

# Amplitude analysis

# Quantum Chromo Dynamics

F. Wilczek, [QCD Made Simple](#)  
 Physics Today **53N8** 22-28, (2000)

$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_f \bar{q}_f (i \gamma^\mu D_\mu + m_f) q_f$$

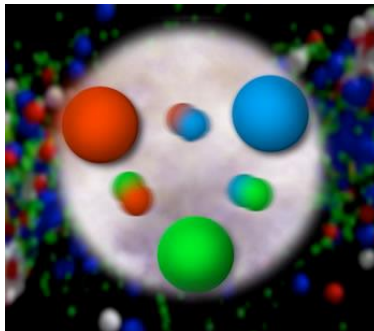
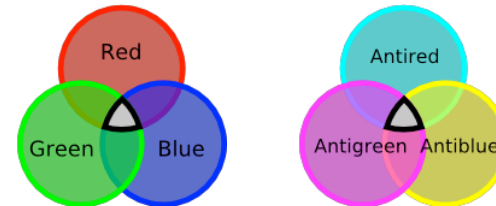
where  $G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c$   
 and  $D_\mu \equiv \partial_\mu + i g_s A_\mu^a$   
 That's it!

The rules that govern how the quarks froze out into hadrons are given by QCD.

Quarks have color charge: red, blue and green.

Antiquarks have anticolors: cyan, yellow and magenta.

Bound states of quarks are color neutral, "white".



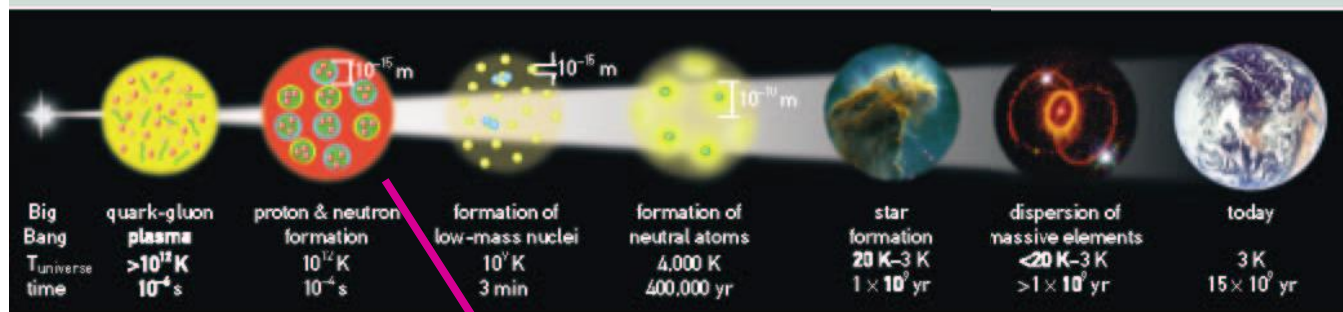
"White" can be one of each color:

red-blue-green, cyan-yellow-magenta

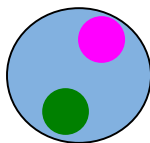
or a color and an anticolor:

red-cyan, blue-yellow, green-magenta

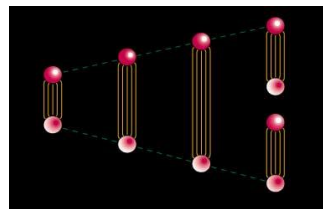
# Confinement



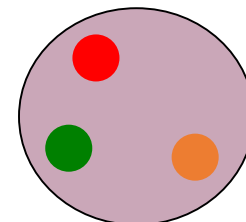
From about  $10^{-6} \text{ s}$  on, the quark and anti quarks became confined inside of Hadronic matter. At the age of  $1 \text{ s}$ , only protons and neutrons remained.



Mesons



The gluons produce the 16 ton force that binds the quarks.



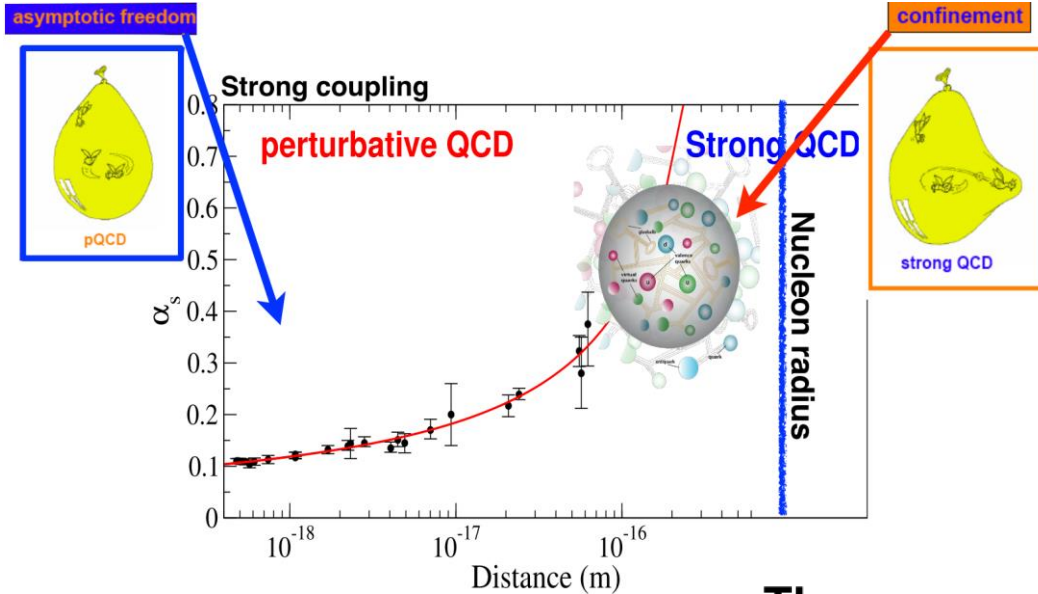
Baryons

Quarks can never be isolated

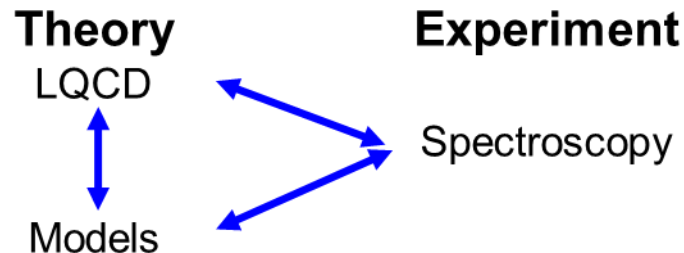
How does QCD give rise to excited hadrons?

- What is the origin of confinement?
- How are confinement and chiral symmetry breaking connected?

# Hadron spectroscopy



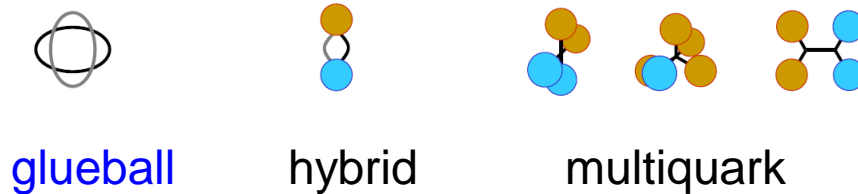
What is the origin of confinement?  
 How are confinement and chiral symmetry breaking connected?



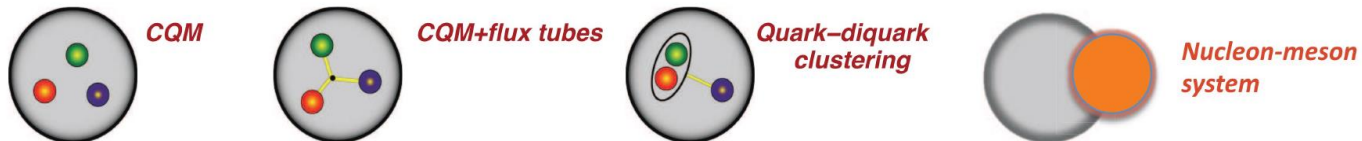
- Testing QCD in the confinement regime
- Revealing the fundamental degrees of freedom

# Light hadron spectroscopy at BESIII

- Meson spectroscopy: What are the nature of QCD exotics? **What's the role of gluonic excitation and how does it connect to the confinement?**

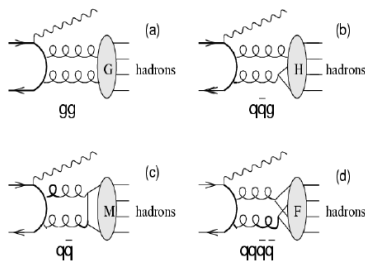


- Baryons spectroscopy: What are the fundamental degrees of freedom inside a nucleon? How do the degrees change with varying quark masses?



