# Baryons in the light－front approach： the three－quark picture 

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arXiv：2304．xxxxx
In collaboration with Fu－Wei Zhang，Xiao－Hui Hu，Yu－Ji Shi

## Outline

Introduction

Framework and some applications

Numerical results

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Introduction

## Observation of CPV in charm decays

PHYSICAL REVIEW LETTERS 122, 211803 (2019)

## Editors' Suggestion

# Observation of CP Violation in Charm Decays 

R. Aaij et al. ${ }^{*}$<br>(LHCb Collaboration)


(Received 21 March 2019; revised manuscript received 2 May 2019; published 29 May 2019)
A search for charge-parity ( $C P$ ) violation in $D^{0} \rightarrow K^{-} K^{+}$and $D^{0} \rightarrow \pi^{-} \pi^{+}$decays is reported, using $p p$ collision data corresponding to an integrated luminosity of $5.9 \mathrm{fb}^{-1}$ collected at a center-of-mass energy of 13 TeV with the LHCb detector. The flavor of the charm meson is inferred from the charge of the pion in $D^{*}(2010)^{+} \rightarrow D^{0} \pi^{+}$decays or from the charge of the muon in $\bar{B} \rightarrow D^{0} \mu^{-} \bar{\nu}_{\mu} X$ decays. The difference between the $C P$ asymmetries in $D^{0} \rightarrow K^{-} K^{+}$and $D^{0} \rightarrow \pi^{-} \pi^{+}$decays is measured to be $\Delta A_{C P}=$ $\left[-18.2 \pm 3.2(\right.$ stat $) \pm 0.9($ syst) $] \times 10^{-4}$ for $\pi$-tagged and $\Delta A_{C P}=[-9 \pm 8($ stat $) \pm 5($ syst $)] \times 10^{-4}$ for $\mu$ tagged $D^{0}$ mesons. Combining these with previous LHCb results leads to $\Delta A_{C P}=(-15.4 \pm 2.9) \times 10^{-4}$, where the uncertainty includes both statistical and systematic contributions. The measured value differs from zero by more than 5 standard deviations. This is the first observation of $C P$ violation in the decay of charm hadrons.

DOI: 10.1103/PhysRevLett. 122.211803
> R. Aaij et al. [LHCb], Phys. Rev. Lett. 122, no.21, 211803 (2019) K. Abe et al. [Belle], Phys. Rev. Lett. 87, 091802 (2001)
> B. Aubert et al. [BaBar], Phys. Rev. Lett. 87, 091801 (2001)
> J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay, Phys. Rev. Lett. 13, 138-140 (1964)

## Observation of $\Xi_{c c}^{++}$

## |일 Selected for a Viewpoint in Physics

# Observation of the Doubly Charmed Baryon $\boldsymbol{\Xi}_{c c}^{++}$ 

$$
\text { R. Aaij et al. }{ }^{*}
$$

(LHCb Collaboration)
(Received 6 July 2017; revised manuscript received 2 August 2017; published 11 September 2017)
A highly significant structure is observed in the $\Lambda_{c}^{+} K^{-} \pi^{+} \pi^{+}$mass spectrum, where the $\Lambda_{c}^{+}$baryon is reconstructed in the decay mode $p K^{-} \pi^{+}$. The structure is consistent with originating from a weakly decaying particle, identified as the doubly charmed baryon $\Xi_{c c}^{++}$. The difference between the masses of the $\Xi_{c c}^{++}$and $\Lambda_{c}^{+}$states is measured to be $1334.94 \pm 0.72$ (stat.) $\pm 0.27$ (syst.) $\mathrm{MeV} / c^{2}$, and the $\Xi_{c c}^{++}$mass is then determined to be $3621.40 \pm 0.72$ (stat.) $\pm 0.27$ (syst.) $\pm 0.14\left(\Lambda_{c}^{+}\right) \mathrm{MeV} / c^{2}$, where the last uncertainty is due to the limited knowledge of the $\Lambda_{c}^{+}$mass. The state is observed in a sample of proton-proton collision data collected by the LHCb experiment at a center-of-mass energy of 13 TeV , corresponding to an integrated luminosity of $1.7 \mathrm{fb}^{-1}$, and confirmed in an additional sample of data collected at 8 TeV .

