

Excitation function of freeze-out parameters in Nucleus-Nucleus and proton-proton collisions at the same collision energy

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In case, the hard process ($p_T < 3 \text{ GeV}/c$) is taken into account, then the power law distribution is found to be more prominent and is given as

➤ Large than 3 or peripheral

$$f(p_T) = \frac{1}{N} \frac{dN}{dp_T} = A p_T \left(1 + \frac{p_T}{p_0} \right)^{-n}, \quad (4)$$

Where A denotes the normalization constant and n and p_0 are the free parameters.

87

We can use the superposition principle when the hard scattering is involved which is as

$$f(p_T) = \frac{1}{N} \frac{dN}{dp_T} = k f_S(p_T) + (1 - k) f_H(p_T). \quad (5)$$

Where f_H and f_S represent soft and hard processes and k is the contribution fraction of the soft process to the hard process.

88

➤ More parameters, better fitting results, consistent phenomenological concepts
➤ Neither part is linear, especially the hard processes (QM ~ linear; QFT₁ ~ non)

89

Fit results (Fig.1, Fig.2, Tab.1)

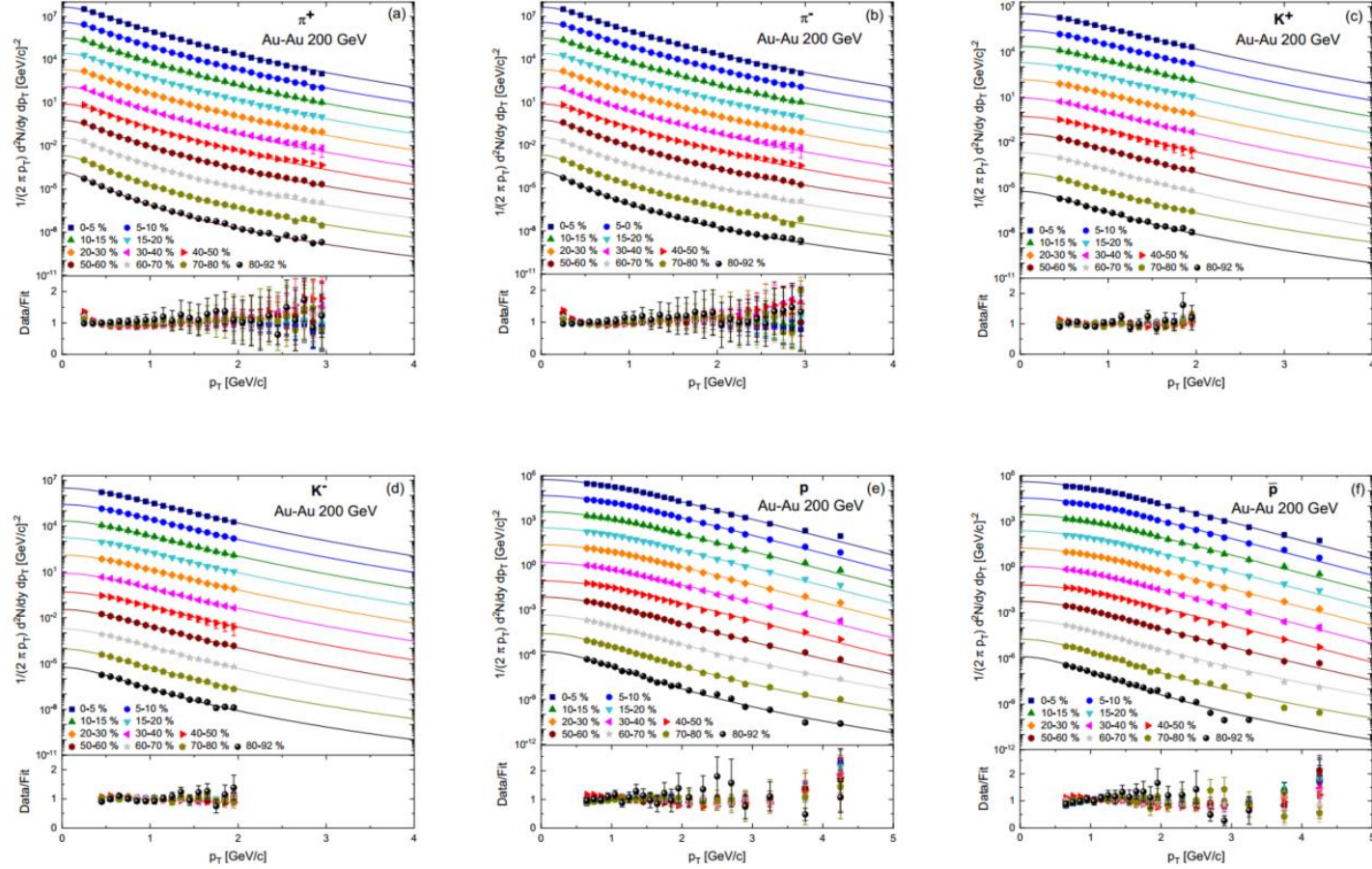
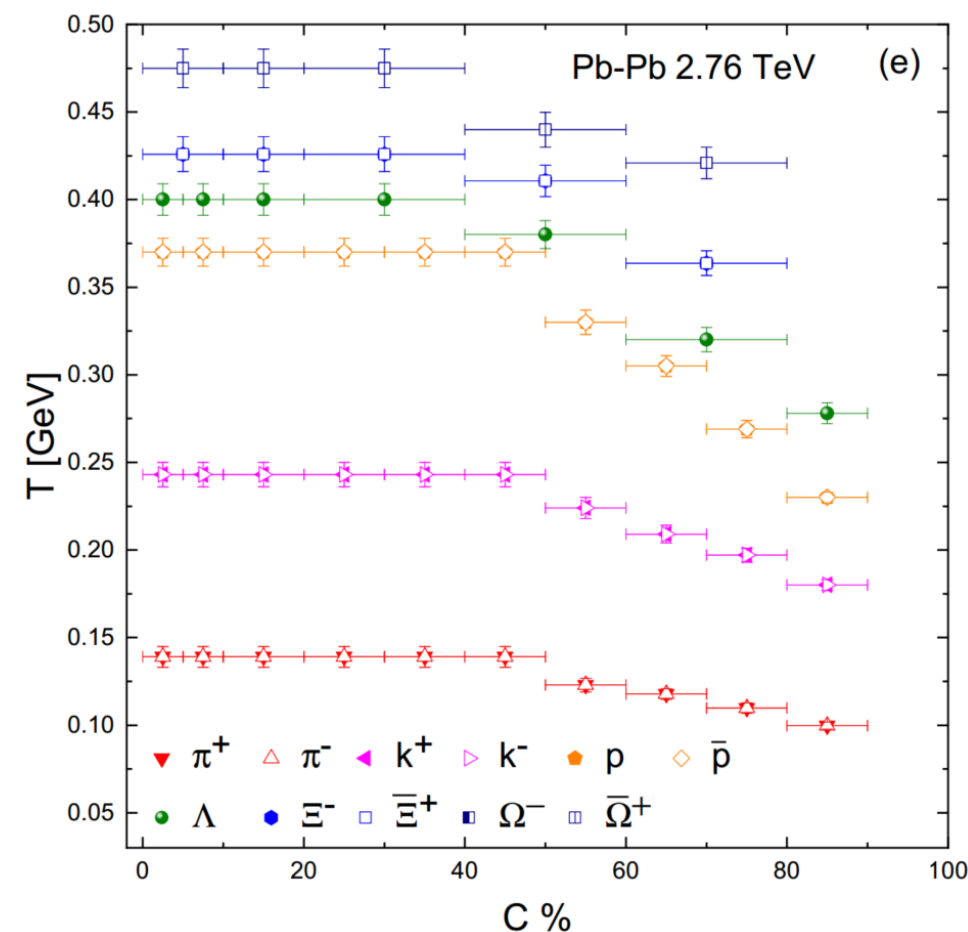
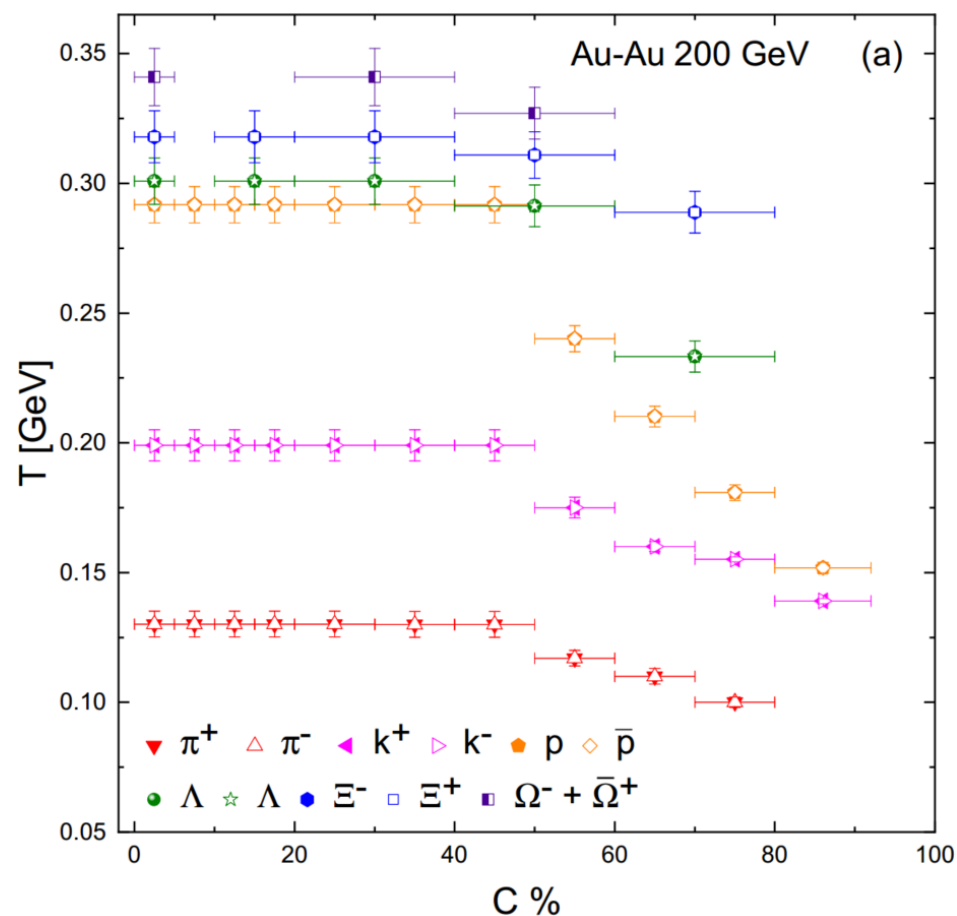


