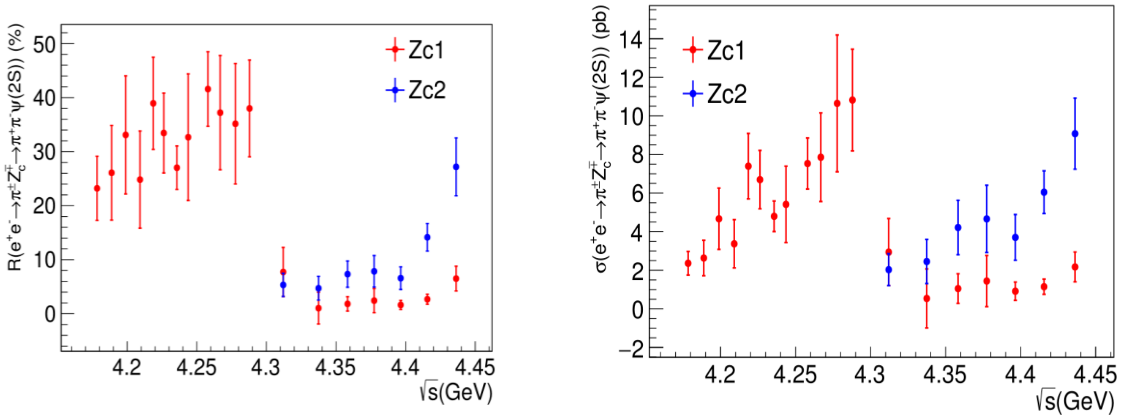
PWA Amplitude Check for helicity formalism and covariant tensor formalism

Xuhong Li, Xinyu Shan, Haiping Peng

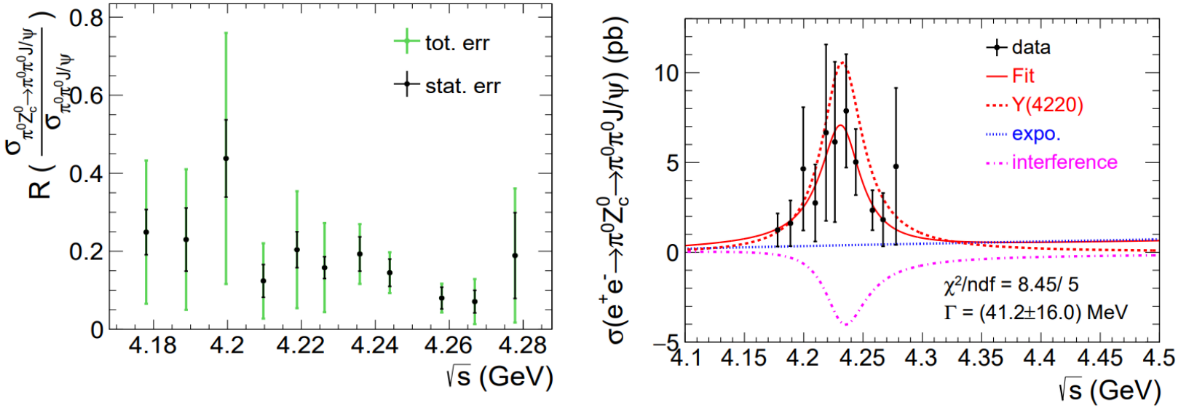
University of Science and Technology of China

Motivation

Covariant tensor formalism



Helicity formalism



- The $Z_c(3900)$ cross section line shapes from $e^+e^- \to \pi^{+/0}\pi^{-/0}J/\psi$ and $e^+e^- \to \pi^+\pi^-\psi(3686)$ are different
- To validate the analysis of $e^+e^- \to \pi^+\pi^-\psi(3686)$, checks on amplitude and program are needed

Helicity amplitude construction

- Two body decay

$$a(J_a,\eta_a) \to b(J_b,\eta_b) + c(J_c,\eta_c)$$

$$A^{J_a}_{\lambda_b,\lambda_c}(\theta,\phi;M) = N_{J_a} F^{J_a}_{\lambda_b,\lambda_c} D^{J_a *}_{M,\lambda}(\phi,\theta,0), (\lambda = \lambda_b - \lambda c)$$

- $F^{J}_{\lambda_b,\lambda_c}$ is helicity decay amplitude

$$F^{J_a}_{\lambda_b,\lambda_c} = \sum_{ls} \left(\frac{2l+1}{2J_a+1}\right)^{1/2} < l0s\lambda|J_a\lambda >< s_b\lambda_b s_c - \lambda_c|s\lambda > G^{J_a}_{ls} r^l$$

$$G^J_{ls} = 4\pi \left(\frac{w}{p}\right)^{\frac{1}{2}} \langle JMls|\mathcal{M}|JM\rangle$$

- ➢ G_{LS} is LS coupling partial wave amplitude
- ➢ With a definite set of helicity of (b,c), G_{LS} should be same
- ➢ In fit, G_{LS} is a float complex parameter

- Helicity coupling amplitudes depend on the relativistic factor for particles with spin 1 or higher

$$\xi_s(\lambda) \equiv f^{(1)}_{\lambda}(\gamma_s) = \begin{cases} [\chi^{(1)*}(\lambda)\cdot\omega(\lambda)] \\ 1 \quad \text{for } \lambda = \pm 1 \\ \gamma_s \quad \text{for } \lambda = 0, \end{cases}$$

$$f^{(2)}_{\lambda}(\gamma_s) = \begin{cases} 1 \quad \text{for } \lambda = \pm 2 \\ \gamma_s \quad \text{for } \lambda = \pm 1 \\ \frac{2}{3}\gamma_s^2 + \frac{1}{3} \quad \text{for } \lambda = 0. \end{cases}$$

Covariant helicity coupling amplitude →

$$F^J_{\lambda\nu} = \sum_{\ell S} g_{\ell S} A_{\ell S}(\lambda\nu),$$

where

$$A_{\ell S}(\lambda\nu) = \left(\frac{2\ell+1}{2J+1}\right)^{1/2} (\ell 0 S\delta|J\delta)(s\lambda\sigma-\nu|S\delta) \times W^n r^{\ell} f^s_{\lambda}(\gamma_s) f^{\sigma}_{\nu}(\gamma_\sigma),$$

PhysRevD.57.431 (1998) by S. U. Chung

Helicity amplitude construction

- Sequential decay: $Y \to \pi^+\pi^- J/\psi$, $J/\psi \to l^+l^-$

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i}} A_{R_i} \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} F_1 D_1 \cdot F_2 D_2 \cdot F_3 D_3 \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} (\sum_{LS} G_{LS} \cdot CG_1 \cdot B_l) D_1 \cdot (\sum_{ls} G_{ls} \cdot CG_2 \cdot B_l) D_2 \cdot F_3 D_3 \right|^2$$

For specific (LS, ls) wave component

$$\sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} (G_{LS} \cdot CG_1 \cdot B_l) D_1 \cdot (G_{ls} \cdot CG_2 \cdot B_l) D_2 \cdot F_3 D_3 \right|^2$$

$$\begin{array}{llll} Decay: Y & \to Z_c & \pi \\ J^{PC}: 1^{--} & \to 1^{+} & 0^{-} \end{array}$$

$L = 0$ (S-wave) $L = 2$ (D-wave)

$$F^1_{1,0} = +g_{01}\sqrt{\frac{1}{3}}r^0 + g_{21}\sqrt{\frac{1}{6}}r^2$$

$$F^1_{0,0} = +g_{01}\sqrt{\frac{1}{3}}r^0\gamma_s - g_{21}\sqrt{\frac{2}{3}}r^2\gamma_s$$

$$\begin{array}{lll} Z_c & \to \psi & \pi \\ 1^{+} & \to 1^{--} & 0^{-} \end{array}$$

$l = 0$ (S-wave) $l = 2$ (D-wave)

$$F^1_{1,0} = +g_{01}\sqrt{\frac{1}{3}}r^0 + g_{21}\sqrt{\frac{1}{6}}r^2$$

$$F^1_{0,0} = +g_{01}\sqrt{\frac{1}{3}}r^0\gamma_s - g_{21}\sqrt{\frac{2}{3}}r^2\gamma_s$$

Four partial waves: SS, SD, DS and DD

Covariant tensor amplitude construction

$Y \to \pi^+\pi^- J/\psi$

$$A = \phi_\mu(m_1)\omega_\nu^*(m_2)A^{\mu\nu} = \phi_\mu(m_1)\omega_\nu^*(m_2)\sum \Lambda_i U_i^{\mu\nu}$$

$J/\psi \to l^+ l^-$

$$\mathcal{B} = ie\omega_\beta(m_2)\bar{u}_{e^-}\gamma^\beta \nu_{e^+}\frac{em_\psi}{f_\psi}$$

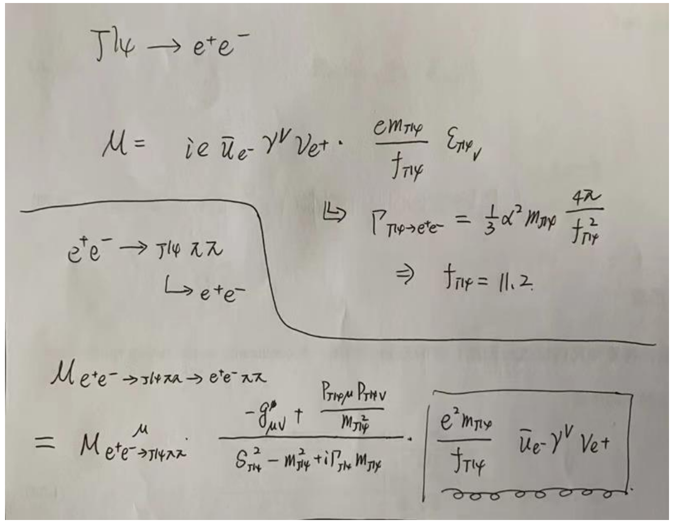
$$\frac{d\sigma}{d\Phi_n} \propto 2|ie\frac{em_\psi}{f_\psi}|^2 \sum_{\mu=1}^{2} \tilde{g}_{\nu\nu'}^{\psi\to ll} A^{\mu\nu} A^{*\mu\nu'}$$

$$= 2|ie\frac{em_\psi}{f_\psi}|^2 \sum_{i,j} \Lambda_i \Lambda_j^* \sum_{\mu=1}^{2} U_i^{\mu\nu} \tilde{g}_{\nu\nu'}^{\psi\to ll} U_j^{*\mu\nu'}$$

$$\tilde{g}_{\nu\nu'}^{\psi\to ll} = \tilde{g}_{\nu\beta}(p_{(\psi)})\tilde{g}_{\nu'\beta'}(p_{(\psi)})\left[p^\beta p'^{\beta'} + p'^\beta p^{\beta'} - g^{\beta\beta'}\left(p\cdot p' + m_l^2\right)\right]$$

- $U^{\mu\nu}$ is the partial wave amplitude constructed according to LS coupling

$$U^{\mu\nu} = (A_{LS})(A_{ls})$$



By Wu Jiajun from UCAS

$$\begin{array}{lllllll} Decay: Y & \to Z_c & \pi & & Z_c \to \psi & \pi \\ J^{PC}: 1^{--} & \to 1^{+} & 0^{-} & & 1^{+} \to 1^{--} & 0^{-} \end{array}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)SS} = \tilde{g}^{\mu\nu}_{(Z_c^+)} f^{(Z_c^+)}_{(01)} + \tilde{g}^{\mu\nu}_{(Z_c^-)} f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)SD} = \tilde{t}^{(2)\mu\nu}_{(\psi\pi^+)} f^{(Z_c^+)}_{(01)} + \tilde{t}^{(2)\mu\nu}_{(\psi\pi^-)} f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)DS} = \tilde{T}^{(2)\mu\lambda}_{(Z_c^+\pi^-)} \tilde{g}_{(Z_c^+)\lambda\sigma} g^{\sigma\nu} f^{(Z_c^+)}_{(01)} + \tilde{T}^{(2)\mu\lambda}_{(Z_c^-\pi^+)} \tilde{g}_{(Z_c^-)\lambda\sigma} g^{\sigma\nu} f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^\pm\pi^\mp)DD} = \tilde{T}^{(2)\mu\lambda}_{(Z_c^+\pi^-)} \tilde{t}^{(2)}_{(\psi\pi^+)\lambda\sigma} g^{\sigma\nu} f^{(Z_c^+)}_{(01)} + \tilde{T}^{(2)\mu\lambda}_{(Z_c^-\pi^+)} \tilde{t}^{(2)}_{(\psi\pi^-)\lambda\sigma} g^{\sigma\nu} f^{(Z_c^-)}_{(02)}$$

