Dalitz Plots I

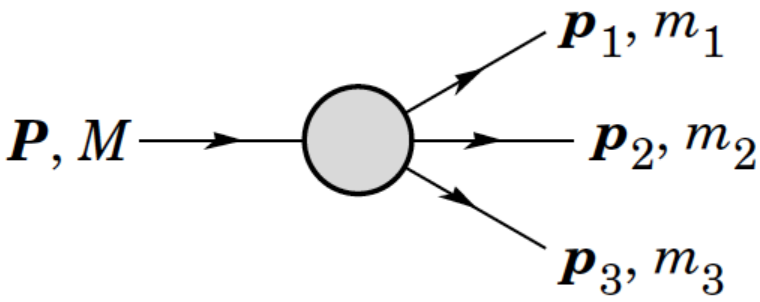
A powerful method for visualizing the dynamics in three-body particle systems


Richard Dalitz 1925-2006

Stephen Lars Olsen

Graphics from Brian Lindquist (SLAC)

Fermi's Golden Rule for 3-body decays

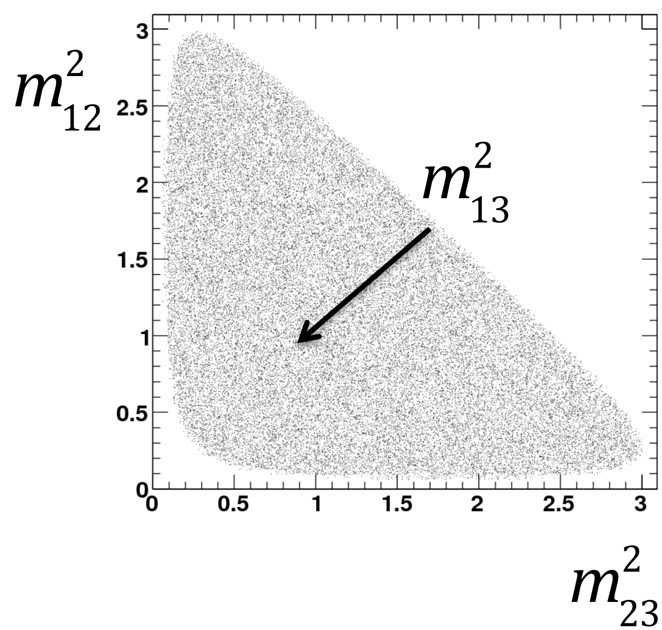


$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{8M} \overline{|\mathscr{M}|^2}\, dE_1\, dE_3$$

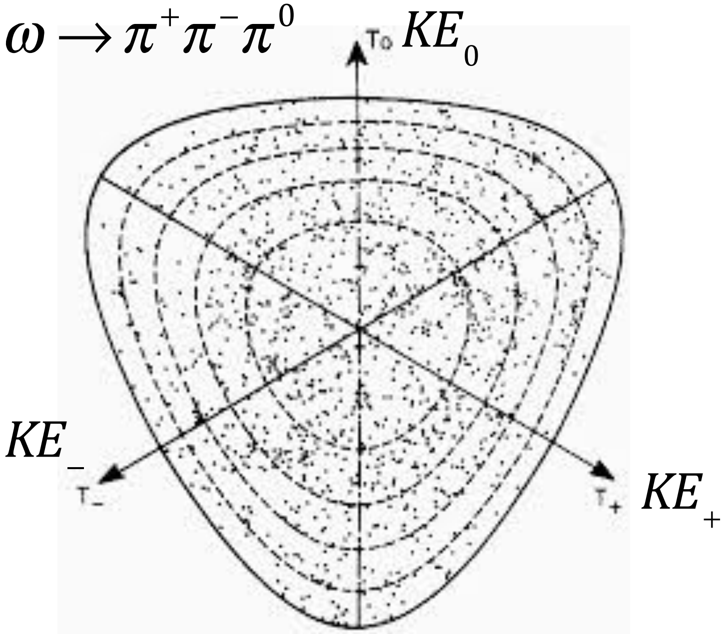
$$= \frac{1}{(2\pi)^3} \frac{1}{32M^3} \overline{|\mathscr{M}|^2}\, dm_{12}^2\, dm_{23}^2$$

if $\left|\mathcal{M}\right|^2 = 0$, i.e., no dynamics,

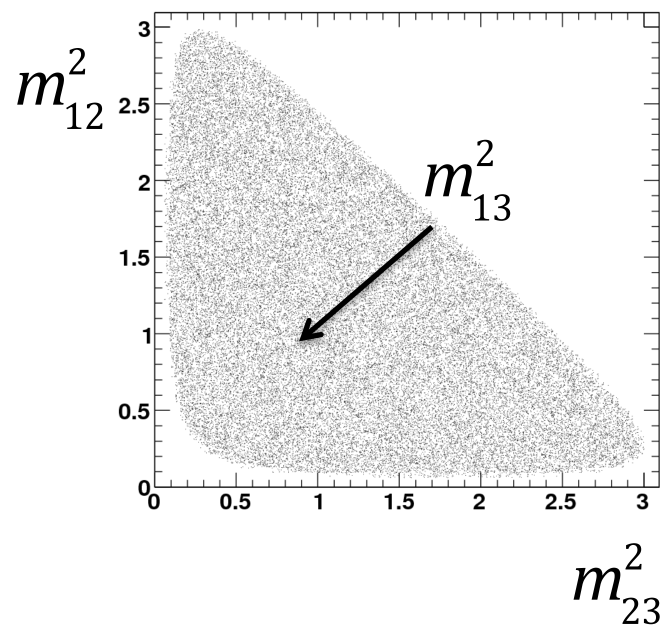
$$\frac{dX}{dm_{12}^2 dm_{23}^2} = \text{constant}$$



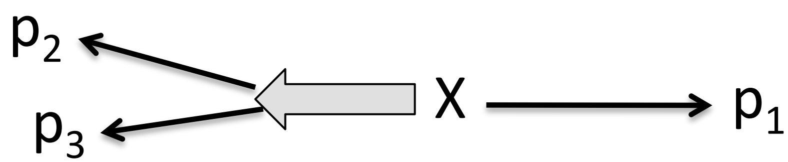
Old-style versus Modern style



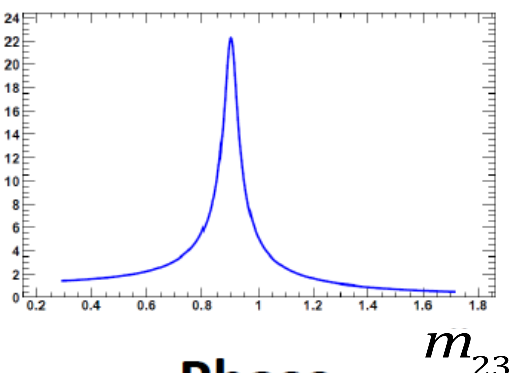
here symmetries are more obvious



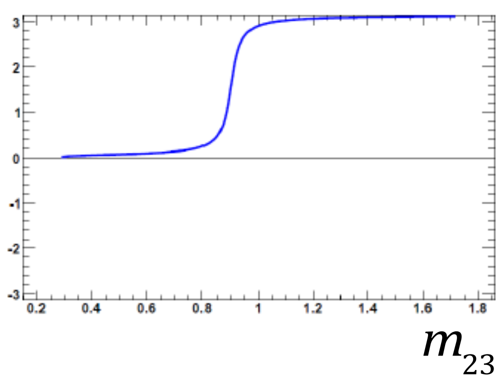
Resonances

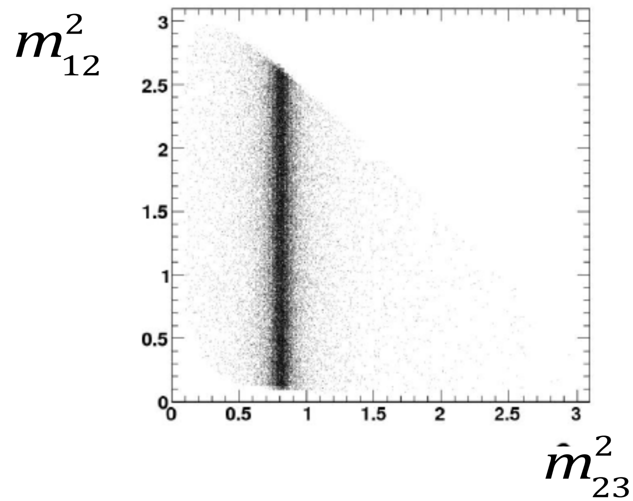


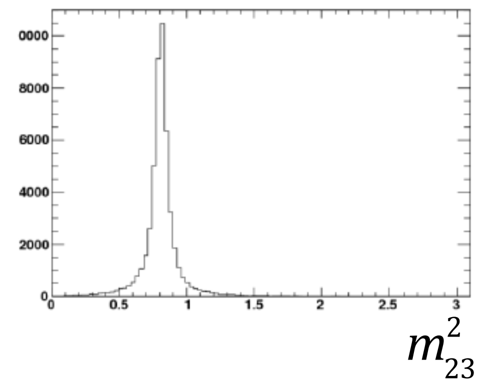
Magnitude



Phase





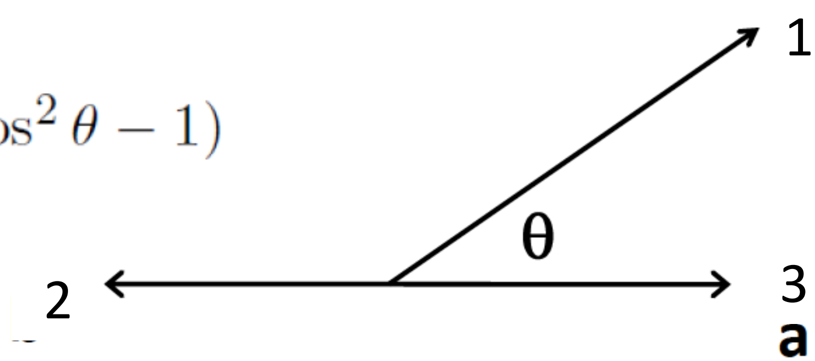


Resonance with spin

If the resonance has spin S, and M, 1, 2 and 3 are spin-0, then decay amplitude is proportional to Legendre polynomial:

