In Experiment and Uncertainty Estimation

- In 2-particle-correlation case,

$$c_2 = \frac{Q_{n,1}^{D_1} Q_{-n,1}^{D_2}}{Q_{0,1}^{D_1} Q_{-0,1}^{D_2}} = \frac{C_2}{W}$$

- What's measured in experiment is

$$\langle c_2 \rangle = \langle \frac{Q_{n,1}^{D_1} Q_{-n,1}^{D_2}}{Q_{0,1}^{D_1} Q_{-0,1}^{D_2}} \rangle = \frac{\Sigma_i (c_2)_i W_i}{\Sigma_i W_i}$$

- The value of the observable

$$\langle c_2 \rangle = \left\langle \frac{Q_{n,1}^{D_1} Q_{-n,1}^{D_2}}{Q_{0,1}^{D_1} Q_{-0,1}^{D_2}} \right\rangle = \frac{\Sigma_i (c_2)_i W_i}{\Sigma_i W_i}$$

$$= \frac{\Sigma_i Q_{n,1}^{D_1^i} Q_{-n,1}^{D_2^i}}{\Sigma_i Q_{0,1}^{D_1^i} Q_{0,1}^{D_2^I}}$$

✓ track-by-track results

- The uncertainty of the observable

$$\sigma(c_2) = \frac{\Sigma_i (c_2)_i^2 W_i}{\Sigma_i W_i} - \left(\frac{\Sigma_i (c_2)_i W_i}{\Sigma_i W_i} \right)^2$$

✗ Not track-by-track results

- The uncertainty above ignores the uncertainty from the track-by-track fluctuation in c_2
- Need to be corrected

Weighted Average and Deviation

- Assuming a distribution $f(x)$, Let the average of $f(x)$ be μ and the standard deviation be σ

- An average estimation

$$\bar{x} = \frac{\Sigma_i w_i x_i}{\Sigma_i w_i} \; (w_i \; is \; weight)$$
$$= \frac{\Sigma_i w_i \mu}{\Sigma_i w_i}$$
$$= \mu \text{(unbiased estimation)}$$

- An standard deviation estimation

$$\overline{x^2} - \bar{x}^2 = \frac{\Sigma_i w_i x_i^2}{\Sigma_i w_i} - \left(\frac{\Sigma_i w_i x_i}{\Sigma_i w_i}\right)^2$$
$$= \frac{\Sigma_i w_i (\sigma^2 + \mu^2)}{\Sigma_i w_i} - \frac{1}{(\Sigma_i w_i)^2} (\Sigma_i w_i^2 x_i^2 + \Sigma_{i \neq j} w_i w_j x_i x_j)$$
$$= (1 - \frac{\Sigma_i w_i^2}{(\Sigma_i w_i)^2}) \sigma^2 + \mu^2 - \frac{1}{(\Sigma_i w_i)^2} (\Sigma_i w_i^2 \mu^2 + \Sigma_{i \neq j} w_i w_j \mu^2)$$
$$= (1 - \frac{\Sigma_i w_i^2}{(\Sigma_i w_i)^2}) \sigma^2$$

(biased estimation)

c_2 uncertainty correction

$$\sigma^2_{measured}(c_2) = \frac{\Sigma_i (c_2)_i^2 W_i}{\Sigma_i W_i} - \left(\frac{\Sigma_i (c_2)_i W_i}{\Sigma_i W_i}\right)^2$$

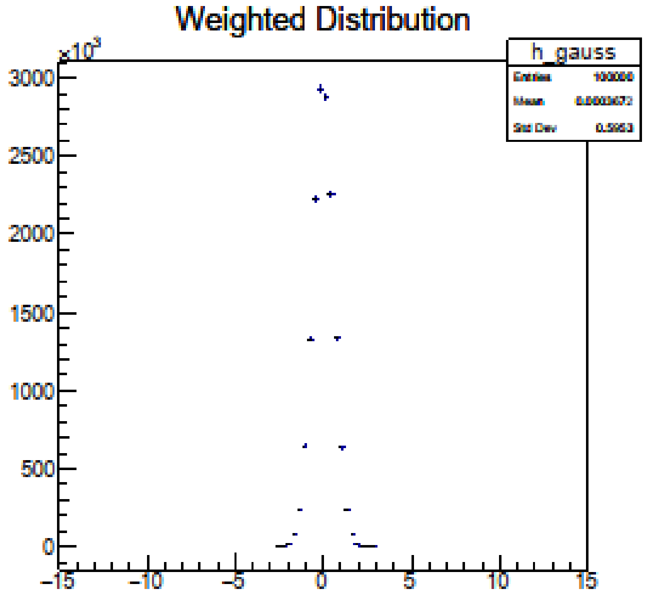
$$\sigma^2_{track}(c_2) = \frac{\Sigma_i ((c_2)_i^2 + \sigma_i^2(c_2)) W_i}{\Sigma_i W_i} - \left(\frac{\Sigma_i (c_2)_i W_i}{\Sigma_i W_i}\right)^2$$