# Charmonium decays provide an ideal lab for light hadron physics

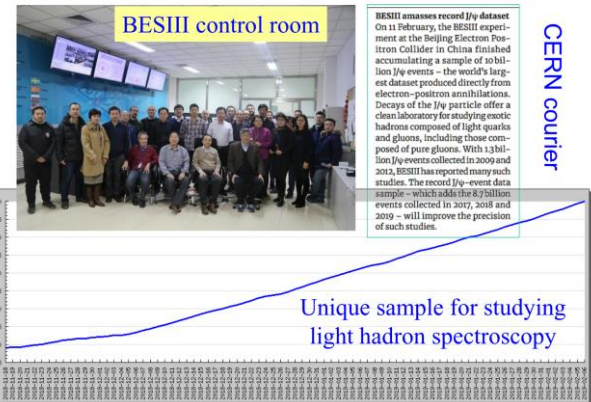
- Clean high statistics data samples
  - High cross sections of  $e^+e^- \rightarrow J/\psi, \psi'$
  - Low background
- Well defined initial and final states
  - Kinematic constraints
  - $I(J^{PC})$  filter
- “Gluon-rich” process



$$\Gamma(J/\psi \rightarrow \gamma G) \sim O(\alpha_s^2), \Gamma(J/\psi \rightarrow \gamma H) \sim O(\alpha_s^3),$$

$$\Gamma(J/\psi \rightarrow \gamma M) \sim O(\alpha_s^4), \Gamma(J/\psi \rightarrow \gamma F) \sim O(\alpha_s^4)$$

10 Billion  $J/\psi$  events by Feb. 2019

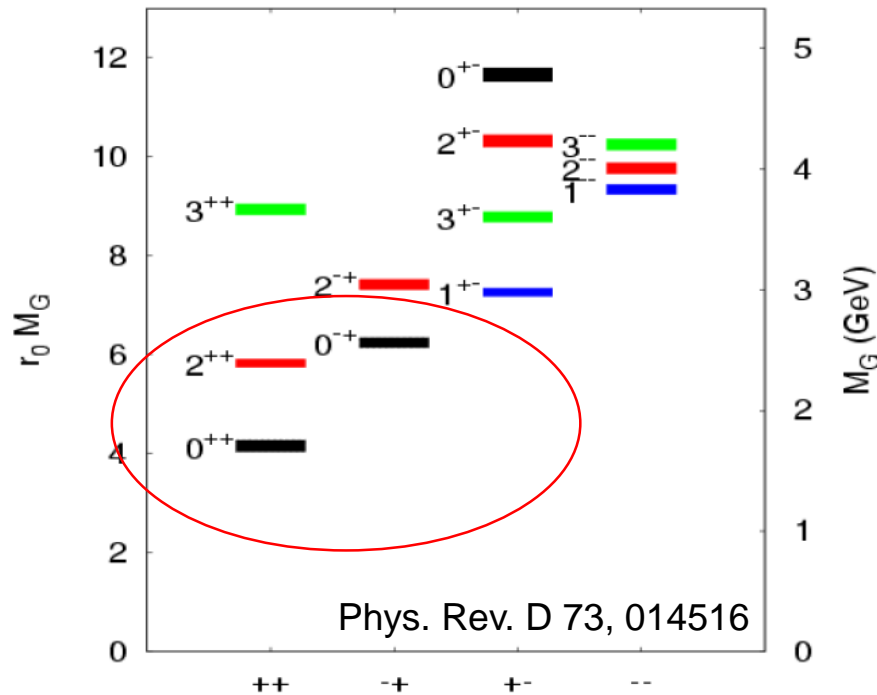


BESIII designed  $J/\psi$  stat. has been achieved  
 —A legacy data set

BESIII remains unique for Light hadron physics

# Glueball

Provide critical information on the gluon field and the quantitative understanding of confinement



	$m_\pi$ (MeV)	$m_{0^{++}}$ (MeV)	$m_{2^{++}}$ (MeV)	$m_{0^{-+}}$ (MeV)
$N_f = 2$	938	1417(30)	2363(39)	2573(55)
	650	1498(58)	2384(67)	2585(65)
$N_f = 2 + 1$ [22]	360	1795(60)	2620(50)	—
quenched [13]	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
quenched [14]	—	1730(50)(80)	2400(25)(120)	2590(40)(130)

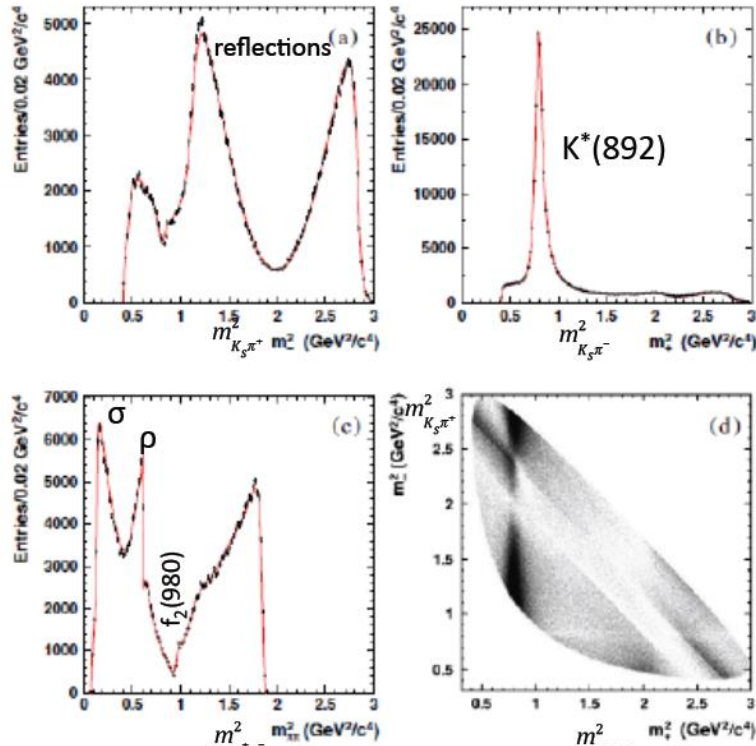
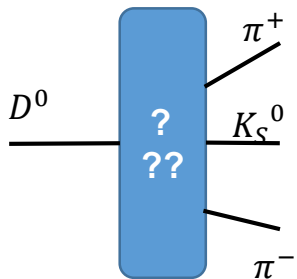
Low lying glueballs with ordinary quantum number  
 → mixing with qqbar mesons

Systematic studies needed

- Outnumbering of conventional QM states
- Abnormal properties

Glueballs from Quenched LQCD

# Amplitude analysis

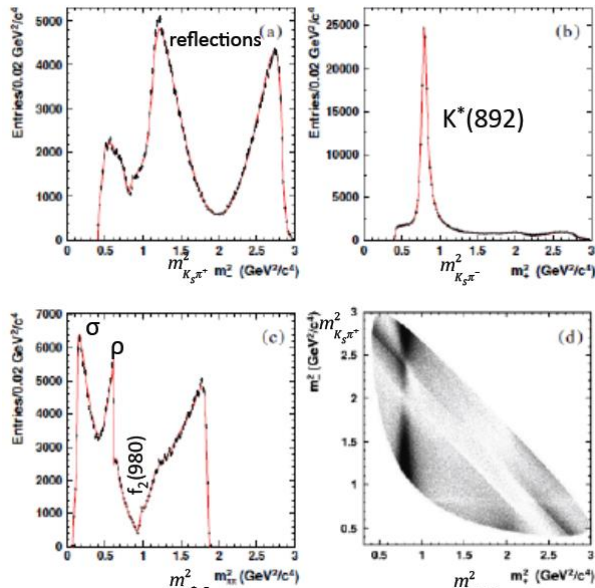
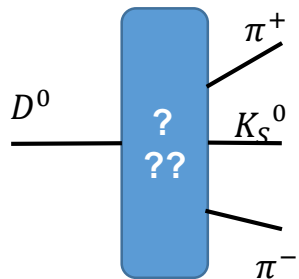


## Tasks:

- Map out the resonances
- Systematic determination of resonance properties:
  - spin-parity,
  - resonance parameters,
  - production properties,
  - decay properties, ...
- ◆ resonances tend to be broad and plentiful, leading to intricate interference patterns, or buried under a background in the same and in other waves.

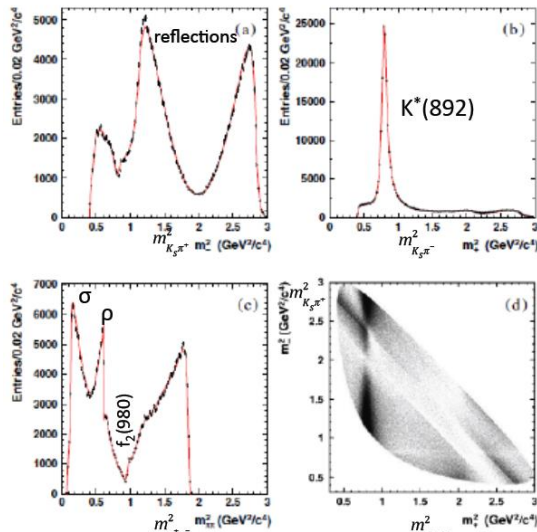
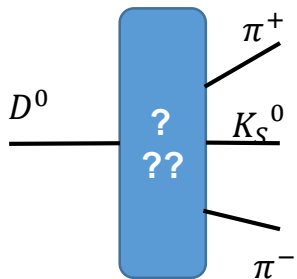


# Amplitude Analysis



- **Production Amplitude** produces a state  $X$  with  $J^{PC}$  quantum numbers
- **Decay Amplitude** describes the decay of  $X$  to final state particles
- **Observables** are the four-momenta of the final-state particles

# Amplitude Analysis



Several different states, all decaying to the same final particles are produced, and they interfere (complex amplitudes)

The probability to observe the event characterized by the measurement  $\xi$

$$P(\xi; \alpha) = \frac{\omega(\xi, \alpha) \epsilon(\xi)}{\int d\xi \omega(\xi, \alpha) \epsilon(\xi)}$$

Differential cross section

$$\omega(\xi, \alpha) = \frac{d\sigma}{d\Phi} = |\sum_i A_i|^2$$

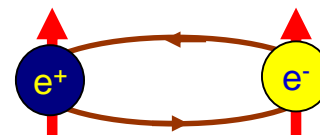
Standard likelihood

$$L = \prod_{i=1}^N P(\xi_i; \alpha)$$

Perform an un-binned log-likelihood fit (fit the data event-by-event to high-dimensional distributions using complex weights) to make our model for  $\omega$  agree with the experimental distribution for  $\omega$  by varying the  $\alpha$ .

