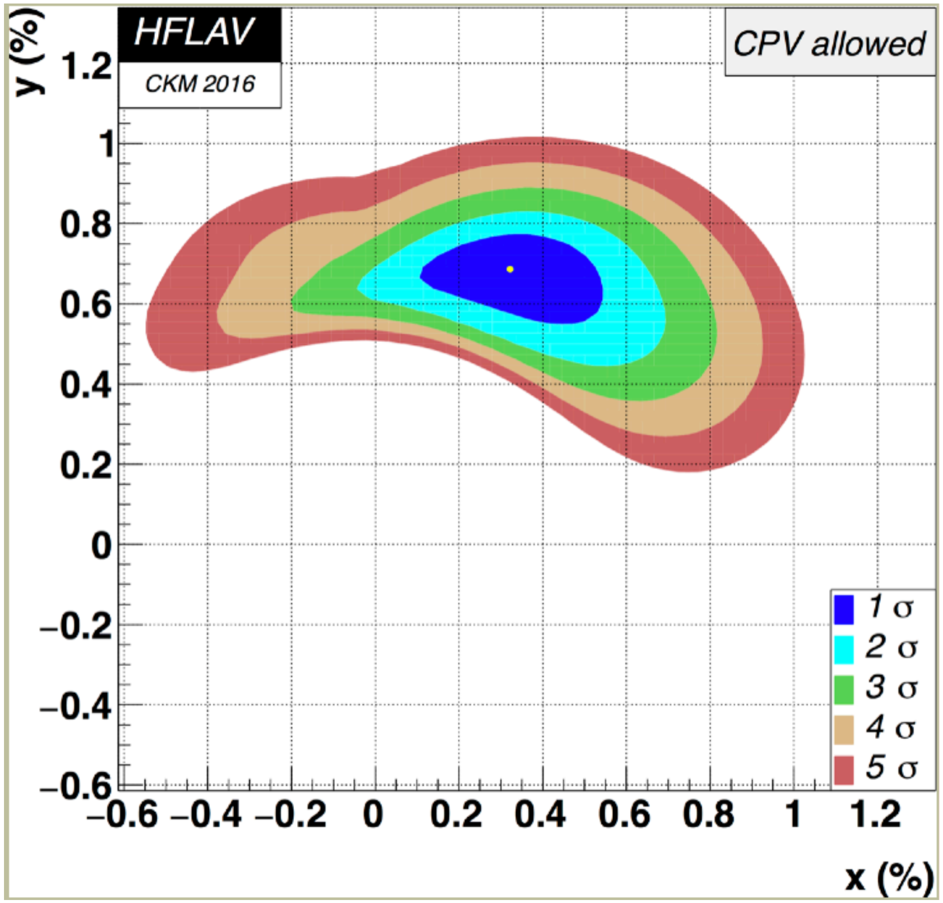
Mixing of charmed mesons: theoretical overview

Alexey A. Petrov
Wayne State University
Michigan Center for Theoretical Physics

Table of Contents:

Introduction

★ Experimental fact: charm mixing parameters are non-zero



★ ... and rather large

- if CP-violation is neglected...

$$x = \left(0.46^{+0.14}_{-0.15}\right)\%$$
$$y = (0.62 \pm 0.08)\%$$

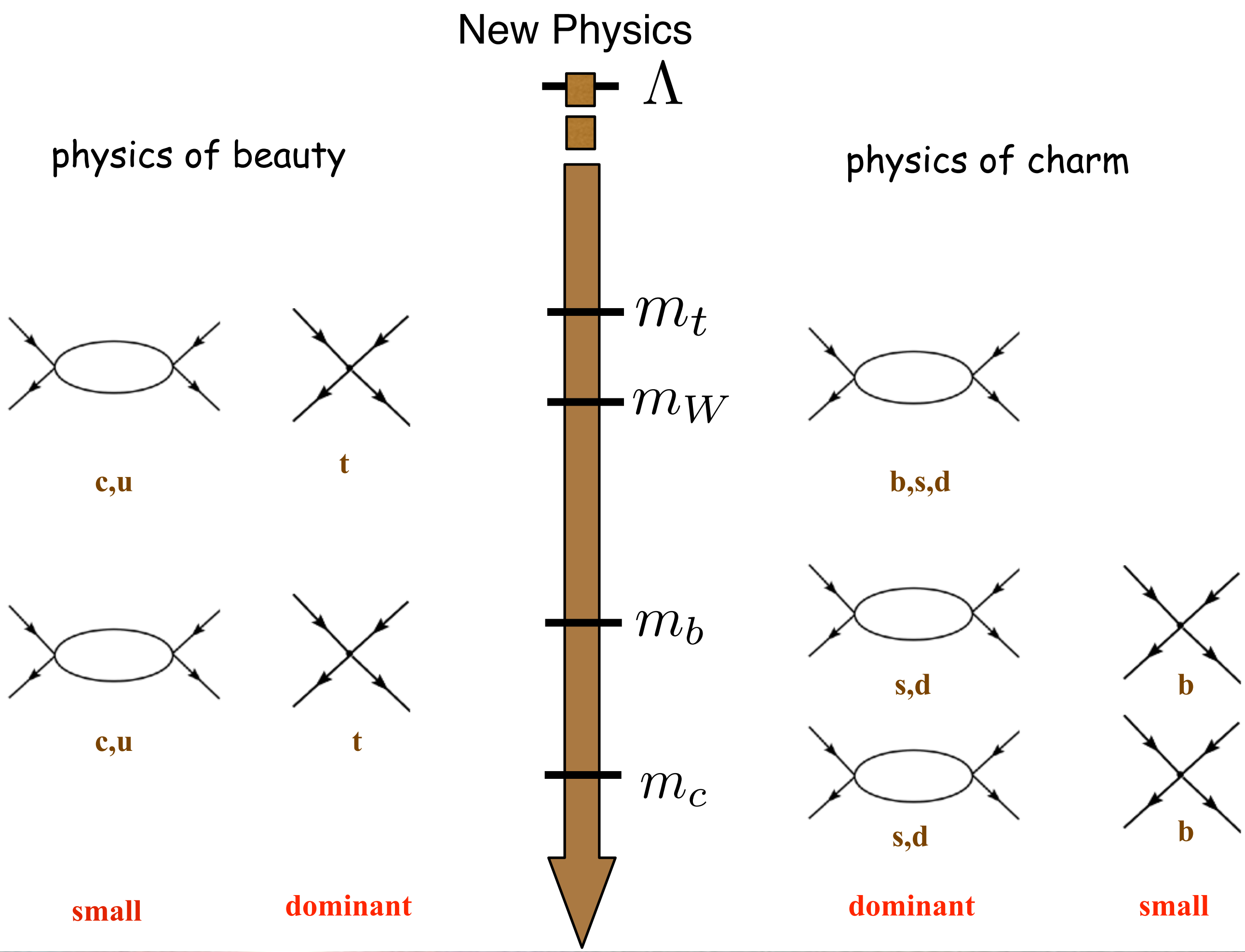
- if CP-violation is allowed

$$x = (0.32 \pm 0.14)\%$$
$$y = \left(0.69^{+0.06}_{-0.07}\right)\%$$

Introduction

★ Main goal of the exercise: understand physics at the most fundamental scale

★ It is important to understand relevant energy scales for the problem at hand



Quark-hadron duality: lifetimes

★ New Physics couples to quark degrees of freedom, we observe hadrons!

