$$\mathcal{M}^J_{\lambda\nu}(\vartheta,\varphi;M) \propto \langle\vartheta,\varphi,\lambda\nu|JM\lambda\nu\rangle\langle JM\lambda\nu|\mathcal{M}|JM\rangle$$
$$\propto D^{J*}_{M\delta}(\varphi,\vartheta,0)F^J_{\lambda\nu},$$
$$F^J_{\lambda\nu}=\sum_{\ell S}\left(\frac{2\ell+1}{2J+1}\right)^{1/2}(\ell 0 S\delta|J\delta)(s\lambda\sigma-\nu|S\delta)G^J_{\ell S},$$

AmpTool中使用的形式是非协变的

covariant helicity-coupling amplitudes

$$F^J_{\lambda\nu}=\sum_{\ell S} g_{\ell S}A_{\ell S}(\lambda\nu),$$

where

$$A_{\ell S}(\lambda\nu)=\left(\frac{2\ell+1}{2J+1}\right)^{1/2}(\ell 0 S\delta|J\delta)(s\lambda\sigma-\nu|S\delta)$$
$$\times W^n r^\ell f^s_\lambda(\gamma_s)f^\sigma_\nu(\gamma_\sigma),$$

子粒子的相对论因子: $\gamma_s=\frac{q_0}{m}$ $\gamma_\sigma=\frac{k_0}{\mu}$

Spin=1

$$\xi_s(\lambda)\equiv f^{(1)}_\lambda(\gamma_s)=\begin{cases}[\chi^{(1)*}(\lambda)\cdot\omega(\lambda)]\\ 1 \quad \text{for } \lambda=\pm1\\ \gamma_s \quad \text{for } \lambda=0,\end{cases}$$

Spin=2

$$f^{(2)}_\lambda(\gamma_s)=\begin{cases}1 \quad \text{for } \lambda=\pm2\\ \gamma_s \quad \text{for } \lambda=\pm1\\ \frac{2}{3}\gamma_s^2+\frac{1}{3} \quad \text{for } \lambda=0.\end{cases}$$

- 根据协变性和洛伦兹不变的原则所构造的
- helicity振幅中包含了末态粒子的相对论因子和母粒子质量项

W^n: 是母粒子质量项,

$s+\sigma+l-J=\text{odd}$, n=1, 否则, n=0

$$D \to \rho^0\pi^0 \to \pi^+\pi^-\pi^0,\ \phi\pi^0 \to K^+K^-\pi^0$$

两个过程都是P波

$$\begin{aligned} Z &= D^0_{0\lambda}(0,\theta_0\phi_0)f_\lambda(\gamma)D^{*1}_{\lambda 0}(0\theta\phi) \\ &= \gamma d^1_{00}(\theta) \qquad \lambda = 0 \\ &= \gamma\cos\theta\ , \end{aligned}$$

$$\begin{aligned} Z &= -2p(p_a - p_b)^3 \\ &= -2p[(\gamma\beta E^* - \gamma q_3) - (\gamma\beta E^* + \gamma q_3)] \\ &= 4p\gamma(q\cos\theta) \\ &= 4pq\gamma\cos\theta\ , \end{aligned}$$

- 两个子过程都是P波的级联过程，振幅中角度部分是相同的
- 两个子过程都是D波的级联过程，振幅不相同
- 对于协变张量机制，相对论因子天然就包含在里面
- 对于helicity机制，即使乘上相对论因子，也和协变张量机制不同
- 只有末态粒子静止时，相对论因子趋于1，两种机制才能给出一致振幅

$$D \to f_2\pi^0 \to \pi^+\pi^-\pi^0$$

两个过程都是D波

$$\begin{aligned} Z &= D^0_{0\lambda}(0,\theta_0\phi_0)f_\lambda(\gamma)D^{*2}_{\lambda 0}(0\theta\phi \\ &= (\frac{2}{3}\gamma^2 + \frac{1}{3})d^2_{00}(\theta) \\ &= \boxed{(\frac{2}{3}\gamma^2 + \frac{1}{3})(\frac{3\cos^2\theta - 1}{2})}, \end{aligned}$$

$$\begin{aligned} Z &= \tilde{T}^{(2)}_{\mu\nu}(p_a + p_b + p_c)\tilde{t}^{\mu\nu(2)}(p_a + p_b) \\ &= \left[(p_a + p_b - p_c)_i(p_a + p_b - p_c)_j - \frac{1}{3}\delta_{ij}(p_a + p_b - p_c)^2\right] \\ &\quad \left[(p_a - p_b)^i(p_a - p_b)^j - \frac{1}{3}\delta^{ij}(p_a - p_b)^2\right] \\ &= [(p_a + p_b - p_c)\cdot(p_a - p_b)]^2 + \frac{1}{3}(p_a + p_b - p_c)^2(p_a - p_b)^2 \\ &\quad -\frac{1}{3}(p_a + p_b - p_c)^2(p_a - p_b)^2 - \frac{1}{3}(p_a + p_b - p_c)^2(p_a - p_b)^2 \\ &= [(p_a + p_b - p_c)\cdot(p_a - p_b)]^2 - \frac{1}{3}(p_a + p_b - p_c)^2(p_a - p_b)^2 \\ &= 16p^2q^2\gamma^2\cos^2\theta - \frac{1}{3}16p^2q^2(\sin^2\theta + \gamma^2\cos^2\theta) \\ &= \boxed{\frac{64}{3}\times p^2q^2\left[(\frac{2}{3}\gamma^2 + \frac{1}{3})(\frac{3\cos^2\theta}{2}) - \frac{1}{2}\right]}\ , \end{aligned}$$

5.3.1 $\psi \to \rho^0\pi^0 \to \pi^+\pi^-\pi^0$

第一个例子，我们采用相空间产生子，产生了$\psi \to \rho^0\pi^0 \to \pi^+\pi^-\pi^0$ 过程的相空间事例，这时Breit-Wigner分布已经包含在内了，但是不包含势垒因子和角分布。

5.3.1.0.1 截面 我们先把每个事例的截面计算出来（去掉Breit-Wigner），看看两种理论描述的计算结果，并求它们的比值，下面是打印出的数值信息：Helicity计算的截面、张量方法计算的截面和它们的比值。它们在计算精度内完全一致。

Helicity: 0.353986	L-S: 0.353986	Ratio: 1.000000
Helicity: 0.795525	L-S: 0.795525	Ratio: 1.000000
Helicity: 0.410571	L-S: 0.410571	Ratio: 1.000000
Helicity: 0.004812	L-S: 0.004812	Ratio: 1.000000
Helicity: 0.704383	L-S: 0.704383	Ratio: 1.000000
Helicity: 0.480020	L-S: 0.480020	Ratio: 1.000000

不考虑能量依赖,$\psi \to \rho^0\pi^0$ 道的完整角分布表达式为

$$
\begin{aligned}
(\frac{d\sigma}{d\Phi})_H &\propto |\mathcal{C}_1|^2 \sum_M \left| D^{1\star}_{M1}(\theta\phi)D^{1\star}_{10}(\theta'\phi') - D^{1\star}_{M-1}(\theta\phi)D^{1\star}_{-10}(\theta'\phi')\right|^2 \\
&\propto \frac{1}{2}[(1+\cos^2\theta)\sin^2\theta' + \sin^2\theta\sin^2\theta'\cos 2\phi']
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\sigma}{d\Phi} &\propto \frac{|\mathcal{C}_2|^2}{M_\psi^2}\sum_{\mu=1}^{2} U_\rho^\mu U_\rho^{\star\mu} \\
&= |\mathcal{C}_2|^2 \epsilon_{ijk} p_1^j p_2^k \epsilon^{imn} p_{1m} p_{2n} \\
&= |\mathcal{C}_2|^2 (\vec{p}_1 \times \vec{p}_2)^2_{xy} .
\end{aligned}
$$