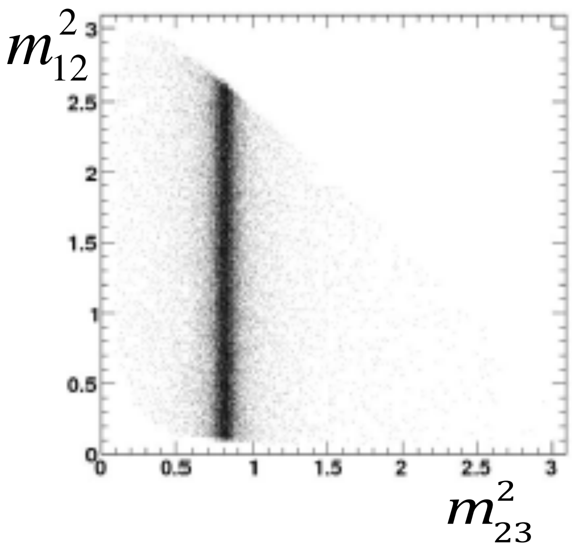
$$A \propto A_{RBW}(m_{ab})P_S(\cos\theta)$$

$$P_0(\cos\theta) = 1$$
$$P_1(\cos\theta) = \cos\theta$$
$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

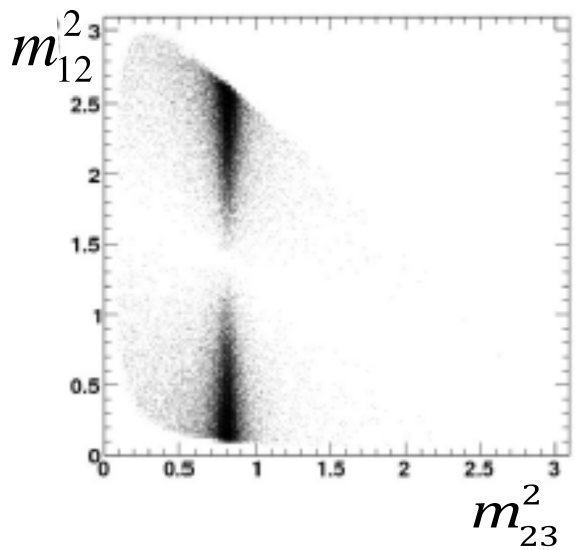


Resonance spin

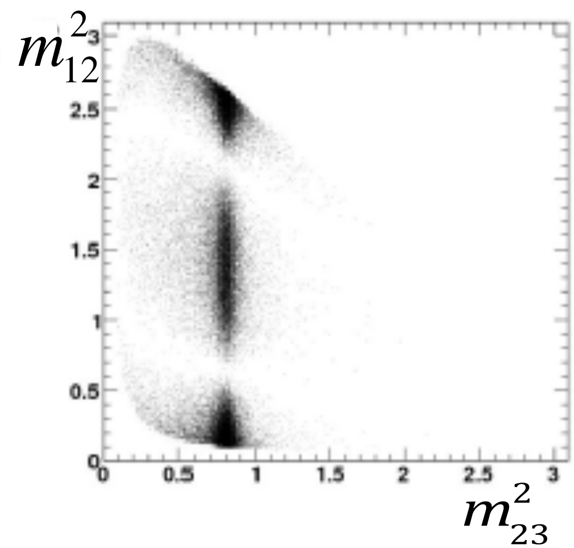
Spin-0



Spin-1

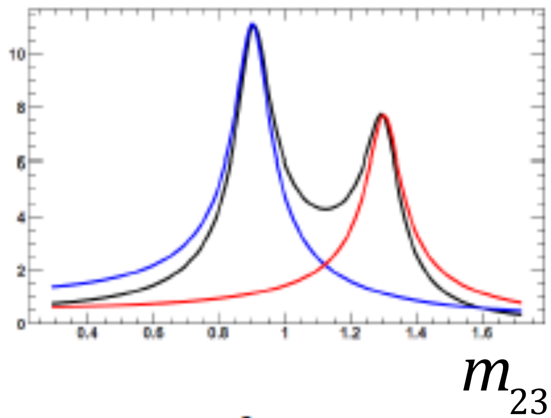


Spin-2

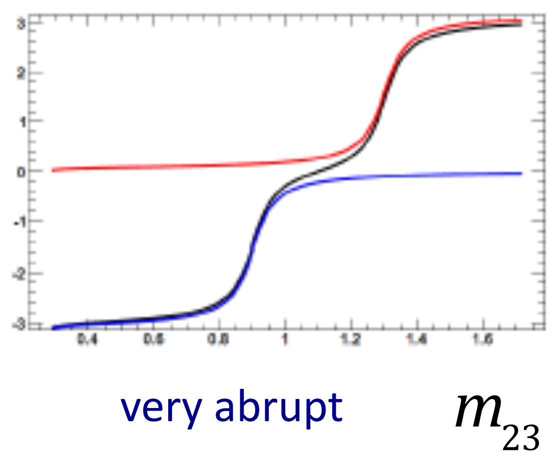


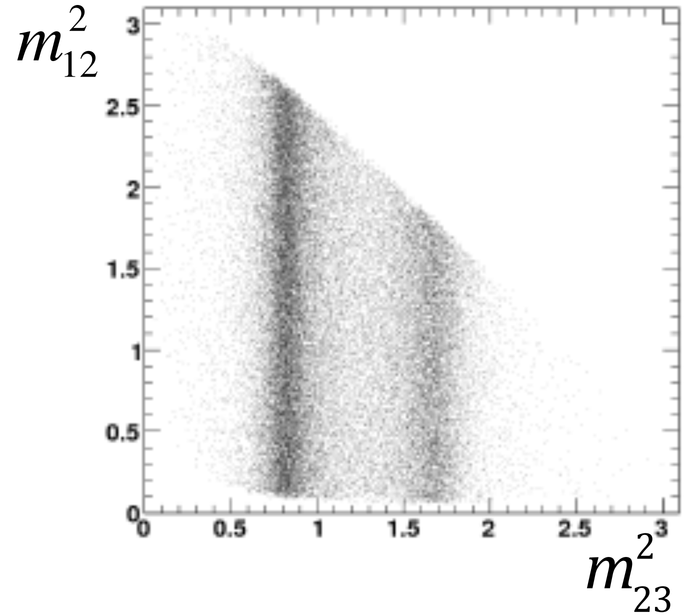
Constructive interference

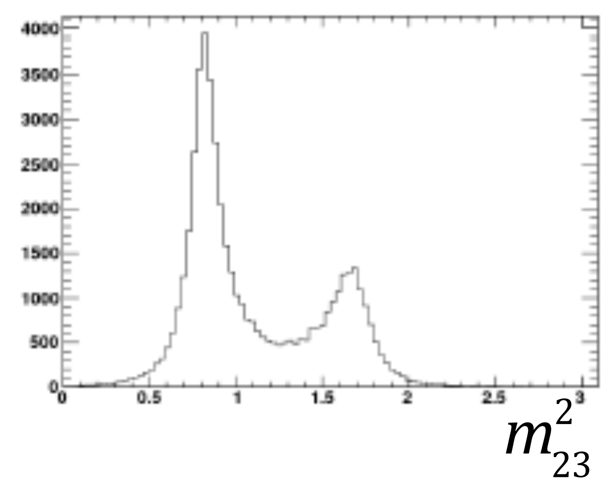
Magnitude



Phase

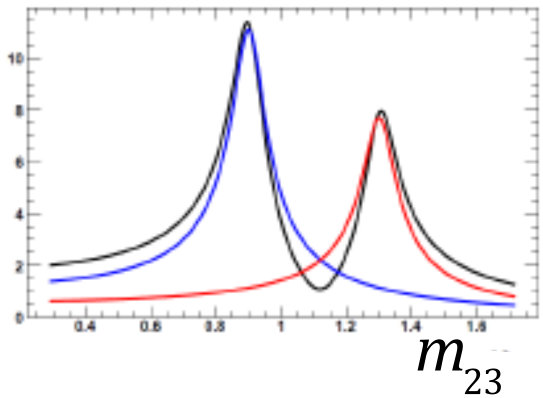




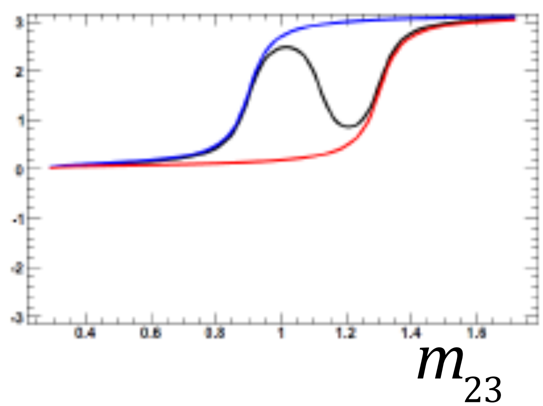


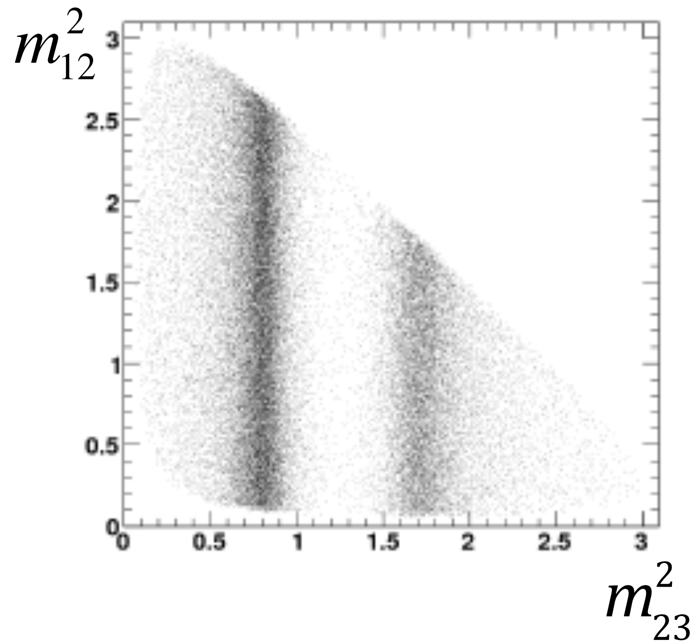
Destructive interference

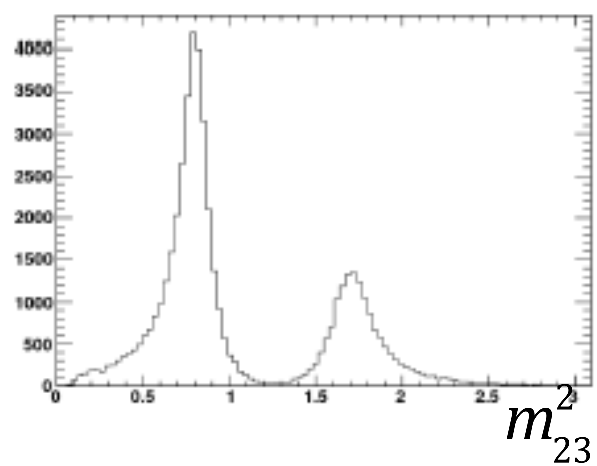
Magnitude



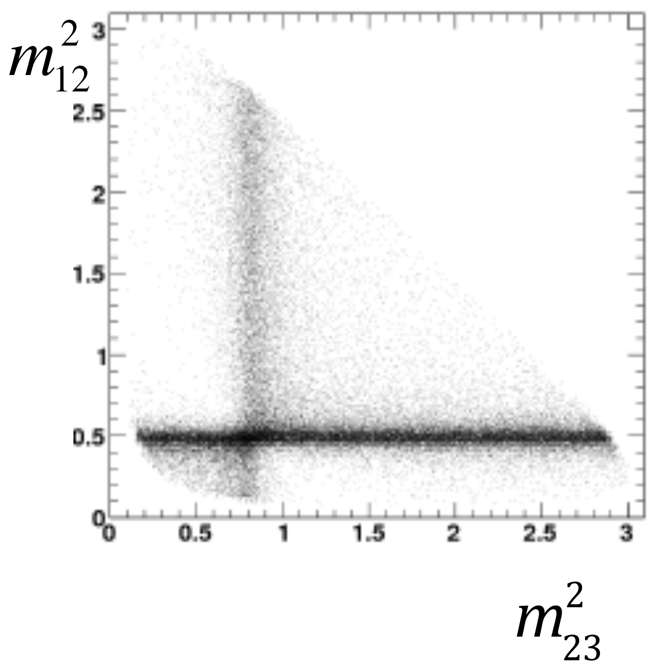
Phase

