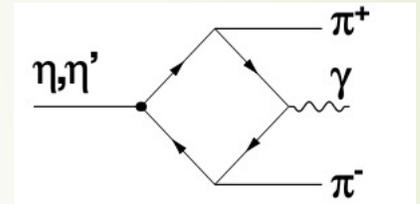


Updated Study of $\eta, \eta' \rightarrow \gamma \pi^+ \pi^-$ Decay Dynamic



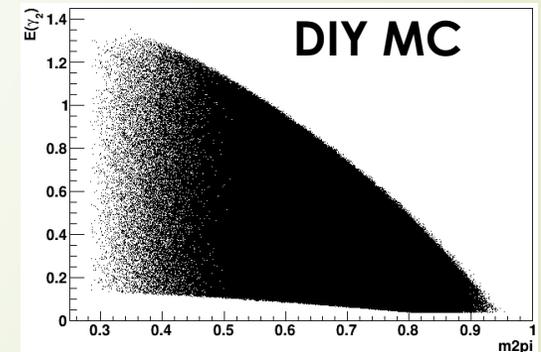
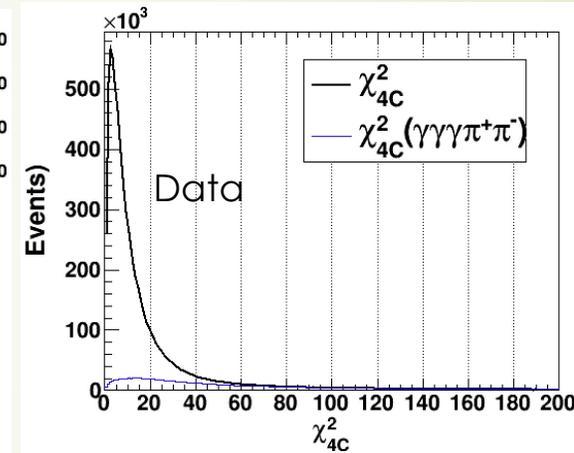
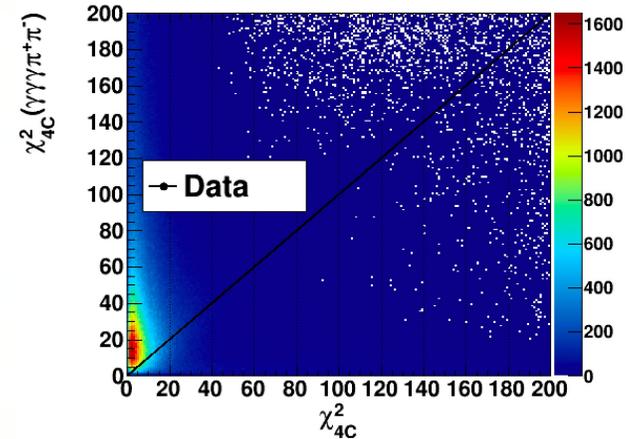
Liqing Qin

06/02, 2022

Selection criteria for $J/\psi \rightarrow \gamma\eta'$, $\eta' \rightarrow \gamma\pi^+\pi^-$

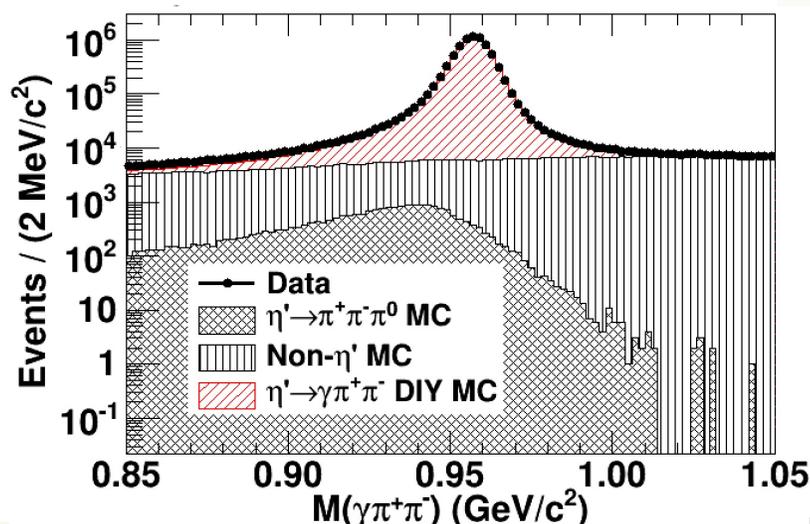
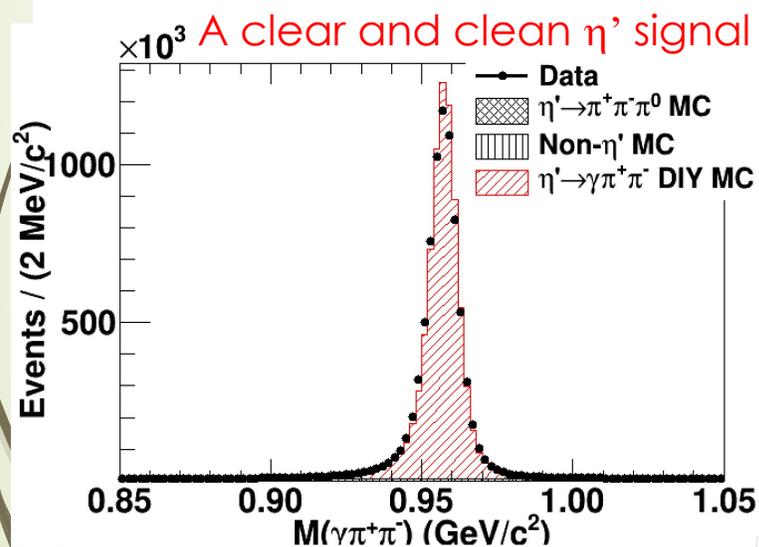
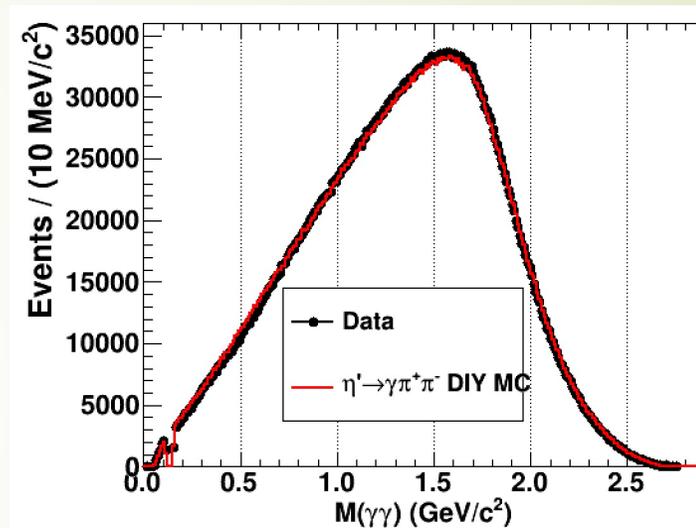
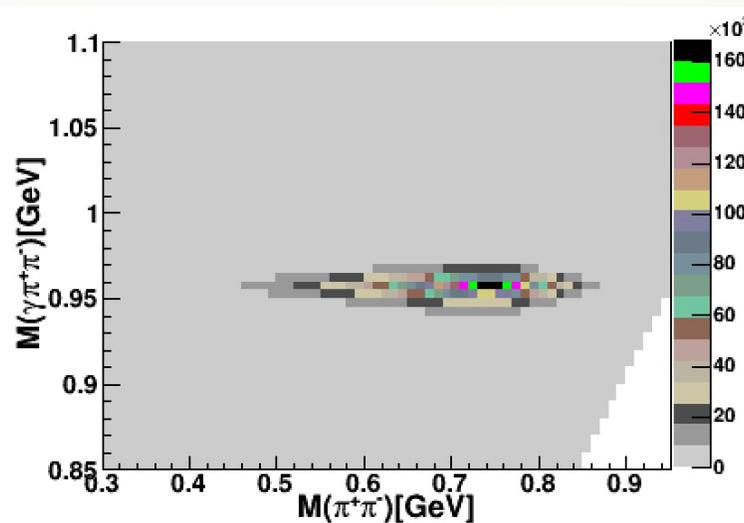
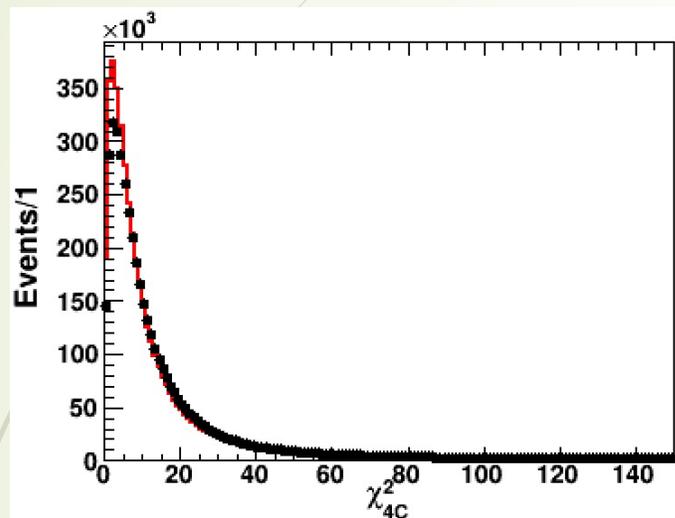
$E_\gamma^{\text{CMS}} = 1.4 \text{ GeV}$

