

# Measurement of $R$ values at BESIII

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# Outline

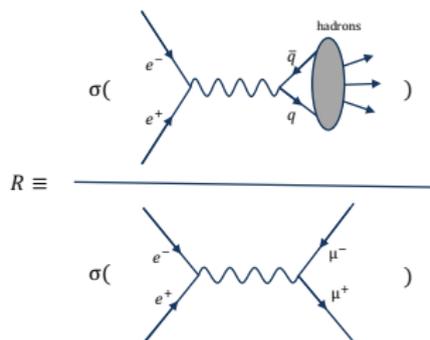
- **Introduction**
- **BEPCII and BESIII**
- **$R$  value measurement**
  - ▶ Signal selection
  - ▶ Background estimation
  - ▶ Signal simulation
  - ▶ Systematic uncertainty study
- **Summary and outlook**

# The definition of $R$ value

$R$  is defined as the ratio of the production rate of hadron and muon pairs:

$$R \equiv \frac{\sigma^0(e^+e^- \rightarrow \text{hadrons})}{\sigma^0(e^+e^- \rightarrow \mu^+\mu^-)} \equiv \frac{\sigma_{\text{had}}^0}{\sigma_{\mu\mu}^0}$$

That is:



According to the QED theory:

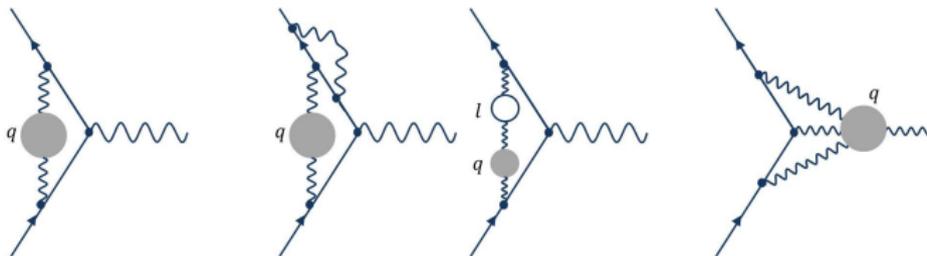
$$\sigma_{\mu\mu}^0(s) = \frac{4\pi\alpha^2}{3s} \frac{\beta_\mu(3 - \beta_\mu^2)}{2}$$

# The anomalous magnetic moment

The abnormal magnetic momentum of lepton  $\ell$  can be not only measured directly but also determined by theoretic calculation based on the SM:

$$a_{\ell}^{\text{SM}} = a_{\ell}^{\text{QED}} + a_{\ell}^{\text{Weak}} + a_{\ell}^{\text{had}}$$

- $a_{\ell}^{\text{QED}}$  and  $a_{\ell}^{\text{Weak}}$  can be analytically calculated in high precision.
- $a_{\ell}^{\text{had}}$  is the contribution of hadronic vacuum polarization:



Above diagrams represents three types of hadronic contribution to  $a_{\ell}^{\text{had}}$

$$a_{\ell}^{\text{had}} = a_{\ell}^{\text{had, LO v.p.}} + a_{\ell}^{\text{had, NLO v.p.}} + a_{\ell}^{\text{had, l-l}}$$

# The anomalous magnetic moment: $a_\ell^{\text{had}}$

At the low energy region ( $\sqrt{s} < 1.0$  GeV), the perturbation theory of QCD is not valid.

Therefore the  $a_\ell^{\text{had, LO v.p.}}$  is calculated in terms of **R value**. Take the  $\mu$  lepton as an example:

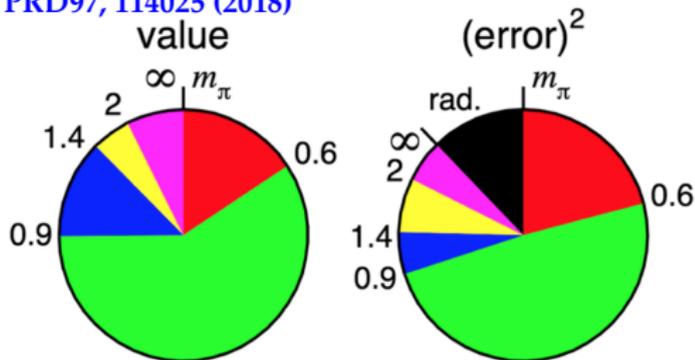
$$a_\mu^{\text{had, LO v.p.}} = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{R(s)K(s)}{s^2}$$

- Only the R values below 1.0 GeV contribute the  $a_\ell^{\text{had, LO v.p.}}$  significantly.
- After sufficient high energy, e.g. 12 GeV, the  $a_\ell^{\text{had, LO v.p.}}$  is calculated via pQCD.

**PRD98, 030001 (2018); 97, 114025 (2018)**

Source	Contribution ( $\times 10^{11}$ )
$a_\mu^{\text{QED}}$	$116584718.95 \pm 0.08$
$a_\mu^{\text{Weak}}$	$153.6 \pm 1.0$
$a_\mu^{\text{had, LO v.p.}}$	$6931 \pm 34$
$a_\mu^{\text{had, NLO v.p.}}$	$-98.7 \pm 0.9$
$a_\mu^{\text{had, NNLO v.p.}}$	$12.4 \pm 0.1$
$a_\mu^{\text{had, l-l}}$	$98 \pm 26$
$a_\mu^{\text{SM}}$	$116591823 \pm 43$
$a_\mu^{\text{exp}}$	$116592091 \pm 63$
$\Delta a_\mu$	$268 \pm 76$

**Fractional contribution to  $a_\mu^{\text{had, LO v.p.}}$ :**  
**PRD97, 114025 (2018)**



# The running coupling constant of QED

In QFT, the exact photon propagator function, which is consisted of inserting all one-particle-irreducible (1PI) diagrams into photon propagator, creates two effects:

- Renormalize the electric charge of the theory: bare charge to physical charge, *i.e.*  $e_0 \mapsto e \equiv e_0 \sqrt{\frac{1}{1-\Pi(0)}}$ . This is a constant shift in the strength of the electric charge.
- The  $q^2$ -dependent running coupling constant:  $\alpha(0) \equiv \frac{e_0^2}{4\pi} \mapsto \alpha(q^2) \equiv \frac{e_0^2}{4\pi} \frac{1}{1-\Pi(q^2)}$ , where the  $\alpha_0$  is the fine structure constant.

In practice, the running coupling constant constant at scale  $\sqrt{s}$  is given by:

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}$$

In the perturbative theory, the leading order corrections to  $\Delta\alpha(s)$  is:

$$\Delta\alpha(s) = \Pi(q^2) \alpha \begin{array}{c} \mu \\ \text{wavy line} \\ \xrightarrow{q} \end{array} \begin{array}{c} \text{1PI} \\ \text{circle} \end{array} \begin{array}{c} \text{wavy line} \\ \xrightarrow{\nu} \end{array} \equiv \begin{array}{c} \mu \\ \text{wavy line} \\ \xrightarrow{q} \end{array} \begin{array}{c} \text{circle} \end{array} \begin{array}{c} \text{wavy line} \\ \xrightarrow{\nu} \end{array} + \dots$$



$E_{\text{beam}}$ : 1.0 - 2.45 GeV  
 $\sigma_E$ :  $5.16 \times 10^{-4}$   
 $L$ :  $1.0 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$  @3770

