Dynamic Composite Higgs: Theory and Status

Haiying Cai

IPNL, Université Lyon 1

Part II :

Anarchic Yukawas and Top partial compositeness

Cacciapaglia, Cai, Flacke, Lee, Parolini and Serodio

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MCHM: $SO(5)/SO(4)$

In MCHM, the Higgs doublet, being identified with the pNGBs in the coset space of $SO(5)/SO(4)$, appears through a matrix U, in the unitary gauge to be:

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos h/f & \sin h/f \\ 0 & 0 & 0 & -\sin h/f & \cos h/f \end{pmatrix}$$

The pion matrix U transforms non-linearly under $\mathbf{g} \in SO(5)$: $U \to \mathbf{g} U \mathbf{h}^\dagger(\mathbf{g}, h)$, where $\mathbf{h} \in SO(4)$. By CCWZ, Cartan-Maurer forms are decomposed in the broken $T^{\hat{a}}$ and unbroken T^a directions:

$$iU^\dagger D_\mu U = d_\mu^{\hat{a}} T^{\hat{a}} + E_\mu^a T^a$$

where d_μ and E_μ are the building blocks for an effective Lagrangian of pNGBs and vector resnances.

Top partners

We embed the left-handed quarks $q_L = (t_L, b_L)^T$ and right-handed one t_R in a fundamental **5** of SO(5):

$$\overline{q}^5_{3L} = \frac{1}{\sqrt{2}}\left(-i\overline{b}_L,\ \overline{b}_L,\ -i\overline{t}_L,\ -\overline{t}_L,\ 0\right)\ , \quad \overline{t}^5_R = (0,\ 0,\ 0,\ 0,\ \overline{t}_R)\ ,$$

The composite sector contains many spin-1/2 fermionic resonances. The minimal set is a four-plet Q and a singlet $\tilde{T}$ of SO(4), in a **5** of SO(5):

$$\psi = \begin{pmatrix} Q \\ \tilde{T} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i\ B - i\ X_{5/3} \\ B + X_{5/3} \\ i\ T + i\ X_{2/3} \\ -T + X_{2/3} \\ \sqrt{2}\ \tilde{T} \end{pmatrix} .$$

General Lagrangian

The general Lagrangian we can write is then:

$$\begin{aligned}
\mathcal{L}_{comp} &= i\overline{Q}_{L,R}\left(\not{D}+\not{E}\right)Q_{L,R}+i\overline{\tilde{T}}_{L,R}\not{D}\tilde{T}_{L,R}-M_4\left(\overline{Q}_LQ_R+\overline{Q}_RQ_L\right)\\
&- M_1\left(\overline{\tilde{T}}_L\tilde{T}_R+\overline{\tilde{T}}_R\tilde{T}_L\right)+ic_L\overline{Q}^i_L\gamma^\mu d^i_\mu\tilde{T}_L+ic_R\overline{Q}^i_R\gamma^\mu d^i_\mu\tilde{T}_R+\text{h.c.}
\end{aligned}$$

$$\begin{aligned}
-\mathcal{L}_{pc} &= y_{L4,1}f\overline{q}^5_{3L}U\psi_R+y_{R4,1}f\overline{t}^5_RU\psi_L+\text{h.c.}\\
&= y_{L4}f\left(\overline{b}_LB_R+c^2_{\theta/2}\overline{t}_LT_R+s^2_{\theta/2}\overline{t}_LX_{2/3R}\right)-\frac{y_{L1}f}{\sqrt{2}}s_\theta\overline{t}_L\tilde{T}_R\\
&+ y_{R4}f\left(\frac{s_\theta}{\sqrt{2}}\overline{t}_RT_L-\frac{s_\theta}{\sqrt{2}}\overline{t}_RX_{2/3L}\right)+y_{R1}fc_\theta\overline{t}_R\tilde{T}_L+h.c.
\end{aligned}$$

with the defination of $s_\theta=\sin\theta=\sin\frac{h+\langle h\rangle}{f}$.The piece of $\mathcal{L}_{pc}$ is a linear mixing between an elementary fermion and one composite fermion, connected by the U matrice, in order to generate a mass for the top via partial compositeness.