Amplitude construction

Two points:

- Relativistic factor is included in covariant helicity formalism
- Blatt-Weisskopf barrier factor

Two test methods:

- Calculate amplitude directly
- Fit to same samples with two formalisms

$$f_\lambda^{(1)}(\gamma_s)=\begin{cases}[\chi^{(1)*}(\lambda)\cdot\omega(\lambda)] \\ 1 \quad \text{for} \quad \lambda=\pm1 \\ \gamma_s \quad \text{for} \quad \lambda=0,\end{cases}$$

$$f_\lambda^{(2)}(\gamma_s)=\begin{cases}1 \quad \text{for} \quad \lambda=\pm2 \\ \gamma_s \quad \text{for} \quad \lambda=\pm1 \\ \frac{2}{3}\gamma_s^2+\frac{1}{3} \quad \text{for} \quad \lambda=0.\end{cases}$$

L	$B_L(q) \quad z=[qd]^2$
0	1
1	$\sqrt{\frac{2z}{1+z}}$
2	$\sqrt{\frac{13z^2}{(z-3)^2+9z}}$
3	$\sqrt{\frac{277z^3}{z^3+6z^2+45z+225}}$
4	$\sqrt{\frac{12746z^4}{z^4+10z^3+135z^2+1575z+11025}}$

$$B_1(Q_{abc})=\sqrt{\frac{2}{Q_{abc}^2+Q_0^2}},$$

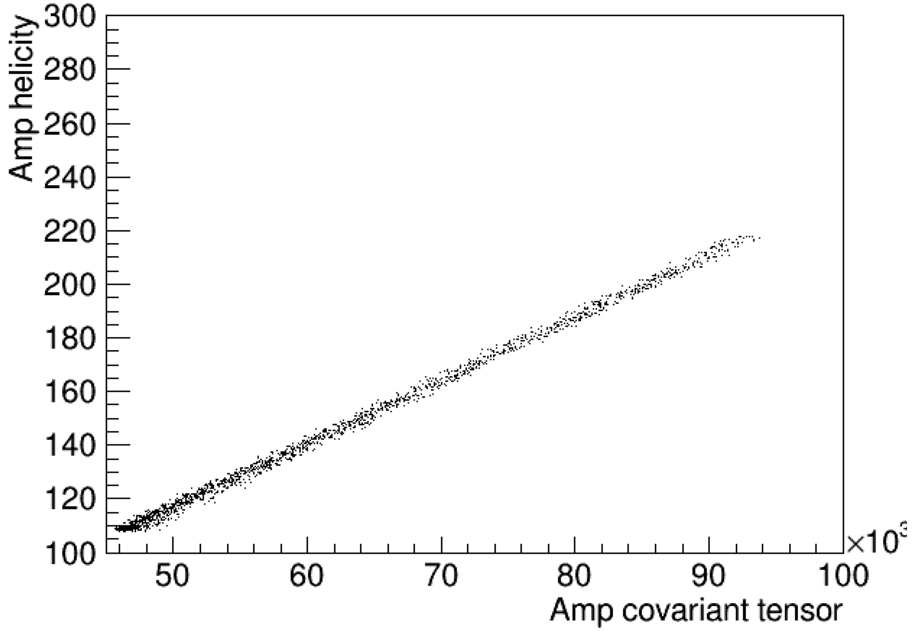
$$B_2(Q_{abc})=\sqrt{\frac{13}{Q_{abc}^4+3Q_{abc}^2Q_0^2+9Q_0^4}},$$

$$B_3(Q_{abc})=\sqrt{\frac{277}{Q_{abc}^6+6Q_{abc}^4Q_0^2+45Q_{abc}^2Q_0^4+225Q_0^6}},$$

$$B_4(Q_{abc})=\sqrt{\frac{12746}{Q_{abc}^8+10Q_{abc}^6Q_0^2+135Q_{abc}^4Q_0^4+1575Q_{abc}^2Q_0^6+11025Q_0^8}}.$$

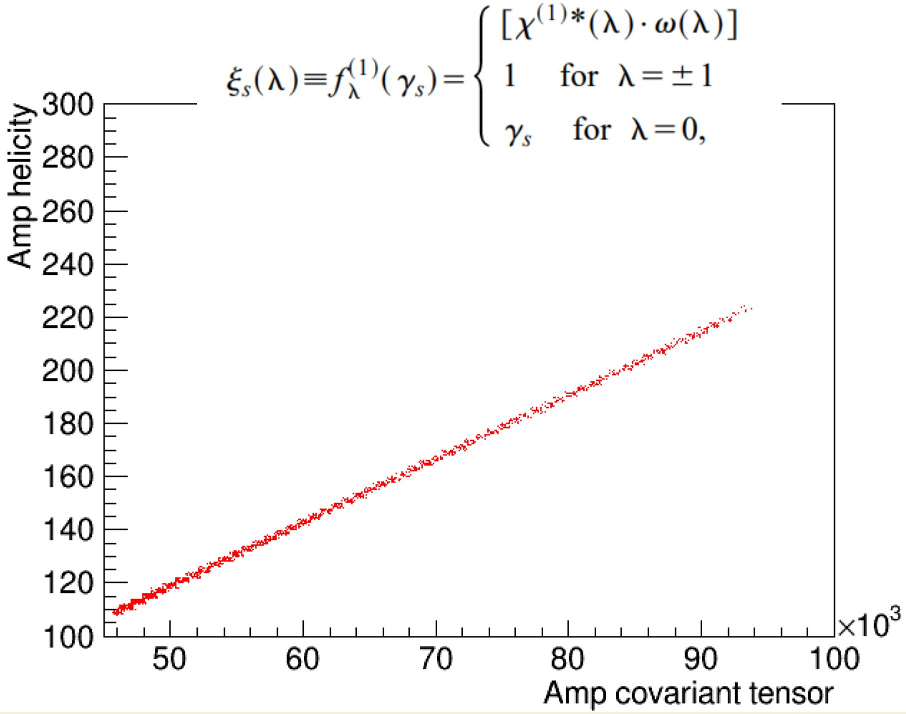
Consistency between two formalisms

MC: $Y \to \pi^+\pi^- J/\psi,\ J/\psi \to l^+ l^-$ phsp MC
Amplitude: only Zc SS component



xsec_cov=69738.5	xsec_hel=161.882	R=430.797
xsec_cov=56658.7	xsec_hel=134.551	R=421.094
xsec_cov=47518.5	xsec_hel=110.925	R=428.384
xsec_cov=91253.7	xsec_hel=212.259	R=429.916
xsec_cov=81966.6	xsec_hel=190.662	R=429.906

$$\xi_s(\lambda) \equiv f_\lambda^{(1)}(\gamma_s) = \begin{cases} [\chi^{(1)*}(\lambda)\cdot\omega(\lambda)] \\ 1 \quad \text{for} \quad \lambda = \pm 1 \\ \gamma_s \quad \text{for} \quad \lambda = 0, \end{cases}$$

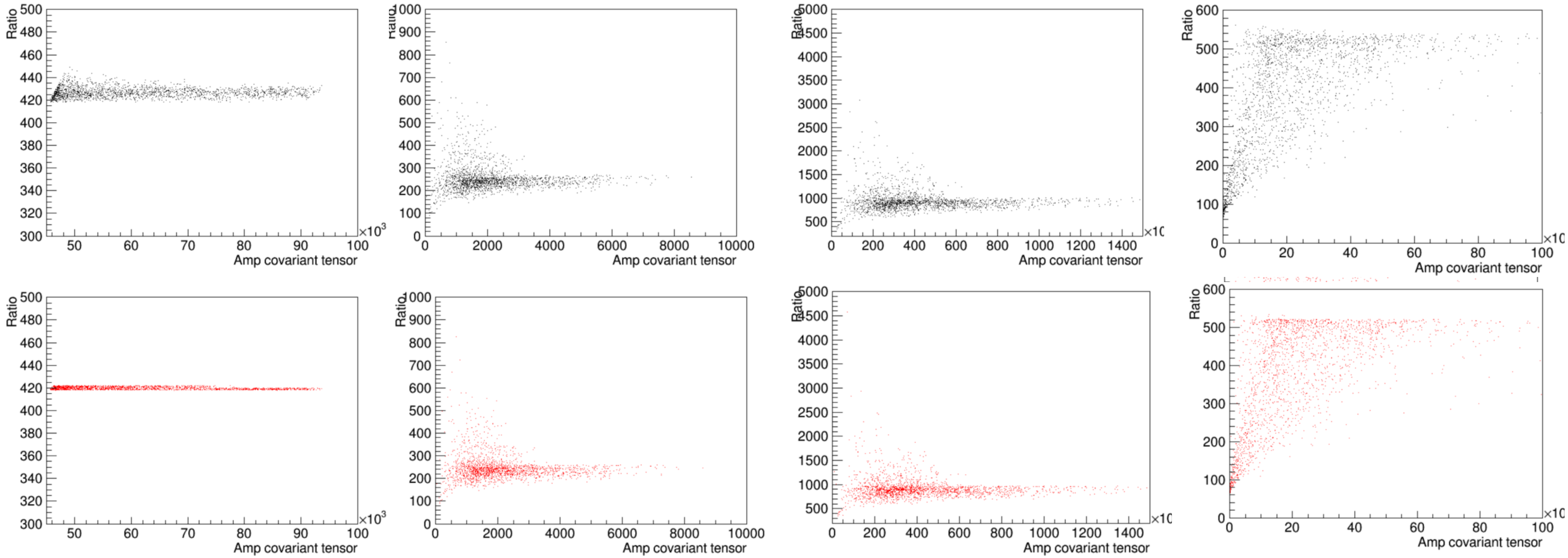


xsec_cov=69738.5	xsec_hel_RF=165.977	R=420.17
xsec_cov=56658.7	xsec_hel_RF=135.051	R=419.537
xsec_cov=47518.5	xsec_hel_RF=112.773	R=421.365
xsec_cov=91253.7	xsec_hel_RF=217.526	R=419.508
xsec_cov=81966.6	xsec_hel_RF=195.262	R=419.777

- The ratio is close to a constant as well
- The relativistic factor also makes the ratio more stable

Consistency between two formalisms

- The black plot represents case without relativistic factors
- The red plot represents case with relativistic factors



Zc SS:
R max = 421.777
R min = 419.452

Zc SD
R max = 820.681
R min = 75.9445

Zc DS
R max = 4593.4
R min = 368.43

Zc DD:
R max = 533.565
R min = 67.3278

Consistency between two formalisms

分类号 ______________ 密级 ______________

UDC ______________ 编号 ______________

中国高等科技中心
博士后研究报告

分波分析方法

李 刚

合作导师 叶铭汉 院士，朱永生 研究员

工作完成日期 2004年7月— 2006年8月

报告提交日期 2006年8月

中国高等科技中心（北京）
2006年8月

$$D \to \rho^0\pi^0 \to \pi^+\pi^-\pi^0$$

Both P wave for two steps

Helicity formalism:

$$\begin{aligned} Z &= D^0_{0\lambda}(0,\theta_0\phi_0) f_\lambda(\gamma) D^{*1}_{\lambda 0}(0\theta\phi) \\ &= \gamma d^1_{00}(\theta) \qquad \lambda = 0 \\ &= \gamma\cos\theta\,, \end{aligned}$$

Covariant tensor formalism:

$$\begin{aligned} Z &= -2p(p_a - p_b)^3 \\ &= -2p[(\gamma\beta E^* - \gamma q_3) - (\gamma\beta E^* + \gamma q_3)] \\ &= 4p\gamma(q\cos\theta). \\ &= 4pq\gamma\cos\theta\,, \end{aligned}$$