DOI: 10.1103/PhysRevLett.119.112001

$$
\Xi_{c c}^{++} \rightarrow \Lambda_{c}^{+} K^{-} \pi^{+} \pi^{+}
$$

```
Observation of the doubly charmed baryon }\mp@subsup{\Xi}{cc}{++
LHCb Collaboration • Roel Aaij (CERN) et al. (Jul 5, 2017)
Published in: Phys.Rev.Lett. }119\mathrm{ (2017) 11, 112001 • e-Print: 1707.01621 [hep-ex]
```

5

## Observation of $\Xi_{c c}^{++}$

Weak decays of doubly heavy baryons：the $1 / 2 \rightarrow 1 / 2$ case
Wei Wang（Shanghai Jiao Tong U．and Shanghai Jiaotong U．），Fu－Sheng Yu（Lanzhou U．），Zhen－Xing
Zhao（Shanghai Jiaotong U．and Shanghai Jiao Tong U．）（Jul 10，2017）
Published in：Eur．Phys．J．C 77 （2017）11， 781 • e－Print： 1707.02834 ［hep－ph］
目 pdf $\quad \rightarrow$ DOI $\quad$ cite $\quad$ 回 claim reference search $\rightarrow 134$ citations

## Discovery Potentials of Doubly Charmed Baryons

Fu－Sheng Yu（Lanzhou U．and Lanzhou，Inst．Modern Phys．），Hua－Yu Jiang（Lanzhou U．），Run－Hui
Li（Neimunggu U．），Cai－Dian Lü（Beijing，Inst．High Energy Phys．and Beijing，GUCAS），Wei Wang（Shanghai Jiao Tong U．and Shanghai Jiaotong U．）et al．（Mar 27，2017）
Published in：Chin．Phys．C 42 （2018）5， 051001 • e－Print： 1703.09086 ［hep－ph］
（4）pdf
（2）DOI
$\square$ cite
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$\rightleftharpoons$
125 citations
－－Test the standard model
－－Search for the origin of CP violation and new physics
－－Understand the strong interactions

## Light-front quark model-the diquark picture


H. W. Ke, X. Q. Li and Z. T. Wei, Phys. Rev. D 77, 014020 (2008)
H. W. Ke, X. H. Yuan, X. Q. Li, Z. T. Wei and Y. X. Zhang, Phys. Rev. D 86, 114005 (2012)

Some defects:
$-\Xi_{b c}(b c q)$,
for c decay, bq -- diquark
for b decay, cq -- diquark
-- more parameters, such as $m_{d i}^{--} m_{[u d]}$ and $m_{\{u d\}}$

## Light-front quark model-the diquark picture

A Lorentz boost between $p_{2}$ and $\bar{P}$

$$
\Gamma=-\frac{1}{\sqrt{3}} \gamma_{5} \xi^{*}\left(p_{2}, \lambda_{2}\right),
$$


H. W. Ke, X. H. Yuan, X. Q. Li, Z. T. Wei and Y. X. Zhang, Phys. Rev. D 86, 114005 (2012)

$$
\begin{aligned}
\Gamma & =\frac{1}{\sqrt{3}} \gamma_{5} \not^{*}\left(\bar{P}, \lambda_{2}\right) \\
& =\frac{1}{\sqrt{3}} \gamma_{5}\left(\not^{*}\left(p_{2}, \lambda_{2}\right)-\frac{M_{0}+m_{1}+m_{2}}{p_{2} \cdot \bar{P}+m_{2} M_{0}} \epsilon^{*}\left(p_{2}, \lambda_{2}\right) \cdot \bar{P}\right),
\end{aligned}
$$

Chun-Khiang Chua, Phys. Rev. D 99, 014023 (2019)

## Light-front quark model - the three-quark picture

S. Tawfiq, P. J. O’Donnell, and J. G. Körner, Phys. Rev. D 58, 054010 (1998)

H.-W. Ke, N. Hao, and X.-Q. Li, Eur. Phys. J. C 79, 540 (2019)<br>C.-Q. Geng, C.-W. Liu, and T.-H. Tsai, Phys.Lett.B 815, 136125 (2021)


-- lack a proof of spin wavefunctions
-- shape parameters cannot be well determined
-- relationship between the diquark picture and the three-quark picture