Covariant tensor： Jpsi -> ll

From GPUPWA

```
47.7413685374 -4.4408920985e-16
10.4568626378 -2.36501246417
58.4460733319 -13.2186580909
21.203377569 -2.21566130381
3.37383239803 -1.11022302463e-16
13.4563560152 0
2.13571964814 0.25385932473
75.2110071338 1.7763568394e-15
26.5711325076 3.15834045052
59.7943684467 1.11022302463e-16
Number of waves to be read from file : 11
Amp_total=1.63063e+06
```

From C++ program

```
i=11 j=11 Fu=(47.7414  ,-5.28442e-16)
i=11 j=12 Fu=(10.4569  ,-2.36501)
i=11 j=13 Fu=(58.4461  ,-13.2187)
i=11 j=14 Fu=(21.2034  ,-2.21566)
i=12 j=12 Fu=(3.37383  ,-1.12757e-16)
i=12 j=13 Fu=(13.4564  ,-4.44306e-16)
i=12 j=14 Fu=(2.13572  ,0.253859)
i=13 j=13 Fu=(75.211  ,-9.70903e-17)
i=13 j=14 Fu=(26.5711  ,3.15834)
i=14 j=14 Fu=(59.7944  ,4.68375e-17)
xsec=1.63063e+06
```

Covariant tensor： Jpsi -> KK

```
259.289537061 0
-37.1523832459 8.40269710848
317.42817103 -71.7922389314
199.057581041 -20.8006567873
5.69008495224 -2.22911966663e-16
-47.8093394825 1.74860126378e-15
-31.5265202057 -3.74735566883
408.480691255 -1.42108547152e-14
249.450133371 29.6505407282
212.834246139 3.5527136788e-15
Number of waves to be read from file : 11
Amp_total=1.49048e+06
```

```
i=11 j=11 Fu=(259.29  ,1.39325e-16)
i=11 j=12 Fu=(-37.1524  ,8.4027)
i=11 j=13 Fu=(317.428  ,-71.7922)
i=11 j=14 Fu=(199.058  ,-20.8006)
i=12 j=12 Fu=(5.69008  ,2.22045e-16)
i=12 j=13 Fu=(-47.8093  ,-1.78977e-15)
i=12 j=14 Fu=(-31.5265  ,-3.74736)
i=13 j=13 Fu=(408.481  ,-1.42299e-14)
i=13 j=14 Fu=(249.45  ,29.6505)
i=14 j=14 Fu=(212.834  ,7.57424e-16)
xsec=1.49048e+06
```

Helicity formalism： Jpsi -> ll

From GPUPWA

```
-------------------------------------------------------
m_MY=-1 m_Ml=-1
coeff=(-0.085568,0.063909)
m_gflag=1  zcp_g1 = (0.1571,-0.09192)
m_gflag=2  zcp_g2 = (-0.0064798,0.014174)
m_gflag=3  zcp_g3 = (-0.070533,-0.09795)
m_gflag=4  zcp_g4 = (0.033269,-0.0059885)
-------------------------------------------------------
m_MY=1 m_Ml=1
coeff=(-0.085568,0.063909)
m_gflag=1  zcm_g1 = (0.15722,0.092952)
m_gflag=2  zcm_g2 = (-0.028133,-0.036909)
m_gflag=3  zcm_g3 = (0.11168,-0.018455)
m_gflag=4  zcm_g4 = (0.1089,0.089588)
```

```
INTENSITY = 24548
```

From C++ program

```
-------------------------------------------------------
m_MY=-1 m_Ml=-1
coeff_Z1=(-0.0855681,0.0639089i)
m_gflag=1  zcp_g1 = (0.1571,-0.0919196i)
m_gflag=2  zcp_g2 = (-0.00647978,0.0141736i)
m_gflag=3  zcp_g3 = (-0.0705334,-0.0979497i)
m_gflag=4  zcp_g4 = (0.0332692,-0.00598851i)
-------------------------------------------------------
m_MY=1 m_Ml=1
coeff_Z1=(-0.0855681,0.0639089i)
m_gflag=1  zcm_g1 = (0.157222,0.0929524i)
m_gflag=2  zcm_g2 = (-0.0281329,-0.0369086i)
m_gflag=3  zcm_g3 = (0.111683,-0.0184547i)
m_gflag=4  zcm_g4 = (0.108899,0.0895882i)
```

```
xsec=24546.2
```

Covariant tensor： Jpsi -> KK

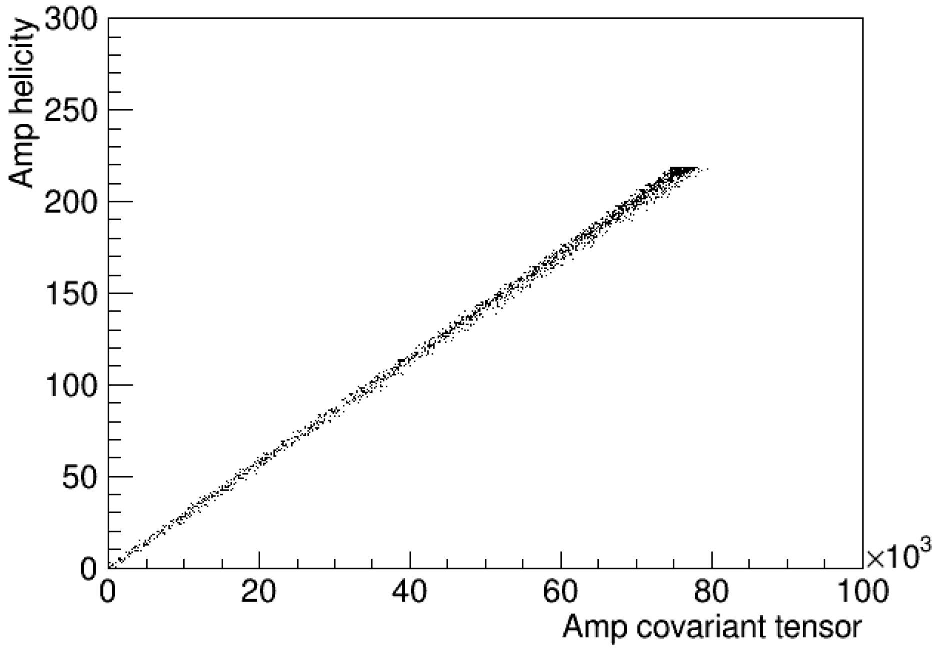
```
-------------------------------------------------------
m_MY=-1 m_Ml=-1
coeff=(-0.085568,0.063909)
m_gflag=1  zcp_g1 = (-0.23417,0.01583)
m_gflag=2  zcp_g2 = (-0.088847,0.075112)
m_gflag=3  zcp_g3 = (-0.11218,-0.012366)
m_gflag=4  zcp_g4 = (-0.028183,0.053312)
-------------------------------------------------------
m_MY=1 m_Ml=1
coeff=(-0.085568,0.063909)
m_gflag=1  zcm_g1 = (0.23402,0.016754)
m_gflag=2  zcm_g2 = (0.059949,0.016432)
m_gflag=3  zcm_g3 = (-1.0362,0.069713)
m_gflag=4  zcm_g4 = (-0.033585,0.17932)
```