Direct Yukawa Interaction

In addition we assume the presence of direct Yukawa interactions of all fermions, generated at a scale $\Lambda_{UV} > \Lambda_{HC}$. e.g. through four-fermion interactions in technicolor theory:

$$\begin{aligned}
\mathcal{L}_Y &= \sqrt{2}\,(\bar{q}^5_{\alpha L}\Sigma) m^u_{\mathrm{UV}\alpha\beta}(\Sigma^T u^5_{\beta R}) + \sqrt{2}\,(\bar{\tilde{q}}^5_{\alpha L}\Sigma) m^d_{\mathrm{UV}\alpha\beta}(\Sigma^T d^5_{\beta R}) \\
&= \frac{s_{2\theta}}{2}\left[\bar{u}_{\alpha L} m^u_{\mathrm{UV}\alpha\beta} u_{\beta R} + \bar{d}_{\alpha L} m^d_{\mathrm{UV}\alpha\beta} d_{\beta R}\right]
\end{aligned}$$

Where the fields q_L and u_R are a generalization to include three families, and $\tilde{q}_L$ and d_R are defined by

$$\tilde{q}^5_{\alpha L} = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} q^5_{\alpha L}\,, \qquad d^5_{\alpha R} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ d_{\alpha R} \end{pmatrix},$$

with $\Sigma = U \cdot (0\,0\,0\,0\,1)^t$, transforming linearly as a **5** under $SO(5)$.

Characteristic energy scale

Due to our model set-up, several energy scales will be present here:

- Flavor scale (Λ_{UV}): additional Yukawa operators are generated.
- Condensation scale (Λ_{HC}): at this scale, the strong dynamics breaks $SO(5)$ into $SO(4)$, and we have $\Lambda_{HC} = 4\pi f$.
- Compositeness scale (f): the strong sector is described by heavy reonances, e.g. spin-1/2 top partners and spin-1 vector resoannces, some of which have a mass of order f.
- EW scale (v): the mass scale of the W and Z gauge bosons.

In order to sufficiently suppress FCNC, we will require $\Lambda_{UV} \gtrsim 10^5$ TeV, and to be consistent with naturalness, the compositeness scale f should be around 1 TeV.

Flavor Structure

The fermionic field content can be split into up and down sectors as

$$\xi_\uparrow = \begin{pmatrix} u & c & t & T & X_{2/3} & \tilde{T} \end{pmatrix}^T , \quad \xi_\downarrow = \begin{pmatrix} d & s & b & B \end{pmatrix}^T .$$

Their Yukawa-mass Lagrangian is given by

$$\begin{aligned} -\mathcal{L}_{\rm yukawa-mass} &= \bar{\xi}_{\uparrow L} \left[M_{\rm up} + Y_{\rm up} h + \cdots \right] \xi_{\uparrow R} \\ &+ \quad \bar{\xi}_{\downarrow L} \left[M_{\rm down} + Y_{\rm down} h + \cdots \right] \xi_{\downarrow R} + \text{h.c.} \end{aligned}$$

Notice that the up-type Yukawa matrice $Y_{\rm up}$ comes from both the differentiation of $M_{\rm up}$ and from the $d^4_\mu \propto \partial_\mu h$ term in the $\mathcal{L}_{comp}$.

We will apply an expansion of $\sin 2\epsilon$, $\epsilon = \langle h \rangle / f$, for a block diagonalization, so that in this new basis the heavy eigenstates are diagonal and can be safely integrated out.

Up-quark sector

After the block diagonalization, we get for the up-quark sector, up to $\mathcal{O}(s_{2\epsilon}^3)$,

$$U_{uL}^\dagger M_{\rm up} U_{uR} \simeq \begin{pmatrix} m_U & 0 \\ 0 & D_M \end{pmatrix} ,$$

with

$$m_U \simeq \tfrac{s_{2\epsilon}}{2}\; m_{\rm UV}^u + m_t \Pi\,, \quad \Pi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D_M \simeq {\rm diag}\left(M_T, M_4, M_{\tilde{T}}\right),$$

where m_t is the contribution to the top mass from partial compositeness (so that $s_{2\epsilon}\; m_{\rm UV} \sim m_c \ll m_t$). The masses are defined as

$$m_t = s_{2\epsilon}\frac{f^2|y_{L1}y_{R1}M_4 - y_{L4}y_{R4}M_1|}{2\sqrt{2}M_T M_{\tilde{T}}}\,,\; M_T = \sqrt{M_4^2 + f^2 y_{L4}^2}\,,\; M_{\tilde{T}} = \sqrt{M_1^2 + f^2 y_{R1}^2}\,.$$

For later use, we define $s_{\phi L} = \frac{y_{L4} f}{M_T}$, and $s_{\phi R} = \frac{y_{R1} f}{M_{\tilde{T}}}$, indicating the degrees of partial compositeness.

