- Two body decay

$$a(J_a,\eta_a) \rightarrow b(J_b,\eta_b) + c(J_c,\eta_c)$$

$$A^{J_a}_{\lambda_b,\lambda_c}(\theta,\phi;M) = N_{J_a} \boxed{F^{J_a}_{\lambda_b,\lambda_c}} D^{J_a *}_{M,\lambda}(\phi,\theta,0), (\lambda = \lambda_b - \lambda c)$$

$F^{J}_{\lambda_b,\lambda_a}$ is helicity decay amplitude

$$F^{J_a}_{\lambda_b,\lambda_c} = \sum_{ls}\left(\frac{2l+1}{2J_a+1}\right)^{1/2} < l0s\lambda|J_a\lambda >< s_b\lambda_b s_c - \lambda_c|s\lambda > G^{J_a}_{ls} r^l B_l(r)$$

$$F^{J}_{\lambda_1\lambda_2} = \eta\eta_1\eta_2(-)^{J-s_1-s_2} F^{J}_{-\lambda_1-\lambda_2}$$

$$G^{J}_{ls} = 4\pi\left(\frac{w}{p}\right)^{\frac{1}{2}} \langle JMls|\mathcal{M}|JM\rangle$$

- Sequential decays: $Y \rightarrow \pi^+\pi^- J/\psi,\ J/\psi \rightarrow l^+l^-$

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y,\Delta\lambda_l}\left|\sum_{\lambda_{R_i}} A_{R_i}\right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l}\left|\sum_{\lambda_{R_i},\lambda_\psi} \boxed{F^J_1 D^J_1}\cdot\boxed{F^J_2 D^J_2}\cdot\boxed{F^J_3 D^J_3}\right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l}\left|\sum_{\lambda_{R_i},\lambda_\psi}\left(\sum_{LS} G_{LS}\cdot CG_1\cdot B_l\right)D^J_1\cdot\left(\sum_{ls} G_{ls}\cdot CG_2\cdot B_l\right)D^J_2\cdot F^J_3 D^J_3\right|^2$$

$$\Longrightarrow \sum_{\lambda_Y,\Delta\lambda_l}\left|\sum_{\lambda_{R_i},\lambda_\psi}(G_{LS}\cdot CG_1\cdot B_l)D^J_1\cdot(G_{ls}\cdot CG_2\cdot B_l)D^J_2\cdot F^J_3 D^J_3\right|^2$$

- $G_{LS}$ is LS coupling partial wave amplitude
- With a definite set of helicity of (b,c), $G_{LS}$ should be same
- In fit, $G_{LS}$ is float parameter
- To obtain the contribution of a LS wave component

For the last step $J/\psi \rightarrow \ell^+\ell^-$, at the relativistic limit, by QED calculation, $F^{J_{J/\psi}}_{1/2,1/2} = F^{J_{J/\psi}}_{-1/2.-1/2} \approx 0$. Here we define $\Delta\lambda_\ell = \lambda_{\ell+} - \lambda_{\ell-}$, we can see only $\Delta\lambda_\ell = \pm 1$ is allowed.

$$\sum_{\lambda_Y,\Delta\lambda_l} | \sum_{\lambda_{R_i},\lambda_\psi} (G_{LS}\cdot CG_1\cdot B_l)D_1^J \cdot (G_{ls}\cdot CG_2\cdot B_l)D_2^J \cdot F_3^J D_3^J|^2$$

$$Decay: Y \to \psi \quad f_0 \qquad f_0 \to \pi^+ \quad \pi^-$$

$$J^{PC}: 1^{--} \to 1^{--} \quad 0^+ \qquad 0^+ \to 0^- \quad 0^-$$

Two components: $SS$ and $DS$

$L=0(S-wave)$ $\quad F^1_{1,0} = +g_{01}\sqrt{\frac{1}{3}}r^0 + g_{21}\sqrt{\frac{1}{6}}r^2$ $\quad S-wave$

$L=2(D-wave)$ $\quad F^1_{0,0} = +g_{01}\sqrt{\frac{1}{3}}r^0\gamma_s - g_{21}\sqrt{\frac{2}{3}}r^2\gamma_s$

$$Decay: Y \to Z_c \quad \pi \qquad Z_c \to \psi \quad \pi$$

$$J^{PC}: 1^{--} \to 1^+ \quad 0^- \qquad 1^+ \to 1^{--}0^-$$

Two components:
$SS$, $SD$, $DS$ and $DD$

$L=0(S-wave)$ $\qquad L=0(S-wave)$

$L=2(D-wave)$ $\qquad L=2(D-wave)$

$$F^1_{1,0} = +g_{01}\sqrt{\frac{1}{3}}r^0 + g_{21}\sqrt{\frac{1}{6}}r^2 \qquad F^1_{1,0} = +g_{01}\sqrt{\frac{1}{3}}r^0 + g_{21}\sqrt{\frac{1}{6}}r^2$$

$$F^1_{0,0} = +g_{01}\sqrt{\frac{1}{3}}r^0\gamma_s - g_{21}\sqrt{\frac{2}{3}}r^2\gamma_s \qquad F^1_{0,0} = +g_{01}\sqrt{\frac{1}{3}}r^0\gamma_s - g_{21}\sqrt{\frac{2}{3}}r^2\gamma_s$$

$$A = \phi_\mu(m_1)\omega_\nu^*(m_2)A^{\mu\nu} = \phi_\mu(m_1)\omega_\nu^*(m_2)\sum_i \Lambda_i U_i^{\mu\nu}$$

$$U^{\mu\nu} = (A_{LS})(A_{ls})$$

- $U^{\mu\nu}$ is the partial wave amplitude constructed according to LS coupling

$$\frac{d\sigma}{d\Phi_n} \propto -\frac{1}{2}\sum_{\mu=1}^{2} \tilde{g}_{\nu\nu'}(p_{(\psi)})A^{\mu\nu}A^{*\mu\nu'}$$

$$= -\frac{1}{2}\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2} U_i^{\mu\nu}\tilde{g}_{\nu\nu'}(p_{(\psi)})U_j^{*\mu\nu'}$$

$$\begin{array}{lllllll} Decay: Y & \to\psi & f_0 & & f_0 & \to\pi^+ & \pi^- \\ J^{PC}: 1^{--} & \to 1^{--} & 0^+ & & 0^+ & \to 0^- & 0^- \end{array}$$

$$\begin{array}{lllllll} Decay: Y & \to Z_c & \pi & & Z_c \to \psi & \pi \\ J^{PC}: 1^{--} & \to 1^+ & 0^- & & 1^+ \to 1^{--} & 0^- \end{array}$$