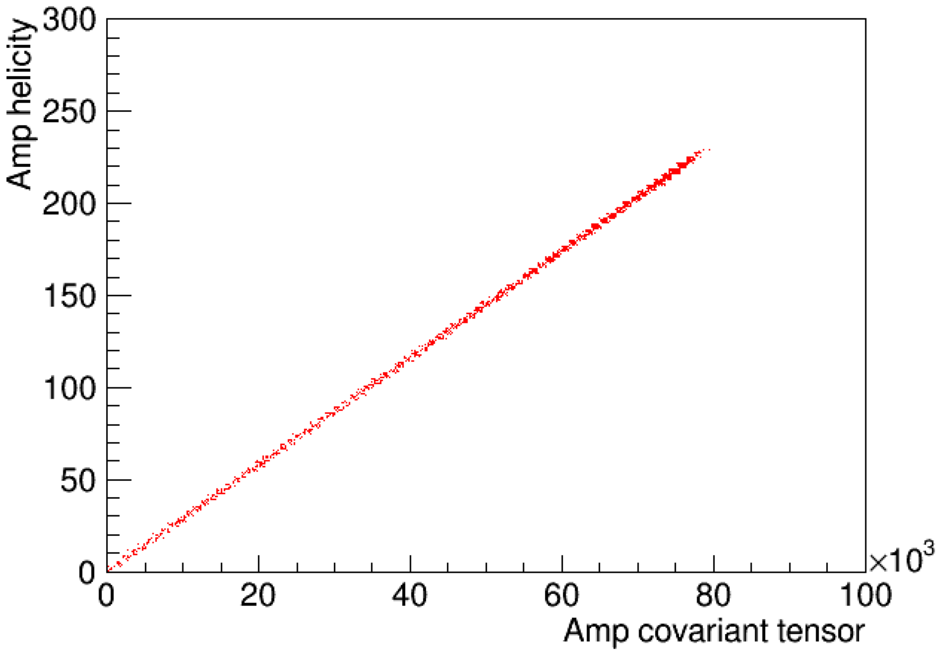
```
INTENSITY = 20700
```

```
m_MY=-1 m_Ml=-1
coeff_Z1=(-0.0855681,0.0639089i)
m_gflag=1  zcp_g1 = (-0.234166,0.0158296i)
m_gflag=2  zcp_g2 = (-0.0888475,0.0751116i)
m_gflag=3  zcp_g3 = (-0.112182,-0.0123659i)
m_gflag=4  zcp_g4 = (-0.0281831,0.0533115i)
-------------------------------------------------------
m_MY=1 m_Ml=1
coeff_Z1=(-0.0855681,0.0639089i)
m_gflag=1  zcm_g1 = (0.234018,0.016754i)
m_gflag=2  zcm_g2 = (0.0599488,0.0164322i)
m_gflag=3  zcm_g3 = (-1.03617,0.0697131i)
m_gflag=4  zcm_g4 = (-0.0335846,0.179321i)
```

```
xsec=20697.2
```

Jpsi -> KK: Only Zc SS component



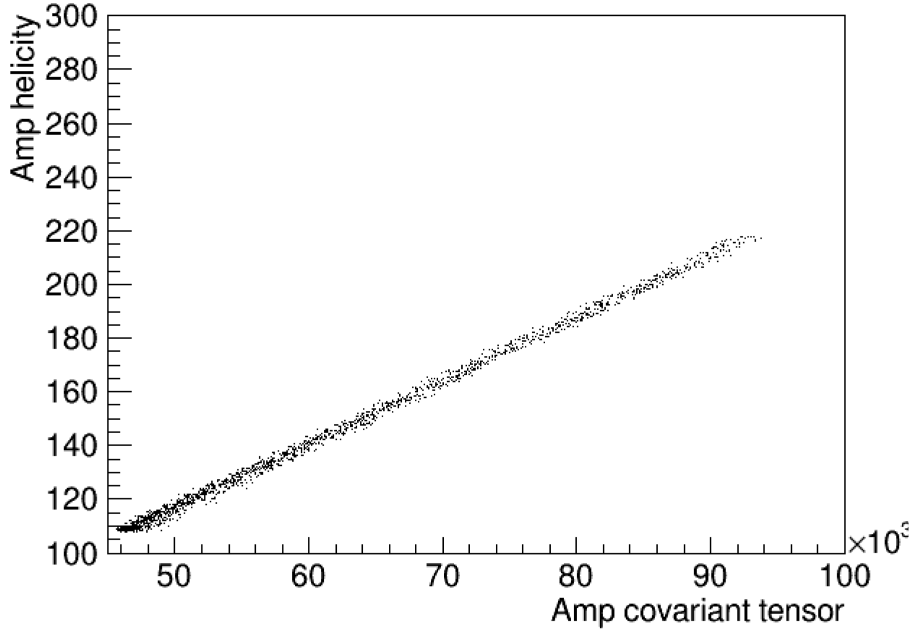


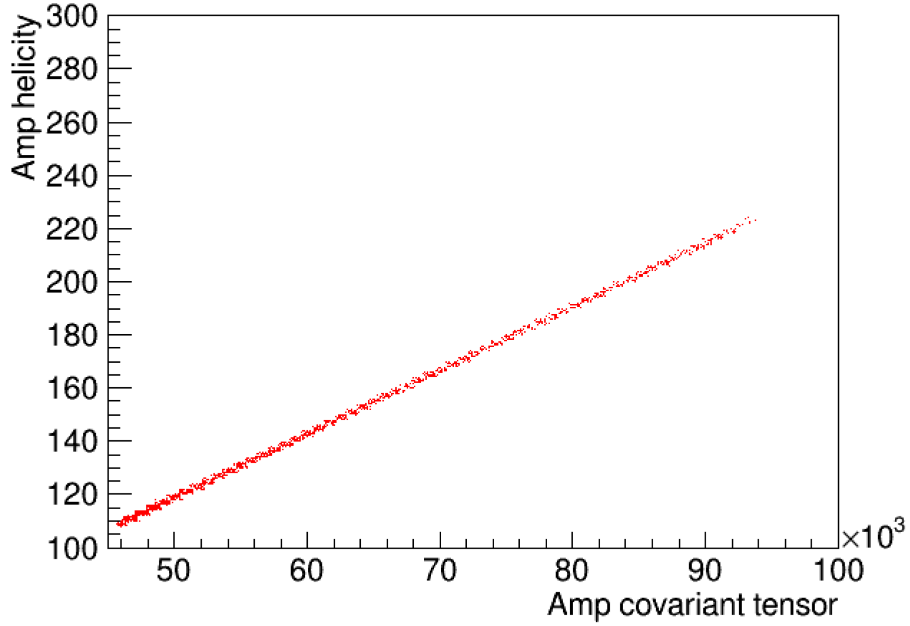
xsec_cov=38535.8	xsec_hel=112.179	R=343.522
xsec_cov=60314.8	xsec_hel=166.832	R=361.531
xsec_cov=73963.2	xsec_hel=214.087	R=345.482
xsec_cov=3927.84	xsec_hel=11.4202	R=343.939
xsec_cov=18518.3	xsec_hel=54.6152	R=339.068

xsec_cov=38535.8	xsec_hel_RF=111.923	R=344.306
xsec_cov=60314.8	xsec_hel_RF=175.165	R=344.332
xsec_cov=73963.2	xsec_hel_RF=214.818	R=344.306
xsec_cov=3927.84	xsec_hel_RF=11.4071	R=344.332
xsec_cov=18518.3	xsec_hel_RF=53.7844	R=344.306

- The ratio of amplitudes from covariant tensor formalism and helicity formalism is almost a constant for only Zc SS wave component
- If the relativistic factor is considered, the amplitude ratio is more stable