The Yukawa interaction is transformed into a non block-diagonal form. For the light quark sector:

$$y_u \simeq \frac{m_U}{f s_{2\epsilon}/2} \left(1 - \tfrac{1}{2} s_{2\epsilon}^2\right) + B_u \,, \quad \text{where} \quad B_u \sim \tfrac{\Sigma_u}{M_*^2} \,.$$

We also define

$$\Sigma_u \sim \begin{pmatrix} m_c^2 & m_c^2 & m_c m_t \\ m_c^2 & m_c^2 & m_c m_t \\ m_c m_t & m_c m_t & m_t^2 \end{pmatrix} .$$

which is only capturing the order of the corrections and should not be considered as true equalities because order one coefficients are neglected.

Down-quark sector

Similar results can be obtained for the down sector:

$$U_{dL}^{\dagger} M_{\mathrm{down}} U_{dR} \simeq \begin{pmatrix} m_D & \\ & M_T \end{pmatrix}, \quad m_D \simeq \tfrac{s_{2\epsilon}}{2}\, m_{\mathrm{UV}}^{d} \,.$$

Note that no partial compositeness for bottom quark. The Yukawa coupling is decomposed as:

$$y_d \simeq \frac{m_D}{f s_{2\epsilon}/2}\left(1 - \frac{s_{2\epsilon}^2}{2}\right) + B_d \,,$$

where in analogy with up-quark sector we have

$$B_d \sim \tfrac{m_b \Sigma_d}{\epsilon M_*^3}\,, \quad \text{where} \quad \Sigma_d \sim \epsilon^2 \left(m_{\mathrm{UV}}^{d}\right)^2.$$

EW gauge currents

The interaction Lagrangian of the EW gauge currents is:

$$\begin{aligned}\mathcal{L}_{\text{gauge}} &= Z_\mu \bar{\xi}_{\uparrow L,R}\gamma^\mu A_{NC}^{tL,R}\xi_{\uparrow L,R} + Z_\mu \bar{\xi}_{\downarrow L,R}\gamma^\mu A_{NC}^{bL,R}\xi_{\downarrow L,R} \\ &+ W_\mu^+ \bar{\xi}_{\uparrow L,R}\gamma^\mu A_{CC}^{L,R}\xi_{\downarrow L,R} + h.c.\end{aligned}$$

Through block diagonalization, the deviations are expressed in terms of squared inverse of M_* (generic top partner mass), Σ_u and Σ_d:

- deviations in the neutral currents

$$\begin{aligned}\delta A_{NC}^{tL}\big|_{3\times3} &\simeq \frac{g}{c_W}\frac{\Sigma_u}{M_*^2}\,, \quad \delta A_{NC}^{tR}\big|_{3\times3} \simeq -\frac{g}{c_W}\frac{\Sigma_u}{M_*^2}\,, \\ \delta A_{NC}^{bL}\big|_{3\times3} &= 0\,, \quad \left(\delta A_{NC}^{bR}\big|_{3\times3}\right)_{ij} \simeq -\frac{g}{2c_W}\frac{\Sigma_d}{M_*^2}\,;\end{aligned}$$

- deviations in the charged currents

$$\left(\delta A_{CC}^{L}\right) \simeq -\frac{g}{\sqrt{2}}\frac{\Sigma_u}{M_*^2}\,, \quad \left(\delta A_{CC}^{R}\right) \simeq -\frac{g}{\sqrt{2}M_*^2}m_b\begin{pmatrix} m_c & m_c & m_c \\ m_c & m_c & m_c \\ m_t & m_t & m_t\,. \end{pmatrix}.$$

True mass eigenbasis

In order to go to the "true" mass eigenbasis we need to perform unitary transformations acting only on the light quark generations:

$$m_U = V_{uL} M_U V_{uR}^\dagger \,, \quad m_D = V_{dL} M_D V_{dR}^\dagger$$

where $M_U = \mathrm{diag}(m_u, m_c, m_t)$ and $M_D = \mathrm{diag}(m_d, m_s, m_b)$ are the masses of the six quarks. Since $m_t \gg m_c$ in automatically settled in our model, the light quarks can be diagonalized through $V_{uL,R}$ of the form

$$V_{uL,R} \sim \begin{pmatrix} O(1) & O(1) & O(\frac{m_c}{m_t}) \\ O(1) & O(1) & O(\frac{m_c}{m_t}) \\ O(\frac{m_c}{m_t}) & O(\frac{m_c}{m_t}) & 1 \end{pmatrix} .$$

Only the mixing between the first or second generation with the third generation is suppressed by a factor of m_c/m_t.

