

Dear Prof. Shen,

Thanks for your questions and comments.

Question 1,2,3:

In fact, they are the same questions caused by typos and the real results in draft are calculated rightly. Sorry for these mistakes.

Question 1,



(5)

chengping
you are calculating Br(jpsi -> eta' K*0 K0), why did you divide by charged K* decay branching fraction ?

$$\begin{aligned}
 & Br(J/\psi \rightarrow \eta' K^*(892)^0 \bar{K}^0 + c.c.) \\
 &= \frac{N_{sig}}{N_{J/\psi} \times Br(\eta' \rightarrow \pi^+ \pi^- \eta) \times Br(\eta \rightarrow \gamma\gamma) \times Br(K^{*\pm} \rightarrow K^0 \pi^\pm) \times Br(K^0 \rightarrow K_S^0) \times Br(K_S^0 \rightarrow \pi^+ \pi^-) \times \epsilon} \\
 &= \frac{8268 \pm 137}{1310.6 \times 10^6 \times 42.9\% \times 39.41\% \times 66.66\% \times 50.0\% \times 0.692\% \times 9.94\%} \\
 &= (1.66 \pm 0.03^{stat.}) \times 10^{-3}
 \end{aligned}$$

After revised, the formula should be:

$$\begin{aligned}
 & Br(J/\psi \rightarrow \eta' K^*(892)^0 \bar{K}^0 + c.c.) \\
 &= \frac{N_{K^*(892)^0}}{N_{J/\psi} \times Br(\eta' \rightarrow \pi^+ \pi^- \eta) \times Br(\eta \rightarrow \gamma\gamma) \times Br(K^*(892)^0 \rightarrow K^+ \pi^-) \times Br(\bar{K}^0 \rightarrow K_S^0) \times Br(K_S^0 \rightarrow \pi^+ \pi^-) \times \epsilon} \\
 &+ \frac{N_{\bar{K}^*(892)^0}}{N_{J/\psi} \times Br(\eta' \rightarrow \pi^+ \pi^- \eta) \times Br(\eta \rightarrow \gamma\gamma) \times Br(\bar{K}^*(892)^0 \rightarrow K^- \pi^+) \times Br(K^0 \rightarrow K_S^0) \times Br(K_S^0 \rightarrow \pi^+ \pi^-) \times \epsilon} \\
 &= \frac{8268 \pm 137}{1310.6 \times 10^6 \times 42.9\% \times 39.41\% \times 66.6\% \times 50.0\% \times 0.692\% \times 9.94\%} \\
 &= (1.66 \pm 0.03^{stat.}) \times 10^{-3}
 \end{aligned}$$

Here, the

$$Br(\bar{K}^0 \rightarrow K_S^0) \times Br(K_S^0 \rightarrow \pi^+ \pi^-) = Br(K^0 \rightarrow K_S^0) \times Br(K_S^0 \rightarrow \pi^+ \pi^-) = 66.6\% \times 50.0\%$$

and

$$N_{K^*(892)^0} + N_{\bar{K}^*(892)^0} = 8268 \pm 137$$

Question 2,3

$$\begin{aligned} & \mathcal{B}(J/\psi \rightarrow \eta' h_1(1380)) \times [\mathcal{B}(h_1(1380) \rightarrow K^*(892)^+ K^- + c.c.) + \mathcal{B}(h_1(1380) \rightarrow K^*(892)^0 \bar{K}^0 + c.c.)] \\ &= \frac{N_{sig}}{N_{J/\psi} \times Br(\eta' \rightarrow \pi^+ \pi^- \eta) \times Br(\eta \rightarrow \gamma \gamma) \times Br(K^{*\pm} \rightarrow K^0 \pi^\pm) \times Br(K^0 \rightarrow K_S^0) \times Br(K_S^0 \rightarrow \pi^+ \pi^-) \times \epsilon} \\ &= \frac{1195 \pm 68}{1310.6 \times 10^6 \times 42.9\% \times 39.41\% \times 66.6\% \times 50.0\% \times 69.2\% \times 11.03\%} \\ &= (2.16 \pm 0.12^{stat.}) \times 10^{-4} \end{aligned}$$

chengping
Sorry I do not understand this calculation. Here, 1195 is the number of h1(1380) signal events from the fits to K*+K- and K*0K0bar ? I mean sum of them ? In the denominator, why I can not find K*0 decay branching fraction ? I missed sth ?