$$U^{\mu\nu}_{(Y\to\psi(2S)f_0)SS} = \langle\psi f_0|01\rangle = g^{\mu\nu}f^{(f_0)}_{(12)}$$

$$U^{\mu\nu}_{(Y\to\psi(2S)f_0)DS} = \langle\psi f_0|21\rangle = \tilde{T}^{(2)\mu\nu}_{(\psi f_0)}f^{(f_0)}_{(12)}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)SS} = \tilde{g}^{\mu\nu}_{(Z_c^+)}f^{(Z_c^+)}_{(01)} + \tilde{g}^{\mu\nu}_{(Z_c^-)}f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)SD} = \tilde{t}^{(2)\mu\nu}_{(\psi\pi^+)}f^{(Z_c^+)}_{(01)} + \tilde{t}^{(2)\mu\nu}_{(\psi\pi^-)}f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)DS} = \tilde{T}^{(2)\mu\lambda}_{(Z_c^+\pi^-)}\tilde{g}_{(Z_c^+)\lambda\sigma}g^{\sigma\nu}f^{(Z_c^+)}_{(01)} + \tilde{T}^{(2)\mu\lambda}_{(Z_c^-\pi^+)}\tilde{g}_{(Z_c^-)\lambda\sigma}g^{\sigma\nu}f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)DD} = \tilde{T}^{(2)\mu\lambda}_{(Z_c^+\pi^-)}\tilde{t}^{(2)}_{(\psi\pi^+)\lambda\sigma}g^{\sigma\nu}f^{(Z_c^+)}_{(01)} + \tilde{T}^{(2)\mu\lambda}_{(Z_c^-\pi^+)}\tilde{t}^{(2)}_{(\psi\pi^-)\lambda\sigma}g^{\sigma\nu}f^{(Z_c^-)}_{(02)}$$

$$A=\phi_\mu(m_1)\omega^*_\nu(m_2)A^{\mu\nu}=\phi_\mu(m_1)\omega^*_\nu(m_2)\sum_i\Lambda_iU_i^{\mu\nu}$$

$$\mathcal{B}=ie\omega_\beta(m_2)\bar{u}_{e^-}\gamma^\beta\nu_{e^+}\frac{em_\psi}{f_\psi}$$

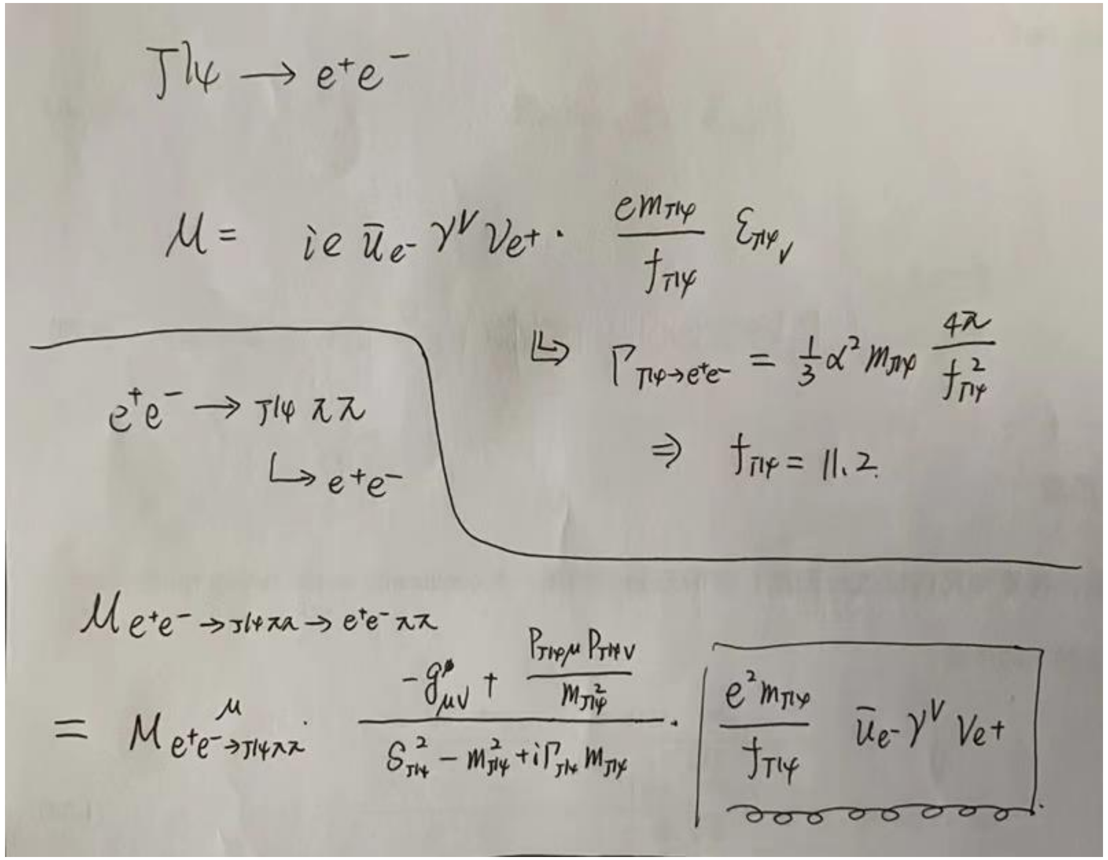
- $\omega_\beta(m_2)$是$\psi$的极化矢量
- $f_\psi=11.2$是常数
- $\bar{u}_{e^-}(\nu_{e^+})$是电子的极化矢量

$$\frac{d\sigma}{d\Phi_n}\propto-\frac{1}{2}\sum_{\mu=1}^{2}\tilde{g}_{\nu\nu'}(p_{(\psi)})A^{\mu\nu}A^{*\mu\nu'}$$

$$=-\frac{1}{2}\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2}U_i^{\mu\nu}\boxed{\tilde{g}_{\nu\nu'}}(p_{(\psi)})U_j^{*\mu\nu'}$$

$$\frac{d\sigma}{d\Phi_n}\propto 2|ie\frac{em_\psi}{f_\psi}|^2\sum_{\mu=1}^{2}\tilde{g}^{\psi\to ll}_{\nu\nu'}A^{\mu\nu}A^{*\mu\nu'}$$

$$=2|ie\frac{em_\psi}{f_\psi}|^2\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2}U_i^{\mu\nu}\boxed{\tilde{g}^{\psi\to ll}_{\nu\nu'}}U_j^{*\mu\nu'}$$




$$\sum_{m_2=1}^{3}\omega_\nu(m_2)\omega^*_{\nu'}(m_2)=-g_{\nu\nu'}+\frac{p_{(\psi)\nu}p_{(\psi)\nu'}}{p_\psi^2}\equiv-\tilde{g}_{\nu\nu'}(p_{(\psi)})$$

$$\tilde{g}^{\psi\to ll}_{\nu\nu'}=\tilde{g}_{\nu\beta}(p_{(\psi)})\tilde{g}_{\nu'\beta'}(p_{(\psi)})\left[p^\beta p'^{\beta'}+p'^\beta p^{\beta'}-g^{\beta\beta'}\left(p\cdot p'+m_l^2\right)\right]$$

