

Future hyperon studies at ~~BES~~ III

Andrzej Kupsc

Prospects for baryon spin physics:

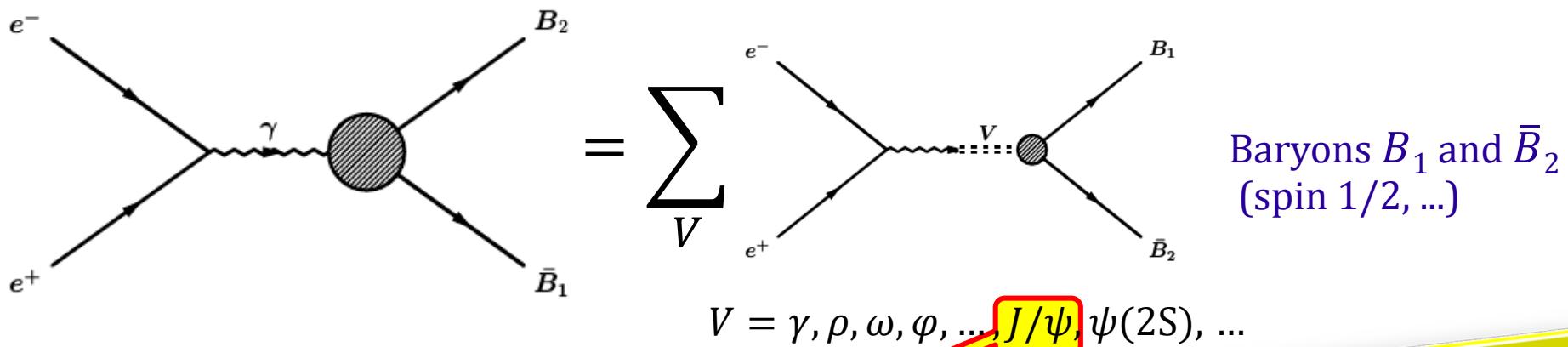
- $e^+e^- \rightarrow J/\psi, \psi' \rightarrow B_1\bar{B}_2$ (ground state hyperons):
polarization, hyperon decay parameters
- $e^+e^- \rightarrow J/\psi, \psi' \rightarrow B_1\bar{B}_2V(P)$:
spectroscopy

Methods:

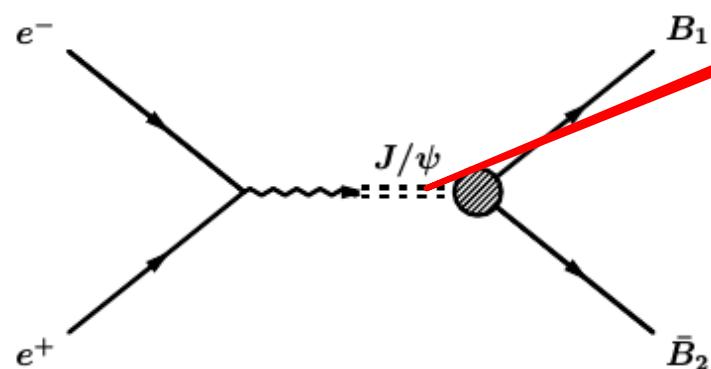
1. G.Fäldt, AK PLB772 (2017) 16
2. E.Perotti,G.Fäldt,AK,S.Leupold,JJ.Song PRD99 (2019)056008
3. G. Fäldt, K. Schönnning arXiv:1908.04157
4. P.Adlarson, AK arXiv:1908.03102

Baryon-antibaryon production in e^+e^- collisions

continuum:



J/ψ decay:



Time like spin $1/2$ baryon FFs:

Dubnickova, Dubnicka, Rekalo

Nuovo Cim. A109 (1996) 241

Gakh, Tomasi-Gustafsson Nucl.Phys. A771 (2006) 169

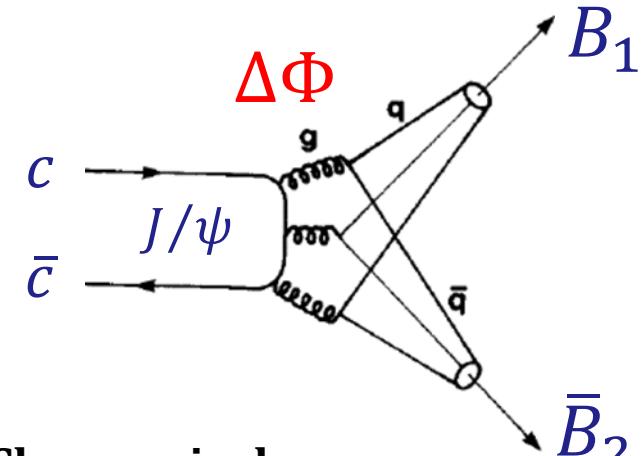
Czyz, Grzelinska, Kuhn PRD75 (2007) 074026

Fäldt EPJ A51 (2015) 74; EPJ A52 (2016) 141

$$V = \gamma, \rho, \omega, \varphi, \dots, J/\psi, \psi(2S), \dots$$

Both processes described by two complex FFs: relative phase $\Delta\Phi$

Cabibbo, Gatto PR124 (1961) 1577



Charmonia decays:

Fäldt, AK PLB772 (2017) 16

Hyperon-hyperon pair production at BESIII

$2.0 \text{ GeV} \leq \sqrt{s} \leq 4.6 \text{ GeV}$

Thresholds:

$\Lambda\bar{\Lambda}$: 2.231 GeV

$\Sigma^+\bar{\Sigma}^-$ 2.379 GeV ($\Omega\bar{\Omega}$ 3.345 GeV)

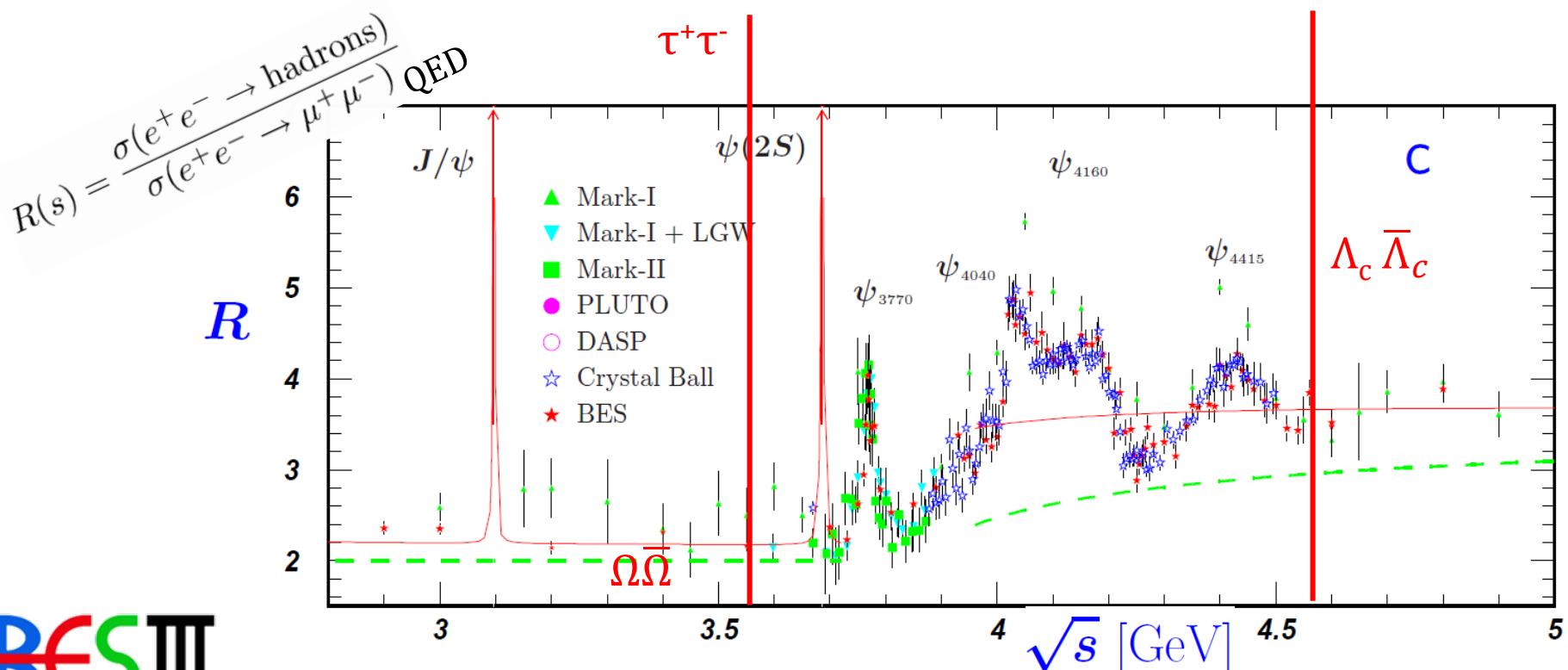
$\Sigma^0\bar{\Sigma}^0$ 2.385 GeV

$\Sigma^-\bar{\Sigma}^+$ 2.395 GeV

$\Xi^0\bar{\Xi}^0$ 2.630 GeV

$\Xi^-\bar{\Xi}^+$ 2.643 GeV

$\Lambda\bar{\Sigma}^0$ 2.308 GeV



$J/\psi, \psi(2S) \rightarrow B\bar{B}$

Expected number of events at BESIII

$$\mathcal{B}(J/\psi \rightarrow p\bar{p}) = (21.21 \pm 0.29) \times 10^{-4}$$

decay mode	$\mathcal{B}(\text{units } 10^{-4})$	α_ψ	eff	events proposal
$J/\psi \rightarrow \Lambda\bar{\Lambda}$	$19.43 \pm 0.03 \pm 0.33$	0.469 ± 0.026	40%	3200×10^3
$\psi(2S) \rightarrow \Lambda\bar{\Lambda}$	$3.97 \pm 0.02 \pm 0.12$	0.824 ± 0.074	40%	650×10^3
$J/\psi \rightarrow \Xi^0\bar{\Xi}^0$	11.65 ± 0.04	0.66 ± 0.03	14%	670×10^3
$\psi(2S) \rightarrow \Xi^0\bar{\Xi}^0$	2.73 ± 0.03	0.65 ± 0.09	14%	160×10^3
$J/\psi \rightarrow \Xi^-\bar{\Xi}^+$	10.40 ± 0.06	0.58 ± 0.04	19%	810×10^3
$\psi(2S) \rightarrow \Xi^-\bar{\Xi}^+$	2.78 ± 0.05	0.91 ± 0.13	19%	210×10^3

$$\mathcal{B}(\psi' \rightarrow \Omega^-\bar{\Omega}^+) = (0.52 \pm 0.04) \times 10^{-4} \quad \text{CLEO-c: PRD 96, 092004}$$

