


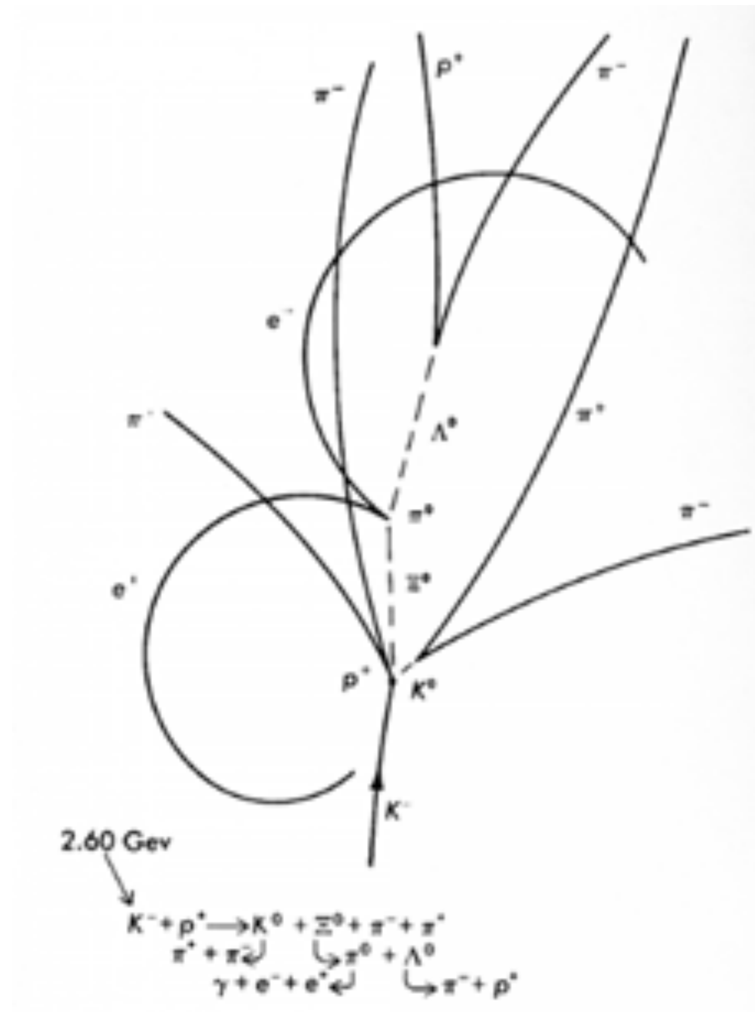
First large-scale applications of digital computers.



On many University campuses, the most powerful computers were in the HEP lab.

$K^-p \rightarrow K^0\ \Xi^0\ \pi^+\ \pi^-$ event

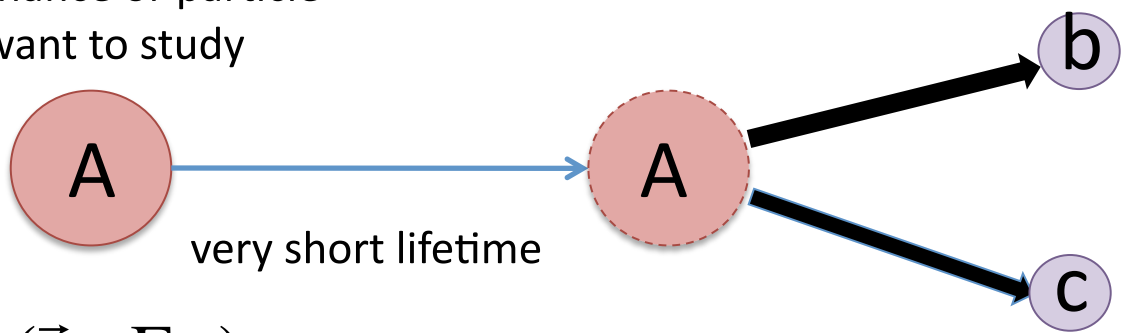




Invariant mass

resonance or particle
we want to study

daughter particles
(the ones we measure)



$$p_{\mathrm{b}} = (\vec{\mathrm{p}}_{\mathrm{b}}, \mathrm{E}_{\mathrm{b}})$$

$$p_{\mathrm{c}} = (\vec{\mathrm{p}}_{\mathrm{c}}, \mathrm{E}_{\mathrm{c}})$$

$$p_{\mathrm{A}} = (\vec{\mathrm{p}}_{\mathrm{A}}, \mathrm{E}_{\mathrm{A}})$$

Einstein rest-mass *vs* energy relation

$$\mathrm{M}_{\mathrm{A}} = \sqrt{p_{\mathrm{A}}^2} = \mathrm{E}_{\mathrm{A}}^2 - \left|\vec{\mathrm{p}}_{\mathrm{A}}\right|^2$$

$+$

energy-momentum conservation

$$p_{\mathrm{A}} = p_{\mathrm{b}} + p_{\mathrm{c}}$$

“invariant mass” of particles a and b

$$\mathrm{M}_{\mathrm{A}} = \sqrt{(p_{\mathrm{b}} + p_{\mathrm{c}})^2} = \sqrt{(\mathrm{E}_{\mathrm{b}} + \mathrm{E}_{\mathrm{c}})^2 - \left|\vec{\mathrm{p}}_{\mathrm{b}} + \vec{\mathrm{p}}_{\mathrm{c}}\right|^2}$$

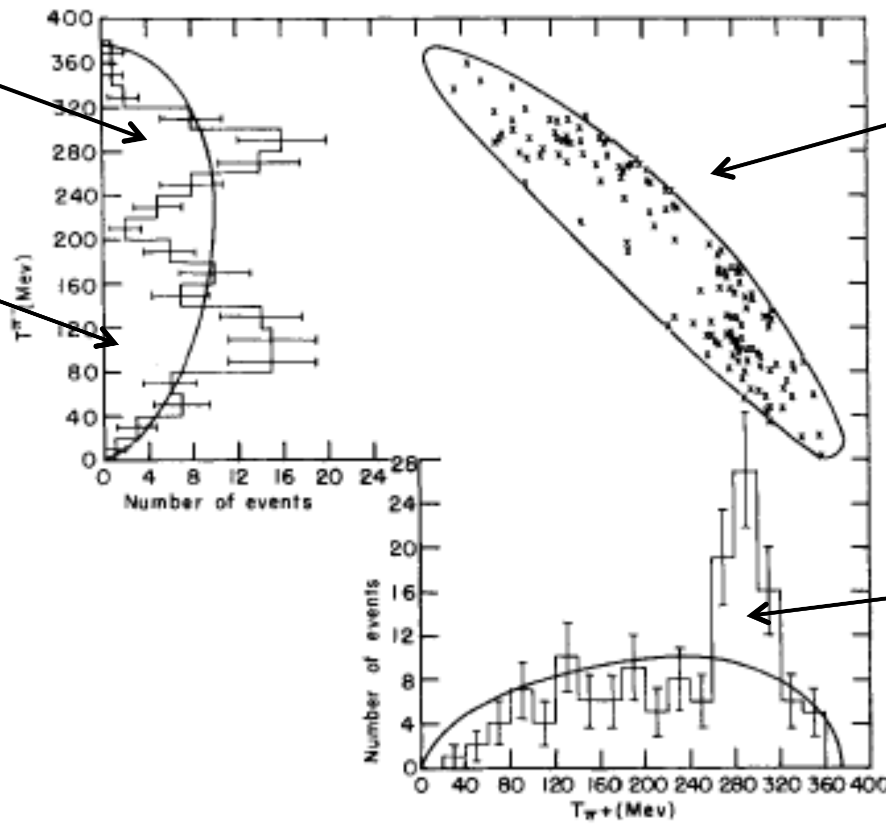
S=-1 resonances

Phys.Rev.Lett. 5, 115 (1960)

$$K^- p \rightarrow \Lambda^0 \pi^+ \pi^- \quad \Leftarrow P_K = 1.15 \text{ GeV}$$