Energy dependent terms are included in covariant tensor amplitude naturally

$$\sqrt{\frac{2z}{1+z}} = \sqrt{\frac{2\frac{Q^2_{abc}}{Q^2_0}}{1+\frac{Q^2_{abc}}{Q^2_0}}} = \sqrt{\frac{2Q^2_{abc}}{Q^2_{abc}+Q^2_0}}$$

$$A_{\ell S}(\lambda\nu) = \left(\frac{2\ell+1}{2J+1}\right)^{1/2} (\ell 0 S\delta|J\delta)(s\lambda\sigma-\nu|S\delta) \times W^n r^\ell f^s_\lambda(\gamma_s) f^\sigma_\nu(\gamma_\sigma),$$

L	$B_L(q)$ $\quad z=[qd]^2$
0	1
1	$\sqrt{\frac{2z}{1+z}}$
2	$\sqrt{\frac{13z^2}{(z-3)^2+9z}}$
3	$\sqrt{\frac{277z^3}{z^3+6z^2+45z+225}}$
4	$\sqrt{\frac{12746z^4}{z^4+10z^3+135z^2+1575z+11025}}$

$$B_1(Q_{abc}) = \sqrt{\frac{2}{Q^2_{abc}+Q^2_0}},$$

$$B_2(Q_{abc}) = \sqrt{\frac{13}{Q^4_{abc}+3Q^2_{abc}Q^2_0+9Q^4_0}},$$

$$B_3(Q_{abc}) = \sqrt{\frac{277}{Q^6_{abc}+6Q^4_{abc}Q^2_0+45Q^2_{abc}Q^4_0+225Q^6_0}},$$

$$B_4(Q_{abc}) = \sqrt{\frac{12746}{Q^8_{abc}+10Q^6_{abc}Q^2_0+135Q^4_{abc}Q^4_0+1575Q^2_{abc}Q^6_0+11025Q^8_0}}.$$

Consistency between two formalisms

$$A_{\ell S}(\lambda\nu)=\left(\frac{2\ell+1}{2J+1}\right)^{1/2}(\ell 0 S\delta|J\delta)(s\lambda\sigma-\nu|S\delta)$$
$$\times W^{\prime}\, r^{\ell} f^{s}_{\lambda}(\gamma_s) f^{\sigma}_{\nu}(\gamma_\sigma),$$

```
)*BarrierF(2,rhoJpsi_zcp*2.0);
)*BarrierF(2,rhoZcp*2.0);
```

Break-up momentum is used in helicity formalism

$$p^{\alpha}=(W;0,0,0),$$

$$q^{\alpha}=(q_0;0,0,q)=(\gamma_s m;0,0,\gamma_s\beta_s m),$$

$$k^{\alpha}=(k_0;0,0,-q)=(\gamma_\sigma\mu;0,0,-\gamma_\sigma\beta_\sigma\mu), \quad (17)$$

$$r^{\alpha}=(q_0-k_0;0,0,2q),$$

where $W=q_0+k_0$, $q_0=\sqrt{m^2+q^2}$, $k_0=\sqrt{\mu^2+q^2}$, and $r=q-k$, the wave functions in the J rest frame are given by

$$\sqrt{\frac{2z}{1+z}}=\sqrt{\frac{2\frac{Q_{abc}^2}{Q_0^2}}{1+\frac{Q_{abc}^2}{Q_0^2}}}=\sqrt{\frac{2Q_{abc}^2}{Q_{abc}^2+Q_0^2}}$$

$$\sqrt{\frac{2z}{1+z}}=\sqrt{\frac{2(\frac{2Q_{abc}}{Q_0})^2}{1+(\frac{2Q_{abc}}{Q_0})^2}}=\sqrt{\frac{2Q_{abc}^2}{Q_{abc}^2+(Q_0/2)^2}}$$

Eur. Phys. J. A 16, 537 – 547 (2003)

$$Q_{abc}^2=\frac{(s_a+s_b-s_c)^2}{4s_a}-s_b$$

Here Q_0 is a hadron "scale" parameter $Q_0=0.197321/R$ GeV/c, where R is the radius of the centrifugal barrier in fm. We remark that in these Blatt-Weisskopf factors, the approximation is made that the centrifugal barrier may be replaced by a square well of radius R.

$$B_1(Q_{abc})=\sqrt{\frac{2}{Q_{abc}^2+Q_0^2}},$$

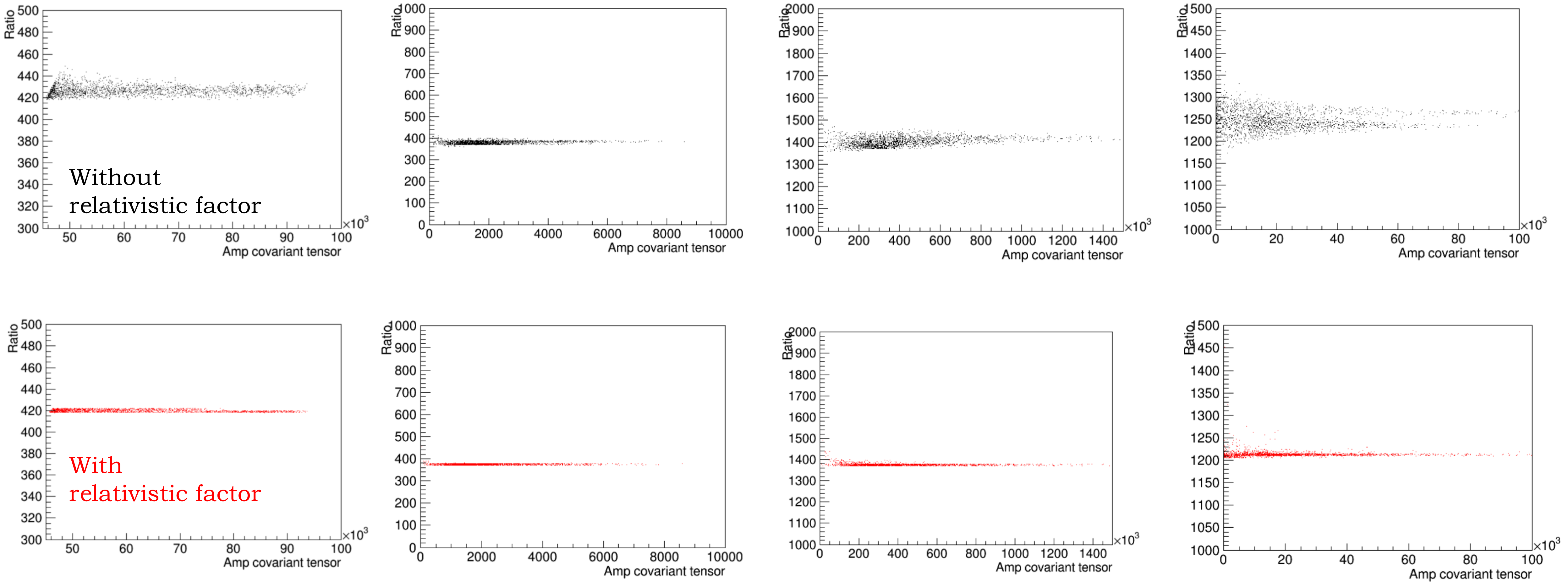
$$B_2(Q_{abc})=\sqrt{\frac{13}{Q_{abc}^4+3Q_{abc}^2Q_0^2+9Q_0^4}},$$

$$B_3(Q_{abc})=\sqrt{\frac{277}{Q_{abc}^6+6Q_{abc}^4Q_0^2+45Q_{abc}^2Q_0^4+225Q_0^6}},$$

$$B_4(Q_{abc})=\sqrt{\frac{12746}{Q_{abc}^8+10Q_{abc}^6Q_0^2+135Q_{abc}^4Q_0^4+1575Q_{abc}^2Q_0^6+11025Q_0^8}}.$$

If R=1fm is used in helicity formalism, R=2fm should be used in covariant tensor formalism

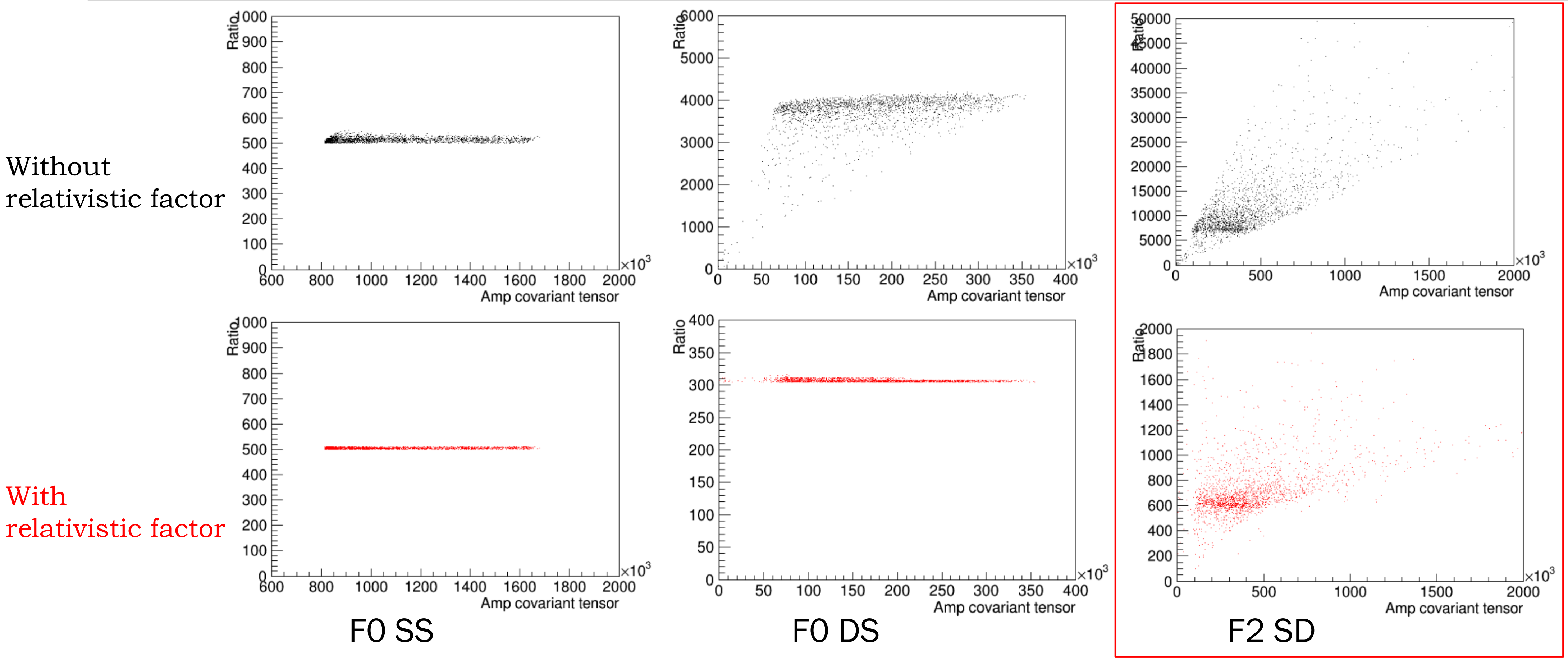
Consistency between two formalisms



Zc SS Zc SD Zc DS Zc DD

- The amplitude ratio of Zc components are almost constants for different partial waves

Consistency between two formalisms



- Two components of F0 have constant ratio for two amplitude formalisms
- Only $|LS> = |01>$ partial wave of F2 is considered, the amplitude ratio is not a constant

Amplitudes of F2

The difference natural exists?

$$Decay: Y \to \psi \quad f_2$$
$$J^{PC}: 1^{--} \to 1^{--} \quad 2^{+}$$

- Check one step decay

$$\frac{d\sigma}{d\Omega} = \sum_{M\lambda\nu} \left| F^1_{\lambda\nu} D^1_{M,\lambda-\nu}(\phi\theta) \right|^2$$

$$F^1_{\lambda\nu} = \begin{pmatrix} F^1_{21} & F^1_{11} & F^1_{01} & F^1_{-11} & F^1_{-21} \\ F^1_{20} & F^1_{10} & F^1_{00} & F^1_{-10} & F^1_{-20} \\ F^1_{2-1} & F^1_{1-1} & F^1_{0-1} & F^1_{-1-1} & F^1_{-2-1} \end{pmatrix}$$