PRD 93, 072003 (2016)

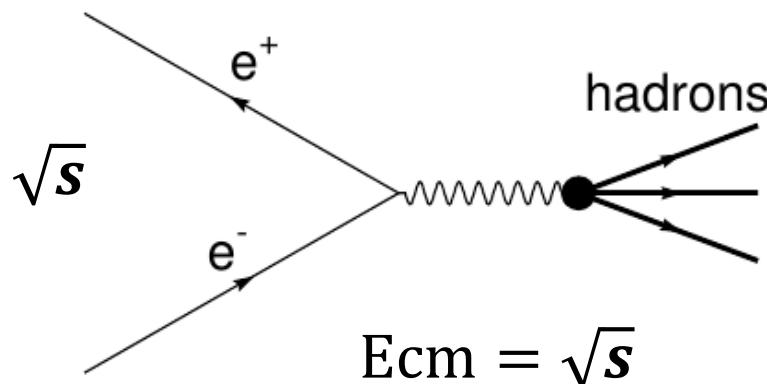
PLB770,217 (2017)

PRD 95, 052003 (2017)

BESIII (Feb 2019): $10^{10} J/\psi$

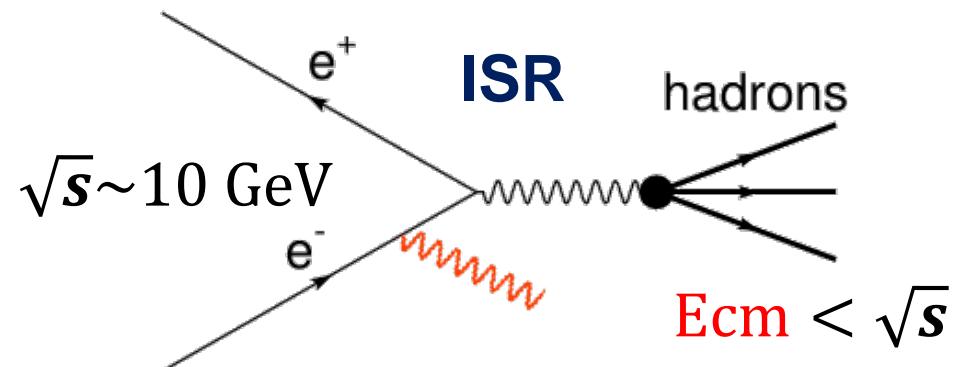
BESIII proposal: $3.2 \times 10^9 \psi(2S)$

Direct scan BESIII

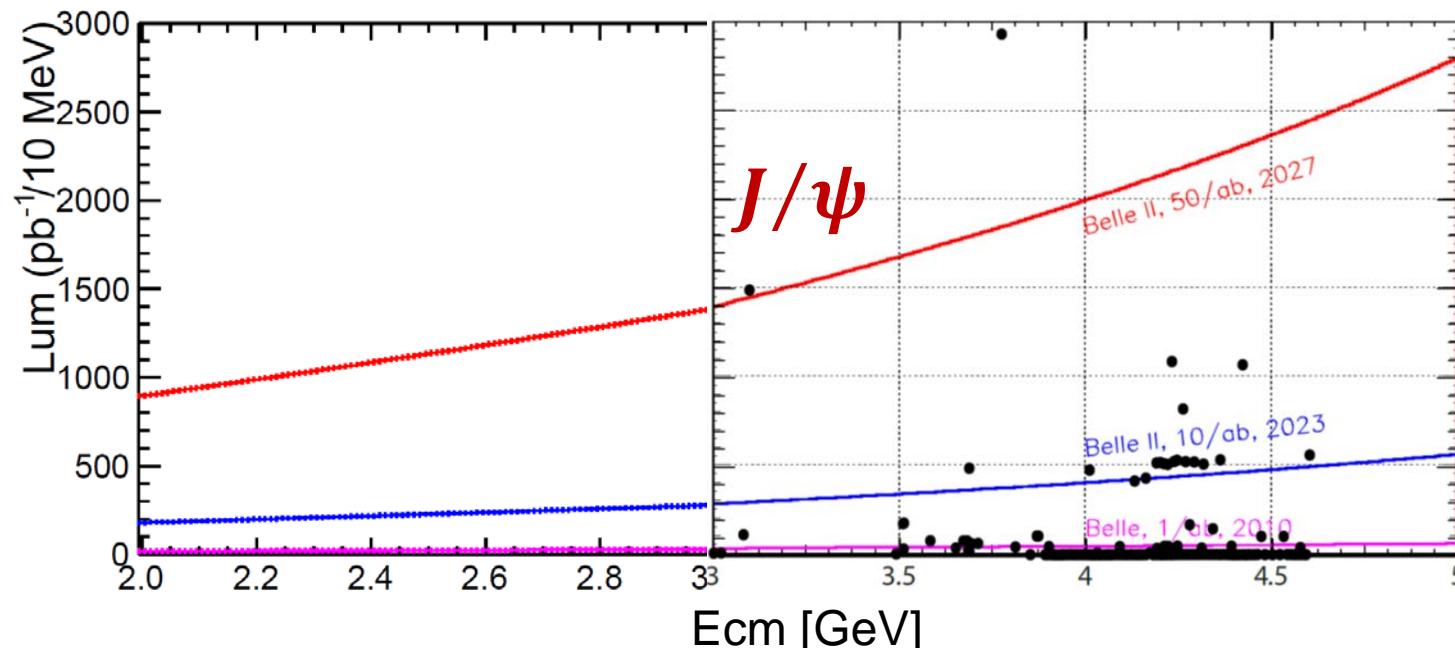


- (very) high luminosity at selected c.m. energies
- better resolution: at J/ψ 0.9 MeV: $10^{10} J/\psi$

ISR Belle II



- many E_{cm} simultaneously
- reduced point-to-point systematics
- mass resolution limited by detector
- boost of hadronic system may help efficiency



Hyperon-antihyperon pairs from J/ ψ and $\psi(2S)$ decays

Motivations: CP violation, QM tests (entangled system) :

CP Asymmetries in Strange Baryon Decays

I. I. Bigi, Xian-Wei Kang, Hai-Bo Li CPC42 (2018) 013101
arXiv:1704.04708 & BESIII Hai-Bo: arXiv:1612.01775

Steve Olsen
presentation on BESIII
30 years

Hyperon decay parameters, hyperon FSI, charmonium decay mechanism,...

Ground state hyperons analyses: MLL fits of angular distributions:

- $\Lambda\bar{\Lambda}, \Sigma^+\bar{\Sigma}^-, (\Sigma^-\bar{\Sigma}^+)$
 - $\Lambda\bar{\Sigma}^0, \Sigma^0\bar{\Sigma}^0$
 - $\Xi\bar{\Xi}$
 - $\Omega\bar{\Omega}$
- Covariant formalism
Ref 1&3
- Jacob-Wick Helicity formalism (1959)

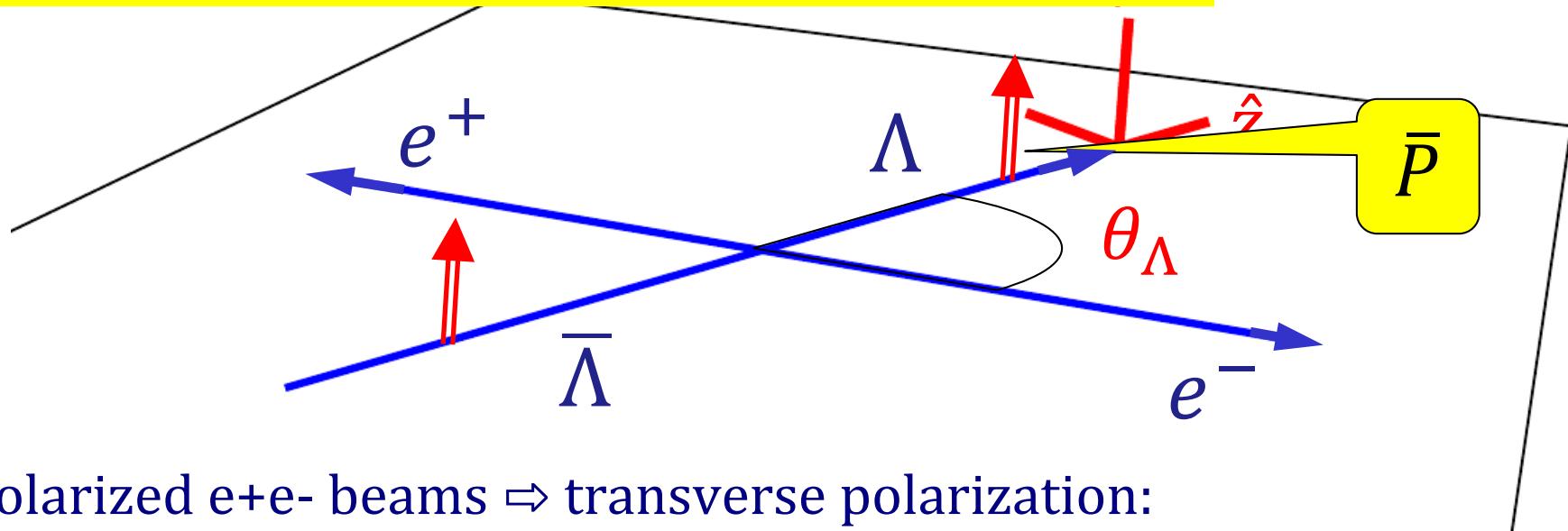
Amplitudes for precision BESIII:

$$e^+ e^- \rightarrow \gamma^* (\rightarrow \psi)$$

$$\begin{aligned} &\rightarrow B_{1/2} \bar{B}_{1/2} \\ &\rightarrow B_{3/2} \bar{B}_{1/2} \\ &\rightarrow B_{3/2} \bar{B}_{3/2} \end{aligned}$$