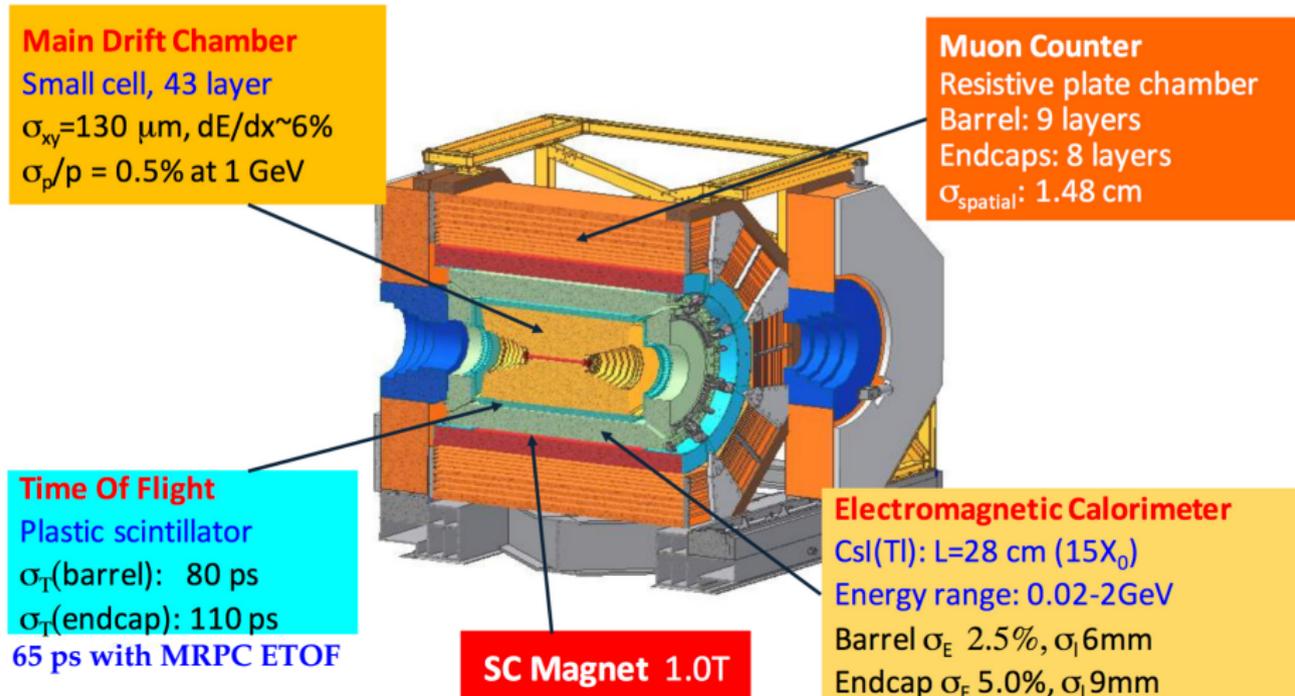
**Linac**

**BES**

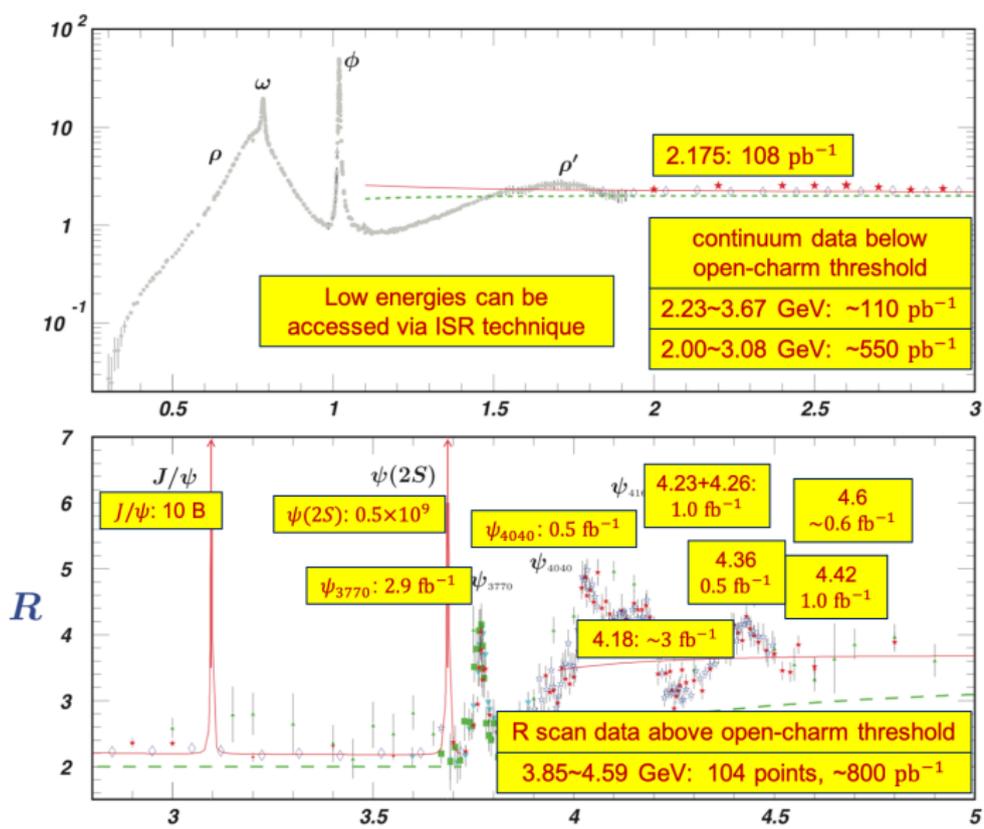
**Storage ring**

<b>Leptons</b>	$u$ <small>up</small>	$c$ <small>charm</small>	$t$ <small>top</small>
	$d$ <small>down</small>	$s$ <small>strange</small>	$b$ <small>bottom</small>
	$\nu_e$ <small>e- neutrino</small>	$\nu_\mu$ <small><math>\mu</math>- neutrino</small>	$\nu_\tau$ <small>t- neutrino</small>
	$e$ <small>electron</small>	$\mu$ <small>muon</small>	$\tau$ <small>tau</small>
	Three Generations of Matter		

BEPC = Beijing Electron Positron Collider

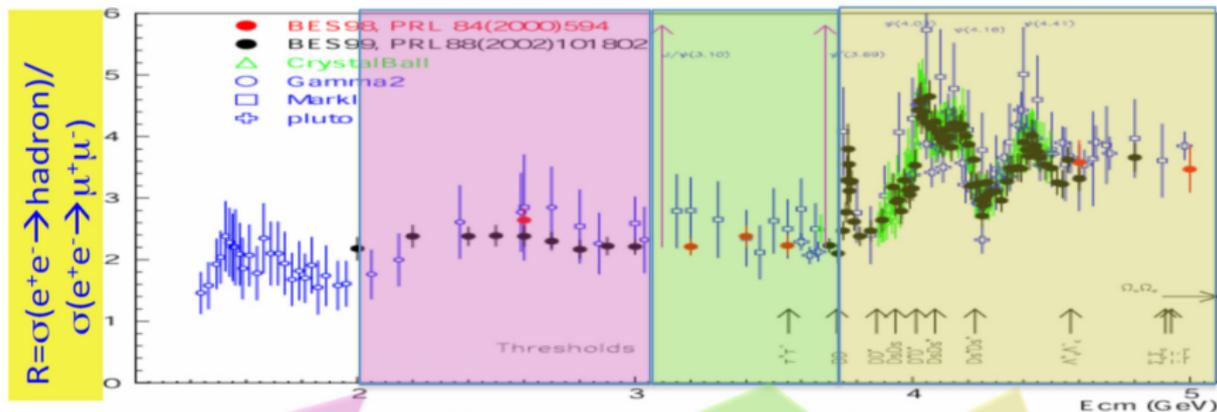


## BESIII = Beijing Spectrometer III



More data samples above open-charm threshold are on the way!

# Physics at $\tau$ -charm energy region



- Hadron form factors
- $Y(2175)$  resonance
- Multiquark states with  $s$  quark,  $Z_s$
- MLLA/LPHD and QCD sum rule predictions

- Light hadron spectroscopy
- Gluonic and exotic states
- Process of LFV and CPV
- Rare and forbidden decays
- Physics with  $\tau$  lepton

- XYZ particles
- D mesons
- $f_D$  and  $f_{D_s}$
- $D_0$ - $D_0$  mixing
- Charm baryons

# Determination of $R$ value in experiment

Experimentally,  $R$  value is determined by

$$R = \frac{N_{\text{had}}^{\text{obs}} - N_{\text{bkg}}}{\mathcal{L}_{\text{int.}} \epsilon_{\text{had}} \epsilon_{\text{trig}} (1 + \delta) \sigma_{\mu\mu}^0}$$

- $N_{\text{had}}^{\text{obs}}$ : Numbers of observed hadronic events.
- $N_{\text{bkg}}$ : Number of the residual background events.
- $\mathcal{L}_{\text{int.}}$ : Integrated luminosity.
- $\epsilon_{\text{trig}}$ : Trigger efficiency.
- $\epsilon_{\text{had}}$ : Detection efficiency of the hadronic events.
- $(1 + \delta)$ : ISR correction factor.
- $\sigma_{\mu\mu}^0(s) = 86.8 \text{ nb/s}$ : Leading order QED cross section for  $e^+e^- \rightarrow \mu^+\mu^-$ .

# Data sets and MC samples

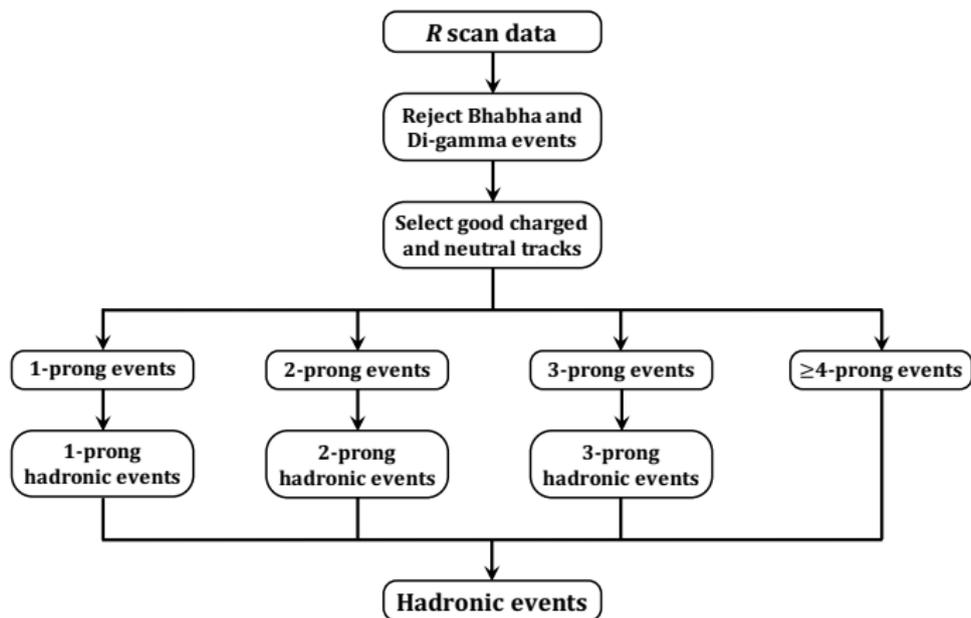
Data sets:

	$\sqrt{s}$ (GeV)	Run range	$\mathcal{L}_{\text{int.}}$ (pb $^{-1}$ )	Purpose
1	2.2324	28624 – 28648	2.645	R scan
2	2.4000	28577 – 28616	3.415	R scan
3	2.8000	28553 – 28575	3.753	R scan
4	3.0500	28312 – 28346	14.893	$J/\psi$ scan
5	3.0600	28347 – 28381	15.040	$J/\psi$ scan
6	3.0800	27147 – 27233, 28241 – 28266	31.019	$J/\psi$ scan
7	3.4000	28543 – 28548	1.733	R scan
8	3.5000	33725 – 33733	3.633	off $\psi$ (3770)
9	3.5424	24983 – 25015, 33734 – 33743	8.693	$\tau$ mass scan
10	3.5538	25016 – 25094	5.562	$\tau$ mass scan
11	3.5611	25100 – 25141	3.847	$\tau$ mass scan
12	3.6002	25143 – 25243	9.502	$\tau$ mass scan
13	3.6500	33747 – 33758	4.760	off $\psi$ (3686)
14	3.6710	33759 – 33764	4.628	off $\psi$ (3770)
15	2.4000	28617 – 28621	...	separated-beam
16	3.4000	28549 – 28552	...	separated-beam

MC samples:

- ▶ **Luarlw** model is used to simulate signals:  $e^+e^- \rightarrow$  hadrons.
- ▶ **Hybrid** model is used as cross check of Luarlw model.
- ▶ Dedicated MC generators are utilized to simulate backgrounds.