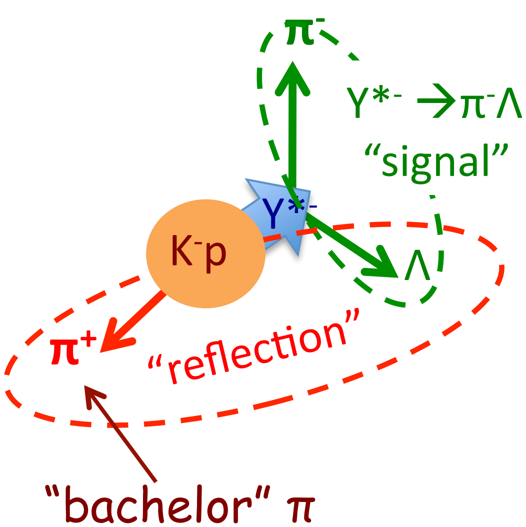
$$K^- p \rightarrow Y^{*+} \pi^-$$
$$\quad \hookrightarrow \Lambda^0 \pi^+$$

“reflection” from Y^{*-}



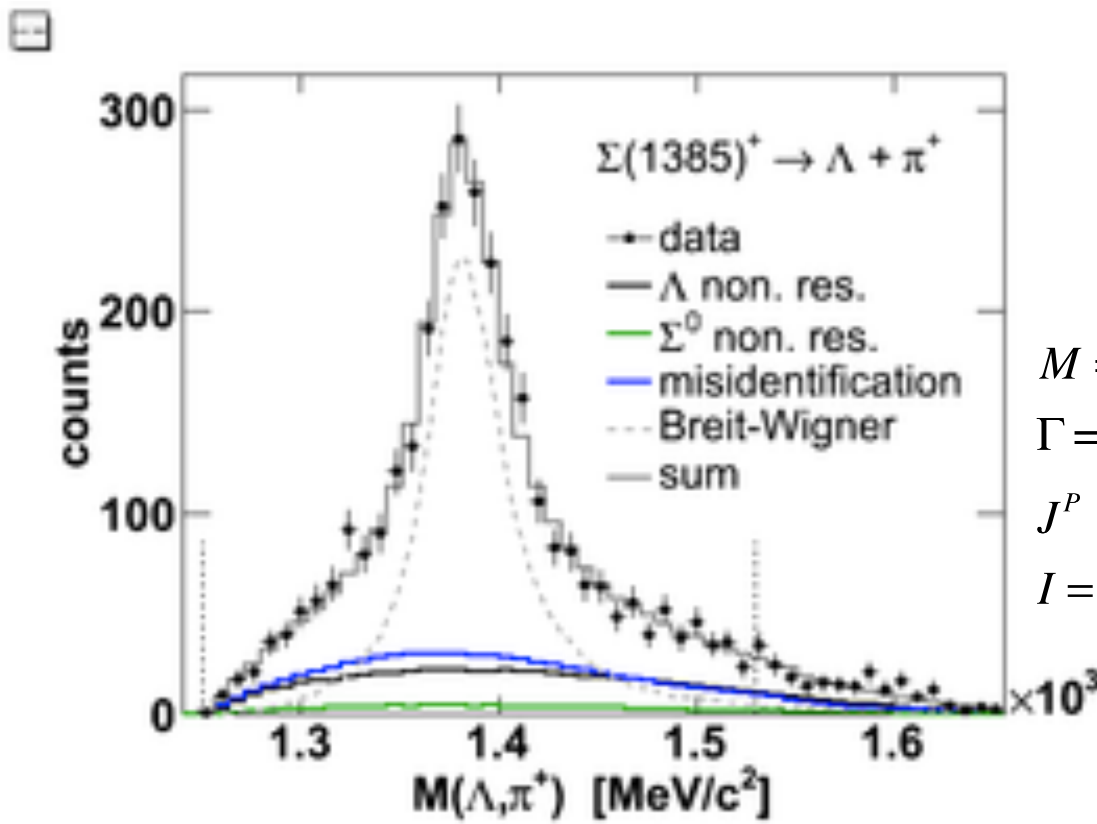
“Dalitz plot”