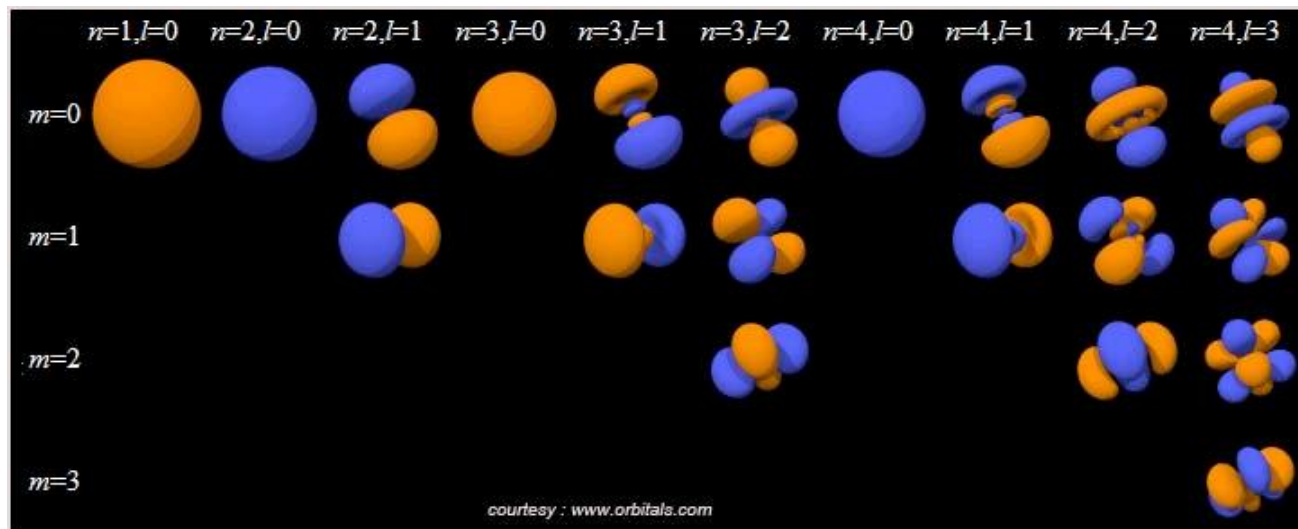
# Partial Wave Amplitude

## Positronium



$$\psi(\vec{r}, S) = R_{nl}(r) Y_{LM}(\theta, \phi) \chi(S)$$

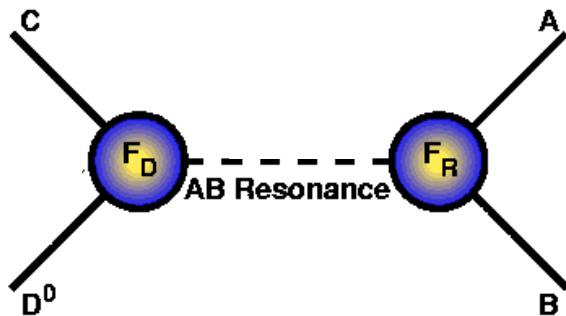
Angular distributions of reactions let you determine the spin and parity of intermediate resonances



Include spin, S, total angular momentum J:  $J = L + S$ :  $(2S+1)L_J$

# Isobar model formalism

$D^0$  three-body decay  $D^0 \rightarrow ABC$  decaying through an  $r=[AB]$  resonance



$D^0$  three-body amplitude

$$\mathcal{A}_D(s_{12}, s_{13}) = a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta_r} \mathcal{A}_r(s_{12}, s_{13})$$

NR term (direct 3 body decay)

$a_0, \delta_0, a_r, \delta_r$  : Free parameters of fit

$$\mathcal{A}_r(s_{12}, s_{13}) = F_D^J F_r^J \times M_r^J \times BW_r^J$$

$J \rightarrow L + l$	Angular distribution
$0 \rightarrow 0+0$	uniform
$0 \rightarrow 1+1$	$(1 + \zeta^2) \cos^2 \theta$
$0 \rightarrow 2+2$	$\left(\zeta^2 + \frac{3}{2}\right)^2 (\cos^2 \theta - 1/3)^2$

Dynamical function, e.g. Breit-Wigner (BW) propagator

Angular distribution: spin formalism

Blatt-Weisskopf centrifugal barrier factor for the D (resonance) decay vertex with radius R,

# Recap

- **Amplitude analysis is a key tool of hadron spectroscopy**: A state-of-the-art way to disentangle contributions from individual, and even small, resonances and to extract the resonance's spin-parity, mass, width and decay properties with high sensitivity and accuracy
- Event-based fits allow one to take into account the full correlation between final-state particles
- Amplitude analysis remains **challenging** in models and techniques

# Challenge: Computing

- PWA is time consuming
  - Large amount of fits (hypothesis tests, systematics)
  - Large amount of data (unbinned likelihood fit)

Event-wise ML fit to **all observables** simultaneously

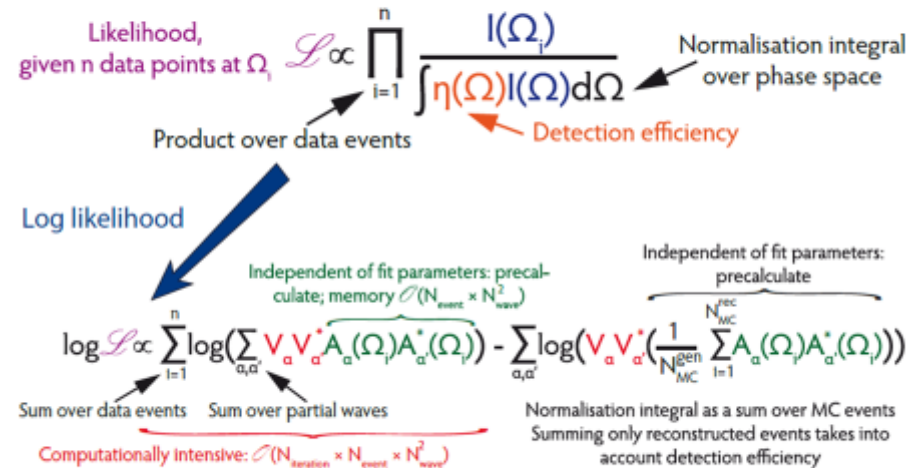
$$\omega(\xi) \equiv \frac{d\sigma}{d\Phi} = \left| \sum_i c_i R_i B(p, q) Z(L) \right|^2$$

dynamic    angular

Event-wise **efficiency** correction

$$P(\xi) = \frac{\omega(\xi)\epsilon(\xi)}{\int \omega(\xi)\epsilon(\xi)}$$

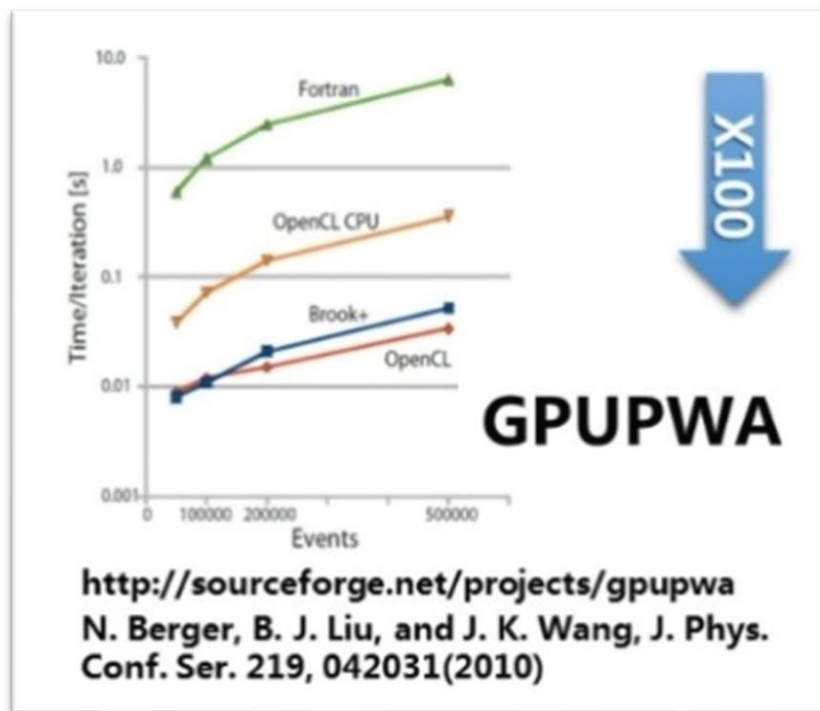
## Likelihood calculation



Data parallelism: do the calculation for every event simultaneously

# Challenge: Computing

- High performance computing with GPU, pioneered by BESIII



**ATHOS White paper: Analysis Tools for Next-Generation Hadron Spectroscopy Experiments [Acta. Phys.Polon. B46 (2015) 257]**

The most promising avenue for PWA is general purpose graphical processor unit (GPGPU) programming. Making use of the many cores on a GPU, likelihood calculations can be performed on many chunks of data at the same time. The pioneer approach of harnessing GPU parallel acceleration in PWA was performed in the framework of BESS-III [171]. Presently there are several hardware-specific programming models (CUDA, OpenCL) but the field is in a state of rapid

# Challenge: Background

- Background modeling

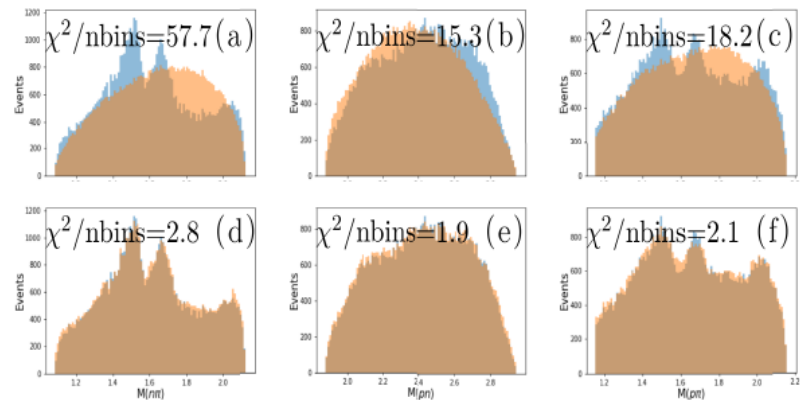
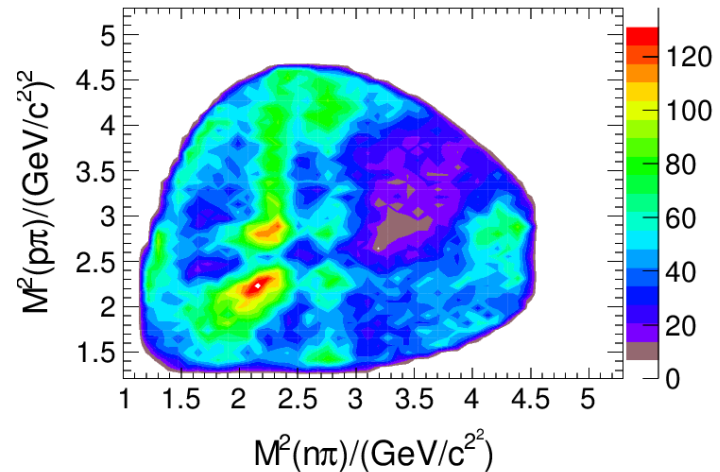
$$L_{S+B}$$