# Analysis strategy



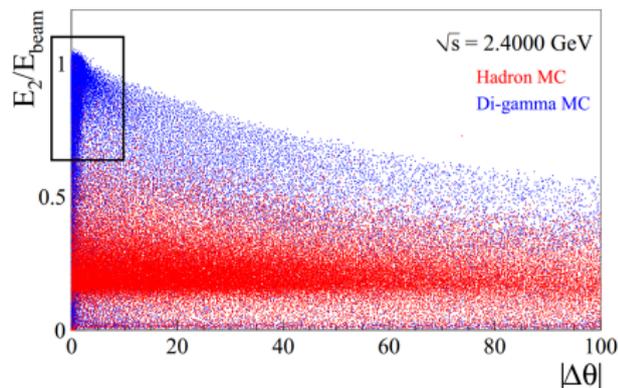
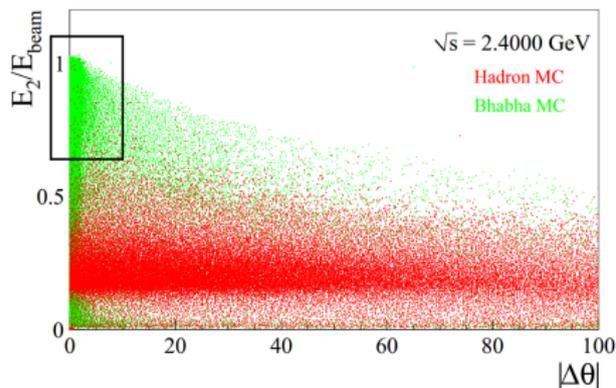
The selection criteria of hadronic events are carefully studied.

# Veto Bhabha and Di-gamma events

Bhabha ( $e^+e^- \rightarrow (\gamma)e^+e^-$ ) and Di-gamma ( $e^+e^- \rightarrow \gamma\gamma$ ) events are dominant backgrounds due to their large cross sections.

► Veto the Bhabha and Di-gamma events:

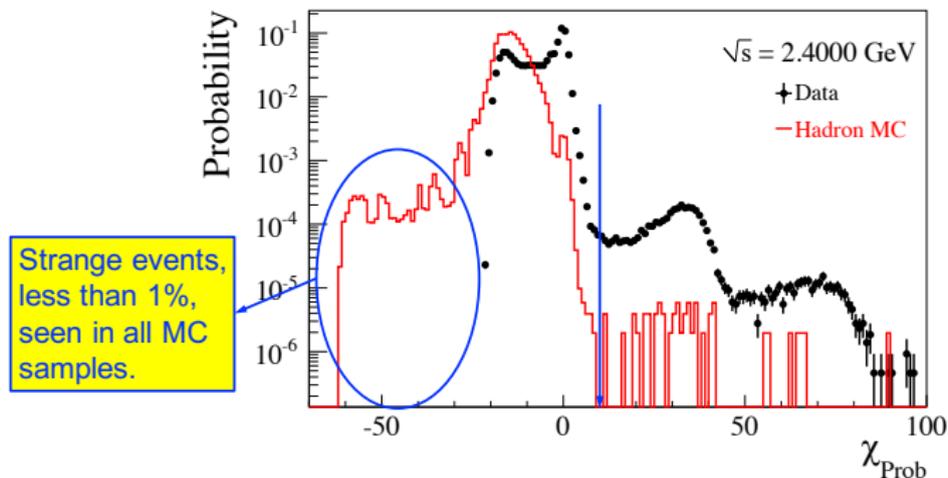
- $N_{shower} \geq 2$ .
- $E_1 \geq E_2 \geq 0.65E_{beam}$  and  $|\Delta\theta| = |\theta_1 + \theta_2 - 180^\circ| < 10^\circ$ .



# Select good charged tracks

## ► Good charged hadronic tracks:

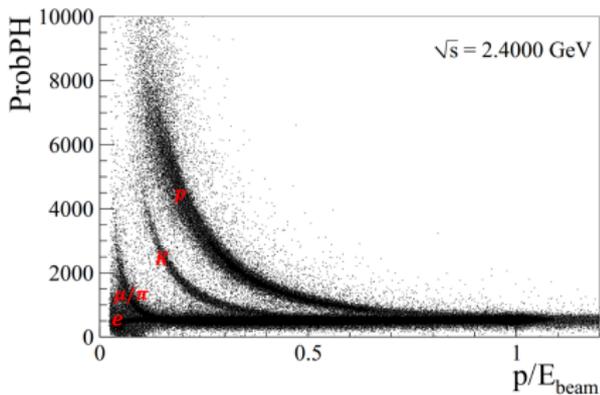
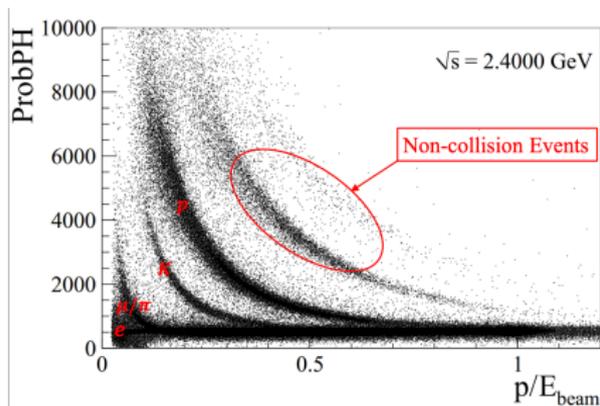
- $|V_r| < 0.5$  cm,  $|V_z| < 5.0$  cm, and  $|\cos \theta| < 0.93$ .
- $\chi_{\text{prob.}} = (dE/dx_{\text{measure}} - dE/dx_{\text{proton}}) / \sigma_{\text{proton}} < 10$ .
- $p_{\text{track}} < 0.94 p_{\text{beam}}$ , where  $p_{\text{beam}} \approx E_{\text{beam}}$ .
- Remove charged tracks when  $E/p > 0.8$  and  $p > 0.65 p_{\text{beam}}$ .
- Veto  $\gamma$ -conversions when  $M(e^+e^-) < 0.1$  GeV and  $\text{Angle}(e^+e^-) < 15^\circ$ .



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## ► Good charged hadronic tracks:

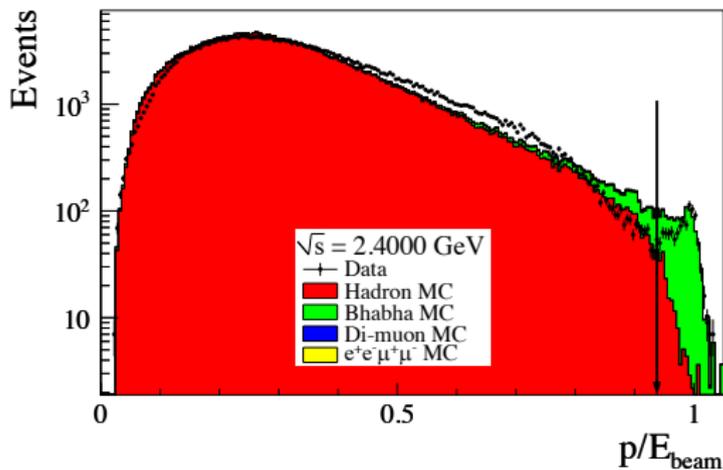
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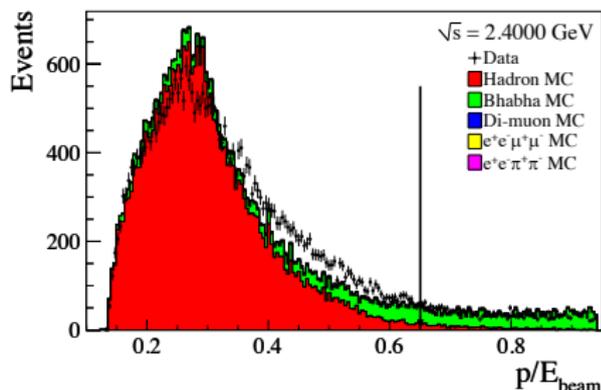
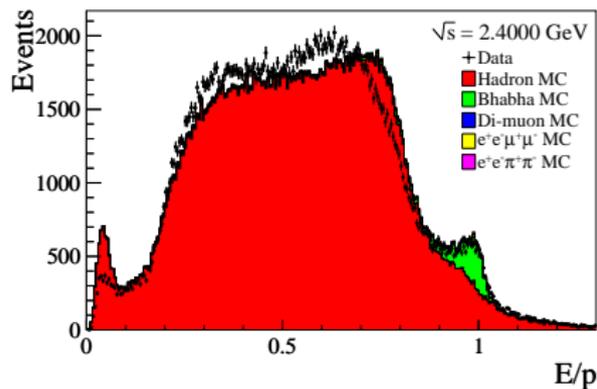
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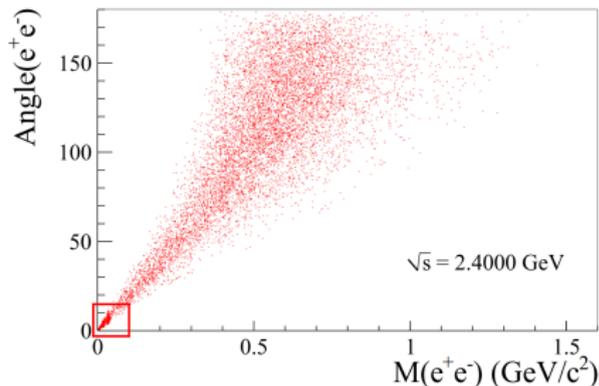
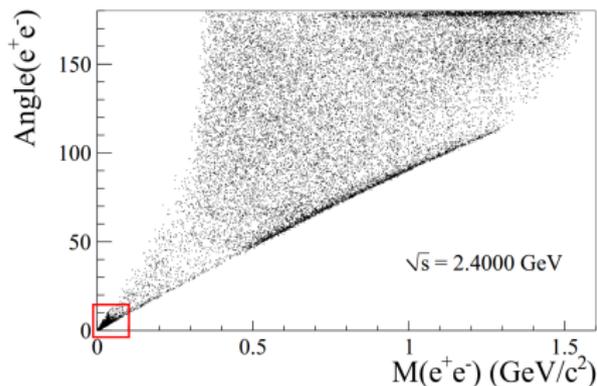
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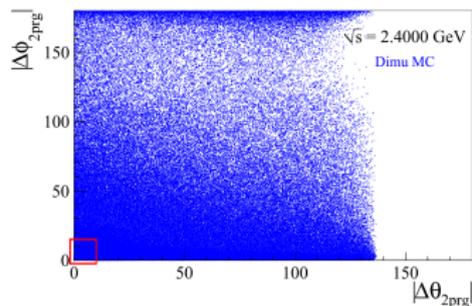
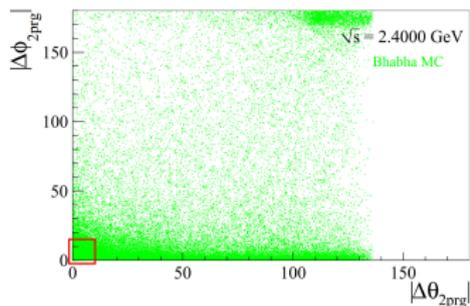
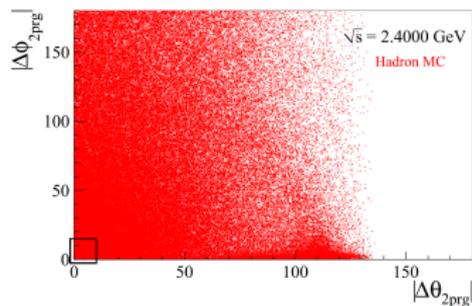
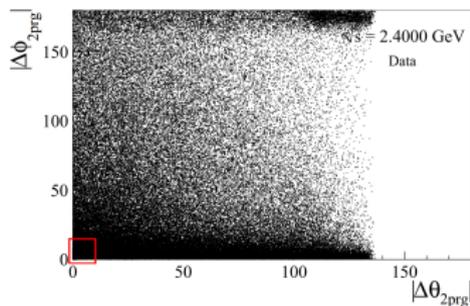
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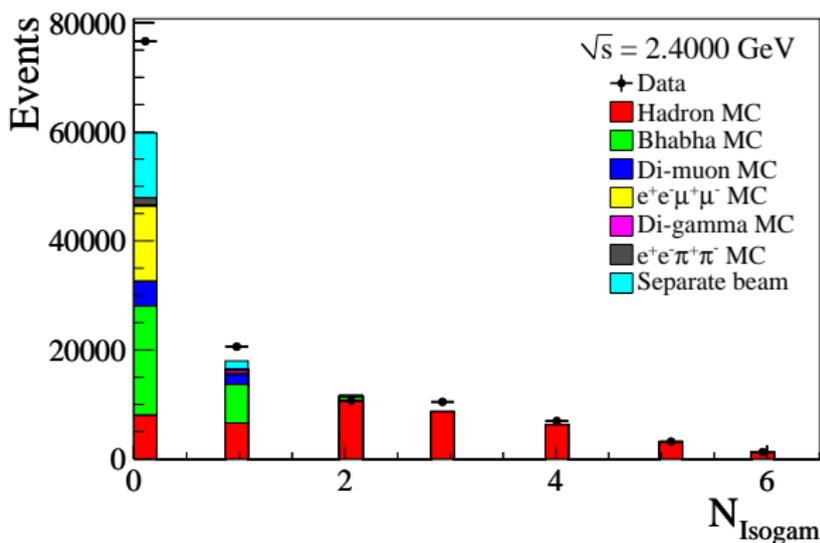
# Select good hadronic events