Jpsi -> ll： Only Zc SS component



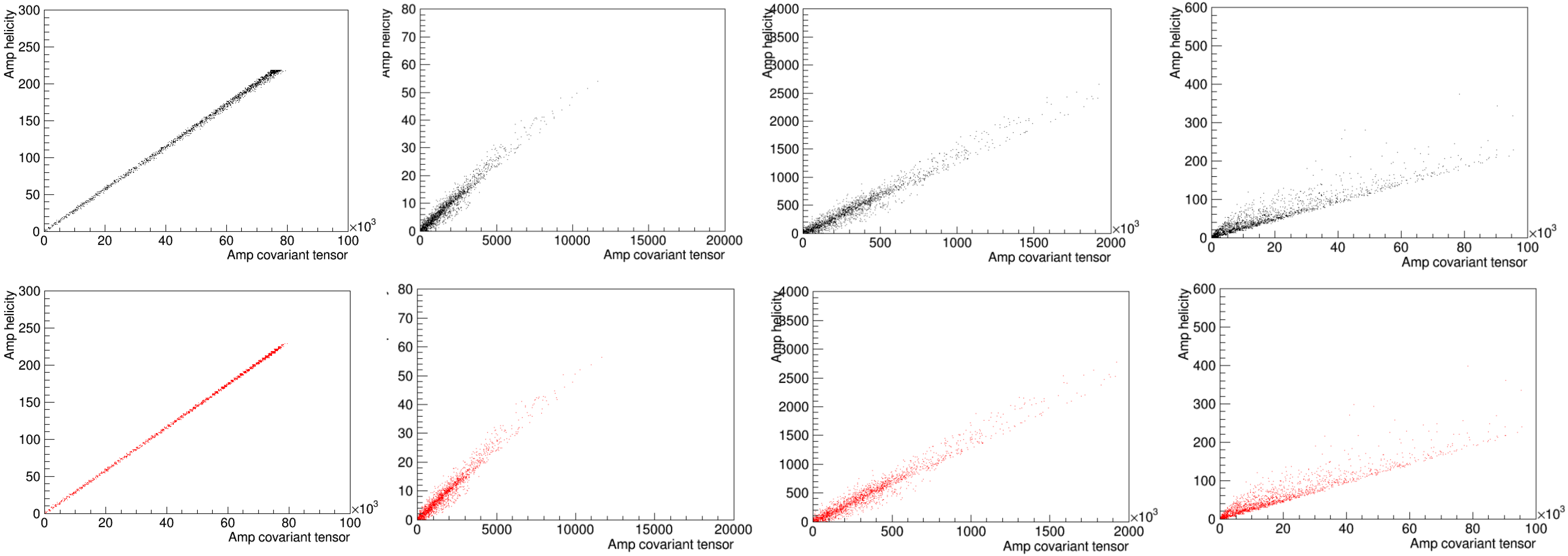


xsec_cov=69738.5	xsec_hel=161.882	R=430.797
xsec_cov=56658.7	xsec_hel=134.551	R=421.094
xsec_cov=47518.5	xsec_hel=110.925	R=428.384
xsec_cov=91253.7	xsec_hel=212.259	R=429.916
xsec_cov=81966.6	xsec_hel=190.662	R=429.906

xsec_cov=69738.5	xsec_hel_RF=165.977	R=420.17
xsec_cov=56658.7	xsec_hel_RF=135.051	R=419.537
xsec_cov=47518.5	xsec_hel_RF=112.773	R=421.365
xsec_cov=91253.7	xsec_hel_RF=217.526	R=419.508
xsec_cov=81966.6	xsec_hel_RF=195.262	R=419.777

- The ratio for $\psi \rightarrow l^+l^-$ is close to a constant as well
- The relativistic factor also makes the ratio more stable