➡ need to know how to compute non-perturbative matrix elements

➡ need to understand how quark-hadron duality works

★ Observables computed in terms of hadronic degrees of freedom…

$$\Gamma_{hadron}\left(H_b\right) = \sum_{\substack{all\ final\ state \\ hadrons}} \Gamma\left(H_b \to h_i\right)$$

Bloom, Gilman;
Poggio, Quinn, Weinberg

★ … must match observables computed in terms of quark degrees of freedom

$$\Gamma\left(H_b\right) = \frac{1}{2M_b}\langle H_b | T | H_b \rangle = \frac{1}{2M_b}\langle H_b | \mathrm{Im}\, i \int d^4x\, T\left\{H_{eff}^{\Delta B=1}(x) H_{eff}^{\Delta B=1}(0)\right\} | H_b \rangle$$

$$\Gamma\left(H_b\right) = \frac{G_F^2 m_Q^5}{192\pi^3}\left[A_0 + \frac{A_2}{m_Q^2} + \frac{A_2}{m_Q^3} + \ldots\right]$$

HQ expansion converges reasonably well…

Quark-hadron duality: lifetimes

★ How to define quark-hadron duality and quantify its violations?

➡ Compute quark correlator in Eucledian space and analytically continue to Minkowski space [exact calculation in ES = exact result in MS]

➡ Expand it in α_S and "$1/Q \sim 1/m_Q$": series truncation

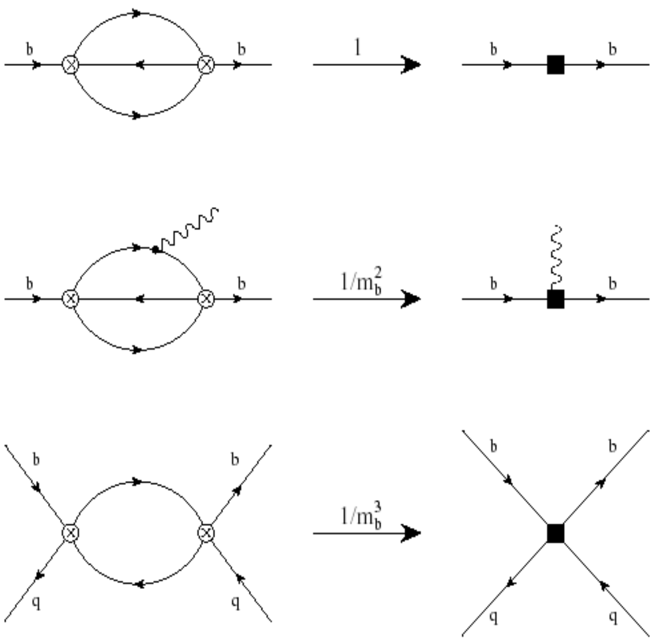
➡ Any deviation beyond "natural uncertainty" is treated as violation of quark-hadron duality [resonances, instantons,…]

This definition is due to M. Shifman

Rob Gonzalves

Quark-hadron duality: lifetimes

★ In case of b-flavored hadrons can compare directly to experiment



Lifetime ratio	Experimental average	HQE prediction
$\tau(B^+)/\tau(B^0)$	1.076 ± 0.004	$1.04^{+0.05}_{-0.01} \pm 0.02 \pm 0.01$
$\tau(B_s^0)/\tau(B^0)$	0.990 ± 0.004	1.001 ± 0.002
$\tau(\Lambda_b^0)/\tau(B^0)$	0.967 ± 0.007	0.935 ± 0.054
$\tau(\Xi_b^0)/\tau(\Xi_b^-)$	0.929 ± 0.028	0.95 ± 0.06

HFLAV, 2017

★ … works surprisingly well…

Channel	Expansion parameter x	Numerical value	$\exp[-1/x]$
$b \to c\bar{c}s$	$\frac{\Lambda}{\sqrt{m_b^2-4m_c^2}} \approx \frac{\Lambda}{m_b}\left(1+2\frac{m_c^2}{m_b^2}\right)$	$0.054 - 0.58$	$9.4 \cdot 10^{-9} - 0.18$
$b \to c\bar{u}s$	$\frac{\Lambda}{\sqrt{m_b^2-m_c^2}} \approx \frac{\Lambda}{m_b}\left(1+\frac{1}{2}\frac{m_c^2}{m_b^2}\right)$	$0.045 - 0.49$	$1.9 \cdot 10^{-10} - 0.13$
$b \to u\bar{u}s$	$\frac{\Lambda}{\sqrt{m_b^2-4m_u^2}} = \frac{\Lambda}{m_b}$	$0.042 - 0.48$	$4.2 \cdot 10^{-11} - 0.12$

Jubb, Kirk, Lenz, Tetlalmatzi-Xolocotzi, 2017

★ How does it work for charmed hadrons?

For the lifetimes, see Prof. H.Y. Cheng's talk from yesterday

Quark-hadron duality: mixing

★ How can one tell that a process is dominated by long-distance or short-distance?

★ To start thing off, mass and lifetime differences of mass eigenstates...

$$x_D = \frac{M_2 - M_1}{\Gamma_D}, \quad y_D = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D}$$

★ ...can be calculated as real and imaginary parts of a correlation function

$$y_{\mathrm{D}} = \frac{1}{2M_{\mathrm{D}}\Gamma_{\mathrm{D}}} \, \mathrm{Im} \, \langle \overline{D^0} | \, i \int \mathrm{d}^4x \, T \left\{ \mathcal{H}_w^{|\Delta C|=1}(x) \, \mathcal{H}_w^{|\Delta C|=1}(0) \right\} | D^0 \rangle$$

bi-local time-ordered product

$$x_{\mathrm{D}} = \frac{1}{2M_{\mathrm{D}}\Gamma_{\mathrm{D}}} \, \mathrm{Re} \left[2\langle \overline{D^0} | H^{|\Delta C|=2} | D^0 \rangle + \langle \overline{D^0} | \, i \int \mathrm{d}^4x \, T \left\{ \mathcal{H}_w^{|\Delta C|=1}(x) \, \mathcal{H}_w^{|\Delta C|=1}(0) \right\} | D^0 \rangle \right]$$

local operator
(b-quark, NP): small?

bi-local time-ordered product

★ ... or can be written in terms of hadronic degrees of freedom...

$$y = \frac{1}{2\Gamma} \sum_n \rho_n \left[\langle D^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | \overline{D}^0 \rangle + \langle \overline{D}^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | D^0 \rangle \right]$$

Mixing: short vs long distance

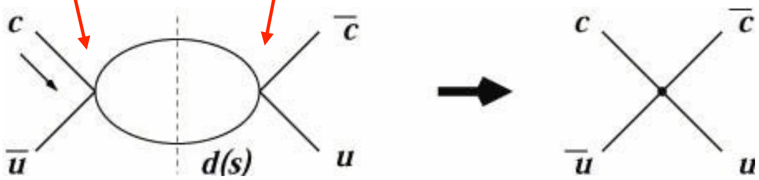
★ How can one tell that a process is dominated by long-distance or short-distance?

$$y = \frac{1}{2\Gamma} \sum_n \rho_n \left[\langle D^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | \overline{D}^0 \rangle + \langle \overline{D}^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | D^0 \rangle \right]$$

★ It is important to remember that the expansion parameter is $1/E_{released}$

$$y_D = \frac{1}{2 M_D \Gamma_D} \, \mathrm{Im} \, \langle \overline{D^0} | \, i \int d^4x \, T \left\{ \mathcal{H}_w^{|\Delta C|=1}(x) \, \mathcal{H}_w^{|\Delta C|=1}(0) \right\} | D^0 \rangle$$

OPE-leading contribution:



★ In the heavy-quark limit $m_c \to \infty$ we have $m_c \gg \sum m_{intermediate\ quarks}$, so $E_{released} \sim m_c$

- the situation is similar to B-physics, where it is "short-distance" dominated
- one can consistently compute pQCD and 1/m corrections

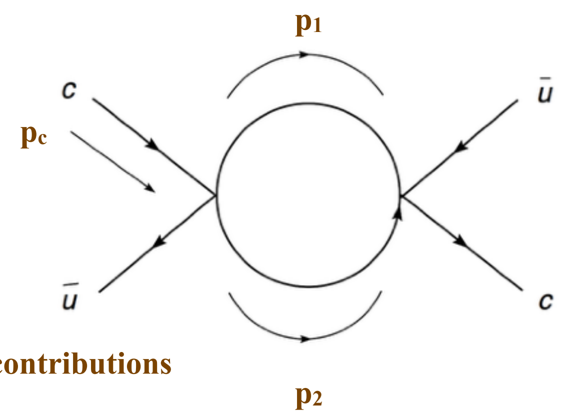
★ But wait, m_c is NOT infinitely large! What happens for finite m_c???