## Framework and some applications

## The baryon state

$$
\begin{aligned}
\left|\mathcal{B}\left(P, S, S_{z}\right)\right\rangle= & \int\left\{d^{3} \tilde{p}_{1}\right\}\left\{d^{3} \tilde{p}_{2}\right\}\left\{d^{3} \tilde{p}_{3}\right\} 2(2 \pi)^{3} \delta^{3}\left(\tilde{P}-\tilde{p}_{1}-\tilde{p}_{2}-\tilde{p}_{3}\right) \frac{1}{\sqrt{P^{+}}} \\
\times & \sum_{\lambda_{1}, \lambda_{2}, \lambda_{3}} \Psi^{S S_{z}}\left(\tilde{p}_{1}, \tilde{p}_{2}, \tilde{p}_{3}, \lambda_{1}, \lambda_{2}, \lambda_{3}\right) C^{i j k}\left|q_{1}^{i}\left(p_{1}, \lambda_{1}\right) q_{2}^{j}\left(p_{2}, \lambda_{2}\right) q_{3}^{k}\left(p_{3}, \lambda_{3}\right)\right\rangle,
\end{aligned}
$$

$$
\Lambda_{Q} \quad A_{0} \bar{u}\left(p_{3}, \lambda_{3}\right)\left(\bar{P}+M_{0}\right)\left(-\gamma_{5}\right) C \bar{u}^{T}\left(p_{2}, \lambda_{2}\right) \bar{u}\left(p_{1}, \lambda_{1}\right) u\left(\bar{P}, S_{z}\right) \Phi\left(x_{i}, k_{i \perp}\right),
$$

$$
\Sigma_{Q} \quad A_{1} \bar{u}\left(p_{3}, \lambda_{3}\right)\left(\bar{P}+M_{0}\right)\left(\gamma^{\mu}-v^{\mu}\right) C \bar{u}^{T}\left(p_{2}, \lambda_{2}\right) \bar{u}\left(p_{1}, \lambda_{1}\right)\left(\frac{1}{\sqrt{3}} \gamma_{\mu} \gamma_{5}\right) u\left(\bar{P}, S_{z}\right) \Phi\left(x_{i}, k_{i \perp}\right),
$$

$$
\Sigma_{Q}^{*} \quad A_{1}^{\prime} \bar{u}\left(p_{3}, \lambda_{3}\right)\left(\bar{P}+M_{0}\right)\left(\gamma^{\mu}-v^{\mu}\right) C \bar{u}^{T}\left(p_{2}, \lambda_{2}\right) \bar{u}\left(p_{1}, \lambda_{1}\right) u_{\mu}\left(\bar{P}, S_{z}\right) \Phi\left(x_{i}, k_{i \perp}\right),
$$

(udQ)
Three different flavors

$$
\left(\frac{1}{2} \otimes \frac{1}{2}\right) \otimes \frac{1}{2}=(0 \oplus 1) \otimes \frac{1}{2}=\frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2} .
$$

## Spin wavefunction

Take $\Sigma_{Q}(u d Q)$ as an example where ud are considered as an axial-vector diquark

Step 1: In the rest frame of quark 2 and 3 - "diquark"

$$
I^{\mu} \equiv \bar{u}\left(p_{3}, s_{3}\right) \frac{\left(\bar{P}+M_{0}\right)}{2 M_{0}} \gamma_{\perp}^{\mu}\left(p_{23}\right)(-C) \bar{u}^{T}\left(p_{2}, s_{2}\right)
$$

$$
\begin{aligned}
\gamma_{\perp}^{\mu}\left(p_{23}\right) & =\gamma_{\perp}^{\mu}(\bar{P})-\frac{M_{0} p_{23}^{\mu}+m_{23} \bar{P}^{\mu}}{m_{23} M_{0}} \frac{\gamma_{\perp}(\bar{P}) \cdot p_{23}}{e_{23}+m_{23}}, \\
p_{23} & =p_{2}+p_{3}, \quad m_{23}^{2}=p_{23}^{2}, \\
\gamma_{\perp}^{\mu}(\bar{P}) & =\gamma^{\mu}-\not \psi v^{\mu}, \quad v^{\mu}=\bar{P}^{\mu} / M_{0} .
\end{aligned}
$$

$$
I^{\mu} \sim\left\langle\frac{1}{2} \frac{1}{2} ; s_{3} s_{2} \left\lvert\, \frac{1}{2} \frac{1}{2}\right. ; 1, s_{23}\right\rangle \epsilon^{* \mu}\left(p_{23}, s_{23}\right) .
$$