Only five independent helicity coupling amplitude:

$$F^1_{21} = F^1_{-2-1}, \quad F^1_{11} = F^1_{-1-1}, \quad F^1_{01} = F^1_{0-1}, \quad F^1_{10} = F^1_{-10}, \quad F^1_{00},$$

$$\begin{aligned}
\sum_{\lambda\nu} \left| F^1_{\lambda\nu} D^1_{1,\lambda-\nu}(\phi\theta) \right|^2 &= \sum_{\lambda\nu} \left| F^1_{\lambda\nu} \left[d^1_{1,\lambda-\nu}(\theta) \right] \right|^2 \\
&= \left| F^1_{21} \right|^2 \left\{ \left[d^1_{1,1}(\theta) \right]^2 + \left[d^1_{1,-1}(\theta) \right]^2 \right\} \\
&+ 2 \left| F^1_{11} \right|^2 \left[d^1_{1,0}(\theta) \right]^2 \\
&+ \left| F^1_{01} \right|^2 \left\{ \left[d^1_{1,-1}(\theta) \right]^2 + \left[d^1_{1,1}(\theta) \right]^2 \right\} \\
&+ \left| F^1_{10} \right|^2 \left\{ \left[d^1_{1,1}(\theta) \right]^2 + \left[d^1_{1,-1}(\theta) \right]^2 \right\} \\
&+ \left| F^1_{00} \right|^2 \left[d^1_{1,0}(\theta) \right]^2 \\
&= \frac{1+\cos^2\theta}{2} \left[\left| F^1_{21} \right|^2 + \left| F^1_{10} \right|^2 + \left| F^1_{01} \right|^2 \right] \\
&+ \frac{\sin^2\theta}{2} \left[2 \left| F^1_{11} \right|^2 + \left| F^1_{00} \right|^2 \right]
\end{aligned}$$

Same results for Y helicity $M = \pm 1$

$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= \sum_{M\lambda\nu} \left| F^1_{\lambda\nu} D^1_{M,\lambda-\nu}(\phi\theta) \right|^2 = 2 \sum_{\lambda\nu} \left| F^1_{\lambda\nu} D^1_{1,\lambda-\nu}(\phi\theta) \right|^2 = 2 \sum_{\lambda\nu} \left| \sum_{\ell S} g_{\ell S} A_{\ell S}(\lambda\nu) d^1_{1,\lambda-\nu} \right|^2 \\
&= 2\left| g_{01} \right|^2 \cdot \left\{ \frac{1+\cos^2\theta}{2} \left[|A_{01}(21)|^2 + |A_{01}(10)|^2 + |A_{01}(01)|^2 \right] + \frac{\sin^2\theta}{2} \left[2|A_{01}(11)|^2 + |A_{01}(00)|^2 \right] \right\} \\
&= 2\left| g_{01} \right|^2 \cdot \left\{ \left[|A_{01}(21)|^2 + |A_{01}(10)|^2 + |A_{01}(01)|^2 \right] + \frac{\sin^2\theta}{2} \left[-|A_{01}(21)|^2 - |A_{01}(10)|^2 - |A_{01}(01)|^2 + 2|A_{01}(11)|^2 + |A_{01}(00)|^2 \right] \right\}
\end{aligned}$$

$$A_{01}(21) = \sqrt{(\tfrac{1}{5})}$$
$$A_{01}(10) = -\sqrt{(\tfrac{1}{10})}\gamma_f\gamma_\psi$$
$$A_{01}(01) = \sqrt{(\tfrac{1}{30})}(\tfrac{2}{3}\gamma_f^2 + \tfrac{1}{3})$$
$$A_{01}(11) = \sqrt{(\tfrac{1}{10})}\gamma_f$$
$$A_{01}(00) = -\sqrt{(\tfrac{2}{15})}((\tfrac{2}{3}\gamma_f^2 + \tfrac{1}{3}))\gamma_\psi$$

Amplitudes of F2

The difference natural exists?

- Check one step decay

$|LS> = |01>$ partial wave

$$
\begin{aligned}
Decay: Y &\rightarrow \psi \quad f_2 \\
J^{PC}: 1^{--} &\rightarrow 1^{--} \quad 2^{+}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{g}_1^{\nu\nu'}\tilde{g}_{2\mu\mu}\tilde{g}_{2\nu\nu'} &= \tilde{g}_{2\mu\mu}\left[g^{\nu\nu'} - \frac{p_1^{\nu}p_1^{\nu'}}{p_1^2}\right]\left[g_{\nu\nu'} - \frac{p_{2\nu}p_{2\nu'}}{p_2^2}\right] \\
&= \tilde{g}_{2\mu\mu}\left[2 + \frac{(p_1p_2)^2}{p_1^2p_2^2}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\sigma}{d\Phi_n} &\propto \frac{1}{2}\sum_{m_1}^{2}\sum_{m_2}^{3}\sum_{m_3}^{5}\xi_{\mu}^{*}(m_1)\omega_{\nu}(m_2)\phi(m_3)^{\mu\nu}\xi_{\mu'}(m_1)\omega_{\nu'}^{*}(m_2)\phi(m_3)^{*\mu'\nu'} \\
&= -\frac{1}{2}\sum_{\mu=1}^{2}\Lambda\Lambda^{*}\cdot\tilde{g}_{\nu\nu'}(p_{(\psi)})P^{(2)\mu\nu\mu\nu'}(p_{(f2)}) \\
&= -\frac{1}{2}\sum_{\mu=1}^{2}\Lambda\Lambda^{*}\cdot\tilde{g}^{\nu\nu'}(p_{(\psi)})P^{(2)}_{\mu\nu\mu\nu'}(p_{(f2)})
\end{aligned}
$$

$$
\begin{aligned}
\tilde{g}_1^{\nu\nu'}\tilde{g}_{2\mu\nu'}\tilde{g}_{2\nu\mu} &= \left[g^{\nu\nu'} - \frac{p_1^{\nu}p_1^{\nu'}}{p_1^2}\right]\left[g_{\mu\nu'} - \frac{p_{2\mu}p_{2\nu'}}{p_2^2}\right]\left[g_{\nu\mu} - \frac{p_{2\nu}p_{2\mu}}{p_2^2}\right] \\
&= \left[g_{\mu}^{\nu} - \frac{p_{2\mu}p_2^{\nu}}{p_2^2} - \frac{p_1^{\nu}p_{1\mu}}{p_1^2} + \frac{(p_1p_2)p_1^{\nu}p_{2\mu}}{p_1^2p_2^2}\right]\left[g_{\nu\mu} - \frac{p_{2\nu}p_{2\mu}}{p_2^2}\right] \\
&= g_{\mu\mu} - \frac{p_{1\mu}p_{1\mu}}{p_1^2} - \frac{p_{2\mu}p_{2\mu}}{p_2^2} + 2\frac{(p_1p_2)p_{1\mu}p_{2\mu}}{p_1^2p_2^2} - \frac{(p_1p_2)^2p_{2\mu}p_{2\mu}}{p_1^2p_2^4}
\end{aligned}
$$

$$
\begin{aligned}
\sum_{\mu=1}^{2}\delta^{\mu\mu'}\tilde{g}^{\nu\nu'}(p_{(\psi)})P^{(2)}_{\mu\nu\mu\nu'}(p_{(f2)}) &= \sum_{\mu=1}^{2}\tilde{g}_1^{\nu\nu'}\left[\frac{1}{2}\left(\tilde{g}_{2\mu\mu}\tilde{g}_{2\nu\nu'} + \tilde{g}_{2\mu\nu'}\tilde{g}_{2\nu\mu}\right) - \frac{1}{3}\tilde{g}_{2\mu\nu}\tilde{g}_{2\mu\nu'}\right] \\
&= \sum_{\mu=1}^{2}\left[\frac{1}{2}\tilde{g}_1^{\nu\nu'}\tilde{g}_{2\mu\mu}\tilde{g}_{2\nu\nu'} + \frac{1}{6}\tilde{g}_1^{\nu\nu'}\tilde{g}_{2\mu\nu'}\tilde{g}_{2\nu\mu}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\sigma}{d\Phi_n} &\propto -\frac{1}{2}\sum_{\mu=1}^{2}\Lambda\Lambda^{*}\cdot\tilde{g}^{\nu\nu'}(p_{(\psi)})P^{(2)}_{\mu\nu\mu\nu'}(p_{(f2)}) \\
&= \frac{1}{2}\Lambda\Lambda^{*}\cdot\{\left[2 + \frac{(p_1p_2)^2}{m_1^2m_2^2}\right]\left[1 + \frac{p^2\sin^2\theta}{2m_2^2}\right] \\
&+ \frac{1}{6}\left[2 + \frac{p^2\sin^2\theta}{m_1^2} + \frac{p^2\sin^2\theta}{m_2^2} + \frac{(p_1p_2)^2p^2\sin^2\theta}{m_1^2m_2^4} + \frac{2(p_1p_2)p^2\sin^2\theta}{m_1^2m_2^2}\right]\} \\
&= \frac{1}{2}\Lambda\Lambda^{*}\cdot\left\{\left[2 + \frac{(p_1p_2)^2}{m_1^2m_2^2} + \frac{1}{3}\right] + \frac{p^2\sin^2\theta}{6}\left[\frac{6}{m_2^2} + \frac{3(p_1p_2)^2}{m_1^2m_2^4} + \frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{(p_1p_2)^2}{m_1^2m_2^4} + \frac{2(p_1p_2)}{m_1^2m_2^2}\right]\right\}
\end{aligned}
$$

Amplitudes of F2

Helicity formalism:

$$\frac{d\sigma}{d\Omega} = \sum_{M\lambda\nu} \left|F^1_{\lambda\nu} D^1_{M,\lambda-\nu}(\phi\theta)\right|^2 = 2\sum_{\lambda\nu} \left|F^1_{\lambda\nu} D^1_{1,\lambda-\nu}(\phi\theta)\right|^2 = 2\sum_{\lambda\nu}\left|\sum_{\ell S} g_{\ell S} A_{\ell S}(\lambda\nu) d^1_{1,\lambda-\nu}\right|^2$$

$$= 2|g_{01}|^2 \cdot \left\{\frac{1+\cos^2\theta}{2}\left[|A_{01}(21)|^2 + |A_{01}(10)|^2 + |A_{01}(01)|^2\right] + \frac{\sin^2\theta}{2}\left[2|A_{01}(11)|^2 + |A_{01}(00)|^2\right]\right\}$$

$$= 2|g_{01}|^2 \cdot \left\{\left[|A_{01}(21)|^2 + |A_{01}(10)|^2 + |A_{01}(01)|^2\right] + \frac{\sin^2\theta}{2}\left[-|A_{01}(21)|^2 - |A_{01}(10)|^2 - |A_{01}(01)|^2 + 2|A_{01}(11)|^2 + |A_{01}(00)|^2\right]\right\}$$

Covariant tensor formalism:

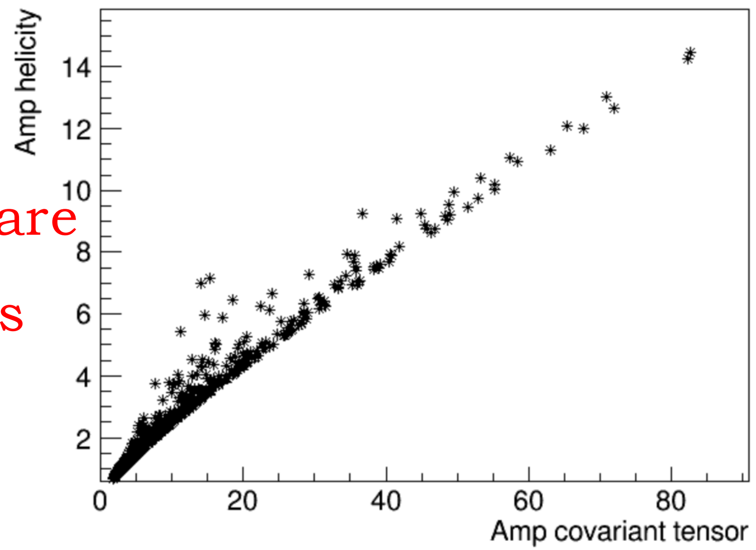
$$\frac{d\sigma}{d\Phi_n} \propto -\frac{1}{2}\sum_{\mu=1}^{2} \Lambda\Lambda^* \cdot \tilde{g}^{\nu\nu'}(p_{(\psi)}) P^{(2)}_{\mu\nu\mu\nu'}(p_{(f2)})$$

$$= \frac{1}{2}\Lambda\Lambda^* \cdot \left\{\left[2 + \frac{(p_1p_2)^2}{m_1^2 m_2^2}\right]\left[1 + \frac{p^2\sin^2\theta}{2m_2^2}\right] + \frac{1}{6}\left[2 + \frac{p^2\sin^2\theta}{m_1^2} + \frac{p^2\sin^2\theta}{m_2^2} + \frac{(p_1p_2)^2 p^2\sin^2\theta}{m_1^2 m_2^4} + \frac{2(p_1p_2)p^2\sin^2\theta}{m_1^2 m_2^2}\right]\right\}$$

$$= \frac{1}{2}\Lambda\Lambda^* \cdot \left\{\left[2 + \frac{(p_1p_2)^2}{m_1^2 m_2^2} + \frac{1}{3}\right] + \frac{p^2\sin^2\theta}{6}\left[\frac{6}{m_2^2} + \frac{3(p_1p_2)^2}{m_1^2 m_2^4} + \frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{(p_1p_2)^2}{m_1^2 m_2^4} + \frac{2(p_1p_2)}{m_1^2 m_2^2}\right]\right\}$$