- If the number of good hadronic tracks = 2:
  - $|\Delta\theta| = |\theta_1 + \theta_2 - 180^\circ| > 10^\circ$  or  $|\Delta\phi| = ||\phi_1 - \phi_2| - 180^\circ| > 15^\circ$
  - At least 2 isolated photons



# Select good hadronic events

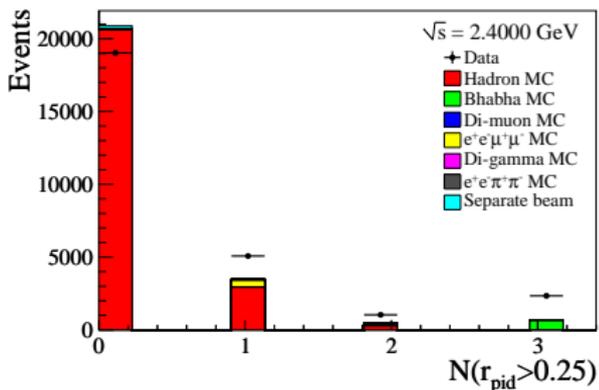
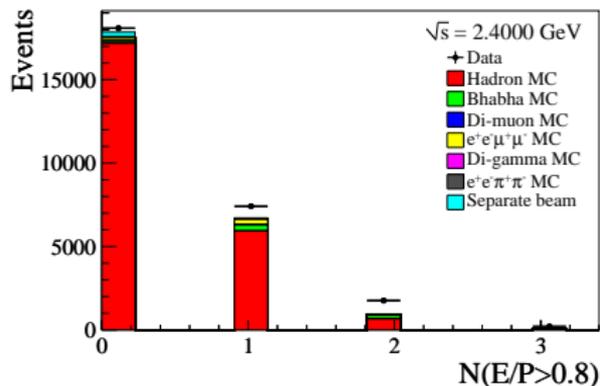
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  - At least 2 isolated photons



**Isolated photon:** energetic photons not originate from charged track.

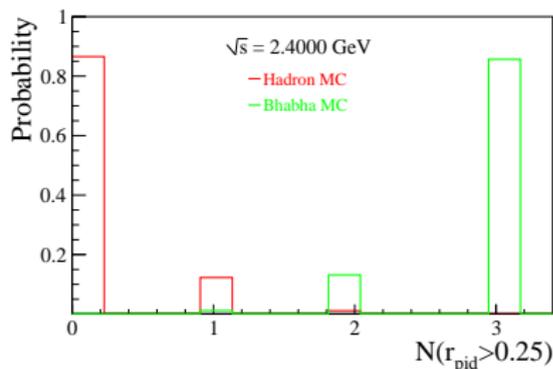
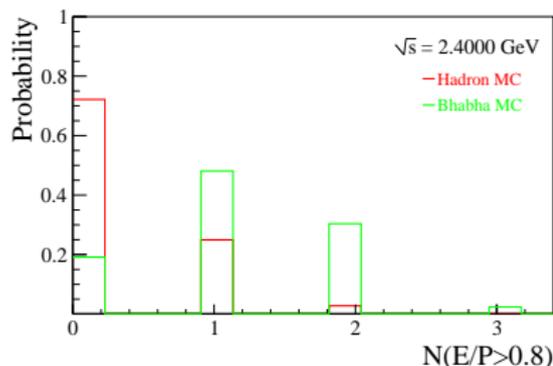
# Select good hadronic events

- If the number of good hadronic tracks = 3:
  - The two highest momentum tracks are required not back-to-back:  $|\Delta\theta| = |\theta_1 + \theta_2 - 180^\circ| < 10^\circ$  or  $|\Delta\phi| = ||\phi_1 - \phi_2| - 180^\circ| < 15^\circ$
  - (number of track with  $E/P > 0.8$ )  $\leq 1$ .
  - (number of track with PID ratio  $> 0.25$ )  $\leq 1$ , where the PID ratio is defined as  $r_{pid} = \text{Prob.}(e) / (\text{Prob.}(p) + \text{Prob.}(K) + \text{Prob.}(\pi) + \text{Prob.}(e))$



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For other multi-prong events, there is no additional selection criterion.

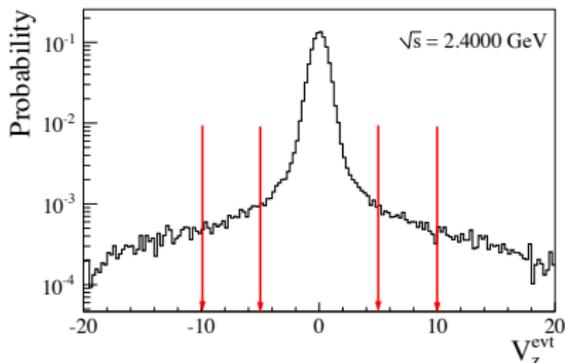
Finally, the number of observed hadron events, i.e.  $N_{\text{had}}^{\text{obs}}$ , will be determined.

# Beam-associated backgrounds: Sideband

The beam-associated background is mainly from **beam-gas** and **beam-wall** reactions.

- ▶ Veto Bhabha and Di-gamma events.
- ▶ Select crude charged hadronic tracks **without** the constraint  $|V_z| < 5.0$  cm ( $N_{\text{crude}}$ ).
- ▶ Calculate the event vertex in z-direction ( $V_z^{\text{evt}}$ ) over all the survived charged tracks.
- ▶ Reserve events with  $|V_z^{\text{evt}}| \in (5, 10)$  cm.
- ▶ The  $N_{\text{good}}$  is then counted only over the tracks satisfy  $|V_z| < 5.0$  cm.
- ▶ Survived events are categorized w.r.t.  $N_{\text{good}}$ , then subsequent criteria are applied.
- ▶ Events survived after above procedure are regarded as beam-associated events.