- Common ones for the **good charged tracks** and **good photons** candidates:
 - ◆ $N_{\text{ch}}=2, \text{net}_{\text{ch}}=0$
 - ◆ $N_\gamma \geq 2$, where the deposited energy is larger than **40 MeV** for barrel rather than 25 MeV.
- PID: without any PID
- **Vertex Fit**
- **4C Kinematic Fit**
 - ◆ $\chi^2(\pi^+\pi^-\gamma\gamma) < 100$
 - ◆ Photon with maximum energy is taken as the radiative one from J/ψ .
 - ◆ $\chi^2(\pi^+\pi^-\gamma\gamma) < \chi^2(\pi^+\pi^-\gamma\pi)$
- **Veto background events with π^0 in the final state: $J/\psi \rightarrow \pi^+\pi^-\pi^0, \gamma\pi^+\pi^-\pi^0, \dots$**
 - ◆ $|M(\gamma\gamma) - m_{\pi^0}| > 20 \text{ MeV}$



Selection criteria

Data vs DIY MC of $\eta' \rightarrow \gamma\pi^+\pi^-$



η' candidates:

$$|M(\gamma\pi^+\pi^-) - m_{\eta'}| < 20 \text{ MeV}$$

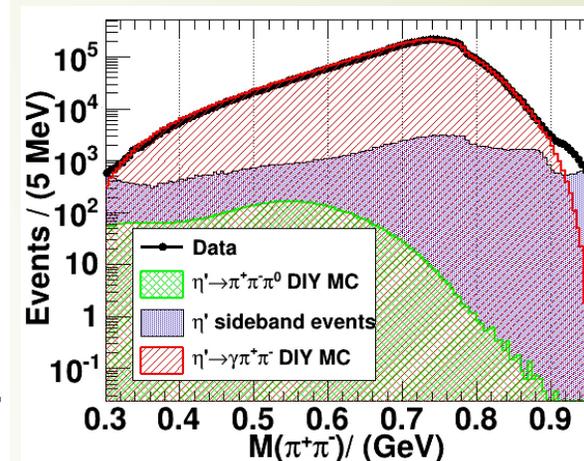
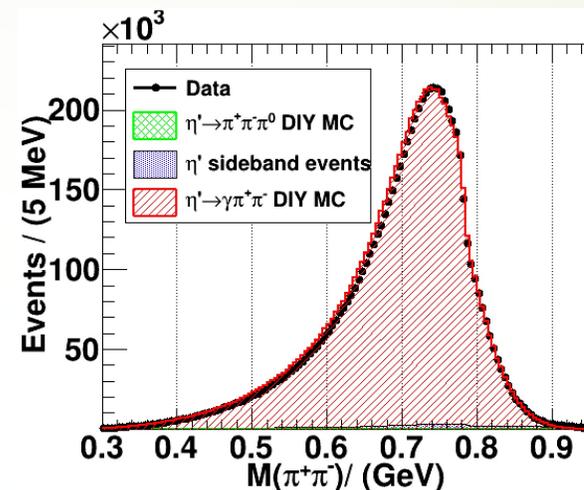
Non- η' :

$$40 \text{ MeV} < |M(\gamma\pi^+\pi^-) - m_{\eta'}| < 60 \text{ MeV}$$

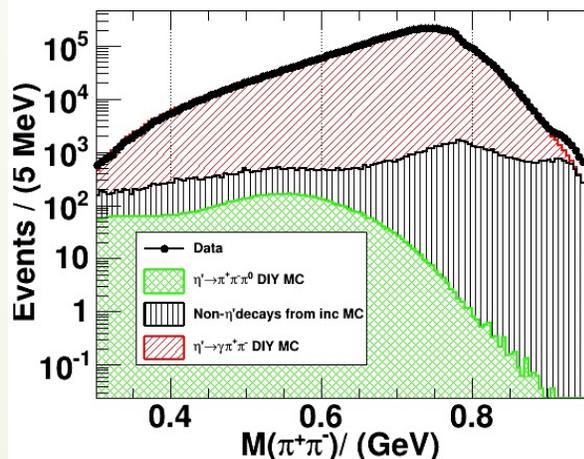
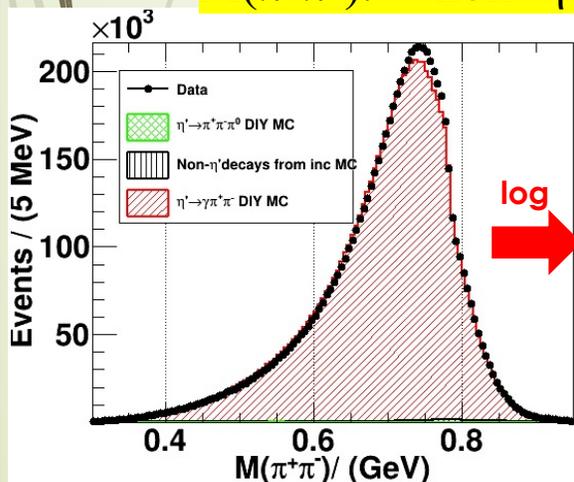
Selection criteria

Peaking bkg: $\eta' \rightarrow \pi^+\pi^-\pi^0$

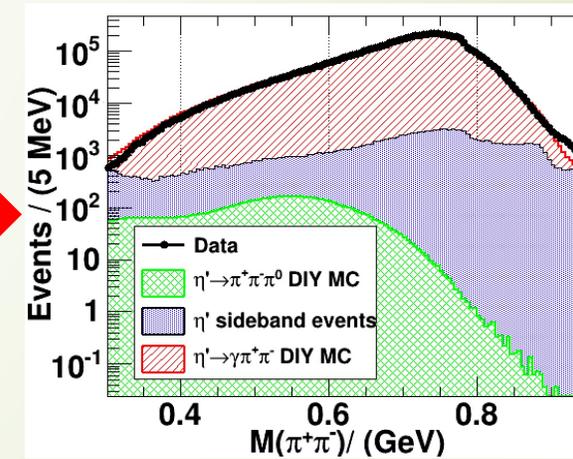
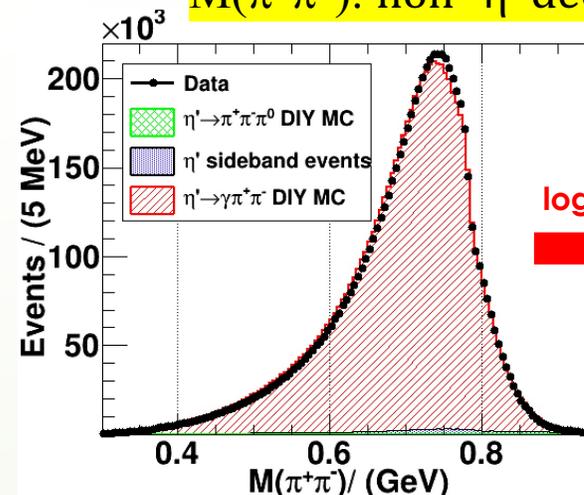
$\eta' \rightarrow \pi^+\pi^-\pi^0$	Cases	N_gen	N_survived	ϵ (%)	N_normalized
09+12 DIY mc	signal region	6,200,000	274,014	4.42	1061
	sideband region		109,901	1.77	426
18+19 DIY mc	signal region	40,000,000	1,688,220	4.22	6777
	sideband region		685,091	1.71	2750



$M(\pi^+\pi^-)$: non- η' decays from inc. mc



$M(\pi^+\pi^-)$: non- η' decays from η' -sideband



Background study with inclusive MC

Without cut: $\chi^2(\pi^+\pi^-\gamma\gamma) < \chi^2(\pi^+\pi^-\gamma\gamma\gamma)$

Table 1: Event trees and their respective initial-final states.