- **F0(500): MC sample contains only DS component**

Without $J/\psi \to l^+l^-$ in covariant tensor formalism

```
Cov: SS wave fraction = 0.588728      Hel: SS wave fraction = 0.00091914
Cov: DS wave fraction = 0.411272      Hel: DS wave fraction = 0.999081
```

With $J/\psi \to l^+l^-$ in covariant tensor formalism

```
Cov: SS wave fraction = 0.0535918     Hel: SS wave fraction = 0.00091914
Cov: DS wave fraction = 0.946408      Hel: DS wave fraction = 0.999081
```

- **Zc3900: MC sample contains only SS component**

Without $J/\psi \to l^+l^-$ in covariant tensor formalism

```
Cov: SS wave fraction = 0.292975      Hel: SS wave fraction = 0.905202
Cov: SD wave fraction = 0.500984      Hel: SD wave fraction = 0.00800091
Cov: DS wave fraction = 0.0744949     Hel: DS wave fraction = 0.0859795
Cov: DD wave fraction = 0.131546      Hel: DD wave fraction = 0.000817607
```

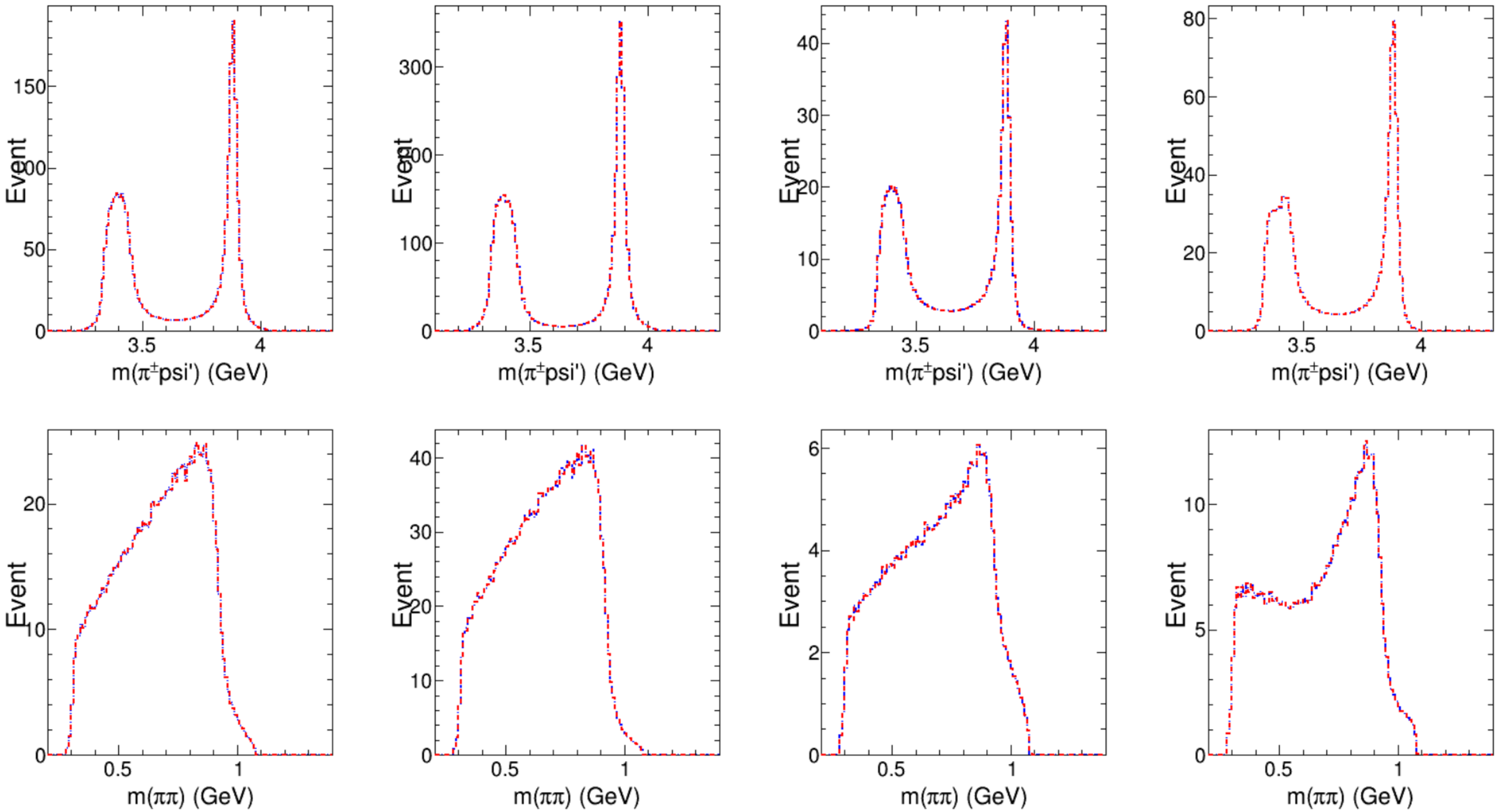
With $J/\psi \to l^+l^-$ in covariant tensor formalism

```
Cov: SS wave fraction = 0.930268      Hel: SS wave fraction = 0.905202
Cov: SD wave fraction = 0.00704404    Hel: SD wave fraction = 0.00800091
Cov: DS wave fraction = 0.062201      Hel: DS wave fraction = 0.0859795
Cov: DD wave fraction = 0.000487272   Hel: DD wave fraction = 0.000817607
```