- Background subtraction

$$L_S(data) - L_S(background)$$

- Recent progress: ML based multi-dimensional reweighting

- to obtain data-like MC for dominate background





# Challenge: Resolution

$$\frac{d\sigma}{d\Phi} = \left| \sum F_i A_i \right|^2 = \sum F_i F_j^* A_i A_j^* = \sum \boxed{F_i F_j^*} T_i T_j^* \boxed{P_i P_j^*}$$

*Magnitude/Phase independent of mass*
*Propagator where mass resolution matters*

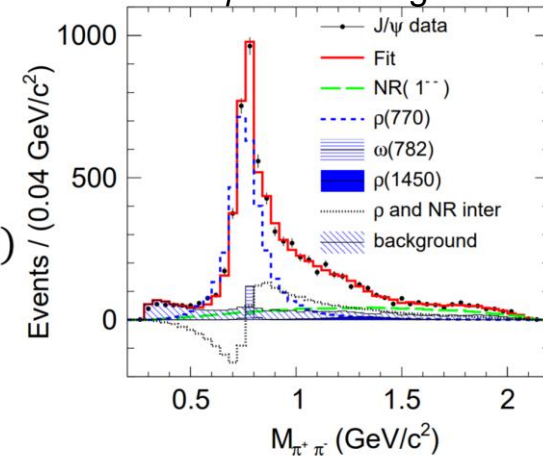
*Orbital Tensor weak dependency on mass*

**w/o mass resolution**  $P_i P_j^* = f_i(x) f_j^*(x) \cdot f_i(y) f_j^*(y) \cdot f_i(z) f_j^*(z) \cdot \dots$

**w/ mass resolution**  $P_i P_j^* = f_i(x) f_j^*(x) \otimes g(x) \cdot f_i(y) f_j^*(y) \otimes g(y) \cdot f_i(z) f_j^*(z) \otimes g(z) \cdot \dots$   
 $\equiv h_{ij}(x) \cdot h_{ij}(y) \cdot h_{ij}(z) \cdot \dots$

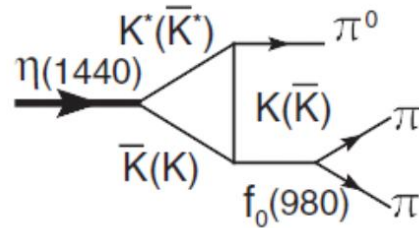
$$h_{ij}(x) = \int f_i(x - m) f_j^*(x - m) g(m) dm \simeq \sum_k w(m_k) f_i(x - m_k) f_j^*(x - m_k)$$

$J/\psi \rightarrow \pi^+ \pi^- \eta'$   
 Phys. Rev. D 96, 112012  
 $\rho - \omega$  mixing

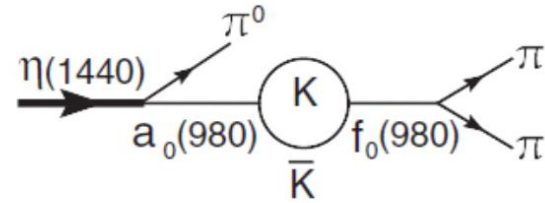


# Challenge: Dynamic models

## —Experiment-theory cooperation



(a)



(b)

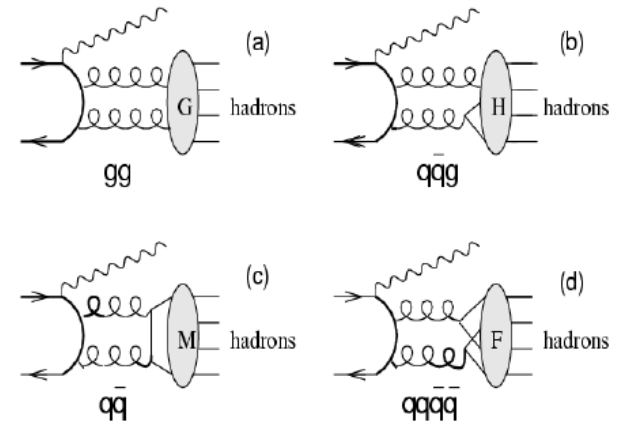
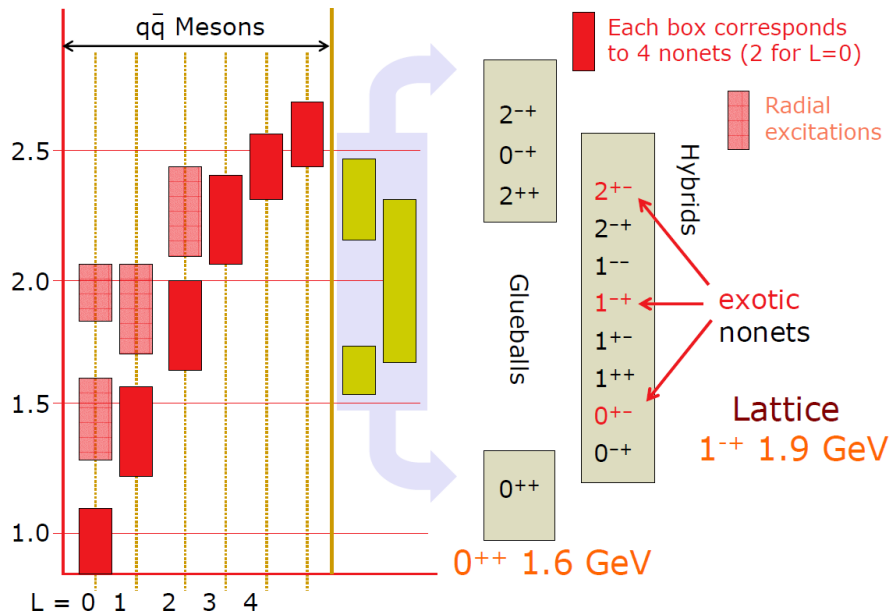
**Triangle singularity**

$$\text{Original: } A_{total} = \epsilon_{\psi\mu} \epsilon_{\gamma\nu} \Lambda U^{\mu\nu}$$

$$\text{Modified: } A_{total} = \epsilon_{\psi\mu} \epsilon_{\gamma\nu} (\Lambda U^{\mu\nu} + \Lambda' I U^{\mu\nu})$$

coefficient to be fitted

Loop integral of coupling



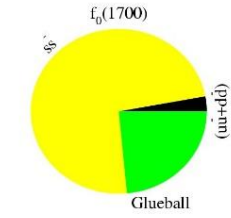
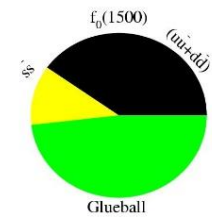
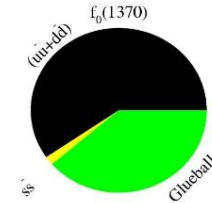
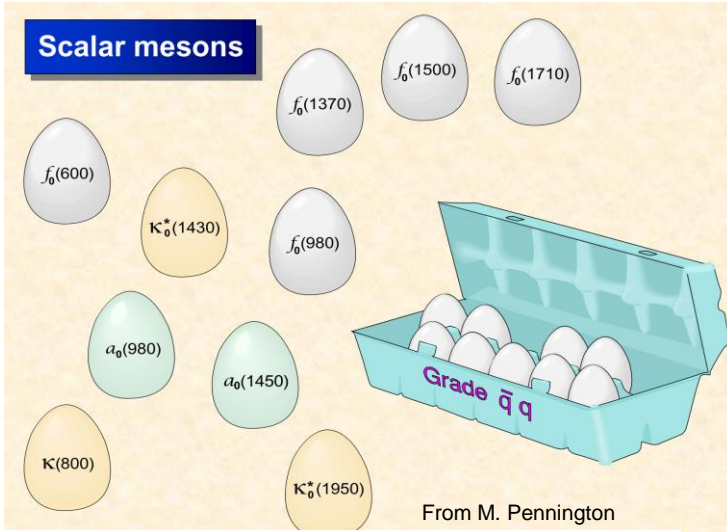
$$\Gamma(J/\psi \rightarrow \gamma G) \sim O(\alpha_s^2), \Gamma(J/\psi \rightarrow \gamma H) \sim O(\alpha_s^3),$$

$$\Gamma(J/\psi \rightarrow \gamma M) \sim O(\alpha_s^4), \Gamma(J/\psi \rightarrow \gamma F) \sim O(\alpha_s^4)$$

# Highlights: Light meson spectroscopy

- Search for glueballs and hybrids

# Overpopulated scalar mesons



Name	Mass [ MeV/c <sup>2</sup> ]	Width [ MeV/c <sup>2</sup> ]
$f_0(600) *$	400 – 1200	600 – 1000
$f_0(980) *$	$980 \pm 10$	40 – 100
$f_0(1370) *$	1200 – 1500	200 – 500
$f_0(1500) *$	$1507 \pm 5$	$109 \pm 7$
$f_0(1710) *$	$1718 \pm 6$	$137 \pm 8$
$f_0(1790)$		
$f_0(2020)$	$1992 \pm 16$	$442 \pm 60$
$f_0(2100)$	$2103 \pm 7$	$206 \pm 15$
$f_0(2200)$	$2189 \pm 13$	$238 \pm 50$

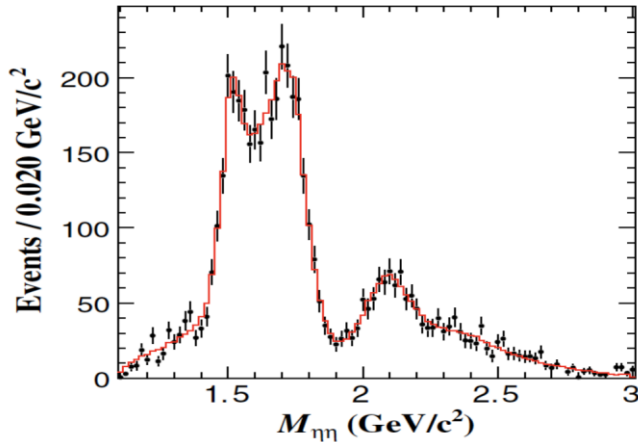
Mixing scheme:

**very controversial and model dependent**

$f_0(1500)$  ,  $f_0(1710)$ , which one has more gluonic component?