However, in the down sector there is no *a priori* hierarchy in the mass matrice.

Confront the data

We confront our model with constraints from flavour conserving/violating processes, which have three distinct origins:

(1) induced solely by the mixing effects due to top partial compositeness and direct Yukawa couplings;

(2) induced by heavy resonances, appearing at the compositeness scale;

(3) the four fermions interactions induced at the UV scale is of the form

$$\mathcal{L} = \frac{1}{\Lambda_{UV}^2}(\bar{q}q)^2 + h.c.$$

For $\Lambda_{UV} \sim 10^5$ TeV, it should not introduce any flavour problem.

Flavour preserving processes

The deviation of $\bar{t}_L \not{W} b_L$ needs to be compatible with $|V_{tb}| = 1.021 \pm 0.032$:

$$|\delta A^L_{CC}|^{1/2} \sim \left| \frac{m_t}{M_*} \frac{(1 - s^2_{\phi R})}{\sqrt{2} s_{\phi R}} \right| \lesssim 10^{-1} \,.$$

This implies that $s_{\phi R} < 1/2$ is disfavoured, unless $M_1 \gg 1$ TeV.

The right-handed coupling $\bar{t}_R \not{W} b_R$,

$$\delta A^R_{CC} \sim \frac{g}{\sqrt{2}} \frac{m_t m_b}{M_4^2}$$

need to satisfy the measurement of $b \to s\gamma$, $\Rightarrow$ we need $M_4 \gtrsim 1$ TeV.
For the couplings of the bottom quark we obtain

$$\delta g_{Zb_L} = 0\,, \quad \delta g_{Zb_R} \simeq -\frac{g}{2c_W} s^2_{\phi L} c^2_{\phi L} \left(\frac{m_b}{M_*} \right)^2 ;$$

δg_{Zb_L} vanishs due to the custodial symmetry, but gets correction from higher order operators; δg_{Zb_R} is suppressed by m_b^2/M_*^2. (EW constrants is satisfied)

Flavour violating processes

The FCNC process with $|\Delta F| = 2$ transitions can place strong constraints. The relevent effective Lagrangian is:

$$\mathcal{L}^{|\Delta F|=2} = \sum_{i=1}^{5} C_i^{q_\alpha q_\beta} \mathcal{Q}_i^{q_\alpha q_\beta} + \sum_{i=1}^{3} \tilde{C}_i^{q_\alpha q_\beta} \tilde{\mathcal{Q}}_i^{q_\alpha q_\beta}$$

with the dimension six operators defined as

$$\begin{aligned}
\mathcal{Q}_1^{q_\alpha q_\beta} &= \left(\overline{q}_{\beta L}\gamma_\mu q_{\alpha L}\right)\left(\overline{q}_{\beta L}\gamma_\mu q_{\alpha L}\right) , & \tilde{\mathcal{Q}}_1^{q_\alpha q_\beta} &= \left(\overline{q}_{\beta R}\gamma_\mu q_{\alpha R}\right)\left(\overline{q}_{\beta R}\gamma_\mu q_{\alpha R}\right) , \\
\mathcal{Q}_2^{q_\alpha q_\beta} &= \left(\overline{q}_{\beta R} q_{\alpha L}\right)\left(\overline{q}_{\beta R} q_{\alpha L}\right) , & \tilde{\mathcal{Q}}_2^{q_\alpha q_\beta} &= \left(\overline{q}_{\beta L} q_{\alpha R}\right)\left(\overline{q}_{\beta L} q_{\alpha R}\right) , \\
\mathcal{Q}_3^{q_\alpha q_\beta} &= \overline{q}_{\beta R}^a q_{\alpha L}^b \overline{q}_{\beta R}^b q_{\alpha L}^a , & \tilde{\mathcal{Q}}_3^{q_\alpha q_\beta} &= \overline{q}_{\beta L}^a q_{\alpha R}^b \overline{q}_{\beta L}^b q_{\alpha R}^a , \\
\mathcal{Q}_4^{q_\alpha q_\beta} &= \left(\overline{q}_{\beta R} q_{\alpha L}\right)\left(\overline{q}_{\beta L} q_{\alpha R}\right) , & & \\
\mathcal{Q}_5^{q_\alpha q_\beta} &= \overline{q}_{\beta R}^a q_{\alpha L}^b \overline{q}_{\beta L}^b q_{\alpha R}^a .
\end{aligned}$$

