1 Systematic uncertainty

Several sources of systematic uncertainties are considered in this analysis

1.1 Luminosity

The integrated luminosity at each energy point is measured using large angle Bhabha events with an uncertainty of 1% following the [?].

1.2 Branching fraction

The uncertainty of branching fraction of intermediate states is taken from the PDG as 0.08%

1.3 ISR and VP correction

The uncertainty of ISR and VP correction factor is obtained from the accuracy of radiation function, which is . This item is estimated by varying the parameters of cross section fitting and then recalculate the ISR and VP correction factor, efficiency and the corresponding cross section. This procedure is repeated 100 times. Take the standard deviation of the cross section distribution as the uncertainty from this item.

1.4 Event selection

The uncertainties from selection include each item described in Chap.3.

- K_S^0 reconstruction:
  The K_S^0 reconstruction is determined from the control sample $J/\psi \to K_S^0 K^{\pm}\pi^{\mp}$ and $J/\psi \to \phi K_S^0 K^{\pm}\pi^{\mp}$. For the momentum of K_S^0 is less than 1.4 GeV/c, the efficiency difference between MC and data is less than 1% for one K_S^0[?].

- Photon detection:
  The photon detection efficiency which is studied with the control sample $e^+e^- \to K^+K^-\pi^+\pi^-\pi^0$[?]. The control samples covers the same angles and momentum region in our analysis, the difference between MC and data is aboud 1% for one photon.

- Kinematic fit:
  The uncertainty of kinematic fit is studied with the control sample $J/\psi \to K_S^0 K_L^0 \pi^0$. This control sample is obtained in same event selection with this analysis and it has similar kinematic behavior with my processes, only the energy is different. The difference between MC and Data in efficiency loss when using the χ^2 cut or not in this procedure is taken as the uncertainty form kinematic fit. The number of events of data sample is obtained by using the same fit method in signal yield extraction and the number of

events of MC sample is obtained by counting, the fit results are shown in Figure 1.

- Helicity cut:
  The uncertainty of helicity cut is studied with the control sample $J/\psi \to K_S^0 K_L^0 \pi^0$. The difference between MC and Data in efficiency loss when using the helicity cut or not in this procedure is taken as the uncertainty form this item.

- π^0 mass window cut:
  The uncertainty of π^0 mass window cut is studied with the control sample $J/\psi \to K_S^0 K_L^0 \pi^0 (\eta$ veto). The difference between MC and Data in efficiency loss when using the π^0 mass window cut or not in this procedure is taken as the uncertainty form this item.

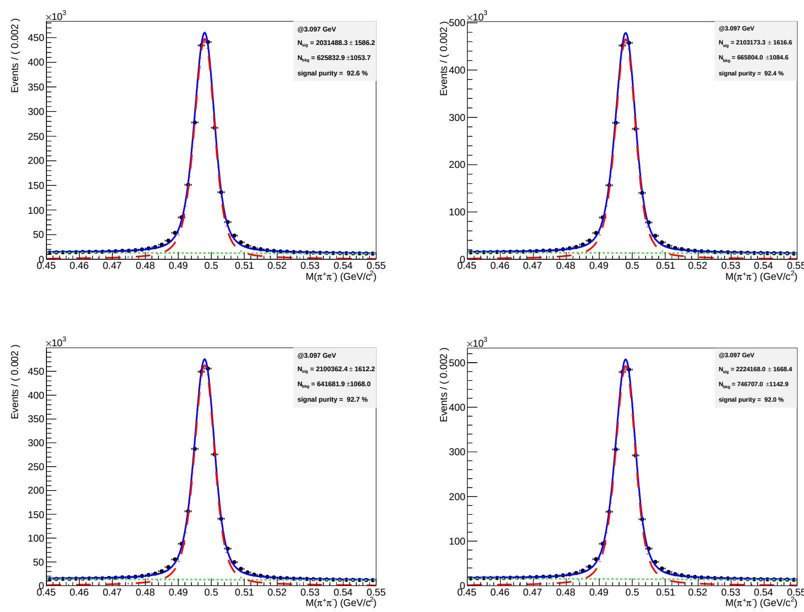

Figure 1: Fit to the $\pi^+\pi^-$ invariant mass distribution for the process $J/\psi \to K_S^0 K_L^0 \pi^0$. (a) With all selection. (b) Without χ^2 cut. (c) Without $|cos\theta|$ cut. (d) Without π^0 mass window cut. Black dots with error bars represent data sample(round11).