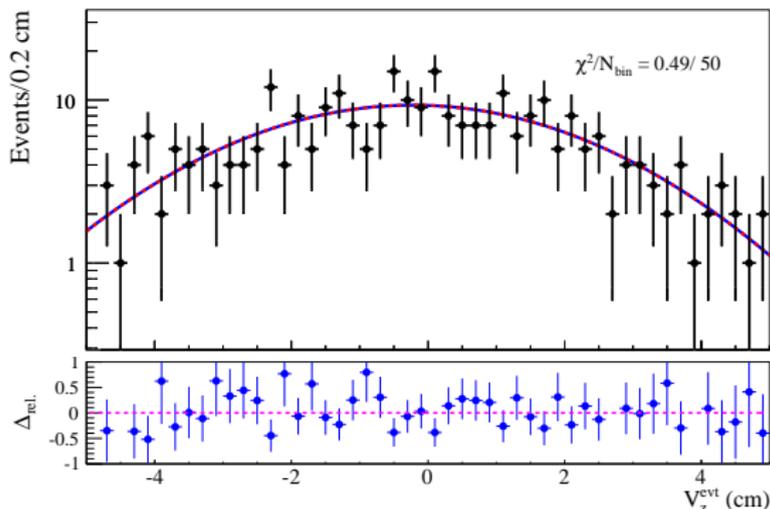
$$V_z^{\text{evt}} = \frac{\sum_1^{N_{\text{crude}}} V_z(i)}{N_{\text{crude}}}$$



# Beam-associated backgrounds: Fitting

Estimate beam-associated backgrounds by fitting  $V_z^{\text{evt}}$  spectra of data:

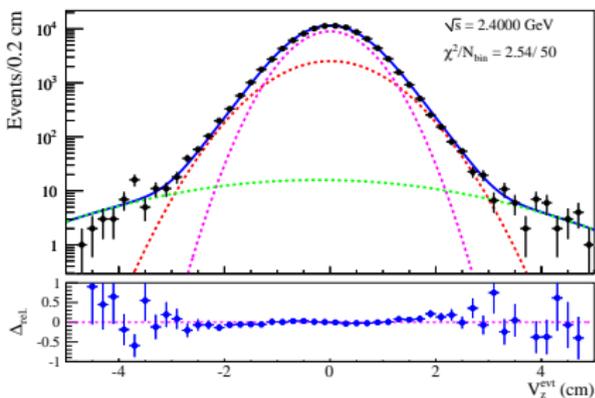
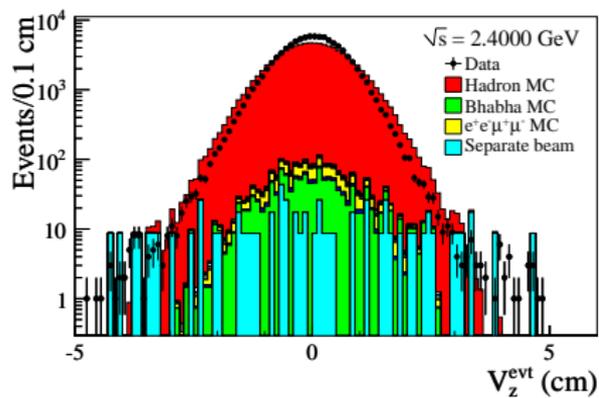
- ▶ Combine separate-beam data at  $\sqrt{s} = 2.4000$  and  $3.4000$  GeV.
- ▶ Fit the  $V_z^{\text{evt}}$  spectrum of combined data via a free Gaussian function.



# Beam-associated backgrounds: Fitting

Estimate beam-associated backgrounds by fitting  $V_z^{\text{evt}}$  spectra of data:

- ▶ QED backgrounds are simulated by dedicated MC generators.
- ▶ They are removed from data after normalizations according to luminosity.
- ▶ The shape of beam-associated background are fixed to the result of previous fit.
- ▶ Signals are described by Gaussian functions (shared mean value).



# Beam-associated backgrounds

Estimated beam-associated backgrounds:

$\sqrt{s}$ (GeV)	separate-beam	sideband method		fitting method		R value
	data	events	ratio (%)	events	ratio (%)	Diff. (%)
2.2324	...	496	0.60	265	0.32	0.28
2.4000	572	624	0.65	478	0.49	0.15
2.8000	...	580	0.69	302	0.36	0.33
3.0500	...	2346	0.83	1201	0.42	0.40
3.0600	...	2394	0.85	1082	0.38	0.46
3.0800	...	4640	0.84	2757	0.50	0.34
3.4000	261	316	0.98	365	1.13	-0.15
3.5000	...	607	0.97	228	0.36	0.60
3.5424	...	1393	0.96	1384	0.95	0.01
3.5538	...	921	0.99	1274	1.37	-0.38
3.5611	...	634	0.98	448	0.69	0.29
3.6002	...	1583	0.99	2156	1.35	-0.36
3.6500	...	711	0.90	212	0.27	0.63
3.6710	...	639	0.85	203	0.27	0.58

- ▶ Separate-beam data are analysed and normalized according to data-taking time.
- ▶ All the three methods are approximate estimations.
- ▶ The estimated background events and their deviations are small comparing to  $N_{\text{had}}^{\text{obs}}$ .

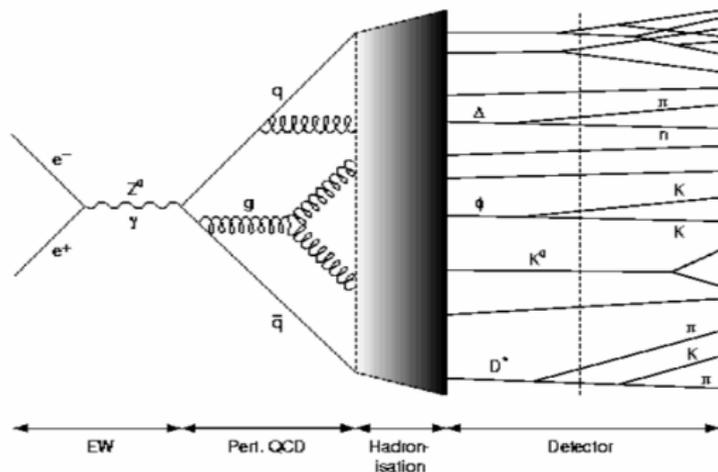
# Estimation of remain QED backgrounds

- ▶ Including **Bhabha**, **Di-gamma**, **Di-tau**, **Di-muon**, and **Two-photon** events.
- ▶ The **Two-photon** events  $e^+e^- \rightarrow e^+e^- + X$ , where  $X = e^+e^-, \mu^+\mu^-, \pi^+\pi^-, K^+K^-, \eta, \eta'$ .
- ▶ A scaling process is applied according to the luminosity of data.
- ▶ Scaled background events which survive above selection procedure are subtracted.

$\sqrt{s}$ (GeV)	beam bkg.	Bhabha	Di-gamma	Di-muon	Di-tau	Two-photon	$N_{\text{bkg}}^{\text{Tot}}$	Bkg. ratio (%)
2.2324	496	847	83	102	—	513	2041	2.45
2.4000	624	899	81	100	—	627	2331	2.41
2.8000	580	698	64	92	—	640	2075	2.48
3.0500	2346	2424	213	330	—	2406	7719	2.72
3.0600	2394	2346	196	335	—	2413	7683	2.72
3.0800	4640	4780	413	719	—	4881	15433	2.79
3.4000	316	220	19	34	—	255	843	2.62
3.5000	607	449	40	68	—	526	1691	2.70
3.5424	1393	1034	83	156	—	1206	3872	2.66
3.5538	921	666	46	99	—	737	2469	2.65
3.5611	634	438	34	68	794	510	2477	3.83
3.6002	1583	1057	86	168	5674	1249	9817	6.15
3.6500	711	532	39	84	4166	636	6168	7.83
3.6710	639	551	43	81	4495	651	6461	8.59

# Luarlw model

Hadronization procedure in Luarlw:



Features of Luarlw:

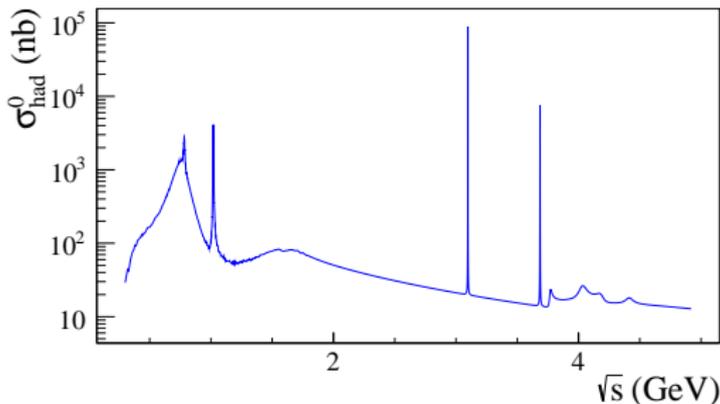
- ▶ Development of **JETSET** for low energy experiments.
- ▶ Simulate production and decay of both **continuum** and **resonance** states.
- ▶ Kinematics of initial hadrons are determined by **Lund Area Law**.
- ▶ **Phenomenological parameters** should be tuned according to experiment data.
- ▶ Integrated the **Initial-state radiation (ISR)** and **Vacuum Polarization (VP)** corrections.

# Input cross section for Luarlw

To correctly simulate **fractions of continuum and resonance states** and **radiation correction**, a line-shape of inclusive hadronic cross section should be input into the generator:

$$\sigma_{\text{had}}^0(s) = \sigma_{\text{con}}^0(s) + \sigma_{\text{res}}^0(s) = \sigma_{\mu\mu}^0(s)R(s) + \sum_i \frac{12\pi\Gamma_{0,i}^{\text{ee}}\Gamma_i^{\text{tot}}}{(s - M_i^2)^2 + (M_i\Gamma_i^{\text{tot}})^2},$$

- ▶  $\sqrt{s} < 1.78 \text{ GeV}$ ,  $\sigma_{\text{had}}^0(s)$  is obtained with  $R$  values cited from PDG summary.
- ▶  $\sqrt{s} > 1.78 \text{ GeV}$ ,  $\sigma_{\text{had}}^0(s)$  is calculated via pQCD and above **BW function**.
- ▶ Interferences between higher charmonium states are considered.