Table 1. Values of free parameters T and q , N_0 , $\langle p_T \rangle$ and χ^2/NDF extracted from the Tsallis-Pareto type function given in Eq.3

Collision	Particle	Centrality	Scaling factor	T [GeV]	q	$\langle p_T \rangle$ [GeV/c]	N_0	χ^2/NDF
Fig.1 Au-Au 200 GeV	π^+	0-5%	10^5	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	18002.8 ± 500	$21.146/25$
		5-10%	10^4	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	15202.8 ± 200	$21.920/25$
		10-15%	10^3	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	12912.8 ± 150	$23.199/25$
		15-20%	10^2	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	10932.7 ± 150	$26.539/25$
		20-30%	10	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	8132.8 ± 140	$35.312/25$
		30-40%	1	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	5842.5 ± 120	$83.160/25$
		40-50%	10^{-1}	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	3472.9 ± 100	$61.810/25$
		50-60%	10^{-2}	0.110 ± 0.004	1.110 ± 0.004	0.232 ± 0.012	2079.8 ± 90	$17.099/25$
		60-70%	10^{-3}	0.103 ± 0.004	1.118 ± 0.004	0.220 ± 0.012	1160.8 ± 30	$9.500/25$
		70-80%	10^{-4}	0.093 ± 0.003	1.125 ± 0.005	0.200 ± 0.011	604.9 ± 10	$5.473/25$
Fig.1 Au-Au 200 GeV	π^-	0-5%	10^5	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	18002.8 ± 500	$25.491/25$
		5-10%	10^4	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	15202.8 ± 200	$27.524/25$
		10-15%	10^3	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	12912.8 ± 150	$32.500/25$
		15-20%	10^2	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	10932.7 ± 150	$33.872/25$
		20-30%	10	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	8132.8 ± 140	$37.682/25$
		30-40%	1	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	5642.5 ± 120	$77.190/25$
		40-50%	10^{-1}	0.120 ± 0.005	1.093 ± 0.003	0.248 ± 0.013	3372.9 ± 100	$63.068/25$
		50-60%	10^{-2}	0.110 ± 0.004	1.110 ± 0.004	0.232 ± 0.012	2079.8 ± 90	$19.137/25$
		60-70%	10^{-3}	0.103 ± 0.004	1.118 ± 0.004	0.220 ± 0.012	1160.8 ± 30	$10.345/25$
		70-80%	10^{-4}	0.093 ± 0.003	1.125 ± 0.005	0.200 ± 0.011	604.9 ± 10	$4.824/25$
Fig.1 Au-Au 200 GeV	K^+	0-5%	10^5	0.199 ± 0.006	1.060 ± 0.003	0.399 ± 0.020	1880.8 ± 90	$40.931/13$
		5-10%	10^4	0.199 ± 0.006	1.060 ± 0.003	0.399 ± 0.020	1540.7 ± 70	$23.196/13$
		10-15%	10^3	0.199 ± 0.006	1.060 ± 0.003	0.399 ± 0.020	1290.8 ± 50	$18.055/13$
		15-20%	10^2	0.199 ± 0.006	1.060 ± 0.003	0.399 ± 0.020	1060.8 ± 30	$17.045/13$
		20-30%	10	0.199 ± 0.006	1.060 ± 0.003	0.399 ± 0.020	779.9 ± 20	$21.988/13$
		30-40%	1	0.199 ± 0.006	1.060 ± 0.003	0.399 ± 0.020	495.8 ± 10	$30.512/13$
		40-50%	10^{-1}	0.199 ± 0.006	1.060 ± 0.003	0.399 ± 0.020	291.5 ± 7	$79.333/13$
		50-60%	10^{-2}	0.175 ± 0.004	1.070 ± 0.004	0.360 ± 0.018	189.1 ± 7	$43.277/13$