$$\begin{aligned} & \mathcal{B}(J/\psi \rightarrow \eta' h_1(1380)) \times [\mathcal{B}(h_1(1380) \rightarrow K^*(892)^+ K^- + c.c.) + \mathcal{B}(h_1(1380) \rightarrow K^*(892)^0 \bar{K}^0 + c.c.)] \\ &= \frac{N_{sig}}{N_{J/\psi} \times Br(\eta' \rightarrow \pi^+ \pi^- \eta) \times Br(\eta \rightarrow \gamma \gamma) \times Br(K^{*\pm} \rightarrow K^0 \pi^\pm) \times Br(K^0 \rightarrow K_S^0) \times Br(K_S^0 \rightarrow \pi^+ \pi^-) \times \epsilon} \\ &= \frac{1195 \pm 68}{1310.6 \times 10^6 \times 42.9\% \times 39.41\% \times 66.6\% \times 50.0\% \times 69.2\% \times 11.03\%} \\ &= (2.16 \pm 0.12^{stat.}) \times 10^{-4} \end{aligned}$$

chengping
As I mentioned above, I do not understand this calculation.

Yes, the 1198 is sum number of K*+K- and K*0 K0 with their conjugate modes. Here, after revised, the calculation should be:

$$\begin{aligned} & \mathcal{B}(J/\psi \rightarrow \eta' h_1(1380)) \times [\mathcal{B}(h_1(1380) \rightarrow K^*(892)^+ K^- + c.c.) + \mathcal{B}(h_1(1380) \rightarrow K^*(892)^0 \bar{K}^0 + c.c.)] \\ &= \frac{N_{h_1 \rightarrow K^{*+} K^-}}{N_{J/\psi} \times Br(\eta' \rightarrow \pi^+ \pi^- \eta) \times Br(\eta \rightarrow \gamma \gamma) \times Br(K^{*+} \rightarrow K^0 \pi^+) \times Br(K^0 \rightarrow K_S^0) \times Br(K_S^0 \rightarrow \pi^+ \pi^-) \times \epsilon} \\ &+ \frac{N_{h_1 \rightarrow K^{*-} K^+}}{N_{J/\psi} \times Br(\eta' \rightarrow \pi^+ \pi^- \eta) \times Br(\eta \rightarrow \gamma \gamma) \times Br(K^{*-} \rightarrow \bar{K}^0 \pi^-) \times Br(\bar{K}^0 \rightarrow K_S^0) \times Br(K_S^0 \rightarrow \pi^+ \pi^-) \times \epsilon} \\ &+ \frac{N_{h_1 \rightarrow K^{*0} \bar{K}^0}}{N_{J/\psi} \times Br(\eta' \rightarrow \pi^+ \pi^- \eta) \times Br(\eta \rightarrow \gamma \gamma) \times Br(K^{*0} \rightarrow K^+ \pi^-) \times Br(\bar{K}^0 \rightarrow K_S^0) \times Br(K_S^0 \rightarrow \pi^+ \pi^-) \times \epsilon} \\ &+ \frac{N_{h_1 \rightarrow \bar{K}^{*0} K^0}}{N_{J/\psi} \times Br(\eta' \rightarrow \pi^+ \pi^- \eta) \times Br(\eta \rightarrow \gamma \gamma) \times Br(\bar{K}^{*0} \rightarrow K^- \pi^+) \times Br(K^0 \rightarrow K_S^0) \times Br(K_S^0 \rightarrow \pi^+ \pi^-) \times \epsilon} \\ &= \frac{1195 \pm 68}{1310.6 \times 10^6 \times 42.9\% \times 39.41\% \times 66.6\% \times 50.0\% \times 69.2\% \times 11.03\%} \\ &= (2.16 \pm 0.12^{stat.}) \times 10^{-4} \end{aligned}$$