Ref 2: Modular framework for entangled **exclusive (DT)** distributions with modifiable decay chains,
Use correct variables vs amplitudes
Weak decays sensitive to the helicity rotation definition

Baryon (spin 1/2) polarization in e+e-



Unpolarized e^+e^- beams \Rightarrow transverse polarization:

$$P_y(\cos \theta_\Lambda) = \frac{\sqrt{1 - \alpha_\psi^2} \cos \theta_\Lambda \sin \theta_\Lambda}{1 + \alpha_\psi \cos^2 \theta_\Lambda} \sin(\Delta\Phi)$$

$\Delta\Phi \neq 0$

Angular distribution:

$$\frac{d\Gamma}{d\Omega} \propto 1 + \alpha_\psi \cos^2 \theta \quad -1 \leq \alpha_\psi \leq 1$$

Notation:

$$R = \left| \frac{G_E}{G_M} \right| \quad \tau = \frac{s}{4M_B^2} \quad \left(\alpha_\psi = \frac{\tau - R^2}{\tau + R^2} \right) \quad G_E = RG_M e^{i\Delta\Phi}$$

Baryon-antibaryon spin density matrix

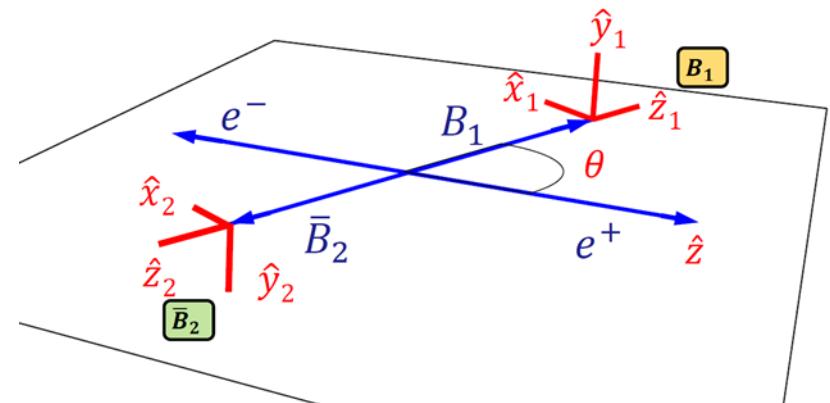
$$e^+ e^- \rightarrow B_1 \bar{B}_2$$

General two spin $\frac{1}{2}$ particle state: $\rho_{1/2,1/2} = \frac{1}{4} \sum_{\mu\bar{\nu}} C_{\mu\bar{\nu}} \sigma_\mu^{B_1} \otimes \sigma_{\bar{\nu}}^{\bar{B}_2}$

($\sigma_0 = \mathbf{1}_2, \sigma_1 = \sigma_x, \sigma_2 = \sigma_y, \sigma_3 = \sigma_z$)

$$C_{\mu\bar{\nu}} = \begin{pmatrix} 1 + \alpha_\psi \cos^2 \theta & 0 & \boxed{\beta_\psi \sin \theta \cos \theta} & 0 \\ 0 & \sin^2 \theta & 0 & \gamma_\psi \sin \theta \cos \theta \\ -\beta_\psi \sin \theta \cos \theta & 0 & \alpha_\psi \sin^2 \theta & 0 \\ 0 & -\gamma_\psi \sin \theta \cos \theta & 0 & -\alpha_\psi - \cos^2 \theta \end{pmatrix}$$

$$\beta_\psi = \sqrt{1 - \alpha_\psi^2} \sin(\Delta\Phi) \quad \gamma_\psi = \sqrt{1 - \alpha_\psi^2} \cos(\Delta\Phi)$$



Hyperon decay parameters

General Partial Wave Analysis of the Decay of a Hyperon of Spin $\frac{1}{2}$

T. D. LEE* AND C. N. YANG

Institute for Advanced Study, Princeton, New Jersey

Phys. Rev. 108 1645 (1957)

s wave parity violating
p wave parity conserving

$$Y \rightarrow B\pi: \quad \frac{1}{2} \rightarrow \frac{1}{2} + 0$$

$$\alpha_Y = \frac{2\text{Re}(s^* p)}{|s|^2 + |p|^2}, \quad \beta_Y = \frac{2\text{Im}(s^* p)}{|s|^2 + |p|^2} = \sqrt{1 - \alpha_Y^2} \sin \phi_Y$$

$$\gamma_Y = \frac{|s|^2 - |p|^2}{|s|^2 + |p|^2} = \sqrt{1 - \alpha_Y^2} \cos \phi_Y$$

α_- or α_Λ

hyperon	decay (BF)	α	ϕ
$\Lambda(uds)$ α_0	$p\pi^-$ (63.9%)	0.642 ± 0.013	$-6.5^\circ \pm 3.5^\circ$
	$n\pi^0$ (35.8%)		
$\bar{\Lambda}(\bar{u}\bar{d}\bar{s})$	$\bar{p}\pi^+$ (63.9%)	-0.71 ± 0.08	$-$
$\Sigma^-(dds)$	$n\pi^-$ (99.8%)	-0.068 ± 0.008	$10^\circ \pm 15^\circ$
$\Sigma^+(uus)$	$p\pi^0$ (51.6%)	-0.980 ± 0.017	$36^\circ \pm 34^\circ$
	$n\pi^+$ (48.3%)	-0.068 ± 0.013	$167 \pm 20^\circ$
$\Xi^0(uss)$	$\Lambda\pi^0$ (99.5%)	-0.406 ± 0.085	$21^\circ \pm 12^\circ$
	$\Lambda\pi^-$ (99.8%)	-0.458 ± 0.012	$-2.1^\circ \pm 0.8^\circ$

CP asymmetry

$$A_{CP} = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}}$$

Obsolete!

PDG 2018

Polarization of daughter baryons:

$\Upsilon \rightarrow B\pi$

$$\mathbf{P}_B = \frac{(\alpha + \mathbf{P}_Y \cdot \hat{\mathbf{n}})\hat{\mathbf{n}} + \beta(\mathbf{P}_Y \times \hat{\mathbf{n}}) + \gamma\hat{\mathbf{n}} \times (\mathbf{P}_Y \times \hat{\mathbf{n}})}{1 + \alpha\mathbf{P}_Y \cdot \hat{\mathbf{n}}} \quad \text{PDG}$$

$$\mathbf{P}_Y = 0 \Rightarrow \mathbf{P}_B = \alpha \hat{\mathbf{n}}$$

Density matrix for a spin $1/2$ particle
in the rest frame:

$$\rho_{1/2} = \frac{1}{2} \sum_{\mu=0}^3 I_\mu \sigma_\mu = \frac{1}{2} I_0 \begin{pmatrix} 1 + P_z & P_x - iP_y \\ P_x + iP_y & 1 - P_z \end{pmatrix}$$

$$\sigma_0 = \mathbf{1}_2, \sigma_1 = \sigma_x, \sigma_2 = \sigma_y, \sigma_3 = \sigma_z$$

Transformation of base matrices:

$$\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + 0^- \quad e.g. \quad \Lambda \rightarrow p + \pi^-$$

Decay matrices

$$\sigma_\mu \rightarrow \sum_{\nu=0}^3 a_{\mu,\nu} \sigma_\nu^d$$

4×4 decay matrix: $a_{\mu,\nu}$

Measuring α , β , γ in the 20th century

Oliver Overseth

James Cronin

1931-2016



PHYSICAL REVIEW

VOLUME 129, NUMBER 4

1928-2008



15 FEBRUARY 1963

Measurement of the Decay Parameters of the Λ^0 Particle*

JAMES W. CRONIN AND OLIVER E. OVERSETH†

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received 26 September 1962)

The decay parameters of $\Lambda^0 \rightarrow \pi^- + p$ have been measured by observing the polarization of the decay protons by scattering in a carbon-plate spark chamber. The experimental procedure is discussed in some detail. A total of 1156 decays with useful proton scatters was obtained. The results are expressed in terms of polarization parameters, α , β , and γ given below:

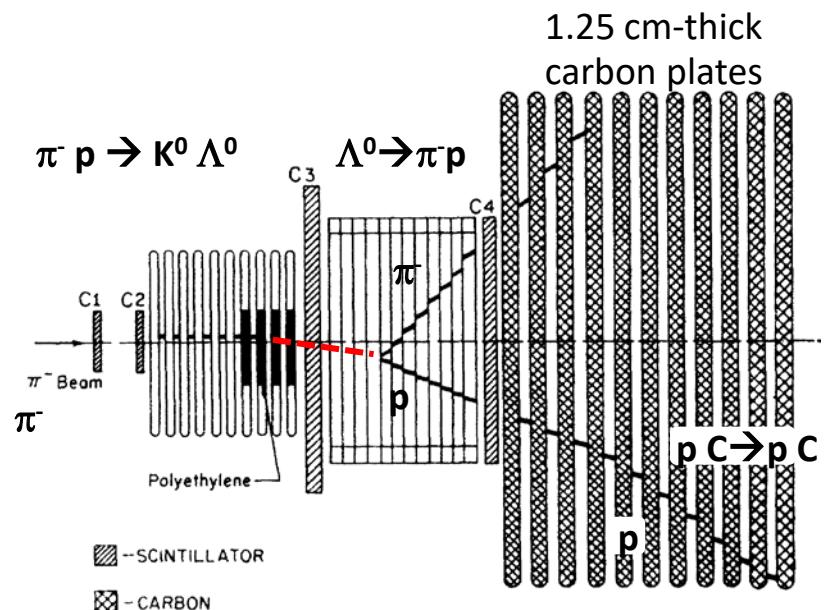
$$\alpha = 2 \operatorname{Re} s^*/(|s|^2 + |p|^2) = +0.62 \pm 0.07,$$

$$\beta = 2 \operatorname{Im} s^*/(|s|^2 + |p|^2) = +0.18 \pm 0.24,$$

$$\gamma = |s|^2 - |p|^2 / (|s|^2 + |p|^2) = +0.78 \pm 0.06,$$

where s and p are the s - and p -wave decay amplitudes in an effective Hamiltonian $s + p\sigma \cdot p/|\mathbf{p}|$, where \mathbf{p} is the momentum of the decay proton in the center-of-mass system of the Λ^0 , and σ is the Pauli spin operator. The helicity of the decay proton is positive. The ratio $|p|/|s|$ is $0.36_{-0.06}^{+0.05}$ which supports the conclusion that the $K\Lambda N$ parity is odd. The result $\beta = 0.18 \pm 0.24$ is consistent with the value $\beta = 0.08$ expected on the basis of time-reversal invariance.

$$P_p = \frac{(\alpha + P_\Lambda \cos \theta) \dot{z} + \beta P_\Lambda \dot{x} + \gamma P_\Lambda \dot{y}}{1 + \alpha P_\Lambda \cos \theta}$$

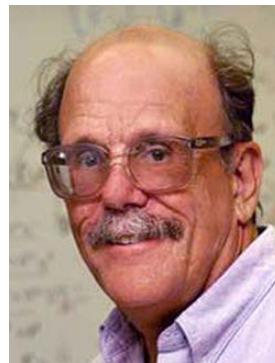
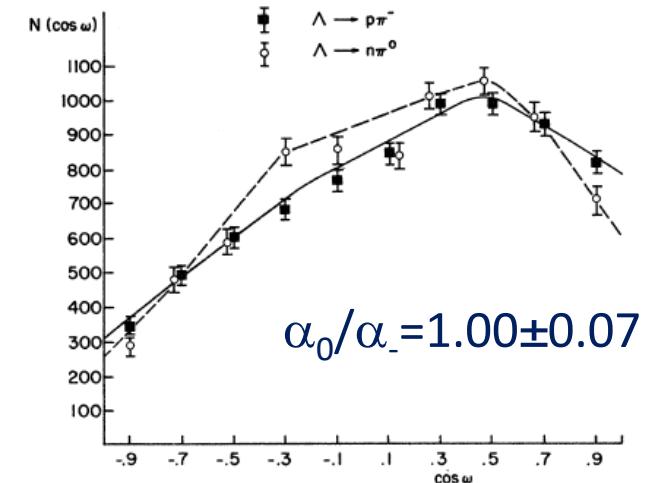
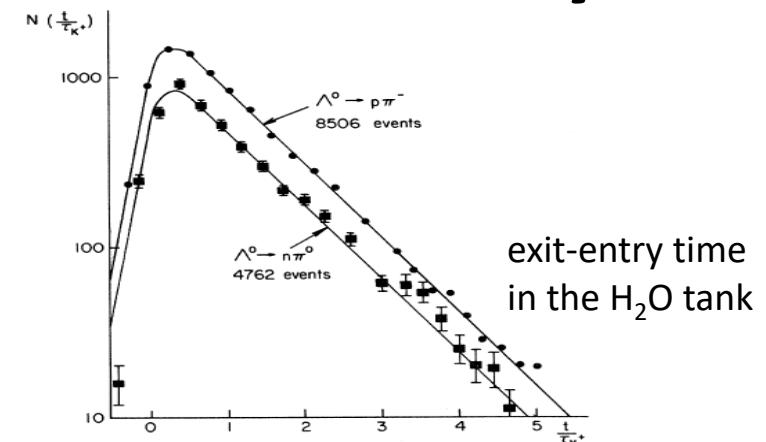
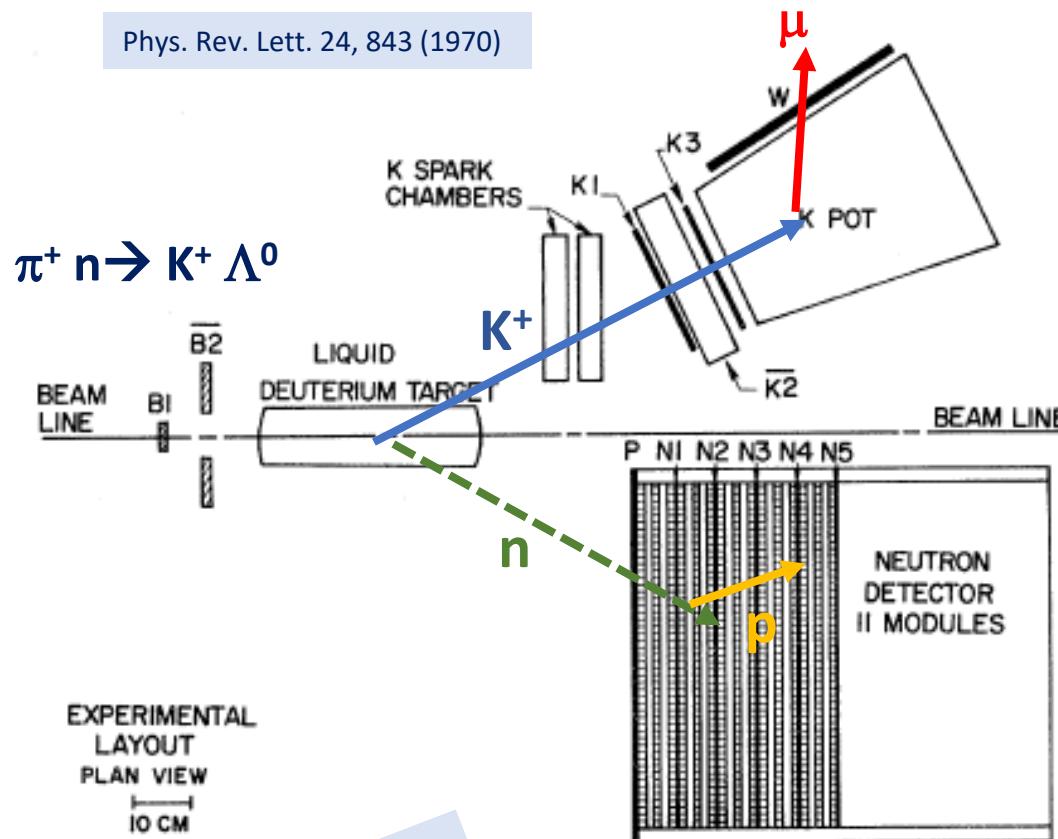


no H₂ target, no magnet;
use kinematics and proton's
range in carbon to infer E_p