Jpsi -> KK



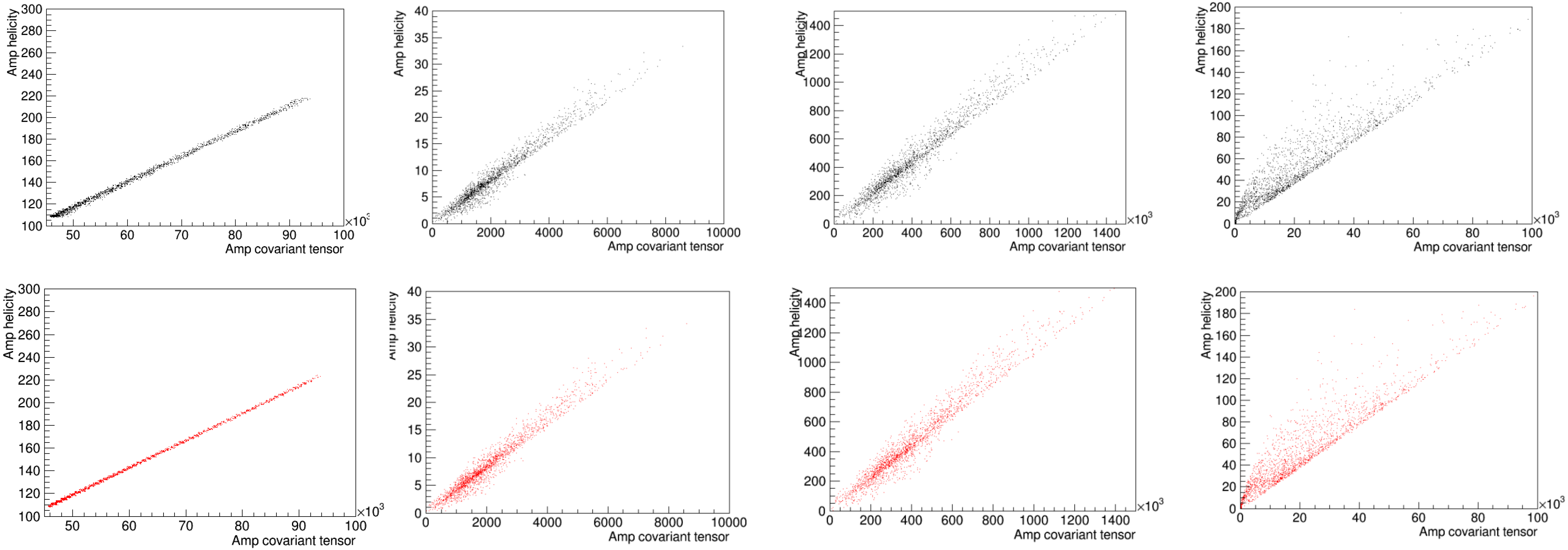
Zc SS:
R max = 344.336
R min = 344.305

Zc SD
R max = 3889.22
R min = 2.96714

Zc DS
R max = 105320
R min = 19.2489

Zc DD:
R max = 644.929
R min = 43.0134

Jpsi -> ll



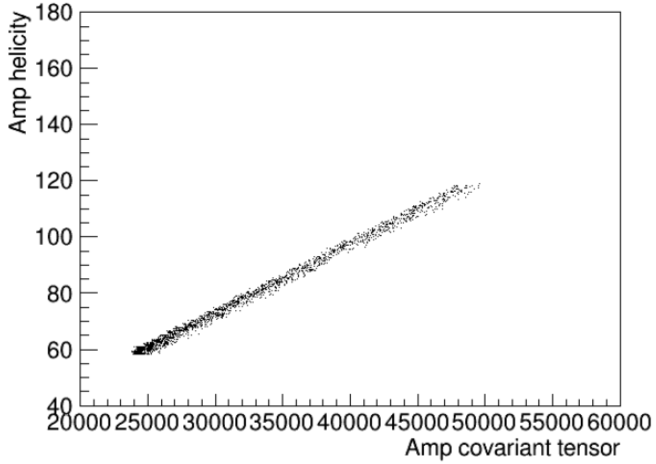
Zc SS:
R max = 421.777
R min = 419.452

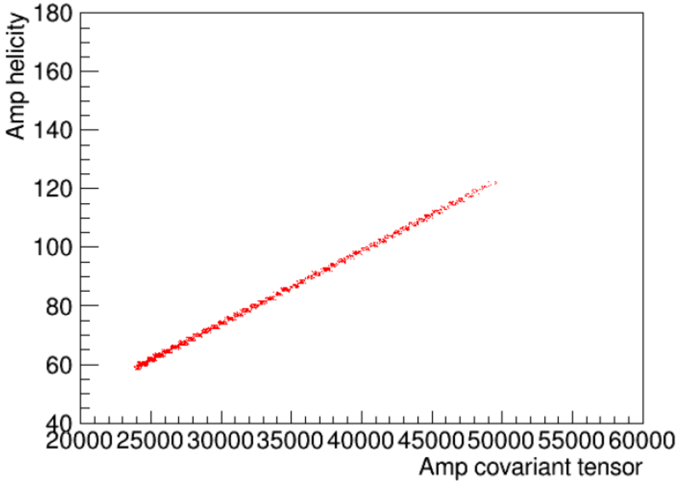
Zc SD
R max = 820.681
R min = 75.9445

Zc DS
R max = 4593.4
R min = 368.43

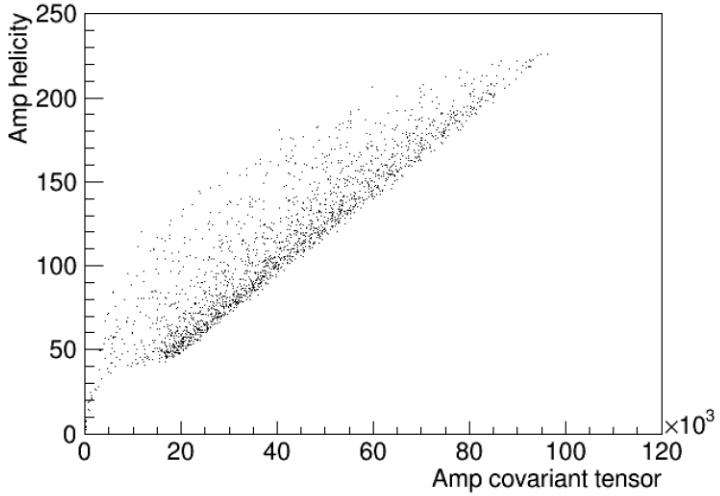
Zc DD:
R max = 533.565
R min = 67.3278

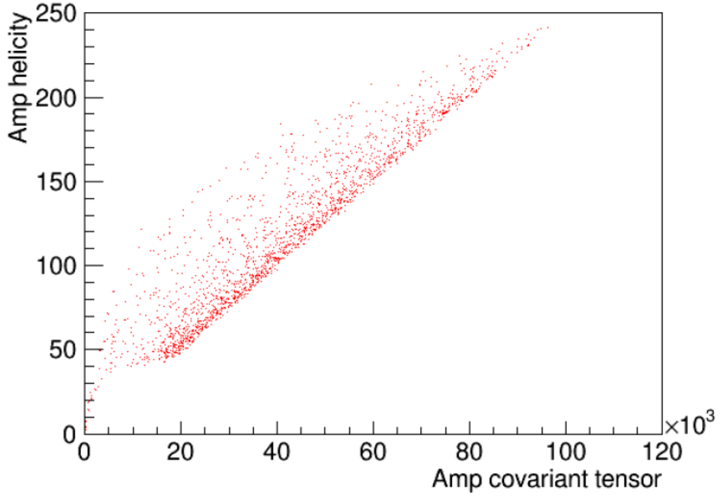
Jpsi -> ll



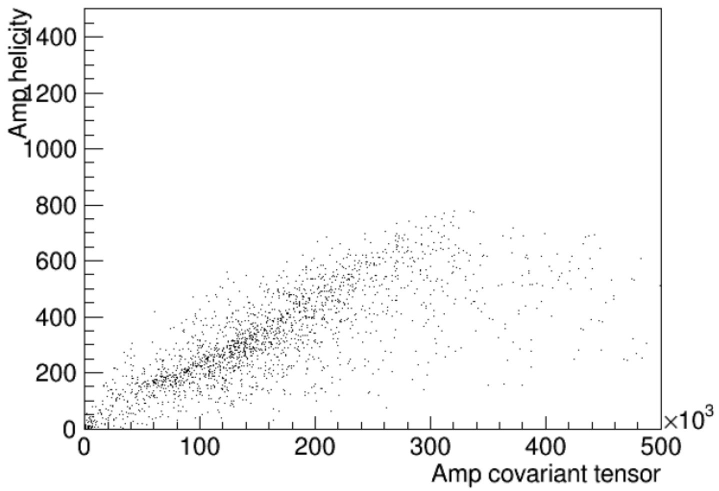


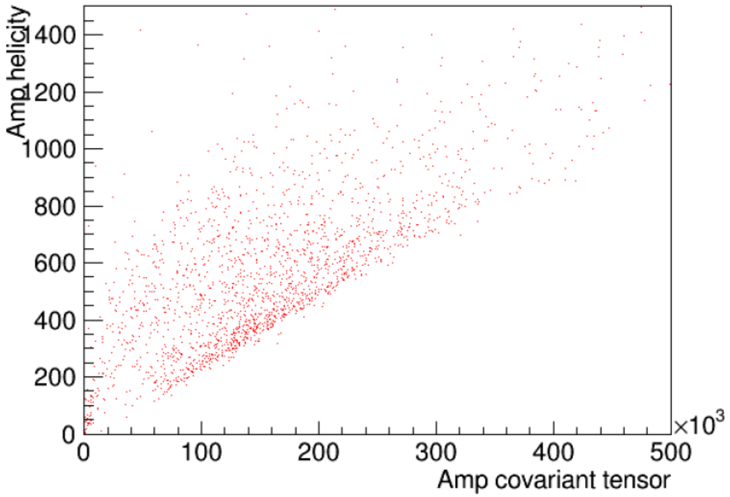
F0500 SS





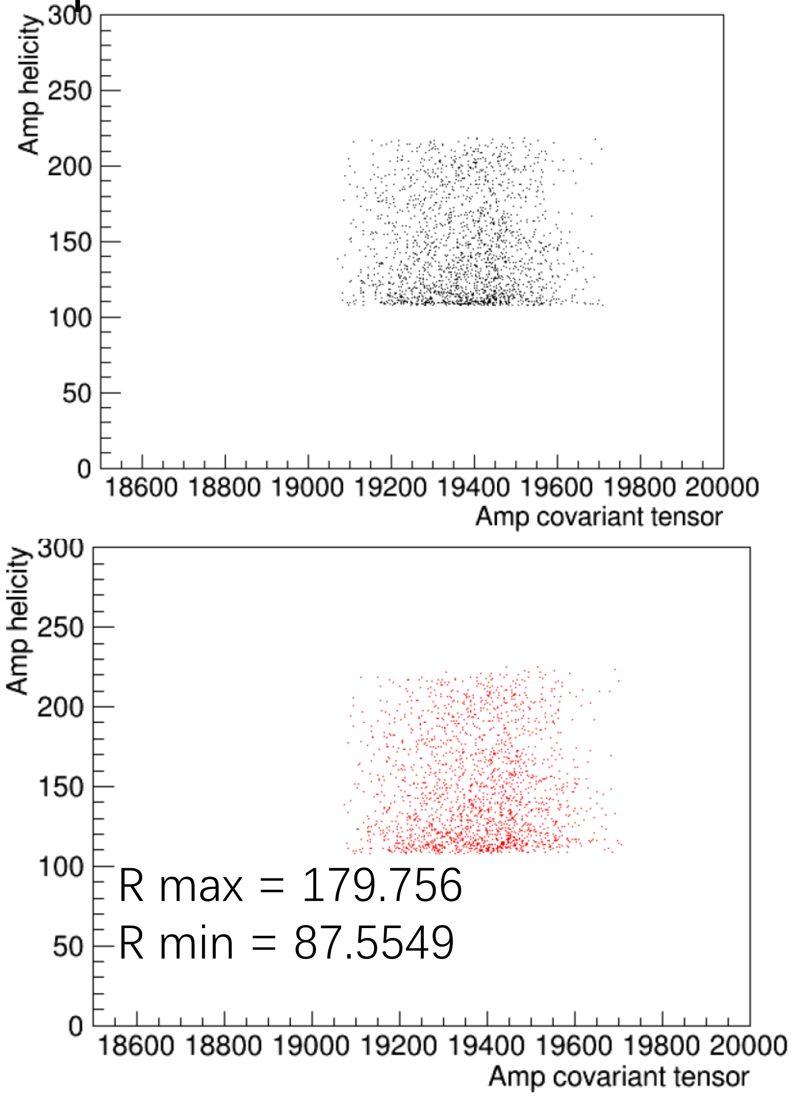
F0500 DS



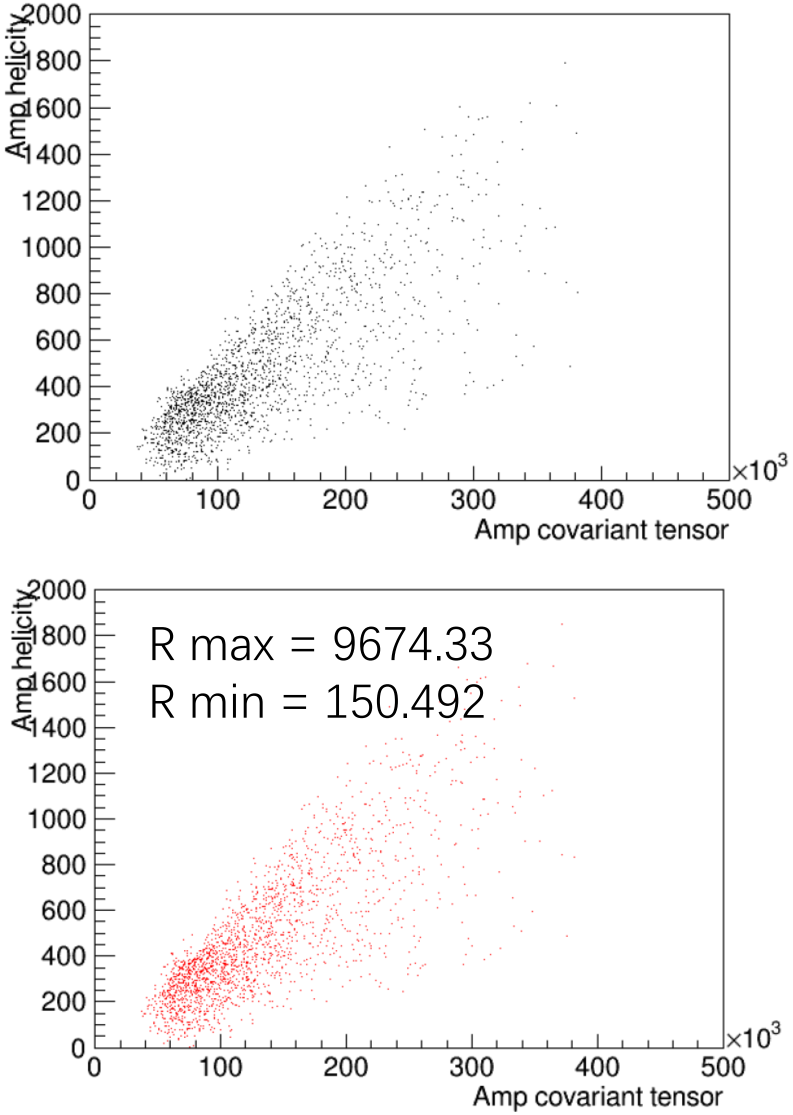


F21270 SD

No Jpsi part



Zc SS



Zc DS

- Without $\psi \to l^+ l^-$, event the amplitude ratio of Zc SS component is not a constant

- Two body decay

$$a(J_a,\eta_a) \rightarrow b(J_b,\eta_b) + c(J_c,\eta_c)$$

$$A^{J_a}_{\lambda_b,\lambda_c}(\theta,\phi;M) = N_{J_a} \boxed{F^{J_a}_{\lambda_b,\lambda_c}} D^{J_a *}_{M,\lambda}(\phi,\theta,0), (\lambda = \lambda_b - \lambda c)$$

$F^{J}_{\lambda_b,\lambda_a}$ is helicity decay amplitude

$$F^{J_a}_{\lambda_b,\lambda_c} = \sum_{ls} (\frac{2l+1}{2J_a+1})^{1/2} < l0s\lambda|J_a\lambda >< s_b\lambda_b s_c - \lambda_c|s\lambda > G^{J_a}_{ls} r^l B_l(r)$$

$$F^{J}_{\lambda_1\lambda_2} = \eta\eta_1\eta_2(-)^{J-s_1-s_2} F^{J}_{-\lambda_1-\lambda_2}$$

$$G^J_{ls} = 4\pi \left(\frac{w}{p}\right)^{\frac{1}{2}} \langle JMls|\mathcal{M}|JM\rangle$$

- Sequential decays: $Y \rightarrow \pi^+\pi^- J/\psi,\ J/\psi \rightarrow l^+l^-$