# Amplitude analysis of $J/\psi \rightarrow \gamma\eta\eta/K_S^0 K_S^0$

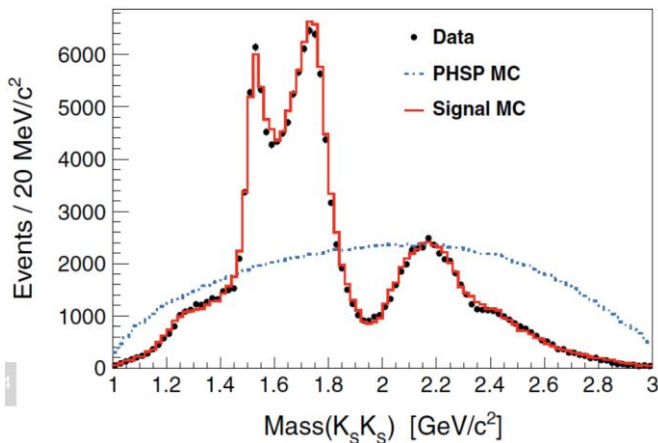
BESIII PRD 87, 092009 (2013)



Resonance	Mass (MeV/ $c^2$ )	Width (MeV/ $c^2$ )	$\mathcal{B}(J/\psi \rightarrow \gamma X \rightarrow \gamma\eta\eta)$	Significance
$f_0(1500)$	$1468^{+14+23}_{-15-74}$	$136^{+41+28}_{-26-100}$	$(1.65^{+0.26+0.51}_{-0.31-1.40}) \times 10^{-5}$	$8.2\sigma$
$f_0(1710)$	$1759 \pm 6^{+14}_{-25}$	$172 \pm 10^{+32}_{-16}$	$(2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$	$25.0\sigma$
$f_0(2100)$	$2081 \pm 13^{+24}_{-36}$	$273^{+27+70}_{-24-23}$	$(1.15^{+0.09+0.51}_{-0.10-0.28}) \times 10^{-4}$	$13.9\sigma$
$f_2'(1525)$	$1513 \pm 5^{+4}_{-10}$	$75^{+12+16}_{-10-8}$	$(3.42^{+0.43+1.37}_{-0.51-1.30}) \times 10^{-5}$	$11.0\sigma$
$f_2(1810)$	$1822^{+29+66}_{-24-57}$	$229^{+52+88}_{-42-155}$	$(5.40^{+0.60+3.42}_{-0.67-2.35}) \times 10^{-5}$	$6.4\sigma$
$f_2(2340)$	$2362^{+31+140}_{-30-63}$	$334^{+62+165}_{-54-100}$	$(5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$	$7.6\sigma$

Br of  $f_0(1710) \sim 10x$  larger than  $f_0(1500)$

BESIII PRD 98, 072003 (2018)



Resonance	$M$ (MeV/ $c^2$ )	$M_{PDG}$ (MeV/ $c^2$ )	$\Gamma$ (MeV/ $c^2$ )	$\Gamma_{PDG}$ (MeV/ $c^2$ )	Branching fraction	Significance
$K^*(892)$	896	$895.81 \pm 0.19$	48	$47.4 \pm 0.6$	$(6.28^{+0.16+0.59}_{-0.17-0.52}) \times 10^{-6}$	$35\sigma$
$K_1(1270)$	1272	$1272 \pm 7$	90	$90 \pm 20$	$(8.54^{+1.07+2.35}_{-1.20-2.13}) \times 10^{-7}$	$16\sigma$
$f_0(1370)$	$1350 \pm 9^{+12}_{-2}$	1200 to 1500	$231 \pm 21^{+28}_{-48}$	200 to 500	$(1.07^{+0.08+0.36}_{-0.07-0.34}) \times 10^{-5}$	$25\sigma$
$f_0(1500)$	1505	$1504 \pm 6$	109	$109 \pm 7$	$(1.59^{+0.16+0.18}_{-0.16-0.56}) \times 10^{-5}$	$23\sigma$
$f_0(1710)$	$1765 \pm 2^{+1}_{-1}$	$1723^{+6}_{-5}$	$146 \pm 3^{+7}_{-1}$	$139 \pm 8$	$(2.00^{+0.03+0.31}_{-0.02-0.10}) \times 10^{-4}$	$\gg 35\sigma$
$f_0(1790)$	$1870 \pm 7^{+2}_{-3}$	...	$146 \pm 14^{+7}_{-15}$	...	$(1.11^{+0.05+0.17}_{-0.06-0.32}) \times 10^{-5}$	$24\sigma$
$f_0(2200)$	$2184 \pm 5^{+4}_{-2}$	$2189 \pm 13$	$364 \pm 9^{+4}_{-7}$	$238 \pm 50$	$(2.72^{+0.08+0.17}_{-0.06-0.47}) \times 10^{-4}$	$\gg 35\sigma$
$f_0(2330)$	$2411 \pm 10 \pm 7$	...	$349 \pm 18^{+23}_{-1}$	...	$(4.95^{+0.21+0.66}_{-0.21-0.72}) \times 10^{-5}$	$35\sigma$
$f_2(1270)$	1275	$1275.5 \pm 0.8$	185	$186.7^{+2.2}_{-2.5}$	$(2.58^{+0.08+0.59}_{-0.09-0.20}) \times 10^{-5}$	$33\sigma$
$f_2'(1525)$	$1516 \pm 1$	$1525 \pm 5$	$75 \pm 1 \pm 1$	$73^{+6}_{-5}$	$(7.99^{+0.03+0.69}_{-0.04-0.50}) \times 10^{-5}$	$\gg 35\sigma$
$f_2(2340)$	$2233 \pm 34^{+9}_{-25}$	$2345^{+50}_{-40}$	$507 \pm 37^{+18}_{-21}$	$322^{+70}_{-60}$	$(5.54^{+0.34+3.82}_{-0.40-1.49}) \times 10^{-5}$	$26\sigma$
$0^{++}$ PHSP	...	...	...	...	$(1.85^{+0.05+0.68}_{-0.05-0.26}) \times 10^{-5}$	$26\sigma$
$2^{++}$ PHSP	...	...	...	...	$(5.73^{+0.99+4.18}_{-1.00-3.74}) \times 10^{-5}$	$13\sigma$

# Scalar glueball candidate

Flavor-blindness of glueball decays

$$\Gamma(J/\psi \rightarrow \gamma G_{0+}) = \frac{4}{27} \alpha \frac{|p|}{M_{J/\psi}^2} |E_1(0)|^2 = 0.35(8) \text{ keV}$$

$$\Gamma/\Gamma_{tot} = 0.33(7)/93.2 = 3.8(9) \times 10^{-3}$$

CLQCD, *Phys. Rev. Lett.* 110, 021601 (2013)



## Experimental results

- $B(J/\psi \rightarrow \gamma f_0(1710) \rightarrow \gamma K\bar{K}) = (8.5_{-0.9}^{+1.2}) \times 10^{-4}$
  - $B(J/\psi \rightarrow \gamma f_0(1710) \rightarrow \gamma \pi\pi) = (4.0 \pm 1.0) \times 10^{-4}$
  - $B(J/\psi \rightarrow \gamma f_0(1710) \rightarrow \gamma \omega\omega) = (3.1 \pm 1.0) \times 10^{-4}$
  - $B(J/\psi \rightarrow \gamma f_0(1710) \rightarrow \gamma \eta\eta) = (2.35_{-0.11}^{+0.13+1.24}_{-0.74}) \times 10^{-4}$
- ⇒  $B(J/\psi \rightarrow \gamma f_0(1710)) > 1.7 \times 10^{-3}$

$$\frac{1}{P.S.} \Gamma(G \rightarrow \pi\pi : K\bar{K} : \eta\eta : \eta\eta' : \eta'\eta') = 3 : 4 : 1 : 0 : 1$$

\*with chiral suppression

PRL 98 149103

$$\Gamma(G \rightarrow \pi\pi) / \Gamma(G \rightarrow K\bar{K}) \approx \frac{f_\pi^4}{f_K^4} \approx 0.48$$



$$\frac{1}{P.S.} \Gamma(G \rightarrow \pi\pi : K\bar{K} : \eta\eta) \approx \underline{1.3 : 3.16 : 1}$$

$f_0(1710)$  largely overlapped with scalar glueball

# Other information

## Two photon couplings

“Stickness”

PDG2018

Citation: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

$f_0(1710) \Gamma(i)\Gamma(\gamma\gamma)/\Gamma(\text{total})$					$\Gamma_1\Gamma_4/\Gamma$
$\Gamma(K\bar{K}) \times \Gamma(\gamma\gamma)/\Gamma_{\text{total}}$	CL%	DOCUMENT ID	TECN	COMMENT	
VALUE (eV)					
$12^{+3}_{-2} + 227_8$		UEHARA	13	BELL $\gamma\gamma \rightarrow K_S^0 K_S^0$	
••• We do not use the following data for averages, fits, limits, etc. •••					
<480	95	ALBRECHT	90G	ARG $\gamma\gamma \rightarrow K^+ K^-$	
<110	95	<sup>1</sup> BEHREND	89C	CELL $\gamma\gamma \rightarrow K_S^0 K_S^0$	
<280	95	<sup>1</sup> ALTHOFF	85B	TASS $\gamma\gamma \rightarrow K\bar{K}\pi$	