Slide from Steve Olsen

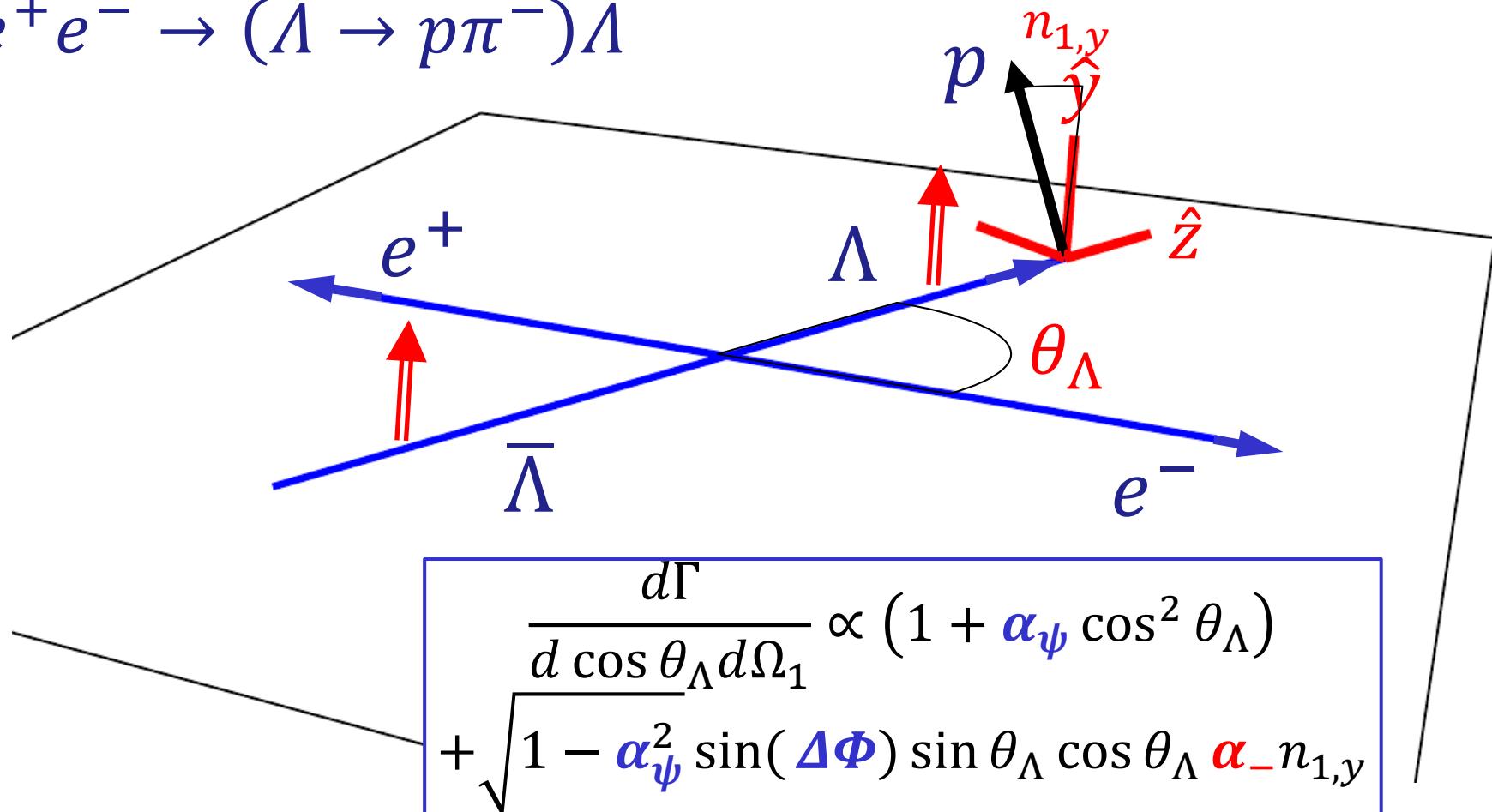
Olsen et al., α_0 parameter in $\Lambda^0 \rightarrow n\pi^0$ decays

Phys. Rev. Lett. 24, 843 (1970)



Inclusive decay angular distributions

$$e^+ e^- \rightarrow (\Lambda \rightarrow p\pi^-) \bar{\Lambda}$$



$$\Lambda \rightarrow p\pi^- : \hat{\mathbf{n}}_1 \rightarrow \Omega_1 = (\cos \theta_1, \phi_1) : \alpha_-$$

⇒ Determine product: $\alpha_- P_y \sim \alpha_- \sin(\Delta\Phi)$

Exclusive joint angular distribution

$$e^+ e^- \rightarrow (\Lambda \rightarrow p\pi^-)(\bar{\Lambda} \rightarrow \bar{p}\pi^+)$$

$$\Lambda \rightarrow p\pi^- : \hat{\mathbf{n}}_1 \rightarrow (\cos \theta_1, \phi_1) : \alpha_- \quad \bar{\Lambda} \rightarrow \bar{p}\pi^+ : \hat{\mathbf{n}}_2 \rightarrow (\cos \theta_2, \phi_2) : \alpha_+$$

$$\xi : (\cos \theta_\Lambda, \hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) \quad \text{5D PhSp}$$

$$d\Gamma \propto W(\xi; \alpha_\psi, \Delta\Phi, \alpha_-, \alpha_+) =$$

$$1 + \alpha_\psi \cos^2 \theta_\Lambda$$

Cross section

$$+ \alpha_- \alpha_+ \left\{ \sin^2 \theta_\Lambda (n_{1,x} n_{2,x} - \alpha_\psi n_{1,y} n_{2,y}) + (\cos^2 \theta_\Lambda + \alpha_\psi) n_{1,z} n_{2,z} \right\}$$

$$+ \alpha_- \alpha_+ \sqrt{1 - \alpha_\psi^2} \cos(\Delta\Phi) \sin \theta_\Lambda \cos \theta_\Lambda (n_{1,x} n_{2,z} + n_{1,z} n_{1,x})$$

$$+ \sqrt{1 - \alpha_\psi^2} \sin(\Delta\Phi) \sin \theta_\Lambda \cos \theta_\Lambda (\alpha_- n_{1,y} + \alpha_+ n_{2,y})$$

Spin correlations

Polarization

$\Delta\Phi \neq 0 \Rightarrow \text{independent}$ determination of α_- and α_+

Exclusive joint angular distribution (modular form)

$$e^+ e^- \rightarrow (\Lambda \rightarrow p\pi^-)(\bar{\Lambda} \rightarrow \bar{p}\pi^+)$$

General two spin $\frac{1}{2}$ particle state: $\rho_{1/2,1/2} = \frac{1}{4} \sum_{\mu\bar{\nu}} C_{\mu\bar{\nu}} \sigma_\mu^\Lambda \otimes \sigma_{\bar{\nu}}^{\bar{\Lambda}}$

$$(\sigma_0 = \mathbf{1}_2, \sigma_1 = \sigma_x, \sigma_2 = \sigma_y, \sigma_3 = \sigma_z)$$

$$C_{\mu\bar{\nu}} = \begin{pmatrix} 1 + \alpha_\psi \cos^2 \theta & 0 & \beta_\psi \sin \theta \cos \theta & 0 \\ 0 & \sin^2 \theta & 0 & \gamma_\psi \sin \theta \cos \theta \\ -\beta_\psi \sin \theta \cos \theta & 0 & \alpha_\psi \sin^2 \theta & 0 \\ 0 & -\gamma_\psi \sin \theta \cos \theta & 0 & -\alpha_\psi - \cos^2 \theta \end{pmatrix}$$

$$\beta_\psi = \sqrt{1 - \alpha_\psi^2} \sin(\Delta\Phi) \quad \gamma_\psi = \sqrt{1 - \alpha_\psi^2} \cos(\Delta\Phi)$$

Apply decay matrices:

$$\sigma_\mu^\Lambda \rightarrow \sum_{\mu'=0}^3 a_{\mu,\mu'}^\Lambda \sigma_{\mu'}^p$$

The angular distribution:

$$W = Tr \rho_{p,\bar{p}} = \sum_{\mu,\bar{\nu}=0}^3 C_{\mu\bar{\nu}} a_{\mu,0}^\Lambda a_{\bar{\nu},0}^{\bar{\Lambda}}$$

BESIII results:

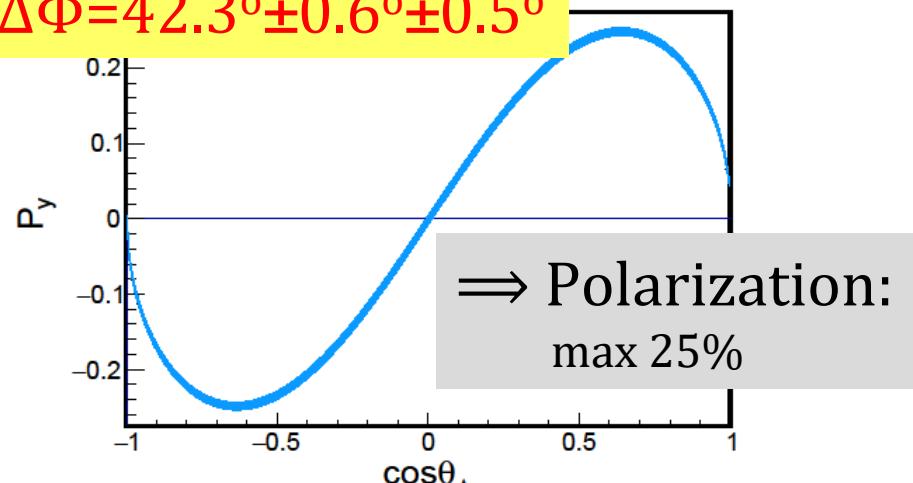
$$e^+ e^- \rightarrow J/\psi \rightarrow \Lambda \bar{\Lambda}$$

BESIII

BESIII Nature Phys. 15,631(2019)