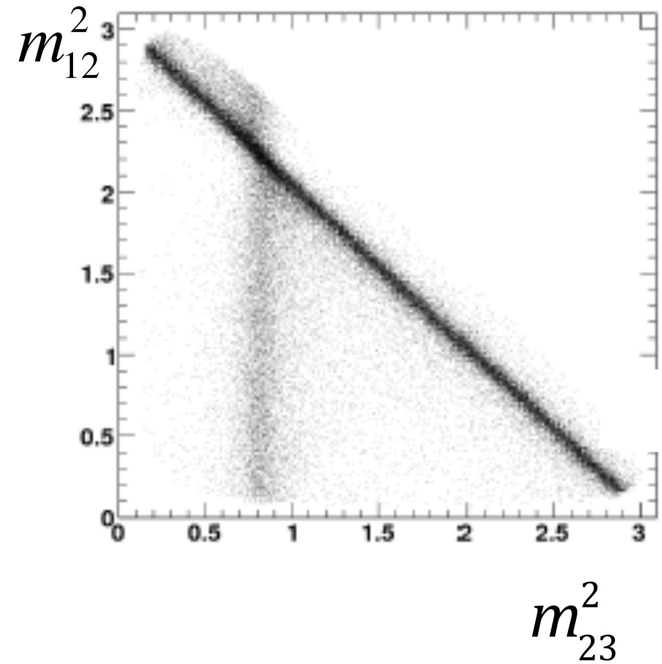






crossing resonances

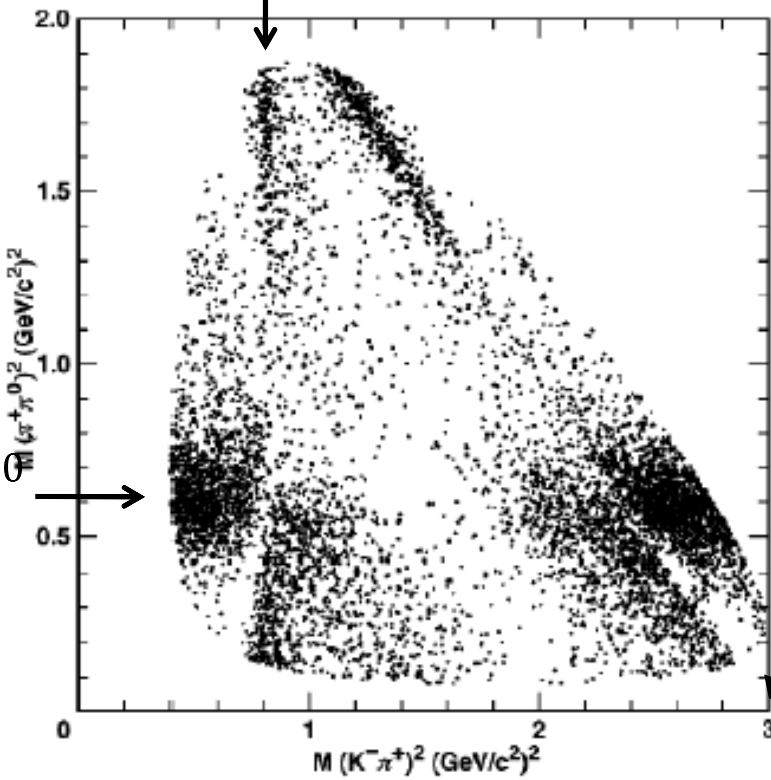




$D^0 \rightarrow K^- \pi^+ \pi^0$ from CLEO

$K^*(892)^0 \rightarrow K^- \pi^+$

$\rho(770)^0 \rightarrow \pi^+ \pi^0$



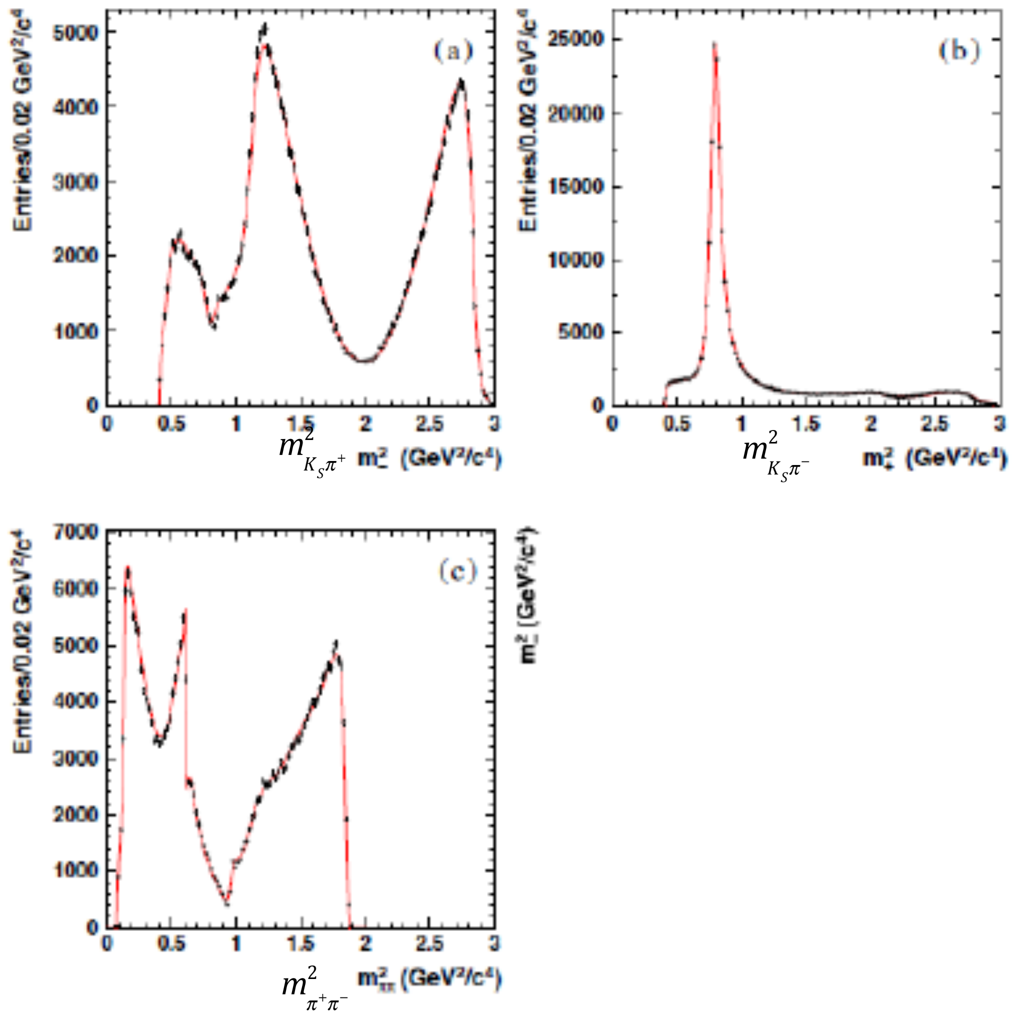
$K^*(892)^- \rightarrow K^- \pi^0$

FIG. 3. The Dalitz distribution of all 7070 $D^0 \rightarrow K^- \pi^+ \pi^0$ candidates in our data sample shown in an unbinned scatter plot.

Kopp et al, Phys. Rev. D **63**, 092001 (2001). "Copyright (2001) by the American Physical Society."

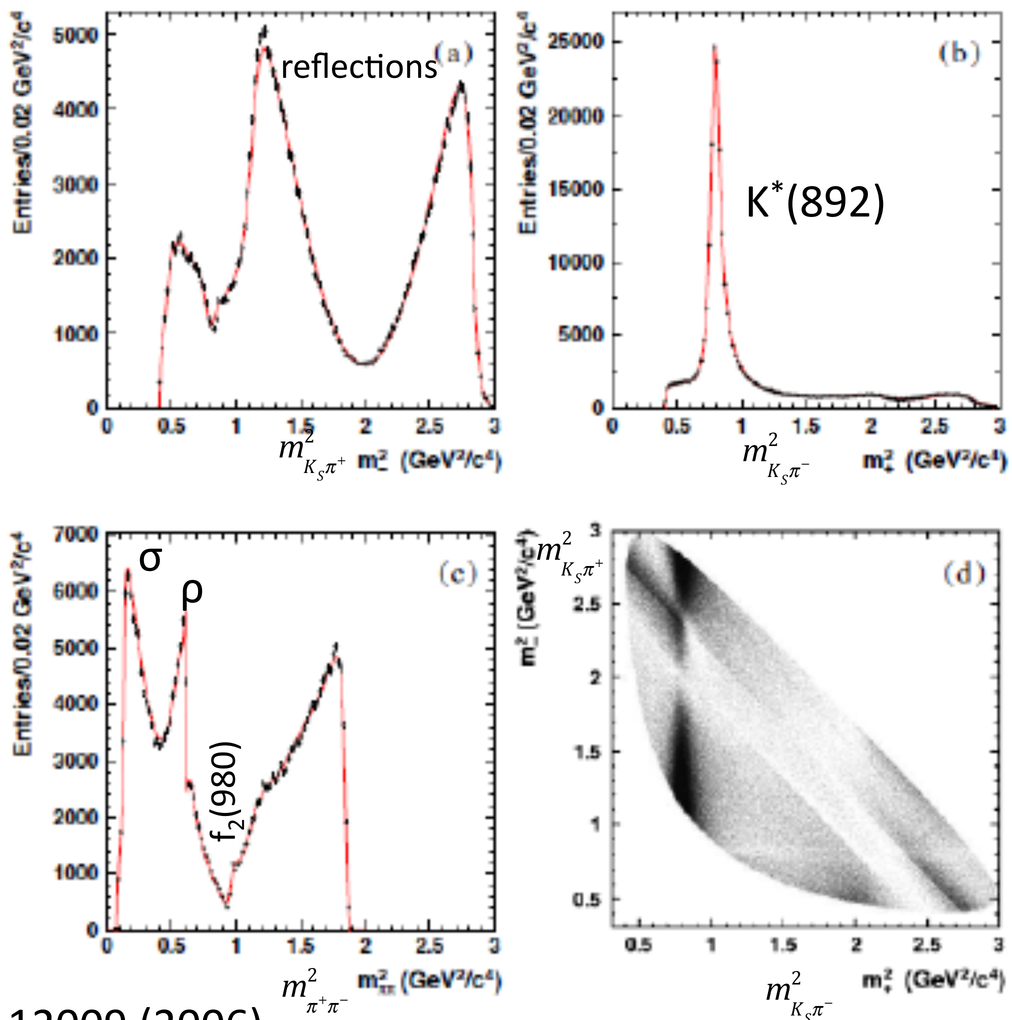
$D^0 \to K_S \pi^+ \pi^-$ from Belle

Invariant mass histograms don't give a clue about the physics



$D^0 \to K_S\pi^+\pi^-$ from Belle

Dalitz plot tells all!



Phys.Rev. D73, 112009 (2006)

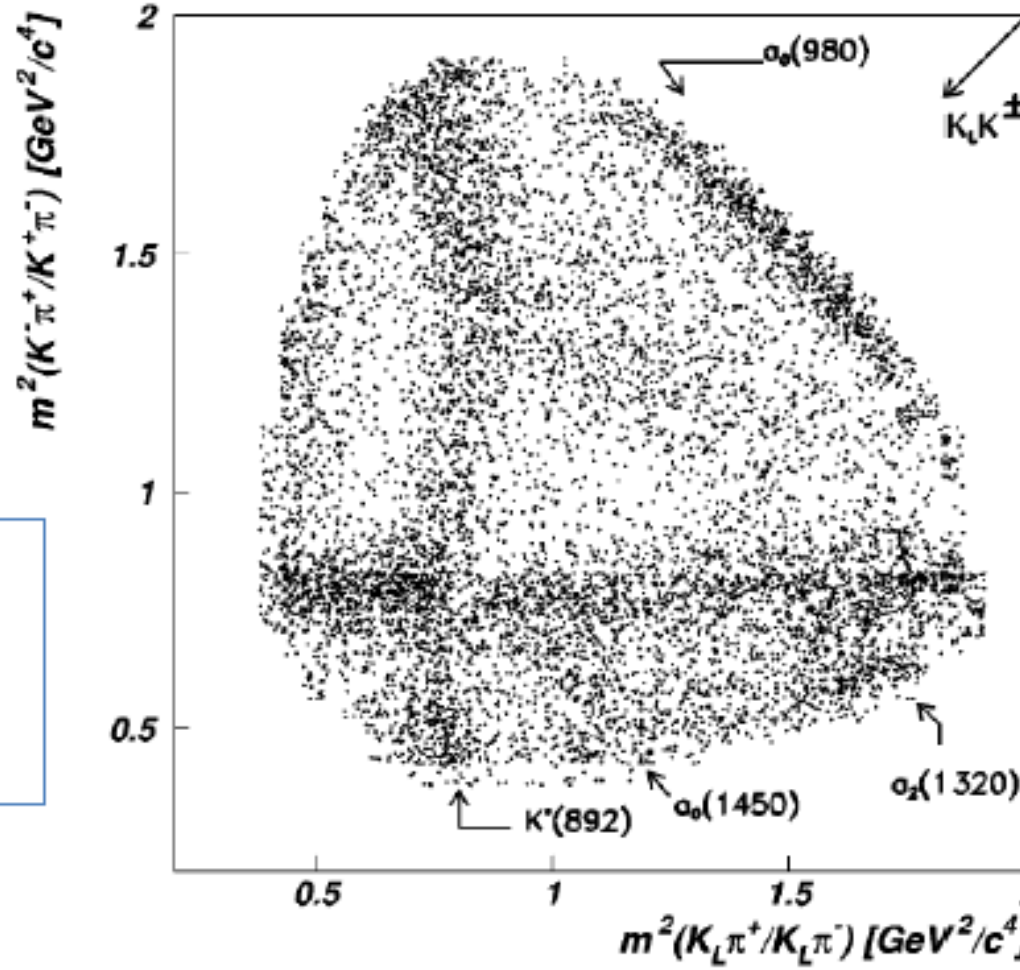