However, a scalar in  $\gamma\gamma \rightarrow \pi^0\pi^0$

Belle PRD 78 052004

TABLE VI: Fitted parameters of the  $f_0(Y)$

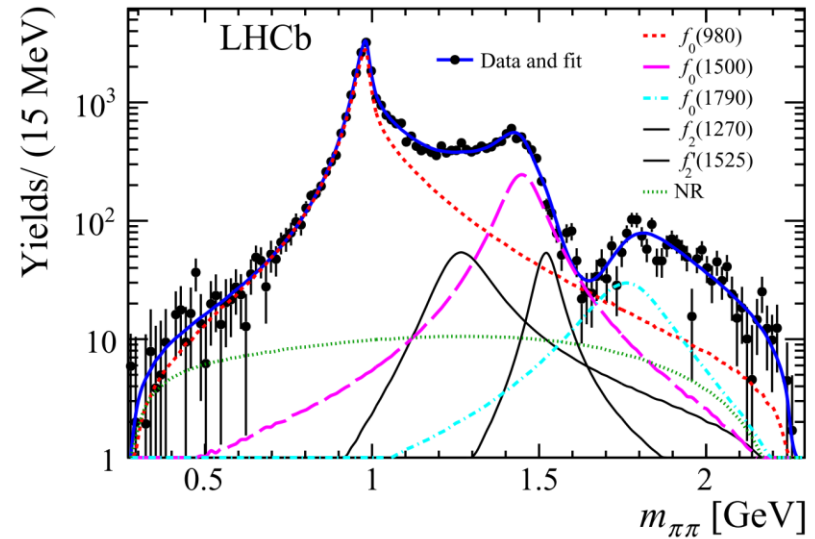
Parameter	Belle( $\pi^0\pi^0$ )	Crystal Ball	$f_0(1370)$ (PDG)	$f_0(1500)$ (PDG)	Unit
Mass	$1470^{+6}_{-7} + 72_{-255}$	1250	1200 - 1500	$1507 \pm 5$	MeV/ $c^2$
$\Gamma_{\text{tot}}$	$90^{+2}_{-1} + 50_{-22}$	$268 \pm 70$	150 - 200	$109 \pm 7$	MeV
$\Gamma_{\gamma\gamma}\mathcal{B}(\pi^0\pi^0)$	$11^{+4}_{-2} + 603_{-7}$	$430 \pm 80$	Unknown	Not seen	eV

$f_0(1370)? f_0(1500)?$

Assignment requires further study with more sophisticated model 23

$B_s \rightarrow J/\psi f_0$   
is selective for  $s\bar{s}$

PLB 797 (2019) 134789



observation of  $f_0(1500)$ ,  
non-observation of  $f_0(1710)$



# Tensor glueball candidate

$$\Gamma(J/\psi \rightarrow \gamma G_{2+}) = 1.01(22) \text{ keV}$$

$$\Gamma(J/\psi \rightarrow \gamma G_{2+})/\Gamma_{tot} = 1.1 \times 10^{-2}$$

*CLQCD, Phys. Rev. Lett. 111, 091601 (2013)*

## Experimental results

$$\text{Br}(J/\psi \rightarrow \gamma f_2(2340) \rightarrow \gamma \eta \eta) = (3.8^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$$

Phys.Rev. D87, 092009 (2013)

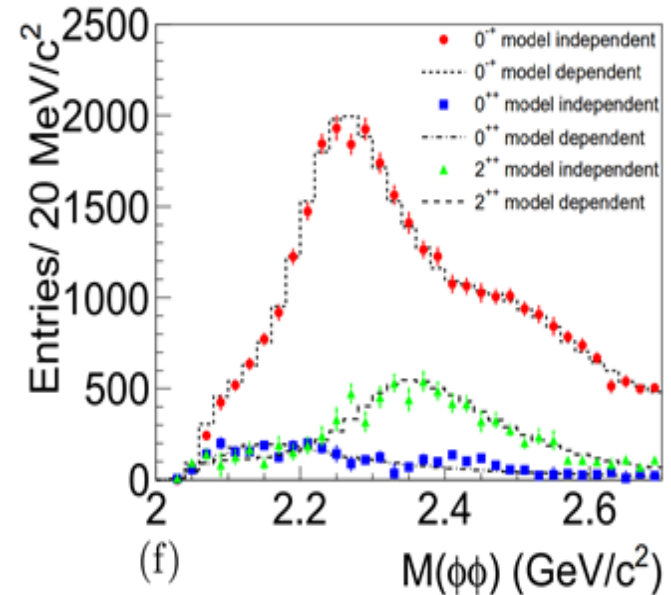
$$\text{Br}(J/\psi \rightarrow f_2(2340) \rightarrow \gamma \phi \phi) = (1.91 \pm 0.14^{+0.72}_{-0.73}) \times 10^{-4}$$

Phys.Rev. D93, 112011 (2016)

$$\text{Br}(J/\psi \rightarrow \gamma f_2(2340) \rightarrow \gamma K_S K_S) = (5.54^{+0.34+3.82}_{-0.40-1.49}) \times 10^{-5}$$

Phys.Rev. D98, 072003 (2018)

BESIII  $J/\psi \rightarrow \gamma \phi \phi$



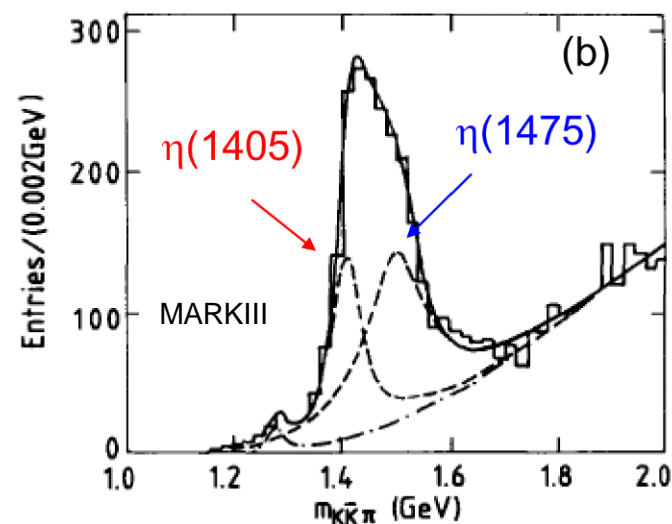
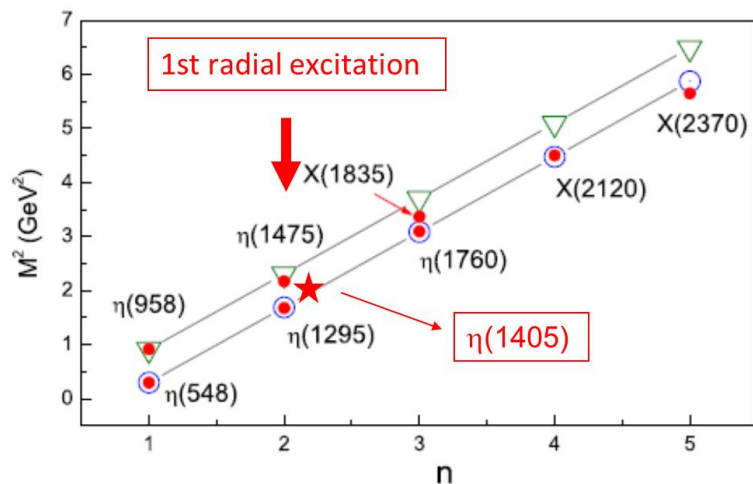
$f_2(2010)$ ,  $f_2(2300)$  and  $f_2(2340)$  stated in  $\pi p$  reactions are observed with a strong production of  $f_2(2340)$  Consist with central exclusion production in WA102

It is desirable to search for more decay modes



# Pseudoscalar glueball

The small number of expected pseudoscalars in the quark model provide a clean and promising environment for the search of glueballs



Where is the  $0^{-+}$  glueball

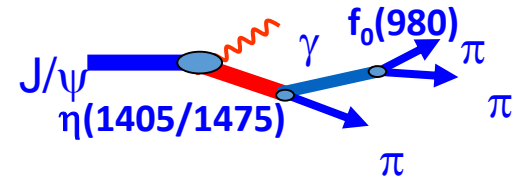
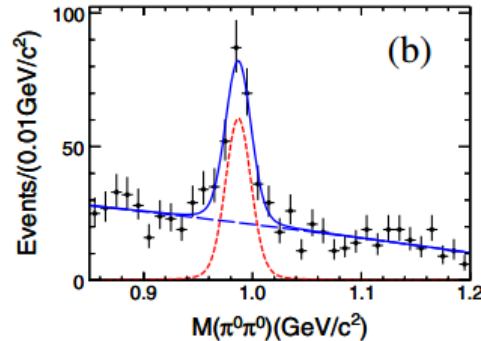
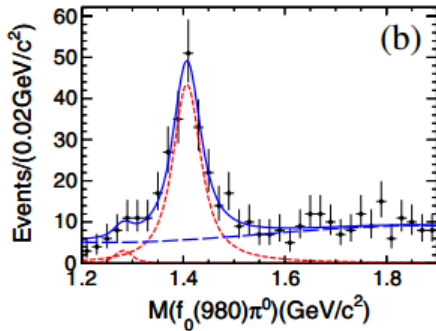
- LQCD:  $0^{-+}(2.3 \sim 2.6 \text{ GeV})$
- Does  $\eta(1295)$  exist?
- What' s the nature of the outnumbered  $\eta(1405)$  ?