- how is large momentum routed in the diagrams?
- are there important hadronization (threshold) effects?

Threshold (and related) effects in OPE

★ How can one tell that a process is dominated by long-distance or short-distance?

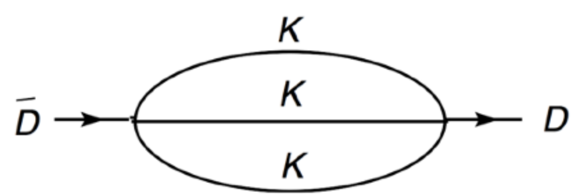
★ Let's look at how the momentum is routed in a leading-order diagram
- injected momentum is $p_c \sim m_c$
- thus, $p_1 \sim p_2 \sim m_c/2 \sim O(\Lambda_{QCD})$?



Still OK with OPE, signals large nonperturbative contributions

★ For a particular example of the lifetime difference, have hadronic intermediate states

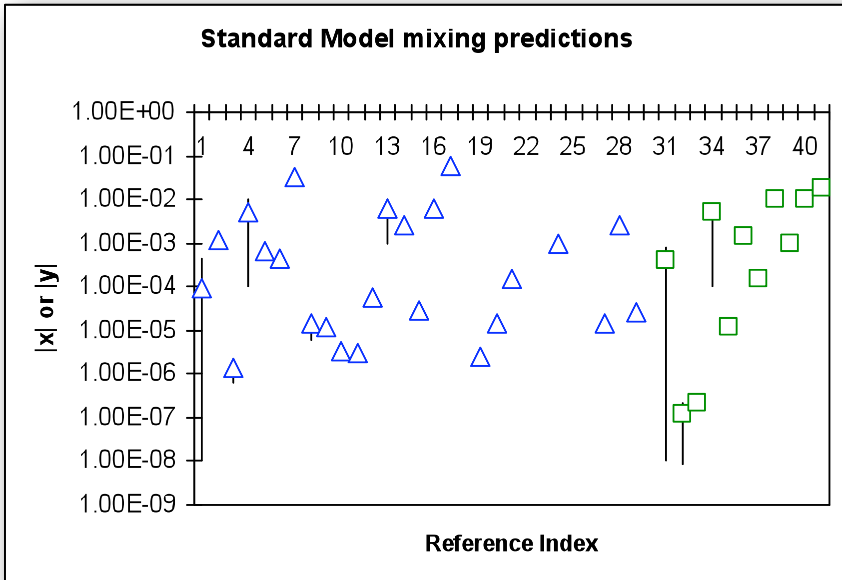
-let's use an example of KKK intermediate state
- in this example, $E_{released} \sim m_D - 3\ m_K \sim O(\Lambda_{QCD})$



★ Similar threshold effects exist in B-mixing calculations
- but $m_b \gg \sum m_{intermediate\ quarks}$, so $E_{released} \sim m_b$ (almost) always
- quark-hadron duality takes care of the rest!

Thus, two approaches: 1. insist on $1/m_c$ expansion, hope for quark-hadron duality
2. saturate correlators by hadronic states

Mixing: Standard Model predictions



*

* Not an actual representation of theoretical uncertainties. Objects might be bigger then what they appear to be...

★ Predictions of x and y in the SM are complicated
- -second order in flavor SU(3) breaking
- $-m_c$ is not quite large enough for OPE
  - $-x, y \ll 10^{-3}$ ("short-distance")
  - $-x, y \sim 10^{-2}$ ("long-distance")

★ Short distance:
- -assume m_c is large
  - -combined m_s, $1/m_c$, α_s expansions
  - -leading order: m_s^2, $1/m_c^6$!
  - -threshold effects?

H. Georgi; T. Ohl, …
I. Bigi, N. Uraltsev;

M. Bobrowski et al

★ Long distance:
- -assume m_c is NOT large
  - -sum of large numbers with alternating signs, SU(3) forces zero!
  - -multiparticle intermediate states dominate

J. Donoghue et. al.
P. Colangelo et. al.

Falk, Grossman, Ligeti, Nir. A.A.P. Phys.Rev. D69, 114021, 2004
Falk, Grossman, Ligeti, and A.A.P. Phys.Rev. D65, 054034, 2002

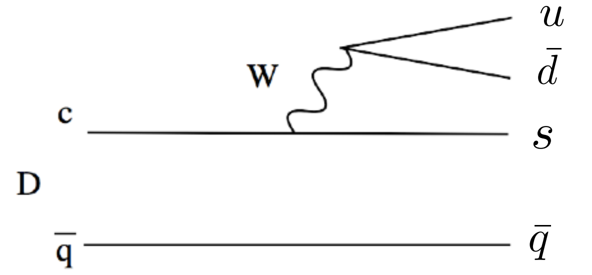
Aside: classification of charm decays

★ Can be classified by SM CKM suppression

★ Cabibbo-favored (CF) decay

- originates from $c \to s\, u\bar{d}$
- examples: $D^0 \to K^- \pi^+$

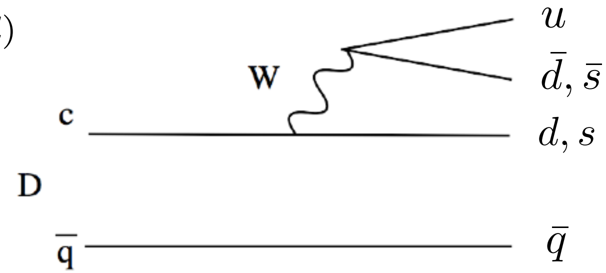
$$V_{cs}V_{ud}^*$$



★ Singly Cabibbo-suppressed (SCS) decay

- originates from $c \to q\, u\bar{q}$
- examples: $D^0 \to \pi\pi$ and $D^0 \to KK$

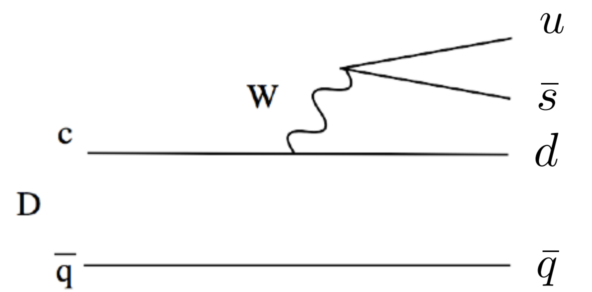
$$V_{cs(d)}V_{us(d)}^*$$



★ Doubly Cabibbo-suppressed (DCS) decay

- originates from $c \to d\, u\bar{s}$
- examples: $D^0 \to K^+ \pi^-$

$$V_{cd}V_{us}^*$$



★ "Common final states" for D and $\bar{D}$ generate mixing in exclusive approach

Exclusive approach to mixing: use data?