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i}} A_{R_i} \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} \boxed{F^J_1 D^J_1} \cdot \boxed{F^J_2 D^J_2} \cdot \boxed{F^J_3 D^J_3} \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} (\sum_{LS} G_{LS} \cdot CG_1 \cdot B_l) D^J_1 \cdot (\sum_{ls} G_{ls} \cdot CG_2 \cdot B_l) D^J_2 \cdot F^J_3 D^J_3 \right|^2$$

$\Rightarrow$

$$\sum_{\lambda_Y,\Delta\lambda_l} | \sum_{\lambda_{R_i},\lambda_\psi} (G_{LS} \cdot CG_1 \cdot B_l) D^J_1 \cdot (G_{ls} \cdot CG_2 \cdot B_l) D^J_2 \cdot F^J_3 D^J_3 |^2$$

- G_{LS} is LS coupling partial wave amplitude
- With a definite set of helicity of (b,c), G_{LS} should be same
- In fit, G_{LS} is float parameter
- To obtain the contribution of a LS wave component

For the last step $J/\psi \rightarrow \ell^+\ell^-$, at the relativistic limit, by QED calculation, $F^{J_{J/\psi}}_{1/2,1/2} = F^{J_{J/\psi}}_{-1/2.-1/2} \approx 0$. Here we define $\Delta\lambda_\ell = \lambda_{\ell+} - \lambda_{\ell-}$, we can see only $\Delta\lambda_\ell = \pm 1$ is allowed.

$$\sum_{\lambda_Y,\Delta\lambda_l} |\sum_{\lambda_{R_i},\lambda_\psi} (G_{LS}\cdot CG_1\cdot B_l)D_1^J\cdot(G_{ls}\cdot CG_2\cdot B_l)D_2^J\cdot F_3^J D_3^J|^2$$

$$Decay: Y \to \psi \quad f_0 \qquad f_0 \to \pi^+ \quad \pi^-$$

$$J^{PC}: 1^{--} \to 1^{--} \quad 0^+ \qquad 0^+ \to 0^- \quad 0^-$$

$L=0(S-wave)$ $\quad F^1_{1,0} = +g_{01}\sqrt{\frac{1}{3}}r^0 + g_{21}\sqrt{\frac{1}{6}}r^2$ $\qquad S-wave$

$L=2(D-wave)$ $\quad F^1_{0,0} = +g_{01}\sqrt{\frac{1}{3}}r^0\gamma_s - g_{21}\sqrt{\frac{2}{3}}r^2\gamma_s$