Here,

$$\begin{aligned} & Br(K^{*+} \rightarrow K^0 \pi^+) \times Br(K^0 \rightarrow K_S^0) \\ &= Br(K^{*-} \rightarrow \bar{K}^0 \pi^-) \times Br(\bar{K}^0 \rightarrow K_S^0) \\ &= Br(K^{*0} \rightarrow K^+ \pi^-) \times Br(\bar{K}^0 \rightarrow K_S^0) \\ &= Br(\bar{K}^{*0} \rightarrow K^- \pi^+) \times Br(K^0 \rightarrow K_S^0) \\ &= 66.6\% \times 50.0\% \end{aligned}$$

and

$$N_{h_1 \rightarrow K^{*+} K^-} + N_{h_1 \rightarrow K^{*-} K^+} + N_{h_1 \rightarrow K^{*0} \bar{K}^0} + N_{h_1 \rightarrow \bar{K}^{*0} K^0} = 1195 \pm 68$$

Question 4,

But isospin-breaking phenomenon have been reported in some process [26]. Such as, the largest isospin-breaking effect in the $D\bar{D}$ production at the $\psi(3770)$ is that due to the mass difference between the charged and the neutral D mesons: $\Delta m_D = 4.78 \pm 0.10$ MeV. So, because of the mass difference between the charged and the neutral K and $K^*(892)$ mesons: $\Delta m_K = (497.614 - 493.677)$ MeV = (3.973) MeV, and $\Delta m_{K^*(892)} = (895.81 - 891.66)$ MeV = (4.15) MeV; the isospin-breaking phenomenon will be certainly not unexpected between $\mathcal{B}(h_1(1380) \rightarrow K^*(892)^+ K^- + c.c.)$ and $\mathcal{B}(h_1(1380) \rightarrow K^*(892)^0 \bar{K}^0 + c.c.)$. 

References for the isospin-breaking effect due to the mass differences:

<https://journals.aps.org/prd/pdf/10.1103/PhysRevD.75.113001>

<https://journals.aps.org/prd/pdf/10.1103/PhysRevD.81.011501>

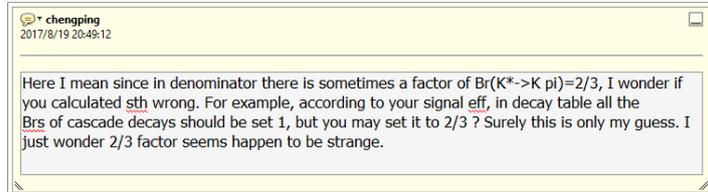
chengping
For psi(3770) to DD:
1. isospin-breaking effect is much much smaller than your result
2. the psi(3770) mass is much more closer to DD mass threshold

So I do not think such argument is a strong reason.

Question 5,

For “Also I noticed $1.51 \times 10^{-4} = 2/3 \times 2.16 \times 10^{-4}$. There is a happened 2/3 factor here :-)”

Sorry for that I don't understand the meaning of the happened 2/3 factor, could you please give more explanations.



Reply:

Just as what you mentioned above, all the branching fractions of the cascade decays in decay table are fixed to be 1.

Also, it seems you expect the happened 2/3 factor to be 1.

In fact, for the sum of **charged mode and neutral mode**, the branching fraction is:

$$\begin{aligned} & \mathcal{B}(J/\psi \rightarrow \eta' h_1(1380)) \times [\mathcal{B}(h_1(1380) \rightarrow K^*(892)^+ K^- + c.c.) + \mathcal{B}(h_1(1380) \rightarrow K^*(892)^0 \bar{K}^0 + c.c.)] \\ & = (2.16 \pm 0.12^{stat.}) \times 10^{-4} \end{aligned}$$

For the **neutral mode alone**, the branching fraction is:

$$\begin{aligned} & \mathcal{B}(J/\psi \rightarrow \eta' h_1(1380)) \times \mathcal{B}(h_1(1380) \rightarrow K^*(892)^+ K^- + c.c.) \\ & = (1.51 \pm 0.09^{stat.}) \times 10^{-4} \end{aligned}$$

It's not reasonable to expect the neutral mode branching fraction equaling to the sum of charged mode and neutral mode branching fraction.

Question 6

The 367th line: did you find the desructive solution ? We should have this solution and show this in the paper.

Reply:

The fit of $h_1(1380)$ is performed with the consideration of interference between $h_1(1380)$ and non-resonant amplitudes, in the range of $[1.250, 1.850]$ GeV/c^2 , as shown in Figure 1.