## Spin wavefunction

Step 2: Couple the "diquark" to quark 1

$$
\begin{gathered}
T \equiv I^{\mu} \cdot \bar{u}\left(p_{1}, s_{1}\right) \Gamma_{1,23 \mu} u\left(\bar{P}, S_{z}\right) \\
\Gamma_{1,23 \mu}=\frac{\gamma_{5}}{\sqrt{3}}\left(\gamma_{\mu}-\frac{M_{0}+m_{1}+m_{23}}{M_{0}\left(e_{23}+m_{23}\right)} \bar{P}_{\mu}\right) . \\
T \sim\left\langle\frac{1}{2} \frac{1}{2} ; s_{3} s_{2} \left\lvert\, \frac{1}{2} \frac{1}{2}\right. ; 1 s_{23}\right\rangle\left\langle\frac{1}{2} 1 ; s_{1} s_{23} \left\lvert\, \frac{1}{2} 1\right. ; \frac{1}{2} S_{z}\right\rangle .
\end{gathered}
$$

Step 3: Tensor simplification

$$
\begin{aligned}
& \bar{u}\left(p_{3}, \lambda_{3}\right)\left(\bar{P}+M_{0}\right) \gamma_{\perp}^{\mu}\left(p_{23}\right)(-C) \bar{u}^{T}\left(p_{2}, \lambda_{2}\right) \bar{u}\left(p_{1}, \lambda_{1}\right) \underline{\Gamma_{1,23 \mu} u\left(\bar{P}, S_{z}\right)} \\
= & \cdots \\
= & \bar{u}\left(p_{3}, \lambda_{3}\right)\left(\bar{P}+M_{0}\right)\left(\gamma^{\mu}-v^{\mu}\right) C \bar{u}^{T}\left(p_{2}, \lambda_{2}\right) \bar{u}\left(p_{1}, \lambda_{1}\right)\left(\frac{1}{\sqrt{3}} \gamma_{\mu} \gamma_{5}\right) u\left(\bar{P}, S_{z}\right)
\end{aligned}
$$

Same method can be applied to multi-quark states!

## Momentum wavefunction

$$
\begin{gathered}
\Phi\left(x_{1}, x_{2}, x_{3}, k_{1 \perp}, k_{2 \perp}, k_{3 \perp}\right)=\sqrt{\frac{e_{1} e_{2} e_{3}}{x_{1} x_{2} x_{3} M_{0}}} \varphi\left(\vec{k}_{1}, \beta_{1}\right) \varphi\left(\frac{\vec{k}_{2}-\vec{k}_{3}}{2}, \beta_{23}\right) \\
\varphi(\vec{k}, \beta) \equiv 4\left(\frac{\pi}{\beta^{2}}\right)^{3 / 4} \exp \left(\frac{-k_{\perp}^{2}-k_{z}^{2}}{2 \beta^{2}}\right)
\end{gathered}
$$

shape parameters
$\int\left(\prod_{i=1}^{3} \frac{d x_{i} d^{2} k_{i \perp}}{2(2 \pi)^{3}}\right) 2(2 \pi)^{3} \delta\left(1-\sum x_{i}\right) \delta^{2}\left(\sum k_{i \perp}\right)\left|\Phi\left(x_{i}, k_{i \perp}\right)\right|^{2}=1$


## To determine the shape parameters

Take $\Lambda_{Q}$ as an example

$$
\langle 0| J_{\Lambda_{Q}}\left|\Lambda_{Q}\left(P, S_{z}\right)\right\rangle
$$

Step 1: Calculate it in LFQM
Step 2: Use the definition of

$$
\langle 0| J_{\Lambda_{Q}}\left|\Lambda_{Q}\left(P, S_{z}\right)\right\rangle=\lambda_{\Lambda_{Q}} u\left(P, S_{z}\right)
$$

Step 3: Extract the pole residue

M can be extracted!