Details

Resonance	Our fit		
	Amplitude	Phase, deg	Br, %
$K^*(892)^-\pi^+$	1.643 ± 0.025	133.8 ± 1.3	60.7 ± 1.8
$K_s\rho^0$	1.0 (fixed)	0 (fixed)	22.5 ± 0.7
$K^*(892)^+\pi^-$	$(10.6 \pm 1.1) \times 10^{-2}$	315 ± 6	0.28 ± 0.06
$K_s\omega$	$(13.5 \pm 2.5) \times 10^{-3}$	104 ± 11	0.14 ± 0.05
$K_sf_0(980)$	0.413 ± 0.014	199.6 ± 2.5	5.3 ± 0.4
$K_sf_0(1370)$	0.52 ± 0.08	42.7 ± 6.6	0.69 ± 0.21
$K_sf_2(1270)$	0.70 ± 0.05	340.0 ± 3.6	0.22 ± 0.03
$K_0^*(1430)^-\pi^+$	2.00 ± 0.09	-5.6 ± 2.8	5.5 ± 0.5
$K_2^*(1430)^-\pi^+$	1.13 ± 0.06	312.4 ± 3.3	3.6 ± 0.4
$K^*(1680)^-\pi^+$	0.91 ± 0.16	197 ± 14	0.34 ± 0.12
$K_s\sigma$	0.93 ± 0.14	192.2 ± 7.8	4.1 ± 1.2
non-resonant	4.29 ± 0.32	311.6 ± 4.3	0.41 ± 0.06

$p\bar{p} \rightarrow K^{\pm}\pi^{\mp}\left(K^{0}\right)$ at rest

- $p\bar{p}$ annihilation at rest.