Firstly we apply the effective approach to the $D^0 - \overline{D}^0$ system. The contribution of a Higgs exchange to the operator $\mathcal{Q}_4^{uc}$ is estimated to be:

$$\frac{1}{m_H^2}\left(\frac{m_c}{M_*}\right)^4 \simeq \frac{10^{-12}}{\mathrm{TeV}^2}\left(\frac{1\ \mathrm{TeV}}{M_*}\right)^4 ,$$

The contribution from exchanging of a Z boson to the operator $\mathcal{Q}_1^{uc}$ is:

$$\frac{g^2}{16 c_W^2 m_Z^2}\left(\frac{m_c}{M_*}\right)^4 \simeq \frac{10^{-11}}{\mathrm{TeV}^2}\left(\frac{1\ \mathrm{TeV}}{M_*}\right)^4 .$$

Therefore, flavour violation in the up sector is well under control.

In the down sector, using the higher order correction to δg_{Zb_L}, we can find effective operators of the following form:

$$
\begin{aligned}
& \frac{1}{m_Z^2}\left(s_{\phi L}\frac{m_Z}{m_V}\right)^4\left[(V^*_{dL33}V_{dL31})^2\mathcal{Q}_1^{db}+(V^*_{dL33}V_{dL32})^2\mathcal{Q}_1^{sb}+(V^*_{dL32}V_{dL31})^2\mathcal{Q}_1^{ds}\right] \\
\simeq \quad & \frac{10^{-4}}{\mathrm{TeV}^2}\left[(V^*_{dL33}V_{dL31})^2\mathcal{Q}_1^{db}+(V^*_{dL33}V_{dL32})^2\mathcal{Q}_1^{sb}+(V^*_{dL32}V_{dL31})^2\mathcal{Q}_1^{ds}\right],
\end{aligned}
$$

setting $m_V \simeq 3$ TeV. These coefficients are too large, therefore one need the mixing angles in the down sector have a hierarchy.

$$
|V^*_{dL33}V_{dL31}| < 10^{-1}, \quad |V^*_{dL33}V_{dL32}| < 10^{-1/2}, \quad |V^*_{dL32}V_{dL31}| < 10^{-5/2}.
$$

Top rare decays

The CMS measurement for the flavour violating top decays, e.g. $t \to ch$, $t \to cZ$ with 19.7 fb^{-1} at $\sqrt{s} = 8$ TeV is:

$$\begin{array}{lcll} \mathcal{B}(t \to ch) & < & 6 \div 8 \times 10^{-3} & 95\%\text{CL}, \\ \mathcal{B}(t \to cZ) & < & 5 \times 10^{-4} & 95\%\text{CL}. \end{array}$$

In our model, we can estimate the corresponding branching ratios to be:

$$\mathcal{B}(t \to ch) \simeq 0.25 \left(|y_{tc,L}|^2 + |y_{tc,R}|^2\right), \quad \mathcal{B}(t \to cZ) \simeq 3.5 \left(\delta A_{NC}^{tL,R}\right)_{32}^2 .$$

The leading contributions to misaligned Yukawas are:

$$y_{tc,L} \simeq y_{tc,R} \sim \frac{m_c m_t}{f M_*} \simeq 10^{-4} ,$$

and the third generation flavour violating Z couplings are:

$$(\delta A_{NC}^{tL,R})_{32} \simeq \tfrac{g}{c_W} \tfrac{m_t m_c}{M_*^2} \simeq 10^{-4} .$$

CKM unitarity

The CKM matrix is defined by the following expression:

$$V_{CKM} = V_{uL}^{\dagger}(1 + \tfrac{\sqrt{2}}{g}\delta A_{CC}^{L})V_{dL}\,.$$