$$K^- p \rightarrow Y^{*-} \pi^+$$
$$\quad \hookrightarrow \Lambda^0 \pi^-$$



$$M^2_{Y*} = E^2_{cm} + m^2_\pi - 2E_{cm} E_{\pi-bachelor}$$

Y*(1385) → Λπ



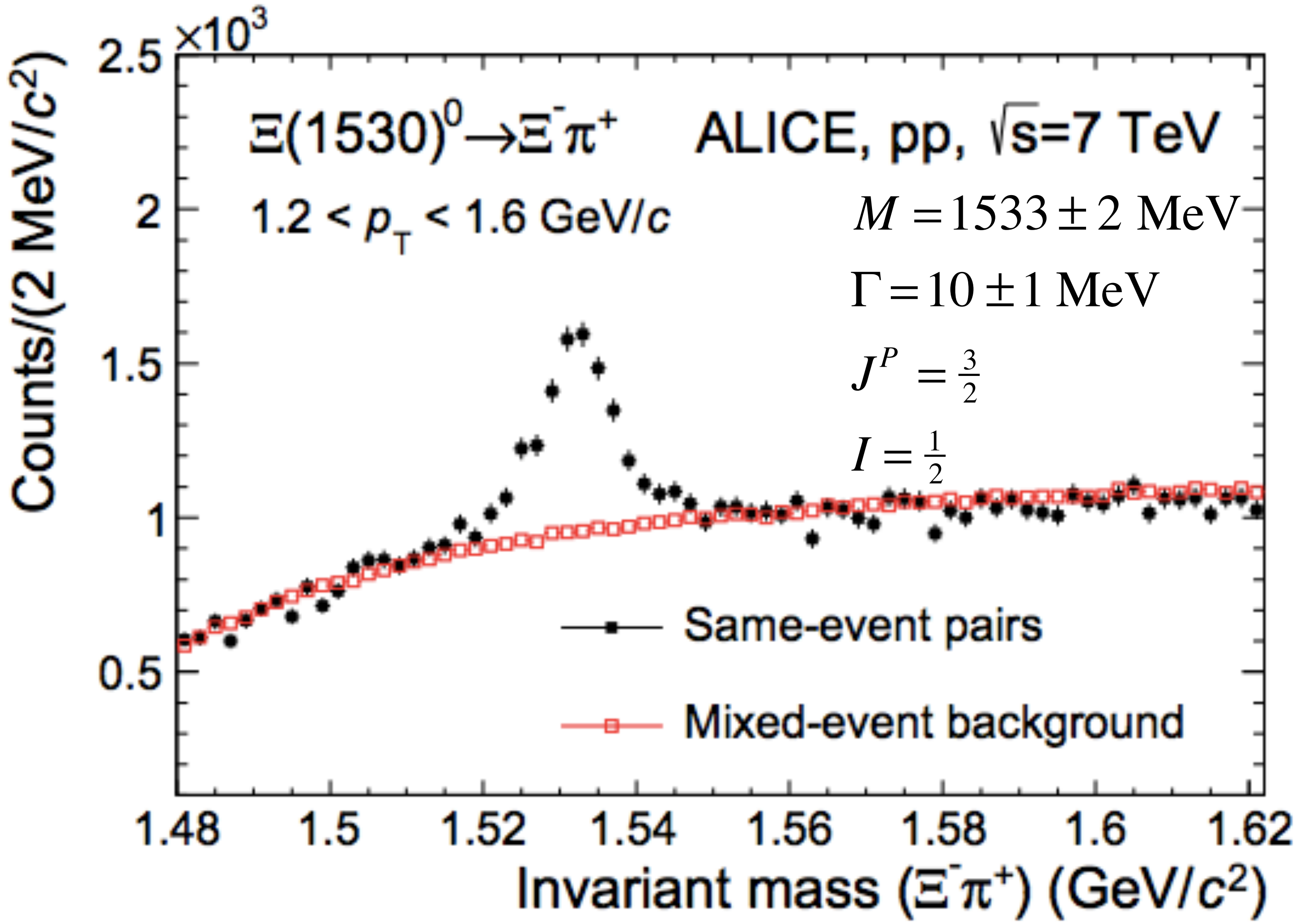
$M = 1384 \pm 2$ MeV

$\Gamma = 36 \pm 2$ MeV

$J^P = \frac{3}{2}^+$

$I = 1$

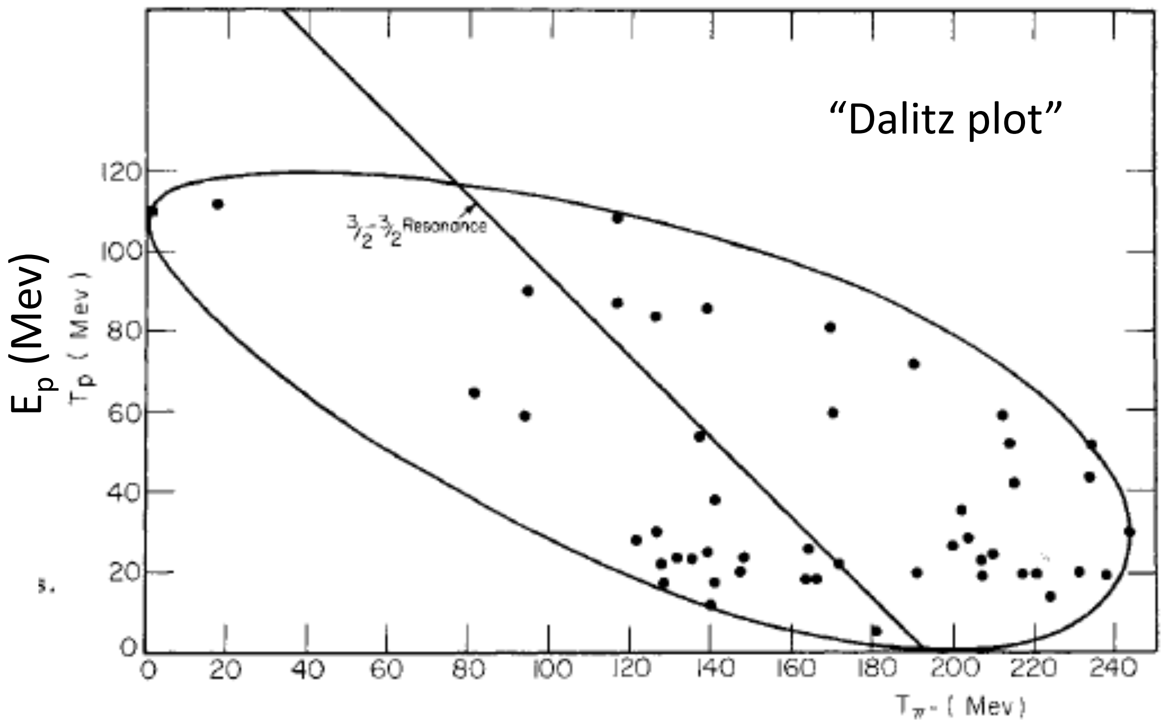
S=-2 resonance: Ξ*(1530)→Ξ π



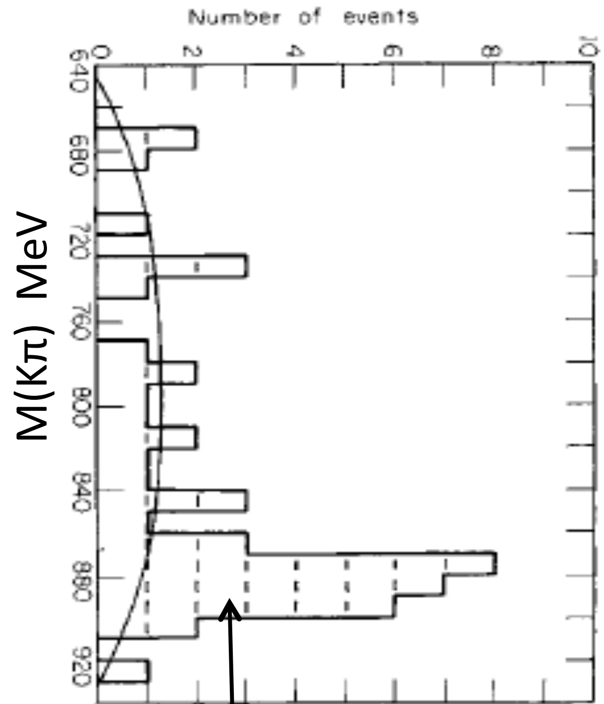
Meson resonance: K*(892)→Kπ

Phys.Rev.Lett. 6, 120 (1961)

$$K^- p \to \overline{K}^0 \pi^- p \quad \Leftarrow P_K = 1.15 \text{ GeV}$$



E_π (Mev)



$$K^- p \to K^{*-}(892)\, p$$

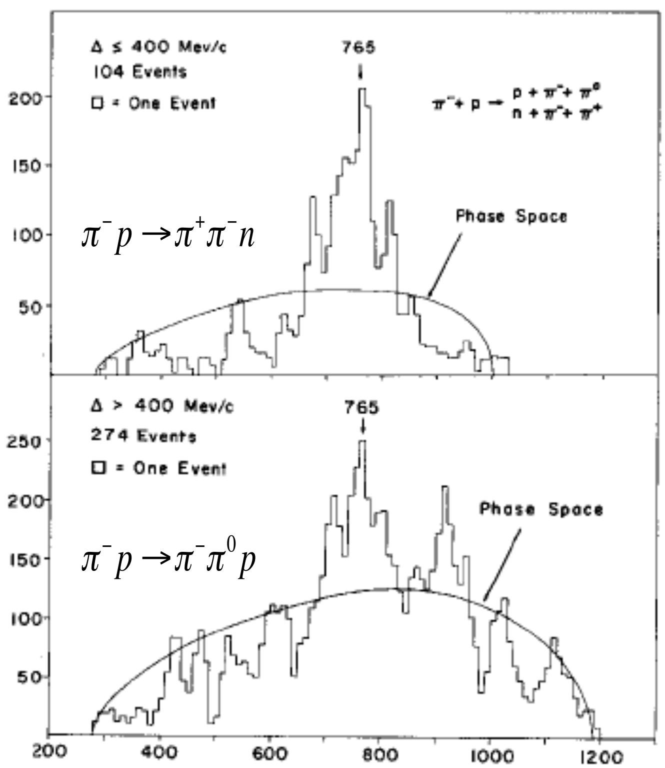
$$\hookrightarrow \overline{K}^0 \pi^-$$

Meson resonance: ρ(770)→ππ

Phys.Rev.Lett. 6, 124 (1961)

$$\pi^- p \rightarrow \pi^- \pi^0 p$$

$$\pi^- p \rightarrow \pi^+ \pi^- n \qquad \Leftarrow P_\pi = 1.89 \text{ GeV}$$



M(ππ) MeV

Table I. Ratios of final states.

	$I=0$	$I=1$	$I=2$	Experiment ($\Delta \leq 400$ Mev/c)
$\pi^-\pi^+n$	2	2	2/9	1.7 ± 0.3
$\pi^-\pi^0p$	0	1	1	1
$\pi^0\pi^0n$	1	0	4/9	$< 0.25 \pm 0.25$

I=1 is favored

$$M_\rho = 775.5 \pm 0.4 \text{MeV}$$

$$\Gamma_\rho = 149 \pm 1 \text{ MeV}$$

$$J^P = 1^-$$

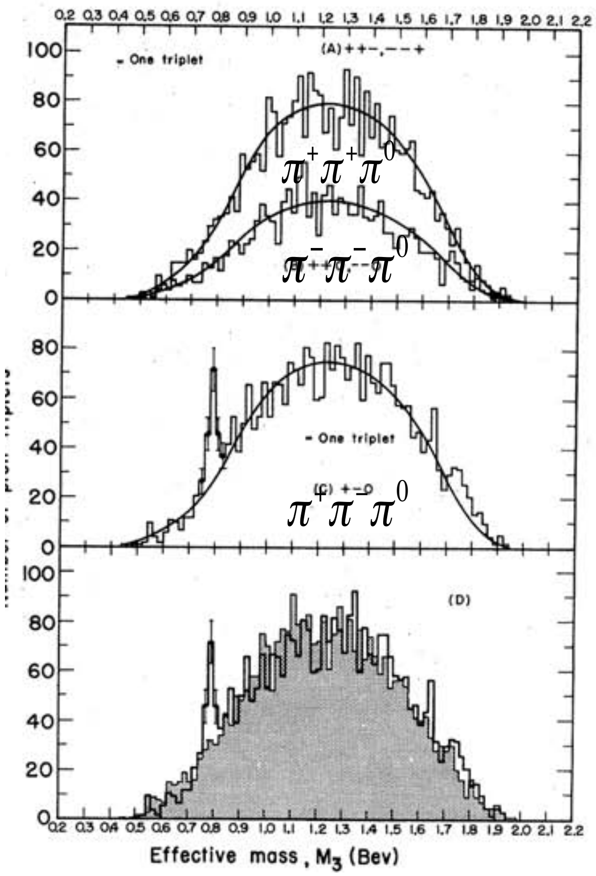
$$I = 1$$

Meson resonance: $\omega(782) \rightarrow \pi^+\pi^-\pi^0$

Phys.Rev.Lett. 7, 126 (1961)