Long standing E- $\iota$  puzzle

$$M = 1416 \pm 8_{-5}^{+7}; \Gamma = 91_{-31-38}^{+67} {}^{+15} \text{ MeV}/c^2$$

$$M = 1490_{-8-6}^{+14+3}; \Gamma = 54_{-21-24}^{+37+13} \text{ MeV}/c^2$$

# Isospin-violating decay of $\eta(1405) \rightarrow f_0(980)\pi^0$



BESIII PRL 108 182001

**$f_0(980)$  is extremely narrow:  $\Gamma \cong 10 \text{ MeV}$ .**

**PDG:  $\Gamma(f_0(980)) \cong 40 \sim 100 \text{ MeV}$ .**

**Anomalously large isospin violation:**

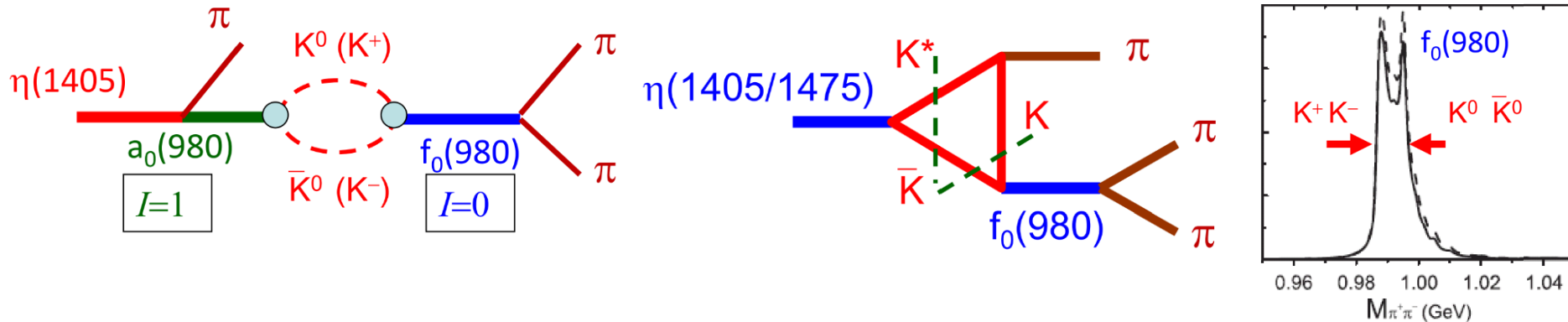
$$\frac{Br(\eta(1405) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{Br(\eta(1405) \rightarrow a_0^0(980)\pi^0 \rightarrow \eta\pi^0\pi^0)} \cong (17.9 \pm 4.2)\%$$

$$\xi_{af} = \frac{Br(\chi_{c1} \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{Br(\chi_{c1} \rightarrow a_0(980)\pi^0 \rightarrow \eta\pi^0\pi^0)} < 1\% (90\% \text{ C.L.}) \quad \text{PRD, 83(2100)032003}$$

# Isospin-violating decay of $\eta(1405) \rightarrow f_0(980)\pi^0$

PDG2012

However, the issue remains controversial as to whether two pseudoscalar mesons really exist. According to Ref. [18] the splitting of a single state could be due to nodes in the decay amplitudes which differ in  $\eta\pi\pi$  and  $K^*(892)\bar{K}$ . Based on the isospin violating decay  $J/\psi(1S) \rightarrow \gamma 3\pi$  observed by BES [19] the splitting could also be due to a triangular singularity mixing  $\eta\pi\pi$  and  $K^*(892)\bar{K}$  [20].

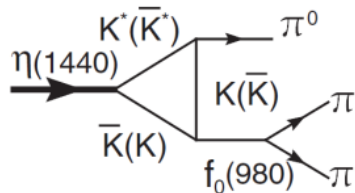


- No need for two pseudoscalars around 1.4 GeV
- Look for pseudoscalar glueball in higher mass region

# Isospin-violating decay of $\eta(1405) \rightarrow f_0(980)\pi^0$

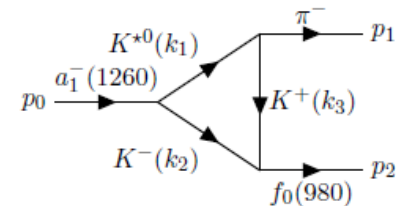
- Inspired by BESIII's observation, the triangle singularity mechanism plays an important role in the study of threshold phenomena

**BESIII**



**COMPASS@CERN**

$a_1(1420)$



## Hadronic molecules

Rev.Mod.Phys. 90 (2018), 015004

*et al.*, 2016). However, two recent experimental observations expose novel features in their decay mechanisms which illustrate the relevance of their couplings to the two-meson continua. The BESIII Collaboration observed an anomalously large isospin symmetry breaking in  $\eta(1405)/\eta(1475) \rightarrow 3\pi$  (Ablikim *et al.*, 2012), which could be accounted for by the so-called triangle singularity (TS) mechanism as studied in Ref. (Aceti *et al.*, 2012; Wu *et al.*, 2012). This special threshold phenomenon arises in triangle (three-point loop) diagrams

## Manifestations of TS in various processes

Phys.Rev.Lett. 108 (2012) 081803

Phys.Rev. D86 (2012) 114007

Phys.Rev. D88 (2013) 014045

Phys.Rev. D87 (2013) 014023

Phys.Rev. D89 (2014), 054038

Phys.Rev. D92 (2015) 034010

Phys.Rev. D91 (2015) 094022

Phys.Rev. D92 (2015) 036003

Phys.Lett. B753 (2016) 297

Phys.Rev. D93 (2016) 114027

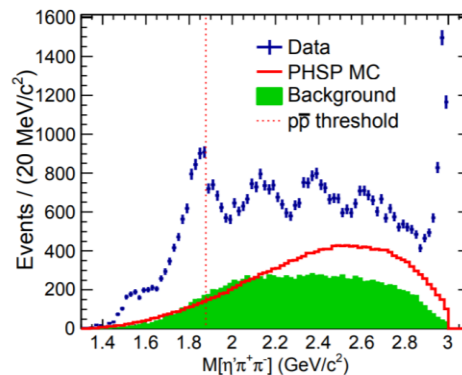
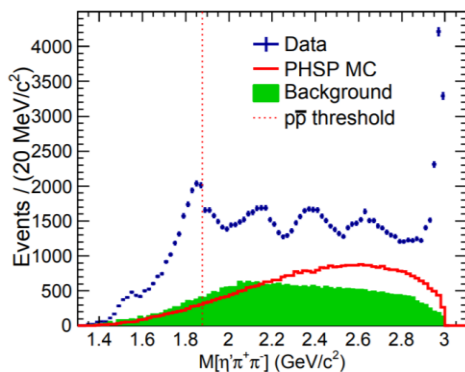
Phys.Rev. D95 (2017) 034015

Phys.Rev. D97 (2018) 096002

# Structures >2 GeV

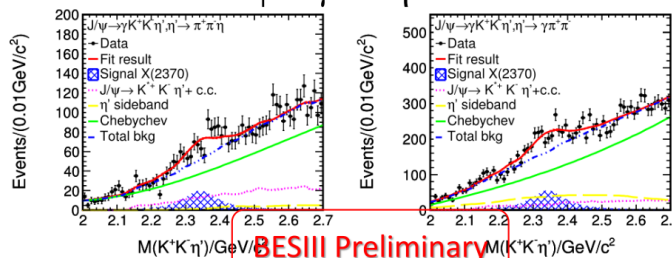
$J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$

PRL 106, 072002, PRL 117, 042002

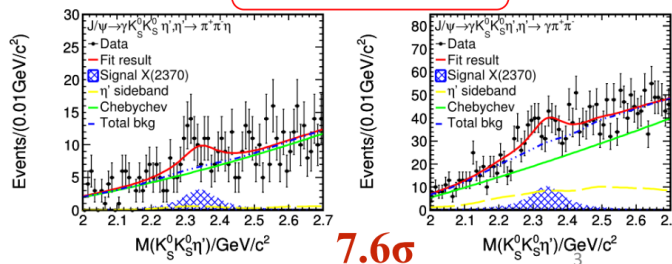


X(2370)

$J/\psi \rightarrow \gamma K K \eta'$



BESIII Preliminary



7.6σ

# Landscape of light glueball has changed

## Scalar: Overpopulation

- LQCD : ground state  $0^+$  glueball  $\sim 1.7$  GeV, first excitation  $\sim 2.1$  GeV



- ✓ **Strong production of  $f_0(1710)/f_0(2100)$  in  $J/\psi \rightarrow \gamma \eta\eta/KK/\pi\pi$** , the pattern consists with LQCD' s prediction

## Tensor: large uncertainty

- LQCD:  $2^{++}(2.3\sim 2.4$  GeV)



- ✓ **Strong production of  $f_2(2340)$  in  $J/\psi \rightarrow \gamma\eta\eta/KK/\pi\pi/\phi\phi$** ; consists with LQCD' s prediction

**Pseudoscalar:** very little known above 2 GeV, puzzles in low mass region

- LQCD:  $0^{-+}(2.3\sim 2.6$  GeV)

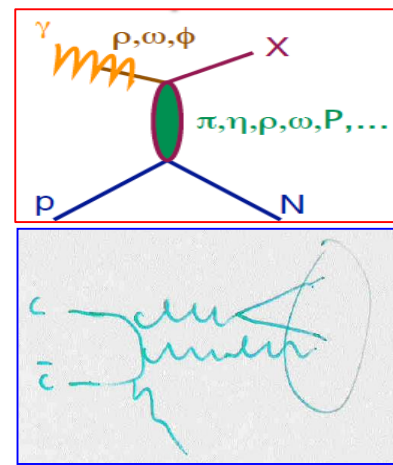


- ✓ **Trajectory:**
  - $f_1(1285)$ , no  $\eta(1295)$
  - $\eta(1405) / \eta(1475)$  can be one resonance