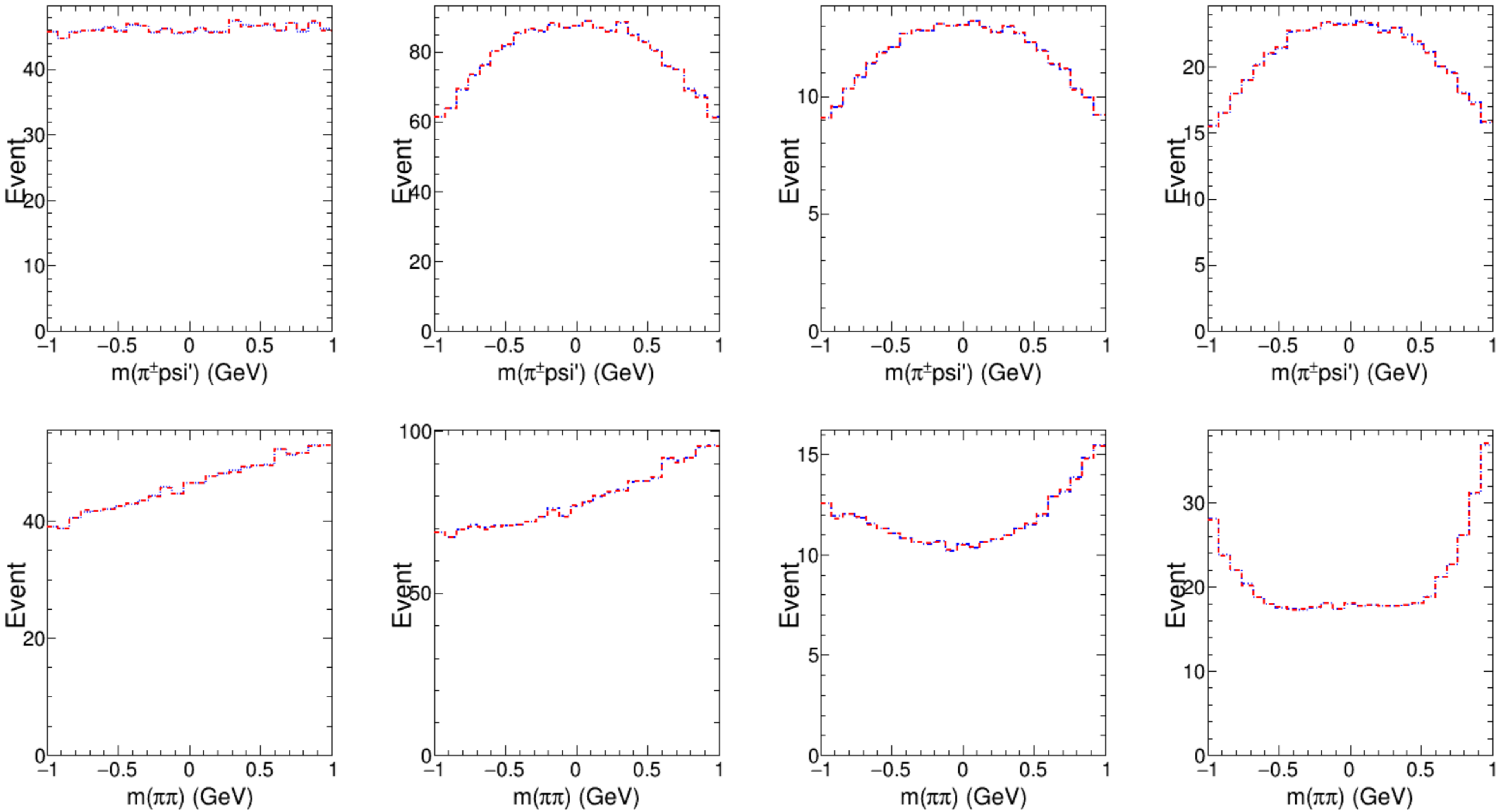
The comparison between cases with or without $J/\psi \to l^+l^-$ in covariant tensor formalism