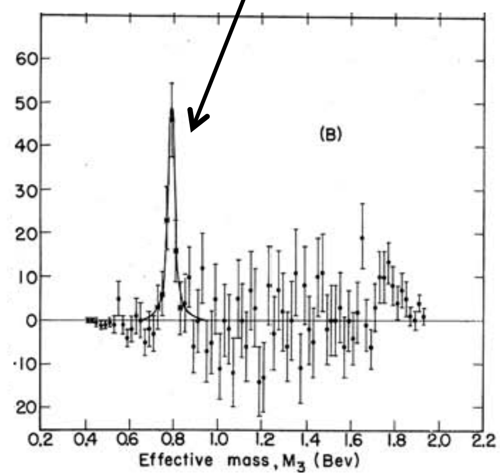
$$\bar{p}p \rightarrow \pi^+\pi^+\pi^-\pi^-\pi^0 \quad \Leftarrow P_{\bar{p}} = 1.61 \text{ GeV}$$

$$M_\omega = 782.6 \pm 0.1 \text{ MeV}$$
$$\Gamma_\omega = 8.5 \pm 0.1 \text{ MeV}$$
$$J^P = 1^-$$
$$I = 0$$



M(πππ) GeV

$$\bar{p}p \rightarrow \omega(782)\pi^+\pi^-$$
$$\quad \hookrightarrow \pi^+\pi^-\pi^0$$



Quantum numbers of the ρ & ω

ρ meson quantum numbers

Known: $\rho^0 \rightarrow \pi^+\pi^-$ and Ispin=1

$$J^P: \quad |\rho^0\rangle = \tfrac{1}{\sqrt{2}}\left[|\pi^+\rangle|\pi^-\rangle - |\pi^-\rangle|\pi^+\rangle\right] \Leftarrow \text{asymmetric in } \pi^+ \leftrightarrow \pi^-$$

to preserve Bose statistics: need asymmetry in parity

$\Rightarrow$ $\pi^+\pi^-$ must be in a *P*-wave, *P*=-1 & $J^P = 1^-$

$$C: \quad C|\rho^0\rangle = \tfrac{1}{\sqrt{2}}\left[C|\pi^+\rangle|\pi^-\rangle - C|\pi^-\rangle|\pi^+\rangle\right] = \tfrac{1}{\sqrt{2}}\left[|\pi^-\rangle|\pi^+\rangle - |\pi^+\rangle|\pi^-\rangle\right] = -|\rho^0\rangle$$

$$\Rightarrow \; C = -1$$

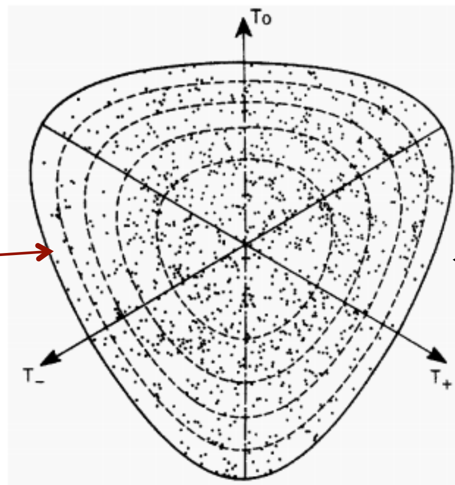
J^{PC}=1^{--}

$$G: \quad G\begin{pmatrix}\rho^+\\ \rho^0\\ \rho^=\end{pmatrix} = Ce^{i\pi I_2}\begin{pmatrix}\rho^+\\ \rho^0\\ \rho^=\end{pmatrix} = +\begin{pmatrix}\rho^+\\ \rho^0\\ \rho^=\end{pmatrix} \Rightarrow G = +$$

ω meson quantum numbers

Known: $\omega^0 \rightarrow \pi^+\pi^-\pi^0$ and Ispin=1

J^P :



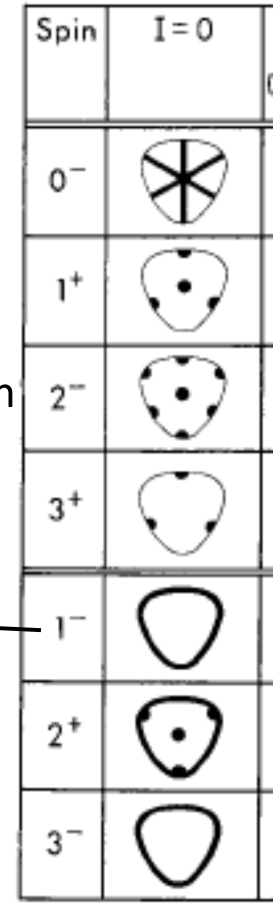
event density is
0 at boundary

0's in the Dalitz plot: C. Zemach
Phys.Rev. 113. B1201 (1964)

$1^- \rightarrow \pi^+\pi^-\pi^0$ decay amplitude

$$\propto (-1)^3 \left|\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}\right|$$

(π parity)^3



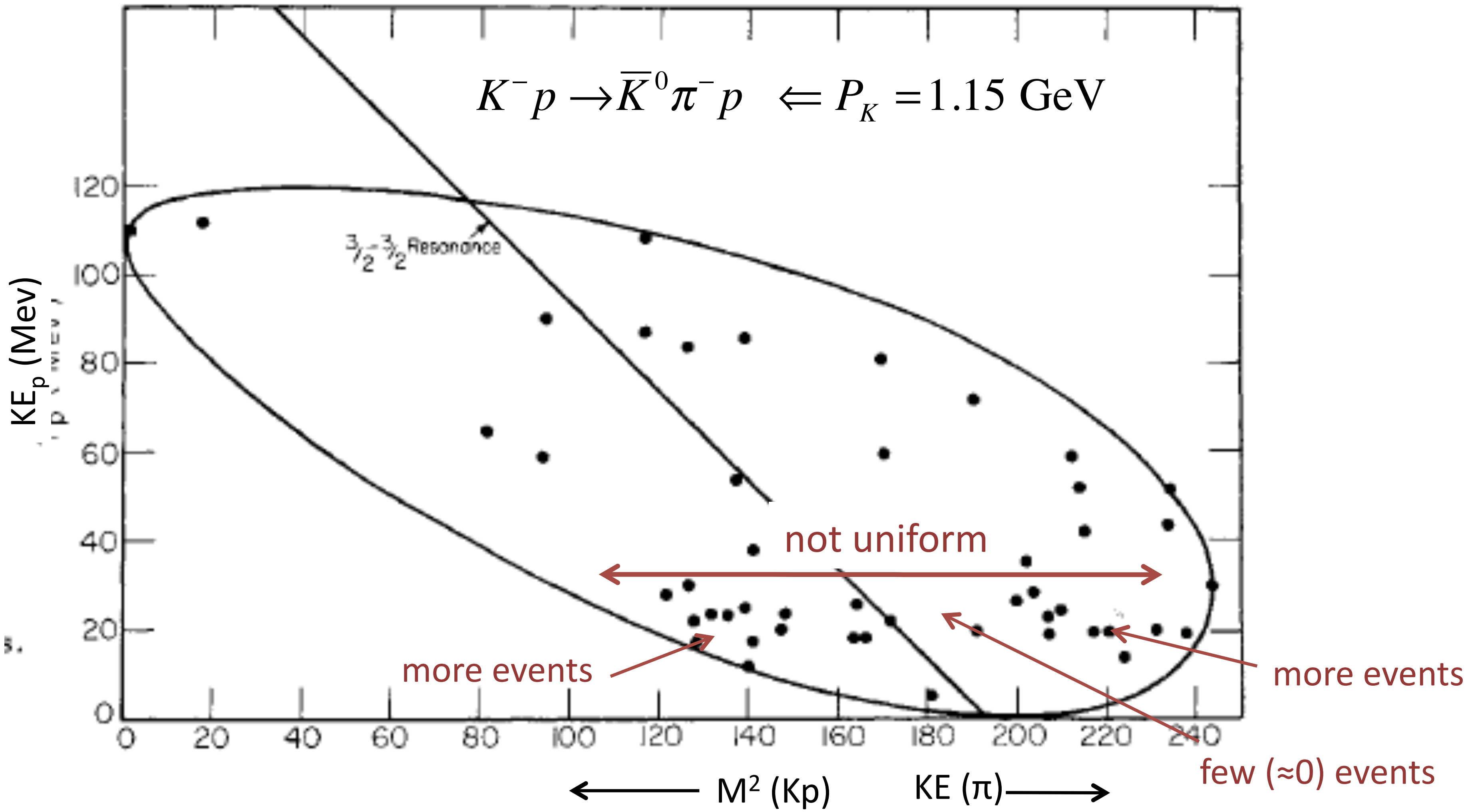
$$G: \quad G|n\pi\rangle = (-1)^n |n\pi\rangle \;\text{ for } \omega \rightarrow 3\pi,\ n = 3 \;\;\&\;\; G_\varpi = -1$$