		60-70%	10^{-3}	0.160 ± 0.004	1.081 ± 0.005	0.337 ± 0.018	98.9 ± 4	$69.408/13$
		70-80%	10^{-4}	0.155 ± 0.003	1.111 ± 0.006	0.336 ± 0.017	41.9 ± 2	$14.914/13$
Fig.1 Au-Au 200 GeV	K^-	0-5%	10^5	0.199 ± 0.006	1.070 ± 0.003	0.402 ± 0.020	1740.7 ± 80	$18.282/13$
		5-10%	10^4	0.199 ± 0.006	1.070 ± 0.003	0.402 ± 0.020	1440.7 ± 50	$16.330/13$
		10-15%	10^3	0.199 ± 0.006	1.070 ± 0.003	0.402 ± 0.020	1180.8 ± 30	$16.330/13$
		15-20%	10^2	0.199 ± 0.006	1.070 ± 0.003	0.402 ± 0.020	971.6 ± 15	$25.125/13$
		20-30%	10	0.199 ± 0.006	1.070 ± 0.003	0.402 ± 0.020	719.8 ± 10	$27.379/13$
		30-40%	1	0.199 ± 0.006	1.070 ± 0.003	0.402 ± 0.020	453.8 ± 7	$71.082/13$
		40-50%	10^{-1}	0.199 ± 0.006	1.070 ± 0.003	0.402 ± 0.020	266.6 ± 6	$99.387/13$
		50-60%	10^{-2}	0.175 ± 0.004	1.080 ± 0.004	0.363 ± 0.018	174.2 ± 4	$22.910/13$
		60-70%	10^{-3}	0.160 ± 0.004	1.088 ± 0.005	0.339 ± 0.018	89.7 ± 3.7	$88.802/13$
		70-80%	10^{-4}	0.155 ± 0.003	1.111 ± 0.006	0.336 ± 0.017	40.9 ± 2	$14.391/13$
Fig.1 Au-Au 200 GeV	p	0-5%	10^5	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	471.8 ± 20	$78.267/19$
		5-10%	10^4	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	394.8 ± 15	$59.704/19$
		10-15%	10^3	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	325.9 ± 13	$39.207/19$
		15-20%	10^2	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	270.3 ± 10	$31.674/19$
		20-30%	10	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	202.2 ± 7	$33.076/19$
		30-40%	1	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	130.3 ± 5	$56.559/19$
		40-50%	10^{-1}	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	76.3 ± 3	$137.801/19$
		50-60%	10^{-2}	0.240 ± 0.005	1.041 ± 0.003	0.513 ± 0.026	54.3 ± 2	$10.924/19$
		60-70%	10^{-3}	0.210 ± 0.004	1.071 ± 0.004	0.472 ± 0.024	31.1 ± 1.5	$5.877/19$
		70-80%	10^{-4}	0.180 ± 0.003	1.089 ± 0.005	0.430 ± 0.022	16.1 ± 0.6	$7.078/19$
Fig.1 Au-Au 200 GeV	$\bar{p}$	0-5%	10^5	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	471.8 ± 20	$78.267/19$
		5-10%	10^4	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	394.8 ± 15	$59.704/19$
		10-15%	10^3	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	325.9 ± 13	$39.207/19$
		15-20%	10^2	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	270.3 ± 10	$31.674/19$
		20-30%	10	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	202.2 ± 7	$33.076/19$
		30-40%	1	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	130.3 ± 5	$56.559/19$
		40-50%	10^{-1}	0.291 ± 0.007	1.011 ± 0.003	0.586 ± 0.029	76.3 ± 3	$137.801/19$
		50-60%	10^{-2}	0.240 ± 0.005	1.041 ± 0.003	0.513 ± 0.026	54.3 ± 2	$10.924/19$
		60-70%	10^{-3}	0.210 ± 0.004	1.071 ± 0.004	0.472 ± 0.024	31.1 ± 1.5	$5.877/19$
		70-80%	10^{-4}	0.180 ± 0.003	1.089 ± 0.005	0.430 ± 0.022	16.1 ± 0.6	$7.078/19$