Thus the correction δA_{CC}^{L} is constrained by unitarity, in particular

$$\begin{aligned}
\left(V_{uL}^{\dagger}V_{dL}\right)^{\dagger}\left(V_{uL}^{\dagger}V_{dL}\right) &= 1 \;\Rightarrow\; V_{dL}^{\dagger}(\delta A_{CC}^{L} + \delta A_{CC}^{L\dagger})V_{dL} = V_{CKM}^{\dagger}V_{CKM} - 1\,,\\
\left(V_{uL}^{\dagger}V_{dL}\right)\left(V_{uL}^{\dagger}V_{dL}\right)^{\dagger} &= 1 \;\Rightarrow\; V_{uL}^{\dagger}(\delta A_{CC}^{L} + \delta A_{CC}^{L\dagger})V_{uL} = V_{CKM}V_{CKM}^{\dagger} - 1\,.
\end{aligned}$$

For the up-type sector, the bound is easily satisfied, while a mild hierarchy in the down-type sector is required, we find

$$|V_{dL13}| < 10^{-1}\,, \quad |V_{dL23}|^{2} < 10^{-1}\,.$$

Scalar resonance

First we consider the scalar interaction: $\mathcal{L} = \Phi(g_B \bar{Q} Q + g_S \bar{\tilde{T}} \tilde{T}) + \frac{1}{2} m_\Phi^2 \Phi^2$. After diagonalization and integrating out Φ, we find:

$$\mathcal{L}_u \simeq \left(\frac{1\ \mathrm{TeV}}{M_*}\right)^2 \left(\frac{\tilde{g}}{m_\Phi/\mathrm{TeV}}\right)^2 \times \frac{10^{-10}}{\mathrm{TeV}^2}\, \mathcal{Q}_4^{uc}\,,$$

$$\mathcal{L}_d \simeq \left(\frac{1\ \mathrm{TeV}}{M_*}\right)^2 \left(\frac{g_B}{m_\Phi/\mathrm{TeV}}\right)^2 \times \frac{10^{-5}}{\mathrm{TeV}^2} \left[z_4^{db} \mathcal{Q}_4^{db} + z_4^{sb} \mathcal{Q}_4^{sb} + z_4^{ds} \mathcal{Q}_4^{ds}\right]$$

with the coefficients given by rotating matrices:

$$z_4^{d_\alpha d_\beta} = V^*_{dL3\alpha} V_{dL3\beta} \sum_{\gamma\delta} V_{dR\gamma\beta} V^*_{dR\delta\alpha}\,.$$

The constraints on $\mathcal{Q}_4$ operators require the coefficients to satisfy:

$$|z_4^{db}| < 10^{-2}\,, \quad |z_4^{sb}|^2 < 10^{-1}\,, \quad |z_4^{ds}| < 10^{-6}\,,$$

by assuming $m_\Phi/g \sim 1$ TeV in a conservative scenario.

Vector resonance

The interaction with a massive vector resonance is:

$$\mathcal{L} = V_\mu(g_B \bar{Q}_L \gamma^\mu Q_L + g_S \bar{\tilde{T}}_L \gamma^\mu \tilde{T}_L) + (L \to R) + \frac{1}{2} m_V^2 V_\mu V^\mu \,.$$

$$C_1^{uc} \sim \left(\frac{g_B}{m_V/\mathrm{TeV}}\right)^2 \times \frac{10^{-9}}{\mathrm{TeV}^2}\,, \quad C_1^{d_\alpha d_\beta} \sim \left(\frac{g_B}{m_V/\mathrm{TeV}}\right)^2 \times \frac{[V^*_{dL3\alpha} V_{dL3\beta}]^2}{\mathrm{TeV}^2}$$

The constraints on C_1^{bd}, C_1^{bs} and C_1^{sd} imply, respectively,

$$|V^*_{dL33} V_{dL31}| < 10^{-3}\,, \quad |V^*_{dL33} V_{dL32}| < 10^{-2}\,, \quad |V^*_{dL32} V_{dL31}| < 10^{-5}\,.$$

$\Rightarrow$ i.e. certain alignment in the down-sector is required.