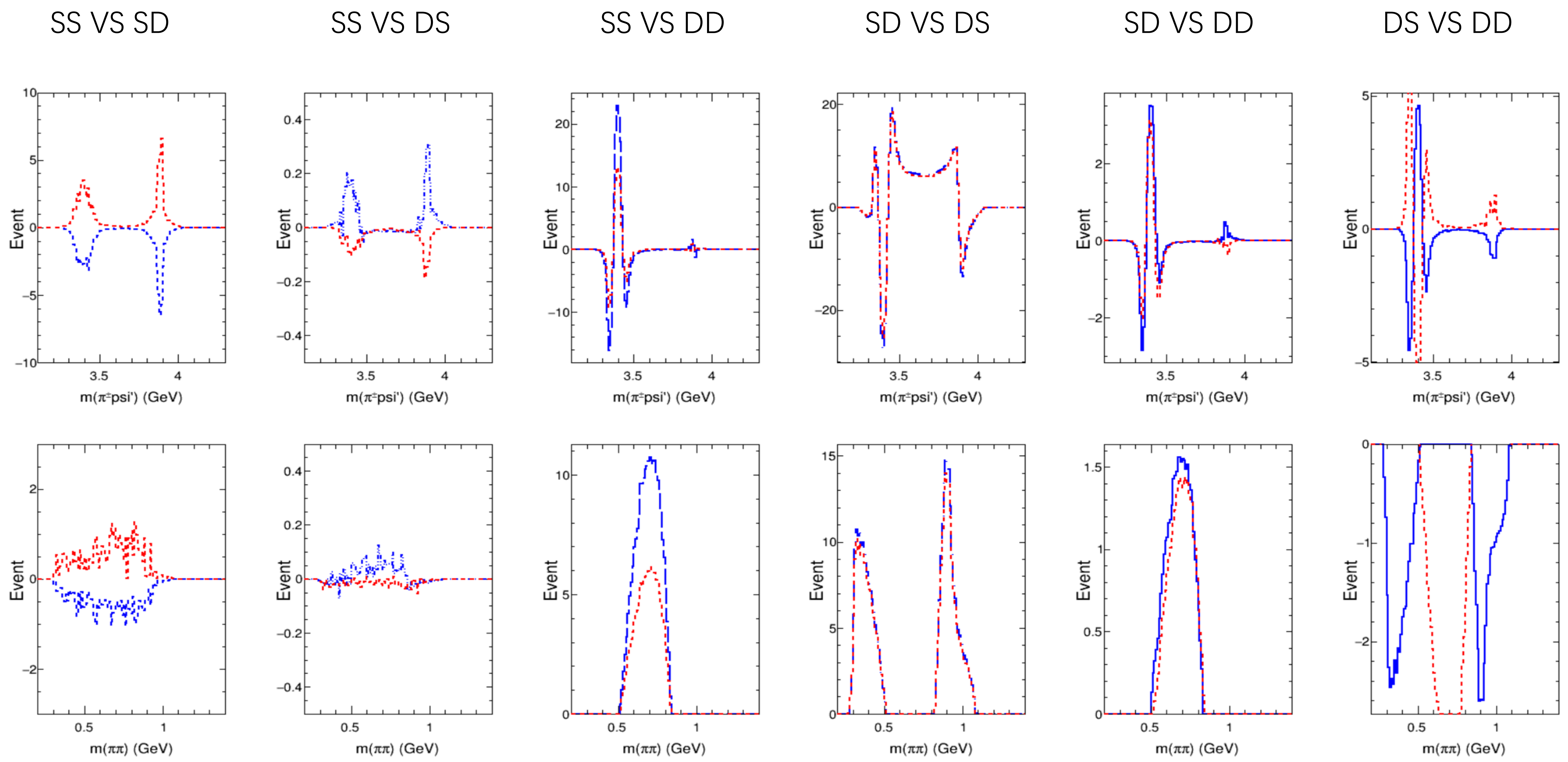
Red: with $J/\psi \to l^+l^-$

Blue: without $J/\psi \to l^+l^-$

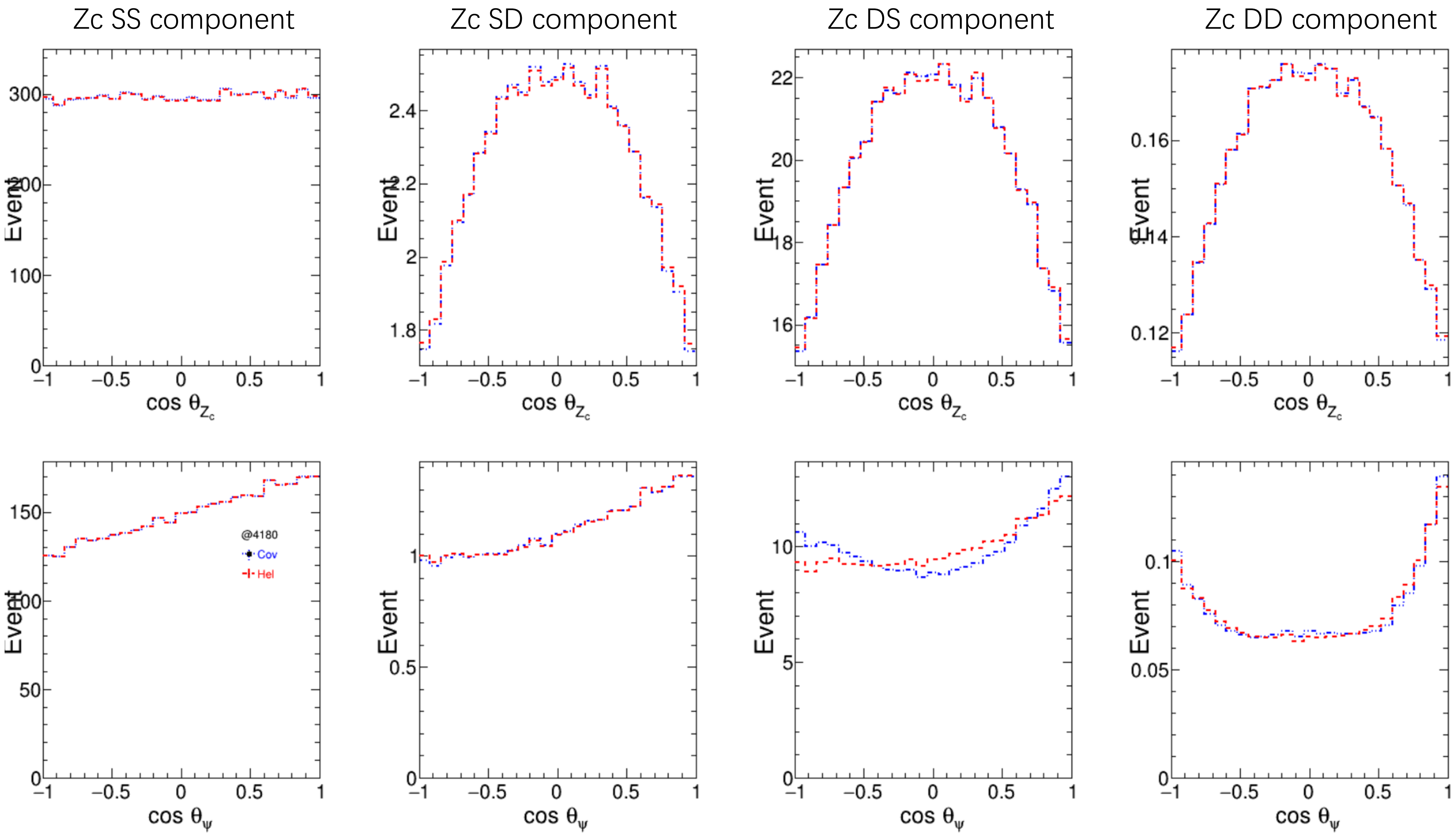


The comparison between cases with or without $J/\psi \rightarrow l^+l^-$ in covariant tensor formalism