$$C: \quad G|\omega\rangle = C_\omega e^{i\pi I_2}|\omega\rangle = -|\omega\rangle \;\text{ for } I = 0,\ e^{i\pi I_2} = 1 \;\Rightarrow\; C_\omega = -1$$

J^{PC}=1^{--}

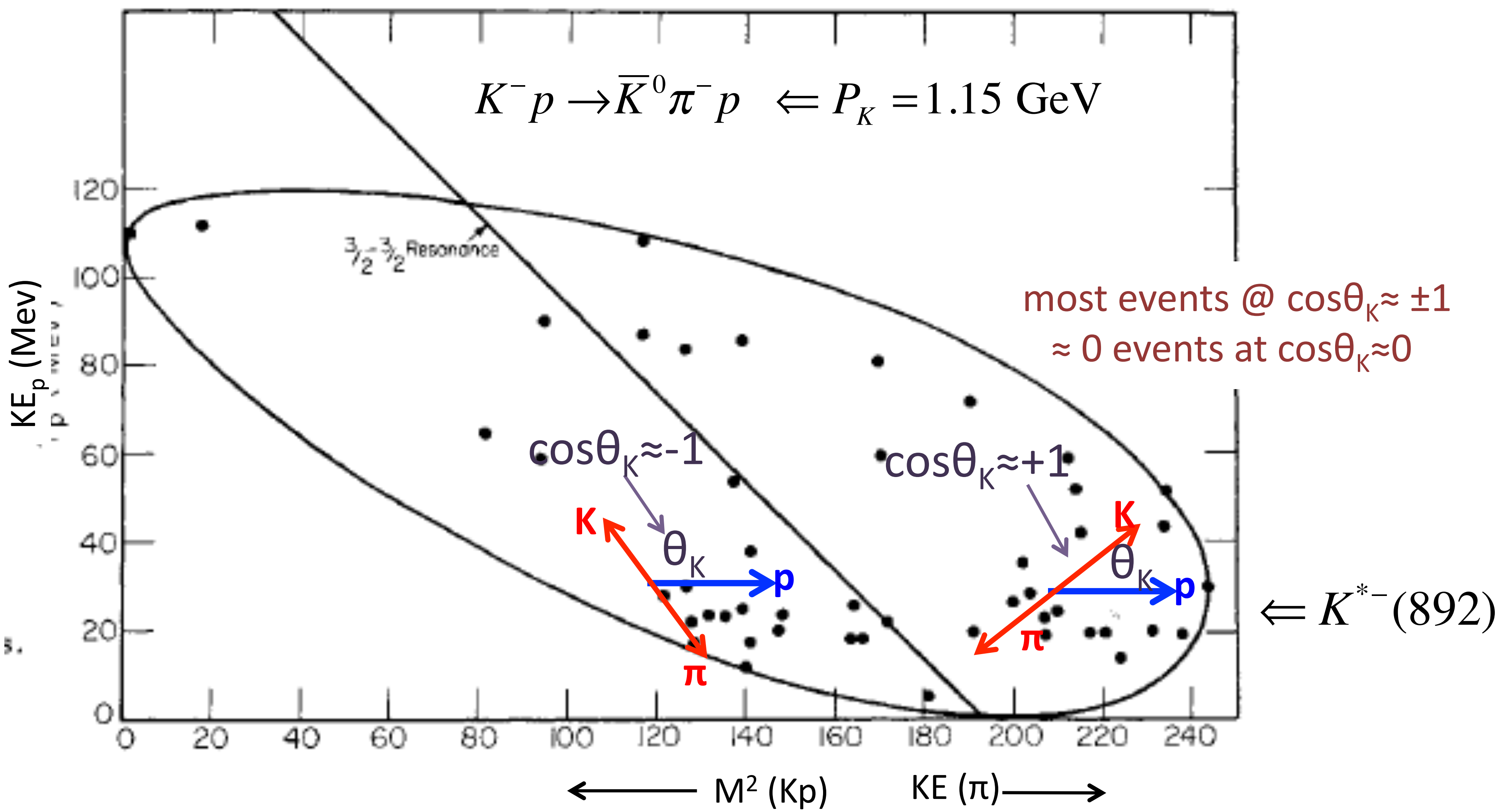
What about K*(892) quantum numbers?

-- examine the Dalitz plot --



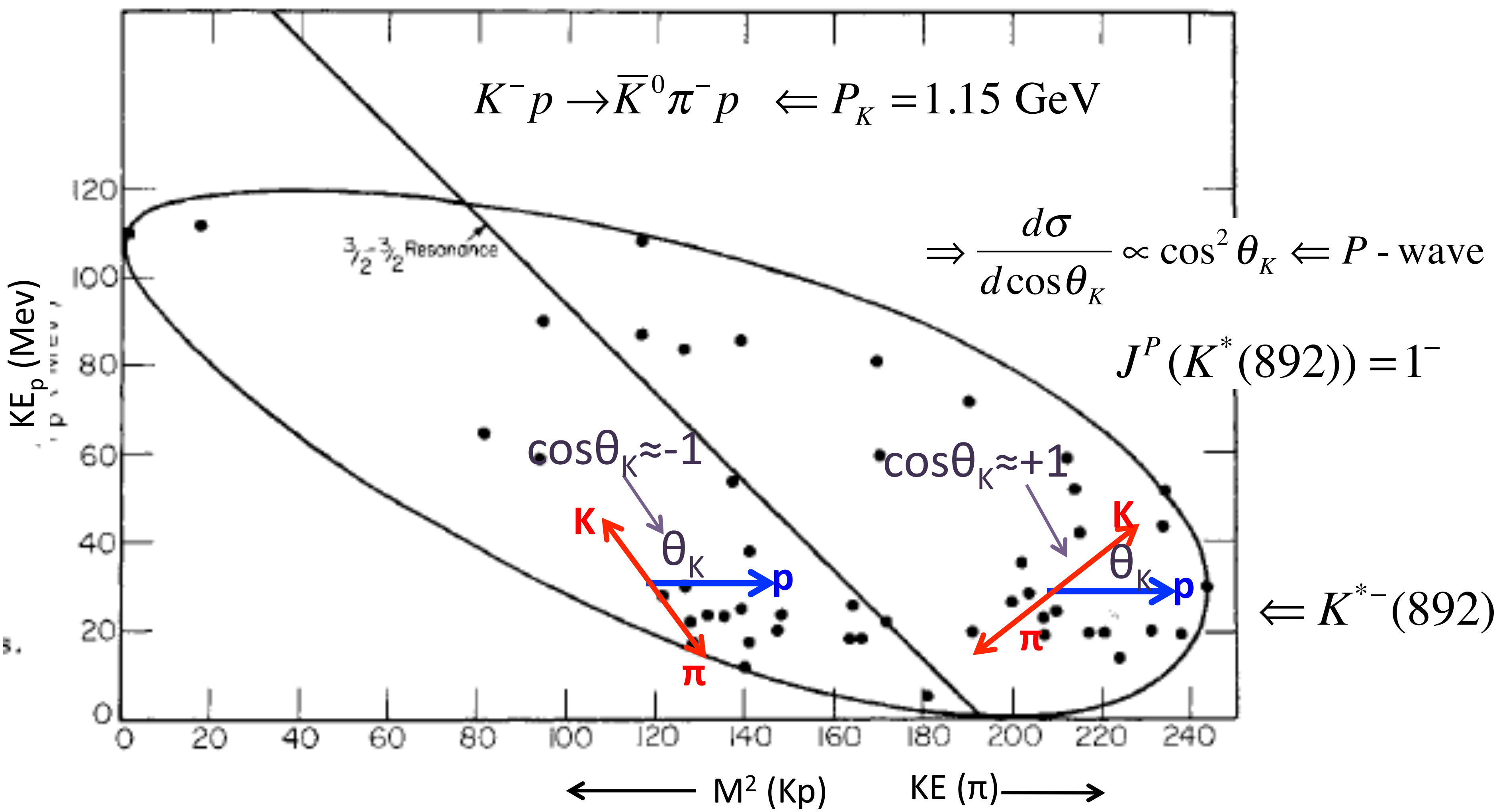
What about K*(892) quantum numbers?