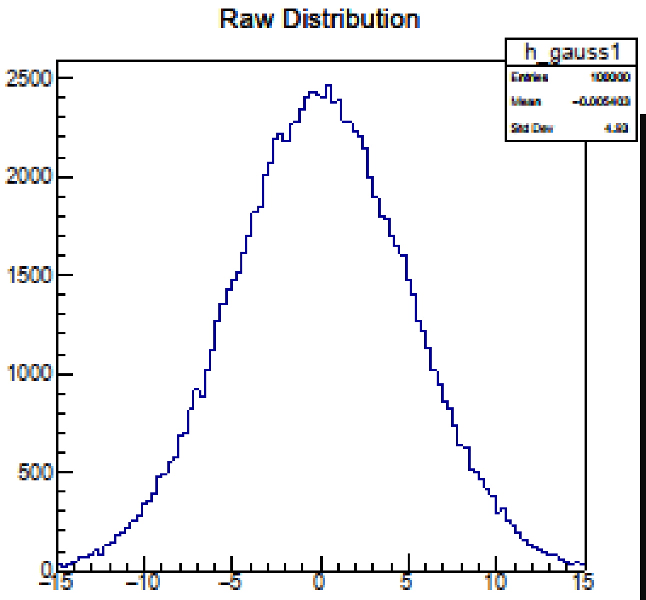
$$= \frac{\Sigma_i \sigma_i^2(c_2) W_i}{\Sigma_i W_i} + \sigma^2_{measured}(c_2) = \sigma^2_{true}$$

$$\frac{\Sigma_i \sigma_i^2(c_2) W_i}{\Sigma_i W_i} = \frac{\Sigma_j ((1 - \frac{\Sigma_i w_{i,j}^2}{\left(\Sigma_i w_{i,j}\right)^2})\sigma^2) W_j}{\Sigma_i W_i} = (1 - \frac{\Sigma_j \frac{\Sigma_i w_{i,j}^2}{W_j}}{\Sigma_i W_i})\sigma^2$$

$$\sigma^2_{measured}(c_2) = \frac{\Sigma_j \frac{\Sigma_i w_{i,j}^2}{W_j}}{\Sigma_i W_i}\sigma^2 = (\Sigma_i \frac{Q_{0,2}^{D_1^i} Q_{0,2}^{D_2^i}}{Q_{0,1}^{D_1^i} Q_{0,1}^{D_2^i}}) \frac{\sigma^2}{\Sigma_i Q_{0,1}^{D_1^i} Q_{0,1}^{D_2^i}}$$

Toy MC Verification






```
double W_total = 0;
double w2OverW_total = 0;
int n = 0;

for (int i = 0; i < 100000; i++) {
  int n_generated = gRandom->Uniform(50, 100);
  double W = 0;
  double w2 = 0;
  double mean = 0;
  for (int j = 0; j < n_generated; j++) {
    double weight = gRandom->Uniform(1, 3);
    mean += f_gauss->GetRandom()*weight;
    W += weight;
    w2 += weight * weight;
  }
  h_gauss->Fill(mean/W, W);
      W_total += W;
      w2OverW_total += w2 / W;
  n++;
}
```

$$\sigma^2_{measured}(c_2) = \frac{\Sigma_j \frac{\Sigma_i w^2_{i,j}}{W_j}}{\Sigma_i W_i}\sigma^2 = \left(\Sigma_i \frac{Q_{0,2}^{D_1^i} Q_{0,2}^{D_2^i}}{Q_{0,1}^{D_1^i} Q_{0,1}^{D_2^i}}\right) \frac{\sigma^2}{\Sigma_i Q_{0,1}^{D_1^i} Q_{0,1}^{D_2^i}}$$

```
W_total = 1.4892e+07
w2OverW_total = 216464
w2OverW_total / W_total = 0.0145356
strandard deviation of weighted distribution: 0.59526
strandard deviation of raw distribution: 4.93023
sigma^2(weighted) / sigma^2(raw) = 0.0145774
```