★ LD calculation: saturate the correlator by hadronic states, e.g.

$$y = \frac{1}{2\Gamma} \sum_n \rho_n \left[\langle D^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | \overline{D}^0 \rangle + \langle \overline{D}^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | D^0 \rangle \right]$$

… with n being all states to which D^0 and $\overline{D}^0$ can decay. Consider $\pi\pi$, πK, KK intermediate states as an example…

J. Donoghue et. al.
L. Wolfenstein
P. Colangelo et. al.

H.Y. Cheng and C. Chiang

$$\begin{aligned} y_2 &= Br(D^0 \to K^+K^-) + Br(D^0 \to \pi^+\pi^-) \\ &- 2\cos\delta \sqrt{Br(D^0 \to K^+\pi^-) Br(D^0 \to \pi^+K^-)} \end{aligned}$$

cancellation expected

If every Br is known up to O(1%) ➡ the result is expected to be O(1%)!

The result here is a series of large numbers with alternating signs, SU(3) forces 0

If experimental data on Br is used, are we only sensitive to exit. uncertainties?

★ Need to "repackage" the analysis: look at complete multiplet contribution

$$y = \sum_{F_R} y_{F,R}\, Br(D^0 \to F_R) = \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma(D^0 \to n)$$

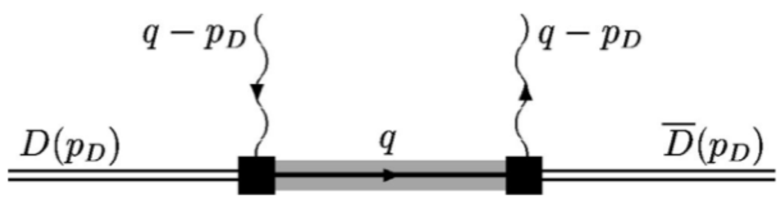
Falk, Grossman, Ligeti, Nir. A.A.P. Phys.Rev. D69, 114021, 2004
Falk, Grossman, Ligeti, and A.A.P. Phys.Rev. D65, 054034, 2002

Exclusive approach to mixing: no data

Falk, Grossman, Ligeti, Nir. A.A.P.
Phys.Rev. D69, 114021, 2004
Falk, Grossman, Ligeti, and A.A.P.
Phys.Rev. D65, 054034, 2002

★ LD calculation: consider the correlation

$$\Sigma_{p_D}(q) = i\int d^4 z \left\langle \overline{D}(p_D) \right| T\left[H_w(z) H_w(0)\right] \left| D(p_D) \right\rangle e^{i(q-p_D)z}$$



$$-\frac{1}{2m_D}\Sigma_{p_D}(p_D) = \left(\Delta m - \frac{i}{2}\Delta\Gamma\right)$$

★ $\Sigma_{p_D}(q)$ is an analytic function of q. To write a disp. relation, go to to HQET:

$$H_w = \frac{4G_F}{\sqrt{2}} V_{cq_1} V^*_{uq_2} \sum_i C_i O_i = \hat{H}_w \left[e^{-im_c v\cdot z} h_v^{(c)} + e^{im_c v\cdot z} \tilde{h}_v^{(c)} \right] + \ldots$$

$$\left| D(p = mv) \right\rangle = \sqrt{m} \left| H(v) \right\rangle + \ldots$$

Now we can interpret $\Sigma_{p_D}(q)$ for all q

Dispersion relations for mixing

★ ...this implies for the correlator

Rapidly oscillates for large m_c

$$\Sigma_{p_D}(q) = i\int d^4z \left\langle \overline{H}(v) \right| T e^{i(q-p_D-m_c v)z} \left[\hat{H}_w h_v^{(c)}(z), \hat{H}_w \tilde{h}_v^{(c)}(0) \right] \left| H(v) \right\rangle +$$
$$i\int d^4z \left\langle \overline{H}(v) \right| T e^{i(q-p_D+m_c v)z} \left[\hat{H}_w \tilde{h}_v^{(c)}(z), \hat{H}_w h_v^{(c)}(0) \right] \left| H(v) \right\rangle + \ldots$$

★ HQ mass dependence drops out for the second term, so for $\overline{\Sigma}_v(q) = \Sigma_{p_D}(q)/m_D$

$$\boxed{\overline{\Sigma}_v(q) = -2\Delta m(E) + i\Delta\Gamma(E)}$$

mass and width difference of a heavy meson with mass E

★ Thus, a dispersion relation

$$\boxed{\Delta m = -\frac{1}{2\pi} P \int_{2m_\pi}^{\infty} dE \left[\frac{\Delta\Gamma(E)}{E-m_D} + O\left(\frac{\Lambda_{QCD}}{E} \right) \right]}$$

Compute $\Delta\Gamma$, then find Δm!

No data: $SU(3)_F$ and phase space

★ "Repackage" the analysis: look at the complete multiplet contribution

$$y = \sum_{F_R} y_{F,R}\, Br(D^0 \to F_R) = \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma(D^0 \to n)$$

y for each SU(3) multiplet

Each is 0 in SU(3)

★ Does it help? If only phase space is taken into account: **mild** model dependence

$$y_{F,R} = \frac{\sum_{n\in F_R} \langle \overline{D}^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}{\sum_{n \in F_R} \Gamma(D^0 \to n)} = \frac{\sum_{n\in F_R} \langle \overline{D}^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}{\sum_{n\in F_R} \langle D^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}$$

Can consistently compute

Example: PP intermediate states

★ Consider PP intermediate state. Note that $(8\times8)_S = 27+8+1$. Look at 8 as an example

Numerator:

$$A_{N,8} = |A_0|^2 s_1^2 \left[\frac{1}{2}\Phi(\eta,\eta) + \frac{1}{2}\Phi(\pi^0,\pi^0) + \frac{1}{3}\Phi(\eta,\pi^0) + \Phi(\pi^+,\pi^-) - \Phi(\overline{K}^0,\pi^0)\right.$$
$$\left. + \Phi(K^+,K^-) - \frac{1}{6}\Phi(\eta,K^0) - \frac{1}{6}\Phi(\eta,\overline{K}^0) - \Phi(K^+,\pi^-) - \Phi(K^-,\pi^+)\right]$$

Denominator:

phase space function

$$A_{D,8} = |A_0|^2 \left[\frac{1}{6}\Phi(\eta,K^0) + \Phi(K^+,\pi^-) + \frac{1}{2}\Phi(K^0,\pi^0) + O(s_1^2)\right]$$

★ This contribution is calculable....

$$y_{2,8} = \frac{A_{N,8}}{A_{D,8}} = -0.038 s_1^2 = -1.8\times10^{-4}$$

1. Repeat for other states
2. Multiply by $\mathrm{Br}_{\mathrm{Fr}}$ to get y

.... but completely negligible!

Old results

★ Repeat for other intermediate states:

Final state representation		$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
PP	8	−0.0038	−0.018
	27	−0.00071	−0.0034
PV	8_S	0.031	0.15
	8_A	0.032	0.15
	10	0.020	0.10
	$\overline{10}$	0.016	0.08
	27	0.040	0.19
$(VV)_{s\text{-wave}}$	8	−0.081	−0.39
	27	−0.061	−0.30
$(VV)_{p\text{-wave}}$	8	−0.10	−0.48
	27	−0.14	−0.70
$(VV)_{d\text{-wave}}$	8	0.51	2.5
	27	0.57	2.8

Final state representation		$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
$(3P)_{s\text{-wave}}$	8	−0.48	−2.3
	27	−0.11	−0.54
$(3P)_{p\text{-wave}}$	8	−1.13	−5.5
	27	−0.07	−0.36
$(3P)_{\text{form-factor}}$	8	−0.44	−2.1
	27	−0.13	−0.64
$4P$	8	3.3	16
	27	2.2	9.2
	27′	1.9	11

- Product is naturally O(1%)
- No (symmetry-enforced) cancellations
- Disp relation: compute x (model-dependence)

Final state	fraction
PP	5%
PV	10%
$(VV)_{s\text{-wave}}$	5%
$(VV)_{d\text{-wave}}$	5%
$3P$	5%
$4P$	10%

naturally implies that x,y ~ 1% is expected in the Standard Model

A.F., Y.G., Z.L., Y.N. and A.A.P. Phys.Rev. D69, 114021, 2004

E.Golowich and A.A.P. Phys.Lett. B427, 172, 1998

Note dominance of near-threshold states!