index	event tree (event initial-final states)	iEvtTr	iEvtIFSts	nEvts	nCmltEvts
1	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \eta'\gamma, \eta' \rightarrow \pi^+\pi^-\gamma$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$)	0	0	7777179	7777179
2	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \eta'\gamma, \eta' \rightarrow \pi^+\pi^-\gamma\gamma_{FSR}$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma_{FSR}$)	3	2	117891	7895070
3	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^0\rho^0, \rho^0 \rightarrow \pi^+\pi^-$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$)	1	0	25019	7920089
4	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^0h_1(1170), h_1(1170) \rightarrow \pi^0\rho^0, \rho^0 \rightarrow \pi^+\pi^-$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma$)	7	1	13696	7933785
5	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^0h_1(1170), h_1(1170) \rightarrow \pi^-\rho^+, \rho^+ \rightarrow \pi^0\pi^+$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma$)	2	1	11085	7944870
6	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^0h_1(1170), h_1(1170) \rightarrow \pi^+\rho^-, \rho^- \rightarrow \pi^0\pi^-$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma$)	5	1	11027	7955897
7	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \eta'\gamma, \eta' \rightarrow \pi^0\pi^+\pi^-$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma$)	4	3	8415	7964312
8	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \rho^0\eta, \rho^0 \rightarrow \pi^+\pi^-, \eta \rightarrow \gamma\gamma$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$)	12	0	5408	7969720
9	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow e^+e^-\gamma_{FSR}\gamma_{FSR}$ ($e^+e^- \rightarrow e^+e^-\gamma_{FSR}\gamma_{FSR}$)	16	7	4594	7974314
10	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow e^+e^-\gamma_{FSR}$ ($e^+e^- \rightarrow e^+e^-\gamma_{FSR}$)	25	11	4185	7978499
11	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^+\pi^-\omega, \omega \rightarrow \pi^0\gamma$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma$)	15	3	3388	7981887
12	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^+\pi^-\eta\gamma, \eta \rightarrow \gamma\gamma$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma$)	9	3	2725	7984612
13	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^0\rho^0, \rho^0 \rightarrow \pi^+\pi^-\gamma_{FSR}$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma_{FSR}$)	17	2	2475	7987087
14	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^+\pi^-\eta, \eta \rightarrow \gamma\gamma$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$)	18	0	2454	7989541
15	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \eta_c\gamma, \eta_c \rightarrow \pi^0\pi^+\pi^-$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma$)	34	3	2093	7991634
16	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^0\rho^0, \rho^0 \rightarrow \pi^+\pi^-\gamma$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma$)	23	3	1629	7993263
17	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow e^+e^-$ ($e^+e^- \rightarrow e^+e^-$)	13	5	1243	7994506

Background level: 1.51%

where the ratio of peaking bkg $\eta' \rightarrow \pi^+\pi^-\pi^0$:
~0.1% (7.0%)

Background study with inclusive MC

With cut: $\chi^2(\pi^+\pi^-\gamma\gamma) < \chi^2(\pi^+\pi^-\gamma\gamma\gamma)$

index	event tree (event initial-final states)	iEvtTr	iEvtIFSts	nEvts	nCmltEvts
1	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \eta'\gamma, \eta' \rightarrow \pi^+\pi^-\gamma$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$)	0	0	7604955	7604955
2	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \eta'\gamma, \eta' \rightarrow \pi^+\pi^-\gamma\gamma_{FSR}$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma_{FSR}$)	4	3	91480	7696435
3	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^0\rho^0, \rho^0 \rightarrow \pi^+\pi^-$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$)	1	0	23565	7720000
4	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \rho^0\eta, \rho^0 \rightarrow \pi^+\pi^-, \eta \rightarrow \gamma\gamma$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$)	8	0	5338	7725338
5	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow e^+e^-\gamma_{FSR}\gamma_{FSR}$ ($e^+e^- \rightarrow e^+e^-\gamma_{FSR}\gamma_{FSR}$)	10	5	4403	7729741
6	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow e^+e^-\gamma_{FSR}$ ($e^+e^- \rightarrow e^+e^-\gamma_{FSR}$)	16	7	4118	7733859
7	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \eta'\gamma, \eta' \rightarrow \pi^0\pi^+\pi^-$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma$)	2	1	3460	7737319
8	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^0h_1(1170), h_1(1170) \rightarrow \pi^0\rho^0, \rho^0 \rightarrow \pi^+\pi^-$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma\gamma$)	5	2	2980	7740299
9	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^+\pi^-\eta, \eta \rightarrow \gamma\gamma$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$)	11	0	2422	7742721
10	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^0h_1(1170), h_1(1170) \rightarrow \pi^+\rho^-, \rho^- \rightarrow \pi^0\pi^-$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma\gamma$)	3	2	2187	7744908
11	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^0h_1(1170), h_1(1170) \rightarrow \pi^-\rho^+, \rho^+ \rightarrow \pi^0\pi^+$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma\gamma$)	19	2	2091	7746999
12	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow e^+e^-$ ($e^+e^- \rightarrow e^+e^-$)	9	4	1224	7748223
13	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^+\pi^-\omega, \omega \rightarrow \pi^0\gamma$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma$)	24	1	1046	7749269
14	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^+\pi^-\eta\gamma, \eta \rightarrow \gamma\gamma$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma\gamma$)	7	1	997	7750266
15	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \pi^0\rho^0, \rho^0 \rightarrow \pi^+\pi^-\gamma_{FSR}$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma_{FSR}$)	14	3	958	7751224
16	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \eta\omega, \eta \rightarrow \gamma\gamma, \omega \rightarrow \pi^+\pi^-$ ($e^+e^- \rightarrow \pi^+\pi^-\gamma\gamma$)	13	0	855	7752079
17	$e^+e^- \rightarrow J/\psi, J/\psi \rightarrow \eta'\gamma, \eta' \rightarrow \mu^+\mu^-\gamma$ ($e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$)	32	12	807	7752886

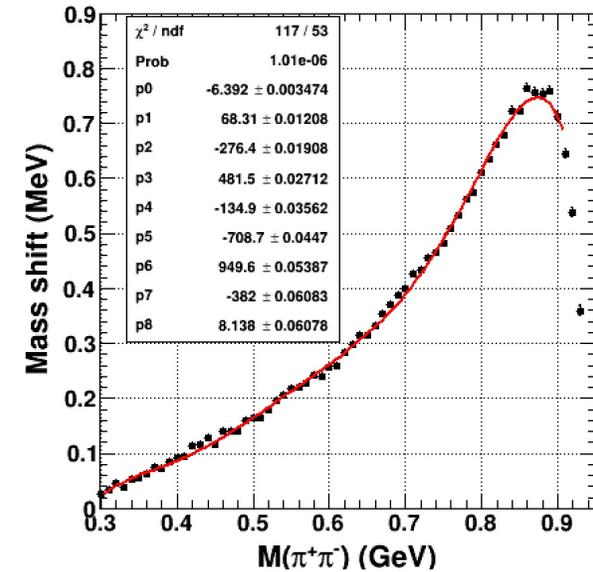
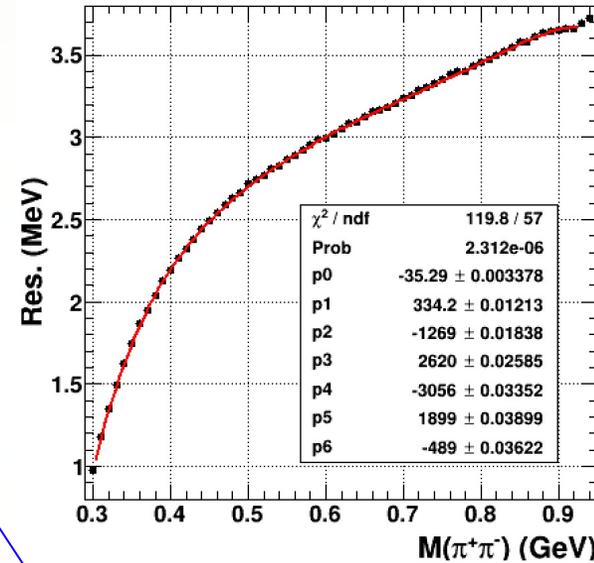
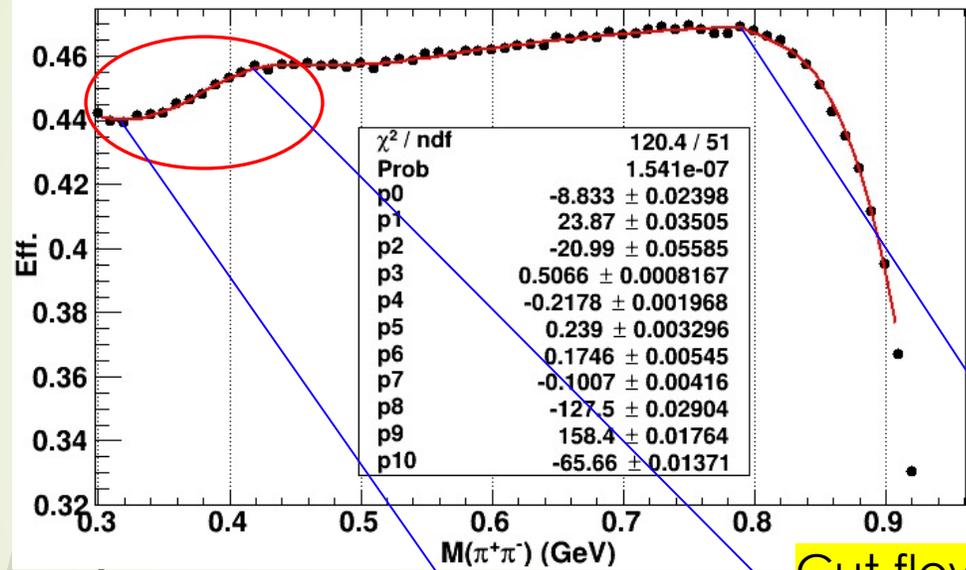
Background level: 0.87%

where the ratio of
peaking bkg $\eta' \rightarrow \pi^+\pi^-\pi^0$:
~0.045% (5.1%)