$$
\begin{array}{rlrl}
\lambda_{\Lambda_{Q}}= & \int \frac{d x_{2} d^{2} k_{2 \perp}}{2(2 \pi)^{3}} \frac{d x_{3} d^{2} k_{3 \perp}}{2(2 \pi)^{3}} \frac{1}{\sqrt{x_{1} x_{2} x_{3}}} \Phi\left(x_{i}, k_{i \perp}\right) \sqrt{6} A_{0} & & \text { Here } \\
& \times \frac{\operatorname{Tr}[\ldots] \operatorname{Tr}\left[\gamma^{+}\left(\not p_{1}+m_{1}\right)\left(\nmid P+M_{0}\right)\right]}{\operatorname{Tr}\left[\gamma^{+}(\not P+M)\right]}, & \text { includes } \beta_{1} \\
\lambda_{\Lambda_{Q}}= & \text { the above equation with } \gamma^{+} \rightarrow \gamma^{+} \gamma^{-} & \text {and } \beta_{23}
\end{array}
$$

$$
\operatorname{Tr}[\ldots]=\operatorname{Tr}\left[C \gamma_{5}\left(\not p_{3}+m_{3}\right)\left(\bar{P}+M_{0}\right)\left(-\gamma_{5}\right) C\left(\not p_{2}+m_{2}\right)^{T}\right]
$$

## Form factors

## $\Lambda_{b} \rightarrow \Lambda_{c}$ Form factors

Step 1: Calculate the matrix elements in LFQM

## Step 2: Write the matrix elements in terms of form factors

$$
\begin{gathered}
\left\langle\Lambda_{c}\left(P^{\prime}, S_{z}^{\prime}\right)\right| \overline{q^{\mu}}{ }^{\mu}\left|\Lambda_{b}\left(P, S_{z}\right)\right\rangle=\bar{u}\left(P^{\prime}, S_{z}^{\prime}\right)\left[\gamma^{\mu} f_{1}\left(q^{2}\right)+i \sigma^{\mu \nu} \frac{q_{\nu}}{M} f_{2}\left(q^{2}\right)+\frac{q^{\mu}}{M} f_{3}\left(q^{2}\right)\right] u\left(P, S_{z}\right), \\
\left\langle\Lambda_{c}\left(P^{\prime}, S_{z}^{\prime}\right)\right| \overline{\gamma^{\mu}} \gamma_{5} b\left|\Lambda_{b}\left(P, S_{z}\right\rangle\right\rangle=\bar{u}\left(P^{\prime}, S_{z}^{\prime}\right)\left[\gamma^{\mu} g_{1}\left(q^{2}\right)+i \sigma^{\mu \nu} \frac{q_{\nu}}{M} g_{2}\left(q^{2}\right)+\frac{q^{\mu}}{M} g_{3}\left(q^{2}\right)\right] \gamma_{5} u\left(P, S_{z}\right),
\end{gathered}
$$

Step 3: Extract the form factors

$$
\begin{aligned}
f_{1}= & \frac{1}{8 P^{+} P^{\prime+}} \int\left\{d^{3} \tilde{p}_{2}\right\}\left\{d^{3} \tilde{p}_{3}\right\} \frac{\Phi^{\prime *} \Phi}{\sqrt{P^{+} P^{\prime+} p_{1}^{+} p_{1}^{\prime+}}} A_{0}^{\prime} A_{0} \operatorname{Tr}[\ldots] \\
& \times \operatorname{Tr}\left[\left(\overline{\not P}+M_{0}\right) \gamma^{+}\left(\bar{p}^{\prime}+M_{0}^{\prime}\right)\left(\not p_{1}^{\prime}+m_{1}^{\prime}\right) \gamma^{+}\left(\not p_{1}+m_{1}\right)\right], \\
\operatorname{Tr}[\ldots]= & \operatorname{Tr}\left[\left(\bar{P}+M_{0}\right)\left(-\gamma_{5}\right) C\left(\not p_{2}+m_{2}\right)^{T} C \gamma_{5}\left(\bar{p}^{\prime}+M_{0}^{\prime}\right)\left(\not p_{3}+m_{3}\right)\right]
\end{aligned}
$$

f2, g1, g2 can also be obtained in a similar way

$$
\Sigma_{b} \rightarrow \Sigma_{c}, \Xi_{c c} \rightarrow \Lambda_{c} \text { Form factors }
$$