Exclusive approach to mixing: use data!

★ What if we insist on using experimental data anyway?

★ Ex., one can employ Factorizaton-Assisted Topological Amplitudes

in units of 10^{-3}

Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)
$\pi^0\overline{K}^0$	24.0 ± 0.8	24.2 ± 0.8	$\pi^0\overline{K}^{*0}$	37.5 ± 2.9	35.9 ± 2.2	$\overline{K}^0\rho^0$	$12.8^{+1.4}_{-1.6}$	13.5 ± 1.4
π^+K^-	39.3 ± 0.4	39.2 ± 0.4	π^+K^{*-}	54.3 ± 4.4	62.5 ± 2.7	$K^-\rho^+$	111.0 ± 9.0	105.0 ± 5.2
$\eta\overline{K}^0$	9.70 ± 0.6	9.6 ± 0.6	$\eta\overline{K}^{*0}$	9.6 ± 3.0	6.1 ± 1.0	$\overline{K}^0\omega$	22.2 ± 1.2	22.3 ± 1.1
$\eta'\overline{K}^0$	19.0 ± 1.0	19.5 ± 1.0	$\eta'\overline{K}^{*0}$	<1.10	0.19 ± 0.01	$\overline{K}^0\phi$	$8.47^{+0.66}_{-0.34}$	8.2 ± 0.6
$\pi^+\pi^-$	1.421 ± 0.025	1.44 ± 0.02	$\pi^+\rho^-$	5.09 ± 0.34	4.5 ± 0.2	$\pi^-\rho^+$	10.0 ± 0.6	9.2 ± 0.3
K^+K^-	4.01 ± 0.07	4.05 ± 0.07	K^+K^{*-}	1.62 ± 0.15	1.8 ± 0.1	K^-K^{*+}	4.50 ± 0.30	4.3 ± 0.2
$K^0\overline{K}^0$	0.36 ± 0.08	0.29 ± 0.07	$K^0\overline{K}^{*0}$	0.18 ± 0.04	0.19 ± 0.03	$\overline{K}^0K^{*0}$	0.21 ± 0.04	0.19 ± 0.03
$\pi^0\eta$	0.69 ± 0.07	0.74 ± 0.03	$\eta\rho^0$		1.4 ± 0.2	$\pi^0\omega$	0.117 ± 0.035	0.10 ± 0.03
$\pi^0\eta'$	0.91 ± 0.14	1.08 ± 0.05	$\eta'\rho^0$		0.25 ± 0.01	$\pi^0\phi$	1.35 ± 0.10	1.4 ± 0.1
$\eta\eta$	1.70 ± 0.20	1.86 ± 0.06	$\eta\omega$	2.21 ± 0.23	2.0 ± 0.1	$\eta\phi$	0.14 ± 0.05	0.18 ± 0.04
$\eta\eta'$	1.07 ± 0.26	1.05 ± 0.08	$\eta'\omega$		0.044 ± 0.004			
$\pi^0\pi^0$	0.826 ± 0.035	0.78 ± 0.03	$\pi^0\rho^0$	3.82 ± 0.29	4.1 ± 0.2			
π^0K^0		0.069 ± 0.002	π^0K^{*0}		0.103 ± 0.006	$K^0\rho^0$		0.039 ± 0.004
π^-K^+	0.133 ± 0.009	0.133 ± 0.001	π^-K^{*+}	$0.345^{+0.180}_{-0.102}$	0.40 ± 0.02	$K^+\rho^-$		0.144 ± 0.009
ηK^0		0.027 ± 0.002	ηK^{*0}		0.017 ± 0.003	$K^0\omega$		0.064 ± 0.003
$\eta'K^0$		0.056 ± 0.003	$\eta'K^{*0}$		0.00055 ± 0.00004	$K^0\phi$		0.024 ± 0.002

Jiang, Yu, Qin, Li, and Lu, 2017

★ … but it appears to yield a smaller result, $y_{PP+PV} = (0.21\pm0.07)\%$.

Exclusive approach to mixing: use data!

★ What if we insist on using experimental data anyway?

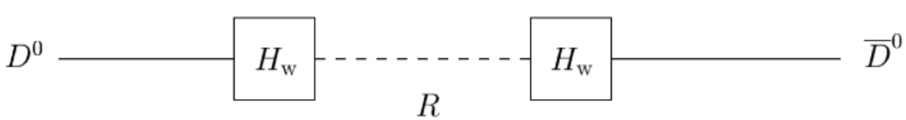
A.A.P. and R. Briere
arXiv:1804.xxxx

★ Possible additional contributions?

- each intermediate state has a finite width, i.e. is not a proper asymptotic state
- within each multiplet widths experience (incomplete) SU(3) cancelations
- this effect already happens for the simplest intermediate states!

★ Consider, for illustration, a set of single-particle intermediate states:

$$-\Sigma_{p_D}(p_D)\Big|^{\text{res}}_{\text{tot}} = \frac{1}{2m_D}\sum_R Re\, \frac{\langle D_L|\mathcal{H}_W|R\rangle\langle R|\mathcal{H}_W^\dagger|D_L\rangle}{m_D^2 - m_R^2 + i\Gamma_R m_D} \quad - \quad (D_L \to D_S)$$



$$\Delta m_D\Big|^{\text{res}}_{\text{R}} \propto \frac{m_D^2 - m_R^2}{(m_D^2 - m_R^2)^2 + \Gamma_R^2 m_D^2}$$

$$\Delta\Gamma_D\Big|^{\text{res}}_{\text{R}} \propto -\frac{\Gamma_R m_D}{(m_D^2 - m_R^2)^2 + \Gamma_R^2 m_D^2}$$

★ Each resonance contributes to $\Delta\Gamma$ only because of its finite width!

Finite width effects and exclusive approach

★ Multiplet effects for (single-particle) intermediate states
- in this simple example: heavy pion, kaon and eta/eta'
- each single-particle intermediate state has a rather large width

$$\Delta\Gamma_D\Big|_{\rm octet}^{\rm res} = \Delta\Gamma_D^{(K_H)} - \frac{1}{4}\Delta\Gamma_D^{(\pi_H)} - \frac{3\cos^2\theta_{\rm H}}{4}\Delta\Gamma_D^{(\eta_H)} - \frac{1\sin^2\theta_{\rm H}}{4}\Delta\Gamma_D^{(\eta'_H)}$$

- where for each state $\Delta\Gamma_D^{\rm res} = -Cf_R^2\,\dfrac{\mu_R\gamma_R}{(1-\mu_R)^2+\gamma_R^2}$
- … and a model calculation gives $C \equiv 2m_D(G_F a_2 f_D \xi_d/\sqrt{2})^2$,
- SU(3) forces cancellations between members: a new SU(3) breaking effect!

Table: Magnitudes of Pseudoscalar Resonance Contributions.