-- examine the Dalitz plot --



What about K*(892) quantum numbers?

-- examine the Dalitz plot --

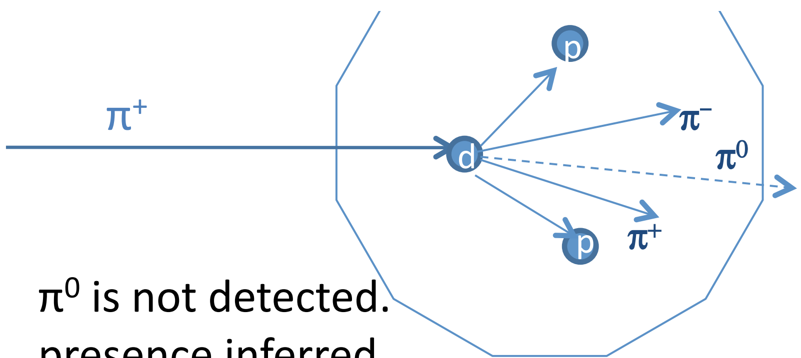


Discovery of the η meson (1961)

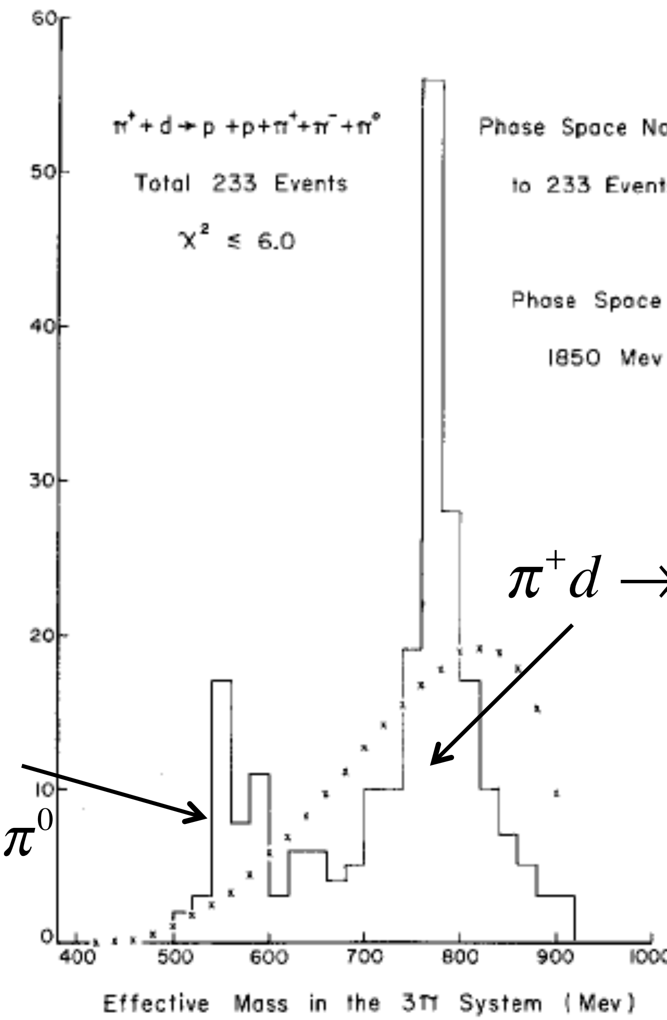
Phys.Rev.Lett. 7, 131 (1961)

$$\pi^+ d \rightarrow pp\pi^+\pi^-\pi^0 \quad \Leftarrow P_{\pi^+} = 1.23\ \mathrm{GeV}$$

Deuterium –filled
80” bubble chamber



π^0 is not detected.
presence inferred
by kinematics



$$\pi^+ d \rightarrow \omega(782)\,pp$$
$$\hookrightarrow \pi^+\pi^-\pi^0$$

$$\pi^+ d \rightarrow \eta(548)\,pp$$
$$\hookrightarrow \pi^+\pi^-\pi^0$$

η is also seen in η→γγ decay mode

-- η→γγ seen ← Charge Parity=+1

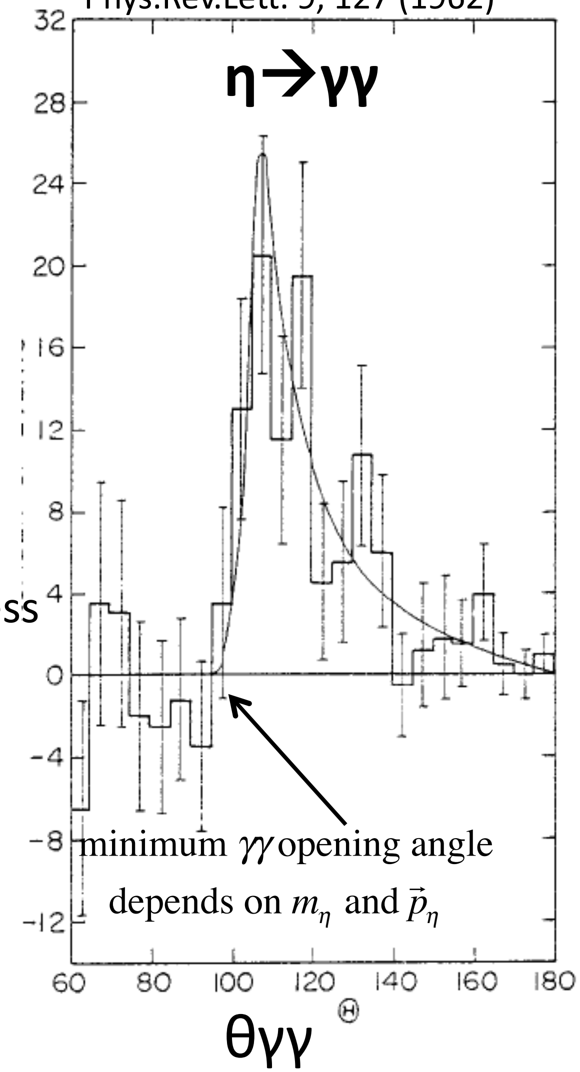
-- by Yang's theorem, spin 1 is not allowed ← Spin=0

-- η→ππ not seen ← Parity=-1

-- η→πππ violates G-parity ← Elec-Mag process

$J^{PC}=0^{-+}$

Phys.Rev.Lett. 9, 127 (1962)