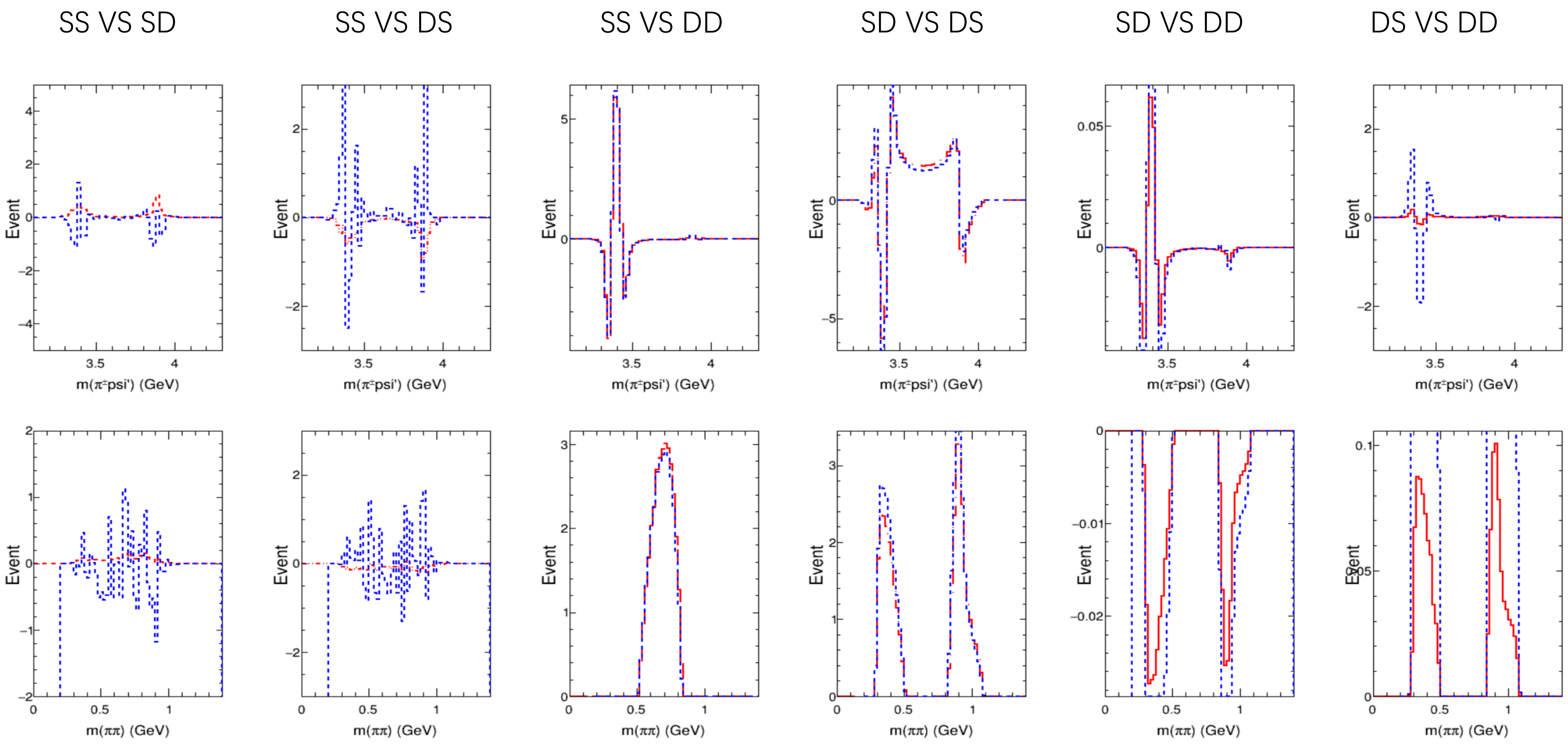




SS VS SD
SS VS DS
SS VS DD
SD VS DS
SD VS DD
DS VS DD
Event
m(π±psi') (GeV)
m(ππ) (GeV)



Zc SS component
Zc SD component
Zc DS component
Zc DD component
Event
cos θ_Zc
cos θ_ψ
@4180
Cov
Hel


SS VS SD
SS VS DS
SS VS DD
SD VS DS
SD VS DD
DS VS DD
Event
m(π±psi') (GeV)
m(ππ) (GeV)

- Sequential decays: $Y \to \pi^+\pi^- J/\psi,\ J/\psi \to l^+l^-$
  - Assume leptons have 0 spin

$$\psi \to \pi^+\pi^-\ ,\ K^+K^-$$

这个过程是一个$1^- \to 0^-0^-$的过程，因为$\eta_a\eta_b\eta_c = -1$ 和角动量守恒，因此只有1个分波：P-wave，所以独立的Helicity振幅也只有一个。即$F = F^1_{00}$，不变振幅：

$$\mathcal{M}(M) = F D^{1*}_{M,0}(\phi,\theta,0)\ ,$$

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y,\Delta\lambda_l}\left|\sum_{\lambda_{R_i}} A_{R_i}\right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l}\left|\sum_{\lambda_{R_i},\lambda_\psi} \boxed{F_1^J D_1^J}\cdot\boxed{F_2^J D_2^J}\cdot\boxed{F_3^J D_3^J}\right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l}\left|\sum_{\lambda_{R_i},\lambda_\psi} (\sum_{LS} G_{LS}\cdot CG_1\cdot B_l)D_1^J\cdot(\sum_{ls} G_{ls}\cdot CG_2\cdot B_l)D_2^J\cdot F_3^J D_3^J\right|^2$$

- Come from the boost of frame
- $\Delta\lambda_l = 0$

$$\frac{d\sigma}{d\Phi} = \sum_{\lambda_Y,\Delta\lambda_l} |\sum_{\lambda_{Z_c},\lambda_{R_j},\lambda_{J/\psi}} [A_{R_j}(\lambda_Y,\lambda_{R_j},\lambda_{J/\psi},\lambda_{l^+},\lambda_{l^-}) + \boxed{e^{i\Delta\lambda_l\alpha_l}} A_{Z_c^0}(\lambda_Y,\lambda_{Z_c^0},\lambda_{J/\psi},\lambda_{l^+},\lambda_{l^-})]\ |^2$$

In covariant tensor formalism, just like:

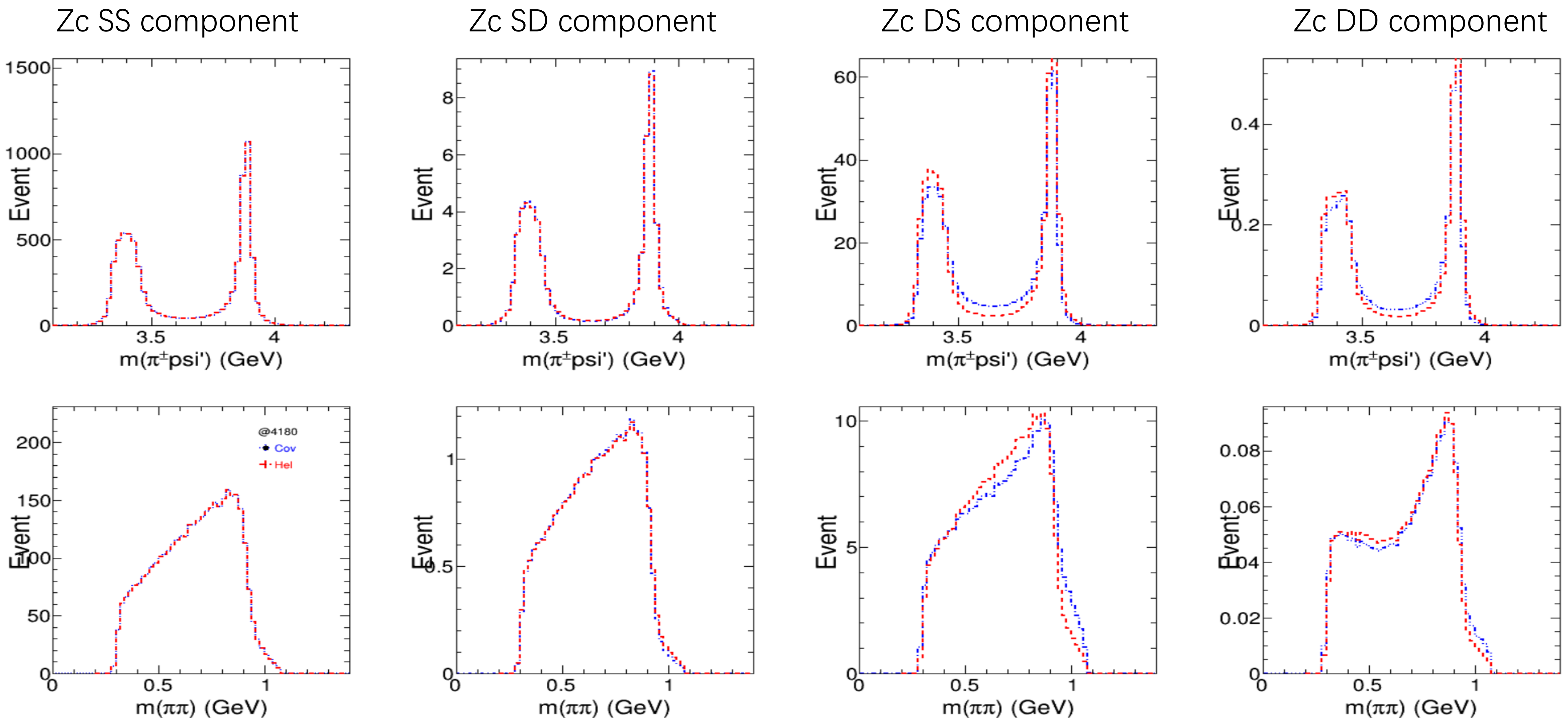
$$\psi \to \phi\pi^+\pi^- \to \mathrm{K}^+\mathrm{K}^-\pi^+\pi^-$$