Summary of bounds

In composite Higgs models, deviations in its couplings to quarks are given by non linearities:

$$\frac{y_{SM}-y}{m/v} \simeq 1 - \frac{1-2s_\epsilon^2}{\sqrt{1-s_\epsilon^2}} \simeq 0.15\,.$$

This value is for the $h\bar{b}b$ coupling is allowed by the experiment measurement. A combined analysis for all the results is in order:

$$\begin{aligned}
&\text{FCNCs} : |V^*_{dL33}V_{dL13}| < 10^{-1}\,, |V^*_{dL33}V_{dL23}| < 10^{-1/2}\,, |V^*_{dL13}V_{dL23}| < 10^{-5/2}\,,\\
&\text{CKM unitarity} \;\; : |V_{dL13}| < 10^{-1}\,, |V_{dL23}| < 10^{-1/2}\,,\\
&\text{Scalar resonance} \;\; : |z_4^{db}| < 1 \div 10^{-2}\,, |z_4^{sb}| < 1 \div 10^{-1/2}\,, |z_4^{ds}| < 10^{-4} \div 10^{-6}\,,\\
&\text{Vector resonance} : |V^*_{dL33}V_{dL31}| < 10^{-1} \div 10^{-3}\,, |V^*_{dL33}V_{dL32}| < 1 \div 10^{-2}\,,\\
&\qquad\qquad\qquad\qquad\quad |V^*_{dL32}V_{dL31}| < 10^{-3} \div 10^{-5}\,.
\end{aligned}$$

The $\mathcal{O}(10^2)$ magnitude variation is that we may times a $(4\pi)^2$ factor for the resonance mass, thus the contraint will be relieved.

Partial composite bottom

We discuss the possibility for b_R to linearly couple to composite operator, containing B_L, without introducing additional bottom partner:

$$\mathcal{L} \supseteq \bar{q}_{3L}\mathcal{O}_{q_L} + \bar{b}_R\mathcal{O}_{b_R} + h.c.$$

The difference is that the mixing with b_R requires EWSB. Explicitly we can write down the Lagrangian to be:

$$\mathcal{L} = y_R f\, \bar{\psi}_L U^t d_{3R}^{14}\Sigma + h.c. = \tfrac{1}{2} y_R f s_\theta \bar{B}_L b_R + h.c.\,,$$

where ψ is the quark partner five-plet that contains the bottom partner B, and d_{3R}^{14} is a spurion formally transforming as the **14** of SO(5), whose only dynamical component is the b_R:

$$d_{3R}^{14} = \tfrac{b_R}{2\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & i & 0 \\ 0 & 0 & -i & 1 & 0 \\ 1 & -i & 0 & 0 & 0 \\ i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Down sector structure

From the partial compositeness and the elementary Yukawas we obtain:

$$\mathcal{L} = \bar{d}_{\alpha L} m^d_{\alpha\beta} d_{\beta R} + h.c. \,, \quad m^d = m^d_{UV} \frac{s_{2\epsilon}}{2} + \Pi \frac{f y_R s_{\phi L}}{2} s_\epsilon \,.$$

We require $f y_R s_{\phi L} s_\epsilon / 2 \simeq m_b$, and $|m^d_{UV}| \sim m_s \ll m_b$. This hierarchy generates the following rotating matrices:

$$V_{dL,R} \sim \begin{pmatrix} O(1) & O(1) & O(\frac{m_s}{m_b}) \\ O(1) & O(1) & O(\frac{m_s}{m_b}) \\ O(\frac{m_s}{m_b}) & O(\frac{m_s}{m_b}) & 1 \end{pmatrix} \,.$$

In analogy to the top partial compositeness, we generate an additional mass hierarchy m_s/m_b, which relieves the flavor constraint in the down-quark sector.

Full composite t_R

Another minimal variation is to assume that t_R is a full composite state. This assumption leads to changes in the up-sector mass, Yukawa matrices and gauge interactions.

$$
\begin{aligned}
\mathcal{L}_{comp} &= i\overline{Q}_{L,R}\left(\not{D}+\not{E}\right)Q_{L,R} + i\overline{\tilde{T}}_{L,R}\not{D}\tilde{T}_{L,R} - M_4\left(\overline{Q}_L Q_R + \overline{Q}_R Q_L\right) \\
&- M_1\left(\overline{\tilde{T}}_L\tilde{T}_R + \overline{\tilde{T}}_R\tilde{T}_L\right) + ic_L\overline{Q}^i_L\gamma^\mu d^i_\mu\tilde{T}_L \\
&+ ic_R\overline{Q}^i_R\gamma^\mu d^i_\mu\tilde{T}_R + ic_t\overline{Q}^i_R\gamma^\mu d^i_\mu t_R + \text{h.c.}
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{L}_{mix} &= y_{L4,1} f\overline{q}^5_{3L}U\psi_R + y_{Lt}f\overline{q}^5_{3L}Ut_R + \text{h.c.} \\
&= y_{L4}f\left(\overline{b}_L B_R + c^2_{\theta/2}\overline{t}_L T_R + s^2_{\theta/2}\overline{t}_L X_{2/3R}\right) \\
&- \frac{y_{L1}f}{\sqrt{2}}s_\theta\overline{t}_L\tilde{T}_R - \frac{y_{Lt}f}{\sqrt{2}}s_\theta\overline{t}_L\tilde{t}_R + h.c.
\end{aligned}
$$