## The relationship between the two pictures

$$
\begin{aligned}
& \psi_{0}(321) \equiv \bar{u}\left(\underline { p _ { 3 } , \lambda _ { 3 } ) } ( \overline { \not P } + M _ { 0 } ) ( - \gamma _ { 5 } ) C \overline { u } ^ { T } \left(\underline{\left.p_{2}, \lambda_{2}\right)} \bar{u}\left(p_{1}, \lambda_{1}\right) u\left(\bar{P}, S_{z}\right)\right.\right. \\
& \psi_{1}(321) \equiv \bar{u}\left(\underline{p_{3}, \lambda_{3}}\right)\left(\overline{\not P}+M_{0}\right)\left(\gamma^{\mu}-v^{\mu}\right) C \bar{u}^{T}\left(p_{2}, \lambda_{2}\right) \bar{u}\left(p_{1}, \lambda_{1}\right)\left(\frac{1}{\sqrt{3}} \gamma_{\mu} \gamma_{5}\right) u\left(\bar{P}, S_{z}\right)
\end{aligned}
$$

Quark 3 and 2 form a diquark $\left\{\begin{array}{l}\psi_{0}(321)=-\psi_{0}(231), \\ \psi_{1}(321)=\psi_{1}(231)\end{array}\right.$
A diquark bases $\left\{\begin{array}{l}\text { normalizatio } \\ \text { orthogonal }\end{array}\right.$

$$
\begin{aligned}
& \frac{1}{4 \sqrt{M_{0}^{3}\left(e_{1}+m_{1}\right)\left(e_{2}+m_{2}\right)\left(e_{3}+m_{3}\right)}} \\
& \sum_{\lambda_{1} \lambda_{2} \lambda_{3}} \psi_{0}^{\dagger}\left(321^{\prime}\right) \psi_{1}(321)=0
\end{aligned}
$$

$\operatorname{T}$ matirx $\quad\binom{\psi_{0}(312)}{\psi_{1}(312)}=T\binom{\psi_{0}(321)}{\psi_{1}(321)} \quad T=\left(\begin{array}{cc}\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{array}\right)$

$$
T^{-1}=T
$$

## The relationship between the two pictures

1. Calculate the overlap factor of $\Xi_{b c}^{+}(c b u) \rightarrow \Lambda_{b}(d b u)$

$$
\begin{aligned}
& \psi_{1}(b c u)=-\frac{\sqrt{3}}{2} \psi_{0}(b u c)-\frac{1}{2} \psi_{1}(b u c), \\
& \psi_{0}(u d b)=\frac{1}{2} \psi_{0}(u b d)-\frac{\sqrt{3}}{2} \psi_{1}(u b d) \\
&=-\frac{1}{2} \psi_{0}(b u d)-\frac{\sqrt{3}}{2} \psi_{1}(b u d) \\
&\left\langle\psi_{0}(u d b) \mid \psi_{1}(b c u)\right\rangle=\frac{\sqrt{3}}{4}\left\langle\psi_{0}(b u d) \mid \psi_{0}(b u c)\right\rangle+\frac{\sqrt{3}}{4}\left\langle\psi_{1}(b u d) \mid \psi_{1}(b u c)\right\rangle,
\end{aligned}
$$

2. Calculate the overlap factor of $\Xi_{c c}^{++}(c c u) \rightarrow \Lambda_{c}(d c u)$

$$
\frac{2}{\sqrt{2}}\left\{\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{4}\right\}=\left\{\frac{\sqrt{6}}{4}, \frac{\sqrt{6}}{4}\right\}
$$

W. Wang, F.-S. Yu, and Z.-X. Zhao, Eur. Phys. J. C 77, 781 (2017)

## Possibly better definitions of interpolating currents

Hermite conjugate $\psi_{0,1}$

$$
\begin{aligned}
& J_{\Lambda_{Q}}^{\mathrm{new}}=\epsilon_{a b c}\left[u_{a}^{T} C \gamma_{5}(1+\psi) d_{b}\right] Q_{c}, \\
& J_{\Sigma_{Q}}^{\mathrm{new}}=\epsilon_{a b c}\left[u_{a}^{T} C\left(\gamma^{\mu}-v^{\mu}\right)(1+\psi) d_{b}\right] \frac{1}{\sqrt{3}} \gamma_{\mu} \gamma_{5} Q_{c} v^{\mu} \equiv p^{\mu} / \sqrt{p^{2}}
\end{aligned}
$$