Fig. 1 Au-Au 200 GeV	Λ	0-5%	10^2	0.300 ± 0.009	1.009 ± 0.003	0.622 ± 0.031
		10-20%	10	0.300 ± 0.009	1.009 ± 0.003	0.622 ± 0.031
		20-40%	1	0.300 ± 0.009	1.009 ± 0.003	0.622 ± 0.031
		40-60%	10^{-1}	0.291 ± 0.008	1.019 ± 0.004	0.611 ± 0.031
		60-80%	10^{-2}	0.233 ± 0.006	1.055 ± 0.005	0.522 ± 0.026
Collision	Particle	Centrality	Scaling factor	T [GeV]	q	$\langle p_T \rangle$ [GeV/c]
Fig. 1 Au-Au 200 GeV	$\bar{\Lambda}$	0-5%	10^2	0.300 ± 0.009	1.009 ± 0.003	0.622 ± 0.031
		10-20%	10	0.300 ± 0.009	1.009 ± 0.003	0.622 ± 0.031
		20-40%	1	0.300 ± 0.009	1.009 ± 0.003	0.622 ± 0.031
		40-60%	10^{-1}	0.291 ± 0.008	1.019 ± 0.004	0.611 ± 0.031
		60-80%	10^{-2}	0.233 ± 0.006	1.055 ± 0.005	0.522 ± 0.026
Fig. 1 Au-Au 200 GeV	Ξ^-	0-5%	10^2	0.317 ± 0.010	1.007 ± 0.003	0.675 ± 0.034
		10-20%	10	0.317 ± 0.010	1.007 ± 0.003	0.675 ± 0.034
		20-40%	1	0.317 ± 0.010	1.007 ± 0.004	0.675 ± 0.034
		40-60%	10^{-1}	0.310 ± 0.009	1.009 ± 0.005	0.665 ± 0.033
		60-80%	10^{-2}	0.288 ± 0.008	1.039 ± 0.006	0.637 ± 0.032
Fig. 1 Au-Au 200 GeV	$\bar{\Xi}^+$	0-5%	10^2	0.317 ± 0.010	1.007 ± 0.003	0.675 ± 0.034
		10-20%	10	0.317 ± 0.010	1.007 ± 0.003	0.675 ± 0.034
		20-40%	1	0.317 ± 0.010	1.007 ± 0.003	0.675 ± 0.034
		40-60%	10^{-1}	0.310 ± 0.009	1.009 ± 0.004	0.665 ± 0.033
		60-80%	10^{-2}	0.288 ± 0.008	1.039 ± 0.005	0.637 ± 0.032
Fig. 1 Au-Au 200 GeV	$\Omega^- + \bar{\Omega}^+$	0-5%	10	0.340 ± 0.011	1.005 ± 0.003	0.756 ± 0.038
		20-40%	1	0.340 ± 0.011	1.005 ± 0.003	0.756 ± 0.038
		40-60%	10^{-1}	0.326 ± 0.010	1.020 ± 0.004	0.740 ± 0.037

Fig.4



- L138 “**more collision energy is deposited in a larger volume rather than higher temperature**” (40-50% to the most central collisions show a smooth behavior for T . It tends to be in thermal equilibrium and reaches a maximum temperature in 40-50%)
- $p_T \sim (0,3)$ hadronization together for $\pi/k/p$, so in principle fit simultaneously (In particular, it is necessary to obtain changes with the center, and the control variable method should also be fitted simultaneously). If perform fits like this, then the above can be drawn for AuAu and PbPb systems and correspond energies.
- T extracted from pion spectra is consistent between auau and pbpb, soft hadrons can directly reflect system evolution. However, the other particle cases are significantly larger in PbPb than in AuAu.