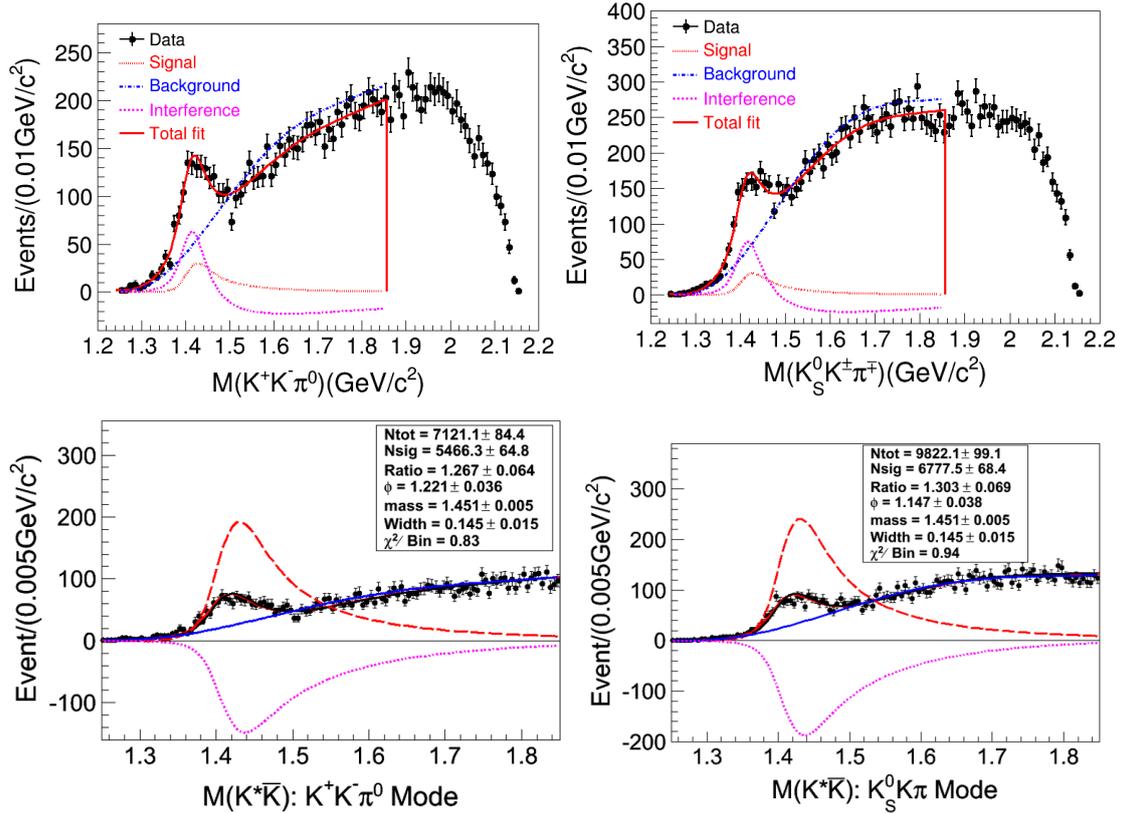


Figure 1: Fit results of $h_1(1380)$ with the consideration of interference between $h_1(1380)$ and non-resonant component.

The negative log Likelihood value as a function of the phase angle is shown in Figure 2.

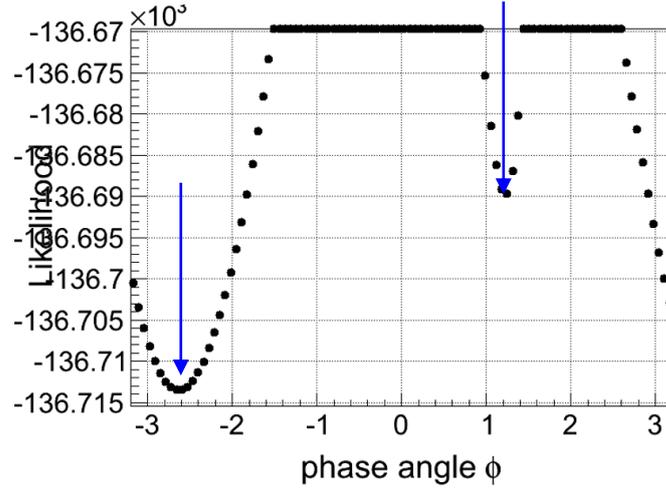


Figure 2: The negative log Likelihood value as a function of the phase angle.