Efficiency loss: ~ 2.65%
Veto background: ~44.2%

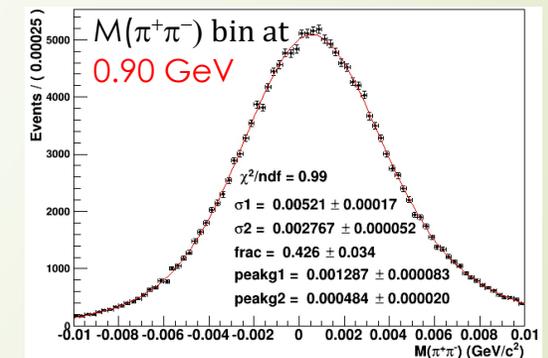
Corrections for the dipion mass spectrum

➤ $M(\pi^+\pi^-)$ -dependent **detection efficiency**, **mass resolution**, and the **mass shift**



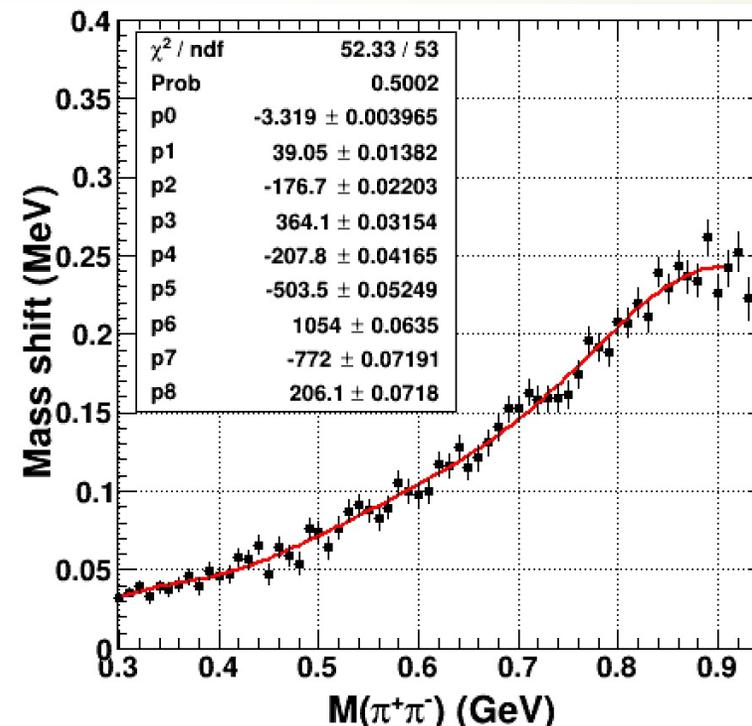
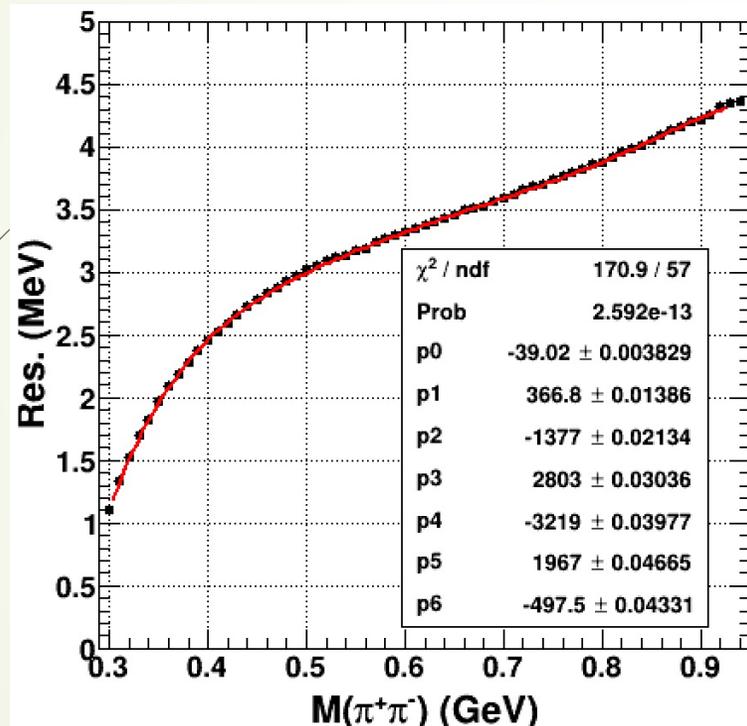
Cut flow

cuts	ϵ of $M(\pi^+\pi^-)$ bin at 0.32 GeV	ϵ of $M(\pi^+\pi^-)$ bin at 0.42 GeV	ϵ of $M(\pi^+\pi^-)$ bin at 0.79 GeV	ϵ of $M(\pi^+\pi^-)$ bin at 0.90 GeV
after pre-sel.	47.53%	49.50%	50.97%	47.93%
$ M(\gamma\gamma) - m_{\pi_0} > 20$ MeV	99.57%	99.66%	99.45%	99.64%
$E_{\gamma_2} > 40$ MeV	99.97%	99.97%	99.64%	89.77%
$\chi^2(\pi^+\pi^-\gamma\gamma) < 100$	97.89%	97.71%	96.93%	95.64%
$ M(\gamma\pi^+\pi^-) - m_{\eta'} < 20$ MeV	94.90%	94.86%	95.83%	96.35%
$\chi^2(\pi^+\pi^-\gamma\gamma) < \chi^2(\pi^+\pi^-\gamma\gamma\gamma)$	97.69%	97.59%	97.66%	98.00%



A Further Check on The Corrections

- $M(\pi^+\pi^-)$ -dependent **mass resolution**, and the **mass shift**, where the $M(\pi^+\pi^-)$ is reconstructed with info. before 4C-kinematic fit.



Similar line-shape as those obtained after 4C-kinematic fit

➔ with 4c-fit: **Res. is improved**, while the mass shift is getting larger.