L139-145

“The region from 0 - 40% centrality is meson dominated region where QGP is produced while the region beyond this to the most periphery is baryon dominated. ”

(pion+) 2.18+ 1.36+ 8.36E-01+ 5.29E-+01+ 3.51E-01+ 2.21E-01+ 1.51E-01+ 1.10E-01+ 7.17

(pion-) 2.16E+00+ 1.30E+00+ 8.30E-01+ 5.26E-01+ 3.45E-01+ 2.32E-01+ 1.47E-01+ 1.05E-

(k+) 4.81E-01+ 3.40E-01+ 2.33E-01+ 1.69E-01+ 1.19E-01+ 7.84E-02+ 5.43E-02+ 3.85E-02+

(k-) 4.43E-01+ 3.16E-01+ 2.31E-01+ 1.56E-01+ 1.09E-01+ 7.06E-02+ 5.72E-02+ 3.67E-02+ 2.38E-02

(p+) 2.04E-01+ 1.65E-01+ 1.27E-01+ 1.00E-01+ 7.43E-02+ 5.88E-02+ 3.98E-02+ 3.41E-02+ 2.41E-02

(p-) 1.58E-01+ 1.25E-01+ 9.50E-02+ 7.38E-02+ 5.50E-02+ 4.34E-02+ 3.19E-02+ 2.40E-02+ 1.90E-02

(60-70% pT~(0.6, 1.5))

(60-80%) ($\wedge$ $|y|<1$) 0.168534+0.143039+0.082781+0.0542705+0.0333924+ 0.0199942

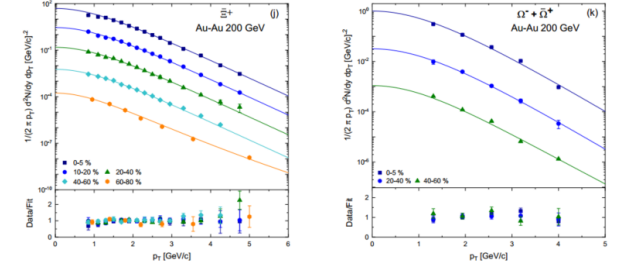


Figure 1. Plots (a) - (k) represent the event centrality-dependant double differential p_T spectra of identified particles at $|\eta| < 0.35$ measured by PHENIX Collaboration at RHIC [51] and strange, at $|y| < 1$ for Λ and $\bar{\Lambda}$ and $|y| < 0.5$ for Ξ , $\bar{\Xi}$ and $\Omega^- + \bar{\Omega}^+$ measured by STAR at RHIC [52] hadrons at $\sqrt{s} = 200$ GeV in Au-Au collision. The data for these particles have been analyzed at all available centrality events indicated in each plot. Different symbols with different colors are used for different centralities, while the solid lines are the results of our fit by using Eq. 3. Each plot has the Data/Fit ratio at its bottom, which shows the fit quality.