$$\Lambda \rightarrow p\pi^- : \alpha_- = 0.750 \pm 0.009 \pm 0.004$$

$$\Delta\Phi = 42.3^\circ \pm 0.6^\circ \pm 0.5^\circ$$



$$\bar{\alpha}_0 / \alpha_+ \quad 0.913 \pm 0.028 \pm 0.012$$

$$\Delta I = \frac{1}{2} \text{ rule violation}$$

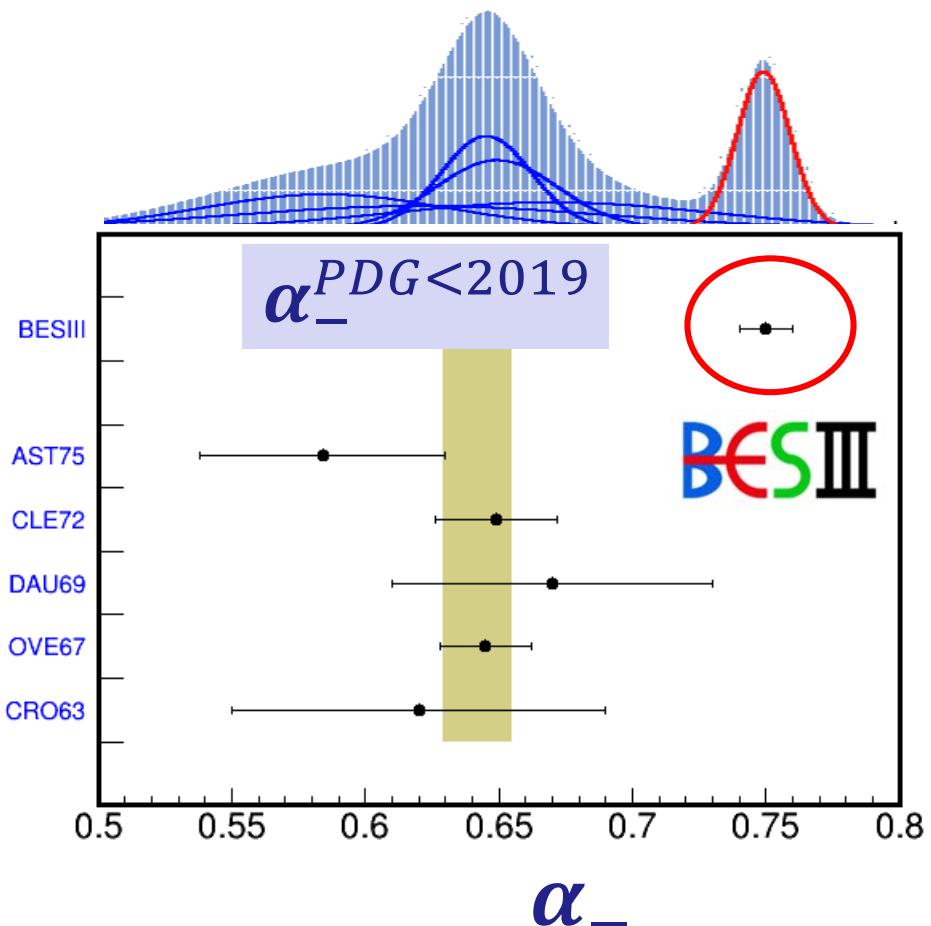
CP test:

$$A_\Lambda = \frac{\alpha_- + \alpha_+}{\alpha_- - \alpha_+}$$

$$A_\Lambda = -0.006 \pm 0.012 \pm 0.007$$

$$A_\Lambda = 0.013 \pm 0.021$$

PS185 PRC54(96)1877



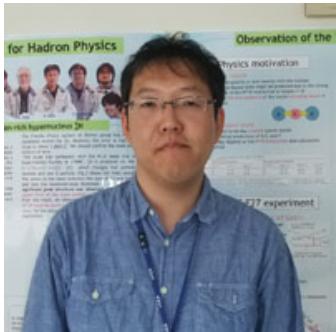
17(3)% larger

Liang's talk $J/\psi, \psi' \rightarrow \Sigma^+ \bar{\Sigma}^-$

2) Why the big change in α ?

Why different?

from: Kiyoshi Tanida
JAEA Japan



- **Multiple scattering:**
 - E.g., at 95 MeV with 3 cm scatterer (target), θ_0 becomes as large as 1.5 degree.
→ 5 degree multiple scattering occurs with a probability of 1 % order and dominates over single scattering
 - Actual scatterer thickness is even larger
 - Of course, analyzing power for multiple Coulomb scattering is almost 0
→ Can explain the difference
- Note: effective A_N depends on target thickness
 - This is why target thickness is explicit in the new data.
 - We have to be careful!!

Also: in PDG \leq 2018 syst uncertainty was not included

Slide from Steve Olsen

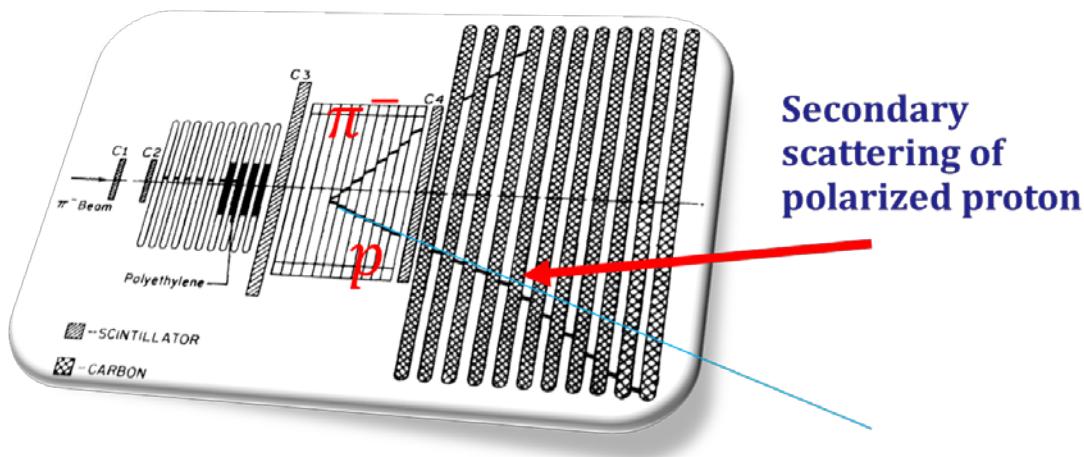
How to verify the result?

$$\vec{\gamma}p \rightarrow K^+ \Lambda$$

$$\alpha_- = 0.721(6)(5)$$

D. Ireland et al arXiv:1904.07616

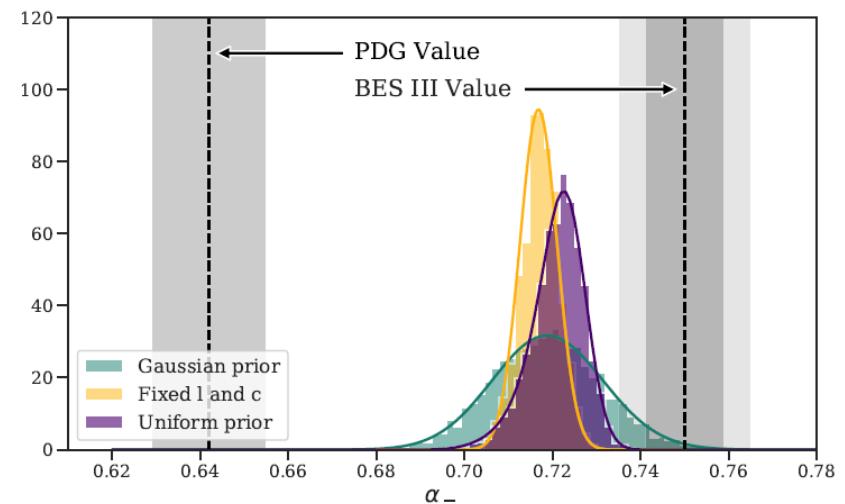
Measure proton polarization?



Independent verifications at BESIII:

$$J/\psi \rightarrow \gamma \eta_c \rightarrow \gamma \Lambda \bar{\Lambda}$$

$$BF = 1.7\% \times 1.1 \times 10^{-3}$$



$$\langle \alpha_- \rangle_{\text{BESIII}} = \frac{\alpha_- - \alpha_+}{2} = 0.754(3)(2)$$

Since $\rho(\text{stat}) = 0.82$ and using quoted syst uncertainties for α_- , α_+ , A_Λ to deduce $\rho(\text{syst}) = 0.835$

ie 4% difference with 3.8σ
new puzzle?...

$$\eta_c \rightarrow \Lambda \bar{\Lambda}$$

$$W = (1 - \alpha_- \alpha_+ \cos \theta_{p\bar{p}})$$

$$e^+ e^- \rightarrow J/\psi \rightarrow \Sigma^0 \bar{\Sigma}^0 \rightarrow \Lambda \gamma \bar{\Lambda} \gamma \rightarrow p \pi^- \gamma \bar{p} \pi^+ \gamma$$

$$W = \sum_{\mu, \bar{\nu}} C_{\mu \bar{\nu}} \sum_{\mu', \bar{\nu}'} \check{a}_{\mu, \mu'}^{\Sigma^0} \check{a}_{\bar{\nu}, \bar{\nu}'}^{\bar{\Sigma}^0}, a_{\mu', 0}^{\Lambda} a_{\bar{\nu}', 0}^{\bar{\Lambda}}$$