The amplitudes $Y \to f_2\, J/\psi$ are different for two formalisms

$Y \to \pi^+\pi^- J/\psi$ phsp MC




```
double Ecms = 4.4;
TLorentzVector mother(0.0, 0.0, 0.0, Ecms);
double masses[2] = { 3.0969, 1.275} ;

TGenPhaseSpace event;
event.SetDecay(mother, 2, masses);
double maxWeight = event.GetWtMax();
```

Two body decay phsp MC

The parts with/without angle have different amplitude ratio

R0=10.0366 R1=16.9879

R0 is the ratio of term in blue box
R1 is the ratio of term in green box

Amplitudes of F2

$Y \ p_0 = (0,0,0,M)$

$\psi \ p_1^\mu = (p_x, p_y, p_z, E_1), \gamma_1 = \frac{E_1}{m_1} = \frac{1}{\sqrt{1-\beta_1^2}}$

$f_2 \ p_2^\mu = (-p_x, -p_y, -p_z, E_2), \gamma_2 = \frac{E_2}{m_2} = \frac{1}{\sqrt{1-\beta_2^2}}$

$|\vec{p_1}| = |\vec{p_2}| = p$

$p_1 p_2 = E_1 E_2 + p^2 = (p_0 - p_2)p_2 = ME_2 - m_2^2$

- Even for the "constant" term, amplitudes are not consistent
- Single partial wave may be different for high spin case

Constant term of covariant formalism

$$2 + \frac{(p_1p_2)^2}{m_1^2m_2^2} + \frac{1}{3} = \frac{7}{3} + (\frac{E_1E_2+p^2}{m_1m_2})^2$$
$$= \frac{7}{3} + (\gamma_1\gamma_2 + \gamma_1\beta_1\gamma_2\beta_2)^2$$
$$= \frac{7}{3} + (\gamma_1\gamma_2 + \frac{\gamma_1\gamma_2}{\sqrt{1-\gamma_1^2}\sqrt{1-\gamma_2^2}})^2$$

$$2 + \frac{(p_1p_2)^2}{m_1^2m_2^2} + \frac{1}{3} = \frac{7}{3} + (\frac{ME_2 - m_2^2}{m_1m_2})^2$$
$$= \frac{7}{3} + (\frac{M}{m_1}\gamma_2 - \frac{m_2}{m_1})^2$$

Constant term of helicity formalism

$$|A_{01}(21)|^2 + |A_{01}(10)|^2 + |A_{01}(01)|^2 = \frac{1}{5} + \frac{1}{10}\gamma_1^2\gamma_2^2 + \frac{1}{30}(\frac{2}{3}\gamma_2^2 + \frac{1}{3})^2$$

Test with MC

- The fractions are consistent between covariant tensor formalism and helicity formalism
- Different (LS) wave components for two formalisms are in match

- MC sample:
  - Generated in helicity formalism
  - Four components: SS+SD+DS+DD
  - BW: different Zc mass and 1 MeV width
- **Fit tools:**
  - AmpTool for helicity formalism
  - GPUPWA for covariant tensor formalism

Zc mass=3880 MeV
Width = 1 MeV

SS wave:	Cov = 0.11943	Hel = 0.117659
SD wave:	Cov = 0.0223932	Hel = 0.0220939
DS wave:	Cov = 0.723714	Hel = 0.725169
DD wave:	Cov = 0.134462	Hel = 0.135078

Zc mass=3885 MeV
Width = 1 MeV

SS wave:	Cov = 0.140743	Hel = 0.137783
SD wave:	Cov = 0.0218467	Hel = 0.0217328
DS wave:	Cov = 0.724729	Hel = 0.725696
DD wave:	Cov = 0.112681	Hel = 0.114788

Zc mass=3890 MeV
Width = 1 MeV

SS wave:	Cov = 0.126908	Hel = 0.122224
SD wave:	Cov = 0.0227674	Hel = 0.0222862
DS wave:	Cov = 0.722514	Hel = 0.724973
DD wave:	Cov = 0.127811	Hel = 0.130517

Test with real data

Fraction of partial waves:

@4180

	covT	helicity
frac: #pi Zc(3900) SS	0.05937	0.059636
frac: #pi Zc(3900) SD	0.03389	0.0338373
frac: #pi Zc(3900) DS	0.19788	0.198336
frac: #pi Zc(3900) DD	0.10910	0.108666
frac: f_{0}(500) #psi' SS	0.09794	0.0988305
frac: f_{0}(500) #psi' DS	0.31474	0.314776
frac: f_{0}(980) #psi' SS	0.23784	0.237843
frac: f_{0}(980) #psi' DS	0.02304	0.0230202
frac: f_{0}(1370) #psi' SS	0.15029	0.150156
frac: f_{0}(1370) #psi' DS	0.27593	0.276179

@4260

	covT	helicity
frac: #pi Zc(3900) SS	0.07413	0.0742924
frac: #pi Zc(3900) SD	0.00228	0.00210837
frac: #pi Zc(3900) DS	0.00748	0.00702821
frac: #pi Zc(3900) DD	0.00022	0.00019002
frac: f_{0}(500) #psi' SS	0.28964	0.289882
frac: f_{0}(500) #psi' DS	0.00649	0.00603729
frac: f_{0}(980) #psi' SS	0.15282	0.153087
frac: f_{0}(980) #psi' DS	0.00620	0.00607647
frac: f_{0}(1370) #psi' SS	1.10608	1.10493
frac: f_{0}(1370) #psi' DS	0.01058	0.00986463

@4360

	covT	helicity
frac: #pi Zc(3900) SS	0.05103	0.0533287
frac: #pi Zc(3900) SD	0.06241	0.0589134
frac: #pi Zc(3900) DS	0.01218	0.0132631
frac: #pi Zc(3900) DD	0.01403	0.0137886
frac: f_{0}(500) #psi' SS	0.25368	0.256105
frac: f_{0}(500) #psi' DS	0.10026	0.0991094
frac: f_{0}(980) #psi' SS	0.14102	0.14692
frac: f_{0}(980) #psi' DS	0.22127	0.215695
frac: f_{0}(1370) #psi' SS	0.72456	0.718424
frac: f_{0}(1370) #psi' DS	0.25803	0.273384

Fraction of components:

@4180

	covT	helicity
frac: #pi Zc(3900)	0.366296	0.366526
frac: f_{0}(500) #psi'	0.374027	0.374723
frac: f_{0}(980) #psi'	0.263715	0.263706
frac: f_{0}(1370) #psi'	0.443862	0.443934

@4260

	covT	helicity
frac: #pi Zc(3900)	0.08349	0.0830393
frac: f_{0}(500) #psi'	0.29203	0.29174
frac: f_{0}(980) #psi'	0.159667	0.159803
frac: f_{0}(1370) #psi'	1.12102	1.1192

@4360

	covT	helicity
frac: #pi Zc(3900)	0.13799	0.137832
frac: f_{0}(500) #psi'	0.310156	0.311393
frac: f_{0}(980) #psi'	0.377258	0.377848
frac: f_{0}(1370) #psi'	1.00454	1.0128

Test with real data

After include f2(1270)

- The fractions for each partial wave are different for two formalisms
- For some specific component, fraction deviates a lot

@4180

	covT	helicity
frac: #pi Zc(3900) SS	0.05129	0.038663 +- 0.0193146
frac: #pi Zc(3900) SD	0.01958	0.0173968 +- 0.00869078
frac: #pi Zc(3900) DS	0.23595	0.22107 +- 0.0350437
frac: #pi Zc(3900) DD	0.08700	0.0960525 +- 0.0152261
frac: f_{0}(500) #psi' SS	0.09348	0.120354 +- 0.0432465
frac: f_{0}(500) #psi' DS	0.37179	0.340525 +- 0.0588701
frac: f_{0}(980) #psi' SS	0.17493	0.189217 +- 0.0473867
frac: f_{0}(980) #psi' DS	0.00593	0.0496333 +- 0.0432182
frac: f_{0}(1370) #psi' SS	0.15850	0.191395 +- 0.062643
frac: f_{0}(1370) #psi' DS	0.20624	0.14879 +- 0.0652218
frac: f_{2}(1270) #psi' SD	0.04638	0.0746205 +- 0.0305894

	covT	helicity
frac: #pi Zc(3900)	0.364086	0.346914 +- 0.0602814
frac: f_{0}(500) #psi'	0.421575	0.414233 +- 0.0554919
frac: f_{0}(980) #psi'	0.179906	0.235208 +- 0.0621272
frac: f_{0}(1370) #psi'	0.379839	0.352955 +- 0.0856871
frac: f_{2}(1270) #psi'	0.0463755	0.0746205 +- 0.0305894