Traditional definitions

$$
\begin{aligned}
& J_{\Lambda_{Q}}=\epsilon_{a b c}\left[u_{a}^{T} C \gamma_{5} d_{b}\right] Q_{c}, \\
& J_{\Sigma_{Q}}=\epsilon_{a b c}\left[u_{a}^{T} C \gamma^{\mu} d_{b}\right] \gamma_{\mu} \gamma_{5} Q_{c}
\end{aligned}
$$

Comments:

1. The factor $1 / \sqrt{3}$

$$
\lambda_{\Sigma_{Q}} \approx 2 \lambda_{\Lambda_{Q}}
$$

2. Let $v \rightarrow 0$ in $J^{\text {new }} \rightarrow J \quad--$ in fact, we cannot do that.

## Numerical results

## Inputs and shape parameters

Inputs $\quad m_{u}=m_{d}=0.25 \mathrm{GeV}, \quad m_{c}=1.4 \mathrm{GeV}, \quad m_{b}=4.8 \mathrm{GeV}$.

$$
\begin{aligned}
& \lambda_{\Lambda_{b}}=0.030 \pm 0.009, \quad \lambda_{\Lambda_{c}}=0.022 \pm 0.008, \\
& \lambda_{\Sigma_{b}}=0.062 \pm 0.018, \quad \lambda_{\Sigma_{c}}=0.045 \pm 0.015, \\
& \lambda_{\Xi_{c c}}=0.115 \pm 0.027 \text {. } \\
& \text { Z.-G. Wang, Eur. Phys. J. C 68, } 479 \text { (2010) } \\
& \text { Z.-G. Wang, Phys. Lett. B 685, } 59 \text { (2010) } \\
& \text { Z.-G. Wang, Eur. Phys. J. A 45, } 267 \text { (2010) } \\
& \Lambda_{b} \rightarrow \Lambda_{c} \quad \beta_{b,[u d]}=0.63 \pm 0.05 \mathrm{GeV}, \quad \beta_{[u d]}=0.27 \pm 0.03 \mathrm{GeV}, \\
& \beta_{c,[u d]}=0.45 \pm 0.05 \mathrm{GeV} \text {; } \\
& \Sigma_{b} \rightarrow \Sigma_{c} \quad \beta_{b,\{u d\}}=0.66 \pm 0.04 \mathrm{GeV}, \quad \beta_{\{u d\}}=0.28 \pm 0.03 \mathrm{GeV}, \\
& \beta_{c,\{u d\}}=0.49 \pm 0.04 \mathrm{GeV} \text {; } \\
& \Xi_{c c} \rightarrow \Lambda_{c} \quad \beta_{u,\{c c\}}=0.490 \pm 0.040 \mathrm{GeV}, \quad \beta_{\{c c\}}=0.400 \pm 0.025 \mathrm{GeV} . \\
& \lambda_{1} \approx \lambda_{2} \approx \lambda_{\mathrm{QCDSR}}
\end{aligned}
$$

## Form factors and comparison

TABLE II: Our form factors are compared with other results in the literature. The asterisk on Ref. [Shi19a] indicates that, in this literature, we made a mistake in the calculation of the axial-vector form factors, which led us to get the wrong symbol, and here we have corrected it.