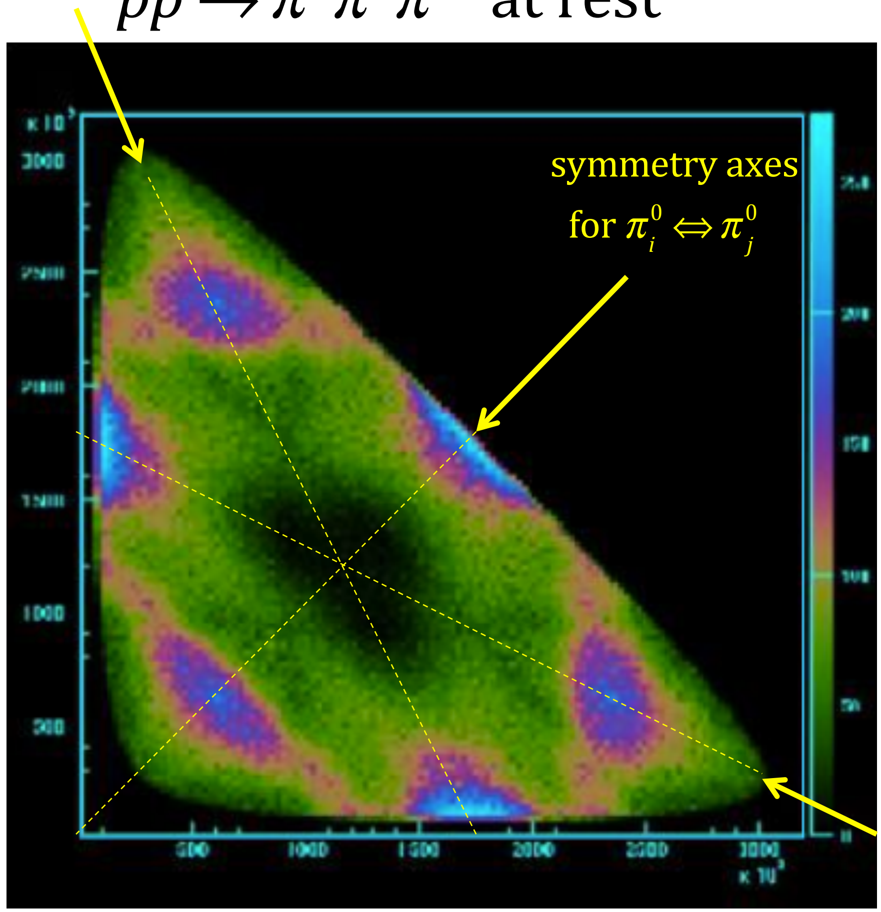
Abele et al, Phys. Rev. D **57**, 3860 (1998).
"Copyright (1998) by the American Physical Society."



Dalitz plot symmetries

only Ispin=1 allowed

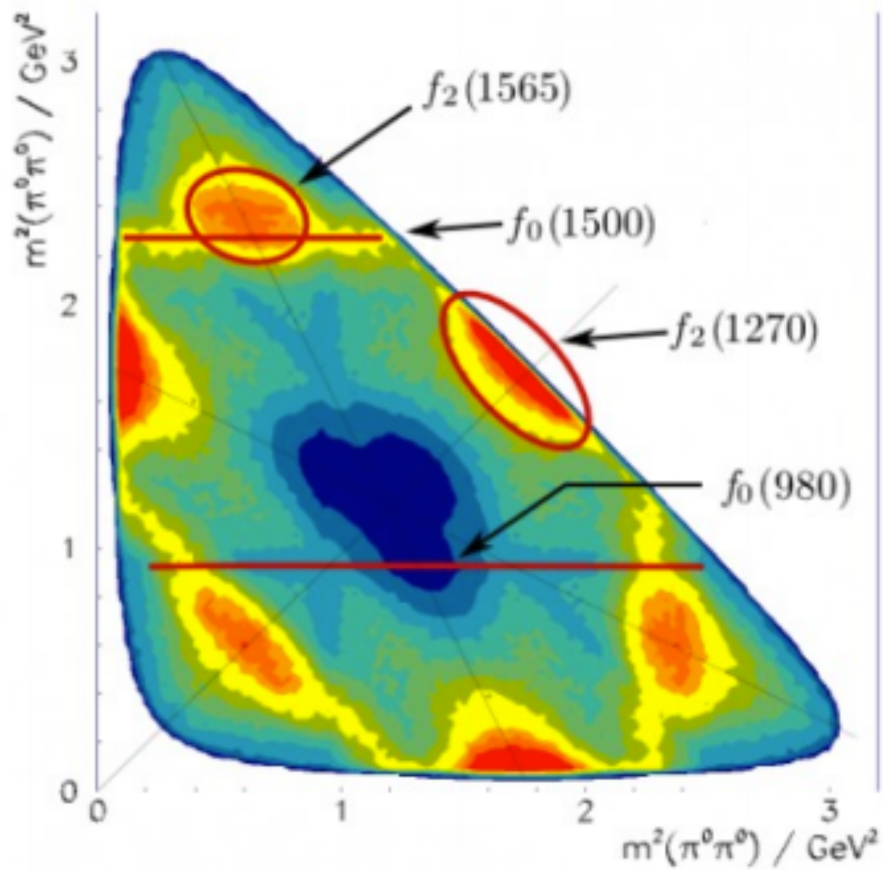
$p\bar{p} \rightarrow \pi^0\pi^0\pi^0$ at rest



C.Amsler et al., EPJC 23 (2002) 29

Interpretation

$J^P=0^-$



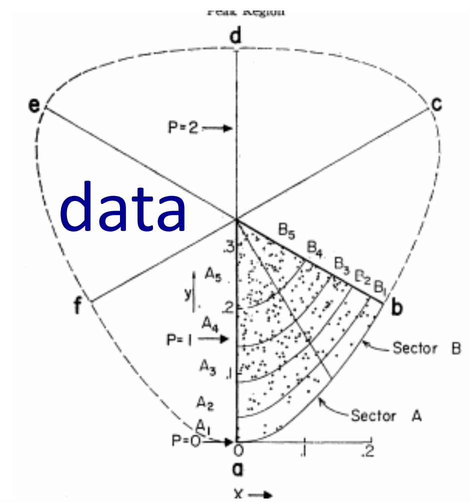
Example:

$$\omega \rightarrow \pi^+\pi^-\pi^0$$

$$1^- \rightarrow 0^- 0^- 0^-$$

$$\text{Ispin=0} \;\Rightarrow (\vec{I}^+ \times \vec{I}^-)\bullet\vec{I}^o = \begin{vmatrix} I_x^+ & I_y^+ & I_z^+ \\ I_x^- & I_y^- & I_z^- \\ I_x^0 & I_y^0 & I_z^0 \end{vmatrix} = \varepsilon_{ijk} I_i I_j I_k \;\Leftarrow i \leftrightarrow j \text{ asymmetric}$$

Phys. Rev. 125, 687 (1962)



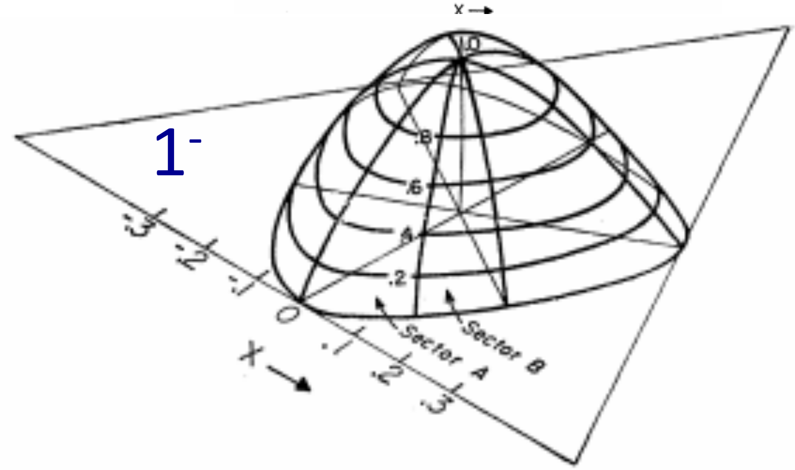
available vectors: $\vec{p}_+, \vec{p}_-, \vec{p}_0;\; \vec{p}_+ + \vec{p}_- + \vec{p}_0 = 0$

axial vector: $\vec{q} = \vec{p}_+ \times \vec{p}_- = \vec{p}_- \times \vec{p}_0 = \vec{p}_0 \times \vec{p}_+$