Resonance	$\lvert\Delta m_D\rvert \times 10^{-16}$ (GeV)	$\lvert\Delta\Gamma_D\rvert \times 10^{-16}$ (GeV)
$K(1460)$	$\sim 1.24\ (f_{K(1460)}/0.025)^2$	$\sim 0.88\ (f_{K(1460)}/0.025)^2$
$\eta(1760)$	$(0.77 \pm 0.27)\ (f_{\eta(1760)}/0.01)^2$	$(0.43 \pm 0.53)\ (f_{\eta(1760)}/0.01)^2$
$\pi(1800)$	$(0.13 \pm 0.06)\ (f_{\pi(1800)}/0.01)^2$	$(0.41 \pm 0.11)\ (f_{\pi(1800)}/0.01)^2$
$K(1830)$	$\sim 0.29\ (f_{K(1830)}/0.01)^2$	$\sim 1.86\ (f_{K(1830)}/0.01)^2$

★ Similar effect for PP', PV, PA, … intermediate states!

A.A.P. and R. Briere
arXiv:1804.xxxx

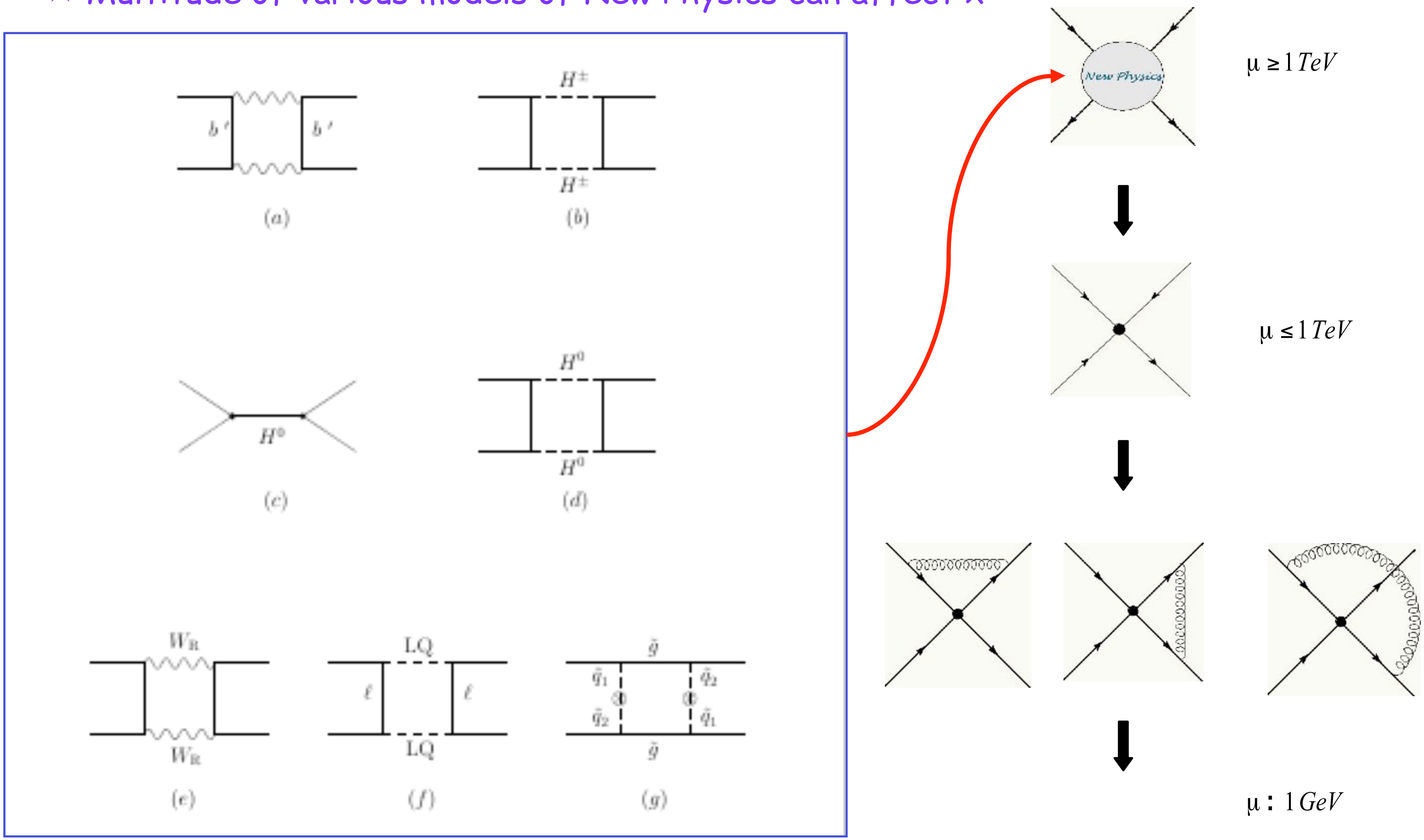
Finite width effects: (near) future

To counteract the effects of finite widths and avoid double counting, work directly with Dalitz plot decays of D-mesons

A.A.P. and R. Briere
arXiv:1804.xxxx

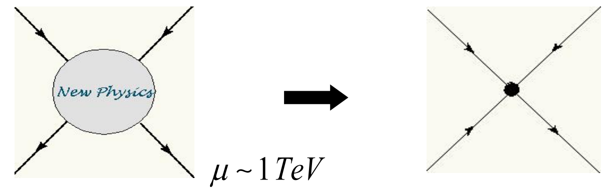
New Physics in charm mixing

★ Multitude of various models of New Physics can affect x



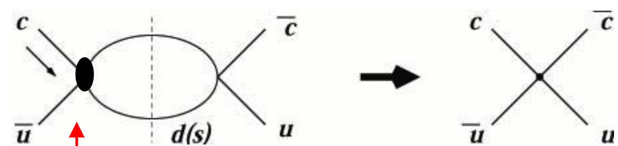
How would New Physics affect charm mixing?

- Local $\Delta C=2$ piece of the mass matrix affects x:



$$\left(M-\frac{i}{2}\Gamma\right)_{ij} = m_D^{(0)}\delta_{ij} + \boxed{\frac{1}{2m_D}\left\langle D_i^0 \left| H_W^{\Delta C=2} \right| D_j^0 \right\rangle} + \boxed{\frac{1}{2m_D}\sum_I \frac{\left\langle D_i^0 \left| H_W^{\Delta C=1} \right| I \right\rangle \left\langle I \left| H_W^{\Delta C=1} \right| D_j^0 \right\rangle}{m_D^2 - m_I^2 + i\varepsilon}}$$

- Double insertion of $\Delta C=1$ affects x and y:



Amplitude $A_n = \left\langle D^0 \left| \left(H_{SM}^{\Delta C=1} + H_{NP}^{\Delta C=1} \right) \right| n \right\rangle \equiv A_n^{SM} + A_n^{NP}$

Suppose $\left|A_n^{NP}\right| / \left|A_n^{SM}\right| \sim O(\text{exp. uncertainty}) \le 10\%$

Example: $$y = \frac{1}{2\Gamma}\sum_n \rho_n \left(\overline{A}_n^{SM} + \overline{A}_n^{NP}\right)\left(A_n^{SM} + A_n^{NP}\right) \approx \underbrace{\frac{1}{2\Gamma}\sum_n \rho_n \overline{A}_n^{SM} A_n^{SM}}_{\text{Zero in the SU(3) limit}} + \underbrace{\frac{1}{2\Gamma}\sum_n \rho_n \left(\overline{A}_n^{SM} A_n^{NP} + \overline{A}_n^{NP} A_n^{SM}\right)}_{\text{Can be significant!!!}}$$

phase space

Zero in the SU(3) limit

Falk, Grossman, Ligeti, and A.A.P.
Phys.Rev. D65, 054034, 2002
2nd order effect!!!

Can be significant!!!