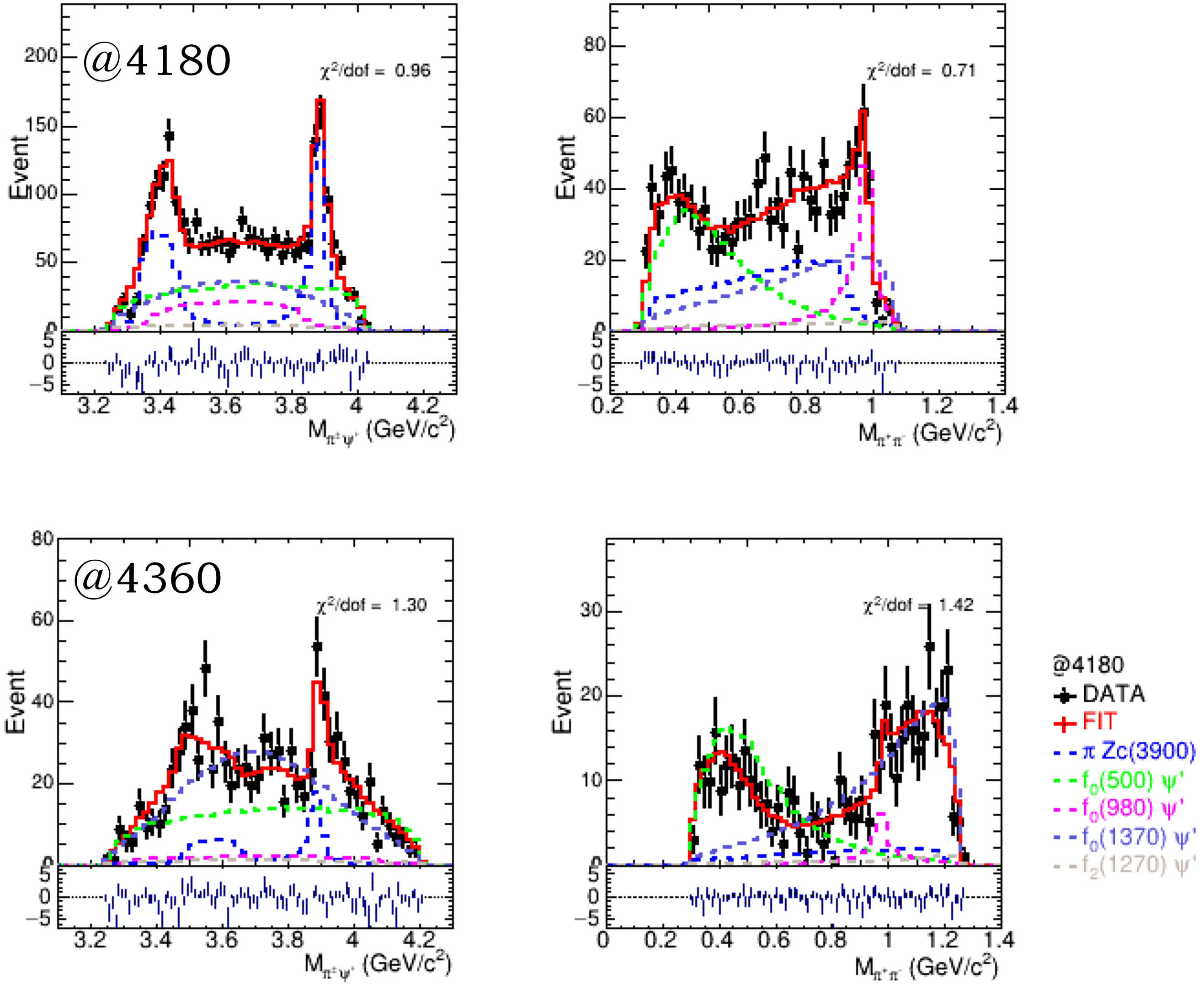
@4360

	covT	helicity
frac: #pi Zc(3900) SS	0.08436	0.0446973 +- 0.0368669
frac: #pi Zc(3900) SD	0.01034	0.077733 +- 0.0641152
frac: #pi Zc(3900) DS	0.02473	0.00606039 +- 0.00616546
frac: #pi Zc(3900) DD	0.00285	0.00991856 +- 0.0100905
frac: f_{0}(500) #psi' SS	0.48054	0.0761289 +- 0.111609
frac: f_{0}(500) #psi' DS	0.06607	0.55959 +- 0.174446
frac: f_{0}(980) #psi' SS	0.05899	0.20192 +- 0.0875702
frac: f_{0}(980) #psi' DS	0.01060	0.200417 +- 0.0680772
frac: f_{0}(1370) #psi' SS	0.69655	0.458141 +- 0.306158
frac: f_{0}(1370) #psi' DS	0.11364	0.543817 +- 0.266504
frac: f_{2}(1270) #psi' SD	0.03990	0.0291616 +- 0.0205761

	covT	helicity
frac: #pi Zc(3900)	0.120783	0.133984 +- 0.09672
frac: f_{0}(500) #psi'	0.570377	0.615523 +- 0.0999847
frac: f_{0}(980) #psi'	0.0690711	0.41935 +- 0.0938778
frac: f_{0}(1370) #psi'	0.811512	1.02619 +- 0.183353
frac: f_{2}(1270) #psi'	0.0398985	0.0291616 +- 0.0205761

Test with real data

- The consideration of f2(1270) seems make more influence to fit of 4360 data
- The fit model can’t describe the 4360 data as good as 4180 data



Test with real data

- Without $J/\psi \rightarrow l^+l^-$, fractions of partial waves and components are not consistent for two formalisms

Sequential decay: $Y \rightarrow \pi^+\pi^- J/\psi,\ J/\psi \rightarrow l^+l^-$

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y, \Delta\lambda_l} \left| \sum_{\lambda_{R_i}} A_{R_i} \right|^2$$

$$= \sum_{\lambda_Y, \Delta\lambda_l} \left| \sum_{\lambda_{R_i}, \lambda_\psi} F_1 D_1 \cdot F_2 D_2 \cdot F_3 D_3 \right|^2$$

$$\frac{d\sigma}{d\Phi} = |\sum_\lambda F(\lambda) \cdot c_\lambda|^2 \xrightarrow{c_\lambda = c} |\sum_\lambda F(\lambda)|^2 \cdot |c|^2$$

$$\frac{d\sigma}{d\Phi} = \sum_\lambda |F(\lambda)|^2$$

with or without Jpsi->ll	without	with
frac: #pi Zc(3900) SS	0.09501	0.059636 +- 0.0196226
frac: #pi Zc(3900) SD	0.05539	0.0338373 +- 0.0111338
frac: #pi Zc(3900) DS	0.17447	0.198336 +- 0.0309891
frac: #pi Zc(3900) DD	0.10545	0.108666 +- 0.0169785
frac: f_{0}(500) #psi' SS	0.01392	0.0988305 +- 0.0403377
frac: f_{0}(500) #psi' DS	0.20531	0.314776 +- 0.0592865
frac: f_{0}(980) #psi' SS	0.05321	0.237843 +- 0.0453384
frac: f_{0}(980) #psi' DS	0.32550	0.0230202 +- 0.0241228
frac: f_{0}(1370) #psi' SS	0.10281	0.150156 +- 0.0538406
frac: f_{0}(1370) #psi' DS	0.41013	0.276179 +- 0.0741884

with or without Jpsi->ll	without	with
frac: #pi Zc(3900)	0.424332	0.366526 +- 0.0589376
frac: f_{0}(500) #psi'	0.210252	0.374723 +- 0.0550911
frac: f_{0}(980) #psi'	0.37617	0.263706 +- 0.052797
frac: f_{0}(1370) #psi'	0.522306	0.443934 +- 0.106097

Summary

- In this report, tests are performed to demonstrate the consistency between two commonly used formalisms: helicity formalism and covariant tensor formalism

- Amplitudes constructed from two formalisms are consistent for some cases(Zc case, f_0 case)

Barrier factor

Relativistic factor

- In f_2 case, amplitudes are not consistent for two formalisms
  - All the partial waves should be considered
  - Is it possible to search a better amplitude construction method?

Not just use part of all partial waves

BACKUP

Helicity amplitude construction

Sequential decays: $Y \to \pi^+\pi^- J/\psi, J/\psi \to l^+l^-$

- Helicity formalism

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i}} A_{R_i} \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} \boxed{F_1^J D_1^J} \cdot \boxed{F_2^J D_2^J} \cdot \boxed{F_3^J D_3^J} \right|^2$$

$$= \sum_{\lambda_Y,\Delta\lambda_l} \left| \sum_{\lambda_{R_i},\lambda_\psi} (\sum_{LS} G_{LS} \cdot CG_1 \cdot B_l) D_1^J \cdot (\sum_{ls} G_{ls} \cdot CG_2 \cdot B_l) D_2^J \cdot F_3^J D_3^J \right|^2$$

Decay : $Y \to Z_c \pi$ | $Z_c \to \psi \pi$

$$A_{Z_c}(\lambda_Y, \lambda_{Z_c}, \lambda_{\ell^+}, \lambda_{\ell^-}) = F^{J_Y}_{\lambda_{Z_c},0} D^{J_Y}_{\lambda_Y,\lambda_{Z_c}}(\theta_{Z_c}, \phi_{Z_c}) \cdot BW(Z_c) \cdot F^{J_{Z_c}}_{\lambda_{J/\psi},0} D^{J_{Z_c}}_{\lambda_{Z_c},\lambda_{J/\psi}}(\theta_{J/\psi}, \phi_{J/\psi}) \cdot F^{J_{J/\psi}}_{\lambda_{\ell^+},\lambda_{\ell^-}} D^{J_{J/\psi}}_{\lambda_{J/\psi},\lambda_{\ell^+}-\lambda_{\ell^-}}(\theta_{\ell^+}, \phi_{\ell^+}),$$

Decay : $Y \to \psi f$ | $f \to \pi^+ \pi^-$

$$A_{R_f}(\lambda_Y, \lambda_{R_f}, \lambda_{\ell^+}, \lambda_{\ell^-}) = F^{J_Y}_{\lambda_{R_f},\lambda_{J/\psi}} D^{J_Y}_{\lambda_Y,\lambda_{R_f}-\lambda_{J/\psi}}(\theta_{R_f}, \phi_{R_f}) \cdot BW(R_f) \cdot F^{J_{R_f}}_{0,0} D^{J_{R_f}}_{\lambda_{R_f},0}(\theta_{\pi^+}, \phi_{\pi^+}) \cdot F^{J_{J/\psi}}_{\lambda_{\ell^+},\lambda_{\ell^-}} D^{J_{J/\psi}}_{\lambda_{J/\psi},\lambda_{\ell^+}-\lambda_{\ell^-}}(\theta_{\ell^+}, \phi_{\ell^+}),$$

- $J/\psi \to l^+l^-$ is included in helicity formalism

For the last step $J/\psi \to \ell^+\ell^-$, at the relativistic limit, by QED calculation, $F^{J_{J/\psi}}_{1/2,1/2} = F^{J_{J/\psi}}_{-1/2.-1/2} \approx 0$. Here we define $\Delta\lambda_\ell = \lambda_{\ell^+} - \lambda_{\ell^-}$, we can see only $\Delta\lambda_\ell = \pm 1$ is allowed.

$$\frac{d\sigma}{d\phi} = \sum_{\lambda_Y,\Delta\lambda_l} | \sum_{\lambda_{Z_c},\lambda_{R_f}} (A_{R_f} + e^{i\Delta\lambda_\ell \alpha_\ell(Z_c^+)} A_{Z_c^+} + e^{i\Delta\lambda_\ell \alpha_\ell(Z_c^-)} A_{Z_c^-})|^2$$

Amplitude construction

Sequential decays: $Y \to \pi^+\pi^- J/\psi$

- Covariant tensor formalism

$$A = \phi_\mu(m_1)\omega_\nu^*(m_2)A^{\mu\nu} = \phi_\mu(m_1)\omega_\nu^*(m_2)\sum_i \Lambda_i U_i^{\mu\nu}$$

$$\sum_{m_1}^{2} \phi_\mu(m_1)\phi_{\mu'}^*(m_1) = \delta_{\mu\mu'}(\delta_{\mu 1} + \delta_{\mu 2})$$

$$\sum_{m_2=1}^{3} \omega_\nu(m_2)\omega_{\nu'}^*(m_2) = -g_{\nu\nu'} + \frac{p_{(\psi)\nu}p_{(\psi)\nu'}}{p_\psi^2} \equiv -\tilde{g}_{\nu\nu'}(p_{(\psi)})$$

$$\frac{d\sigma}{d\Phi_n} \propto \frac{1}{2}\sum_{m_1}^{2}\sum_{m_2}^{3} \phi_\mu(m_1)\omega_\nu^*(m_2)A^{\mu\nu}\phi_{\mu'}^*(m_1)\omega_{\nu'}(m_2)A^{*\mu'\nu'}$$

$$\frac{d\sigma}{d\Phi_n} \propto -\frac{1}{2}\sum_{\mu=1}^{2} \tilde{g}_{\nu\nu'}(p_{(\psi)})A^{\mu\nu}A^{*\mu\nu'}$$

$$= -\frac{1}{2}\sum_{i,j}\Lambda_i\Lambda_j^* \sum_{\mu=1}^{2} U_i^{\mu\nu}\tilde{g}_{\nu\nu'}(p_{(\psi)})U_j^{*\mu\nu'}$$

- $U^{\mu\nu}$ is the partial wave amplitude constructed according to LS coupling

$$U^{\mu\nu} = (A_{LS})(A_{ls})$$

$$\begin{array}{llll} Decay: Y & \to Z_c & \pi & \qquad Z_c \to \psi \quad \pi \\ J^{PC}: 1^{--} & \to 1^{+} & 0^{-} & \qquad 1^{+} \to 1^{--}0^{-} \end{array}$$

$$U^{\mu\nu}_{(Y\to Z_c^{\pm}\pi^{\mp})SS} = \tilde{g}^{\mu\nu}_{(Z_c^+)} f^{(Z_c^+)}_{(01)} + \tilde{g}^{\mu\nu}_{(Z_c^-)} f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^{\pm}\pi^{\mp})SD} = \tilde{t}^{(2)\mu\nu}_{(\psi\pi^+)} f^{(Z_c^+)}_{(01)} + \tilde{t}^{(2)\mu\nu}_{(\psi\pi^-)} f^{(Z_c^-)}_{(02)}$$

$$U^{\mu\nu}_{(Y\to Z_c^{\pm}\pi^{\mp})DS} = \tilde{T}^{(2)\mu\lambda}_{(Z_c^+\pi^-)} \tilde{g}_{(Z_c^+)\lambda\sigma} g^{\sigma\nu} f^{(Z_c^+)}_{(01)} + \tilde{T}^{(2)\mu\lambda}_{(Z_c^-\pi^+)} \tilde{g}_{(Z_c^-)\lambda\sigma} g^{\sigma\nu} f^{(Z_c^-)}_{(02)}$$