$$U^\mu_{b_1 SS} = \tilde{g}^{\mu\nu}_{(123)}\tilde{t}^{(1)}_{(12)\nu} f^{(\phi)}_{(12)} f^{(b_1)}_{(123)} + \tilde{g}^{\mu\nu}_{(124)}\tilde{t}^{(1)}_{(12)\nu} f^{(\phi)}_{(12)} f^{(b_1)}_{(124)},$$

$$U^\mu_{b_1 SD} = \tilde{t}^{(2)\mu\nu}_{(\phi 3)}\tilde{t}^{(1)}_{(12)\nu} f^{(\phi)}_{(12)} f^{(b_1)}_{(123)} + \tilde{t}^{(2)\mu\nu}_{(\phi 4)}\tilde{t}^{(1)}_{(12)\nu} f^{(\phi)}_{(12)} f^{(b_1)}_{(124)},$$

$$U^\mu_{b_1 DS} = \tilde{T}^{(2)\mu\lambda}_{(b_1 4)}\tilde{g}_{(123)\lambda\nu}\tilde{t}^{(1)\nu}_{(12)} f^{(\phi)}_{(12)} f^{(b_1)}_{(123)} + \tilde{T}^{(2)\mu\lambda}_{(b_1 3)}\tilde{g}_{(124)\lambda\nu}\tilde{t}^{(1)\nu}_{(12)} f^{(\phi)}_{(12)} f^{(b_1)}_{(124)},$$

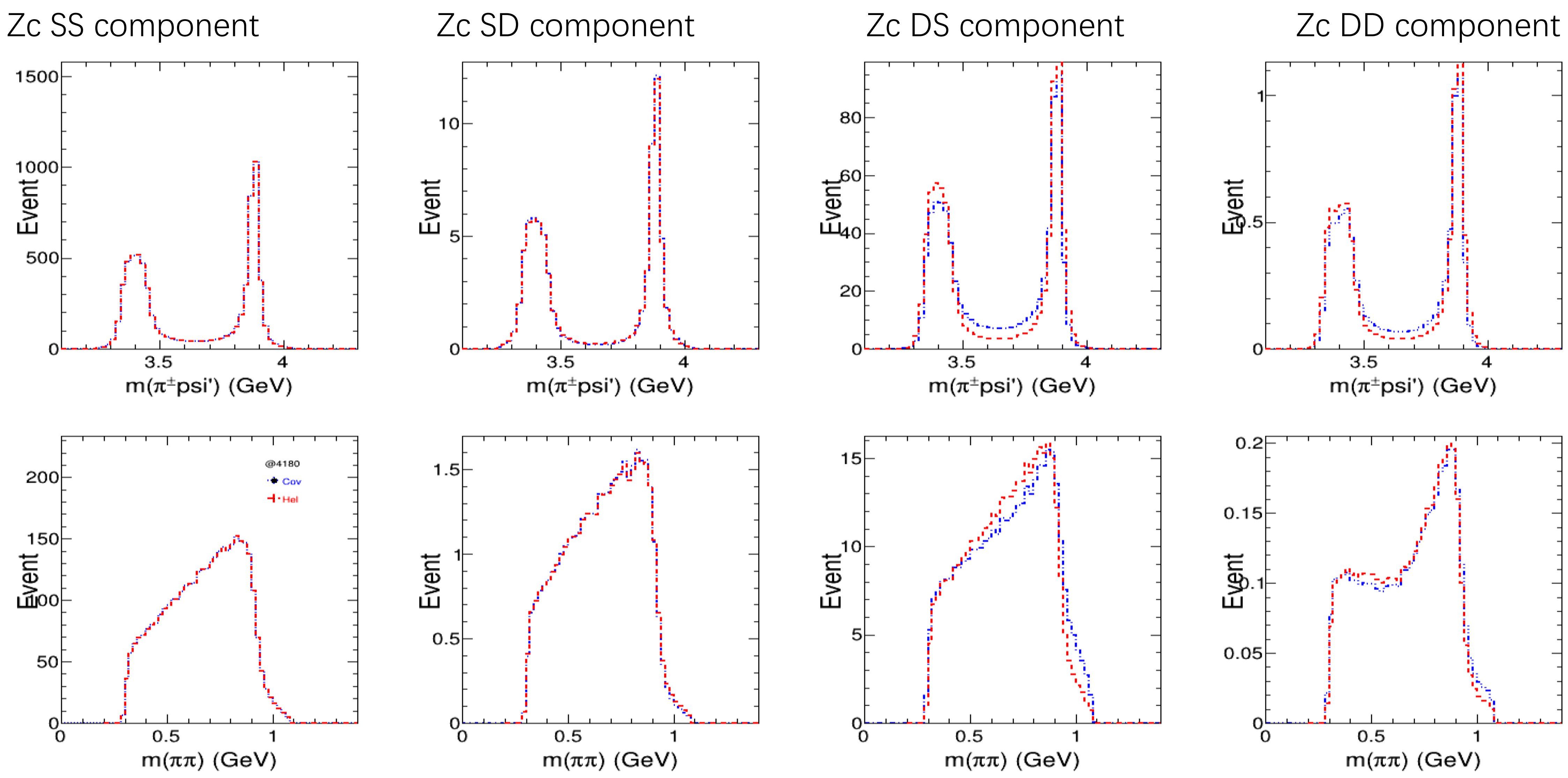
$$U^\mu_{b_1 DD} = \tilde{T}^{(2)\mu\lambda}_{(b_1 4)}\tilde{t}^{(2)}_{(\phi 3)\lambda\nu}\tilde{t}^{(1)\nu}_{(12)} f^{(\phi)}_{(12)} f^{(b_1)}_{(123)} + \tilde{T}^{(2)\mu\lambda}_{(b_1 3)}\tilde{t}^{(2)}_{(\phi 4)\lambda\nu}\tilde{t}^{(1)\nu}_{(12)} f^{(\phi)}_{(12)} f^{(b_1)}_{(124)}.$$




```
Cov: SS wave fraction = 0.930268      Hel: SS wave fraction = 0.905202
Cov: SD wave fraction = 0.00704404    Hel: SD wave fraction = 0.00800091
Cov: DS wave fraction = 0.062201      Hel: DS wave fraction = 0.0859795
Cov: DD wave fraction = 0.000487272   Hel: DD wave fraction = 0.000817607
```

```
Cov: SS wave fraction = 0.292975
Cov: SD wave fraction = 0.500984
Cov: DS wave fraction = 0.0744949
Cov: DD wave fraction = 0.131546
```