For EM decay $\Sigma \rightarrow \Lambda \gamma$

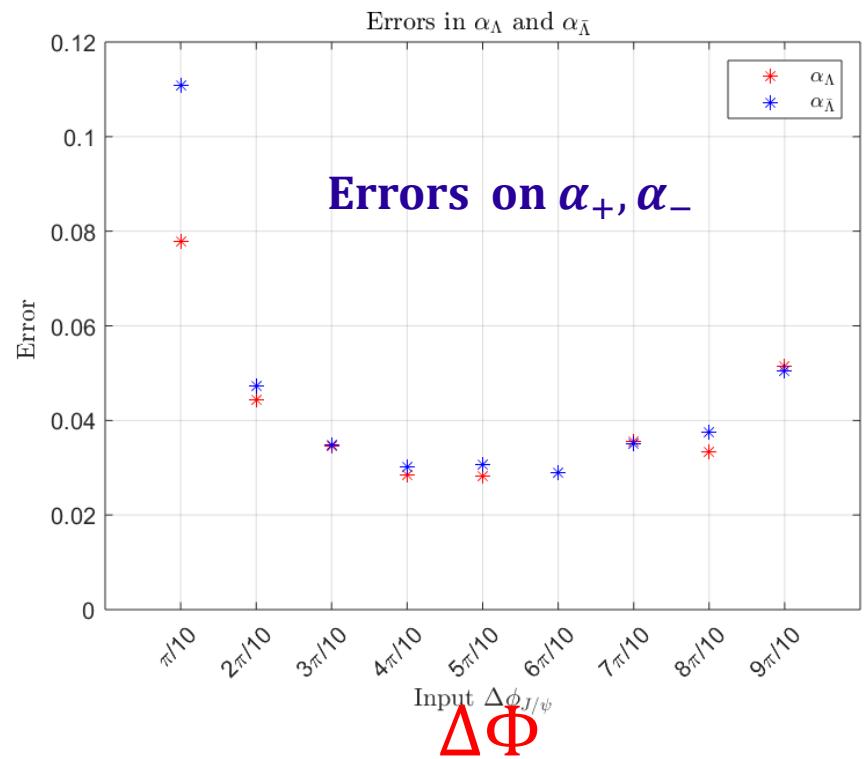
$$\check{a}_{00} = 1,$$

$$\check{a}_{13} = -\sin \theta \cos \phi,$$

$$\check{a}_{23} = -\sin \theta \sin \phi,$$

$$\check{a}_{33} = -\cos \theta,$$

G. Fäldt, K. Schönnung arXiv:1908.04157

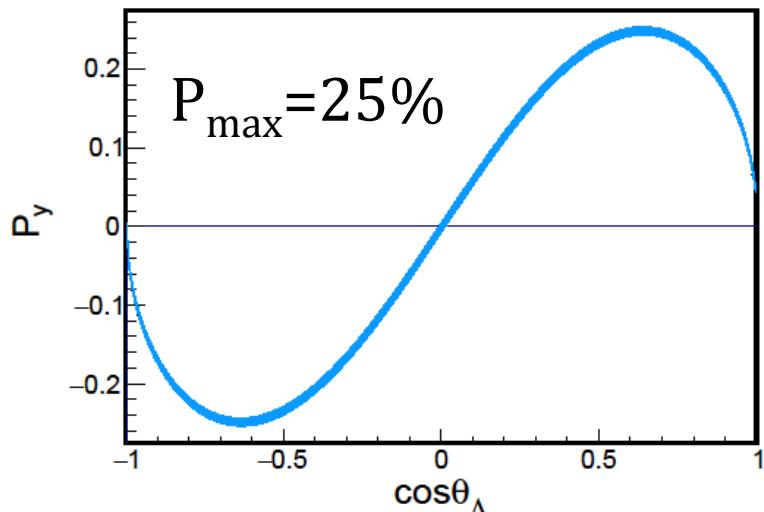


Annele Heikkila MSc Thesis UU

Comparison of $\Lambda\bar{\Lambda}$ and $\Lambda\bar{\Lambda}$ (simplified)

$$e^+e^- \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda}$$

$$e^+e^- \rightarrow J/\psi \rightarrow \Xi^-\bar{\Xi}^+ \rightarrow \Lambda\pi^-\bar{\Lambda}\pi^+$$



$$P_{\text{avg}} = 11\%$$

Λ from $\Xi^- \rightarrow \Lambda\pi^-$ is polarized even if Ξ^- unpolarized:
 $P_\Lambda = |\alpha_\Xi| \approx 39\%$

$$W \propto 1 + \color{red}{\alpha_\Lambda \alpha_\Xi} \cos \theta_p$$

Question: Can one determine α_Λ in unique way?

$$e^+ e^- \rightarrow J/\psi \rightarrow \Xi^- \bar{\Xi}^+ \rightarrow \Lambda \pi^- \bar{\Lambda} \pi^+ \rightarrow p \pi^- \pi^- \bar{p} \pi^+ \pi^+$$

$d\Gamma \propto W(\xi; \omega)$ ξ 9 kinematical variables 9D PhSp

Parameters: 2 production + 6 for decay chains

$$\omega = (\alpha_\psi, \Delta\Phi, \alpha_{\Xi}, \phi_{\Xi}, \alpha_{\Lambda}, \bar{\alpha}_{\Xi}, \bar{\phi}_{\Xi}, \bar{\alpha}_{\Lambda})$$

$$W = \sum_{\mu, \bar{\nu}} C_{\mu \bar{\nu}} \sum_{\mu', \bar{\nu}'} a_{\mu, \mu'}^\Xi a_{\bar{\nu}, \bar{\nu}'}^{\bar{\Xi}} a_{\mu', 0}^\Lambda a_{\bar{\nu}', 0}^{\bar{\Lambda}}$$

$\Delta\Phi \neq 0$ is not needed!

Variables and parameters factorize:

$$W(\xi; \omega) = \sum_{k=1}^M f_k(\omega) T_k(\xi)$$

$$\Xi^- \bar{\Xi}^+ \quad \Lambda \bar{\Lambda}$$

$$\Delta\Phi \neq 0 : \quad M = 72 \quad (7)$$

$$\Delta\Phi = 0 : \quad M = 56 \quad (5)$$

Asymptotic likelihood method

$$V_{kl}^{-1} = E \left(-\frac{\partial^2 \ln \mathcal{L}}{\partial \omega_k \partial \omega_l} \right) = N \int \frac{1}{\mathcal{P}} \frac{\partial \mathcal{P}}{\partial \omega_k} \frac{\partial \mathcal{P}}{\partial \omega_l} d\xi$$

Tool to determine:

- Best possible (ultimate) sensitivity and correlations for parameters
- Structure of complicated angular distribution:
e.g. V_{kl}^{-1} singular – parameters cannot be determined separately

V_{kl} – covariance matrix

$$\mathcal{L}(\omega) = \prod_{i=1}^N \mathcal{P}(\xi_i, \omega) \equiv \prod_{i=1}^N \frac{\mathcal{W}(\xi_i, \omega)}{\int \mathcal{W}(\xi, \omega) d\xi},$$

Validation of the method



	$\bar{\alpha}_\Lambda$	α_ψ	$\Delta\Phi$
α_Λ	0.87	-0.05	-0.07
$\bar{\alpha}_\Lambda$		0.05	0.07
α_ψ			0.28

$$\sigma(\alpha_\Lambda) = \frac{7}{\sqrt{N}} \quad (0.011)$$

$$\sigma(A_\Lambda) = \frac{9}{\sqrt{N}} \quad (0.014)$$

$e^+e^- \rightarrow J/\Psi \rightarrow \Xi\bar{\Xi}$

Correlation matrix:

$$\sigma(\alpha_{\Xi}) = \frac{2}{\sqrt{N}}$$

$$\sigma(\phi_{\Xi}) = \frac{6}{\sqrt{N}}$$

$$\sigma(\alpha_{\Lambda}) = \frac{3}{\sqrt{N}}$$

	$\bar{\alpha}_{\Xi}$	α_{Λ}	$\bar{\alpha}_{\Lambda}$	ϕ_{Ξ}	$\bar{\phi}_{\Xi}$	α_{ψ}	$\Delta\Phi$
α_{Ξ}	0.03	0.37	0.11	0.0	0.0	0.0	0.0
$\bar{\alpha}_{\Xi}$		0.11	0.37	0.0	0.0	0.0	0.0
α_{Λ}			0.43	0.0	0.0	-0.1	0.0
$\bar{\alpha}_{\Lambda}$				0.0	0.0	0.1	0.0
ϕ_{Ξ}		$\Delta\Phi = 0$				0.0	0.0
$\bar{\phi}_{\Xi}$						0.0	0.0
α_{ψ}							0.0

$$\sigma(A_{\Lambda}) = \frac{3.3}{\sqrt{N}}$$

$e^+e^- \rightarrow J/\Psi \rightarrow \Xi\bar{\Xi}$

Correlation matrix:

$$\sigma(\alpha_{\Xi}) = \frac{2}{\sqrt{N}}$$

$$\sigma(\phi_{\Xi}) = \frac{6}{\sqrt{N}}$$

$$\sigma(\alpha_{\Lambda}) = \frac{3}{\sqrt{N}}$$

	$\bar{\alpha}_{\Xi}$	α_{Λ}	$\bar{\alpha}_{\Lambda}$	ϕ_{Ξ}	$\bar{\phi}_{\Xi}$	α_{ψ}	$\Delta\Phi$
α_{Ξ}	0.03	0.37	0.11	0.0	0.0	0.0	0.0
$\bar{\alpha}_{\Xi}$		0.11	0.37	0.0	0.0	0.0	0.0
α_{Λ}			0.43	0.0	0.0	-0.1	0.0
$\bar{\alpha}_{\Lambda}$				0.0	0.0	0.1	0.0
ϕ_{Ξ}	$\Delta\Phi = 0$					0.0	0.0
$\bar{\phi}_{\Xi}$		$\bar{\alpha}_{\Xi}$	α_{Λ}	$\bar{\alpha}_{\Lambda}$		0.0	0.0
α_{ψ}	α_{Ξ}	0.01	0.31	0.07			0.0
	$\bar{\alpha}_{\Xi}$		0.07	0.31			
	α_{Λ}	$\Delta\Phi = \frac{\pi}{2}$					

$$\sigma(A_{\Lambda}) = \frac{3.3}{\sqrt{N}}$$

Spin density matrix for $e^+e^- \rightarrow \Omega^-\Omega^+$

$$\rho_{3/2, \overline{3/2}}^{\lambda_1\lambda_2, \lambda_1'\lambda_2'} = \sum_{\kappa=\pm 1} D_{\kappa, \lambda_1 - \lambda_2}^{1*}(0, \theta_\Omega, 0) D_{\kappa, \lambda_1' - \lambda_2'}^1(0, \theta_\Omega, 0) A_{\lambda_1 \lambda_2} A_{\lambda_1' \lambda_2'}^*$$

$$A = \begin{pmatrix} \mathbf{h}_4 & \mathbf{h}_3 & 0 & 0 \\ \mathbf{h}_3 & \mathbf{h}_1 & \mathbf{h}_2 & 0 \\ 0 & \mathbf{h}_2 & \mathbf{h}_1 & \mathbf{h}_3 \\ 0 & 0 & \mathbf{h}_3 & \mathbf{h}_4 \end{pmatrix}$$

