Update on H3L Errors

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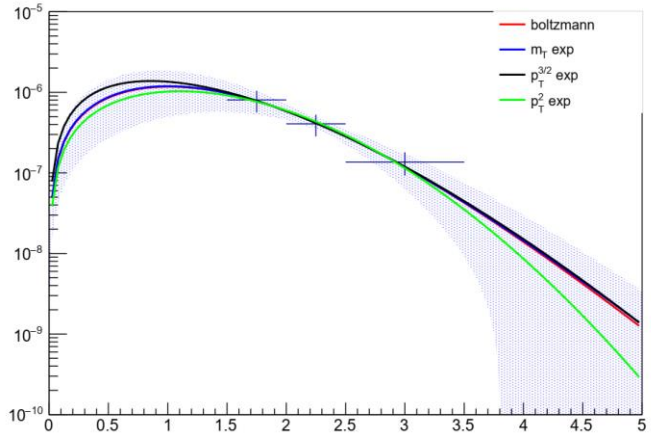
How I combine errors for various components

- dN/dy and its errors
  - In region where we have measurement
    - Use bincontents and binerror (stat. /sys.)
    - To combine stat./sys. err in different bins from measured region, do sqrt(a*a+b*b);
  - Extrapolation to region where we have no measurement
    - Stat. Err calculated with the 68% confidence band returned by TFitResult
    - Sys. Err calculated with scaling method
    - Function fit on dN/dptdy rather than dN/dptdy * 1/pt, with option "I"
    - To combine stat. Err in measured and extrapolated region, do sqrt(a*a+b*b)
    - To combine sys. Err in measured and extrapolated region, do a+b



Both calculated from the 68% confidence level band returned by TFitResult.

Centrality	dN/dy $\pm$ stat. $^{+\text{sys. up}}_{-\text{sys. down}}$	Extrapolation fraction
0-10	4.03e-5 $\pm$ 1.02e−5 $^{+\,4.40e-6}_{-\,4.24e-6}$	0.56
10-20	2.12e-5 $\pm$ 8.30e−6 $^{+\,2.23e-6}_{-\,2.38e-6}$	0.56
20-40	1.21e-5 $\pm$ 3.78e−6 $^{+\,1.43e-7}_{-\,1.48e-6}$	0.62
40-80	2.08e-6 $\pm$ 8.86e−7 $^{+\,2.62e-7}_{-\,2.77e-7}$	0.64

Old (fit dN/dptdy *1/pt with option "I")
Whenever I need to combine errors, do a+b only.

Centrality	dN/dy $\pm$ stat. $^{+\text{sys. up}}_{-\text{sys. down}}$	Extrapolation fraction
0-10	4.03e-5 $\pm$ 7.76e−6 $^{+\,3.96e-6}_{-\,3.12e-6}$	0.56
10-20	2.12e-5 $\pm$ 7.02e−6 $^{+\,2.18e-6}_{-\,1.79e-6}$	0.56
20-40	1.21e-5 $\pm$ 2.70e−6 $^{+\,1.58e-7}_{-\,1.16e-6}$	0.62
40-80	2.08e-6 $\pm$ 8.09e−7 $^{+\,3.09e-7}_{-\,2.23e-7}$	0.64

New (fit dN/dptdy with option "I")
Do a+b only for scaling method. If correlation between a and b is not so clear, I will do sqrt(a*a+b*b)

Discussion on extraction of stat. err for extrapolation

- 3 methods to assign stat. error on the extrapolation component of dn/dy
  - Depend on stat. error from data and the fit performance
    - ~~Confidence band method~~ → Propagation method
    - Bootstrap method
  - ONLY depends on stat. error from data
    - Scaling method