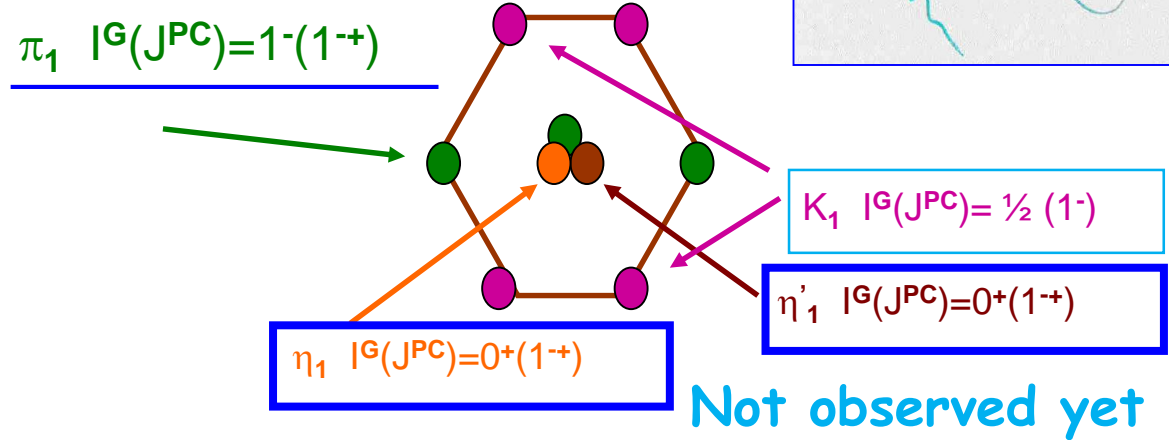
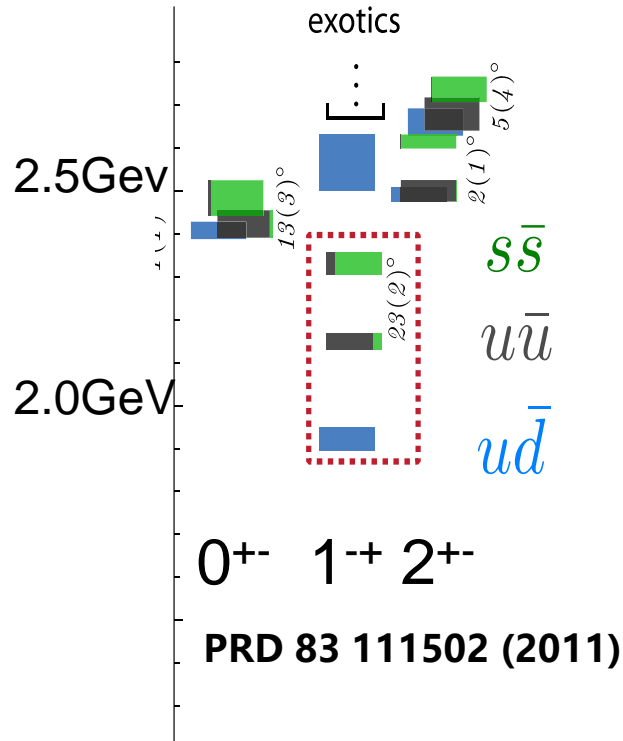
□ **Above 2 GeV:  $X(2370)$ ?**

# Hybrids

GlueX@JLab  
BESIII

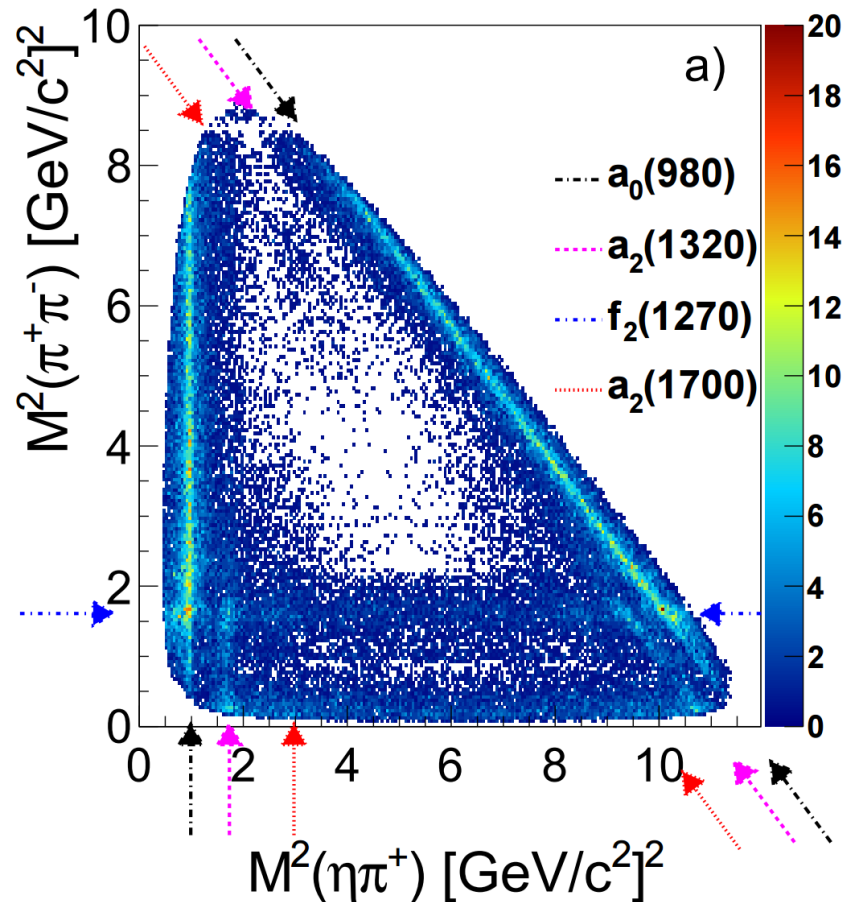


## Lattice QCD Predictions:

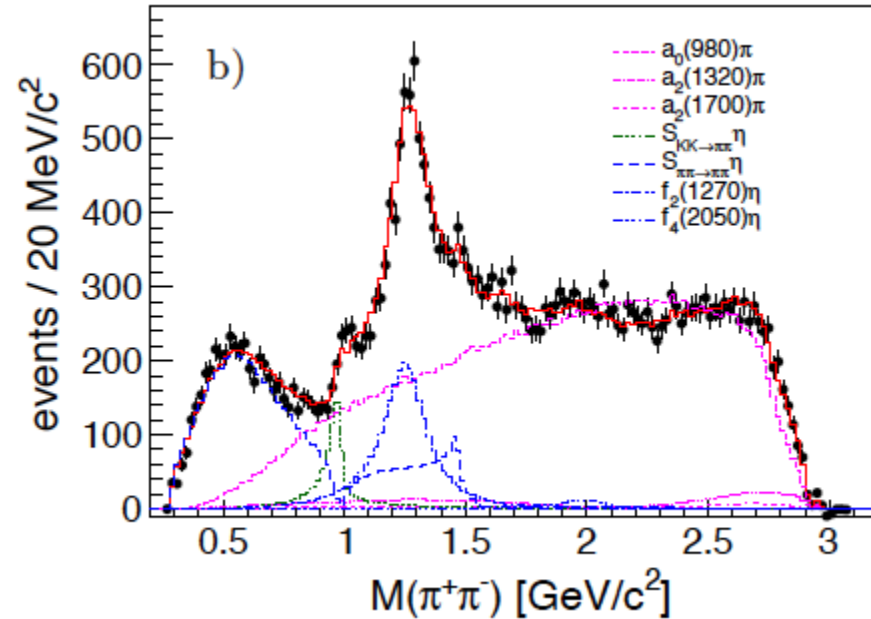
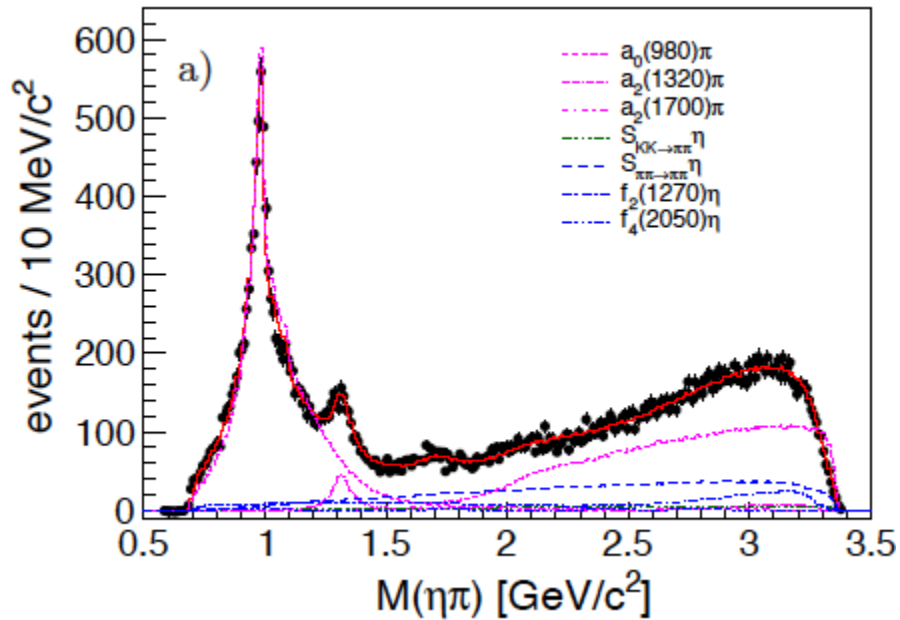


- Lightest hybrid is with exotic  $J^{PC} = 1^{-+}$
- Isoscalar  $1^{-+}$  is critical to establish the nonet
- $J/\psi \rightarrow \gamma + X$ , synergies between other experiments with different production mechanism
- $\chi_{c1}$  decays can be another source

- $\chi_{c1}$  provides another suitable environment to look for  $1^-$ 
  - $\pi_1(1600)$  studied in  $\chi_{c1}$  decays by CLEO-c
  - only  $\pi_1(1400)$  has been reported decays to  $\eta\pi$
- Properties of  $a_0$  and  $a_2$  still need further studies







Most of resonances are BW's

Dispersion integrals for

$\pi\pi$  S-wave: N/D approach PRD84 112009

$a_0(980)$ : PRD78,74023

- Clear evidence for  $a_2(1700)$  in  $\chi_{c1}$  decays
- First measurement of  $g'_{\eta'\pi} \neq 0$  using  $a_0(980) \rightarrow \eta\pi$  line shape
- Measured upper limits for  $\pi_1(1^{-+})$  in 1.4 - 2.0  $\text{GeV}/c^2$  region

# Summary and outlook

- Light hadron spectroscopy: Map out light hadrons as complete/precise as possible
  - Provide critical information on the quantitative understanding of confinement
- Amplitude analysis is a key tool. Experiment-theory cooperation is important
- BESIII collected 10 billions of  $J/\psi$  and will continue to run for more years. Data with unprecedented statistical accuracy provides great opportunities to map out light meson spectroscopy and study QCD exotics

Thank you