The statistical significance of the interference, calculated based on the differences of likelihood and degrees of freedom between fits with interference (Figure 1) and without interference (Figure 3), as shown in Table 1 (last list).

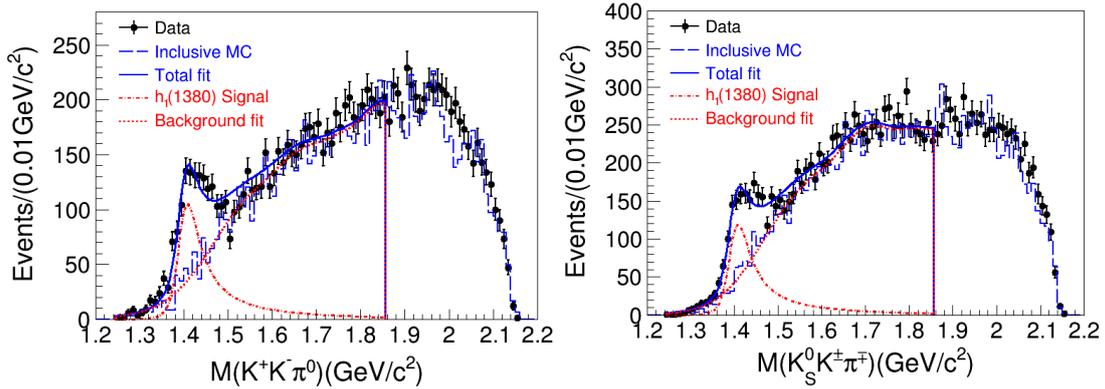


Figure 3: Fit results of $h_1(1380)$ **without** the consideration of interference between $h_1(1380)$ and non-resonant component.

Table 1: Two solutions of the fit to $M(K^*K)$ by taking interference between $h_1(1380)$ signal and non-resonant components.

Fit mode		Mass (MeV/c ²)	Width (MeV/c ²)	FCN/ndf	Significance (Δ FCN/ Δ ndf)
Interference	Constructive	1442 ± 5	111 ± 13	-136714 / 8	5.8σ (19.0/2)
	Destructive	1451 ± 5	144 ± 15	-136695 / 8	(0.0/2)
Non-interference		1423 ± 2	90 ± 10	-136695 / 6	

Table 1: Two solutions of the fit to $M(K^*K)$ by taking interference between $h_1(1380)$ signal and non-resonant components.

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Non-interference		1423 ± 2	90 ± 10	-136695 / 6	

Due to the limited significance for destructive solution, we don't present it in the draft. 

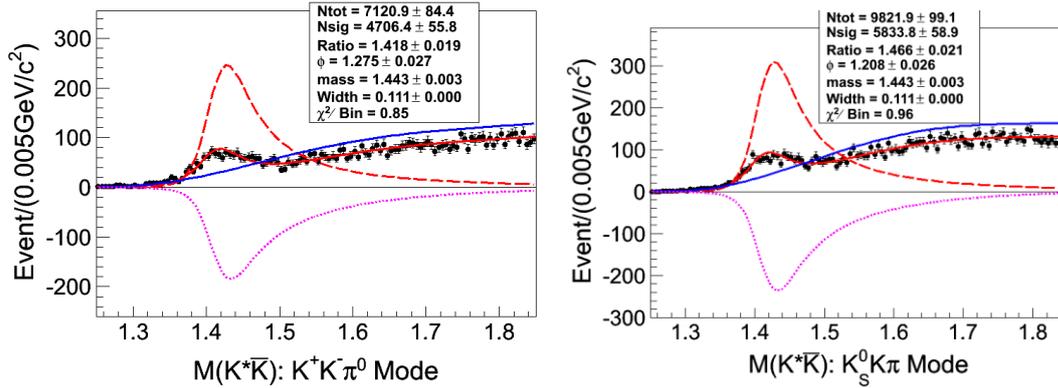
chengping
 Since we know the mass and width from constructive and destructive solutions should be the same. You may fix mass or width to have a check destructive solution? It may be easier to find the correct solution since there is a one more constraint.