- How I get the confidence band
  - By calling FitResult::GetConfidenceIntevals(unsigned int n, unsigned int stride1, unsigned int stride2, const double * x, double * ci, double cl, bool norm)
  - double* x should be organized as a bunch of vectors, with stride1 and stride2 defines the index to access each component of each vector typically, stride1 = ndim, stride2 = 1

```
fitter→GetConfidenceIntervals(gBand[ifunc][icut][ichg][icent]→GetN(), 1, 1, gBand[ifunc][icut][ichg][icent]→GetX(), gBand[if
unc][icut][ichg][icent]→GetEY(), 0.682689492137, false);
```

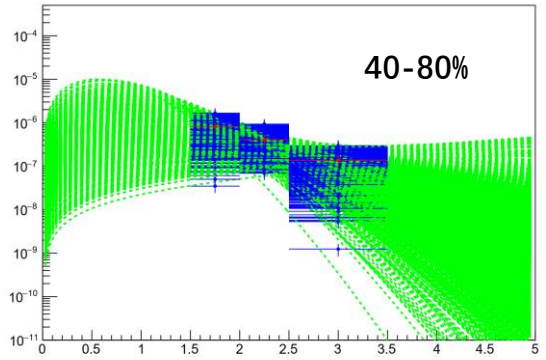
  - https://root.cern/doc/v628/FitResult_8cxx_source.html#l00495 (Line 495 -- 570)
  - Propagate errors for parameters to the function value F(x,par)
    - By calculating gradient and the covariance matrix w.r.t. the parameters
  - Return 1-sigma band for F(x,par)

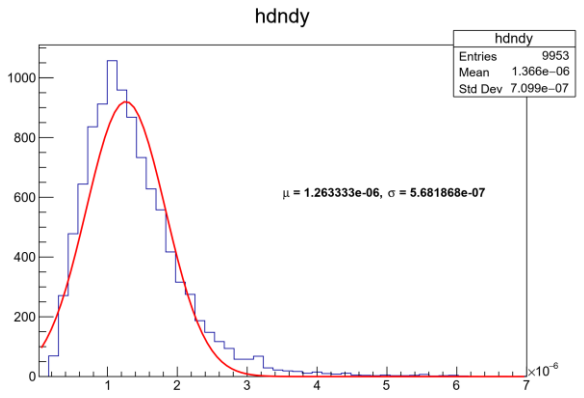
- Confidence band is not good for dN/dy calculation
  - We need 1-sigma band for [s(par) = integral of F(x,par)]
  - We can calculate gradient for [s(par) = integral of F(x,par)] and do the propagation by ourselves
    - Use [s(par+dpar)-s(par-dpar)]/(2 dpar) (central difference) to calculate gradient near the best fit on par
    - Use sqrt($\mathbf{g}^T\mathbf{C}\mathbf{g}$) to do the propagation
  - Mathematically it is very clear

Discussion on extraction of stat. err for extrapolation

- How I do bootstrap
  - For i-th point (xi,yi) on the original spectrum, the stat. err is eyi
  - We get many other spectra by resampling the i-th point with Gaussian distribution (mean = yi, sigma = eyi)
  - Fit each resampling spectrum to get a dNdy in the extrapolated region, they form a distribution
  - Fit the resampling dN/dy distribution with a Gaussian to extract stat. err
  - Potential issue: the fit sometimes not good, how to extract stat. err? Do we use sigma from gaussian fit or use standard deviation of the resampling distribution?

- How I do scaling
  - Fit the spectrum
  - Calculate dNdy_mea and dNdy_ex, dNdy = dNdy_mea + dNdy_ex
  - We assume f = err_dNdy_ex / dNdy_ex = err_dNdy_mea / dNdy_mea
  - Then err_dN/dy = f * dNdy
  - Potential issue: I use an assumption; f might have fluctuation?
  - If we use scaling on systematic error estimation, we face similar problem?