Momentum of π^+ vs π^-

reconstructed after 4C fit

$M(\pi^+\pi^-)$ bin at 300 MeV

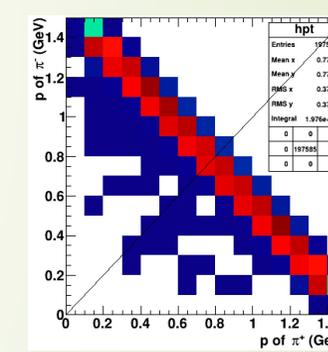
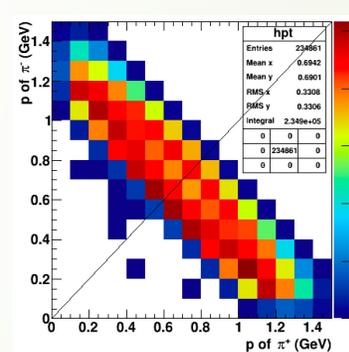
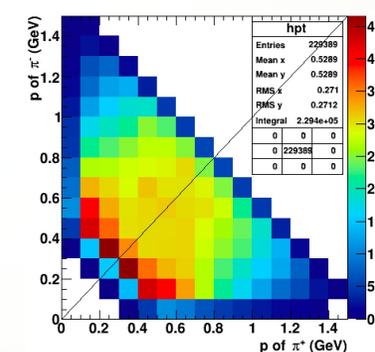
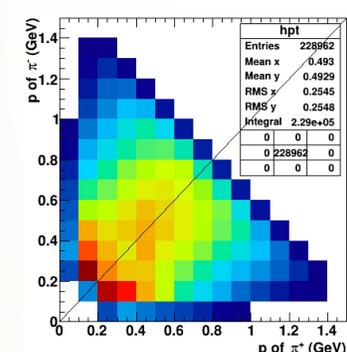
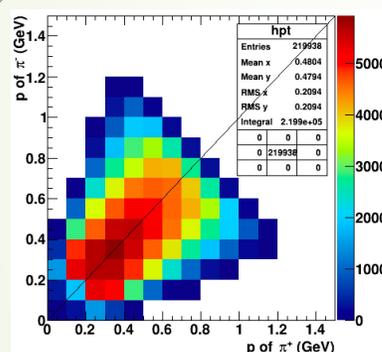
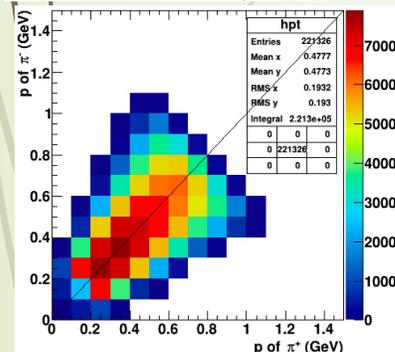
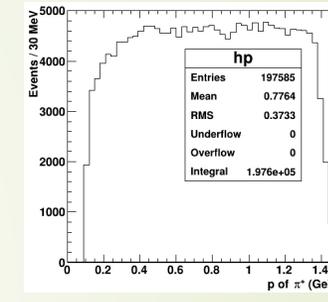
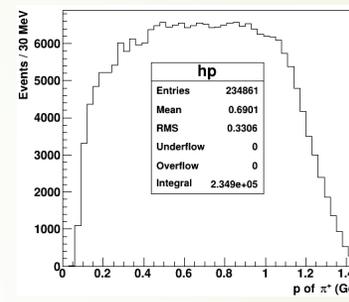
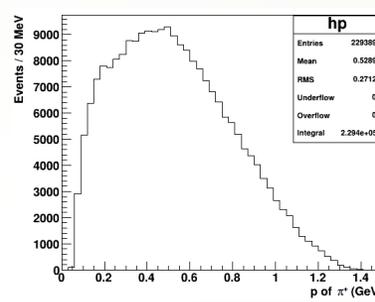
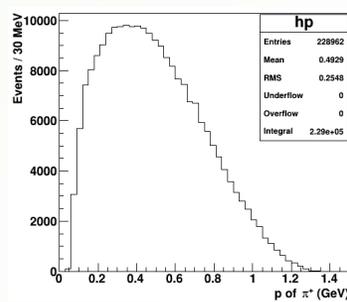
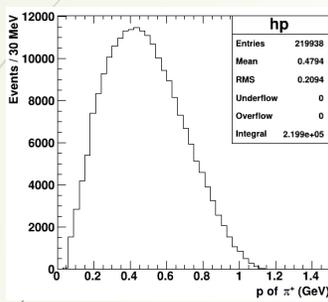
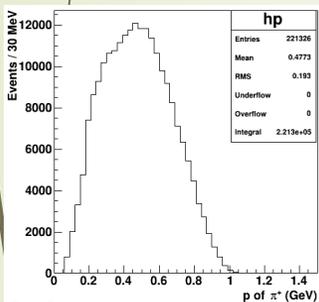
$M(\pi^+\pi^-)$ bin at 320 MeV

$M(\pi^+\pi^-)$ bin at 420 MeV

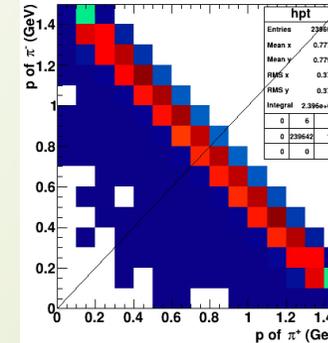
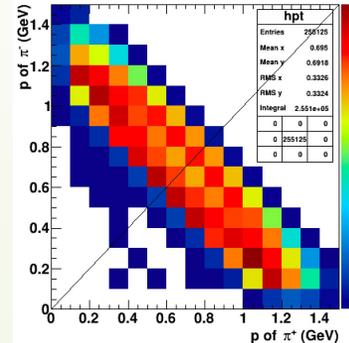
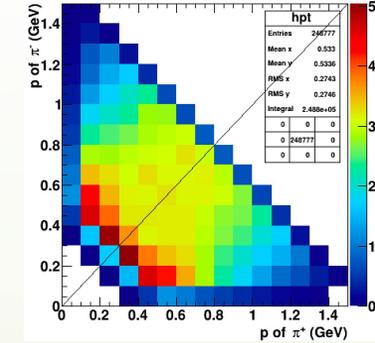
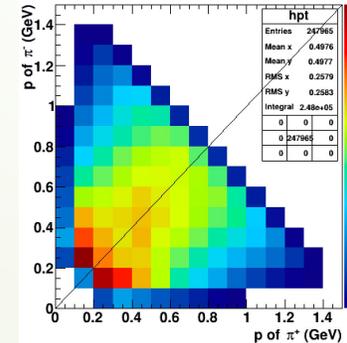
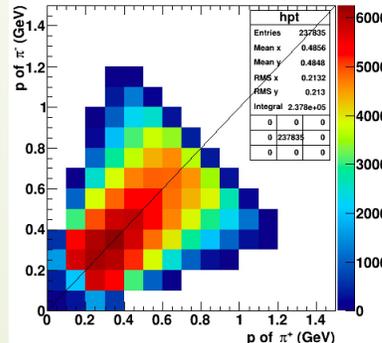
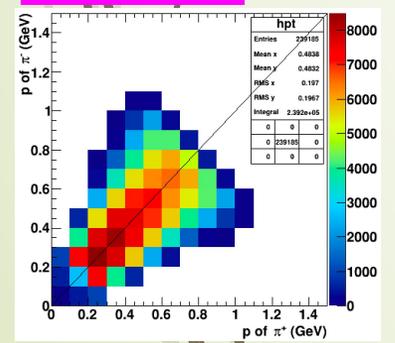
$M(\pi^+\pi^-)$ bin at 520 MeV

$M(\pi^+\pi^-)$ bin at 790 MeV

$M(\pi^+\pi^-)$ bin at 900 MeV



at truth level



Model-dependent study

- Differential rate $[d\Gamma/dM(\pi^+\pi^-)] = [k_\gamma^3 q_\pi^3(s)/48\pi^3] |\mathcal{A}|^2$, $k_\gamma = (m_{\eta'}^2 - s)/(2m_{\eta'})$, $q_\pi(s) = \sqrt{s - 4m_\pi^2}/2$
- Decay amplitude:

$$\mathcal{A} = \frac{BW_\rho^{\text{GS}}(s)(1 + \delta \frac{s}{M_\omega^2} BW_\omega(s)) + \beta BW_{\rho'}^{\text{GS}}(s)}{1 + \beta} \\ \times 2\sqrt{48\pi M_\rho^{-4}} + \alpha,$$

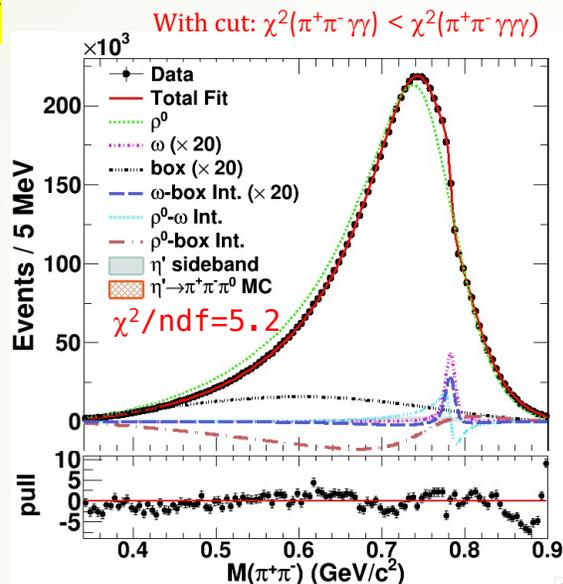
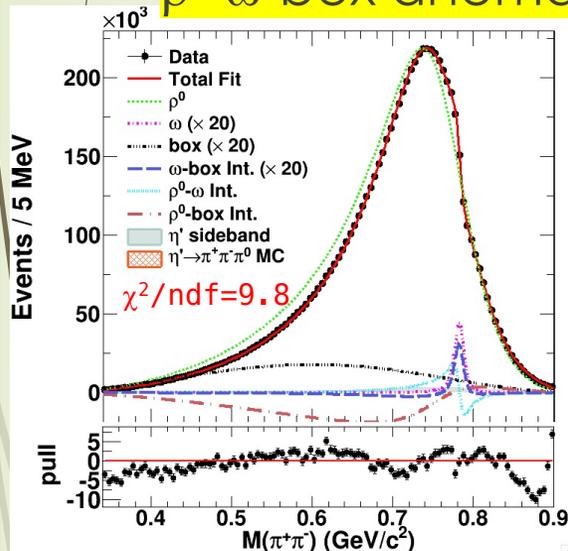
δ, β are complex parameters.

α : a constant accounting for the box anomaly.