10

Reply:

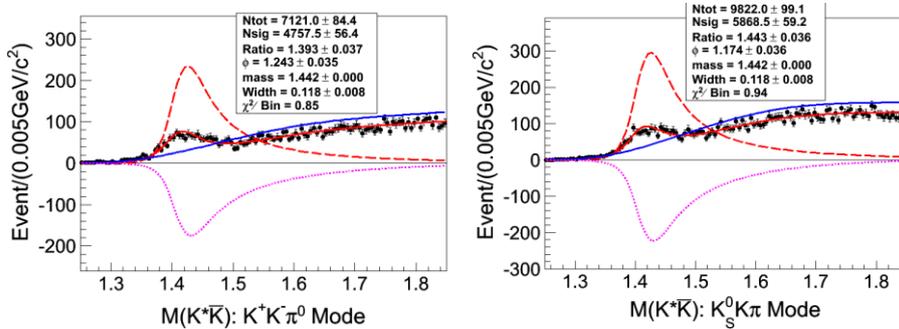
It not always happened that the destructive solution equaling to the constructive solution.

Only width fixed



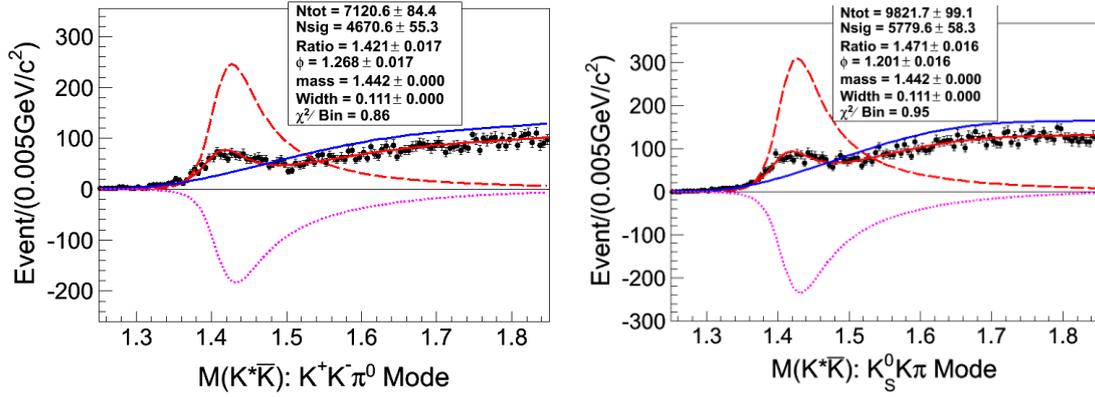
FCN=-136693, ndf=7

Only mass fixed



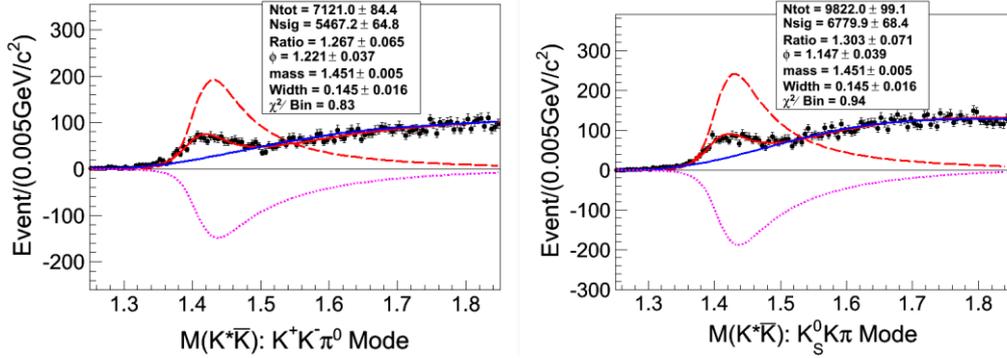
FCN=-136693, ndf=7

Both mass and width are fixed



FCN=-136693, ndf=6

Both mass and width are allowed to vary



FCN=-136695, ndf=8

	Fit methods	FCN	ndf
With interference (Destructive)	Only mass fixed	-136693	7
	Only width fixed	-136693	7
	Both mass and width fixed	-136693	6
	Both mass and width allowed to vary	-136695	8
Without interference	Both mass and width allowed to vary	-136695	6

With the mass or width fixed to the constructive solution, the destructive interference results with FCN value smaller than non-interference result and ndf value larger than the non-interference result are shown in the above table.

Therefore, with the further check, the destructive solution should be discarded.