Discussion on extraction of stat. err for extrapolation

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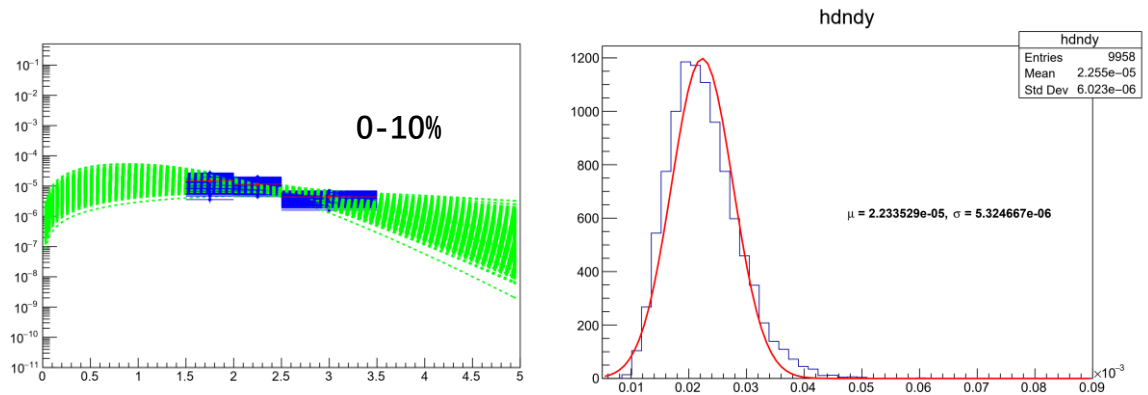