$J^p = 1^- = (-1)^3\,?$

? must be $\vec{q}$: $\Leftarrow$ to get $J = 1$ and $P =$ -1

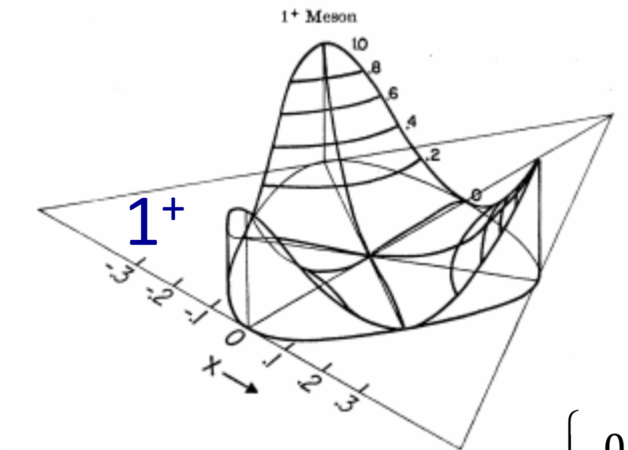
$$\therefore\; M \propto \vec{p}_+ \times \vec{p}_- \begin{cases} \text{max at } \vec{p}_+ = \vec{p}_- = \vec{p}_0 :\Leftarrow \text{center} \\ 0 \text{ when any } |\vec{p}| = 0 :\Leftarrow \text{boundary} \end{cases}$$



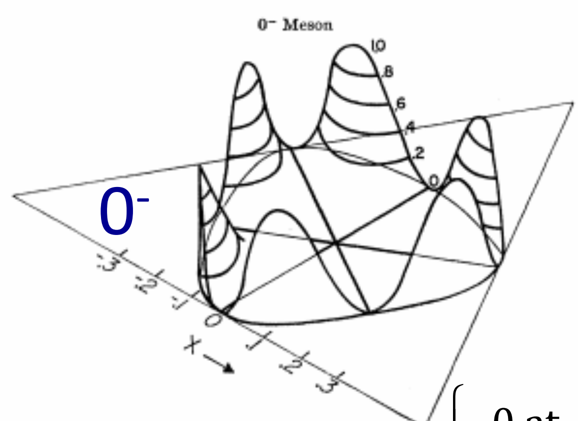
↑

Dalitz-plot boundary

Data rules out 1^+ and 0^-

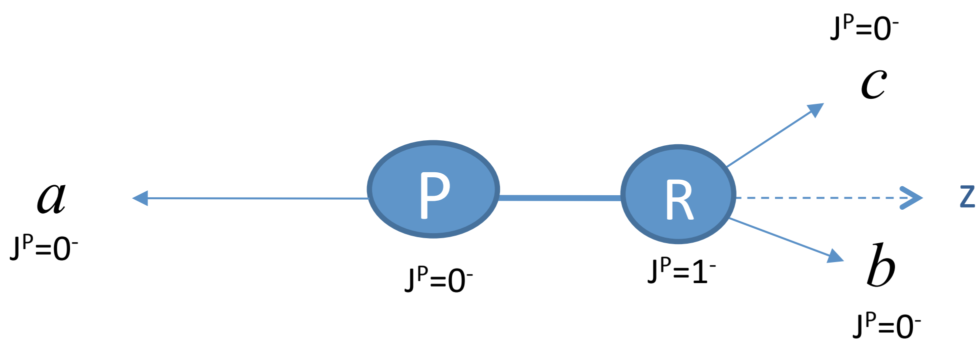


$$1^+: M \propto E_-(\vec{p}_0-\vec{p}_+)+E_0(\vec{p}_+-\vec{p}_-)+E_+(\vec{p}_- - \vec{p}_0) \quad \begin{cases} 0 \text{ at } (\vec{p}_i=\vec{p}_j) \Leftarrow \text{center \& diag lines} \\ \text{max when any } \vec{p}_k=0 \Leftarrow \text{boundary} \end{cases}$$



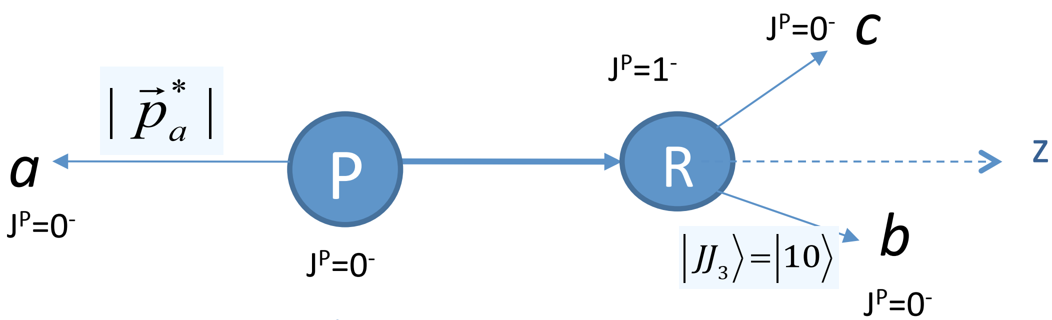
$$0^-: M \propto (E_- - E_0)(E_0 - E_+)(E_+ - E_-) \quad \begin{cases} 0 \text{ at } (\mathrm{E}_i = E_j) \Leftarrow \text{center \& diag lines} \\ \text{max when any } E_k = m_\pi \Leftarrow \text{boundary} \end{cases}$$

simple non-trival case: $P \rightarrow a\ R;\ R \rightarrow b\ c$



3-body decay $P \rightarrow aR;\ R \rightarrow b+c$

where P has $J^P=0^-$ and a, b & c are pions or kaons



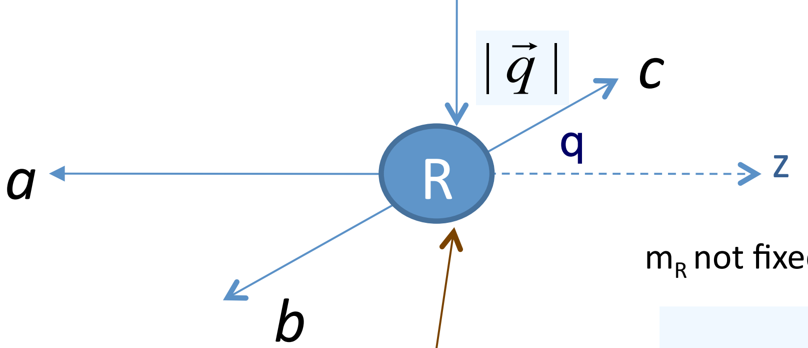
Energy-momentum conservation

$$p_P = p_a + p_R$$

$$p_R = p_b + p_c$$

4-vectors

Transform to the R rest frame:



$$|JJ_3\rangle = |10\rangle = Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

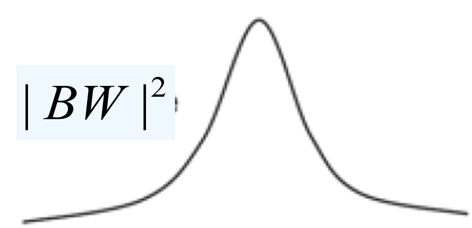
$$\cos\theta = \frac{m_R}{4|\vec{p}_a^{\,*}||\vec{q}\,|m_P}\left[(m_{ac}^2 - m_{ab}^2) + \frac{(m_P^2 - m_a^2)(m_b^2 - m_c^2)}{m_R^2}\right]$$

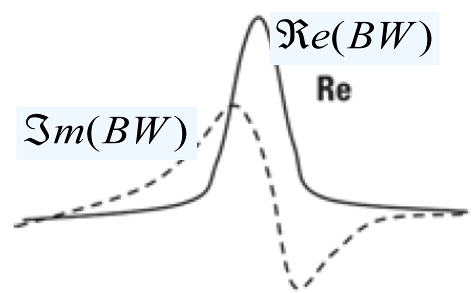
m_R not fixed; has a BW resonance form:

$$BW = \frac{i\sqrt{m_{bc}\Gamma_0}}{(m_0^2 - m_{bd}^2) - im_0\Gamma_0}$$

$$\Re e(BW) = \frac{(m_0\Gamma_0)^{3/2}}{(m_0^2 - m_{bc}^2)^2 + m_0^2\Gamma_0^2}$$

$$\Im m(BW) = \frac{\sqrt{m_0\Gamma_0}(m_0^2 - m_{bc}^2)}{(m_0^2 - m_{bc}^2)^2 + m_0^2\Gamma_0^2}$$