Elementary particle "Zoo" in 1963

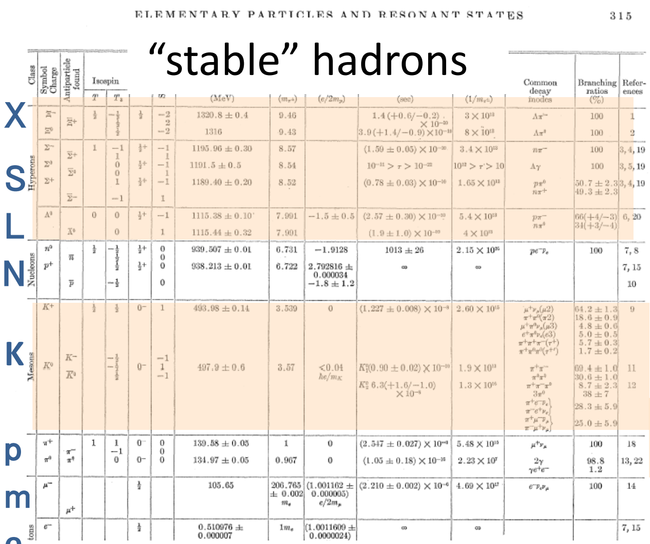
REVIEWS OF MODERN PHYSICS VOLUME 35, NUMBER 2 APRIL 1963

Tables of Elementary Particles and Resonant States

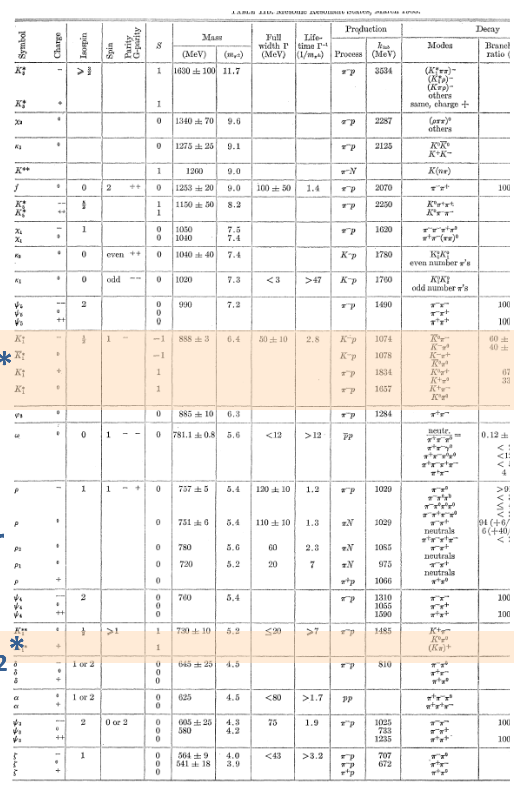
MATTS ROOS
Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

←"wallet cards"

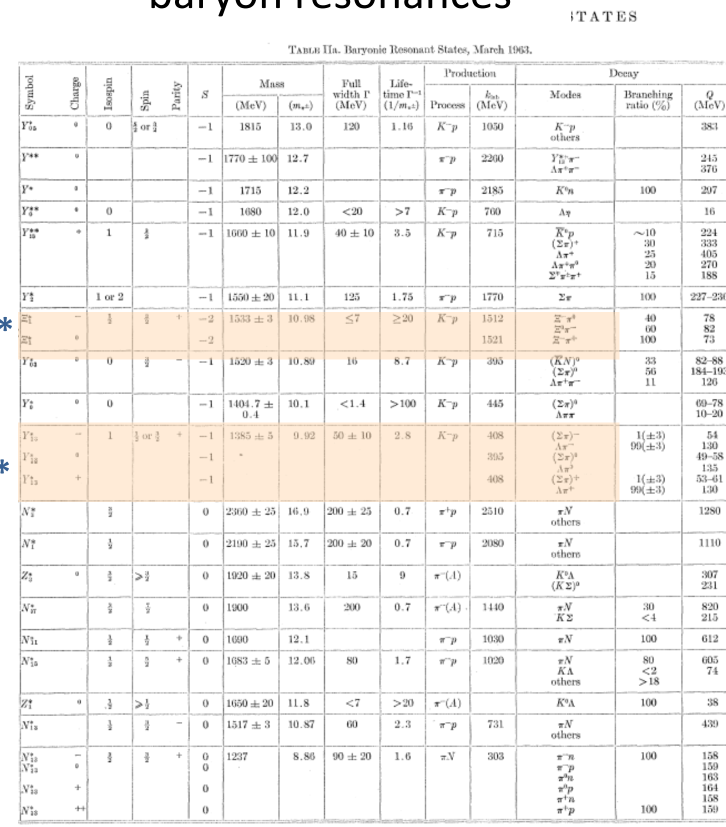
"stable" hadrons



meson resonances



baryon resonances



"flavors"

Two "classes" of hadrons

"non-strange:" n, p, p, r, ...

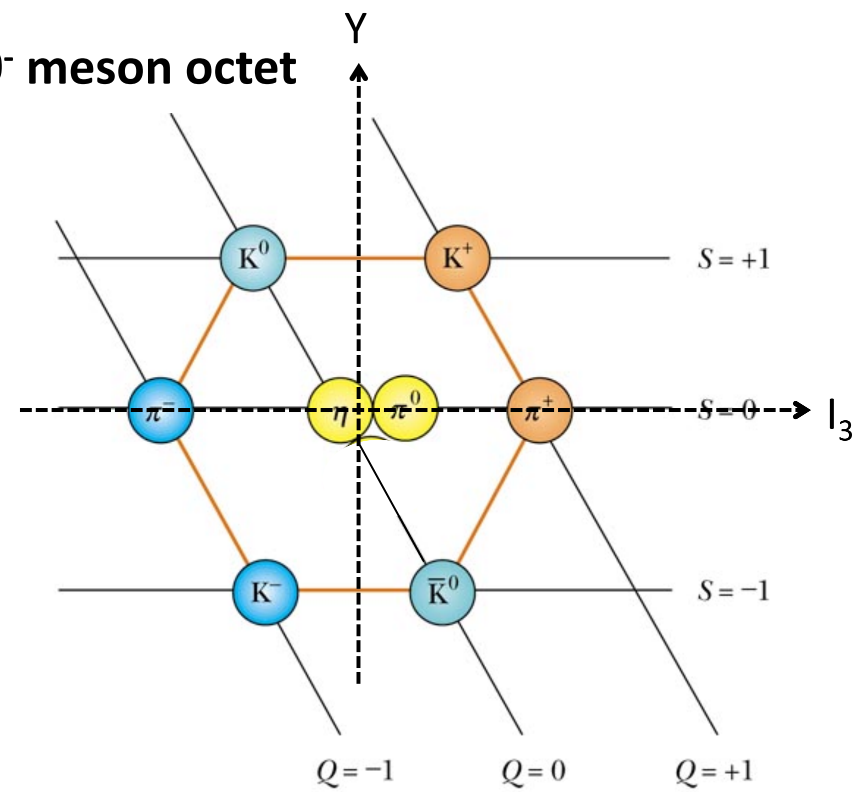
"strange:" L, S, K, K*, ...