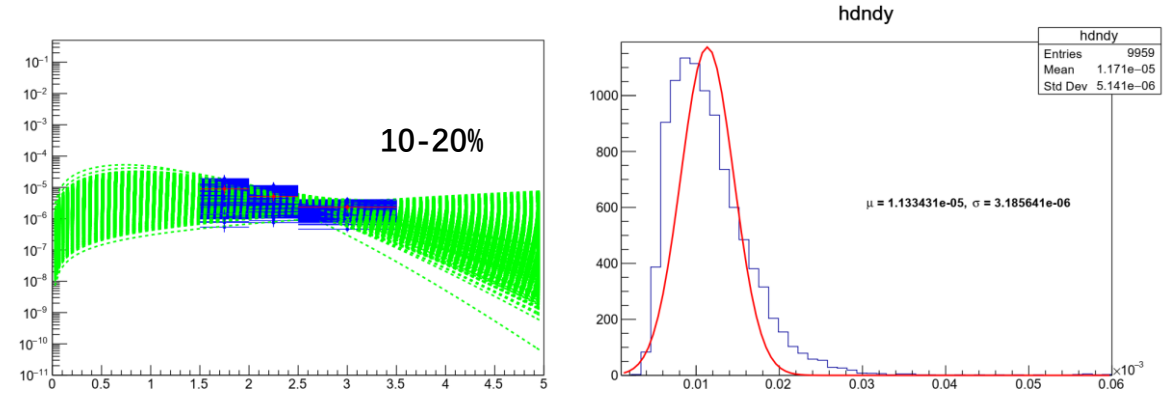

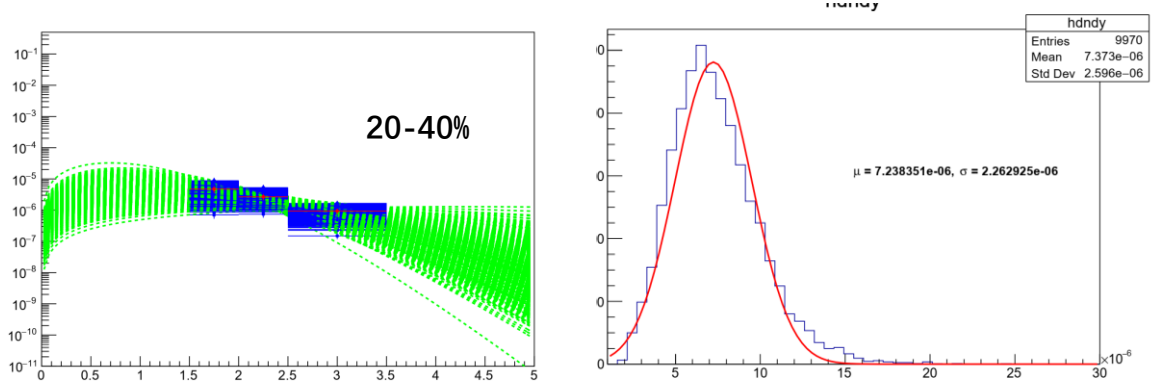

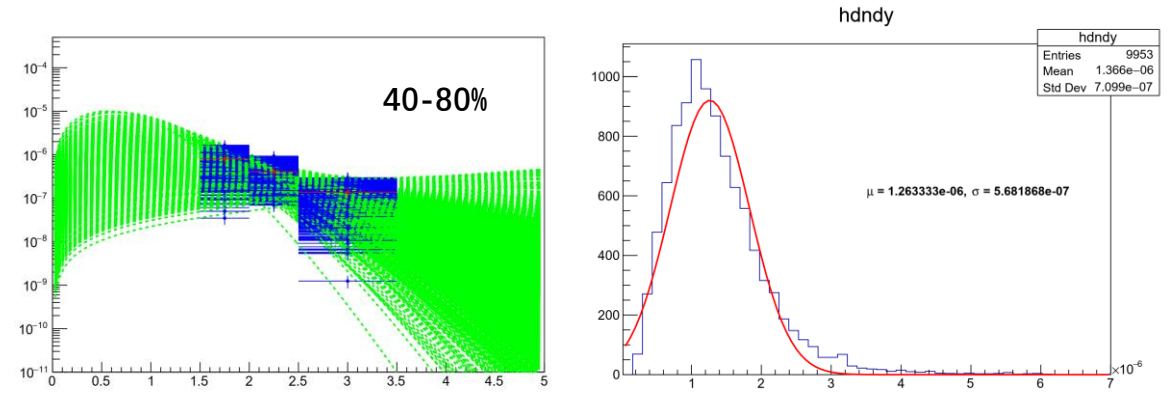

Discussion on extraction of stat. err for extrapolation

- Compare 3 different methods on calculating stat. error for extrapolated dNdy
  - For bootstrap, we report gaus mean (mean of resampling distribution), gaus sigma (standard deviation of resampling distribution)
  - Confidence band method will estimate error larger than propagation
  - Scaling method will give much smaller error compared with other methods
  - Bootstrap will give dNdy and its error similar to propagation method, if we use mean and sigma of the resampling distribution rather than fitting a gaussian to get them.

0-10%	Extrapolated dNdy	Its stat. error
Confidence band	2.267e-5	7.465e-6
Propagation	2.267e-5	6.393e-6
Bootstrap	2.234e-5 (2.255e-5)	5.325e-6 (6.023e-6)
Scaling	2.267e-5	2.709e-6

10-20%	Extrapolated dNdy	Its stat. error
Confidence band	1.182e-5	6.821e-6
Propagation	1.182e-5	5.927e-6
Bootstrap	1.334e-5 (1.171e-5)	3.186e-6 (5.141e-6)
Scaling	1.182e-5	2.107e-6

20-40%	Extrapolated dNdy	Its stat. error
Confidence band	7.466e-6	2.615e-6
Propagation	7.466e-6	2.452e-6
Bootstrap	7.238e-6 (7.373e-6)	2.263e-6 (2.596e-6)
Scaling	7.466e-6	1.094e-6

40-80%	Extrapolated dNdy	Its stat. error
Confidence band	1.332e-6	7.967e-7
Propagation	1.332e-6	7.577e-7
Bootstrap	1.263e-6 (1.366e-6)	5.682e-7 (7.099e-7)
Scaling	1.332e-6	2.541e-7