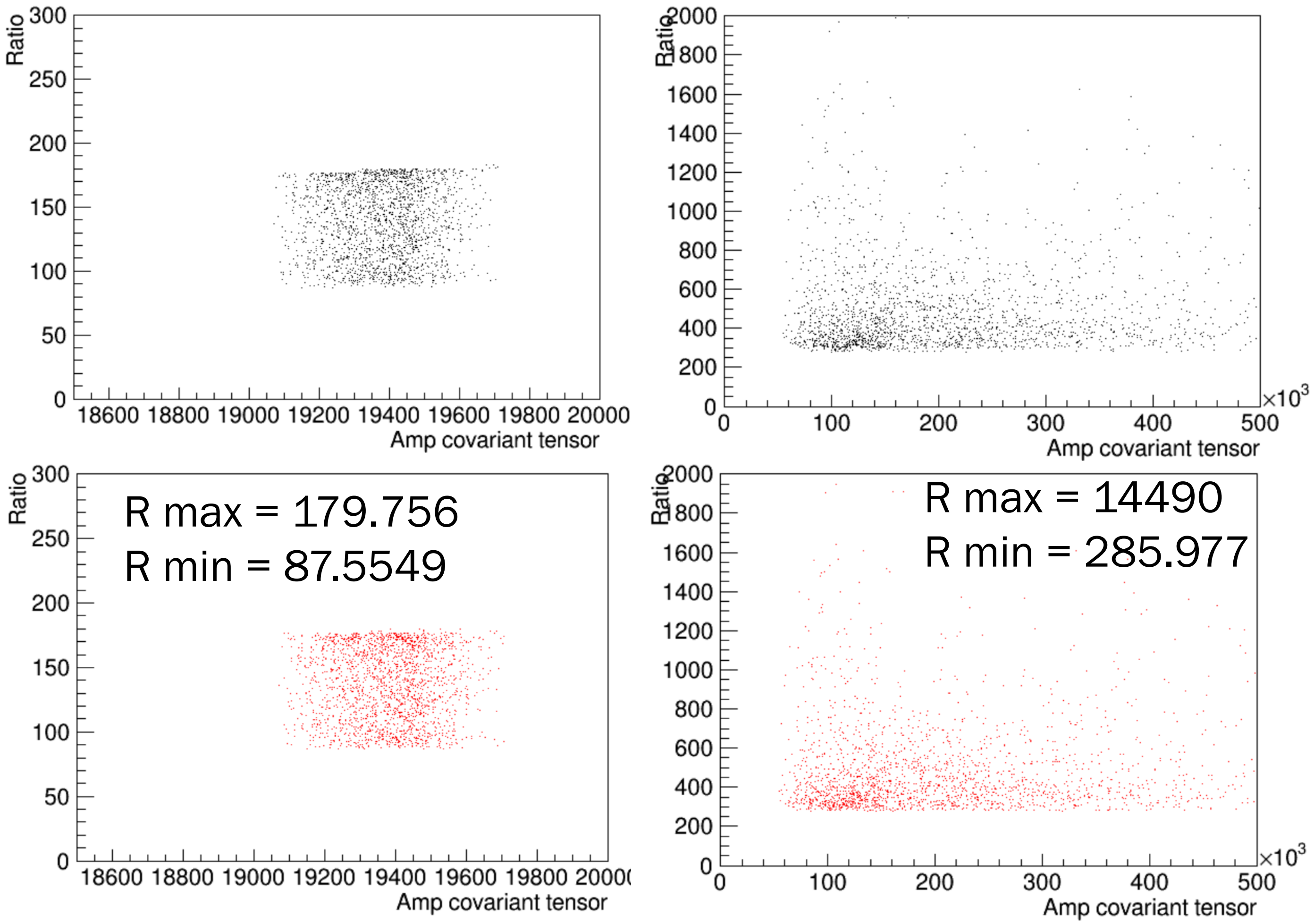
$$U^{\mu\nu}_{(Y\to Z_c^{\pm}\pi^{\mp})DD} = \tilde{T}^{(2)\mu\lambda}_{(Z_c^+\pi^-)} \tilde{t}^{(2)}_{(\psi\pi^+)\lambda\sigma} g^{\sigma\nu} f^{(Z_c^+)}_{(01)} + \tilde{T}^{(2)\mu\lambda}_{(Z_c^-\pi^+)} \tilde{t}^{(2)}_{(\psi\pi^-)\lambda\sigma} g^{\sigma\nu} f^{(Z_c^-)}_{(02)}$$

$$\begin{array}{llll} Decay: Y & \to \psi & f_0 & \qquad f_0 \to \pi^+ \quad \pi^- \\ J^{PC}: 1^{--} & \to 1^{--} & 0^{+} & \qquad 0^{+} \to 0^{-} \quad 0^{-} \end{array}$$

$$U^{\mu\nu}_{(Y\to\psi(2S)f_0)SS} = \langle \psi f_0 | 01 \rangle = g^{\mu\nu} f^{(f_0)}_{(12)}$$

$$U^{\mu\nu}_{(Y\to\psi(2S)f_0)DS} = \langle \psi f_0 | 21 \rangle = \tilde{T}^{(2)\mu\nu}_{(\psi f_0)} f^{(f_0)}_{(12)}$$

Consistency between two formalisms



Zc SS Zc DS

- Without $J/\psi \to l^+l^-$, even the amplitude ratio of Zc SS component is not a constant

Test with MC

Zc3900 MC sample: only SS component, BW function has width

Without $J/\psi \to l^+l^-$ in covariant tensor formalism

Cov: SS wave fraction = 0.292975	Hel: SS wave fraction = 0.905202
Cov: SD wave fraction = 0.500984	Hel: SD wave fraction = 0.00800091
Cov: DS wave fraction = 0.0744949	Hel: DS wave fraction = 0.0859795
Cov: DD wave fraction = 0.131546	Hel: DD wave fraction = 0.000817607

With $J/\psi \to l^+l^-$ in covariant tensor formalism

Cov: SS wave fraction = 0.930268	Hel: SS wave fraction = 0.905202
Cov: SD wave fraction = 0.00704404	Hel: SD wave fraction = 0.00800091
Cov: DS wave fraction = 0.062201	Hel: DS wave fraction = 0.0859795
Cov: DD wave fraction = 0.000487272	Hel: DD wave fraction = 0.000817607

- The test result supports the necessity of $J/\psi \to l^+l^-$
- Differences appear in fractions of two formalisms
- Invariant scattering amplitude has mass-dependent term? This term is treated as a constant?

$$G^J_{ls} = 4\pi \left(\frac{w}{p}\right)^{\frac{1}{2}} \langle JMls|\mathcal{M}|JM\rangle$$

Consistency between two formalisms

分类号 ______________ 密级 ______________

UDC ______________ 编号 ______________

中国高等科技中心
博士后研究报告

分波分析方法

李 刚

合作导师 叶铭汉 院士，朱永生 研究员

工作完成日期 2004年7月— 2006年8月

报告提交日期 2006年8月

中国高等科技中心（北京）
2006年8月

$D \to \rho^0\pi^0 \to \pi^+\pi^-\pi^0,\ \phi\pi^0 \to K^+K^-\pi^0$

Both P wave for two steps

$D \to f_2\pi^0 \to \pi^+\pi^-\pi^0$

Both D wave for two steps

Helicity formalism:

$$
\begin{aligned}
Z &= D^0_{0\lambda}(0,\theta_0\phi_0)f_\lambda(\gamma)D^{*1}_{\lambda 0}(0\theta\phi) \\
&= \gamma d^1_{00}(\theta) \qquad \lambda = 0 \\
&= \gamma\cos\theta\,,
\end{aligned}
$$

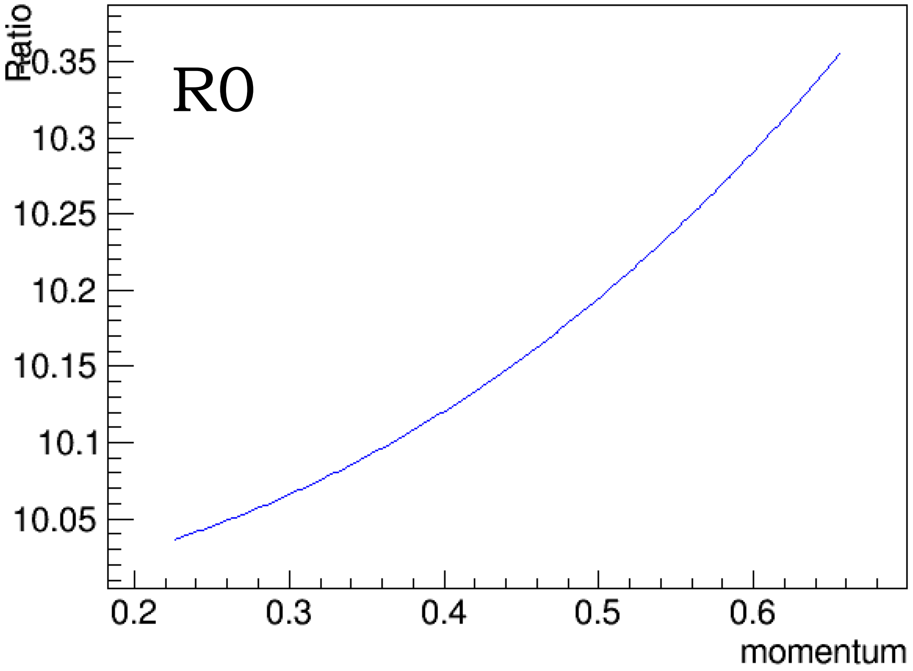
$$
\begin{aligned}
Z &= D^0_{0\lambda}(0,\theta_0\phi_0)f_\lambda(\gamma)D^{*2}_{\lambda 0}(0\theta\phi \\
&= (\frac{2}{3}\gamma^2+\frac{1}{3})d^2_{00}(\theta) \\
&= (\frac{2}{3}\gamma^2+\frac{1}{3})(\frac{3\cos^2\theta-1}{2})\,,
\end{aligned}
$$