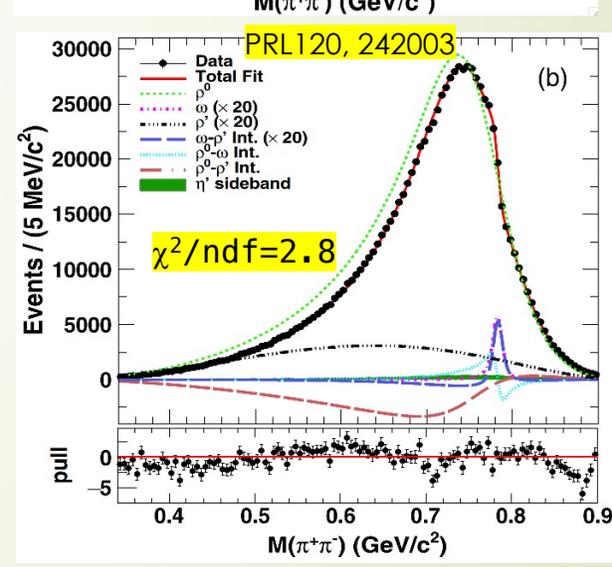
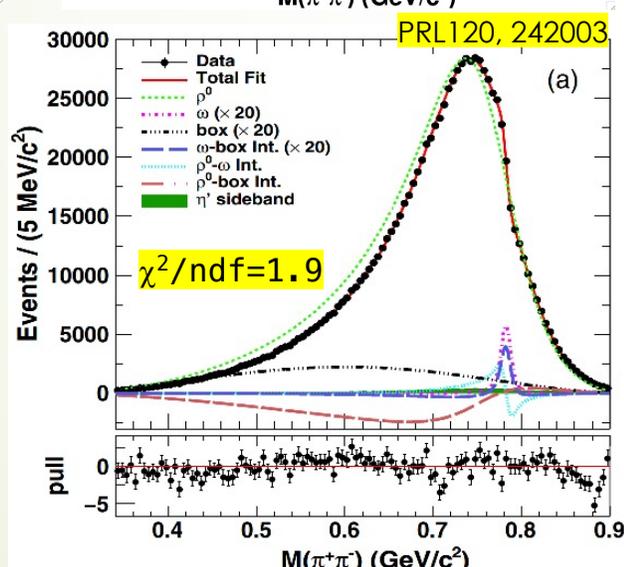
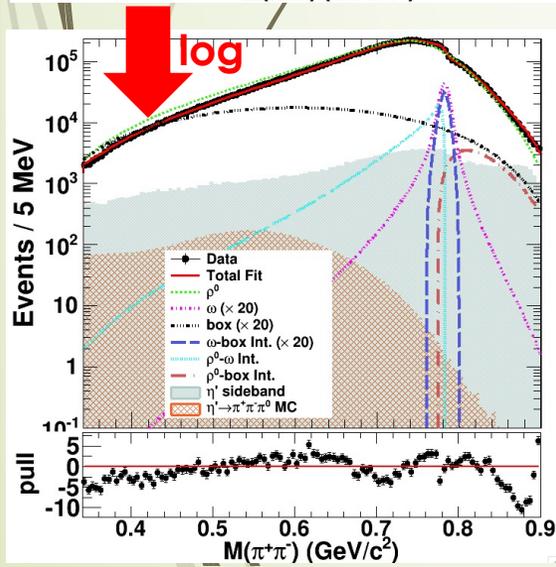
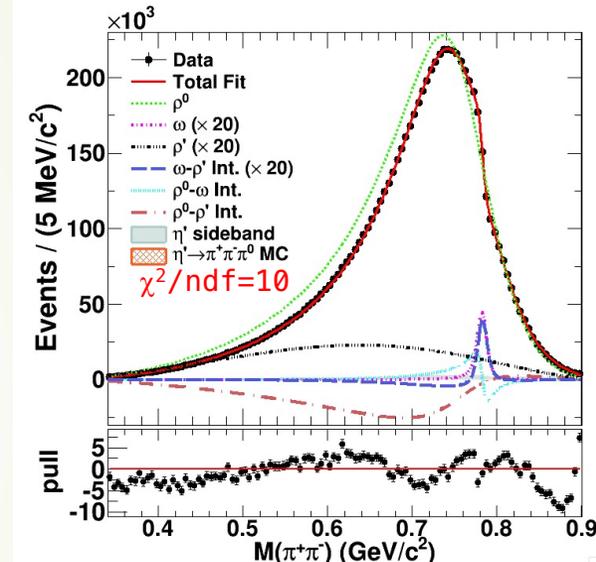
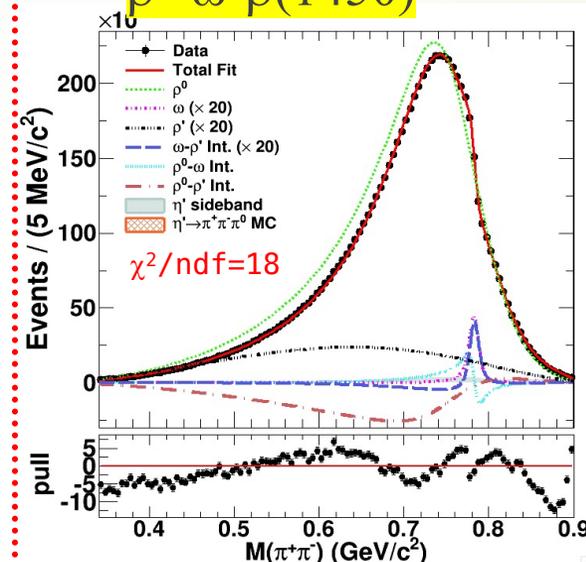
- The PDF is constructed to be: $PDF(m) = \sigma_{\text{res}}(m) \otimes (\epsilon(m) \times \frac{d\Gamma}{dm}) + BKG(m)$

Model-dependent study

ρ^0 - ω -box anomaly



ρ^0 - ω - $\rho(1450)$



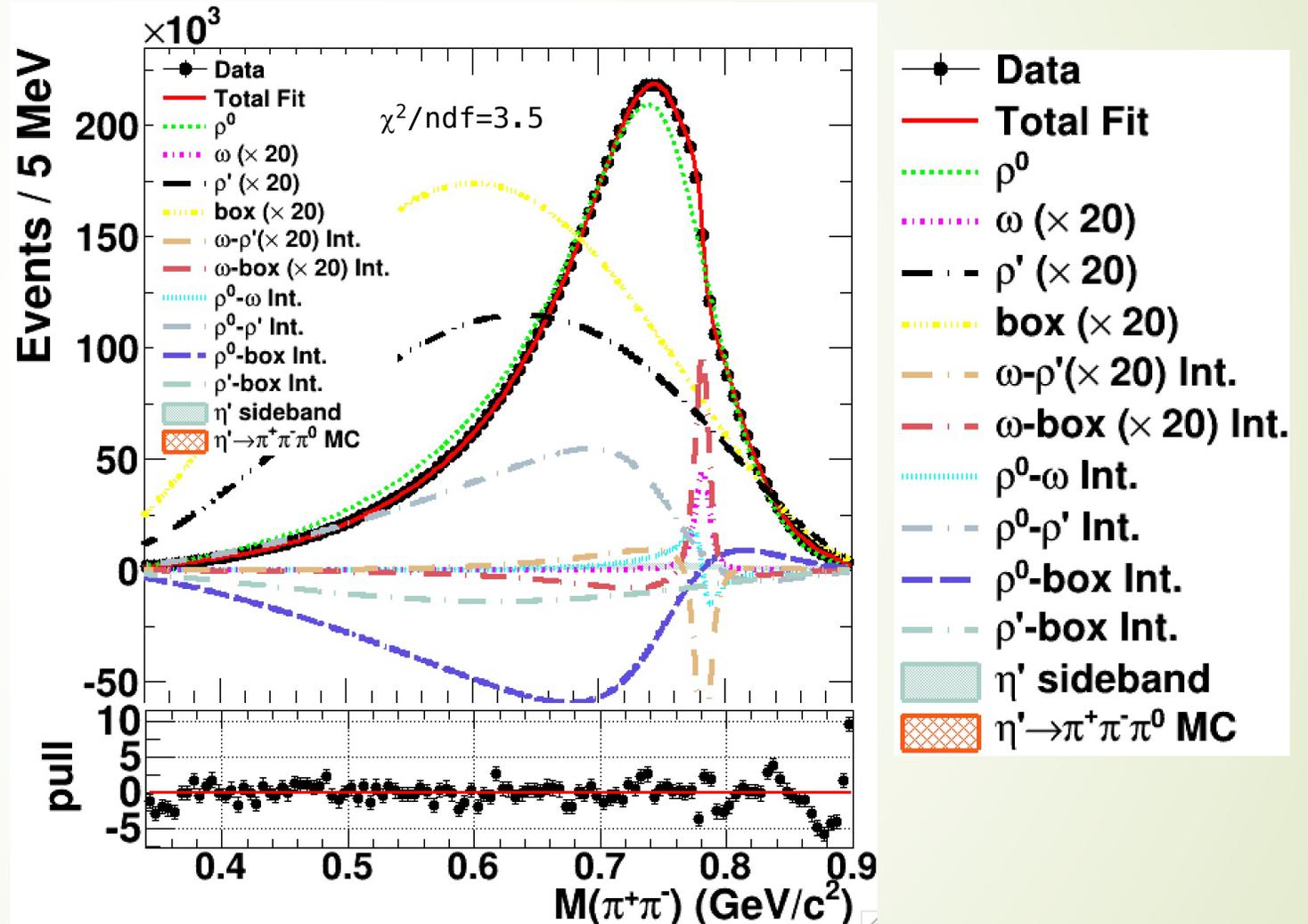
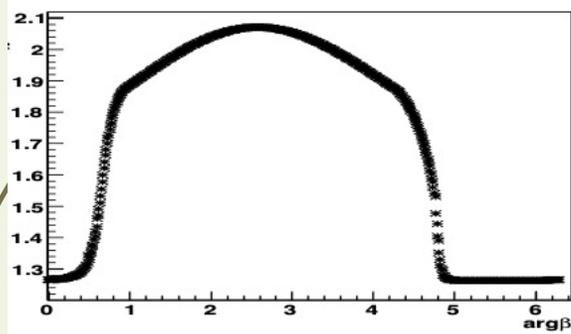
Model-dependent study