Up sector structure

We find that the factorization pattern displayed for m_U continue to hold, and the additional c_t term in the $\mathcal{L}_{comp}$ will give rise to an $\mathcal{O}(1)$ correction in the Yukawa interaction:

$$m_U \simeq \tfrac{s_{2\epsilon}}{2}\; m^u_{\mathrm{UV}} \mp m_t \Pi\,, \quad y_u \simeq \frac{m_U}{f s_{2\epsilon}/2}\left(1-\tfrac{1}{2}s^2_{2\epsilon}\right) + c_t\left(y_{Lt}-y_{L4}\right) c_{\phi L}\Pi + B_u\,,$$

$$\text{where} \quad \Pi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B_u \sim \tfrac{\Sigma_u}{M_*^2}\,, \quad \text{and} \quad c_{\phi L} = \tfrac{M_*}{\sqrt{M_*^2+f^2 y^2_{L4}}}\,.$$

The deviations in charged and neutral right-hand currents can be calculated and are of the same order as in the partial composite case.

Scalar resonance

For scalar interaction, the effective Lagrangian is the same as before:

$$\mathcal{L} = \Phi(g_B \bar{Q} Q + g_S \bar{\tilde{T}} \tilde{T}) + \frac{1}{2} m_\Phi^2 \Phi^2 \,.$$

When we integrate out the scalar resonance, the effective Lagrangian for the dimension-6 operator is,

$$\mathcal{L}_S \simeq \frac{10^{-10}}{\mathrm{TeV}^2} \left(\frac{1 \text{ TeV}}{M_*} \right)^2 \mathcal{Q}_4^{uc} \,.$$

The coefficient C_4^{uc} is well below the experimental bound.

Vector resonance

Larger difference comes from the t_R coupling to a vector resonance V_μ due to the chiral property:

$$\mathcal{L}_V = V_\mu(g_B \bar{Q}_L \gamma^\mu Q_L + g_S \bar{\tilde{T}}_L \gamma^\mu \tilde{T}_L) + (L \to R) + V_\mu g_S' \bar{t}_R \gamma^\mu t_R + \frac{1}{2} m_V^2 V_\mu V^\mu \,.$$

After rotating to the mass eigenstates and intergrating out the vector resonance,, we find the coefficients:

$$\begin{aligned} \mathcal{Q}_1^{uc} &: \quad \frac{1}{f^2}\left(s_{\phi L}^2 \left(\frac{m_c}{m_t}\right)^2 - 2 s_{\phi L}^2 c_{\phi L}^2 \left(\frac{m_c}{M_*}\right)^2\right)^2 \\ \tilde{\mathcal{Q}}_1^{uc} &: \quad \frac{1}{f^2}\left(\frac{g_S'}{g_B}\left(\frac{m_c}{m_t}\right)^2 + \left((1-2c_{\phi L})^2 - \frac{g_S'}{g_B}\right) s_{\phi L}^2 \left(\frac{m_c}{M_*}\right)^2\right)^2 \,. \end{aligned}$$

Therefore numerically, the Wilson coefficients are of order of $10^{-9}/\mathrm{TeV}^2$, below the experimental bound.

Conclusion

- We have explored the possibility that the top quark mass is mainly generated by partial compositeness, while other quarks masses are from four-fermion interaction.
- We showed that this scenario is naturally compatible with bounds from flavour constraints for the up sector. For the down sector, in order to satisfy the bounds, they require a mild hierarchy in the mixing matrice.
- The discussion of flavor violating suppression is quite general, can be generalized to the case with additional top partners and the scenario of non minimal coset.
- The mechanism also applies to the scenario of t_R being a full composite state, as the flavor structure would not be affected.