# The VP correction in Luarlw

The VP corrections are split into three parts:

$$\Pi(s) = \Pi_{\text{QED}}(s) + \Pi_{\text{res}}(s) + \Pi_{\text{con}}(s)$$

and

$$\Pi_{\text{QED}}(s) = \frac{\alpha}{\pi} \sum_{\ell=e,\mu,\tau} f(x_\ell)$$

where  $x_\ell = 4m_\ell^2/s$ , and

$$f(x) = \begin{cases} -\frac{5}{9} - \frac{x}{3} + \frac{\sqrt{x-1}(2+x)}{3} \arctan \frac{1}{\sqrt{x-1}} & 1 < x \\ -\frac{5}{9} - \frac{x}{3} + \frac{\sqrt{1-x}(2+x)}{6} \left[ \ln \frac{1 + \sqrt{1-x}}{1 - \sqrt{1-x}} - i\pi \right] & 0 < x < 1 \end{cases}$$

$$\Pi_{\text{res}}(s) = \frac{3s}{\alpha} \sum_r \frac{\Gamma_{0,r}^{\text{ee}}}{M_r} \frac{1}{s - M_r^2 + iM_r\Gamma_r^{\text{tot}}}$$

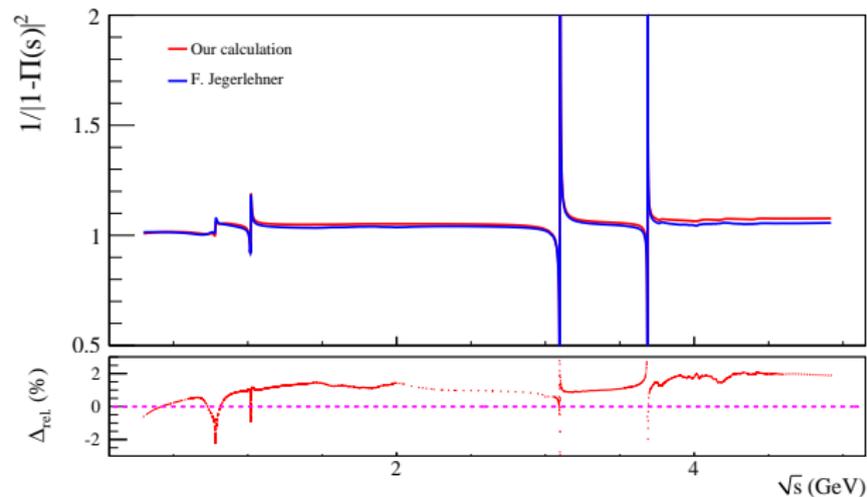
$$\Pi_{\text{con}}(s) = -\frac{\alpha s}{3\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s' - s} \left[ \frac{R_{\text{con}}(s')}{s'} - \frac{R_{\text{con}}(s)}{s} \right]$$

# The VP correction in Luarlw

For simplicity, in Luarlw

$$\Pi_{\text{con}}(s) = \frac{\alpha}{\pi} f_{\text{QCD}} \sum_q N_c Q_q^2 f(x_q)$$

where  $f_{\text{QCD}} \approx 1.2$  and  $q = u, d, s, c, b$ ,  $x_q = 4m_q^2/s$ , where  $m_q$  is the effective mass of quark  $q$ . We use  $m_u = m_d = m_\pi$ ,  $m_s = 0.5 \text{ GeV}/c^2$ ,  $m_c = 1.5 \text{ GeV}/c^2$  while  $m_b = 5.0 \text{ GeV}/c^2$ .



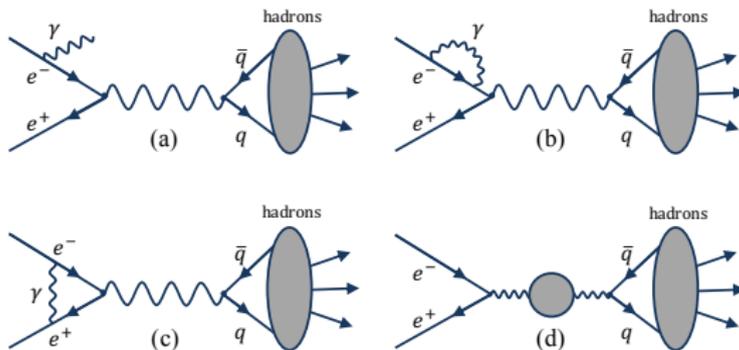
In F. Jegerlehner's calculation:

- ▶  $\Pi_{\text{QED}}(s)$  is calculated up to 3-loops diagrams.
- ▶ For  $0.28 < \sqrt{s} < 40.0 \text{ GeV}$ ,  $R$  values are used when calculating  $\Pi_{\text{con}}(s) + \Pi_{\text{res}}(s)$ .
- ▶ For other energies, pQCD predictions are used.

**The deviations are less than 2%, which will be considered as systematic uncertainty.**

# The ISR correction in Luarlw

In Luarlw, the **Feynman Diagram (FD)** scheme is used to simulate ISR correction and calculate  $(1 + \delta)$ . Feynman diagrams related to ISR procedure are:



The total hadronic cross section measured by experiment is the total effect of all these diagrams:

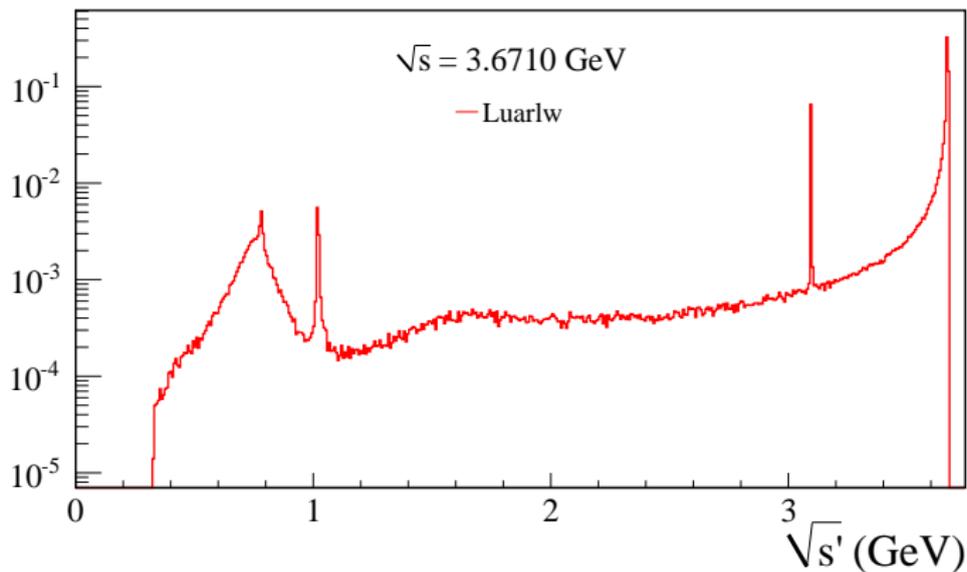
$$\sigma_{\text{had}}^{\text{tot}}(s) = \beta \int_0^{x_m} dx \frac{x^\beta}{x} \left(1 - x + \frac{x^2}{2}\right) \frac{\sigma_{\text{had}}^0(s')}{|1 - \Pi(s')|^2} + \delta_{\text{vert}} \frac{\sigma_{\text{had}}^0(s)}{|1 - \Pi(s)|^2}.$$

Thus

$$1 + \delta \equiv \sigma_{\text{had}}^{\text{tot}} / \sigma_{\text{had}}^0$$

# The ISR correction in Luarlw

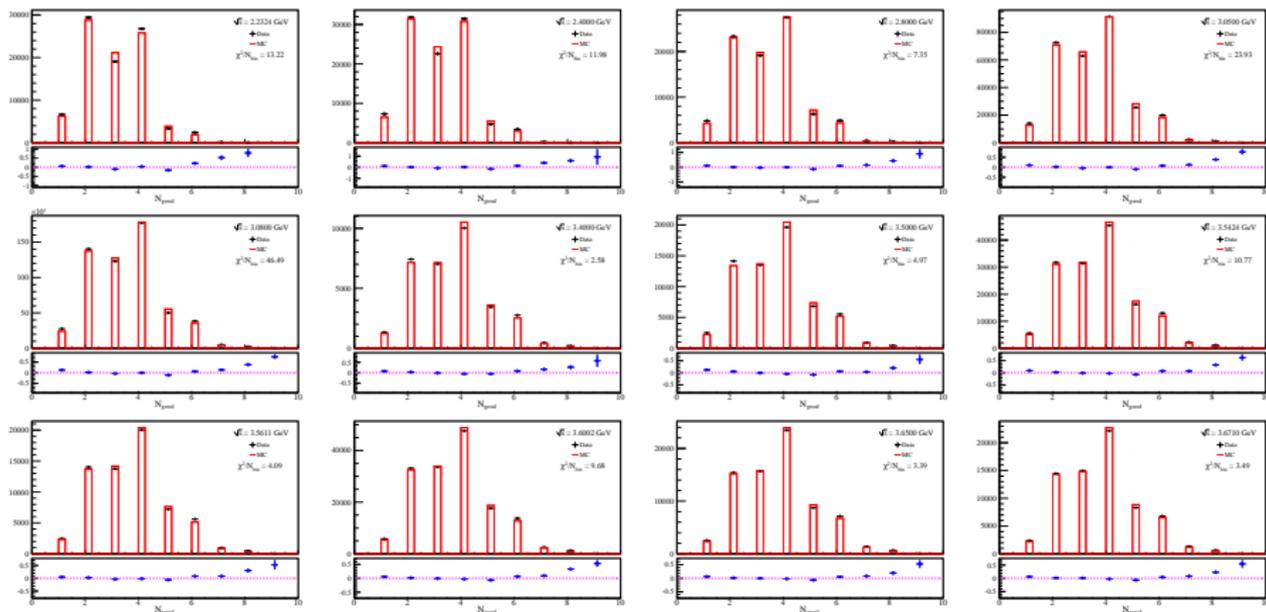
The spectrum of effective energy after ISR correction:



**The resonance states  $\rho$ ,  $\omega$ ,  $\phi$  and  $J/\psi$  can be produced after ISR.**

# Tuning result of the Luarlw model: $N_{\text{good}}$

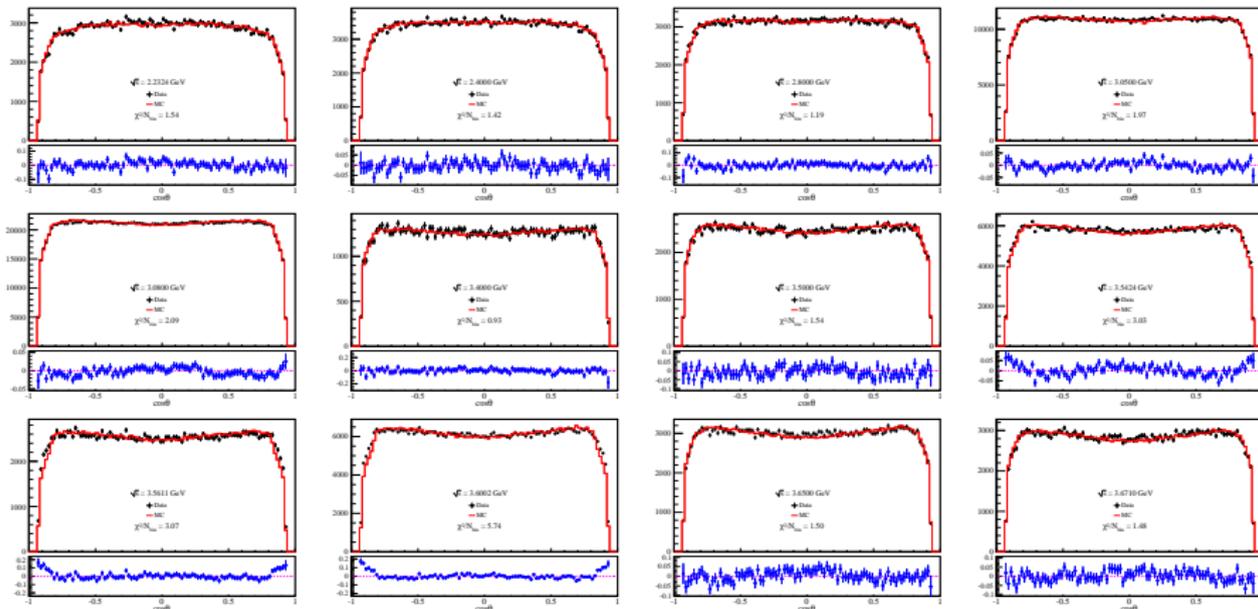
To reliably estimate the detection efficiency of hadronic signal, the Luarlw model should be carefully tuned according to experimental data:



The  $N_{\text{good}}$  is multiplicity of good charged tracks in detector level

# Tuning result of the Luarlw model: $\cos \theta$

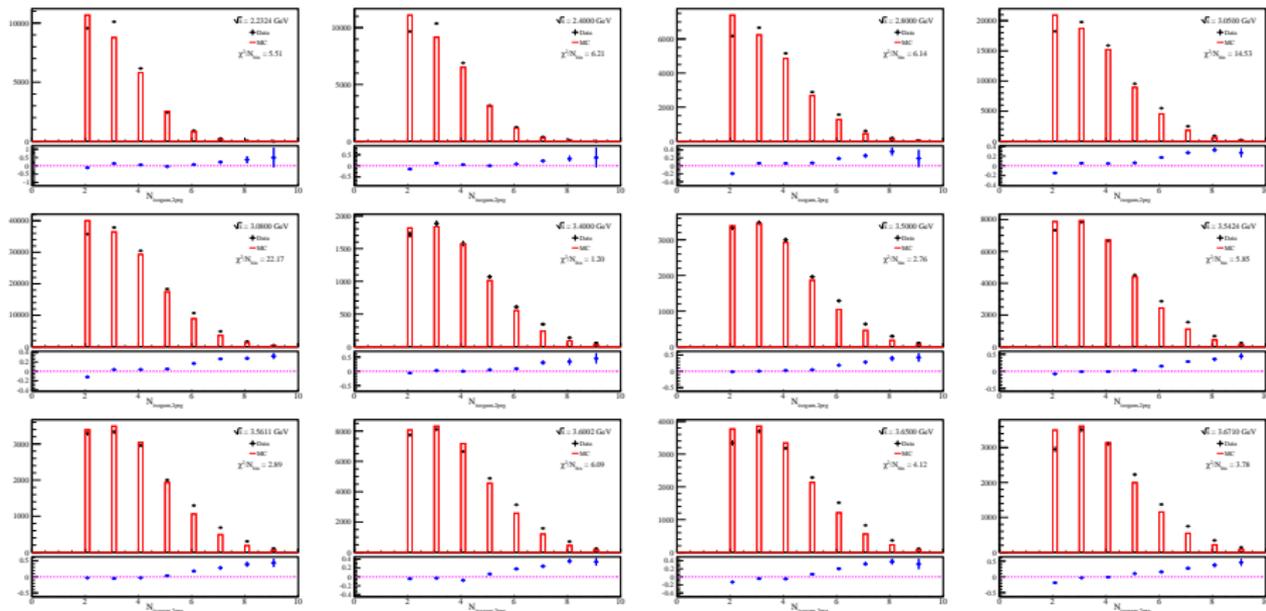
The  $\cos \theta$  is polar angle distribution of charged tracks in detector level



The  $\cos \theta$  will significantly affect the detection efficiency.

# Tuning result of the Luarlw: $N_{\text{isogam},2\text{prg}}$

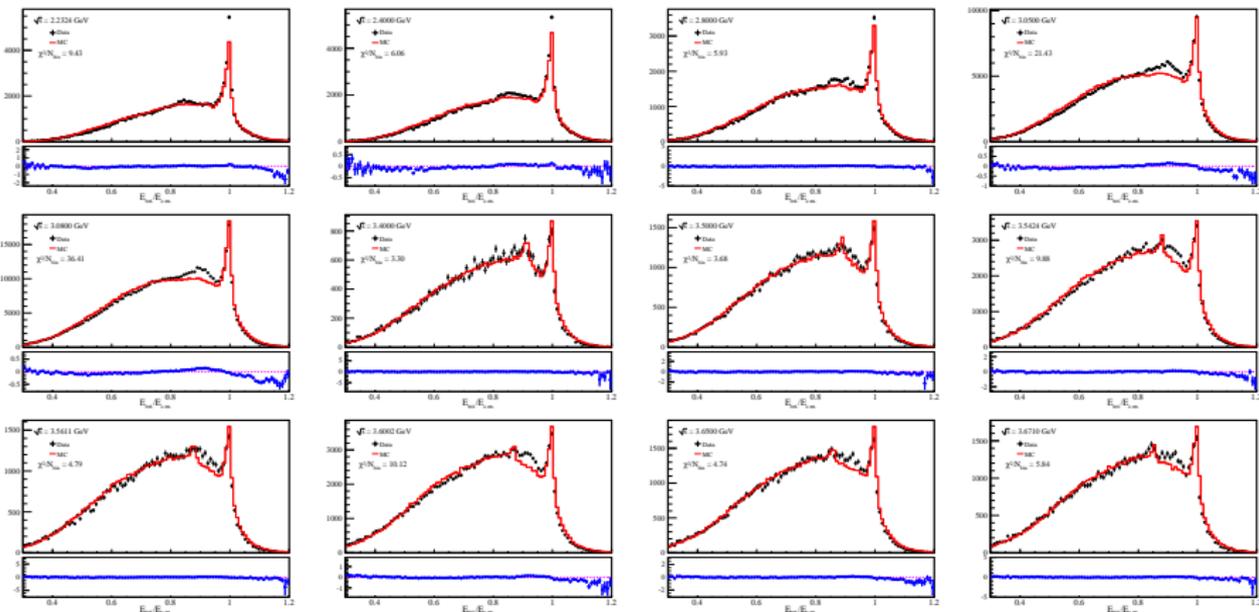
The  $N_{\text{isogam},2\text{prg}}$  is the number of isolated photon in 2-prong events.



The  $N_{\text{isogam},2\text{prg}}$  is crucial to suppress backgrounds in 2-prong events.

# Tuning result of the Luarlw model: $E_{\text{tot}}$ .

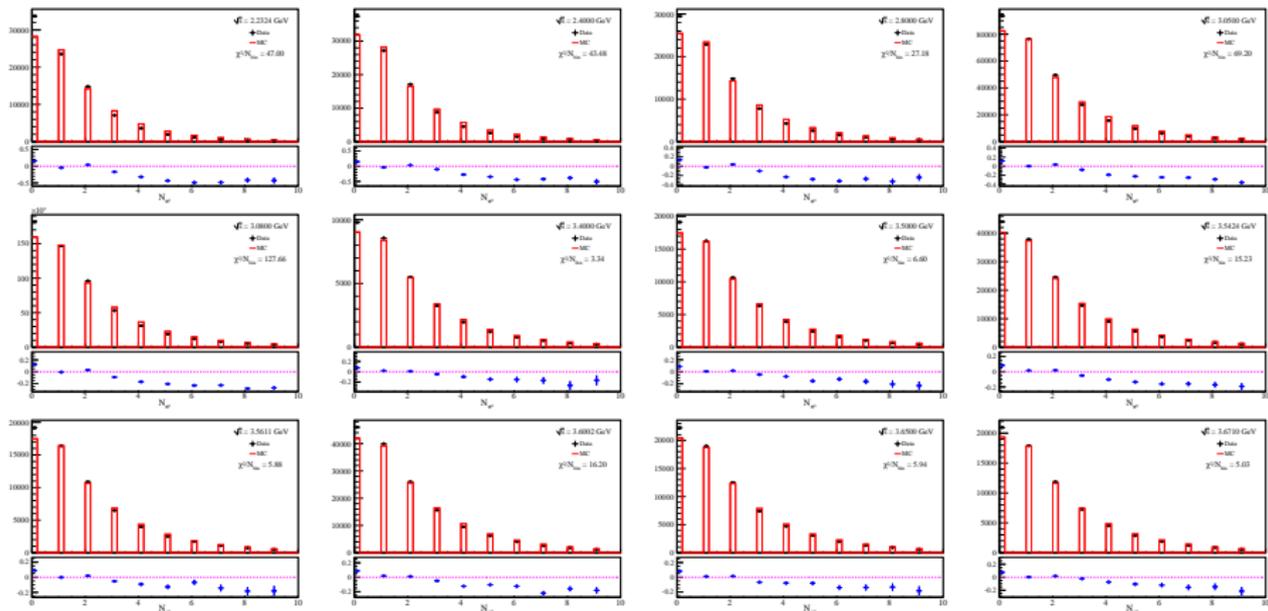
The  $E_{\text{tot}}$  is the total detected energy of each event.



The  $E_{\text{tot}}$  can reflect the reliability of ISR simulation.

# Tuning result of the Luarlw model: $N_{\pi^0}$

The  $N_{\pi^0}$  is the multiplicity of reconstructed  $\pi^0$  in each event.