| $\Lambda_{b} \rightarrow \Lambda_{c}$ | $f_{1}(0)$ | $f_{2}(0)$ | $f_{3}(0)$ | $g_{1}(0)$ | $g_{2}(0)$ | $g_{3}(0)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| This work | $0.469 \pm 0.029$ | $-0.105 \pm 0.011$ | - | $0.461 \pm 0.027$ | $0.006 \pm 0.005$ | - |
| Three-quark picture [Ke19] | 0.488 | -0.180 | - | 0.470 | -0.048 | - |
| Diquark picture [Zhao18] | 0.670 | -0.132 | - | 0.656 | -0.012 | - |
| Diquark picture [Ke07] | 0.506 | -0.099 | - | 0.501 | -0.009 | - |
| QCD sum rules [Zhao20] | 0.431 | -0.123 | 0.022 | 0.434 | 0.036 | -0.160 |
| Lattice QCD [Detmold15] | 0.418 | -0.099 | -0.075 | 0.390 | -0.004 | -0.206 |
| $\Sigma_{b} \rightarrow \Sigma_{c}$ | $f_{1}(0)$ | $f_{2}(0)$ | $f_{3}(0)$ | $g_{1}(0)$ | $g_{2}(0)$ | $g_{3}(0)$ |
| This work | $0.490 \pm 0.018$ | $0.467 \pm 0.006$ | - | $-0.163 \pm 0.005$ | $0.007 \pm 0.001$ | - |
| Three-quark picture [Ke19] | 0.494 | 0.407 | - | -0.156 | -0.0529 | - |
| Diquark picture [Ke12] | 0.466 | 0.736 | - | -0.130 | -0.0898 | - |
| $\Xi_{c c} \rightarrow \Lambda_{c}$ | $f_{1}(0)$ | $f_{2}(0)$ | $f_{3}(0)$ | $g_{1}(0)$ | $g_{2}(0)$ | $g_{3}(0)$ |
| This work | $0.517 \pm 0.071$ | $-0.036 \pm 0.007$ | - | $0.155 \pm 0.019$ | $-0.072 \pm 0.012$ | - |
| Diquark picture [Wang17] | 0.790 | -0.008 | - | 0.224 | -0.050 | - |
| QCD sum rules [Shi19a] | 0.63 | -0.05 | -0.81 | 0.24 | -0.11 | -0.84 |
| Light-cone sum rules [Shi19b] | $0.81 \pm 0.01$ | $0.32 \pm 0.01$ | $-0.90 \pm 0.07$ | $1.09 \pm 0.02$ | $-0.86 \pm 0.02$ | $0.76 \pm 0.01$ |
| NRQM [Perez-Marcial89] | 0.36 | 0.14 | 0.08 | 0.20 | 0.01 | -0.03 |
| MBM [Perez-Marcial89] | 0.45 | 0.01 | -0.28 | 0.15 | 0.01 | -0.70 |

## Diquark picture

| $\beta_{u[c q]}$ | $\beta_{d[c q]}$ | $\beta_{s[c q]}$ | $\beta_{c[c q]}$ | $\beta_{b[c q]}$ | $\beta_{u[b q]}$ | $\beta_{d[b q]}$ | $\beta_{s[b q]}$ | $\beta_{c[b q]}$ | $\beta_{b[b q]}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.470 | 0.470 | 0.535 | 0.753 | 0.886 | 0.562 | 0.562 | 0.623 | 0.886 | 1.472 |

## Decay widths and comparison

TABLE III: Our decay widths (in units of $10^{-14} \mathrm{GeV}$ ) are compared with other results in the literature.

| $\Lambda_{b} \rightarrow \Lambda_{c} e^{-} \bar{\nu}_{e}$ | Decay width | $\Sigma_{b} \rightarrow \Sigma_{c} e^{-} \bar{\nu}_{e}$ | Decay width | $\Xi_{c c} \rightarrow \Lambda_{c} e^{+} \nu_{e}$ | Decay width |
| :---: | :---: | :---: | :---: | :---: | :---: |
| This work | 2.54 | This work | 0.870 | This work | 0.755 |
| Three-quark picture [Ke19] | 2.78 | Three-quark picture [Ke19] | 1.03 | Diquark picture [Wang17] | 1.05 |
| Diquark picture [Zhao18] | 3.96 | Diquark picture [Ke07] | 0.908 | QCD sum rules [Shi19a] ${ }^{*}$ | $0.76 \pm 0.37$ |
| Diquark picture [Ke07] | 3.39 |  |  | Light-cone sum rules [Shi19b] | $3.95 \pm 0.21$ |
| QCD sum rules [Zhao20] | $2.96 \pm 0.48$ |  |  |  |  |
| Lattice QCD [Detmold15] | $2.35 \pm 0.15$ |  |  |  |  |

## Summary and outlook

## Summary

- A three-quark picture in LFQM is built up using the quark spinors and Dirac matrices
- The shape parameters are determined with the help of pole residue
- The relationship between the diquark picture and the three-quark picture is figured out


A small flaw?

```
\lambda
```


## Outlook

- Various applications
- Lorentz boost effect plays an important role in the model building -- Multiquark states?
- Interpolating currents of baryons

Thank you for your attention!