► ρ^0 - ω - $\rho(1450)$ -box anomaly

issues:

Correlation coefficient between α and $|\delta|$:
 -0.9887

multi-solution?



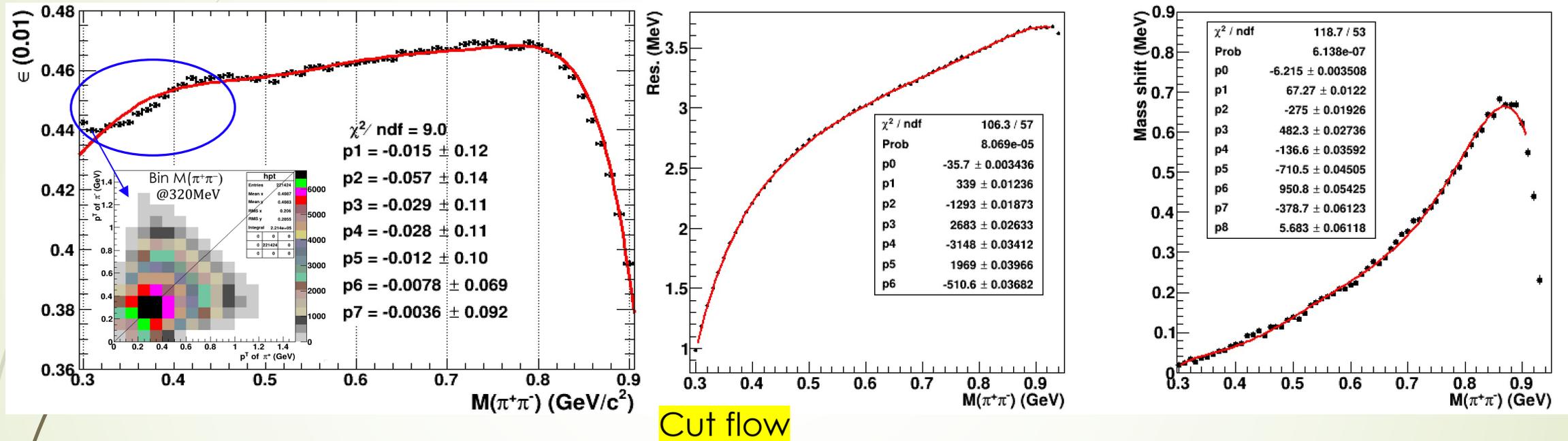
Model-independent study

- ▶ based on a combination of chiral perturbation theory (ChPT)-extended from SU(3) to U(3), and a dispersive analysis.
- ▶ Decay amplitude: $\mathcal{A} = NP(s)F_V(s)$

backup

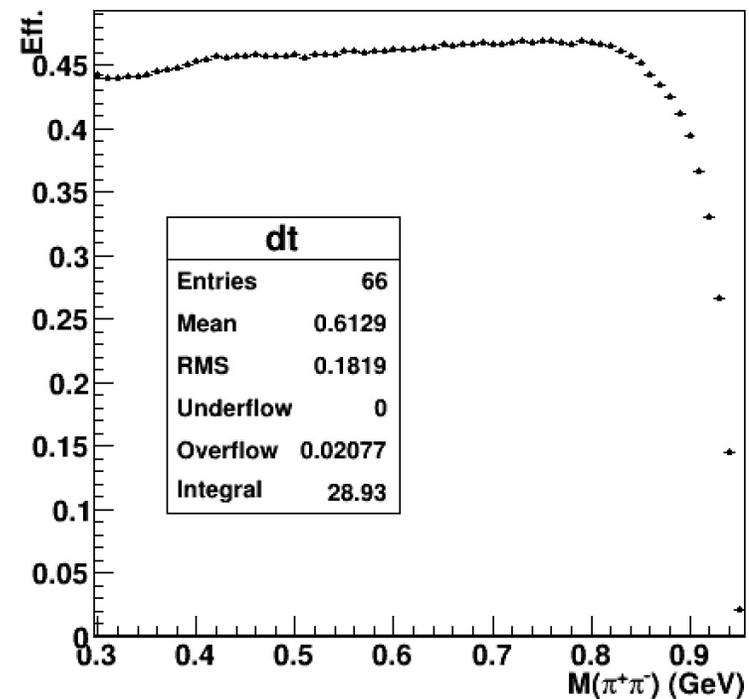
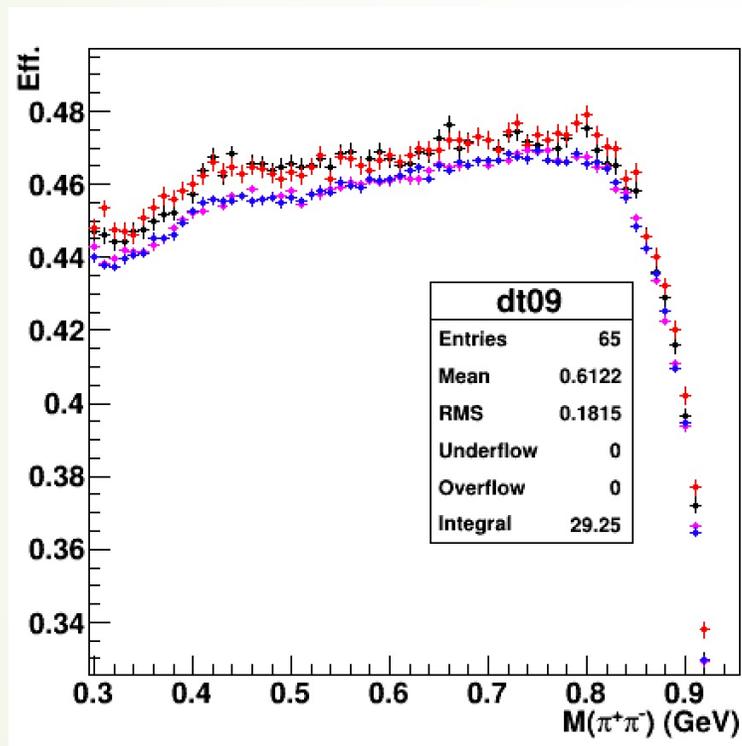
Corrections for the dipion mass spectrum

► $M(\pi^+\pi^-)$ -dependent **detection efficiency**, **mass resolution**, and the **mass shift**



cuts	ϵ of $M(\pi^+\pi^-)$ bin at 0.32 GeV	ϵ of $M(\pi^+\pi^-)$ bin at 0.42 GeV	ϵ of $M(\pi^+\pi^-)$ bin at 0.79 GeV	ϵ of $M(\pi^+\pi^-)$ bin at 0.90 GeV
$E_{\gamma 2} > 40$ MeV	47.40%	49.41%	50.69%	42.89%
$\chi^2(\pi^+\pi^-\gamma\gamma) < 100$	46.40%	48.28%	49.14%	41.01%
$ M(\gamma\pi^+\pi^-) - m_{\eta'} < 20$ MeV	44.01%	45.77%	47.08%	39.51%
$ M(\gamma\gamma) - m_{\pi_0} > 20$ MeV	43.95%	45.71%	46.91%	39.50%
Relative diff. of ϵ	7.12%	7.34%	7.46%	17.28%

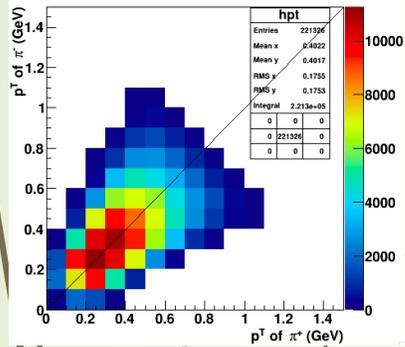
$M(\pi^+\pi^-)$ -dependent detection efficiency for 4 runs