According to definition of  $R$ , its uncertainty is expressed as

$$\left(\frac{\Delta R}{R}\right)_{\text{sys}}^2 = \left(\frac{\Delta \tilde{N}}{\tilde{N}}\right)^2 + \left(\frac{\Delta \mathcal{L}_{\text{int.}}}{\mathcal{L}_{\text{int.}}}\right)^2 + \left(\frac{\Delta \varepsilon_{\text{had}}}{\varepsilon_{\text{had}}}\right)^2 + \left[\frac{\Delta(1+\delta)}{(1+\delta)}\right]^2 + \left(\frac{\Delta \varepsilon_{\text{trig}}}{\varepsilon_{\text{trig}}}\right)^2,$$

where

$$\tilde{N} = \frac{N_{\text{had}}^{\text{net}}}{\varepsilon_{\text{had}}} = \frac{N_{\text{had}}^{\text{obs}} - N_{\text{bkg}}}{\varepsilon_{\text{had}}}$$

In practice, its uncertainties are addressed in different aspects:

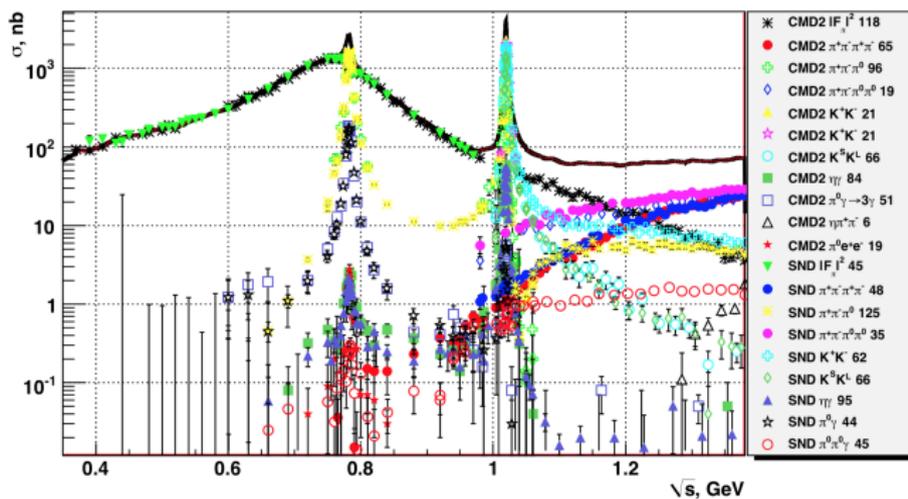
- **Event selection:** all implemented selection criteria are slightly changed.
- **Background estimation:** different methods and background simulation models are used.
- **Integrated luminosity:** uncertainty is directly cited from published results.
- **Signal simulation:** Hybrid model is development and used as a cross check.
- **ISR correction factor:** considered in calculation precision, different  $\sigma_{\text{had}}^0$  and VP schemes.
- **Trigger efficiency:**  $\varepsilon_{\text{trig}}$  approaches to 100% with an uncertainty less than 0.1%.

# Hybrid model

The Hybrid model is developed as an alternative simulation model of hadronic events.

Hybrid model is consisted of **three** components:

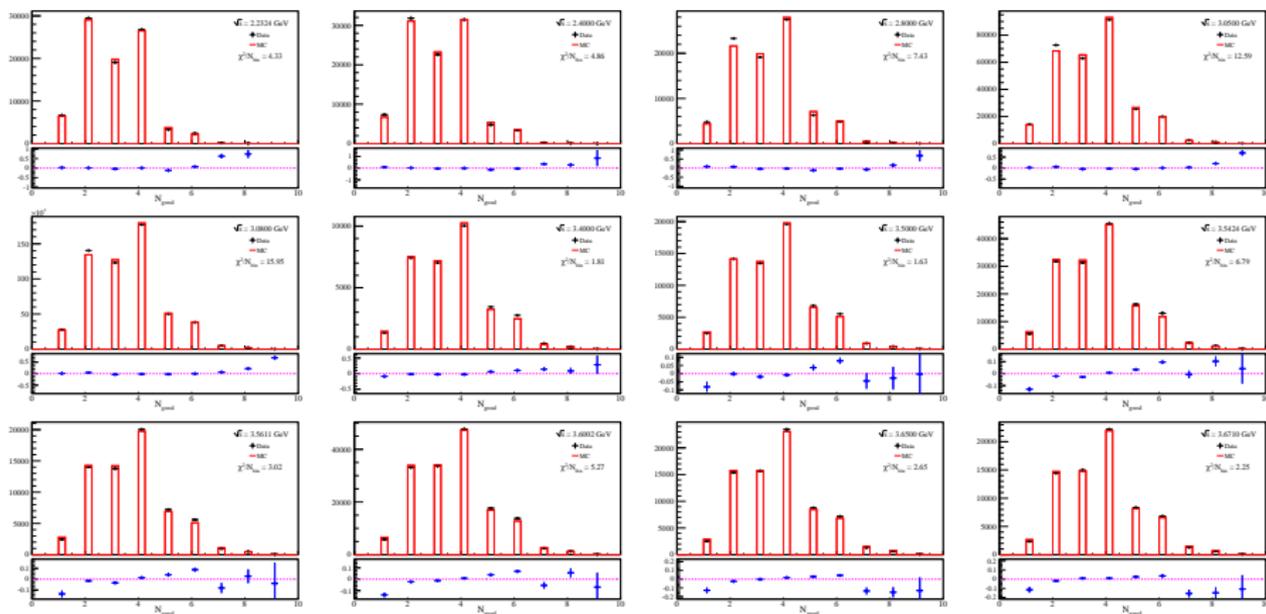
- **Phokhara**: describes 10 well known procedures such as  $e^+e^- \rightarrow 2\pi, 3\pi, 4\pi$  etc..
- **ConExc**: simulates exclusive procedures which are established at least in cross section.
- **Luarlw**: responsible for remain unknown procedures, need further tuning.



# Tuning result of the Hybrid model: $N_{\text{good}}$

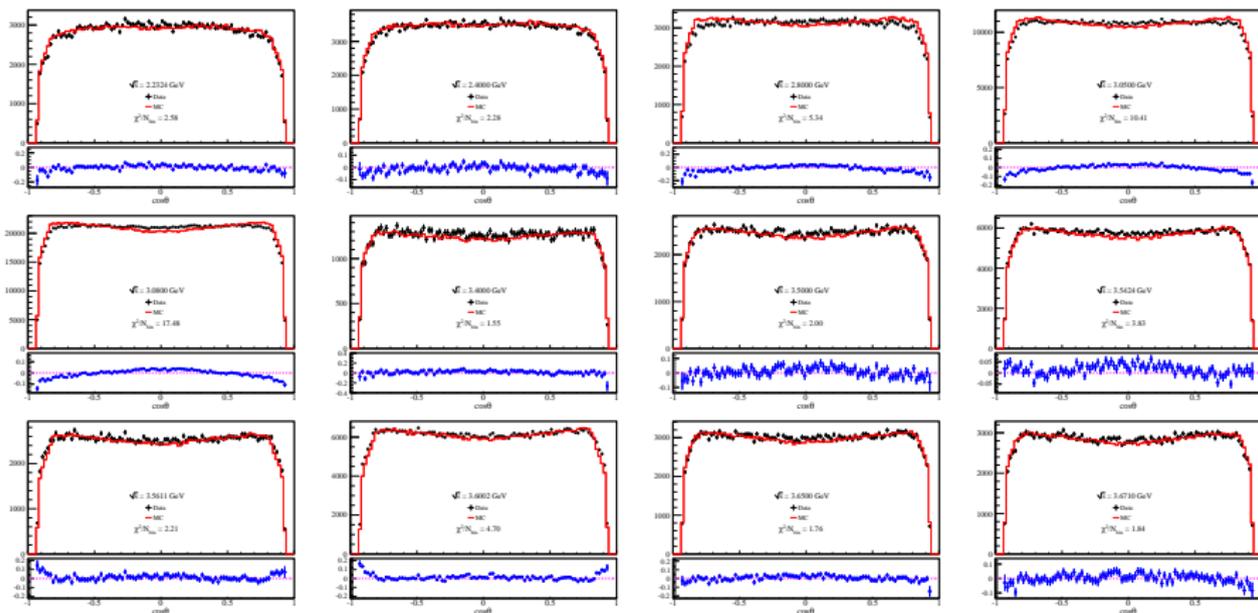
Phenomenology parameters of Hybrid model is tuned at  $\sqrt{s} = 3.08 \text{ GeV}$  and applied at all the energies:

The  $N_{\text{good}}$  is multiplicity of charged tracks in detector level



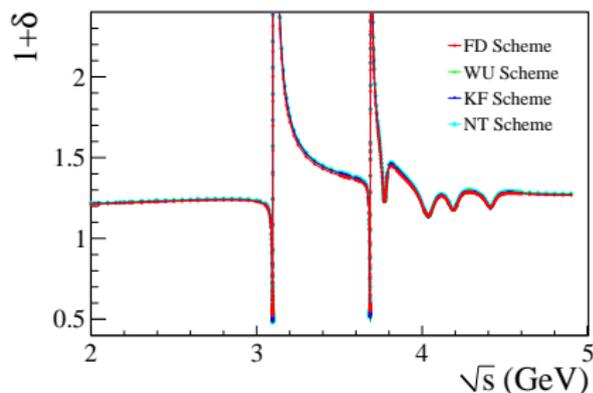
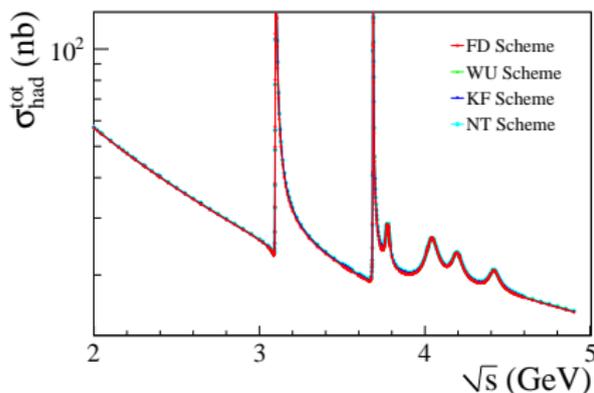
# Tuning result of the Hybrid model: $\cos \theta$

The  $\cos \theta$  is polar angle distribution of charged tracks in detector level



# Calculated ISR factors

ISR factors are calculated by FD scheme and cross checked by **structure function (SF)** scheme:



- ▶ Same input hadronic cross section  $\sigma_{\text{had}}^0$ .
- ▶ Same VP correction scheme  $1/|1 - \Pi(s)|^2$ .
- ▶ The NT scheme is integrated in Hybrid model.

# Summary and outlook

- Measurement of  $R$  values is systematically established at BESIII.
- Simulation of inclusive hadronic events is of great importance in  $R$  measurement.
- The Lualw and Hybrid models need further understanding and fine tuning.
- The  $R$  values will be published after the signal simulation models are justified.

**Thanks for your attention!**