Two components: SS and DS

$$Decay: Y \to Z_c \quad \pi \qquad Z_c \to \psi \quad \pi$$

$$J^{PC}: 1^{--} \to 1^+ \quad 0^- \qquad 1^+ \to 1^{--}0^-$$

$L=0(S-wave)$ $\qquad L=0(S-wave)$

$L=2(D-wave)$ $\qquad L=2(D-wave)$

$$F^1_{1,0} = +g_{01}\sqrt{\frac{1}{3}}r^0 + g_{21}\sqrt{\frac{1}{6}}r^2 \qquad F^1_{1,0} = +g_{01}\sqrt{\frac{1}{3}}r^0 + g_{21}\sqrt{\frac{1}{6}}r^2$$

$$F^1_{0,0} = +g_{01}\sqrt{\frac{1}{3}}r^0\gamma_s - g_{21}\sqrt{\frac{2}{3}}r^2\gamma_s \qquad F^1_{0,0} = +g_{01}\sqrt{\frac{1}{3}}r^0\gamma_s - g_{21}\sqrt{\frac{2}{3}}r^2\gamma_s$$

four components:
SS, SD, DS and DD

$$A = \phi_\mu(m_1)\omega_\nu^*(m_2)A^{\mu\nu} = \phi_\mu(m_1)\omega_\nu^*(m_2)\sum_i \Lambda_i \boxed{U_i^{\mu\nu}}$$

$$U^{\mu\nu} = (A_{LS})(A_{ls})$$

- $U^{\mu\nu}$ is the partial wave amplitude constructed according to LS coupling

$$\frac{d\sigma}{d\Phi_n} \propto -\frac{1}{2}\sum_{\mu=1}^{2} \tilde{g}_{\nu\nu'}(p_{(\psi)})A^{\mu\nu}A^{*\mu\nu'}$$
$$= -\frac{1}{2}\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2} U_i^{\mu\nu}\tilde{g}_{\nu\nu'}(p_{(\psi)})U_j^{*\mu\nu'}$$

$$\begin{array}{llllll} Decay: Y & \to\psi & f_0 & f_0 & \to\pi^+ & \pi^- \\ J^{PC}: 1^{--} & \to 1^{--} & 0^+ & 0^+ & \to 0^- & 0^- \end{array}$$

$$U^{\mu\nu}_{(Y\to\psi(2S)f_0)SS} = \langle\psi f_0|01\rangle = g^{\mu\nu}f^{(f_0)}_{(12)}$$
$$U^{\mu\nu}_{(Y\to\psi(2S)f_0)DS} = \langle\psi f_0|21\rangle = \tilde{T}^{(2)\mu\nu}_{(\psi f_0)}f^{(f_0)}_{(12)}$$

$$\begin{array}{lllllll} Decay: Y & \to Z_c & \pi & Z_c & \to & \psi & \pi \\ J^{PC}: 1^{--} & \to 1^+ & 0^- & 1^+ & \to & 1^{--} & 0^- \end{array}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)SS} = \tilde{g}^{\mu\nu}_{(Z_c^+)}f^{(Z_c^+)}_{(01)} + \tilde{g}^{\mu\nu}_{(Z_c^-)}f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)SD} = \tilde{t}^{(2)\mu\nu}_{(\psi\pi^+)}f^{(Z_c^+)}_{(01)} + \tilde{t}^{(2)\mu\nu}_{(\psi\pi^-)}f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)DS} = \tilde{T}^{(2)\mu\lambda}_{(Z_c^+\pi^-)}\tilde{g}_{(Z_c^+)\lambda\sigma}g^{\sigma\nu}f^{(Z_c^+)}_{(01)} + \tilde{T}^{(2)\mu\lambda}_{(Z_c^-\pi^+)}\tilde{g}_{(Z_c^-)\lambda\sigma}g^{\sigma\nu}f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)DD} = \tilde{T}^{(2)\mu\lambda}_{(Z_c^+\pi^-)}\tilde{t}^{(2)}_{(\psi\pi^+)\lambda\sigma}g^{\sigma\nu}f^{(Z_c^+)}_{(01)} + \tilde{T}^{(2)\mu\lambda}_{(Z_c^-\pi^+)}\tilde{t}^{(2)}_{(\psi\pi^-)\lambda\sigma}g^{\sigma\nu}f^{(Z_c^-)}_{(02)}$$

$$A = \phi_\mu(m_1)\omega^*_\nu(m_2)A^{\mu\nu} = \phi_\mu(m_1)\omega^*_\nu(m_2)\sum_i \Lambda_i U_i^{\mu\nu}$$

$$\mathcal{B} = ie\omega_\beta(m_2)\bar{u}_{e^-}\gamma^\beta \nu_{e^+}\frac{em_\psi}{f_\psi}$$

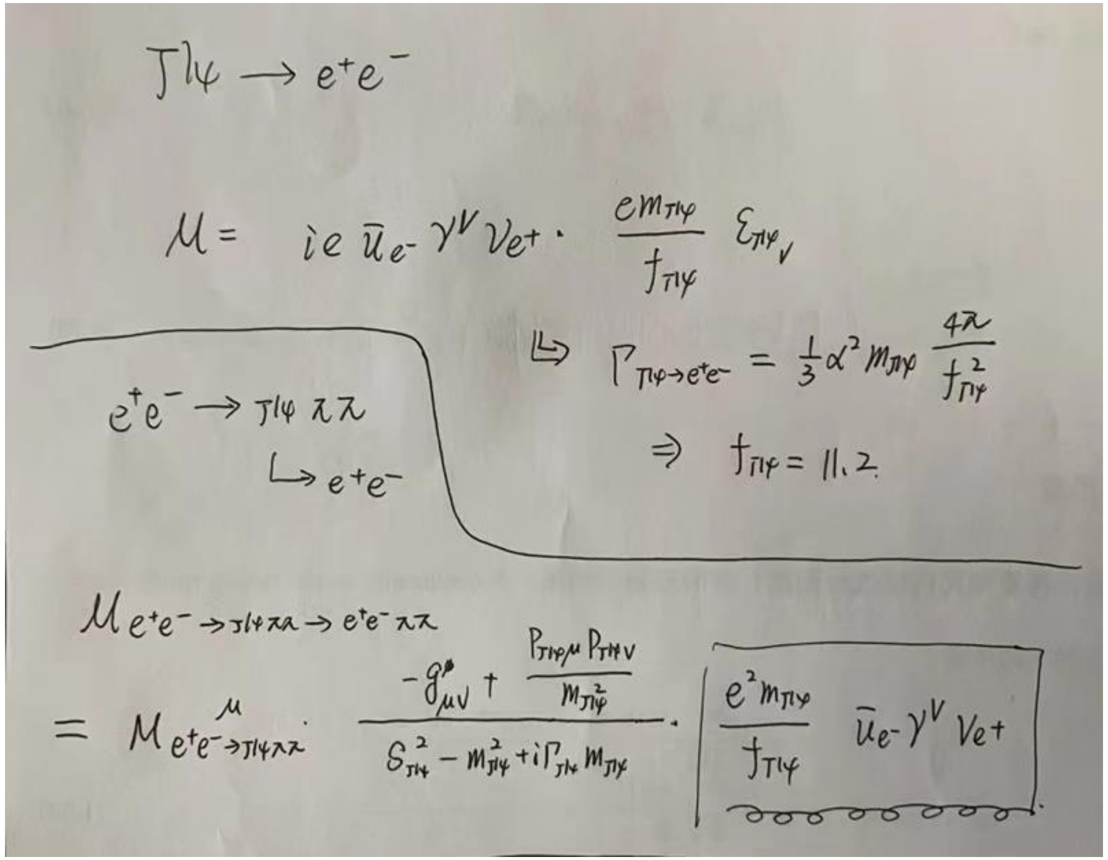
- $\omega_\beta(m_2)$是ψ的极化矢量
- $f_\psi = 11.2$是常数
- $\bar{u}_{e^-}(\nu_{e^+})$是电子的极化矢量

$$\frac{d\sigma}{d\Phi_n} \propto -\frac{1}{2}\sum_{\mu=1}^{2}\tilde{g}_{\nu\nu'}(p_{(\psi)})A^{\mu\nu}A^{*\mu\nu'}$$

$$= -\frac{1}{2}\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2}U_i^{\mu\nu}\tilde{g}_{\nu\nu'}(p_{(\psi)})U_j^{*\mu\nu'}$$

$$\frac{d\sigma}{d\Phi_n} \propto 2|ie\frac{em_\psi}{f_\psi}|^2\sum_{\mu=1}^{2}\tilde{g}^{\psi\to ll}_{\nu\nu'}A^{\mu\nu}A^{*\mu\nu'}$$

$$= 2|ie\frac{em_\psi}{f_\psi}|^2\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2}U_i^{\mu\nu}\tilde{g}^{\psi\to ll}_{\nu\nu'}U_j^{*\mu\nu'}$$



$$\sum_{m_2=1}^{3}\omega_\nu(m_2)\omega^*_{\nu'}(m_2) = -g_{\nu\nu'} + \frac{p_{(\psi)\nu}p_{(\psi)\nu'}}{p_\psi^2} \equiv -\tilde{g}_{\nu\nu'}(p_{(\psi)})$$

$$\tilde{g}^{\psi\to ll}_{\nu\nu'} = \tilde{g}_{\nu\beta}(p_{(\psi)})\tilde{g}_{\nu'\beta'}(p_{(\psi)})\left[p^\beta p'^{\beta'} + p'^\beta p^{\beta'} - g^{\beta\beta'}(p\cdot p' + m_l^2)\right]$$