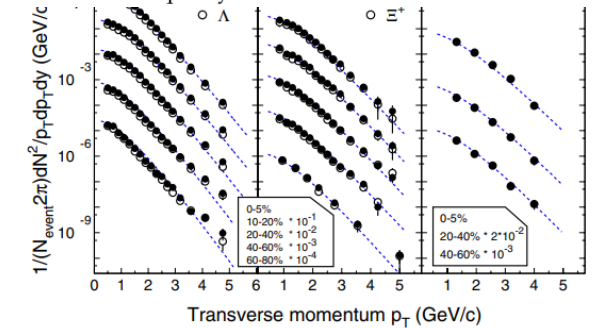
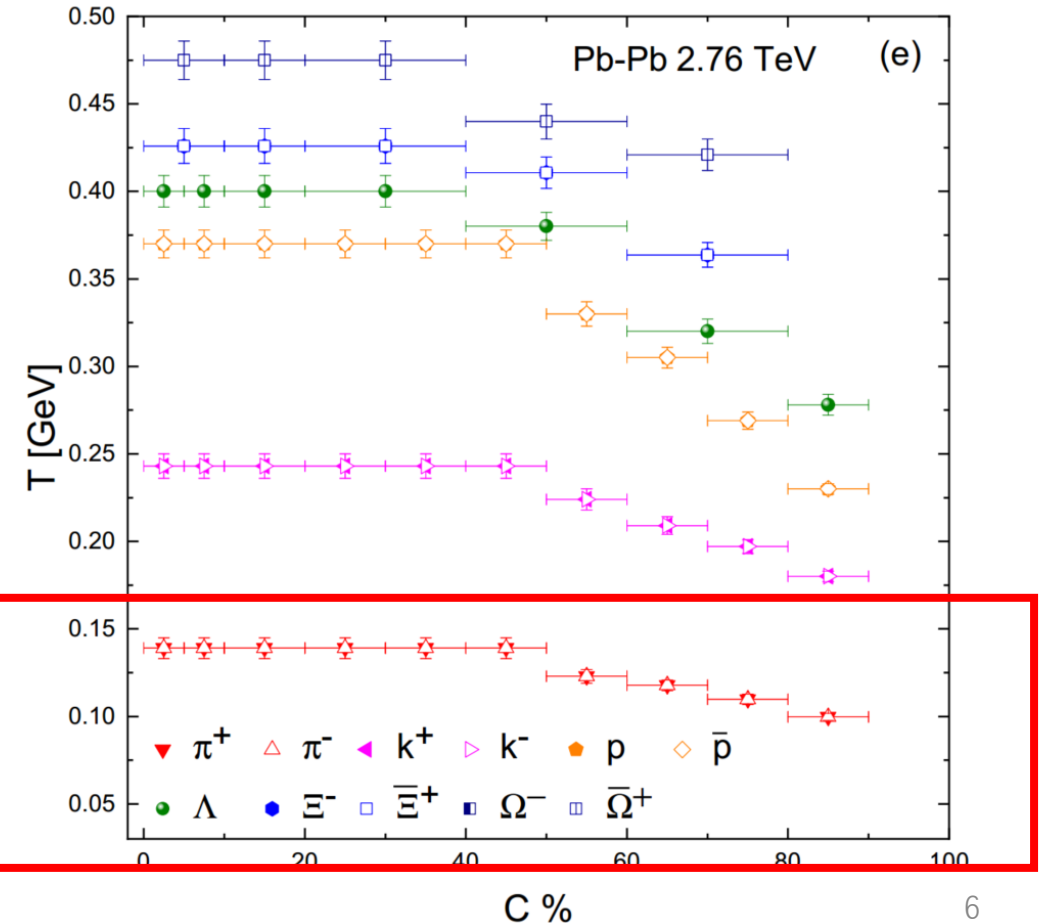
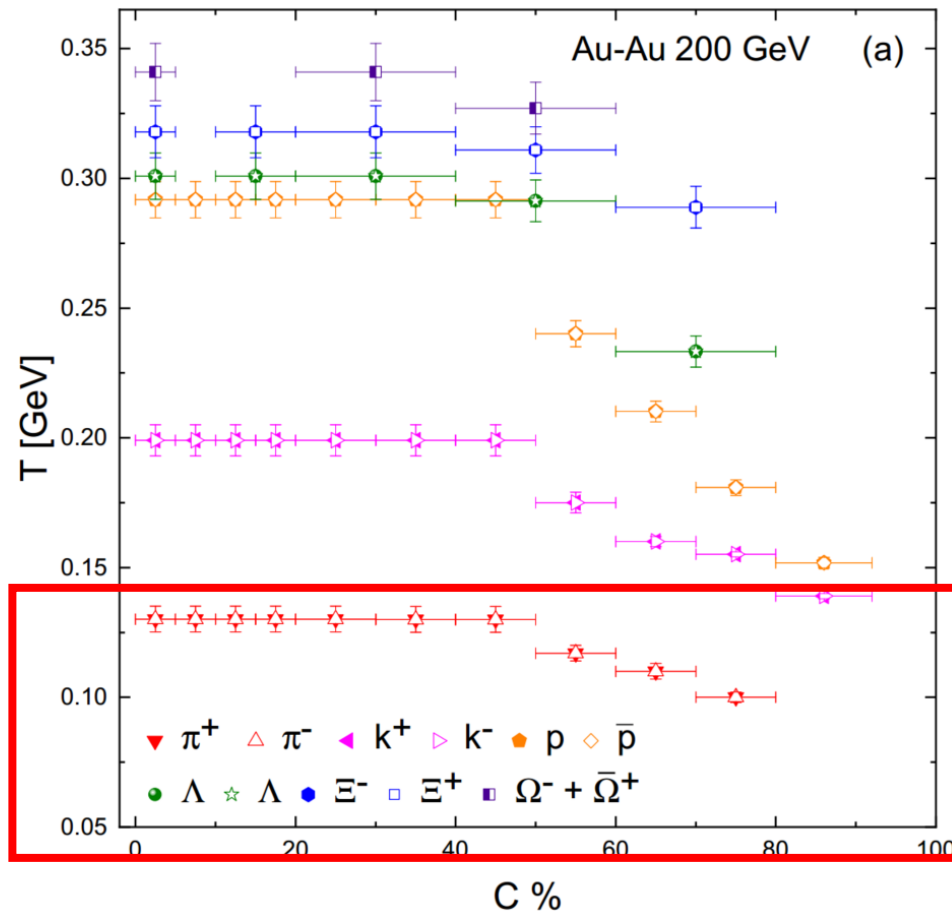


FIG. 1 (color online). Transverse momentum distributions of (a) $\Lambda(\bar{\Lambda})$ for $|y| < 1.0$, (b) $\Xi^-(\bar{\Xi}^+)$ for $|y| < 0.75$, and (c) $\Omega^- + \bar{\Omega}^+$ for $|y| < 0.75$ in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV as a function of centrality. Scale factors were applied to the spectra for clarity. Only statistical errors are shown. The dashed curves show a Boltzmann fit to the Λ , Ξ^- , and $\Omega^- + \bar{\Omega}^+$ data; the fits to the $\bar{\Lambda}$ and $\bar{\Xi}^+$ are omitted for clarity.