(Complex) Form Factors
 $\mathbf{h}_k \rightarrow h_k \exp(i\phi_k)$

Using base 3/2 spin matrices Q:

M.G.Doncel, L.Michel, P.Minnaert Nucl. Phys. B38, 477(1972)

$$\rho_{3/2, \overline{3/2}} = \sum_{\mu=0}^{15} \sum_{\bar{\nu}=0}^{15} C_{\mu, \bar{\nu}} Q_\mu \otimes Q_{\bar{\nu}}$$

Single tag angular distribution

Single 3/2-spin baryon density matrix is

$$\rho_{3/2} = \sum_{\mu=0}^{15} r_\mu Q_\mu = \sum_{\mu=0}^{15} C_{\mu,0} Q_\mu$$

Angular distribution (using decay matrices in helicity frames):

$$W = \sum_{\mu=0}^{15} \sum_{\kappa=0}^3 C_{\mu,0} b_{\mu,\kappa}^\Omega a_{\kappa,0}^\Lambda$$

decay $1/2 \rightarrow 1/2 0$
 $(\Lambda \rightarrow p\pi)$

decay $3/2 \rightarrow 1/2 0$
 $(\Omega \rightarrow \Lambda K)$

$$r_0 = (1 + \cos^2 \theta_\Omega)(h_2^2 + 2h_3^2) + 2 \sin^2 \theta_\Omega (h_1^2 + h_4^2)$$

$$r_1 = 2 \sin 2 \theta_\Omega \frac{2 \Im(\mathbf{h}_1 \mathbf{h}_2^*) + \sqrt{3} \Im(\mathbf{h}_3^*(\mathbf{h}_1 + \mathbf{h}_4))}{\sqrt{30}}$$

$$r_6 = -\frac{2 \sin^2 \theta_\Omega (h_1^2 - h_4^2) + h_2^2 (\cos^2 \theta + 1)}{\sqrt{3}}$$

$$r_7 = \sqrt{2} \sin 2 \theta_\Omega \frac{\Re(\mathbf{h}_3^*(\mathbf{h}_4 - \mathbf{h}_1))}{\sqrt{3}}$$

$$r_8 = 2 \sin^2 \theta_\Omega \frac{\Re(\mathbf{h}_3 \mathbf{h}_2^*)}{\sqrt{3}}$$

$$r_{10} = 2 \sin^2 \theta_\Omega \frac{\Im(\mathbf{h}_3 \mathbf{h}_2^*)}{\sqrt{3}}$$

$$r_{11} = 2 \sin 2 \theta_\Omega \frac{\Im(\sqrt{3} \mathbf{h}_2 \mathbf{h}_1^* + \mathbf{h}_3^*(\mathbf{h}_1 + \mathbf{h}_4))}{\sqrt{15}}$$

$$\alpha_\psi = \frac{h_2^2 - 2(h_1^2 - h_3^2 + h_4^2)}{h_2^2 + 2(h_1^2 + h_3^2 + h_4^2)}$$

$$\frac{d\Gamma}{d \cos \theta_\Omega} = 1 + \alpha_\psi \cos^2 \theta_\Omega$$

Polarization of a spin 3/2 particle:

$$\rho_{3/2} = r_0 \left(Q_0 + \frac{3}{4} \sum_{M=-1}^1 r_M^1 Q_M^1 + \frac{3}{4} \sum_{M=-2}^2 r_M^2 Q_M^2 + \frac{3}{4} \sum_{M=-3}^3 r_M^3 Q_M^3 \right)$$

$r_{-1}^1 \rightarrow P_y \quad r_0^1 \rightarrow P_x \quad r_1^1 \rightarrow P_z$

M.G.Doncel, L.Michel, P.Minnaert Nucl. Phys. B38, 477(1972)

real coefficients,
scalable $J=1/2, 3/2, \dots$

$$\frac{3}{4} Q_M^L \rightarrow Q_\mu, \mu = 1, \dots, 15$$

$$Q_0 = \frac{1}{4} I$$

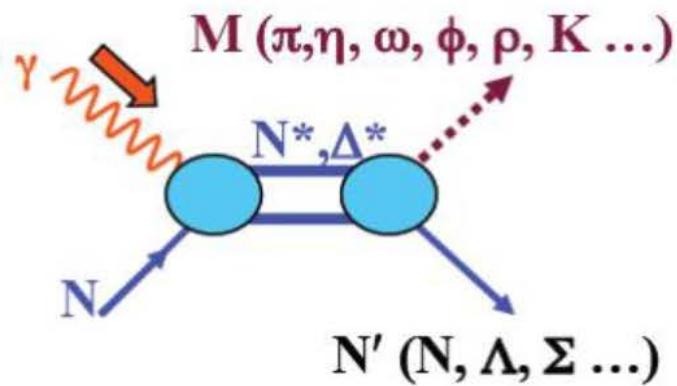
$$\rho_{3/2} = \sum_{\mu=0}^{15} r_\mu Q_\mu$$

Degree of polarization

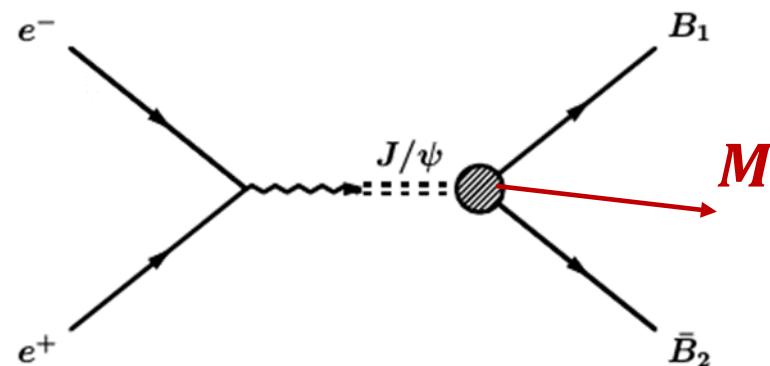
$$d(\rho_{3/2}) = \sqrt{\sum_{L=1}^3 \sum_{M=-L}^L (r_M^L)^2}$$

At threshold: $d(3/2) = 23\%$

Light baryon spectroscopy



$$e^+ e^- \rightarrow J/\psi, \psi' \rightarrow B_1 \bar{B}_2 M$$



If there are (anti)hyperons one can determine spin density matrix for free

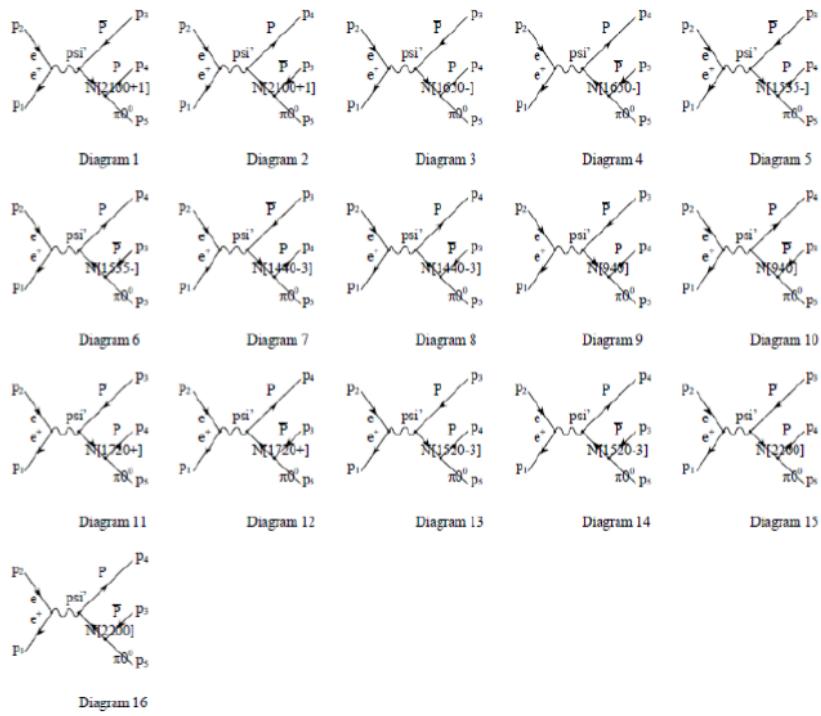
$$e^+ e^- \rightarrow J/\psi, \psi' \rightarrow B_1 \bar{B}_2 V(P)$$

Feynman Diagram Calculation: FDC-PWA Nucl.Instrum.Meth. A534 (2004) 241
Package used for baryon PWA at BESIII

mesons and baryons

$J=(0, 1/2, 1, 3/2, 2, 5/2, 3, 7/2, 4, 9/2)$

Automatically generated
Feynman diagrams in $\psi' \rightarrow \pi^0 p\bar{p}$



From effective Lagrangian Feynman rules are generated and angular distribution is calculated. Fit parameters are:

- Resonance parameters
- Coupling constants (complex)

However hyperons are treated as stable particles...

Conclusions:

BESIII Hyperon group: goal to develop and verify formalism and methods for precision hyperon production and decay studies at BESIII in $e^+e^- \rightarrow B_1\bar{B}_2(M)$ including spin degrees of freedom.

J/ ψ and ψ' decays into hyperon-antihyperon:
unique spin entangled system for CP tests and for determination of
(anti-)hyperon decay parameters.

BESIII in progress: analyses using collected 10^{10} J/ ψ

Plan: more ψ' data ...

Prospects for a CP violation signal at Super Tau Charm Factories.
Methods can be extended for analyses at BelleII and PANDA

Thank you!