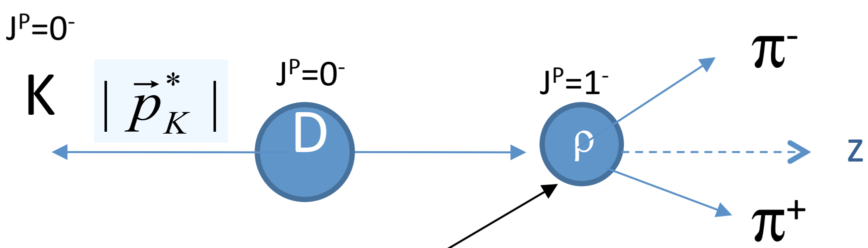


Fermi's Golden Rule

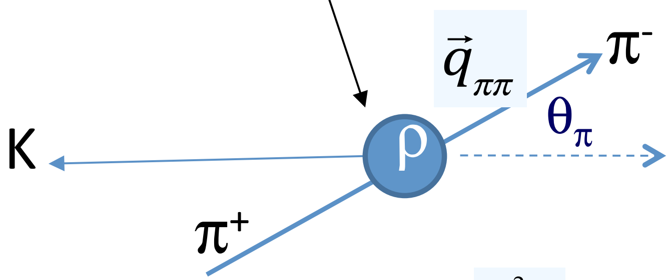
$$d\Gamma(m_{ab}^2, m_{ac}^2) = \frac{1}{(2\pi)^3 32\sqrt{m_P^3}} \left|\mathcal{M}\right|^2 dm_{ab}^2 dm_{ac}^2$$

$$\left|\mathcal{M}\right|^2 = \left|BW \times |10\rangle\right|^2 = \left|\frac{i\sqrt{m_{bc}\Gamma_R}}{(m_R^2 - m_{bd}^2) - i m_0 \Gamma_0} Y_1^0(\cos\theta)\right|^2$$

$D \rightarrow K\rho \quad \rho \rightarrow \pi^+\pi^-$



$$|JJ_3\rangle = |10\rangle = Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta_\pi$$



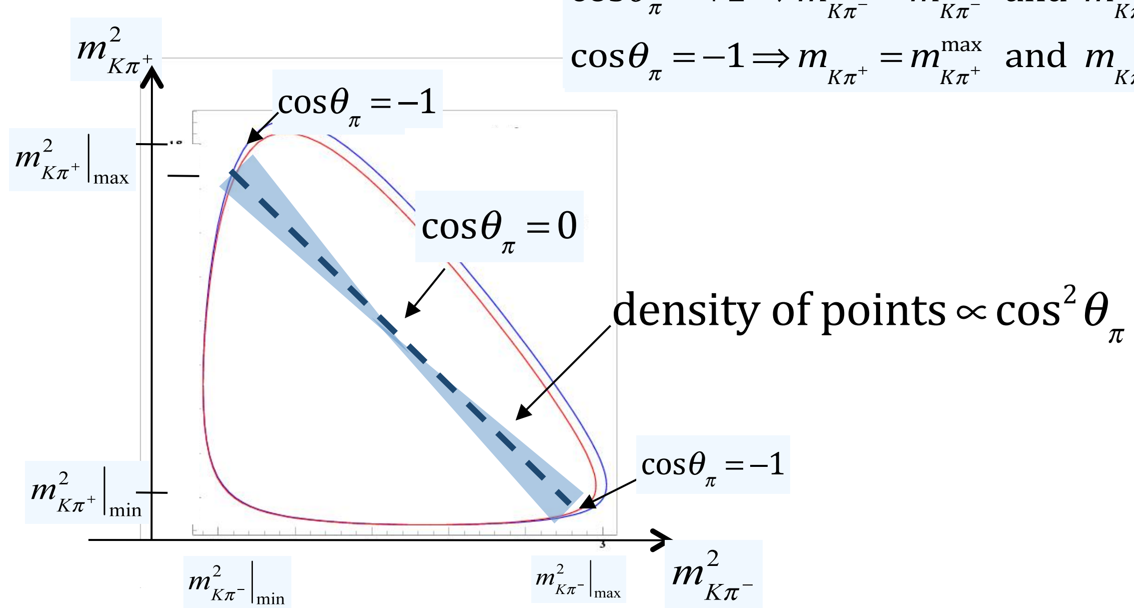
$$\cos\theta_\pi = \frac{m_{\pi^+\pi^-}}{4|\vec{p}_K^{\,*}||\vec{q}_{\pi\pi}|m_D}\left[(m_{K\pi^-}^2 - m_{K\pi^+}^2) + \frac{(m_D^2 - m_K^2)(m_{\pi^+}^2 - m_{\pi^-}^2)}{m_D^2}\right]$$

$$(m_{\pi^+}^2 - m_{\pi^-}^2) = 0 \Rightarrow \qquad \cos\theta_\pi = \frac{m_{\pi^+\pi^-}(m_{K\pi^-}^2 - m_{K\pi^+}^2)}{4|\vec{q}_{\pi\pi}||\vec{p}_K^{\,*}|m_D}$$

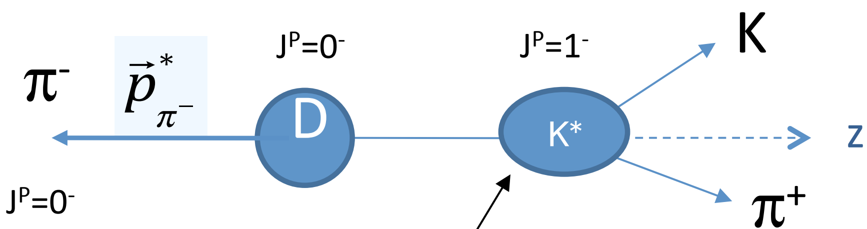
$$\cos\theta_\pi = +1 \Rightarrow m_{K\pi^-} = m_{K\pi^-}^{\max} \text{ and } m_{K\pi^+} = m_{K\pi^+}^{\min}$$

$$\cos\theta_\pi = -1 \Rightarrow m_{K\pi^+} = m_{K\pi^+}^{\max} \text{ and } m_{K\pi^-} = m_{K\pi^-}^{\min}$$

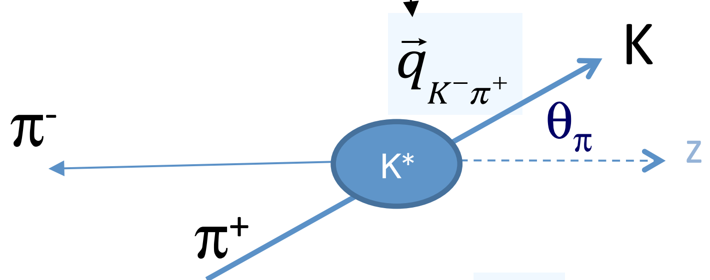
For given $m_{\pi\pi}$:



$D \rightarrow K^{*}\pi^{-}$; $K^{*} \rightarrow K^{-}\pi^{+}$



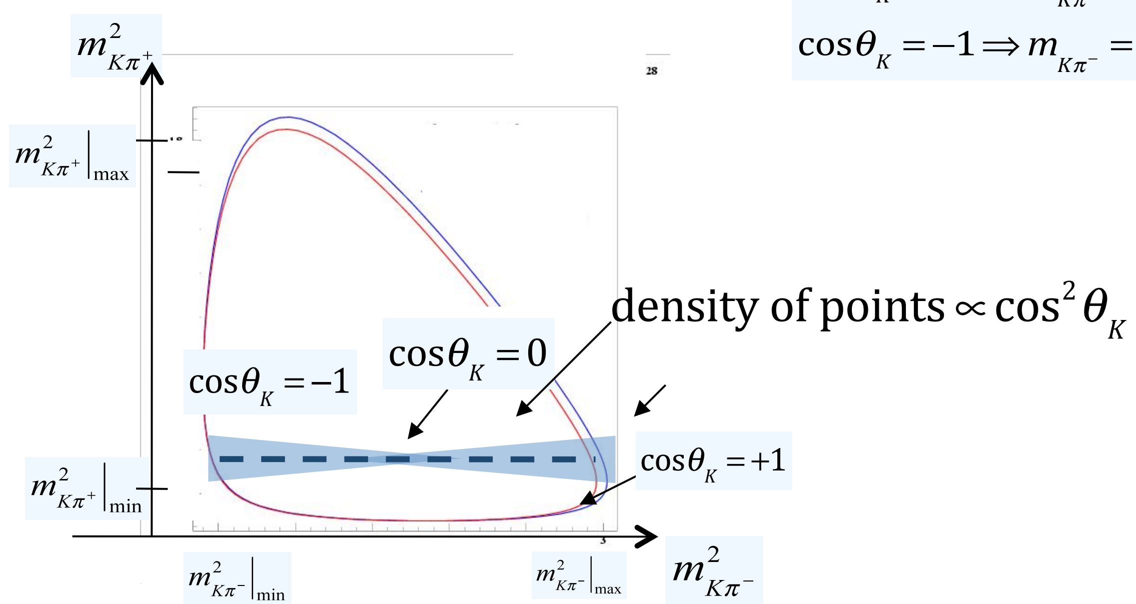
$$|JJ_3\rangle = |1\,0\rangle = Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta_K$$



$$\cos\theta_K = \frac{m_{K^-\pi^+}}{4|\vec{p}^{\,*}_{\pi^-}||\vec{q}_{K^-\pi^+}|m_D}\left[(m^2_{K^-\pi^-} - m^2_{\pi^+\pi^-}) + \frac{(m_D^2 - m_{\pi^-}^2)(m_{\pi^+}^2 - m_{K^-}^2)}{m^2_{K^-\pi^+}}\right]$$

$$\cos\theta_K = +1 \Rightarrow m_{K\pi^-} = m^{\max}_{K\pi^-}$$
$$\cos\theta_K = -1 \Rightarrow m_{K\pi^-} = m^{\min}_{K\pi^-}$$