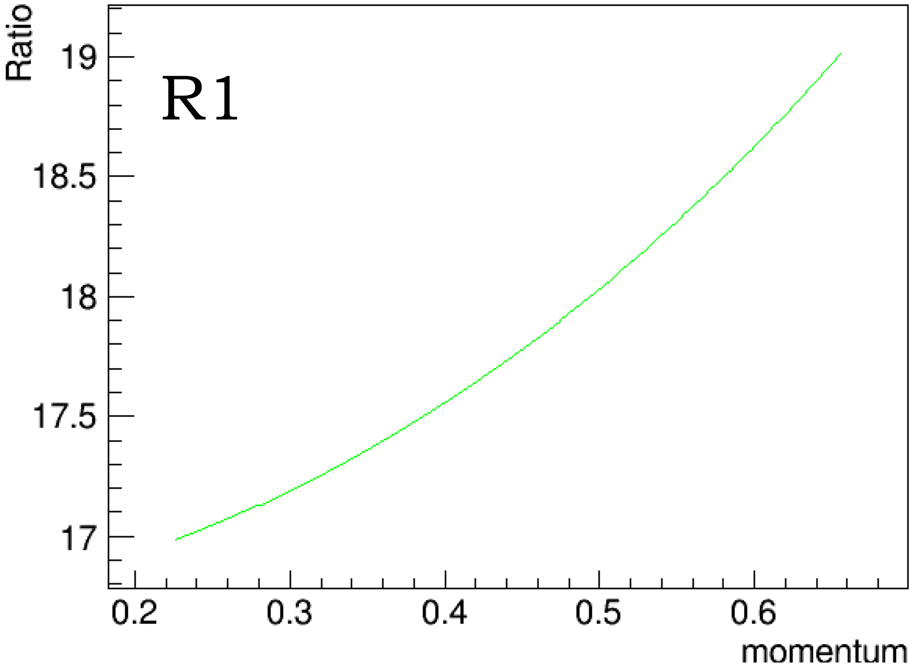
Covariant formalism:

$$
\begin{aligned}
Z &= -2p(p_a-p_b)^3 \\
&= -2p[(\gamma\beta E^*-\gamma q_3)-(\gamma\beta E^*+\gamma q_3)] \\
&= 4p\gamma(q\cos\theta). \\
&= 4pq\gamma\cos\theta\,,
\end{aligned}
$$

$$
\begin{aligned}
Z &= \tilde{T}^{(2)}_{\mu\nu}(p_a+p_b+p_c)\tilde{t}^{\mu\nu(2)}(p_a+p_b) \\
&= \left[(p_a+p_b-p_c)_i(p_a+p_b-p_c)_j-\frac{1}{3}\delta_{ij}(p_a+p_b-p_c)^2\right] \\
&\quad \left[(p_a-p_b)^i(p_a-p_b)^j-\frac{1}{3}\delta^{ij}(p_a-p_b)^2\right] \\
&= [(p_a+p_b-p_c)\cdot(p_a-p_b)]^2+\frac{1}{3}(p_a+p_b-p_c)^2(p_a-p_b)^2 \\
&\quad -\frac{1}{3}(p_a+p_b-p_c)^2(p_a-p_b)^2-\frac{1}{3}(p_a+p_b-p_c)^2(p_a-p_b)^2 \\
&= [(p_a+p_b-p_c)\cdot(p_a-p_b)]^2-\frac{1}{3}(p_a+p_b-p_c)^2(p_a-p_b)^2 \\
&= 16p^2q^2\gamma^2\cos^2\theta-\frac{1}{3}16p^2q^2(\sin^2\theta+\gamma^2\cos^2\theta) \\
&= \boxed{\frac{64}{3}\times p^2q^2\left[(\frac{2}{3}\gamma^2+\frac{1}{3})(\frac{3\cos^2\theta}{2})-\frac{1}{2}\right]}
\end{aligned}
$$

Amplitudes of F2





$$\frac{\left[2+\frac{(p_1p_2)^2}{m_1^2m_2^2}+\frac{1}{3}\right]}{\left[|A_{01}(21)|^2+|A_{01}(10)|^2+|A_{01}(01)|^2\right]}$$

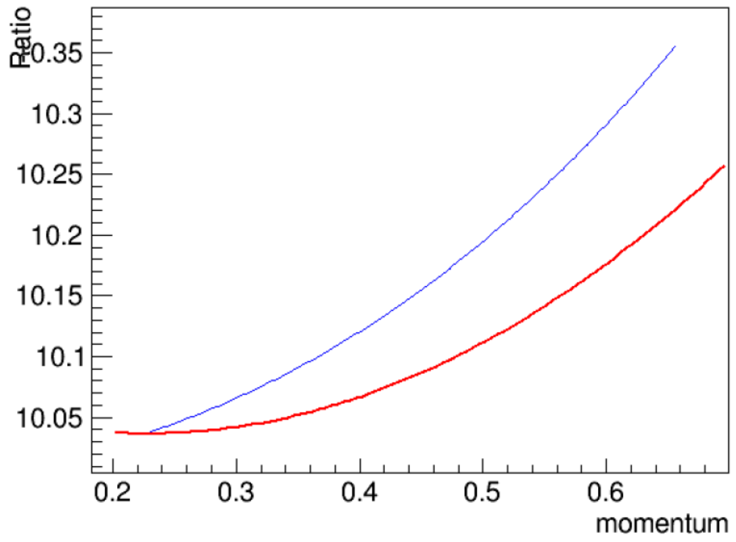
$$\frac{\left[\frac{6}{m_2^2}+\frac{3(p_1p_2)^2}{m_1^2m_2^4}+\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{(p_1p_2)^2}{m_1^2m_2^4}+\frac{2(p_1p_2)}{m_1^2m_2^2}\right]}{-|A_{01}(21)|^2-|A_{01}(10)|^2-|A_{01}(01)|^2+2|A_{01}(11)|^2+|A_{01}(00)|^2]}$$

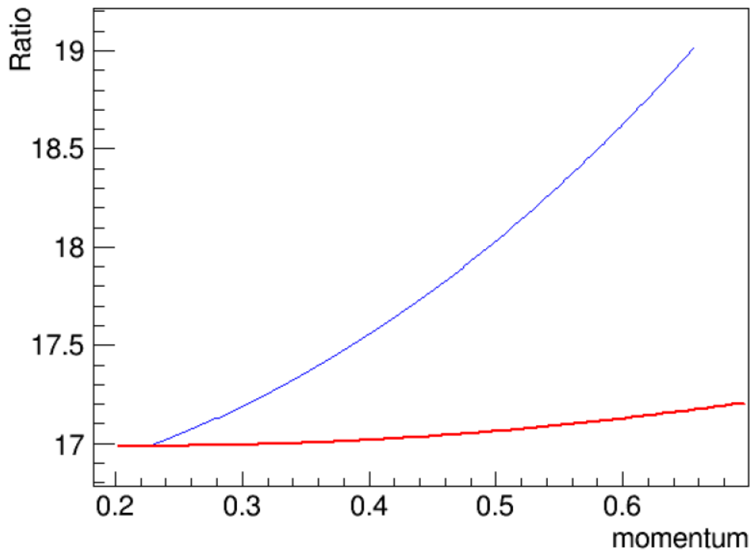
- The tendency of two part amplitude ratio to daughter particle momentum

Amplitudes of F2

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} &= \sum_{M\lambda\nu} \left|F^1_{\lambda\nu} D^1_{M,\lambda-\nu}(\phi\theta)\right|^2 = 2\sum_{\lambda\nu} \left|F^1_{\lambda\nu} D^1_{1,\lambda-\nu}(\phi\theta)\right|^2 = 2\sum_{\lambda\nu} \left|\sum_{\ell S} g_{\ell S} A_{\ell S}(\lambda\nu) d^1_{1,\lambda-\nu}\right|^2 \\
&= 2\,|g_{01}|^2 \cdot \left\{ \frac{1+\cos^2\theta}{2} \left[|A_{01}(21)|^2 + |A_{01}(10)|^2 + |A_{01}(01)|^2\right] + \frac{\sin^2\theta}{2} \left[2|A_{01}(11)|^2 + |A_{01}(00)|^2\right] \right\} \\
&= 2\,|g_{01}|^2 \cdot \left\{ \boxed{\left[|A_{01}(21)|^2 + |A_{01}(10)|^2 + |A_{01}(01)|^2\right]} + \frac{\sin^2\theta}{2} \boxed{\left[-|A_{01}(21)|^2 - |A_{01}(10)|^2 - |A_{01}(01)|^2 + 2|A_{01}(11)|^2 + |A_{01}(00)|^2\right]} \right\}
\end{aligned}
$$

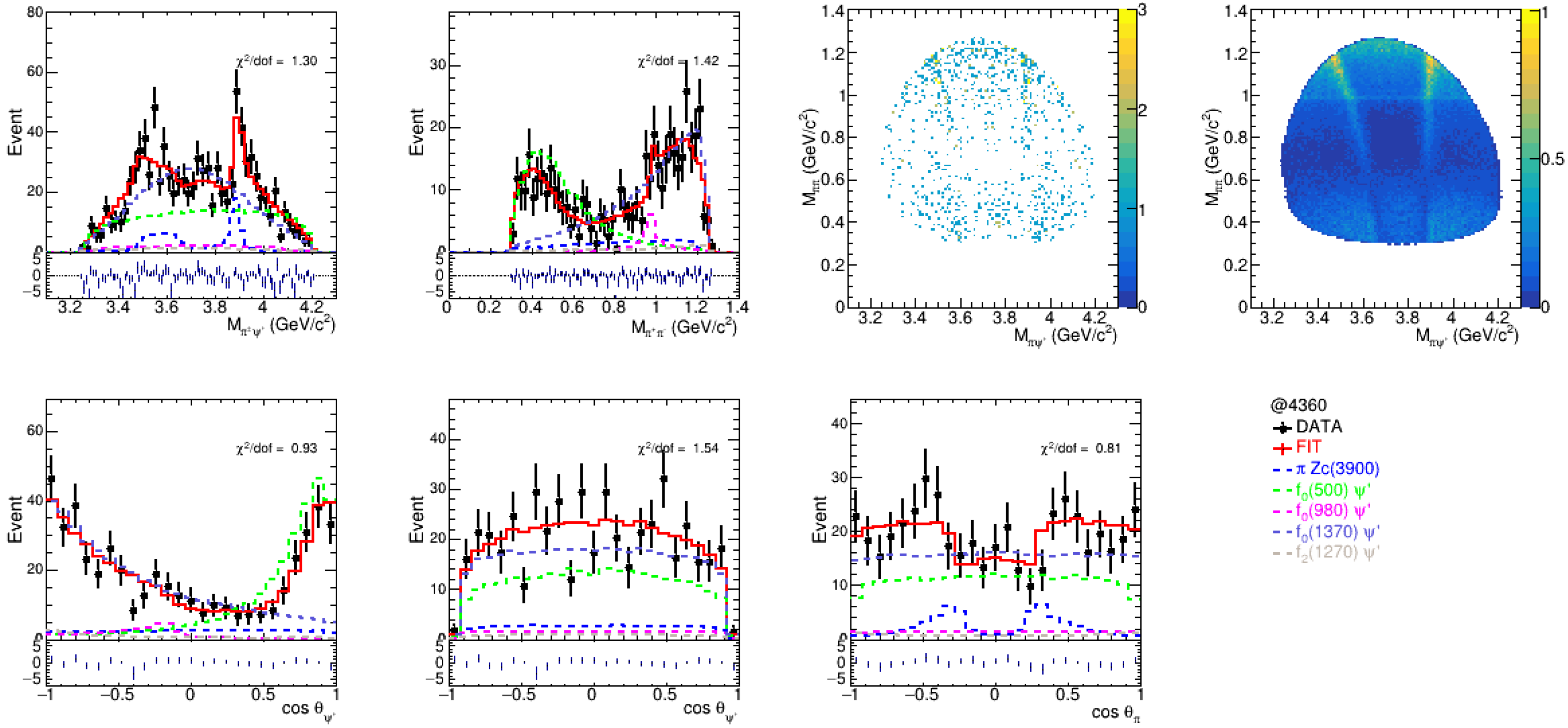
$$
\begin{aligned}
\frac{d\sigma}{d\Phi_n} &\propto -\frac{1}{2}\sum_{\mu=1}^{2} \Lambda\Lambda^* \cdot \tilde{g}^{\nu\nu'}(p_{(\psi)}) P^{(2)}_{\mu\nu\mu\nu'}(p_{(f2)}) \\
&= \frac{1}{2}\Lambda\Lambda^* \cdot \{ \left[2 + \frac{(p_1p_2)^2}{m_1^2 m_2^2}\right] \left[1 + \frac{p^2\sin^2\theta}{2m_2^2}\right] \\
&+ \frac{1}{6}\left[2 + \frac{p^2\sin^2\theta}{m_1^2} + \frac{p^2\sin^2\theta}{m_2^2} + \frac{(p_1p_2)^2 p^2\sin^2\theta}{m_1^2 m_2^4} + \frac{2(p_1p_2)p^2\sin^2\theta}{m_1^2 m_2^2}\right] \} \\
&= \frac{1}{2}\Lambda\Lambda^* \cdot \left\{ \boxed{\left[2 + \frac{(p_1p_2)^2}{m_1^2 m_2^2} + \frac{1}{3}\right]} + \frac{p^2\sin^2\theta}{6} \boxed{\left[\frac{6}{m_2^2} + \frac{3(p_1p_2)^2}{m_1^2 m_2^4} + \frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{(p_1p_2)^2}{m_1^2 m_2^4} + \frac{2(p_1p_2)}{m_1^2 m_2^2}\right]} \right\}
\end{aligned}
$$





- Up plot is the tendency of ratio to momentum for terms in blue box, down plot is for terms in green box
- Blue lines are the ratios for two body decay phsp MC, red lines are second power functions for comparison

Fit result of 4360



Fit result of 4360