Golowich, Pakvasa, A.A.P.
Phys. Rev. Lett.98:181801, 2007

Generic restrictions on NP from $D\bar{D}$-mixing

★ Comparing to experimental value of x, obtain constraints on NP models
- assume x is dominated by the New Physics model
- assume no accidental strong cancellations b/w SM and NP

$$\mathcal{H}_{NP}^{\Delta C=2} = \frac{1}{\Lambda_{NP}^2} \sum_{i=1}^{8} z_i(\mu) Q_i'$$

$$\begin{aligned} Q_1^{cu} &= \bar{u}_L^\alpha \gamma_\mu c_L^\alpha \bar{u}_L^\beta \gamma^\mu c_L^\beta, \\ Q_2^{cu} &= \bar{u}_R^\alpha c_L^\alpha \bar{u}_R^\beta c_L^\beta, \\ Q_3^{cu} &= \bar{u}_R^\alpha c_L^\beta \bar{u}_R^\beta c_L^\alpha, \end{aligned} + \left\{ \begin{matrix} L \\ \updownarrow \\ R \end{matrix} \right\} + \begin{aligned} Q_4^{cu} &= \bar{u}_R^\alpha c_L^\alpha \bar{u}_L^\beta c_R^\beta, \\ Q_5^{cu} &= \bar{u}_R^\alpha c_L^\beta \bar{u}_L^\beta c_R^\alpha, \end{aligned}$$

★ ... which are

$$\begin{aligned} |z_1| &\lesssim 5.7 \times 10^{-7} \left(\frac{\Lambda_{\rm NP}}{1\ TeV}\right)^2, \\ |z_2| &\lesssim 1.6 \times 10^{-7} \left(\frac{\Lambda_{\rm NP}}{1\ TeV}\right)^2, \\ |z_3| &\lesssim 5.8 \times 10^{-7} \left(\frac{\Lambda_{\rm NP}}{1\ TeV}\right)^2, \\ |z_4| &\lesssim 5.6 \times 10^{-8} \left(\frac{\Lambda_{\rm NP}}{1\ TeV}\right)^2, \\ |z_5| &\lesssim 1.6 \times 10^{-7} \left(\frac{\Lambda_{\rm NP}}{1\ TeV}\right)^2. \end{aligned}$$

New Physics is either at a very high scales

tree level: $\Lambda_{NP} \geq (4-10) \times 10^3$ TeV

loop level: $\Lambda_{NP} \geq (1-3) \times 10^2$ TeV

or have highly suppressed couplings to charm!

Gedalia, Grossman, Nir, Perez
Phys.Rev.D80, 055024, 2009

E.Golowich, J. Hewett, S. Pakvasa and A.A.P.
Phys. Rev. D76:095009, 2007

★ Constraints on particular NP models available

New Physics in mixing: particular models

Extra fermions

Extra gauge bosons

Extra scalars

Extra dimensions

SUSY

Model	Approximate Constraint
Fourth Generation (Fig. 2)	$\lvert V_{ub'}V_{cb'}\rvert \cdot m_{b'} < 0.5$ (GeV)
$Q = -1/3$ Singlet Quark (Fig. 4)	$s_2 \cdot m_S < 0.27$ (GeV)
$Q = +2/3$ Singlet Quark (Fig. 6)	$\lvert\lambda_{uc}\rvert < 2.4 \cdot 10^{-4}$
Little Higgs	Tree: See entry for $Q = -1/3$ Singlet Quark
	Box: Region of parameter space can reach observed x_D
Generic Z' (Fig. 7)	$M_{Z'}/C > 2.2 \cdot 10^3$ TeV
Family Symmetries (Fig. 8)	$m_1/f > 1.2 \cdot 10^3$ TeV (with $m_1/m_2 = 0.5$)
Left-Right Symmetric (Fig. 9)	No constraint
Alternate Left-Right Symmetric (Fig. 10)	$M_R > 1.2$ TeV ($m_{D_1} = 0.5$ TeV)
	$(\Delta m/m_{D_1})/M_R > 0.4$ TeV^{-1}
Vector Leptoquark Bosons (Fig. 11)	$M_{VLQ} > 55(\lambda_{PP}/0.1)$ TeV
Flavor Conserving Two-Higgs-Doublet (Fig. 13)	No constraint
Flavor Changing Neutral Higgs (Fig. 15)	$m_H/C > 2.4 \cdot 10^3$ TeV
FC Neutral Higgs (Cheng-Sher ansatz) (Fig. 16)	$m_H/\lvert\Delta_{uc}\rvert > 600$ GeV
Scalar Leptoquark Bosons	See entry for RPV SUSY
Higgsless (Fig. 17)	$M > 100$ TeV
Universal Extra Dimensions	No constraint
Split Fermion (Fig. 19)	$M/\lvert\Delta y\rvert > (6 \cdot 10^2$ GeV)
Warped Geometries (Fig. 21)	$M_1 > 3.5$ TeV
Minimal Supersymmetric Standard (Fig. 23)	$\lvert(\delta_{12}^u)_{\rm LR,RL}\rvert < 3.5 \cdot 10^{-2}$ for $\tilde{m} \sim 1$ TeV
	$\lvert(\delta_{12}^u)_{\rm LL,RR}\rvert < .25$ for $\tilde{m} \sim 1$ TeV
Supersymmetric Alignment	$\tilde{m} > 2$ TeV
Supersymmetry with RPV (Fig. 27)	$\lambda'_{12k}\lambda'_{11k}/m_{\tilde{d}_{R,k}} < 1.8 \cdot 10^{-3}/100$ GeV
Split Supersymmetry	No constraint

E.Golowich, J. Hewett, S. Pakvasa and A.A.P.
Phys. Rev. D76:095009, 2007

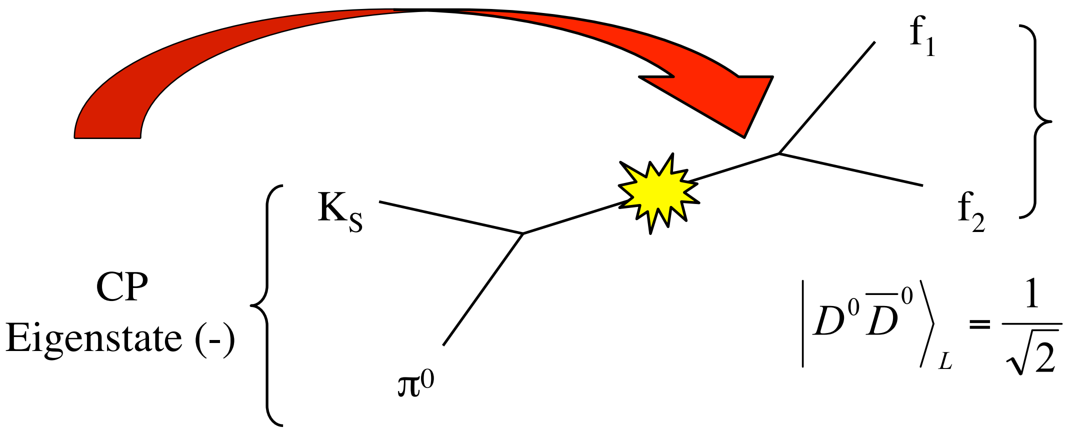
Gedalia, Grossman, Nir, Perez
arXiv:0906.1879 [hep-ph]

Bigi, Blanke, Buras, Recksiegel,
JHEP 0907:097, 2009

Measuring charm mixing with HIEPA

★ If CP violation is neglected: mass eigenstates = CP eigenstates

★ CP eigenstates do NOT evolve with time, so can be used for "tagging"



$$D_{CP(+)} \to f_1 f_2$$

$$\left| D^0 \overline{D}^0 \right\rangle_L = \frac{1}{\sqrt{2}} \left[\left| D^0(k_1) \overline{D}^0(k_2) \right\rangle + (-1)^L \left| D^0(k_2) \overline{D}^0(k_1) \right\rangle \right]$$

★ τ-charm factories have good CP-tagging capabilities

CP anti-correlated $\psi(3770)$: $CP(tag)\,(-1)^L = \underbrace{[CP(K_S)\,CP(\pi^0)]}_{(-)}\,(-1) = +1$

CP correlated $\psi(4140)$

Can measure ($y \cos \phi$): $B^l_{\pm} = \frac{\Gamma(D_{CP\pm} \to X l \nu)}{\Gamma_{tot}}$

$$y \cos \phi = \frac{1}{4} \left(\frac{B^l_+}{B^l_-} - \frac{B^l_-}{B^l_+} \right)$$

D. Atwood, A.A.P., hep-ph/0207165

D. Asner, W. Sun, hep-ph/0507238

No need for time dependence!