- Sequential decays: $Y \to \pi^+\pi^- J/\psi,\ J/\psi \to l^+l^-$
  - Assume leptons have 0 spin

$$\psi \to \pi^+\pi^-\ ,\ K^+K^-$$

这个过程是一个$1^- \to 0^-0^-$的过程，因为$\eta_a\eta_b\eta_c = -1$和角动量守恒，因此只有1个分波：P-wave，所以独立的Helicity振幅也只有一个。即$F = F^1_{00}$，不变振幅：

$$\mathcal{M}(M) = F D^{1*}_{M,0}(\phi,\theta,0)\ ,$$

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i}} A_{R_i} \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} F_1^J D_1^J \cdot F_2^J D_2^J \cdot F_3^J D_3^J \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} (\sum_{LS} G_{LS} \cdot CG_1 \cdot B_l) D_1^J \cdot (\sum_{ls} G_{ls} \cdot CG_2 \cdot B_l) D_2^J \cdot F_3^J D_3^J \right|^2$$

- Come from the boost of frame
- $\Delta\lambda_l = 0$

$$\frac{d\sigma}{d\Phi} = \sum_{\lambda_Y,\Delta\lambda_l} \Big| \sum_{\lambda_{Z_c},\lambda_{R_j},\lambda_{J/\psi}} [A_{R_j}(\lambda_Y,\lambda_{R_j},\lambda_{J/\psi},\lambda_{l^+},\lambda_{l^-}) + e^{i\Delta\lambda_l\alpha_l} A_{Z_c^0}(\lambda_Y,\lambda_{Z_c^0},\lambda_{J/\psi},\lambda_{l^+},\lambda_{l^-})] \Big|^2$$

In covariant tensor formalism:

$$\psi \rightarrow \phi\pi^+\pi^- \rightarrow \mathrm{K}^+\mathrm{K}^-\pi^+\pi^-$$

$$\begin{aligned}
U^{\mu}_{b_1SS} &= \tilde{g}^{\mu\nu}_{(123)}\tilde{t}^{(1)}_{(12)\nu}f^{(\phi)}_{(12)}f^{(b_1)}_{(123)} \\
&+\tilde{g}^{\mu\nu}_{(124)}\tilde{t}^{(1)}_{(12)\nu}f^{(\phi)}_{(12)}f^{(b_1)}_{(124)}, \\
U^{\mu}_{b_1SD} &= \tilde{t}^{(2)\mu\nu}_{(\phi3)}\tilde{t}^{(1)}_{(12)\nu}f^{(\phi)}_{(12)}f^{(b_1)}_{(123)} \\
&+\tilde{t}^{(2)\mu\nu}_{(\phi4)}\tilde{t}^{(1)}_{(12)\nu}f^{(\phi)}_{(12)}f^{(b_1)}_{(124)}, \\
U^{\mu}_{b_1DS} &= \tilde{T}^{(2)\mu\lambda}_{(b_14)}\tilde{g}_{(123)\lambda\nu}\tilde{t}^{(1)\nu}_{(12)}f^{(\phi)}_{(12)}f^{(b_1)}_{(123)} \\
&+\tilde{T}^{(2)\mu\lambda}_{(b_13)}\tilde{g}_{(124)\lambda\nu}\tilde{t}^{(1)\nu}_{(12)}f^{(\phi)}_{(12)}f^{(b_1)}_{(124)}, \\
U^{\mu}_{b_1DD} &= \tilde{T}^{(2)\mu\lambda}_{(b_14)}\tilde{t}^{(2)}_{(\phi3)\lambda\nu}\tilde{t}^{(1)\nu}_{(12)}f^{(\phi)}_{(12)}f^{(b_1)}_{(123)} \\
&+\tilde{T}^{(2)\mu\lambda}_{(b_13)}\tilde{t}^{(2)}_{(\phi4)\lambda\nu}\tilde{t}^{(1)\nu}_{(12)}f^{(\phi)}_{(12)}f^{(b_1)}_{(124)}.
\end{aligned}$$

$$\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_n} &= \frac{1}{2}\sum_{\mu=1}^{2}A^{\mu}A^{*\mu} = \\
&\frac{1}{2}\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2}U_i^{\mu}U_j^{*\mu} \equiv \sum_{i,j}P_{ij}\cdot F_{ij}
\end{aligned}$$

where

$$P_{ij} = P_{ji}^* = \Lambda_i\Lambda_j^*,$$

$$F_{ij} = F_{ji}^* = \frac{1}{2}\sum_{\mu=1}^{2}U_i^{\mu}U_j^{*\mu}.$$

第一级振幅:

$$A = \phi_\mu(m_1)\omega_\nu^*(m_2)A^{\mu\nu} = \phi_\mu(m_1)\omega_\nu^*(m_2)\sum_i \Lambda_i U_i^{\mu\nu}$$

第二级振幅:

$$B = \omega_\beta\tilde{t}^{(1)\beta}$$

考虑两级过程后总的振幅为:

$$\begin{aligned}
\mathcal{M} &= [\phi_\mu(m_1)\omega_\nu^*(m_2)A^{\mu\nu}][\omega_\beta\tilde{t}_\phi^{(1)\beta}] \\
&= \phi_\mu(m_1)A^{\mu\nu}(\omega_\nu^*\omega_\beta)\tilde{t}_\phi^{(1)\beta} \\
&= \phi_\mu(m_1)A^{\mu\nu}\tilde{g}_{(\phi)\nu\beta}\tilde{t}_\phi^{(1)\beta}
\end{aligned}$$

对上式求模方:

$$\begin{aligned}
\frac{d\sigma}{d\Phi_n} \propto &\frac{1}{2}\sum_{m_1}^{2}\phi_\mu(m_1)\phi^*_{\mu'}(m_1)A^{\mu\nu}A^{*\mu'\nu'}\tilde{g}_{\nu\beta}(p_{(\phi)})\tilde{g}_{\nu'\beta'}(p_{(\phi)})\tilde{t}_\phi^{(1)\beta}\tilde{t}_\phi^{(1)\beta'} \\
&\frac{1}{2}\sum_{\mu=1}^{2}A^{\mu\nu}A^{*\mu\nu'}\tilde{g}_{\nu\beta}(p_{(\phi)})\tilde{g}_{\nu'\beta'}(p_{(\phi)})\tilde{t}_\phi^{(1)\beta}\tilde{t}_\phi^{(1)\beta'}
\end{aligned}$$

- **令** $\tilde{g}^{\phi\to KK}_{\nu\nu'} = \tilde{g}_{\nu\beta}(p_{(\phi)})\tilde{g}_{\nu'\beta'}(p_{(\phi)})\tilde{t}_\phi^{(1)\beta}\tilde{t}_\phi^{(1)\beta'} = \tilde{t}^{(1)}_{(\phi)\nu}\tilde{t}^{(1)}_{(\phi)\nu'}$

最终在程序中计算的公式为:

$$\frac{d\sigma}{d\Phi_n} \propto -\frac{1}{2}\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2}U_i^{\mu\nu}\tilde{g}^{\phi\to KK}_{\nu\nu'}U_j^{*\mu\nu'}$$