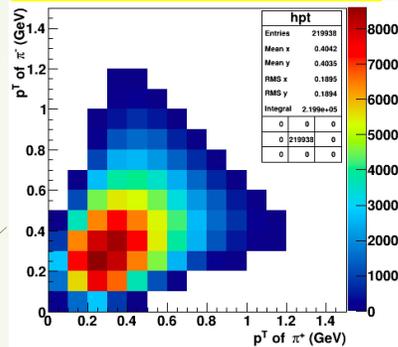
Transverse Momentum of π^+ vs π^-

reconstructed after 4C fit: p_T

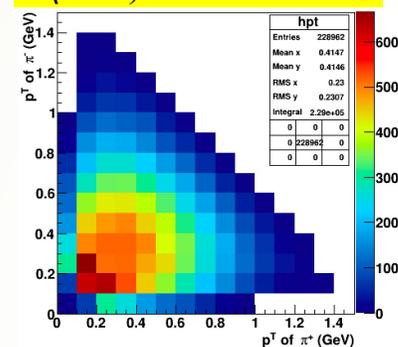
$M(\pi^+\pi^-)$ bin at 300MeV



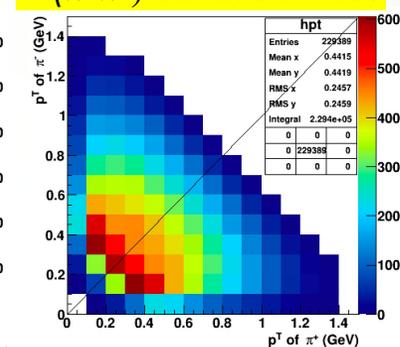
$M(\pi^+\pi^-)$ bin at 320MeV



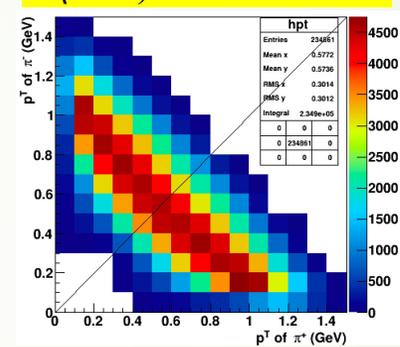
$M(\pi^+\pi^-)$ bin at 420MeV



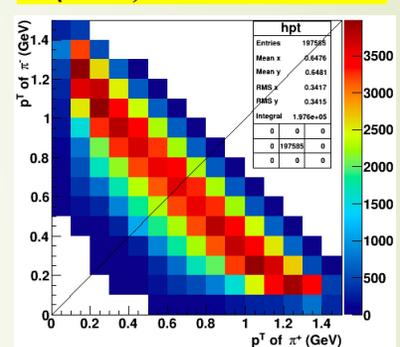
$M(\pi^+\pi^-)$ bin at 520MeV



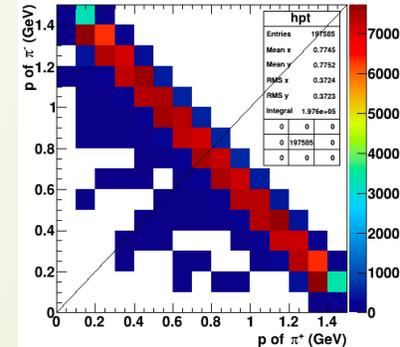
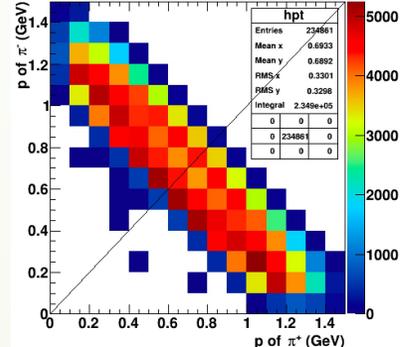
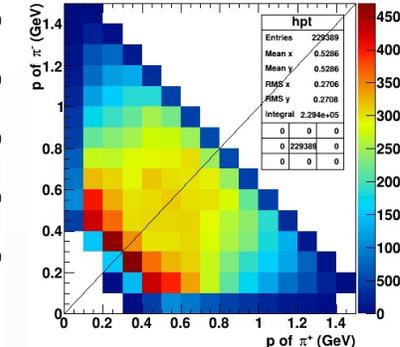
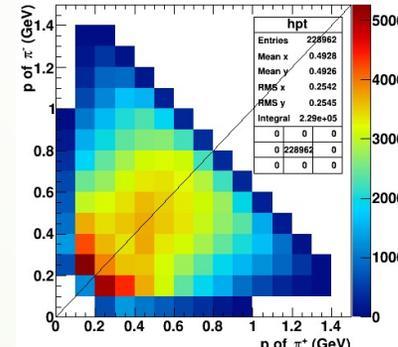
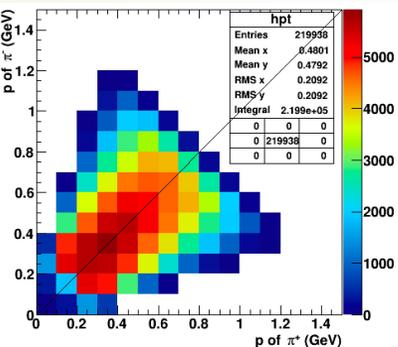
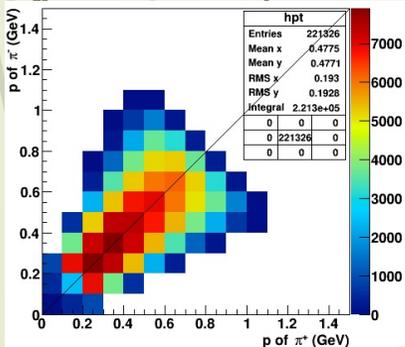
$M(\pi^+\pi^-)$ bin at 790MeV

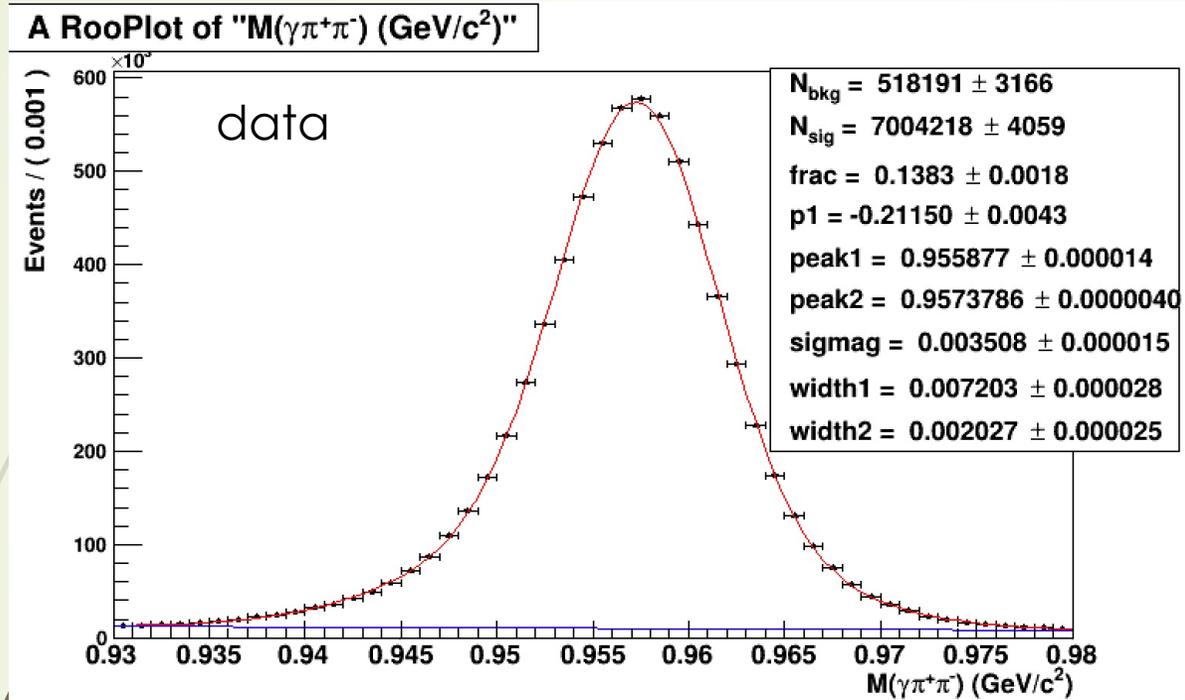


$M(\pi^+\pi^-)$ bin at 900MeV



at truth level: p



Check on the bkg level: a fit to $M(\gamma\gamma)$ 

Background level: 6%