Meson and Baryon Octets

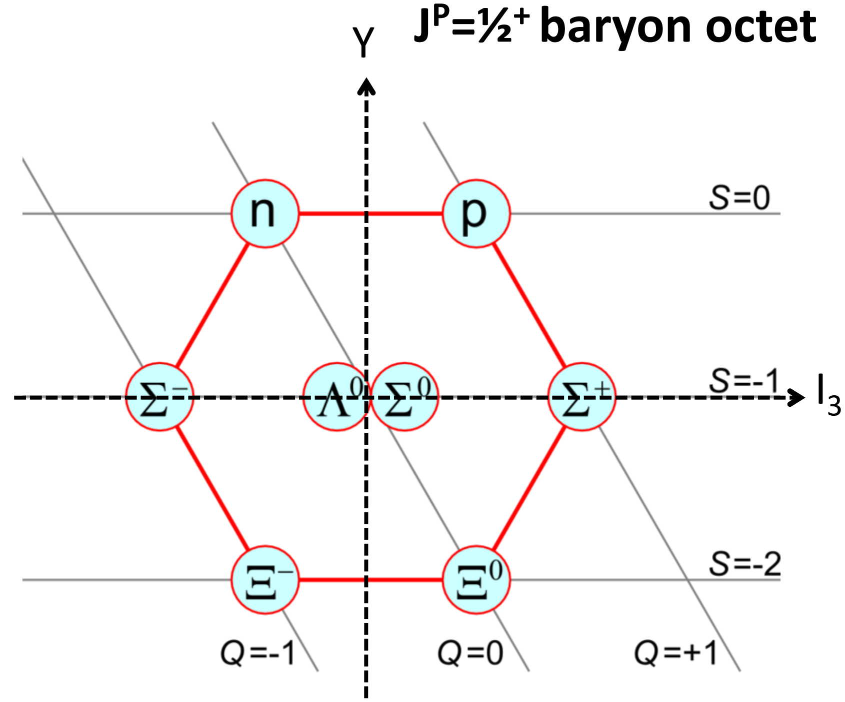
strangeness baryon number

Y = "hypercharge" = S+B; I_3= 3rd I-spin component

$J^P=0^-$ meson octet

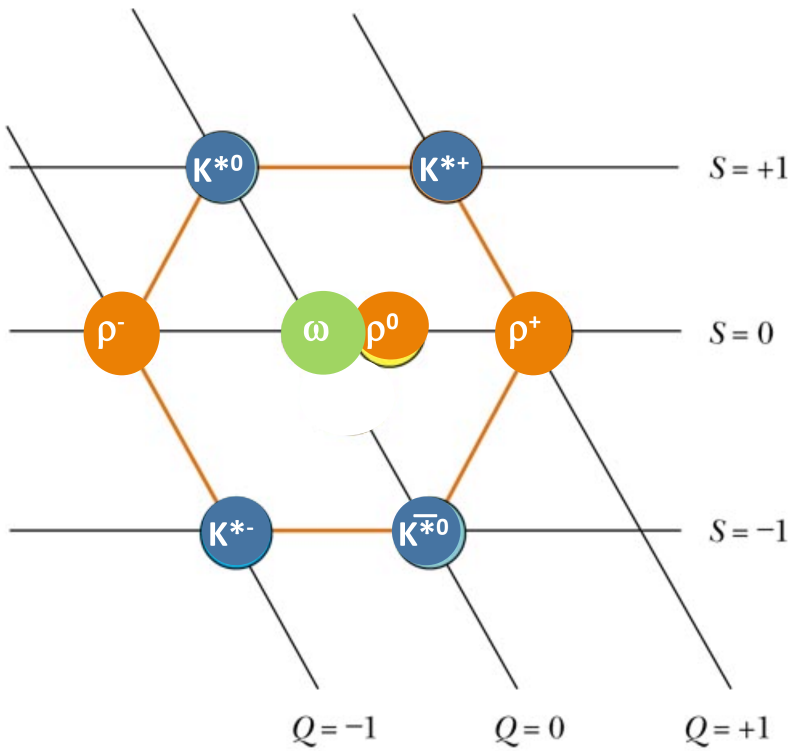


$J^P=½^+$ baryon octet

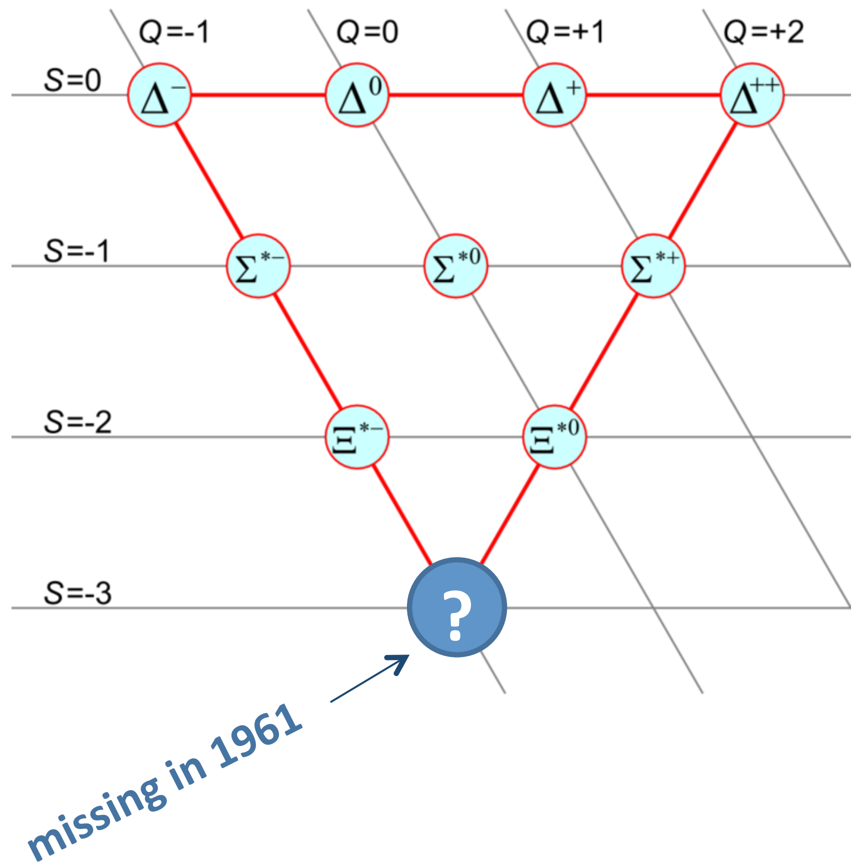


resonances

$J^P=1^-$ vector meson octet

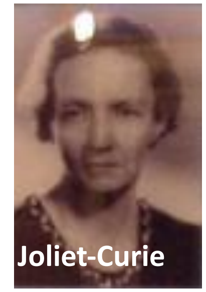


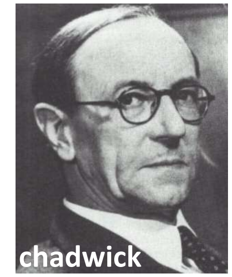
$J^P=3/2^+$ baryon "decuplet"



History:

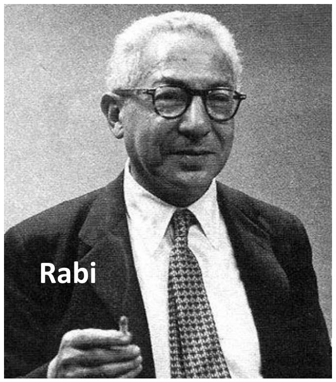
(sub-atomic particles)





1932: proton & neutron

..all we need???

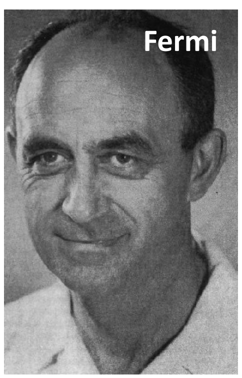


1937: muon

"Who ordered that?"

1947: pion

predicted in 1935



1950's: π,K,Λ,…

"Had I foreseen that, I would have gone into botany" – Fermi

Summary (lecture 1)

- The (expected) discovery of the π-meson and the (unexpected) discovery of the K =meson & Λ baryon in 1947 marked the beginning of hadron physics.
- The fact that K-mesons were produced in association with Λ-baryons led to the discovery of Strangeness, the 1st flavor. S is conserved in strong and electro-magnetic, but not in weak interactions.
- Experiments showed that the spin and parity of the π-, K- and mesons are all $J^P=0^-$.
- A matching set of meson resonances with $J^P=1^-$., the ρ-, K*- and ω-mesons, were found in bubble chamber experiments.
- A set of spin=3/2 baryon resonances, the Δ(1232), Y*(1385) (now called the Σ(1385)) and Ξ(1530) was also discovered.
- Mesons come in octets; baryons come in octets and decuplets.

some discussion items/questions

Show that for $f\bar{f}$ states, $C=(-1)^{L+S}$, where f is a fermion.

Prove Yang's theorem.

Why would the decay $\eta(547)\rightarrow\pi\pi$ violate parity conservation?

Although the $\eta(547)$ mass is well above $3m_\pi$, the partial decay width $\Gamma(\eta\rightarrow 3\pi)$ is only ≈1 keV, which means it is not an allowed strong Interaction process. Why is $\eta\rightarrow 3\pi$ not an allowed strong Interaction process?

Why doesn't the $\rho(770)^0$ decay to $\pi^0\pi^0$?

Why doesn't the $\omega(782)^0$ decay to $\pi^0\pi^0\pi^0$?