4. Things to take home

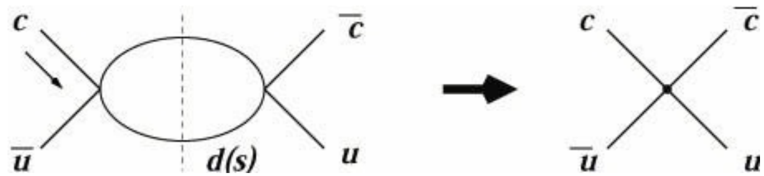
- Computation of charm mixing amplitudes is a difficult task
  - no dominant heavy dof, as in beauty decays
  - light dofs give no contribution in the flavor SU(3) limit
- Charm quark is neither heavy nor light enough for a clean application of well-established techniques
  - "heavy-quark-expansion" techniques miss threshold effects
  - "heavy-quark" techniques give numerically leading contribution that is parametrically suppressed by $1/m^6$
  - "hadronic" techniques need to sum over large number of intermediate states, AND cannot use current experimental data on D-decays
  - "hadronic" techniques currently neglect some sources of SU(3) breaking
- Finite width effects complicate use of experimental data in exclusive analyses to obtain mass and lifetime differences
  - instead, direct use of Dalitz decays of D-mesons is desirable
- Quantum-coherent initial states allow for unique measurements
  - lifetime differences, hadronic and CP-violating observables

Rob Gonzalves


EXTRA!

Mixing: short-distance estimates

★ SD calculation: expand the operator product in $1/m_c$, e.g.



E. Golowich and A.A.P.
Phys. Lett. B625 (2005) 53

★ Note that $1/m_c$ is not small, while factors of m_s make the result small

- keep $V_{ub} \neq 0$, so the leading SU(3)-breaking contribution is suppressed by $\lambda_b^2 \sim \lambda^{10}$
- ... but it is tiny, so look for SU(3)-breaking effects that come from mass insertions and quark condensates

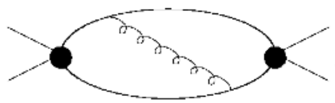
H. Georgi, …
I. Bigi, N. Uraltsev

M. Bobrowski et al
JHEP 1003 (2010) 009

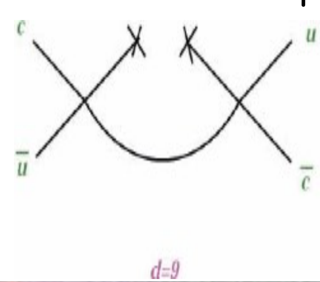
$$\Gamma_{12} = -\lambda_s^2 \left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}\right) + 2\lambda_s\lambda_b \left(\Gamma_{12}^{sd} - \Gamma_{12}^{dd}\right) - \lambda_b^2\, \Gamma_{12}^{dd}$$

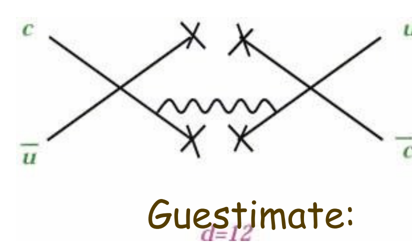
LO:	$O(m_s^4)$	$O(m_s^2)$	$O(1)$
NLO:	$O(m_s^3)$	$O(m_s^1)$	$O(1)$

- ... main contribution comes from dim-12 operators!!!



(c)





Guestimate: $x \sim y \sim 10^{-3}$?

Correlate rare decays with D-mixing?

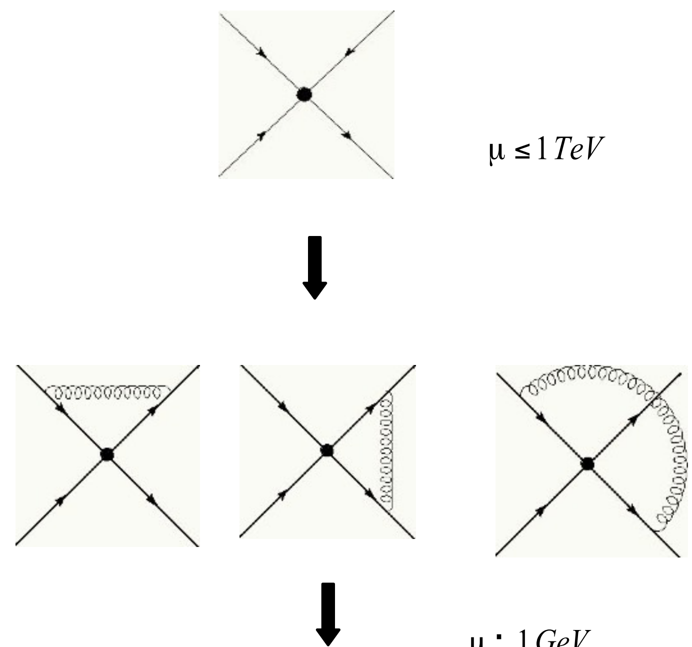
★ Let's write the most general ΔC=2 Hamiltonian

E.Golowich, J. Hewett, S. Pakvasa and A.A.P. (07)
Gedalia, Grossman, Nir, Perez (09)

$$\mathcal{H}_{NP}^{\Delta C=2} = \frac{1}{\Lambda_{NP}^2} \sum_{i=1}^{8} C_i(\mu) Q_i$$

… with the following set of 8 independent operators…

$$\begin{aligned}
Q_1 &= (\overline{u}_L \gamma_\mu c_L)\ (\overline{u}_L \gamma^\mu c_L)\ , & Q_5 &= (\overline{u}_R \sigma_{\mu\nu} c_L)\ (\overline{u}_R \sigma^{\mu\nu} c_L)\ , \\
Q_2 &= (\overline{u}_L \gamma_\mu c_L)\ (\overline{u}_R \gamma^\mu c_R)\ , & Q_6 &= (\overline{u}_R \gamma_\mu c_R)\ (\overline{u}_R \gamma^\mu c_R)\ , \\
Q_3 &= (\overline{u}_L c_R)\ (\overline{u}_R c_L)\ , & Q_7 &= (\overline{u}_L c_R)\ (\overline{u}_L c_R)\ , \\
Q_4 &= (\overline{u}_R c_L)\ (\overline{u}_R c_L)\ , & Q_8 &= (\overline{u}_L \sigma_{\mu\nu} c_R)\ (\overline{u}_L \sigma^{\mu\nu} c_R)\ .
\end{aligned}$$



RG-running relate $C_i(m)$ at NP scale to the scale of m ~ 1 GeV, where ME are computed (on the lattice)

$$\frac{d}{d\log\mu}\vec{C}(\mu) = \hat{\gamma}^T(\mu)\vec{C}(\mu)$$

Each model of New Physics provides unique matching condition for $C_i(\Lambda_{NP})$

★ Comparing to experimental value of x, obtain constraints on NP models

- assume x is dominated by the New Physics model
- assume no accidental strong cancellations b/w SM and NP