All PWA-unrelated systematic uncertainties are summarized in Table 2 and Table 2. The total systematic uncertainty is obtained by adding all individual contributions in quadrature.

Table 1: The selection efficiencies of control sample $J/\psi \to K_S^0 K_L^0 \pi^0$

Cut condition	**Efficiency(Data)**	**Efficiency(MC)**	**Uncertainty**
χ^2	96.59%	96.49%	0.10%
Helicity	97.80%	98.76%	0.96%
π^0 mass window	91.33%	91.56%	0.21%

Table 2: Summary of PWA-unrelated systematic uncertainties and the method. Ones that marked with * mean the correlated uncerntainties at different energies, others are not. These two kinds of uncertaintes are treated differently in cross section line shape fitting.

Uncertainty	**Method**
Luminosity*	1%(Luminosity measurement paper[**?**])
Branching fraction*	0.08%
ISR and VP correction	Toy line shape and recalculate the factor
K_S reconstruction*	Control sample $J/\psi \to K_S^0 K^\pm \pi^\mp$ and $J/\psi \to \phi K_S^0 K^\pm \pi^\mp$
Photon detection*	Control sample $e^+e^- \to K^+K^-\pi^+\pi^-\pi^0$
χ^{2*}	Control sample $J/\psi \to K_S^0 K_L^0 \pi^0$
Helicity*	Control sample $J/\psi \to K_S^0 K_L^0 \pi^0$
π^0 mass window*	Control sample $J/\psi \to K_S^0 K_L^0 \pi^0$
Signal shape	MC shape $\bigotimes$ Gaussian $\to$ Breit $-$ Wigner $\bigotimes$ Gaussian
Background shape	1st order polynomial $\to$ 2nd order polynomial
Fitting range	$M(\pi\pi)$ from (0.45,0.55) $\to$ (0.46,0.54) GeV/c^2

Table 3: PWA-unrelated uncerntainties

$\sqrt{s}$(GeV)	$\mathcal{L}$	K_S reconstruction	Photon detection	Kinematic fit	Branching fraction	$1+\delta$	Helicity	π^0 mass window	Signal shape	Bkg shape	Fitting range	total
2.0000	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	0.9%	0.1%	0.6%	%
2.0500	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	0.4%	0.6%	0.7%	%
2.1000	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	0.3%	0.2%	0.7%	%
2.1250	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	0.5%	0.8%	0.8%	%
2.1500	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	2.0%	2.0%	1.9%	%
2.1750	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	0.9%	0.7%	1.0%	%
2.2000	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	0.7%	1.2%	0.7%	%
2.2324	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	1.1%	1.8%	0.3%	%
2.3094	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	1.6%	2.2%	1.7%	%
2.3864	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	1.2%	0.8%	0.7%	%
2.3960	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	1.1%	0.8%	0.7%	%
2.6444	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	1.4%	1.6%	1.1%	%
2.6464	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	1.4%	1.5%	1.1%	%
2.9000	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	1.2%	0.9%	0.4%	%
2.9500	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	3.5%	4.0%	3.6%	%
2.9810	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	4.0%	1.5%	2.9%	%
3.0000	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	3.8%	2.5%	2.9%	%
3.0200	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	4.1%	0.5%	1.2%	%
3.0800	1.0%	1.0%	2.0%	0.1%	0.08%	%	1.0%	0.2%	0.9%	0.4%	0.9%	%

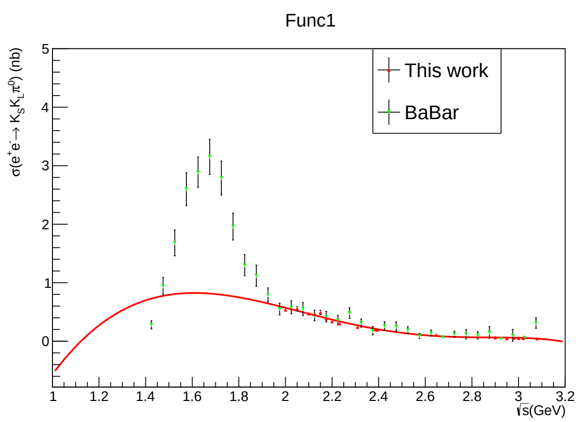

Figure 2: Polynomial