Without $J/\psi \to l^+ l^-$



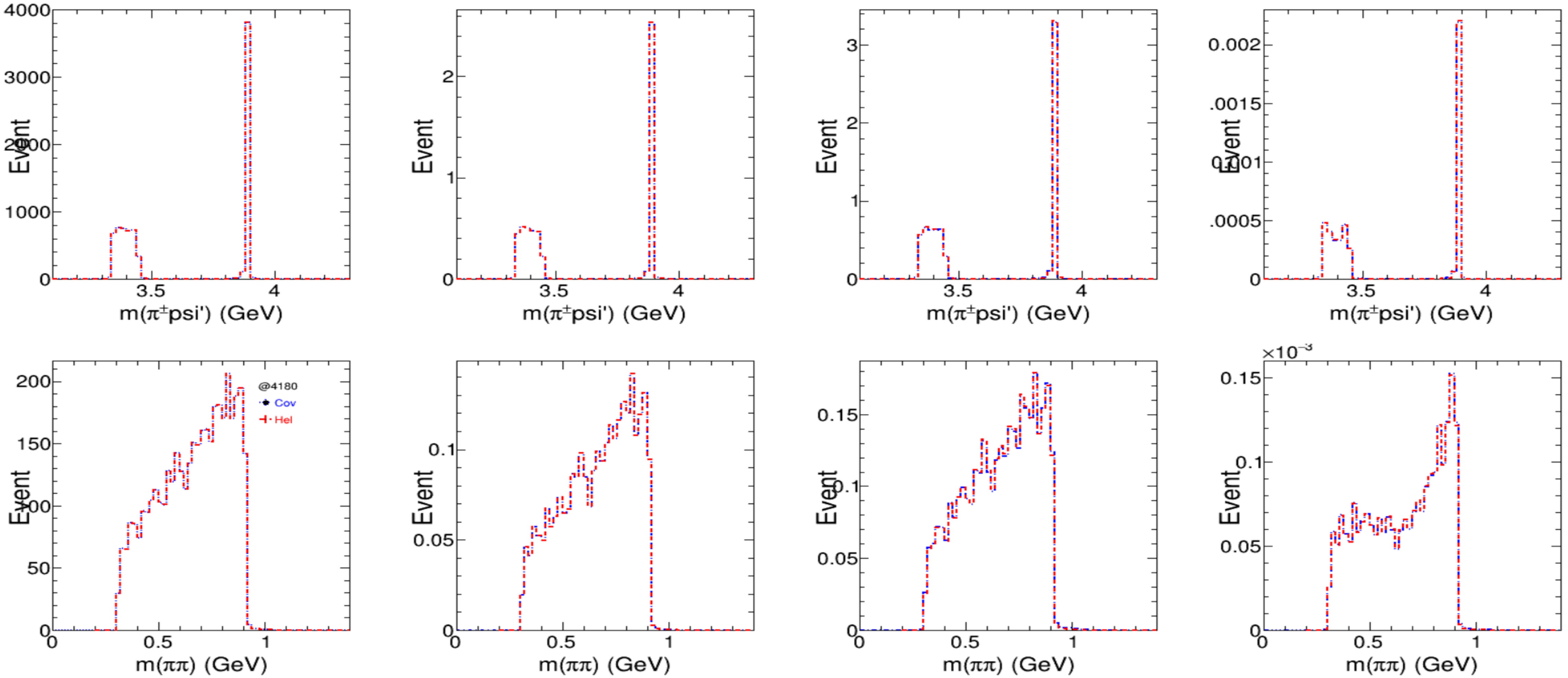

```
Cov: SS wave fraction = 0.894325      Hel: SS wave fraction = 0.918925
Cov: SD wave fraction = 0.00955142    Hel: SD wave fraction = 0.0646136
Cov: DS wave fraction = 0.0950784     Hel: DS wave fraction = 0.0153109
Cov: DD wave fraction = 0.00104474    Hel: DD wave fraction = 0.00115088
```

With $J/\psi \to l^+l^-$, leptons have 0 spin

```
Cov: SS wave fraction = 0.734788
Cov: SD wave fraction = 0.0656007
Cov: DS wave fraction = 0.182761
Cov: DD wave fraction = 0.0168496
```

Without $J/\psi \to l^+l^-$

Zc SS component　　Zc SD component　　Zc DS component　　Zc DD component




```
Cov: SS wave fraction = 0.998473      Hel: SS wave fraction = 0.998094
Cov: SD wave fraction = 0.00066174    Hel: SD wave fraction = 0.00136856
Cov: DS wave fraction = 0.000864748   Hel: DS wave fraction = 0.000536306
Cov: DD wave fraction = 5.75326e-07   Hel: DD wave fraction = 7.39178e-07
```

```
Cov: SS wave fraction = 0.0643843
Cov: SD wave fraction = 0.586987
Cov: DS wave fraction = 0.0343617
Cov: DD wave fraction = 0.314267
```

Without $J/\psi \to l^+l^-$