Line 176

From the above results, we also reported that $T_\perp < p_T$ and N_0 are dependent on the size of the system. Larger the system, the larger they are. As a larger collision system has large participants involved in the interaction, it experiences an intense reaction, where more energy is deposited in the system, which corresponds to a large transfer of momentum in the system which naturally leads the system to be highly excited, and due to large transfer of energy (momentum), further multiple scattering happens and results in larger N_0 (multiplicity).



- Fig.7 does not appear in the text
- Kinematic Freeze-out Temperature is different between previous results.
- T is different from temperature in Boltzmann's thermodynamic statistics. How does the $T_{thermal}$ depend on centrality classes?

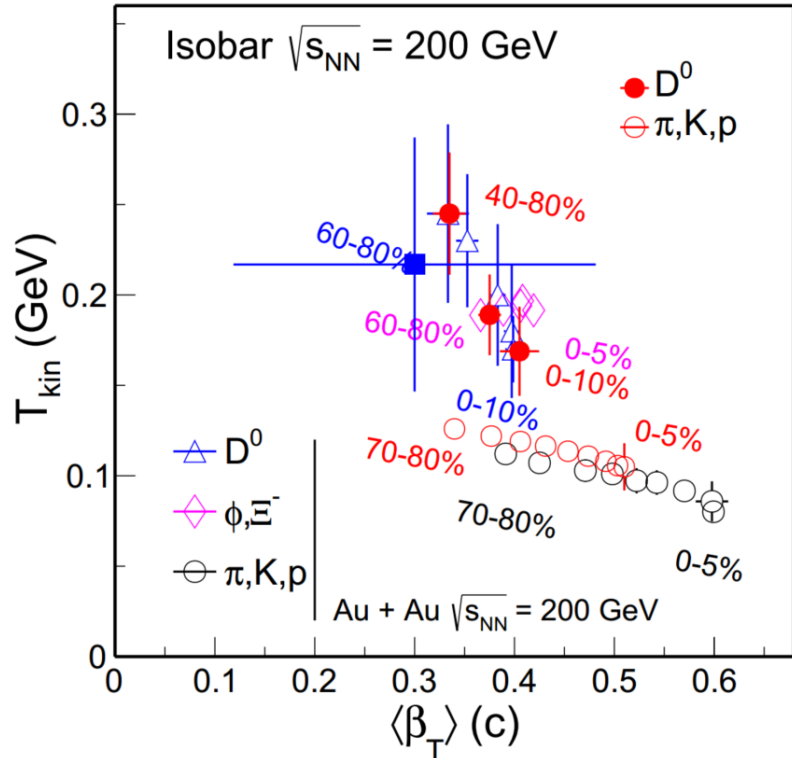


Figure 7. Dependence of (a) $\langle T_0 \rangle$ and (b) $\langle \beta_T \rangle$ on centrality in Au-Au and Pb-Pb collisions respectively.

Gábor Bíró et al 2020 J. Phys. G: Nucl. Part. Phys. 47 105002

$$T = T_{fro} + m \langle u_t \rangle^2, \quad (33)$$

where T_{fro} is the hadron kinetic freeze-out temperature and $\langle u_t \rangle$ is the measure of the strength of the average radial transverse flow, which is connected to the averaged transverse velocity via

$$\langle v_t \rangle = \frac{\langle u_t \rangle}{\sqrt{1 + \langle u_t \rangle^2}}. \quad (34)$$

$$T_{thermal} = T \sqrt{\frac{1 - v_t}{1 + v_t}}$$

Some other conflicts

work. But to show the extraction of the kinetic freeze-out temperature T_0 and transverse
flow velocity β_T from T , we took the particles with same centrality and extracted the two

The following abbreviations are used in this manuscript:

p_T	transverse momentum
T_{eff}	<u>kinetic freeze-out temperature</u>
$\langle T_{eff} \rangle$	<u>effective temperature</u>
β_T	transverse flow velocity
$\langle \beta_T \rangle$	average transverse flow velocity

from these systems. The present work extracts the effective temperature T , non-extensivity parameter
(q), the mean transverse momentum spectra ($\langle p_T \rangle$), the multiplicity parameter (N_0), kinetic
freeze-out temperature (T_0) and transverse flow velocity (β_T). We reported a plateau structure of p_T ,