For given $m_{\pi\pi}$:

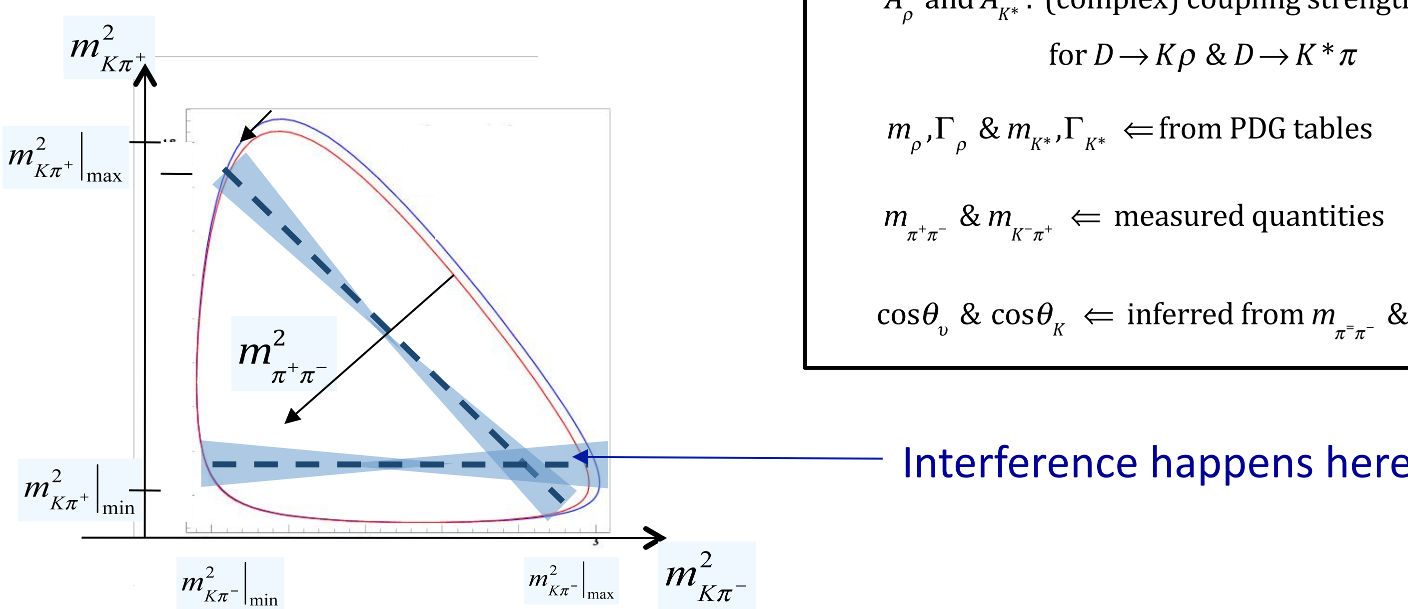


Fermi's Golden Rule

$$d\Gamma(m_{ab}^2, m_{ac}^2) = \frac{1}{(2\pi)^3 32\sqrt{m_P^3}} \left|\mathcal{M}\right|^2 dm_{ab}^2 dm_{ac}^2$$

$$\left|\mathcal{M}\right|^2 = \left|A_\rho BW_\rho \times Y_1^0(\cos\theta_\pi) + A_{K^*} BW_{K^*} Y_1^0(\cos\theta_K)\right|^2$$

$$= \left|A_\rho \frac{i\sqrt{m_{\pi^+\pi^-}\Gamma_\rho}}{(m_\rho^2 - m_{\pi^+\pi^-}^2) - im_\rho\Gamma_\rho} Y_1^0(\cos\theta_\pi) + A_{K^*} \frac{i\sqrt{m_{K^-\pi^+}\Gamma_{K^*}}}{(m_{K^*}^2 - m_{K^-\pi^+}^2) - im_{K^*}\Gamma_{K^*}} Y_1^0(\cos\theta_K)\right|^2$$



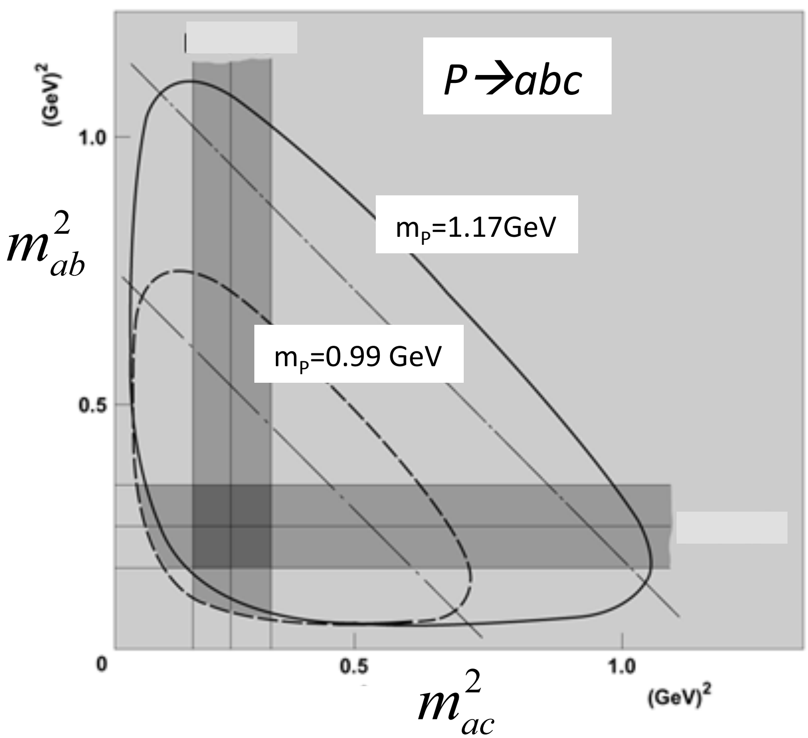
A_ρ and A_{K^*}: (complex) coupling strengths for $D \to K\rho$ & $D \to K^*\pi$

m_ρ, Γ_ρ & m_{K^*}, Γ_{K^*} $\Leftarrow$ from PDG tables

$m_{\pi^+\pi^-}$ & $m_{K^-\pi^+}$ $\Leftarrow$ measured quantities

$\cos\theta_\upsilon$ & $\cos\theta_K$ $\Leftarrow$ inferred from $m_{\pi^=\pi^-}$ & $m_{K^-\pi^+}$

Some Dalitz Plot relations



$$\vec{p}_a+\vec{p}_b+\vec{p}_c=0$$

$$E_a+E_b+E_c=M_P$$

$$M_P^2+m_a^2+m_b^2+m_c^2=m_{ab}^2+m_{ac}^2+m_{bc}^2$$

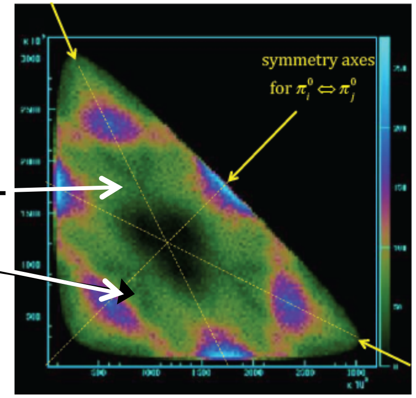
Boundary corresponds to $|\cos^2\theta|=1$

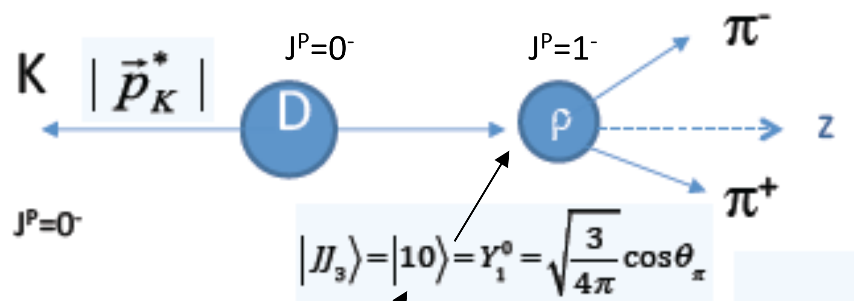
$$1=\left[\frac{m_R}{4|\vec{p}_a^{\,*}||\vec{q}|\,m_P}\left[(m_{ac}^2-m_{ab}^2)+\frac{(m_P^2-m_a^2)(m_b^2-m_c^2)}{m_R^2}\right]\right]^2 \Rightarrow$$

$$m^{2\ \max}_{ac\ \min}=\frac{1}{2}\Big[\ m_P^2+m_a^2+m_b^2+m_c^2-m_{ac}^2$$
$$\pm\sqrt{1-\left((m_b+m_c)/m_{bc}\right)^2}\times\sqrt{m_P^2-(m_a+m_b+m_c)^2}\times\sqrt{m_P^2-(m_a-m_b-m_c)^2}\ \Big]$$

questions/items for discussion

- What are these dark lines





- Why only $|JJ_3> = |10>$? … why no $|JJ_3> = |0{\pm}1>$?

- Check the formulae in the preceding slides
  -- report errors to me